Deep Neural Networks Are Our Friends



Wang Ling



Outline

- Part I Neural Networks are our friends
 - Numbers are our friends
 - Variables are our friends
 - Operators are our friends
 - Functions are our friends
 - Parameters are our friends
 - Cost Functions are our friends
 - Optimizers are our friends
 - Gradients are our friends

Outline

- Part I Neural Networks are our friends
- Part 2 Into Deep Learning
 - Nonlinear Neural Models
 - Multilayer Perceptrons
 - Using Discrete Variables
 - Example Applications

Numbers are our friends



Numbers are our friends



Variables are our friends





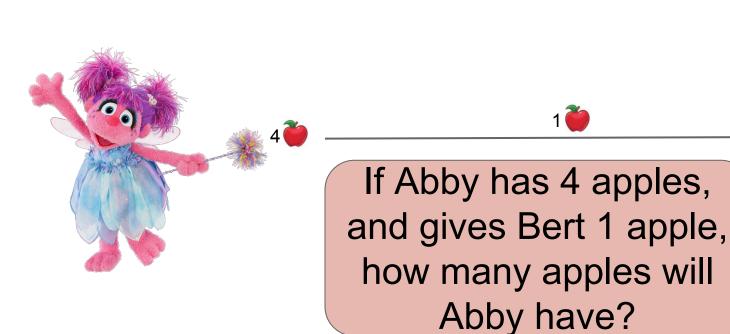
Variables are our friends



5 **y**

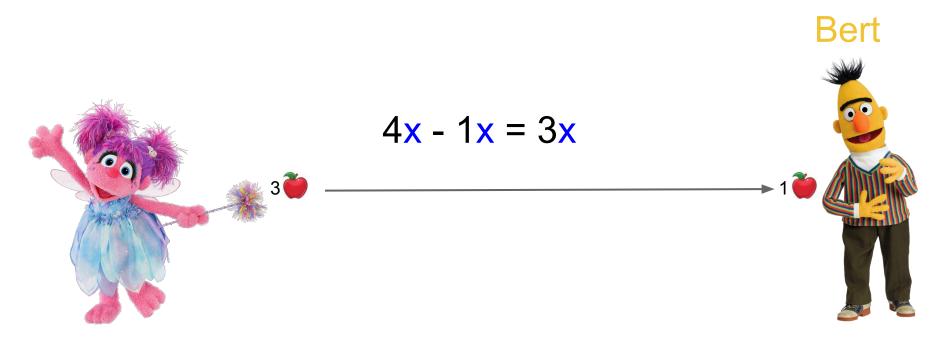


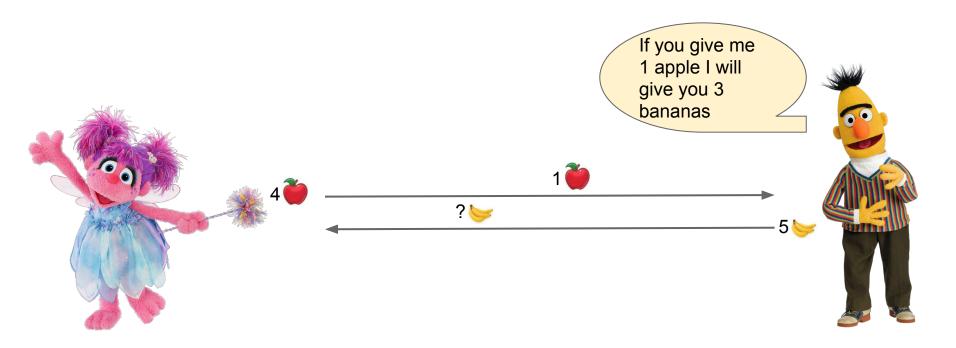
Operators are our friends





Operators are our friends



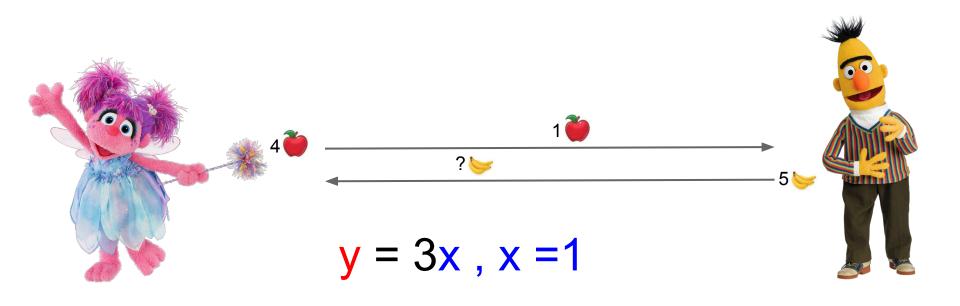


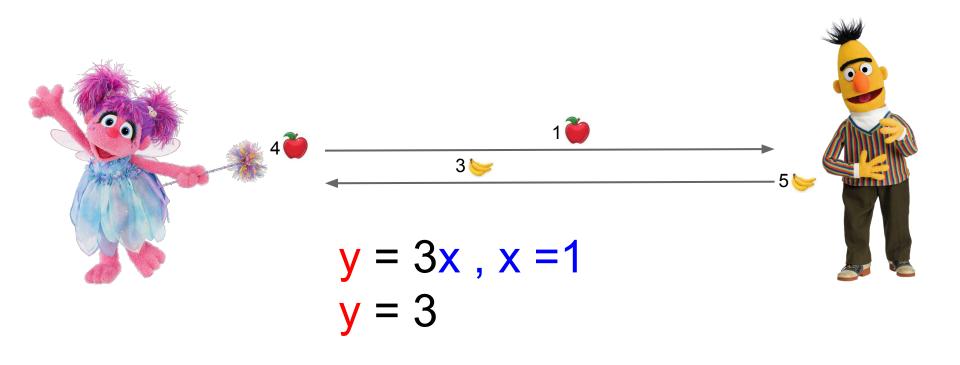
$$y = 3x$$

 Input, x - Number of Apples given by Abby

$$y = 3x$$

- Input, x Number of Apples given by Abby
- Output, y Number of Bananas received by Abby





$$y = 3x$$



x : English Sentence

Translate

Break through language barriers.



y: Move

x : Image



y: Category

x: Board

?????????????????????????

y: Move

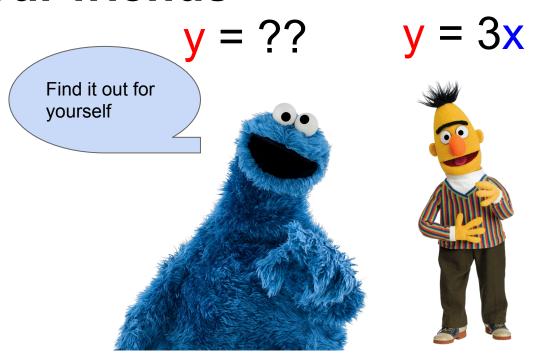




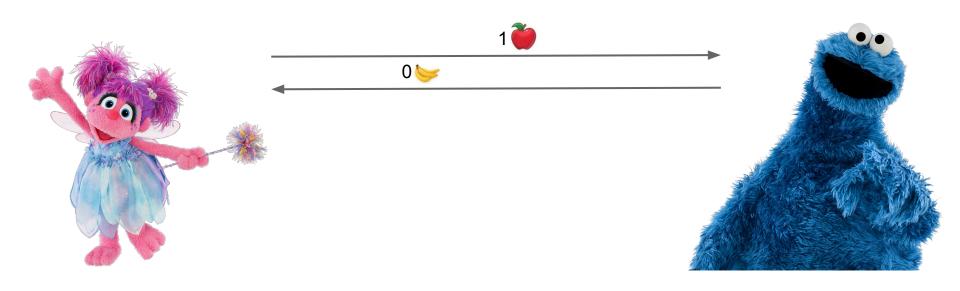




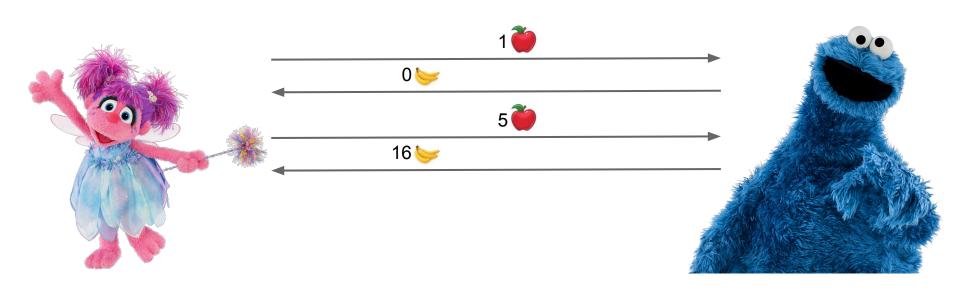




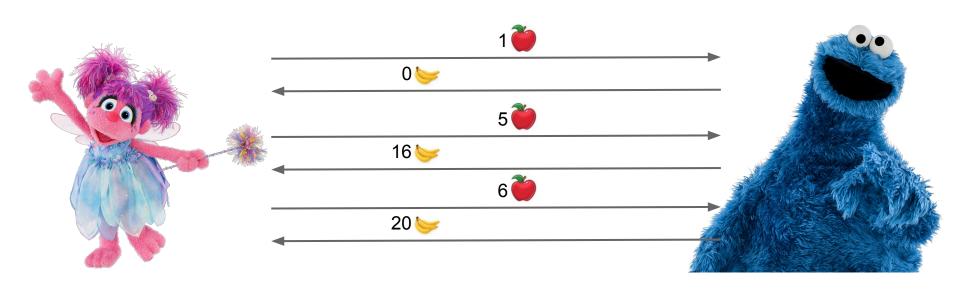
y = ??

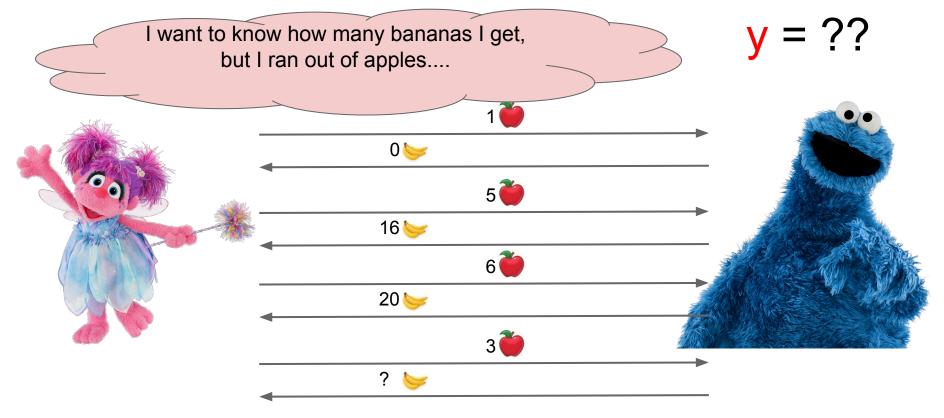


y = ??



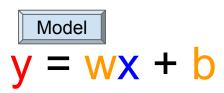
y = ??





$$y = 3x + 1$$

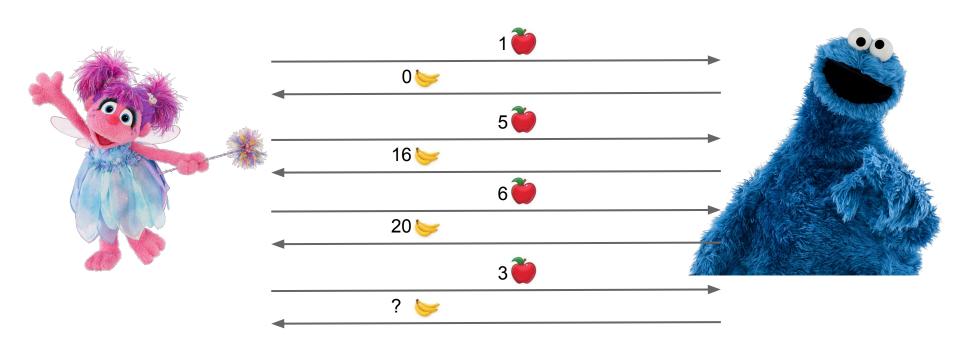
- Input
- Output

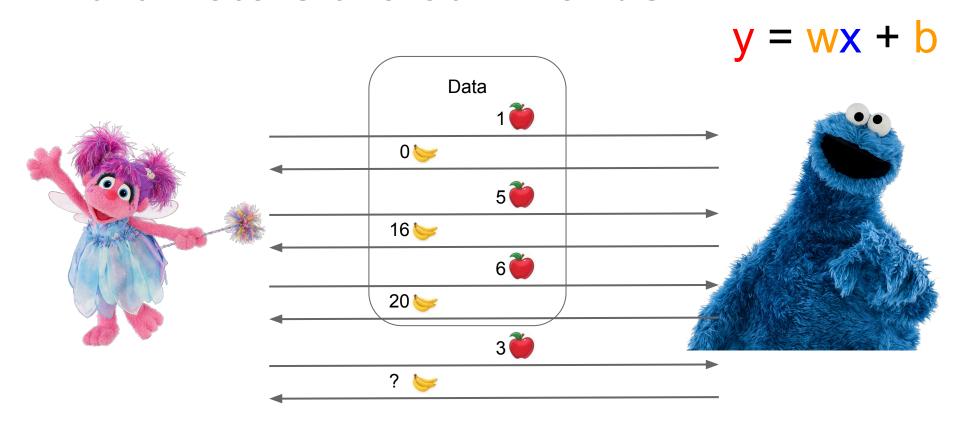


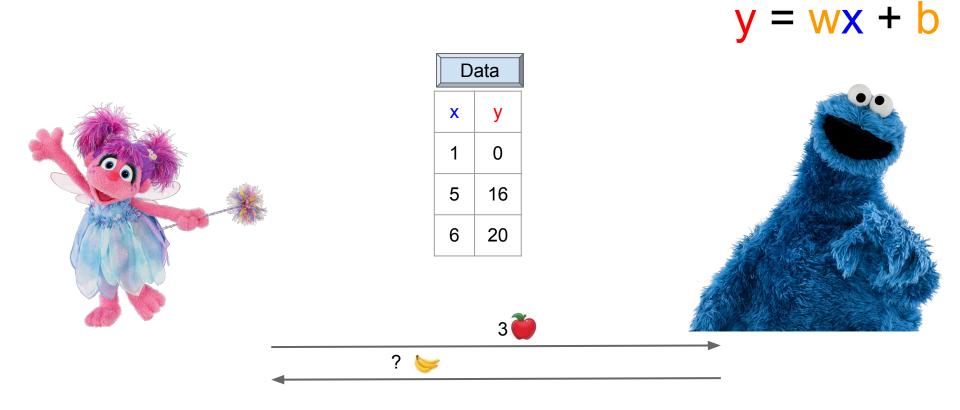
- Input
- Output
- Parameters

Input - Fixed, comes from data
Parameters - Need to be estimated

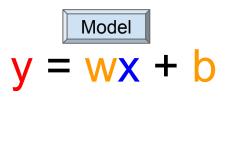
$$y = wx + b$$

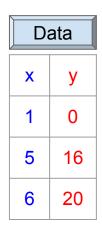




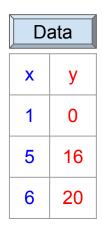


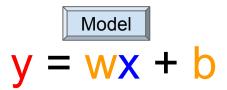
Data	
X	у
1	0
5	16
6	20





How to find the parameters w and b?



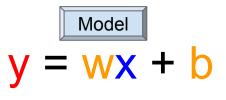




$$y = 1x + 0$$

X	y
1	0
5	16
6	20

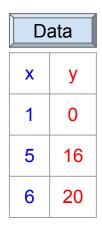
Data	
X	у
1	0
5	16
6	20



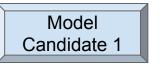


$$y = 1x + 0$$
 $1 = 1*1 + 0$
 $5 = 1*5 + 0$
 $6 = 1*6 + 0$

X	у	ŷ
1	0	1
5	16	5
6	20	6



$$y = wx + b$$



$$y = 1x + 0$$

X	y	ŷ
1	0	1
5	16	5
6	20	6

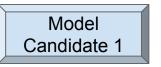
Model Candidate 2

$$y = 2x + 2$$

X	y	ŷ
1	0	4
5	16	12
6	20	14

Data	
X	у
1	0
5	16
6	20

$$y = wx + b$$



$$y = 1x + 0$$

X	у	ŷ
1	0	1
5	16	5
6	20	6

Model Candidate 2

$$y = 2x + 2$$

Which one is better?

X	у	ŷ
1	0	4
5	16	12
6	20	14

Parameters are our friends

Data	
x y	
1	0
5	16
6	20

$$y = wx + b$$



$$y = 1x + 0$$

X	у	ŷ
1	0	1
5	16	5
6	20	6

$$y = 2x + 2$$

X	y	ŷ
1	0	4
5	16	12
6	20	14

	Da	ata
n	X	у
0	1	0
1	5	16
2	6	20

$$y_n = wx_n + b$$

Model	
Candidate 1	

$$y = 1x + 0$$

X	у	ŷ
1	0	1
5	16	5
6	20	6

$$y = 2x + 2$$

X	у	ŷ
1	0	4
5	16	12
6	20	14

	Da	ata
n	X	у
0	1	0
1	5	16
2	6	20

$$y_n = wx_n + b$$

Model	
Candidate 1	

$$y = 1x + 0$$

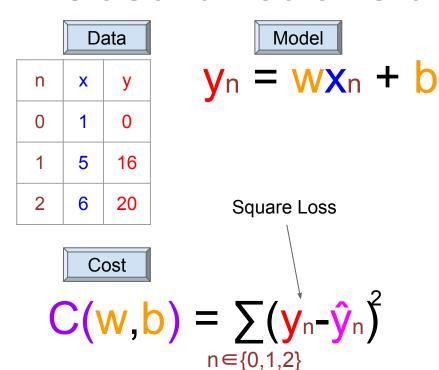
X	у	ŷ
1	0	1
5	16	5
6	20	6

Cost

C(w,b)

$$y = 2x + 2$$

X	y	ŷ
1	0	4
5	16	12
6	20	14



	Model
	Candidate 1
L	Carididate 1

$$y = 1x + 0$$

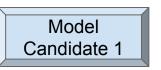
X	у	ŷ
1	0	1
5	16	5
6	20	6

$$y = 2x + 2$$

X	y	ŷ
1	0	4
5	16	12
6	20	14

	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	

$$y_n = wx_n + b$$



$$y = 1x + 0$$

n	X	y	ŷ	(y-ŷ) ²
0	1	0	1	
1	5	16	5	
2	6	20	6	

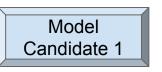
$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$

$$y = 2x + 2$$

X	y	ŷ
1	0	4
5	16	12
6	20	14

	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	

$$y_n = wx_n + b$$



$$y = 1x + 0$$

n	X	у	ŷ	(y -ŷ) ²
0	1	0	1	1
1	5	16	5	
2	6	20	6	

$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$

$$y = 2x + 2$$

X	y	ŷ
1	0	4
5	16	12
6	20	14

	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	

$$y_n = wx_n + b$$

$$y = 1x + 0$$

n	X	у	ŷ	(y-ŷ) ²
0	1	0	1	1
1	5	16	5	121
2	6	20	6	

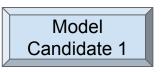
$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$

$$y = 2x + 2$$

X	y	ŷ
1	0	4
5	16	12
6	20	14

	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	

$$y_n = wx_n + b$$



$$y = 1x + 0$$

n	X	у	ŷ	(y-ŷ) ²
0	1	0	1	1
1	5	16	5	121
2	6	20	6	196

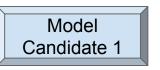
$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$

$$y = 2x + 2$$

X	у	ŷ
1	0	4
5	16	12
6	20	14

	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	

$$y_n = wx_n + b$$



$$y = 1x + 0$$

n	X	У	Ŷ	$(y-\hat{y})^2$
0	1	0	1	1
			_	101
1	5	16	5	121
2	6	20	6	196
	C(1,0)		318	

$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$

$$y = 2x + 2$$

y	ŷ
0	4
16	12
20	14
	0

	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	

$$y_n = wx_n + b$$

Model	
Candidate 1	

$$y = 1x + 0$$

n	X	y	ŷ	(<mark>y-ŷ</mark>) ²
0	1	0	1	1
1	5	16	5	121
2	6	20	6	196
C(1,0)			318	

$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$

Model Candidate 2

$$y = 2x + 2$$

n	X	y	ŷ	$(y-\hat{y})^2$
0	1	0	4	16
1	5	16	12	16
2	6	20	14	36

C(2,2)

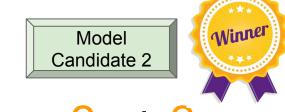
68

	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	

$$y_n = wx_n + b$$

$$y = 1x + 0$$

$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$



$$y = 2x + 2$$

318

	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	

$$y_n = wx_n + b$$

Cost

How to find the parameters w and b?

$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$

	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	

$$y_n = wx_n + b$$

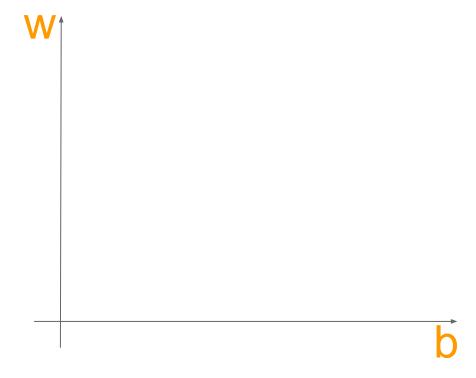
$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$

$$\underset{w,b \in [-\infty,\infty]}{\text{arg min } C(w,b)}$$

```
optimizer

arg min C(w,b)

w,b \in [-\infty,\infty]
```

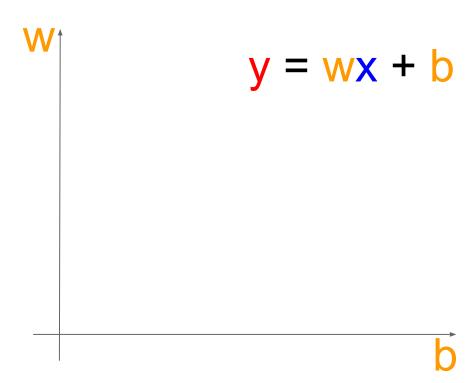


```
Optimizer

arg min C(w,b)

w,b \in [-\infty,\infty]

w_0,b_0 = 2,2 : C(w_0,b_0) = 68
```

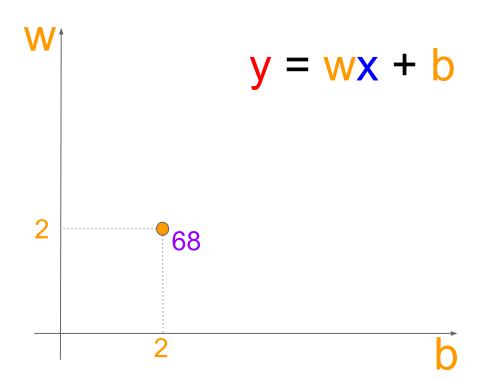


```
Optimizer

arg min C(w,b)

w,b \in [-\infty,\infty]

w_0,b_0 = 2,2 : C(w_0,b_0) = 68
```



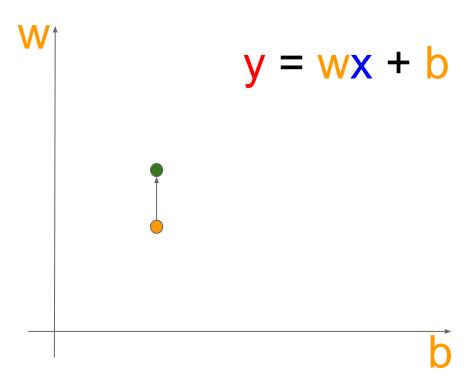
```
Optimizer

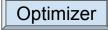
arg min C(w,b)

w,b \in [-\infty,\infty]

w_0,b_0 = 2,2 : C(w_0,b_0) = 68

w_1,b_1 = 3,2 : C(w_1,b_1) = ?
```

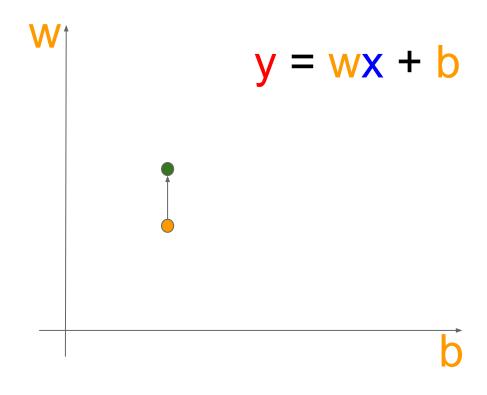


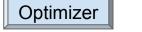


$$w_0,b_0 = 2,2 : C(w_0,b_0) = 68$$

$$w_1,b_1 = 3,2 : C(w_1,b_1) = 26$$

n	X	у	ŷ	(y -ŷ) ²
0	1	0	5	25
1	5	16	17	1
2	6	20	20	0
	26			

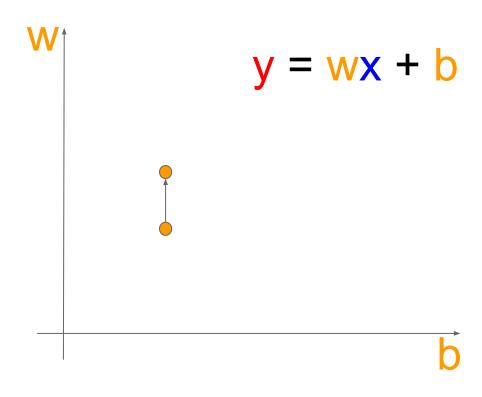




$$w_0,b_0 = 2,2 : C(w_0,b_0) = 68$$

$$w_1,b_1 = 3,2 : C(w_1,b_1) = 26$$

n	X	у	ŷ	(y -ŷ) ²
0	1	0	5	25
1	5	16	17	1
2	6	20	20	0
	26			



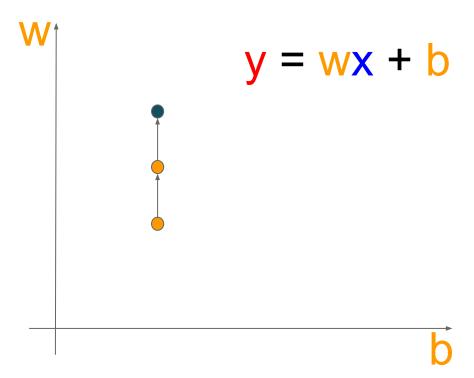
```
Optimizer

arg min C(w,b)

w,b \in [-\infty,\infty]

w_1,b_1 = 3,2 : C(w_1,b_1) = 26

w_2,b_2 = 4,2 : C(w_2,b_2) = ??
```



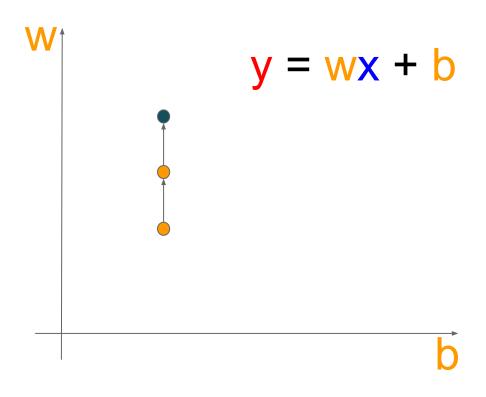
Optimizer

$$w,b \in [-\infty,\infty]$$

$$w_1,b_1 = 3,2 : C(w_1,b_1) = 26$$

$$w_2,b_2 = 4,2 : C(w_2,b_2) = 136$$

n	X	у	ŷ	(y -ŷ) ²
0	1	0	6	36
1	5	16	22	64
2	6	20	26	36
	136			

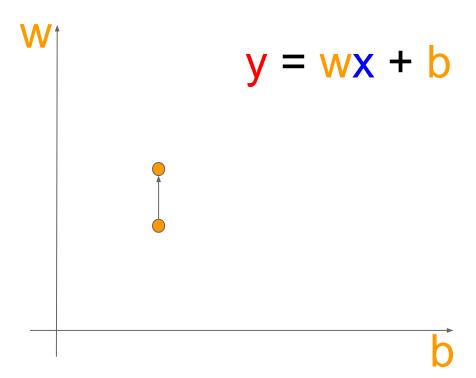


```
Optimizer

arg min C(w,b)

w,b \in [-\infty,\infty]

w_1,b_1 = 3,2 : C(w_1,b_1) = 26
```

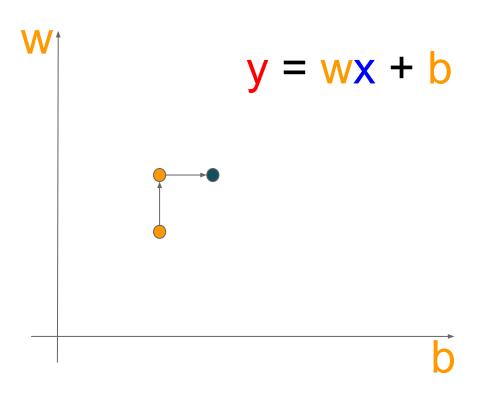


Optimizer

$$w_1,b_1 = 3,2 : C(w_1,b_1) = 26$$

$$w_2,b_2 = 3,3 : C(w_2,b_2) = 41$$

n	X	у	ŷ	(y -ŷ) ²
0	1	0	6	36
1	5	16	18	4
2	6	20	21	1
	41			

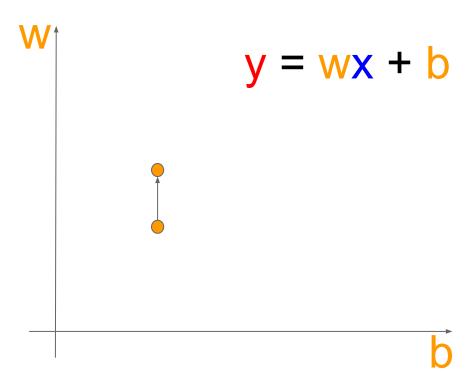


```
Optimizer

arg min C(w,b)

w,b \in [-\infty,\infty]

w_1,b_1 = 3,2 : C(w_1,b_1) = 26
```



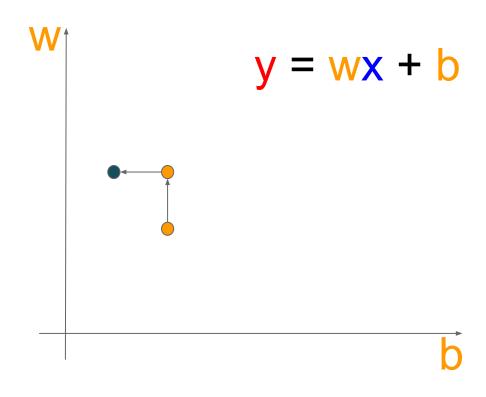
Optimizer

$$w,b \in [-\infty,\infty]$$

$$w_1,b_1 = 3,2 : C(w_1,b_1) = 26$$

$$w_2,b_2 = 3,1 : C(w_2,b_2) = 17$$

n	X	y	ŷ	(y -ŷ) ²
0	1	0	4	16
1	5	16	16	0
2	6	20	19	1
	17			

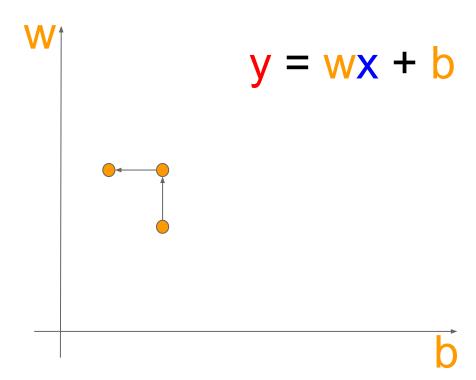


```
Optimizer

arg min C(w,b)

w,b \in [-\infty,\infty]

w_2,b_2 = 3,1 : C(w_2,b_2) = 17
```



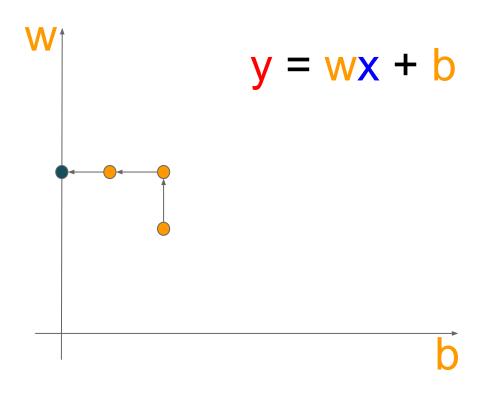
Optimizer

$$w,b \in [-\infty,\infty]$$

$$w_2,b_2 = 3,1 : C(w_2,b_2) = 17$$

 $w_3,b_3 = 3,0 : C(w_3,b_3) = 13$

n	X	y	ŷ	(y-ŷ) ²
0	1	0	3	9
1	5	16	15	1
2	6	20	18	4
	13			

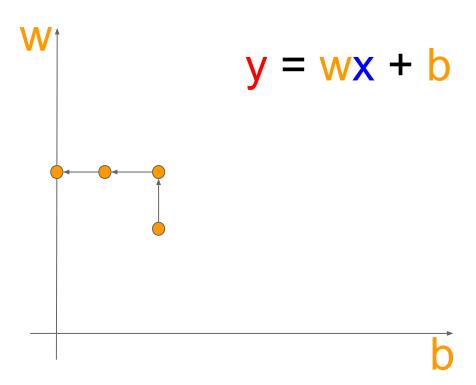


```
Optimizer

arg min C(w,b)

w,b \in [-\infty,\infty]

w<sub>3</sub>,b<sub>3</sub> = 3,0 : C(w<sub>3</sub>,b<sub>3</sub>) = 13
```



Optimizer

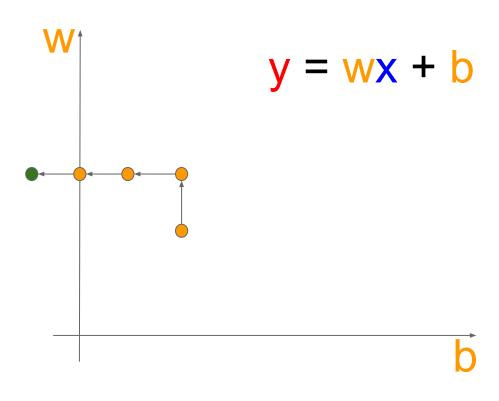
arg min C(w,b)

 $w,b \in [-\infty,\infty]$

$$w_3,b_3=3,0:C(w_3,b_3)=13$$

$$w_4,b_4=3,-1:C(w_4,b_4)=17$$

n	X	у	ŷ	(y -ŷ) ²
0	1	0	2	4
1	5	16	14	4
2	6	20	17	9
	17			



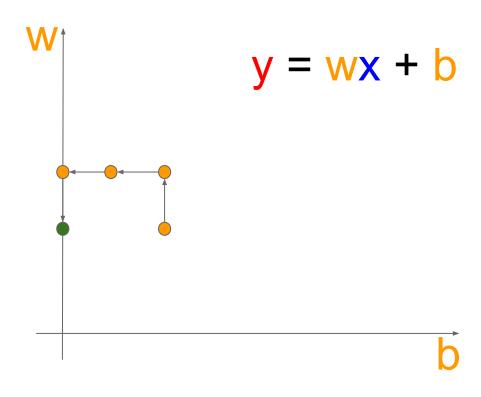
Optimizer

$$w,b \in [-\infty,\infty]$$

$$w_3,b_3=3,0:C(w_3,b_3)=13$$

$$w_4,b_4 = 2,0 : C(w_4,b_4) = 104$$

n	X	у	ŷ	(y -ŷ) ²
0	1	0	2	4
1	5	16	10	36
2	6	20	12	64
	104			



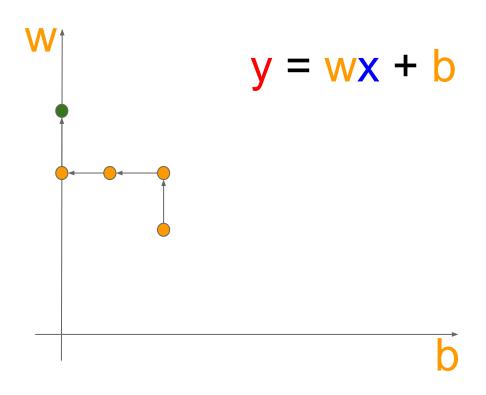
Optimizer

$$w,b \in [-\infty,\infty]$$

$$w_3,b_3=3,0:C(w_3,b_3)=13$$

$$w_4,b_4 = 4,0 : C(w_4,b_4) = 104$$

n	X	у	ŷ	(y -ŷ) ²
0	1	0	4	16
1	5	16	20	16
2	6	20	24	16
	54			



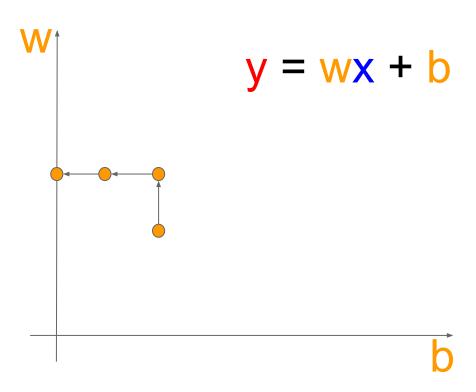
```
Optimizer

arg min C(w,b)

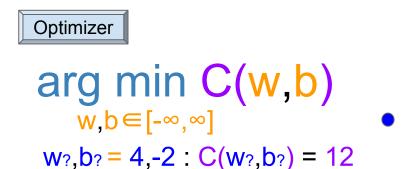
w,b \in [-\infty,\infty]

w_3,b_3 = 3,0 : C(w_3,b_3) = 13
```

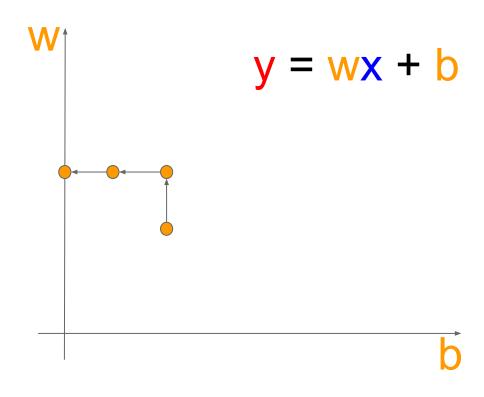
The End?

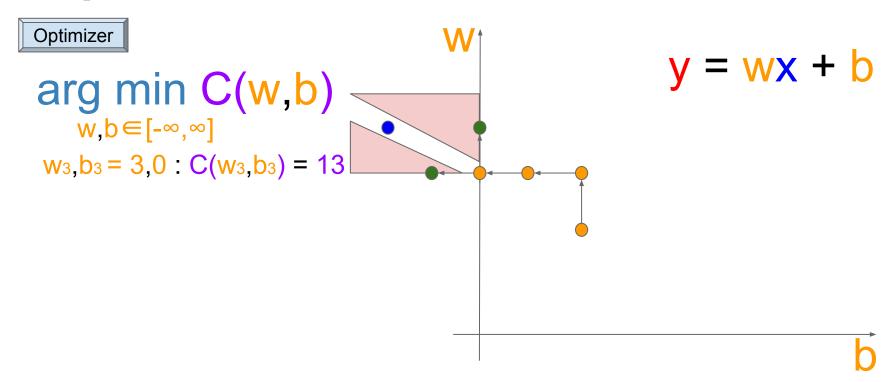


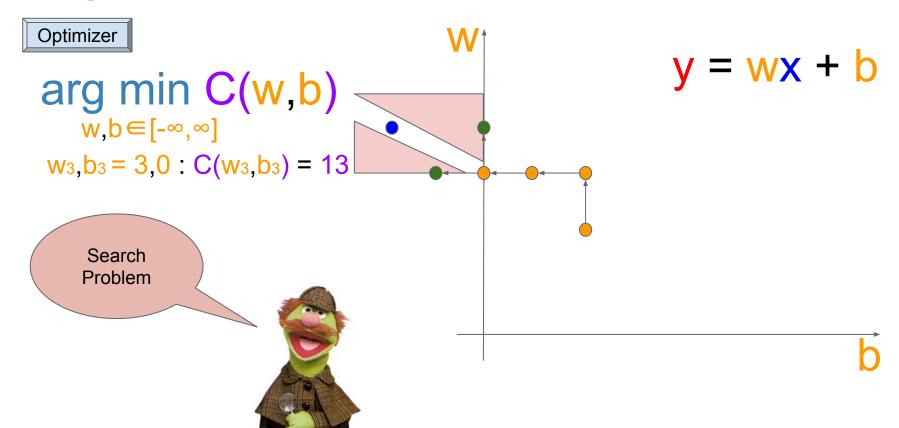
```
Optimizer
                                               y = wx + b
arg min C(w,b)
   w,b∈[-∞,∞]
w_?,b_? = 4,-2 : C(w_?,b_?) = ??
```

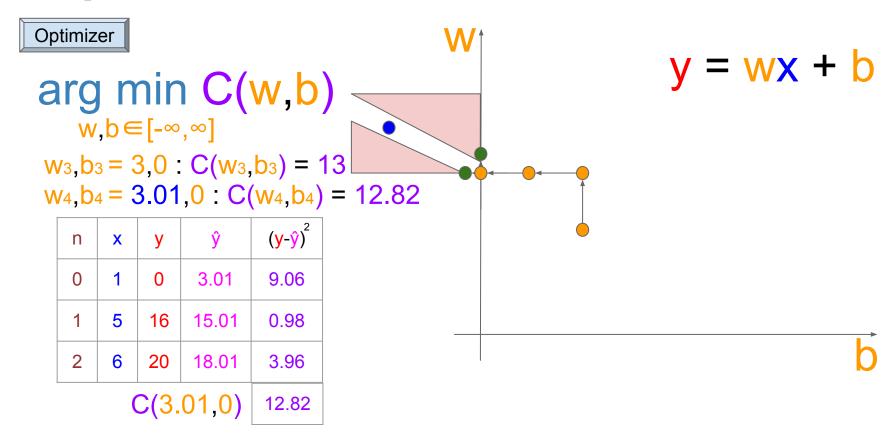


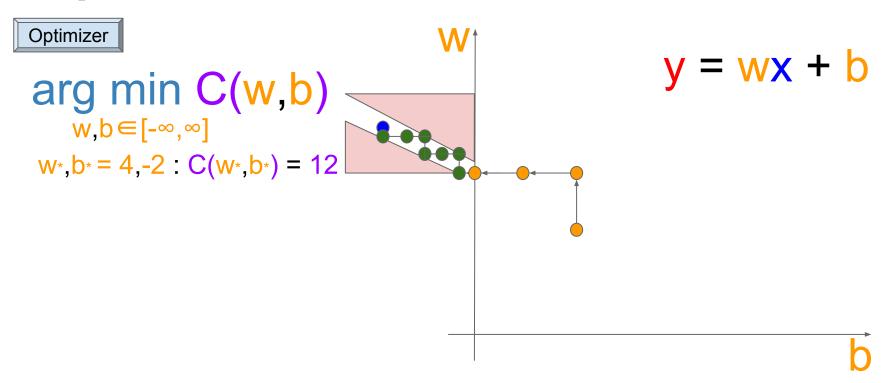
n	X	у	ŷ	(y -ŷ) ²
0	1	0	2	4
1	5	16	18	4
2	6	20	22	4
	12			

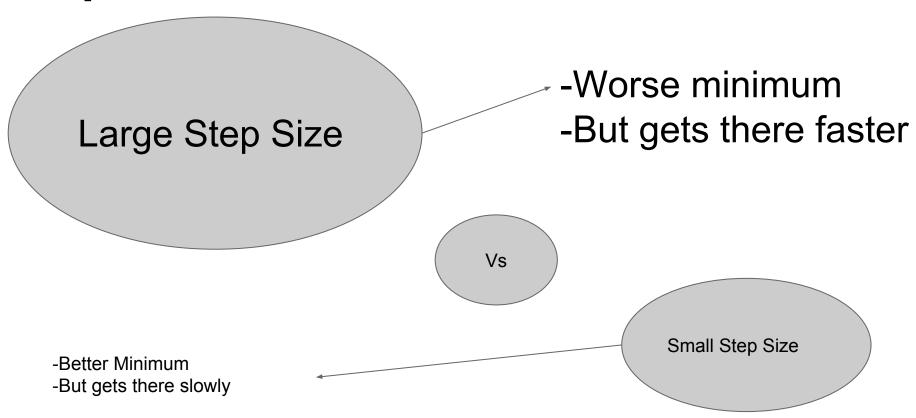


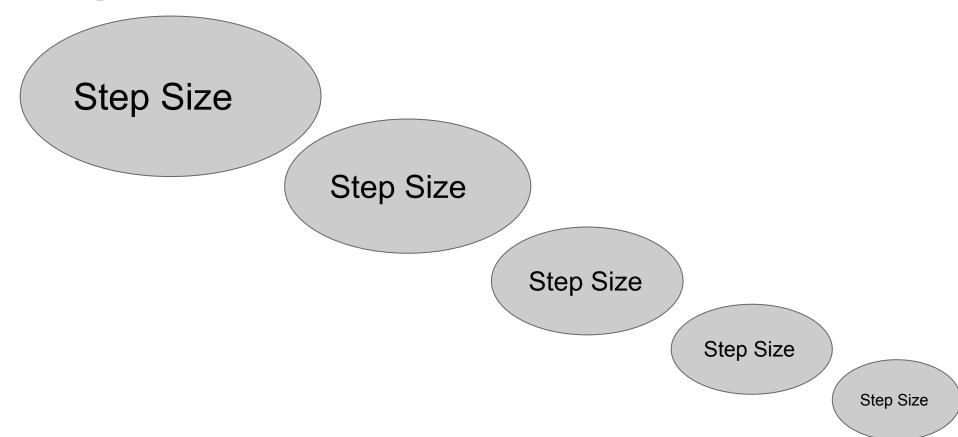










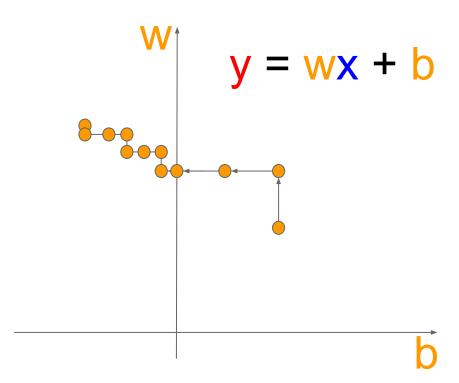


```
Optimizer

arg min C(w,b)

w,b \in [-\infty,\infty]

w,b^* = 4,-2 : C(w^*,b^*) = 12
```

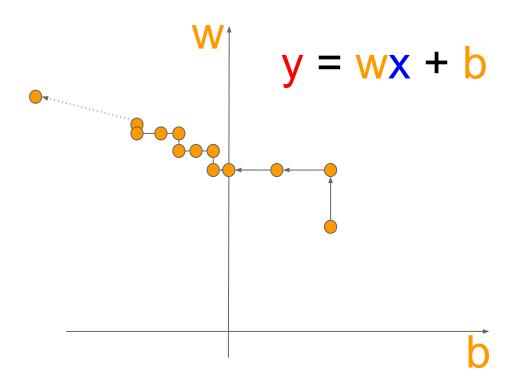


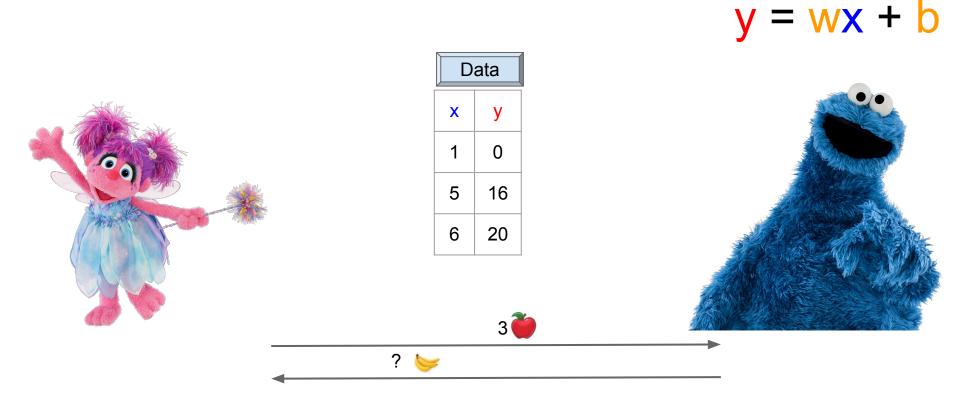
```
Optimizer

arg min C(w,b)

w,b \in [-\infty,\infty]

w*,b* = 4,-4 : C(w*,b*) = 0
```









Data		
X	у	
1	0	
5	16	
6	20	











Data		
x y		
1	0	
5	16	
6	20	









Functions are our friends

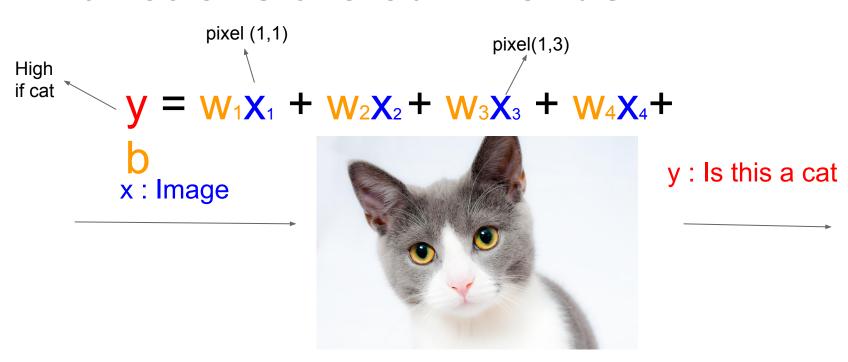
$$y = wx + b$$

x : Image

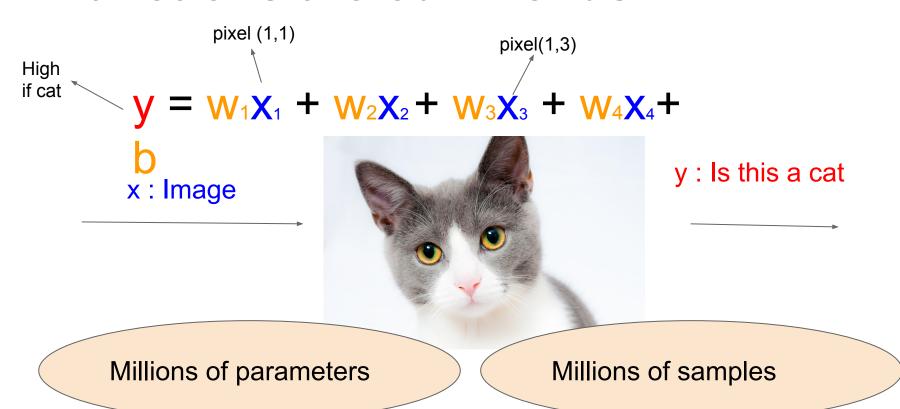


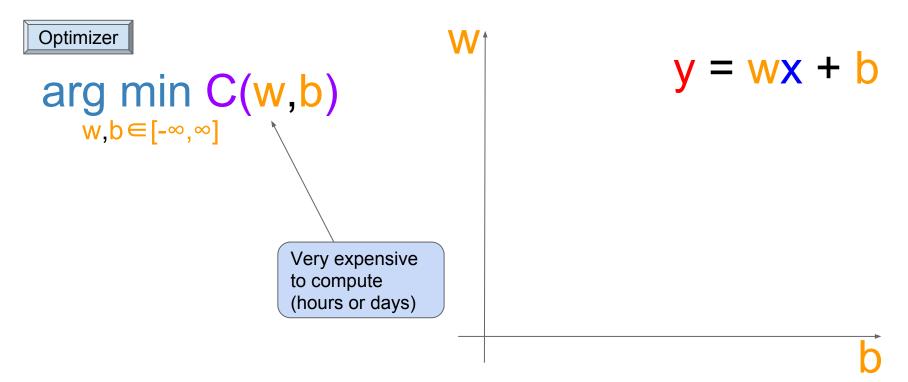
y: Is this a cat

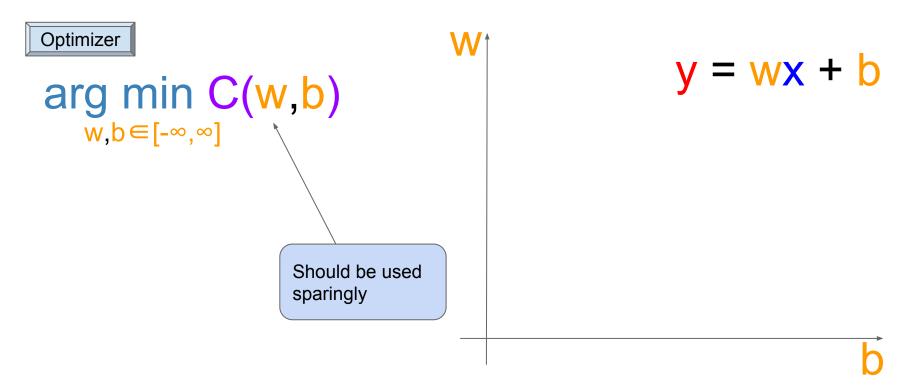
Functions are our friends



Functions are our friends





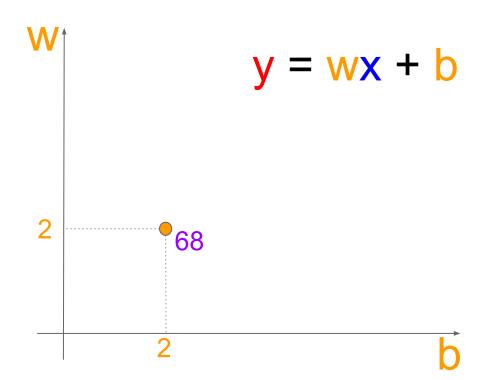


```
Optimizer

arg min C(w,b)

w,b \in [-\infty,\infty]

w<sub>0</sub>,b<sub>0</sub> = 2,2 : C(w<sub>0</sub>,b<sub>0</sub>) = 68
```



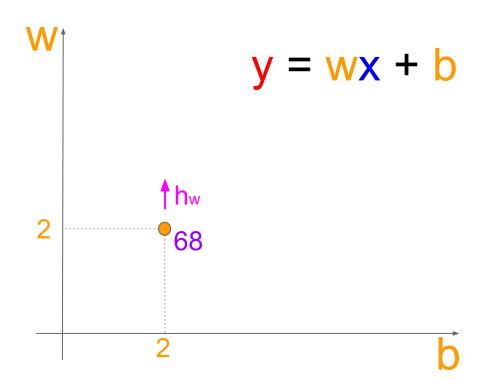
```
optimizer

arg min C(w,b)

w,b \in [-\infty,\infty]

w_0,b_0 = 2,2 : C(w_0,b_0) = 68

h_w = 1
```



```
Optimizer

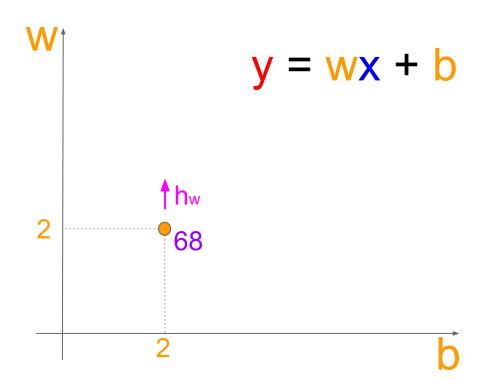
arg min C(w,b)

w,b \in [-\infty,\infty]

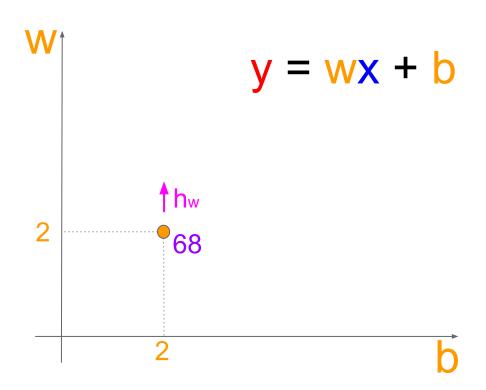
w_0,b_0 = 2,2 : C(w_0,b_0) = 68

h_w = 1

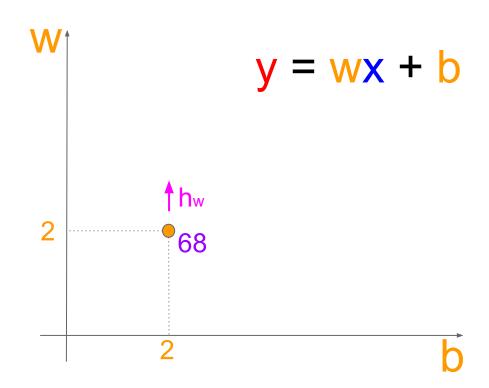
C(w_0+h_w,b_0) = C(3,2) = 26
```



```
Optimizer
arg min C(w,b)
    w,b \in [-\infty,\infty]
 w_0,b_0 = 2,2 : C(w_0,b_0) = 68
 h_{w} = 1
 C(w_0+h_w,b_0)=C(3,2)=26
r = \frac{(C(w_0+1,b_0)-C(w_0,b_0))}{(w_0+1,b_0)-C(w_0,b_0)}
```



```
Optimizer
arg min C(w,b)
   w,b \in [-\infty,\infty]
 w_0,b_0 = 2,2 : C(w_0,b_0) = 68
 h_w = 1, r = -42
 h_w = 0.1, r = -98
 h_w = 0.01, r = -104
 h_w = 0.001, r = -104
```



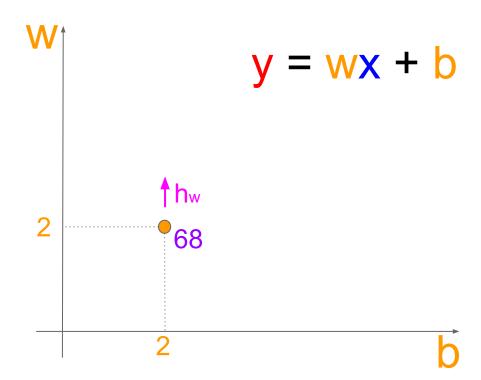
Optimizer y = wx + barg min C(w,b) $w,b \in [-\infty,\infty]$ $w_0,b_0 = 2,2 : C(w_0,b_0) = 68$ $h_w = 1$, r = -42 $h_w = 0.1$, r = -98 $h_w = 0.01$, r = -104 $h_w = 0.001$, r = -104 $h_w \rightarrow 0$, $r = \frac{\partial C}{\partial w}$ (w₀,b₀) $D_{\mathbf{u}}f(\mathbf{a}) = \lim_{h \to 0} \frac{f(\mathbf{a} + h\mathbf{u}) - f(\mathbf{a})}{h}$

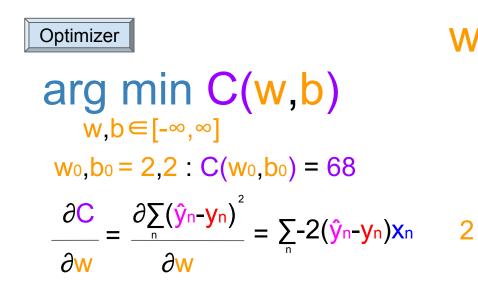
Optimizer

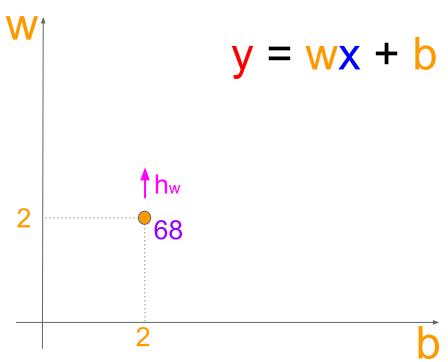
arg min C(w,b)

$$w_0,b_0 = 2,2 : C(w_0,b_0) = 68$$

$$\frac{\partial \mathbf{C}}{\partial \mathbf{W}} = \frac{\partial \sum_{\mathbf{n}} (\hat{\mathbf{y}}_{\mathbf{n}} - \mathbf{y}_{\mathbf{n}})^{2}}{\partial \mathbf{W}}$$







Optimizer

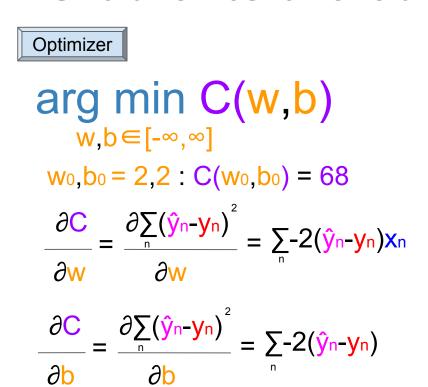
arg min C(w,b)

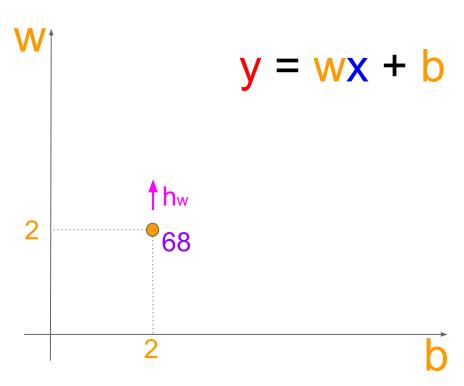
$$w_0,b_0 = 2,2 : C(w_0,b_0) = 68$$

$$\frac{\partial C}{\partial w} = \frac{\partial \sum_{n} (\hat{y}_{n} - y_{n})^{2}}{\partial w} = \sum_{n} -2(\hat{y}_{n} - y_{n}) x_{n}$$

$$h_w \rightarrow 0$$
, $r = \frac{\partial C}{\partial w} (w_0, b_0) = -104$

n	X	у	ŷ	(ŷ-y)	-2(ŷ-y)x
0	1	0	4	4	8
1	5	16	12	-4	-40
2	6	20	14	-6	-72





Optimizer

arg min C(w,b)

$$w_0,b_0 = 2,2 : C(w_0,b_0) = 68$$

$$h_w \rightarrow 0$$
, $r_w = \frac{\partial C}{\partial w} (w_0, b_0) = -104$
 $h_b \rightarrow 0$, $r_b = \frac{\partial C}{\partial w} (w_0, b_0) = -12$

n	X	у	ŷ	(ŷ-y)	-2(ŷ- y)
0	1	0	4	4	8
1	5	16	12	-4	-8
2	6	20	14	-6	-12

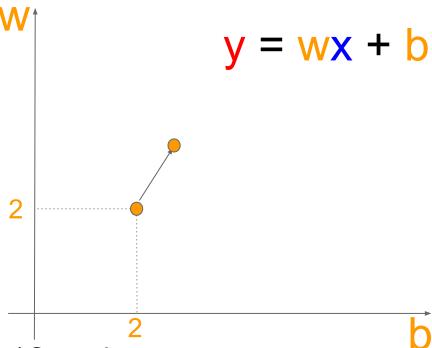
Optimizer

arg min C(w,b)

$$w_0,b_0 = 2,2 : C(w_0,b_0) = 68$$

$$h_w \rightarrow 0$$
, $r_w = \frac{\partial C}{\partial w} (w_0, b_0) = -104$
 $h_b \rightarrow 0$, $r_b = \frac{\partial C}{\partial w} (w_0, b_0) = -12$

$$b_1 = b_0 - r_b a$$



a → Learning Rate/ Step size

Summary

	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	

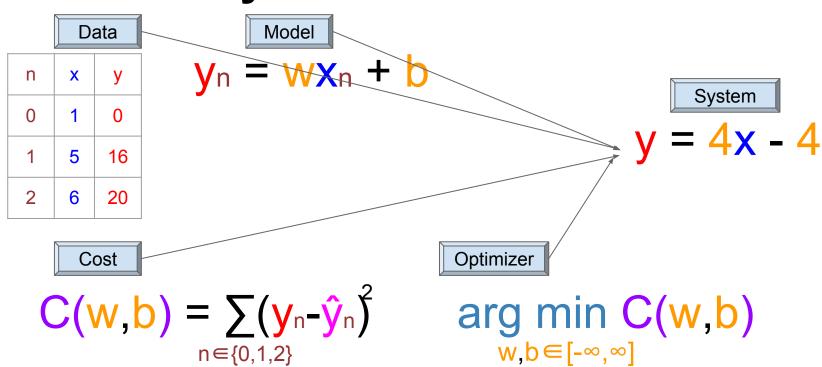
$$y_n = wx_n + b$$

$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$

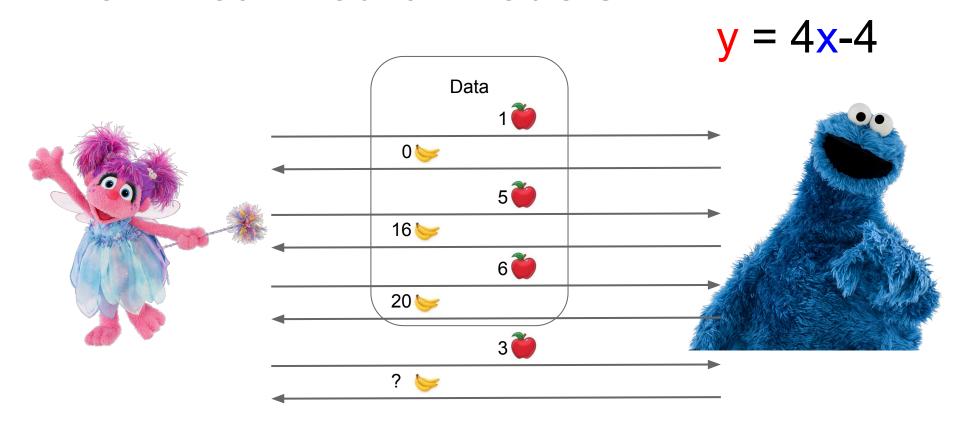
Optimizer

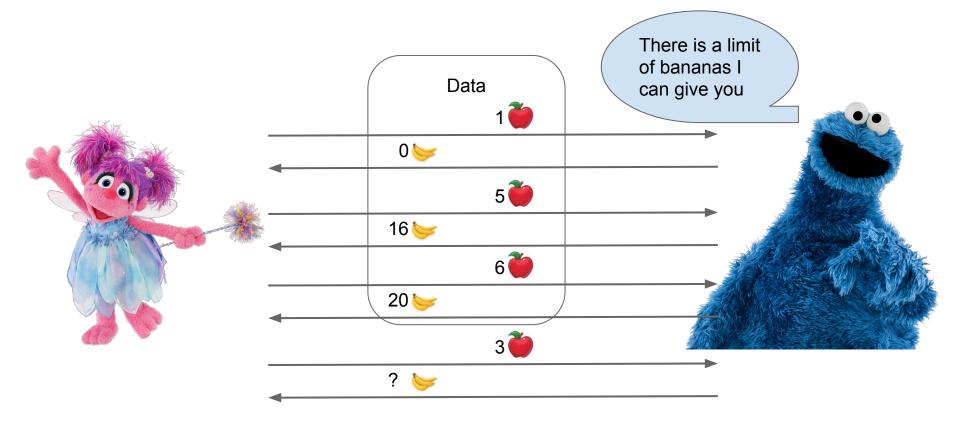
$$\underset{w,b \in [-\infty,\infty]}{\text{arg min } C(w,b)}$$

Summary

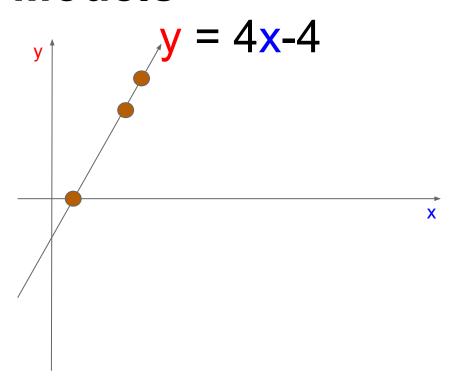


Into Deep Learning

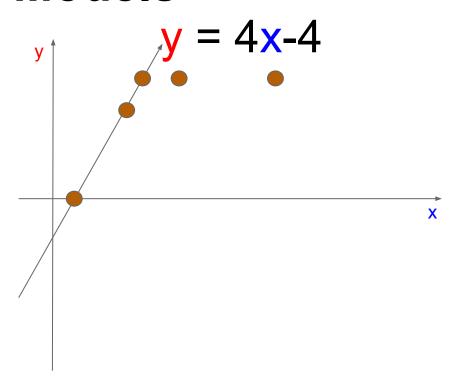


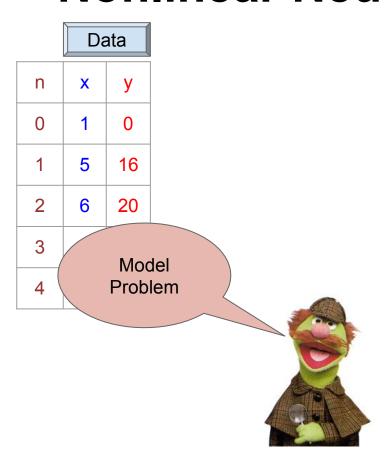


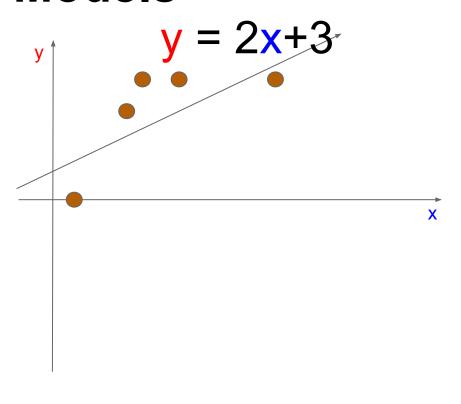
	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	

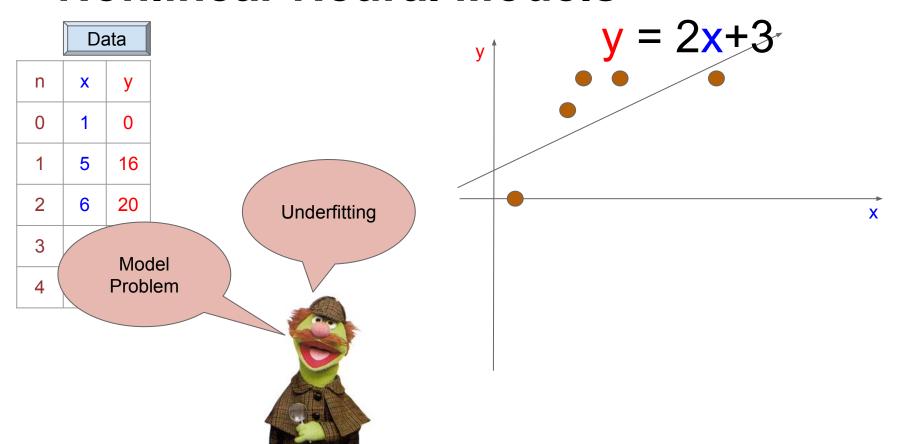


	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	
3	9	20	
4	11	20	

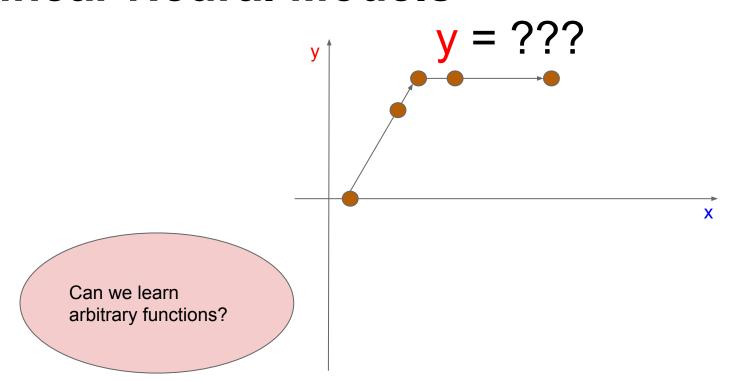








	Data		
n	X	у	
0	1	0	
1	5	16	
2	6	20	
3	9	20	
4	11	20	



$$y = (w_1x + b_1)s_1 + (w_2x+b_2)s_2$$

Use different linear functions depending on the value of x?

$$y = (W_1X + b_1)S_1 + (W_2X + b_2)S_2$$

$$S_1 - 1 \text{ if } x < 6 \text{ and } 0 \text{ otherwise}$$

$$S_2 - 1 \text{ if } x >= 6 \text{ and } 0 \text{ otherwise}$$

$$y = (w_1x + b_1)s_1 + (w_2x+b_2)s_2$$

S1 - 1 if x < 6 and 0 otherwise

 S_2 - 1 if x >= 6 and 0 otherwise

	Data	
n	X	V

n	X	y
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (4x - 4)s_1 + (0x+20)s_2$$

$$y = (w_1x + b_1)s_1 + (w_2x+b_2)s_2$$

S1 - 1 if x < 6 and 0 otherwise

 S_2 - 1 if x >= 6 and 0 otherwise

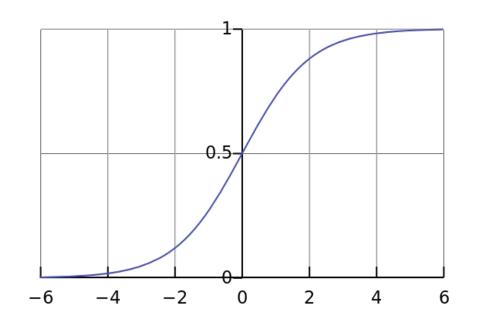
	/	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

Data

$$y = (4x - 4)s_1 + (0x+20)s_2$$

$$s = \sigma(wx + b)$$

$$\sigma(t) = \frac{1}{1 + e^{-t}}$$



$$s = \sigma(1000x)$$

$$s = \sigma(1000x)$$

$$x = 0.1$$
 then $\sigma(1000x) = 1$

$$x = -0.1$$
 then $\sigma(1000x) = 0$



$$s = \sigma(1000x - 6000)$$

$$x = 6.1$$
 then $\sigma(1000x - 6000) = 1$

$$x = 5.9$$
 then $\sigma(1000x - 6000) = 0$

$$y = (w_1x + b_1)s_1 + (w_2x+b_2)s_2$$

 $s_1 = \sigma(w_3x + b_3)$
 $s_2 = \sigma(w_4x + b_4)$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (4x - 4)s_1 + (0x+20)s_2$$

$$s1 = \sigma(-1000x + 6000)$$

$$s2 = \sigma(1000x - 6000)$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (4x - 4)s_1 + (0x+20)s_2$$

$$s1 = \sigma(-1000x + 6000)$$

$$s2 = \sigma(1000x - 6000)$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (16)s_1 + (0x+20)s_2$$

$$s1 = \sigma(-1000x + 6000)$$

$$s2 = \sigma(1000x - 6000)$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (16)s_1 + (20)s_2$$

$$s_1 = \sigma(-1000x + 6000)$$

$$s_2 = \sigma(1000x - 6000)$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (16)s_1 + (20)s_2$$

$$s1 = \sigma(1000)$$

$$s2 = \sigma(1000x - 6000)$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (16)s_1 + (20)s_2$$

$$s1 = \sigma(1000)$$

$$s2 = \sigma(-1000)$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (16)1 + (20)0$$

$$s1 = \sigma(1000)$$

$$s2 = \sigma(-1000)$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = 16$$

$$s1 = \sigma(1000)$$

$$s2 = \sigma(-1000)$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (4x - 4)s_1 + (0x+20)s_2$$

$$s1 = \sigma(-1000x + 6000)$$

$$s2 = \sigma(1000x - 6000)$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (32)s_1 + (0x+20)s_2$$

$$s_1 = \sigma(-1000x + 6000)$$

$$s_2 = \sigma(1000x - 6000)$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (32)s_1 + (20)s_2$$

$$s1 = \sigma(-1000x + 6000)$$

$$s2 = \sigma(1000x - 6000)$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (32)s_1 + (20)s_2$$

s1 =
$$\sigma$$
(-3000)
s2 = σ (1000x - 6000)

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (32)s_1 + (20)s_2$$

$$s1 = \sigma(-3000)$$

$$s2 = \sigma(3000)$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

$$y = (32)0 + (20)1$$

$$s1 = \sigma(-3000)$$

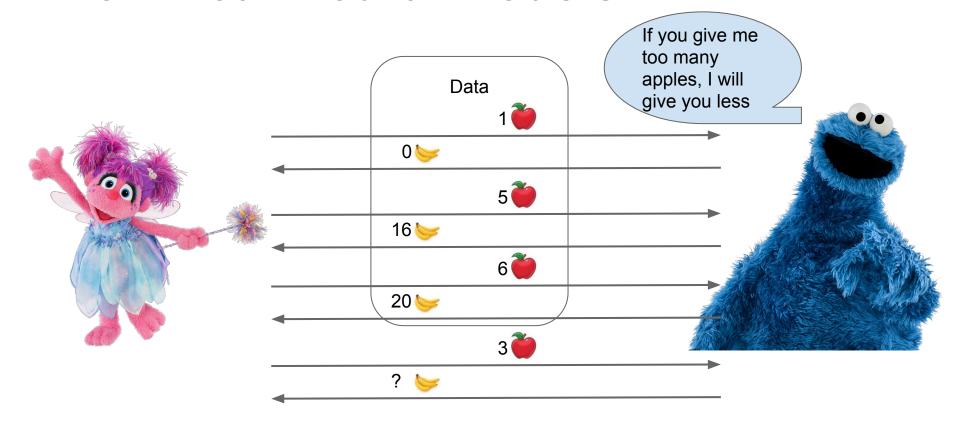
$$s2 = \sigma(3000)$$

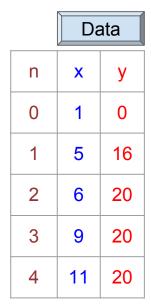
	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20

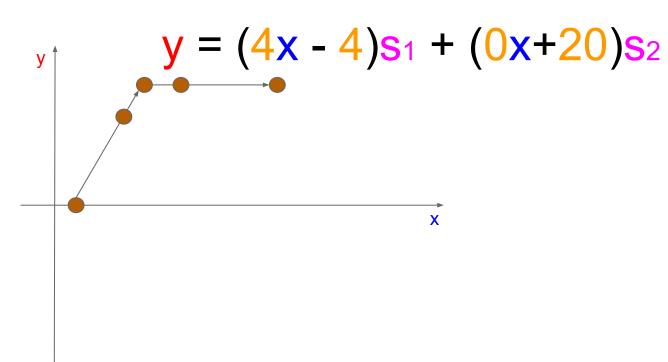
$$y = 20$$

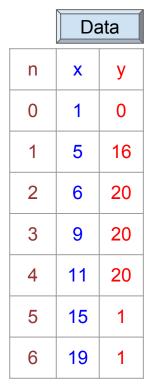
$$s1 = \sigma(-3000)$$

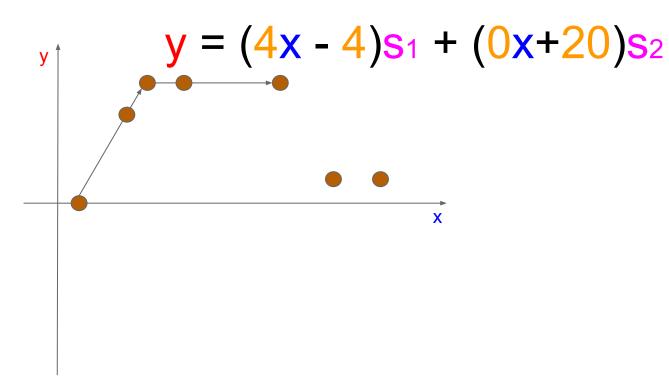
$$s2 = \sigma(3000)$$











	Da	ata
n	X	y
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

```
y = (4x - 4)s_1 + (0x+20)s_2 + (0x+1)s_3
s_1 = \sigma(-1000x + 6000)
s_2 = ????
s_3 = \sigma(1000x - 15000)
```

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

```
y = (4x - 4)s_1 + (0x+20)s_2 + (0x+1)s_3
s_1 = \sigma(-1000x + 6000)
s_2 = \text{not } s_1 \text{ and not } s_3
s_3 = \sigma(1000x - 15000)
```

$$y = (w_{1}x + b_{1})s_{1} + (w_{2}x+b_{2})s_{2} + (w_{3}x+b_{3})s_{3}$$

$$s_{1} = \sigma(w_{4}x + b_{4})$$

$$s_{2} = \sigma(w_{5}s_{1} + w_{6}s_{3} + b_{5})$$

$$s_{3} = \sigma(w_{7}x + b_{6})$$

$$y = (W_1X + b_1)S_1 + (W_2X + b_2)S_2 + (W_3X + b_3)S_3$$

$$S_1 = \sigma(W_4X + b_4)$$

$$S_2 = \sigma(W_5S_1 + W_6S_3 + b_5)$$

$$S_3 = \sigma(W_7X + b_6)$$
Layer 1 Perceptron

$$y = (W_1X + b_1)S_1 + (W_2X + b_2)S_2 + (W_3X + b_3)S_3$$

$$S_1 = \sigma(W_4X + b_4)$$

$$S_2 = \sigma(W_5S_1 + W_6S_3 + b_5)$$

$$S_3 = \sigma(W_7X + b_6)$$
Layer 1 Perceptron
$$S_3 = \sigma(W_7X + b_6)$$
Layer 1 Perceptron

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

```
y = (4x - 4)s_1 + (0x+20)s_2 + (0x+1)s_3
s_1 = \sigma(-1000x + 6000)
s_2 = \text{not } s_1 \text{ and not } s_3
s_3 = \sigma(1000x - 15000)
```

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

```
y = (4x - 4)s_1 + (0x+20)s_2 + (0x+1)s_3
s_1 = \sigma(-1000x + 6000)
s_2 = \sigma(-1000s_1 - 1000s_3 + 500)
s_3 = \sigma(1000x - 15000)
```

	Data	
n	X	y
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

```
y = (4x - 4)s_1 + (0x+20)s_2 + (0x+1)s_3
s_1 = \sigma(-1000x + 6000)
s_2 = \sigma(-1000s_1 - 1000s_3 + 500)
s_3 = \sigma(1000x - 15000)
```

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

```
y = (40)s_1 + (20)s_2 + (1)s_3
s_1 = \sigma(-1000x + 6000)
s_2 = \sigma(-1000s_1 - 1000s_3 + 500)
s_3 = \sigma(1000x - 15000)
```

	Data	
n	X	y
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

$$y = (40)s_1 + (20)s_2 + (1)s_3$$

$$s_1 = \sigma(-5000) = 0$$

$$s_2 = \sigma(-1000s_1 - 1000s_3 + 500)$$

$$s_3 = \sigma(-4000) = 0$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

$$y = (40)s_1 + (20)s_2 + (1)s_3$$

$$s_1 = \sigma(-5000) = 0$$

$$s_2 = \sigma(-0 - 0 + 500)$$

$$s_3 = \sigma(-4000) = 0$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

$$y = (40)s_1 + (20)s_2 + (1)s_3$$

 $s_1 = \sigma(-5000) = 0$
 $s_2 = \sigma(500)$
 $s_3 = \sigma(-4000) = 0$

	Data	
n	X	y
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

$$y = (40)s_1 + (20)s_2 + (1)s_3$$

 $s_1 = \sigma(-5000) = 0$
 $s_2 = \sigma(500) = 1$
 $s_3 = \sigma(-4000) = 0$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

$$y = (40)0 + (20)1 + (1)0$$

 $s_1 = \sigma(-5000) = 0$
 $s_2 = \sigma(500) = 1$
 $s_3 = \sigma(-4000) = 0$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

$$y = 20$$

 $s_1 = \sigma(-5000) = 0$
 $s_2 = \sigma(500) = 1$
 $s_3 = \sigma(-4000) = 0$

	Data	
n	X	y
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

```
y = (4x - 4)s_1 + (0x+20)s_2 + (0x+1)s_3
s_1 = \sigma(-1000x + 6000)
s_2 = \sigma(-1000s_1 - 1000s_3 + 500)
s_3 = \sigma(1000x - 15000)
```

	Data	
n	X	y
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

```
y = (772)s_1 + (20)s_2 + (1)s_3
s_1 = \sigma(-1000x + 6000)
s_2 = \sigma(-1000s_4 - 1000s_5 + 500)
s_3 = \sigma(1000x - 15000)
```

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

$$y = (772)s_1 + (20)s_2 + (1)s_3$$

$$s_1 = \sigma(-13000) = 0$$

$$s_2 = \sigma(-1000s_4 - 1000s_5 + 500)$$

$$s_3 = \sigma(4000) = 1$$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

$$y = (772)s_1 + (20)s_2 + (1)s_3$$

 $s_1 = \sigma(-13000) = 0$
 $s_2 = \sigma(-1000 + 0 + 500)$
 $s_3 = \sigma(4000) = 1$

	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

$$y = (772)s_1 + (20)s_2 + (1)s_3$$

 $s_1 = \sigma(-13000) = 0$
 $s_2 = \sigma(-500) = 0$
 $s_3 = \sigma(4000) = 1$

	Da	ata
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

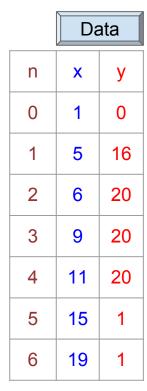
$$y = (772)0 + (20)0 + (1)1$$

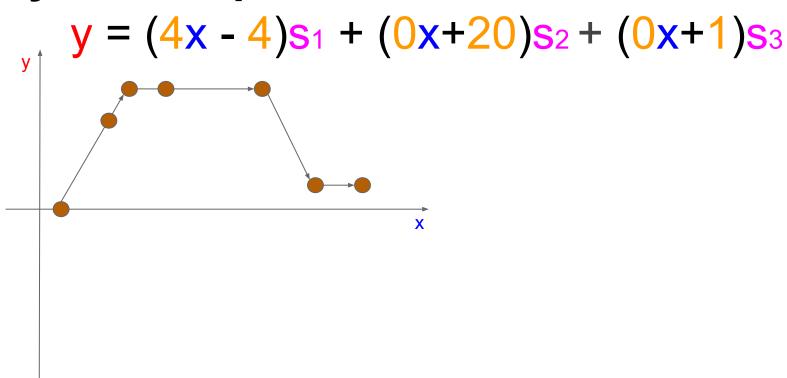
 $s_1 = \sigma(-13000) = 0$
 $s_2 = \sigma(-500) = 0$
 $s_3 = \sigma(4000) = 1$

	Data	
n	X	y
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1

$$y = 1$$

 $s_1 = \sigma(-13000) = 0$
 $s_2 = \sigma(-500) = 0$
 $s_3 = \sigma(4000) = 1$





$$y = (w_{1}x + b_{1})s_{1} + (w_{2}x+b_{2})s_{2} + (w_{3}x+b_{3})s_{3}$$

$$s_{1} = \sigma(w_{4}x + b_{4})$$

$$s_{2} = \sigma(w_{5}s_{1} + w_{6}s_{3} + b_{5})$$

$$s_{3} = \sigma(w_{7}x + b_{6})$$

$$y = (w_{1}x + b_{1})s_{1} + (w_{2}x+b_{2})s_{2} + (w_{3}x+b_{3})s_{3}$$

$$s_{1} = \sigma(w_{4}x + b_{4})$$

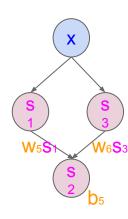
$$s_{2} = \sigma(w_{5}s_{1} + w_{6}s_{3} + b_{5})$$

$$s_{3} = \sigma(w_{7}x + b_{6})$$

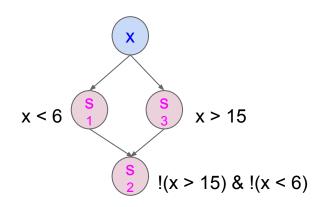
$$y = (w_1x + b_1)s_1 + (w_2x+b_2)s_2 + (w_3x+b_3)s_3$$

$$s_1 = \sigma(w_4x + b_4)$$

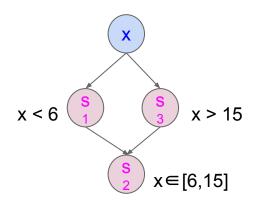
 $s_2 = \sigma(w_5s_1 + w_6s_3 + b_5)$
 $s_3 = \sigma(w_7x + b_6)$

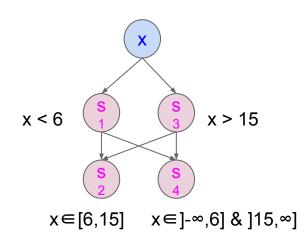


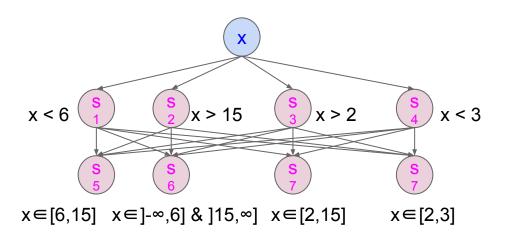
$$y = (w_1x + b_1)s_1 + (w_2x+b_2)s_2 + (w_3x+b_3)s_3$$

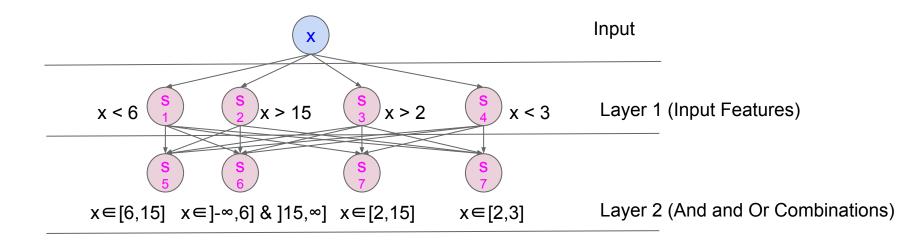


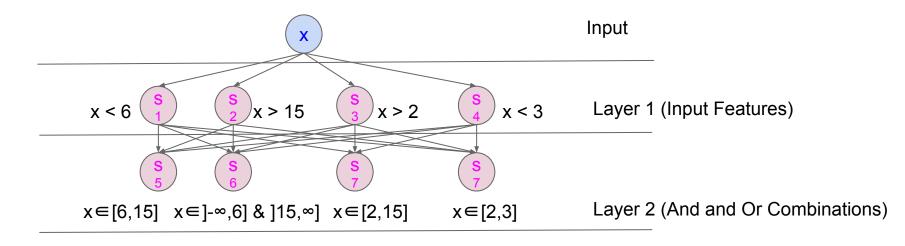
$$y = (w_1x + b_1)s_1 + (w_2x+b_2)s_2 + (w_3x+b_3)s_3$$



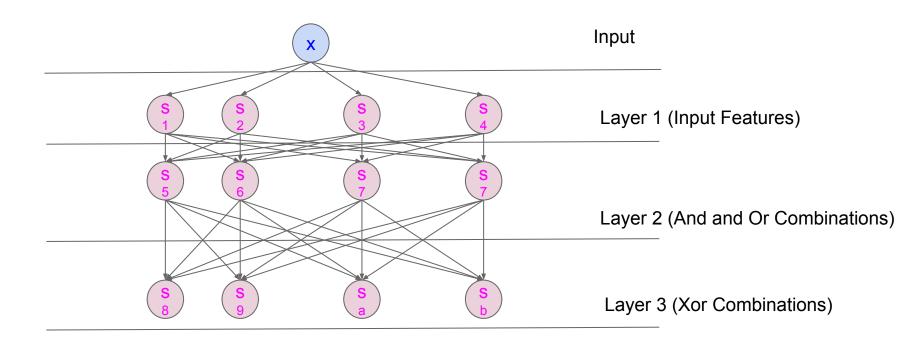


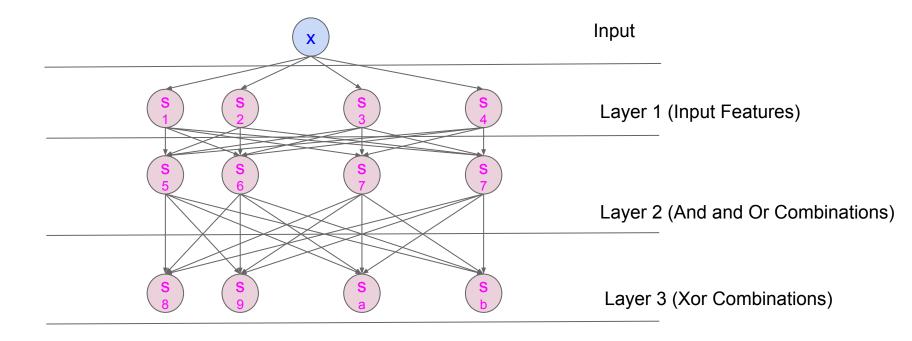






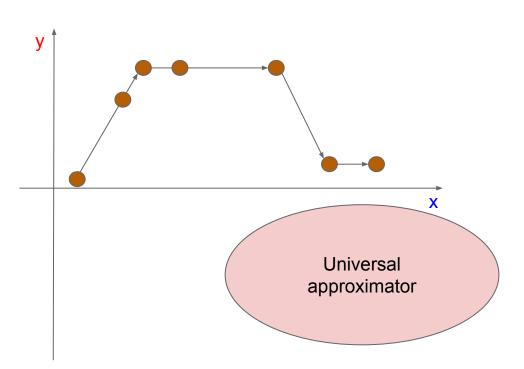
And(s₁,s₂) =
$$\sigma$$
(1000s₁ + 1000s₃ - 1500)
Or(s₁,s₂) = σ (1000s₁ + 1000s₃ - 500)



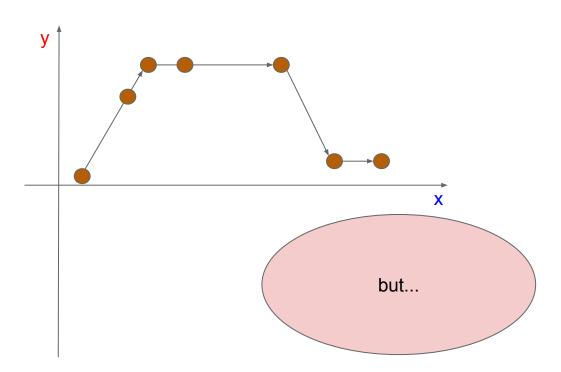


 $Xor(s_1,s_2) = Or(And(s_1,!s_2), And(!s_1,s_2))$

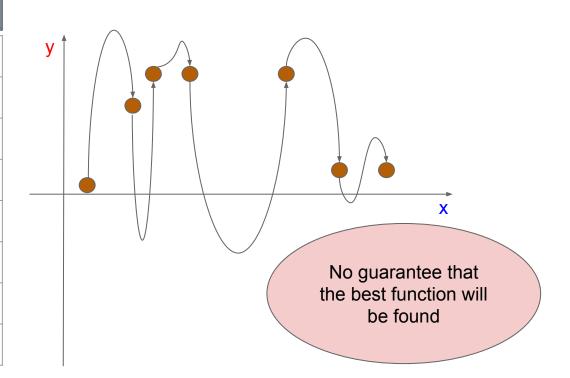
	Data	
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1



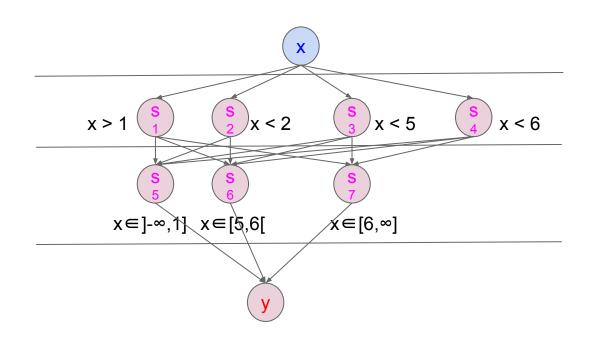
	Da	ata
n	X	y
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1



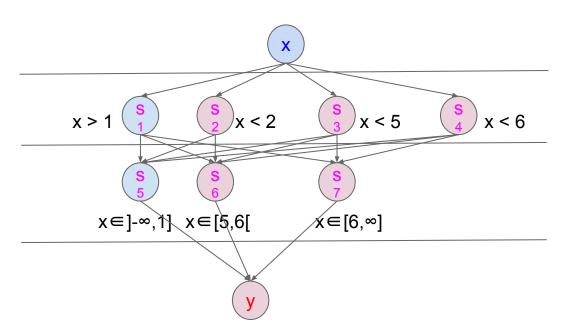
	Da	ata
n	X	у
0	1	0
1	5	16
2	6	20
3	9	20
4	11	20
5	15	1
6	19	1



n	X	y
0	1	0
1	5	16
2	6	20

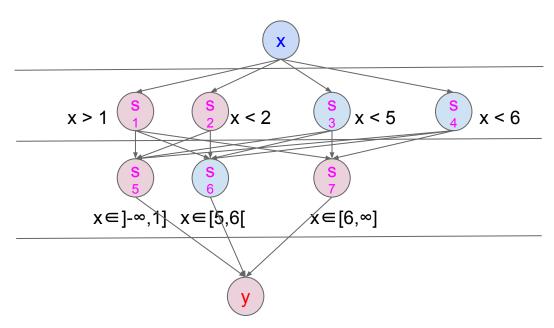


n	X	y
0	1	0
1	5	16
2	6	20



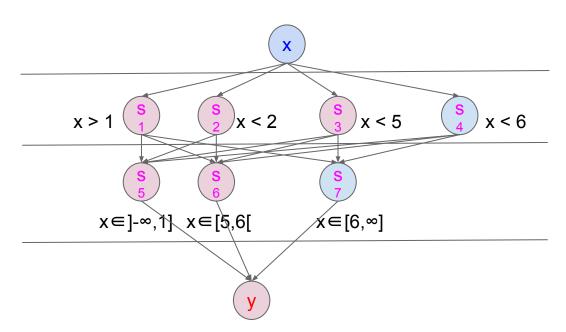
$$y = 0s_5 + 16s_6 + 20s_7$$

n	X	у
0	1	0
1	5	16
2	6	20



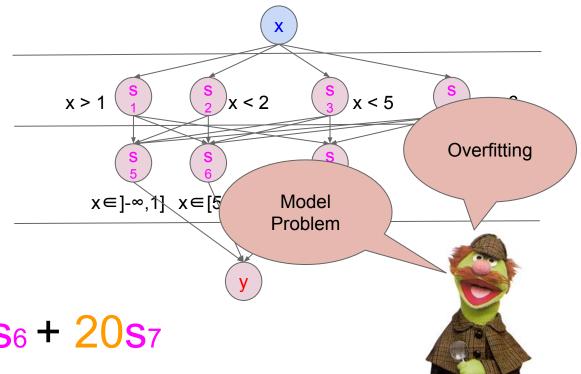
$$y = 0s_5 + 16s_6 + 20s_7$$

n	X	y
0	1	0
1	5	16
2	6	20



$$y = 0s_5 + 16s_6 + 20s_7$$

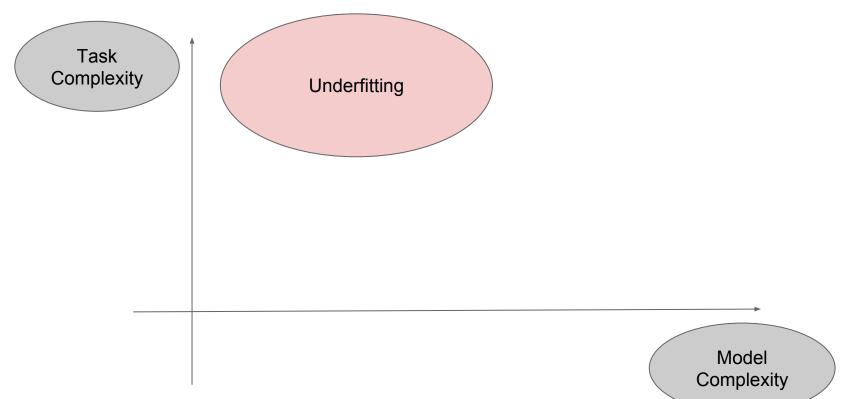
n	X	у
0	1	0
1	5	16
2	6	20

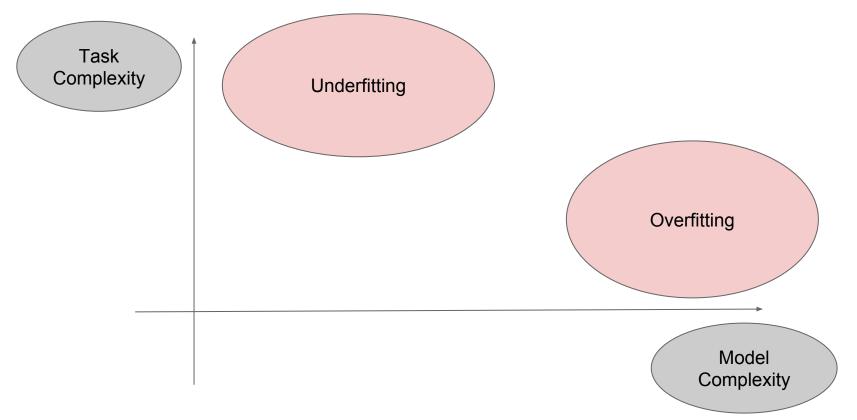


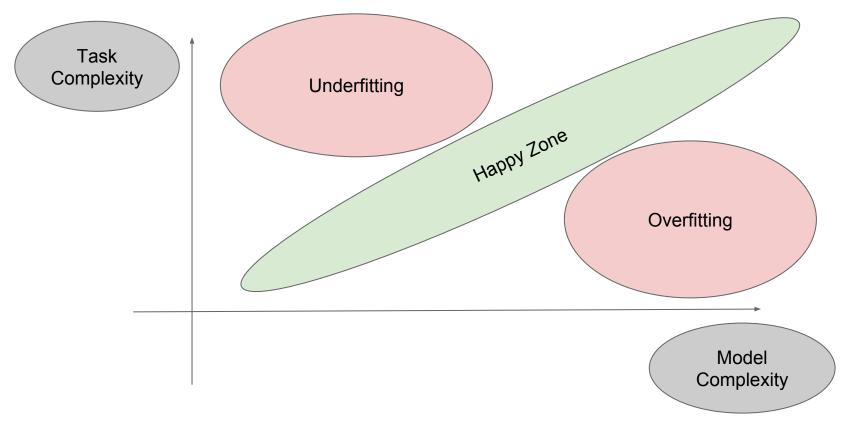
$$y = 0s_5 + 16s_6 + 20s_7$$

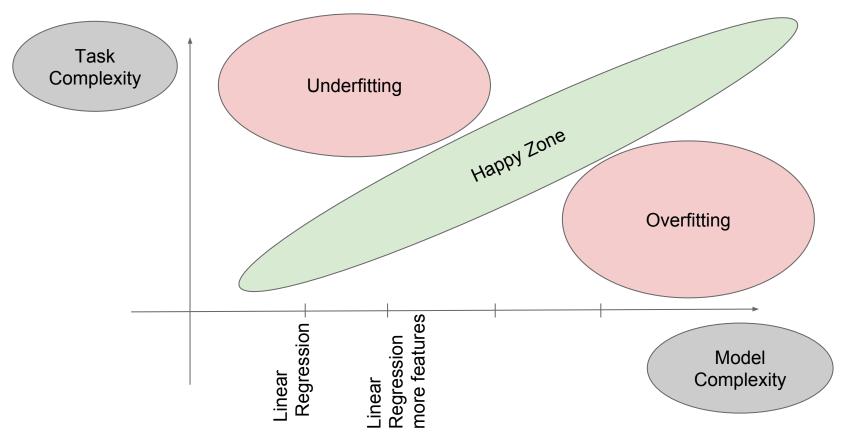


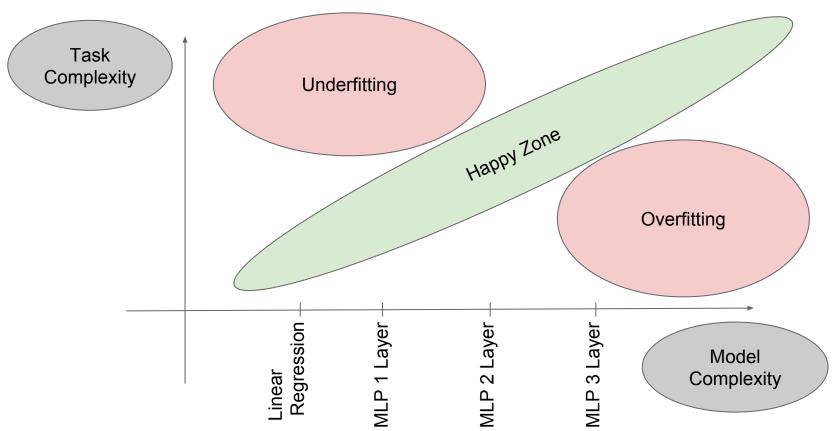
Complexity

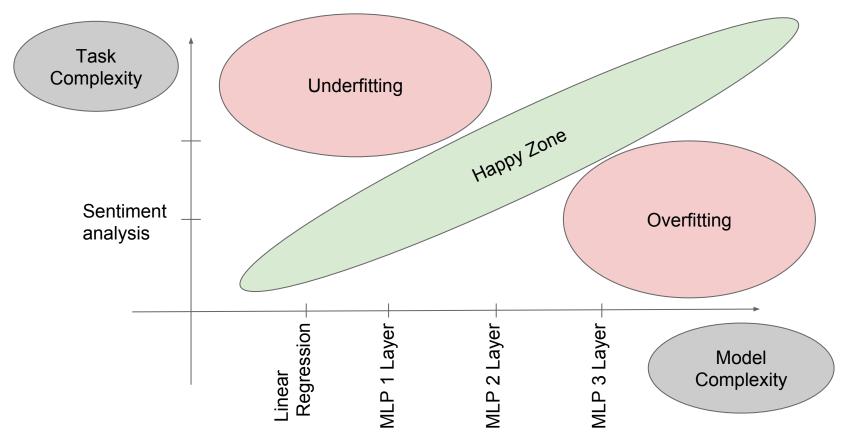


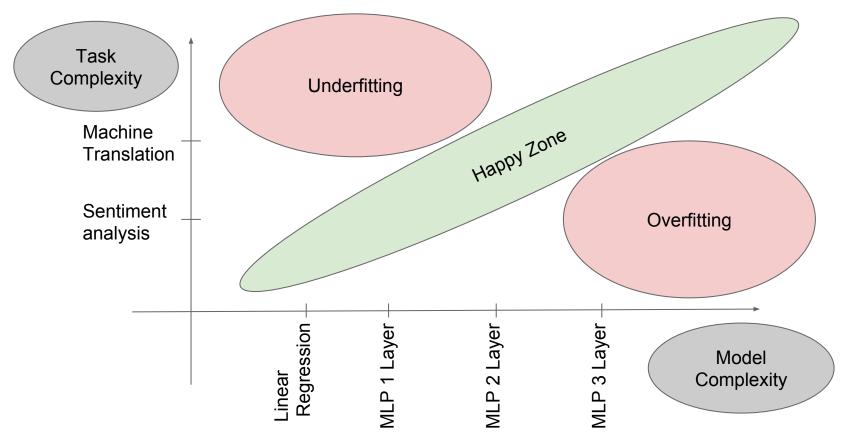


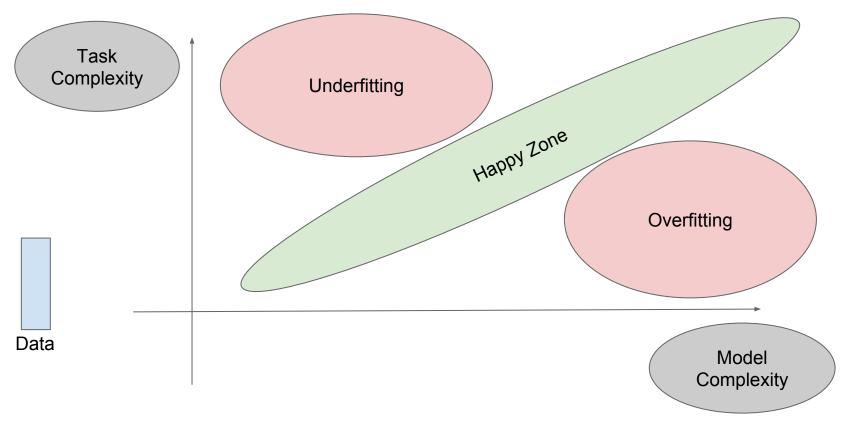


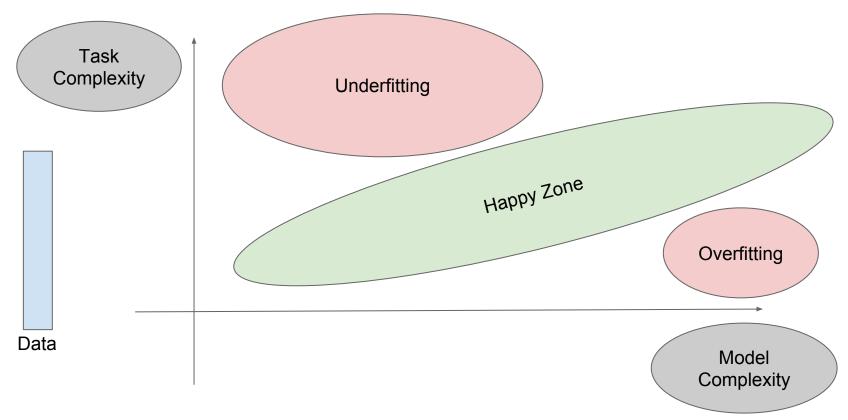


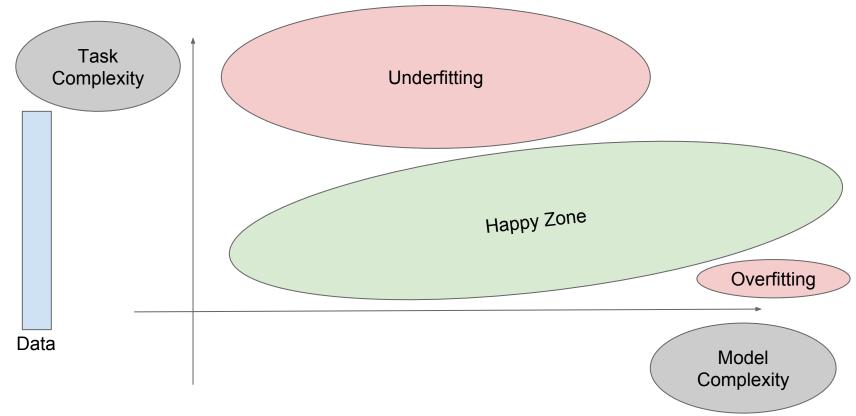




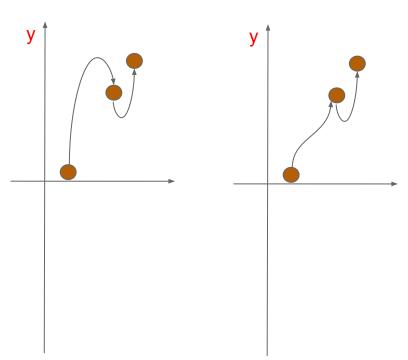


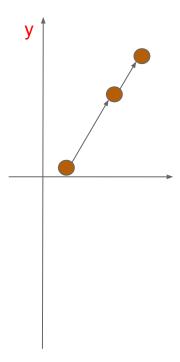




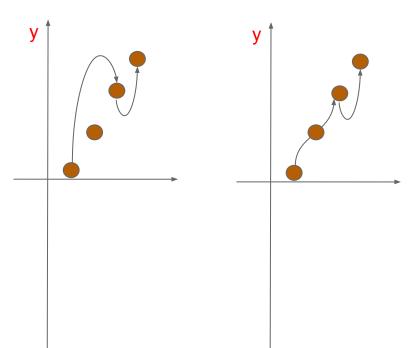


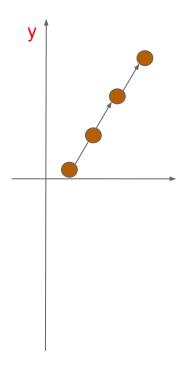
n	X	y
0	1	0
1	5	16
2	6	20



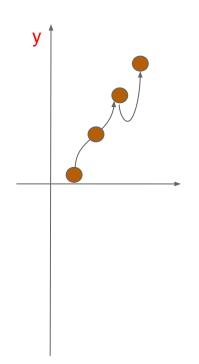


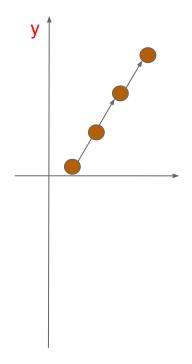
n	X	y
0	1	0
1	5	16
2	6	20
3	2	4

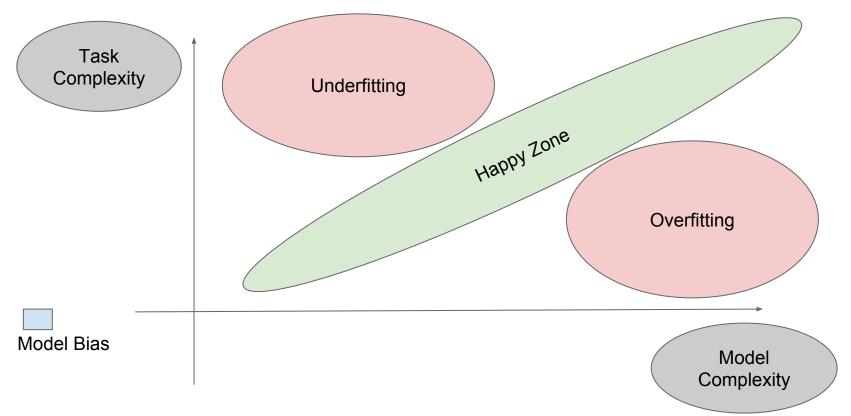


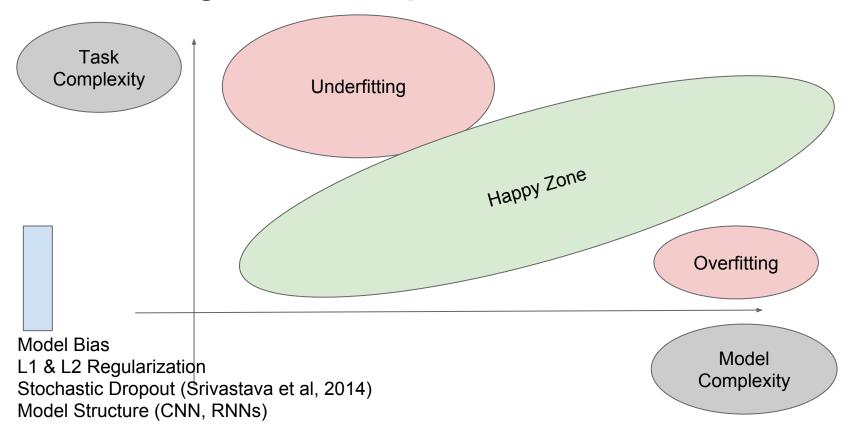


n	X	y
0	1	0
1	5	16
2	6	20
3	2	4







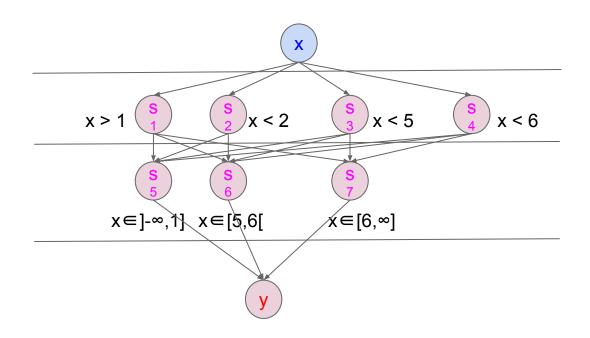


Regularization

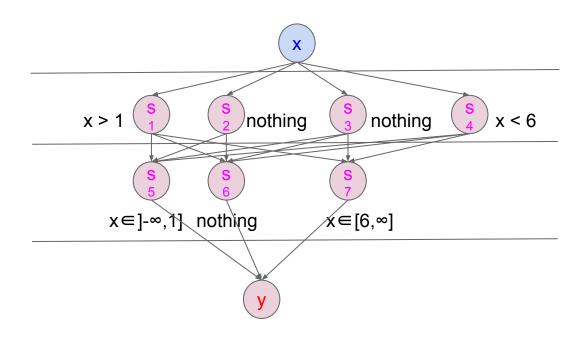
$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2 + (w+b) \beta$$

ß = Regularization constant

Regularization

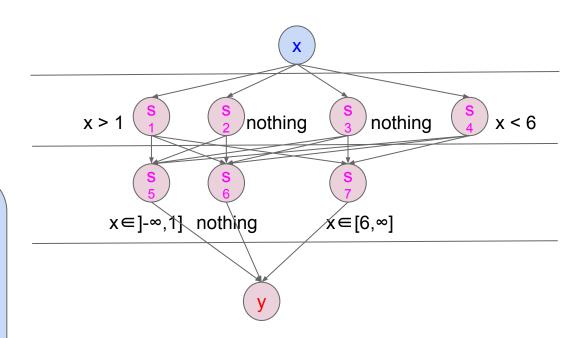


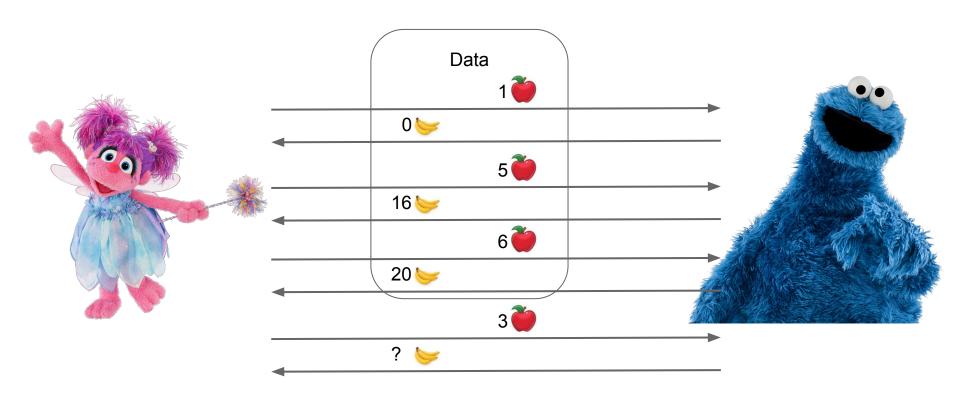
Regularization

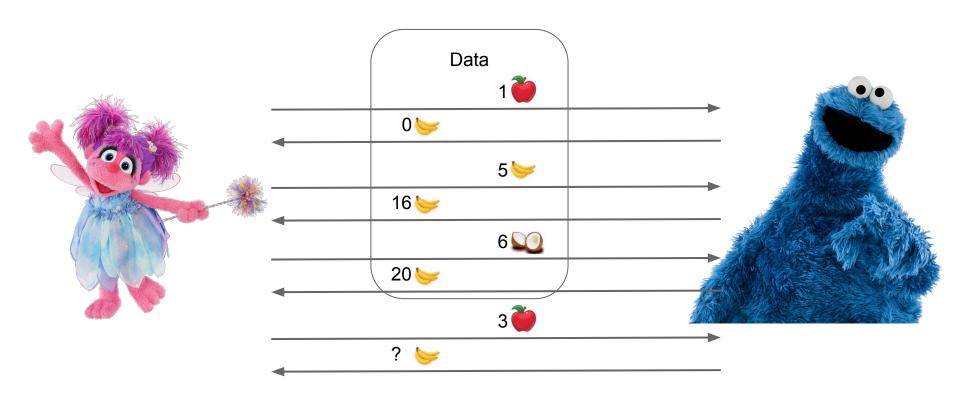


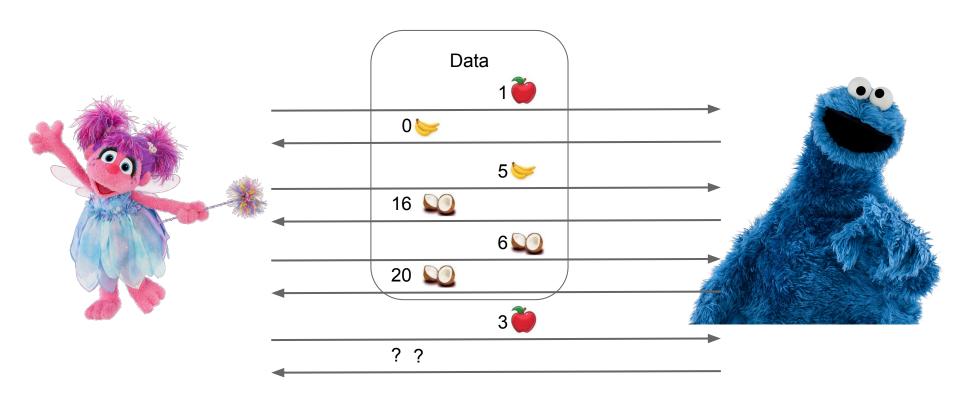
Regularization

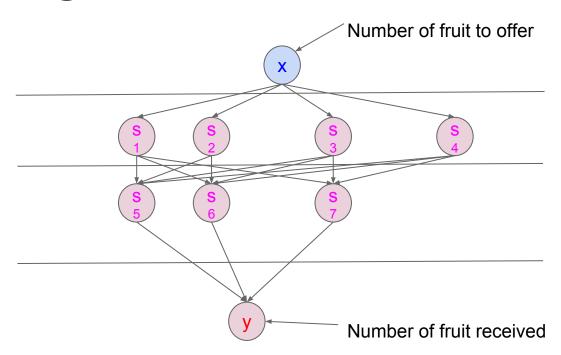
Find solutions that require less effort

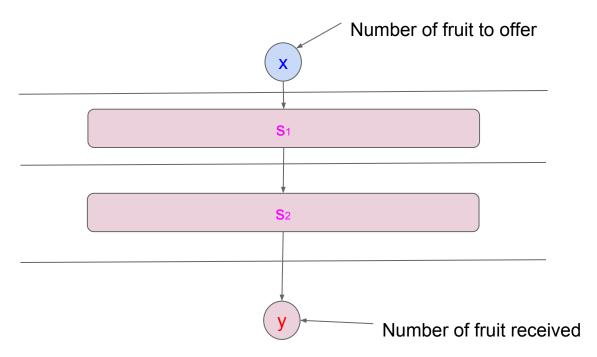


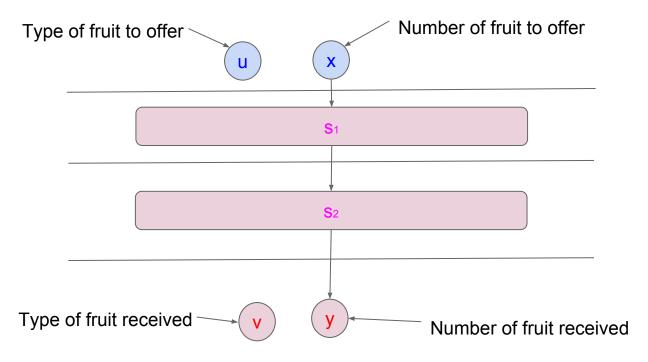


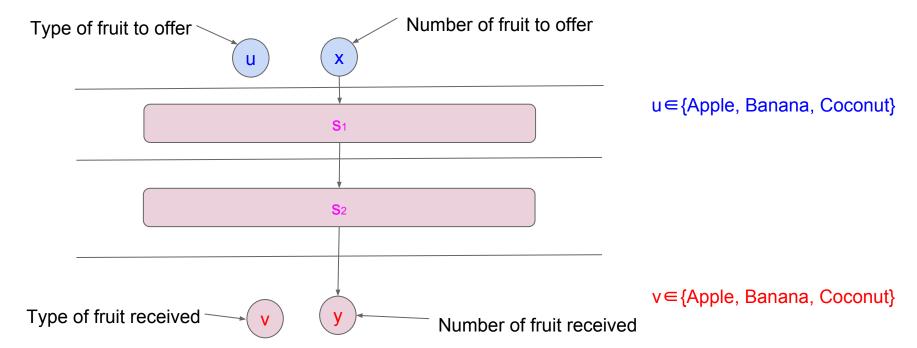








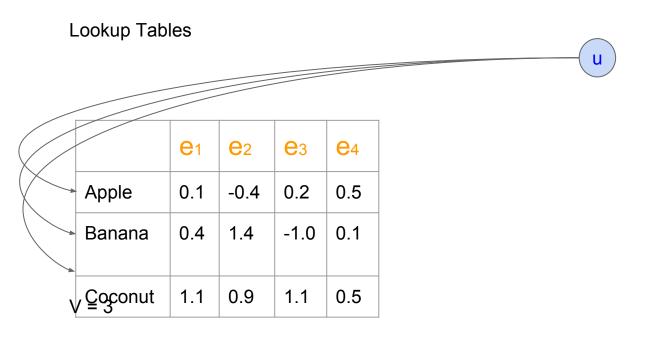


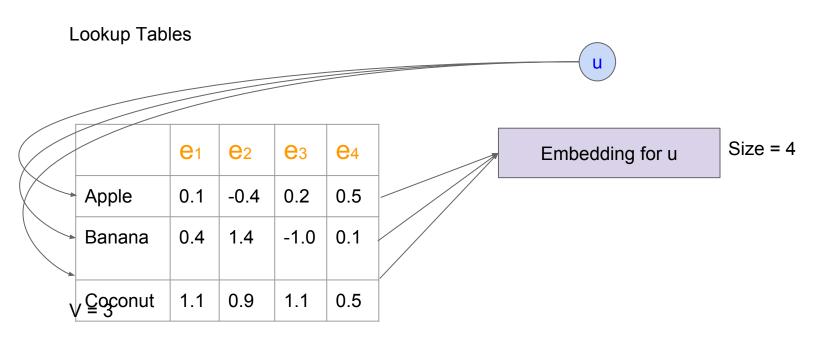


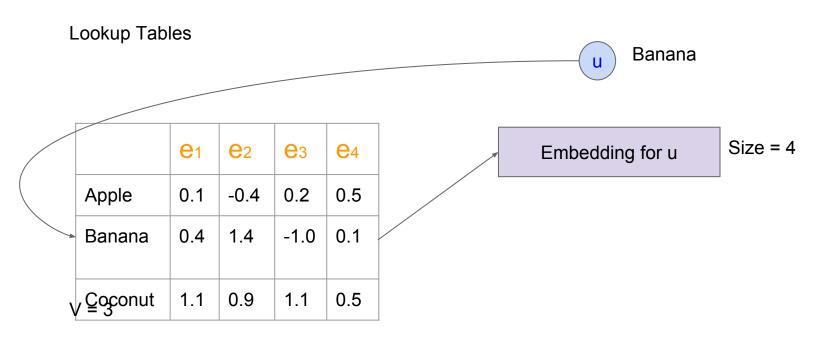
Lookup Tables

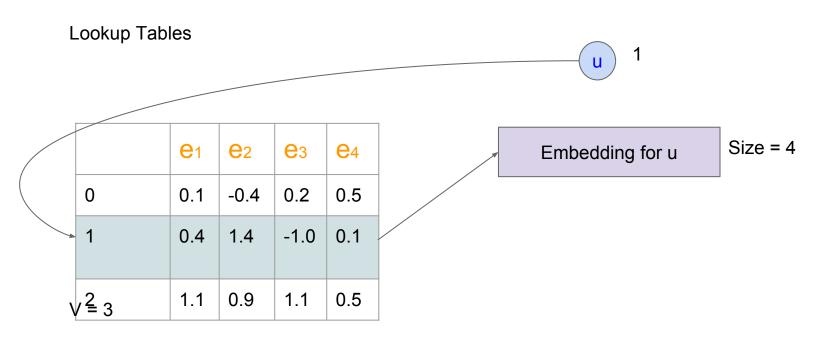
u

		e 1	e 2	e 3	e 4
	Apple	0.1	-0.4	0.2	0.5
	Banana	0.4	1.4	-1.0	0.1
V	, <u>C</u> ogconut	1.1	0.9	1.1	0.5

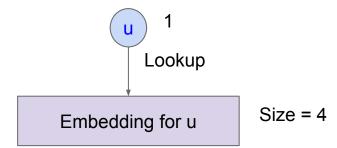


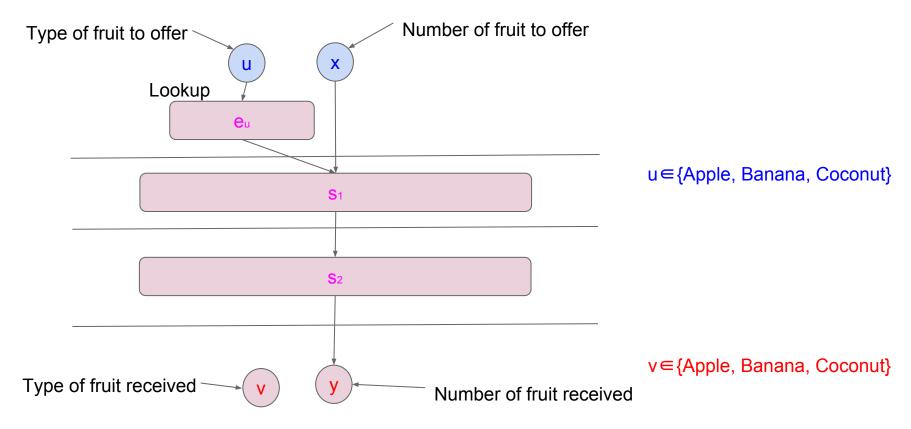






Lookup Tables





Softmax

$$V = 3$$

	Apple	Banana	Coconut
W 1	0.1	-0.4	0.2
W ₂	0.4	1.4	-1.0
W 3	1.1	0.9	1.1
W4	1.3	0.1	0.4

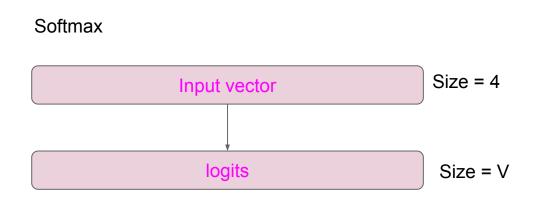
Softmax

Input vector

Size = 4

V = 3

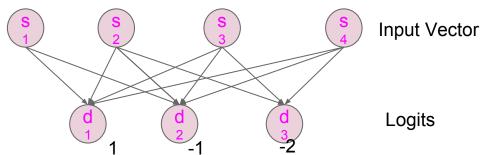
	Apple	Banana	Coconut
W 1	0.1	-0.4	0.2
W ₂	0.4	1.4	-1.0
W 3	1.1	0.9	1.1
W4	1.3	0.1	0.4



V	=	3
---	---	---

	Apple	Banana	Coconut
W 1	0.1	-0.4	0.2
W2	0.4	1.4	-1.0
W 3	1.1	0.9	1.1
W4	1.3	0.1	0.4

Softmax

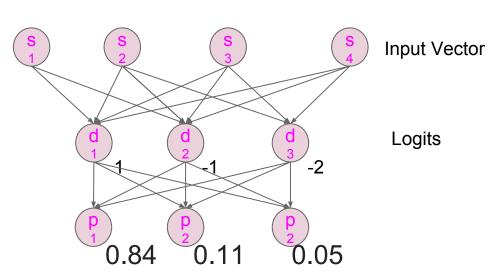


V = 3

Logits

	Apple	Banana	Coconut
W 1	0.1	-0.4	0.2
W ₂	0.4	1.4	-1.0
W 3	1.1	0.9	1.1
W 4	1.3	0.1	0.4

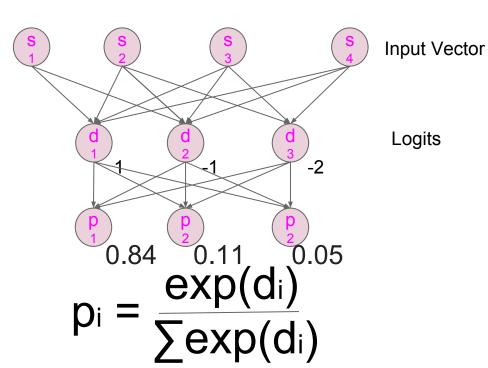
Softmax



V = 3

	Apple	Banana	Coconut
W 1	0.1	-0.4	0.2
W ₂	0.4	1.4	-1.0
W 3	1.1	0.9	1.1
W 4	1.3	0.1	0.4

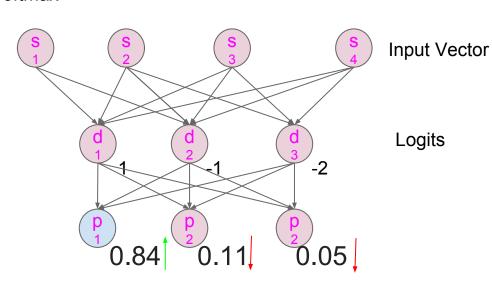
Softmax



$$V = 3$$

	Apple	Banana	Coconut
W 1	0.1	-0.4	0.2
W2	0.4	1.4	-1.0
W 3	1.1	0.9	1.1
W4	1.3	0.1	0.4

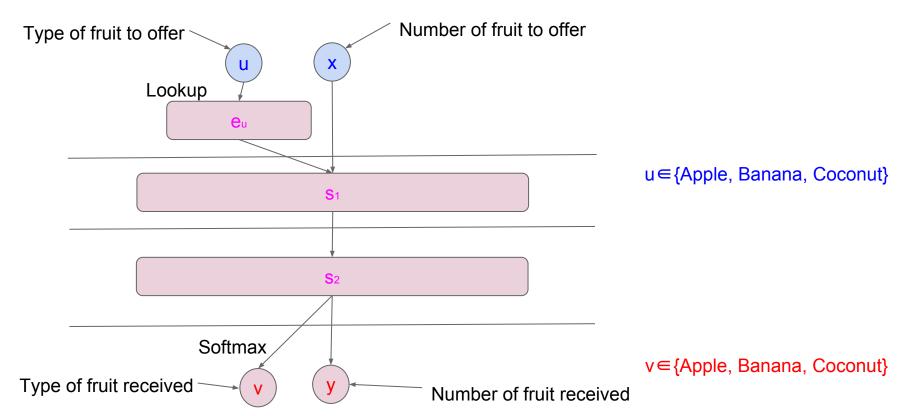
Softmax

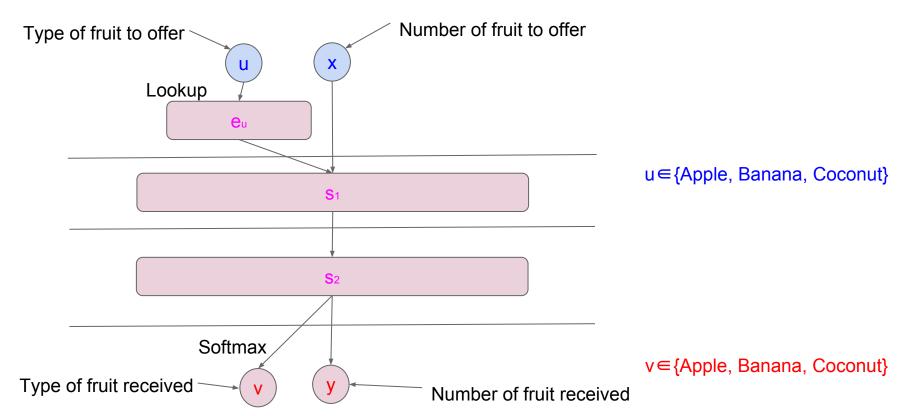


Apple

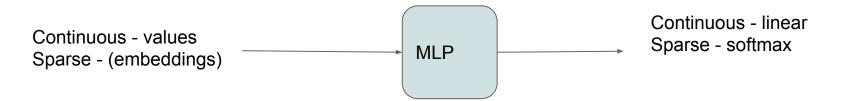
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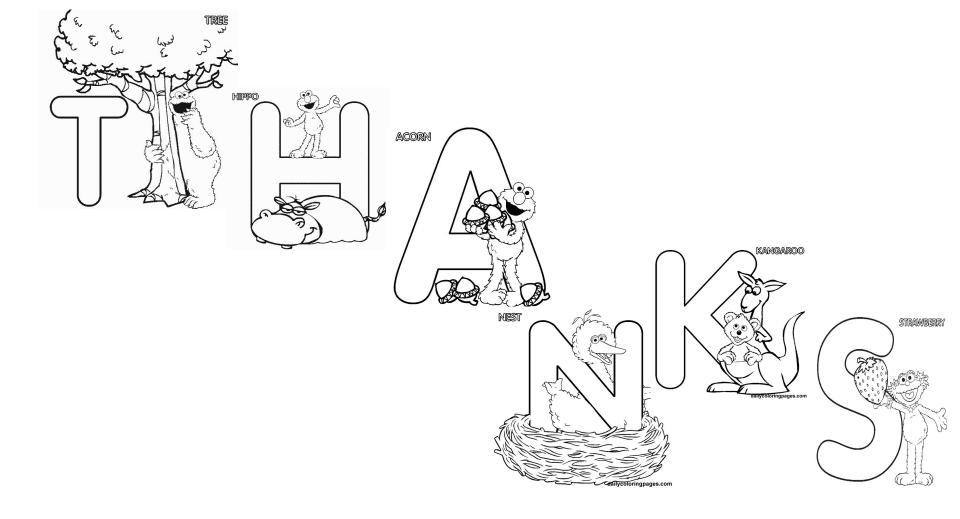
	Apple	Banana	Coconut
W 1	0.1	-0.4	0.2
W2	0.4	1.4	-1.0
W 3	1.1	0.9	1.1
W4	1.3	0.1	0.4





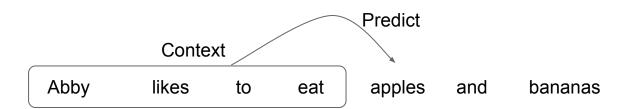
Summary

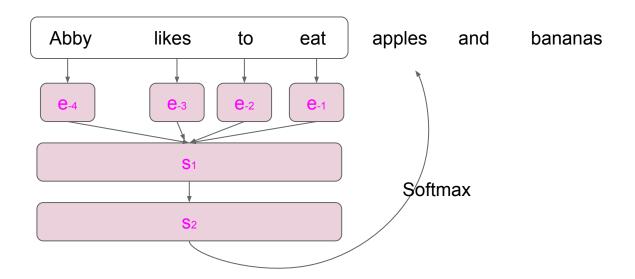


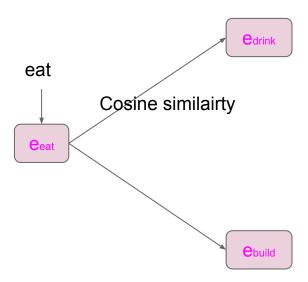


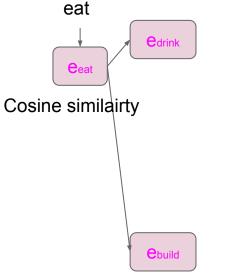
Embedding Pretraining (Collobert et al, 2011)

Abby likes to eat apples and bananas

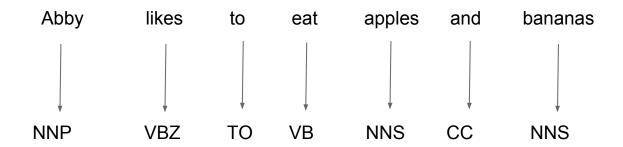


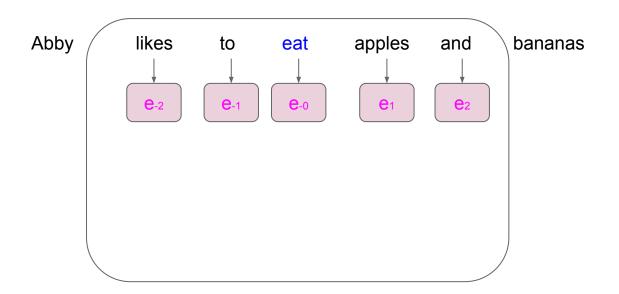


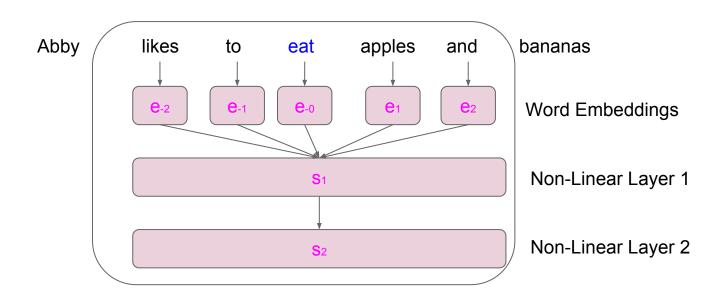


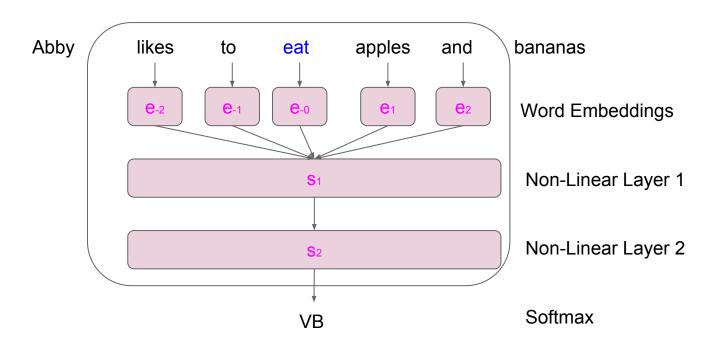


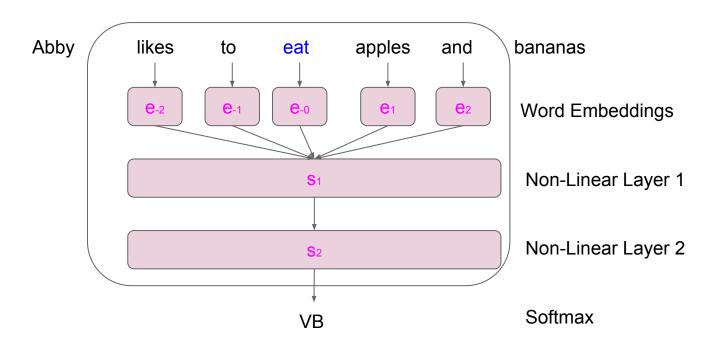
FRANCE	JESUS	XBOX	REDDISH	SCRATCHED	MEGABITS
454	1973	6909	11724	29869	87025
AUSTRIA	GOD	AMIGA	GREENISH	NAILED	OCTETS
BELGIUM	SATI	PLAYSTATION	BLUISH	SMASHED	MB/S
GERMANY	CHRIST	MSX	PINKISH	PUNCHED	$_{ m BIT/S}$
ITALY	SATAN	IPOD	PURPLISH	POPPED	BAUD
GREECE	KALI	\mathbf{SEGA}	BROWNISH	CRIMPED	CARATS
SWEDEN	INDRA	PSNUMBER	GREYISH	SCRAPED	$_{ m KBIT/S}$
NORWAY	VISHNU	HD	GRAYISH	SCREWED	MEGAHERTZ
EUROPE	ANANDA	DREAMCAST	WHITISH	SECTIONED	MEGAPIXELS
HUNGARY	PARVATI	GEFORCE	SILVERY	SLASHED	$_{ m GBIT/S}$
SWITZERLAND	GRACE	CAPCOM	YELLOWISH	RIPPED	AMPERES











Approach	POS	CHUNK	NER	SRL
	(PWA)	(F1)	(F1)	(F1)
Benchmark Systems	97.24	94.29	89.31	77.92
NN+WLL	96.31	89.13	79.53	55.40
NN+SLL	96.37	90.33	81.47	70.99
NN+WLL+LM1	97.05	91.91	85.68	58.18
NN+SLL+LM1	97.10	93.65	87.58	73.84
NN+WLL+LM2	97.14	92.04	86.96	58.34
NN+SLL+LM2	97.20	93.63	88.67	74.15

Translation Rescoring (Devlin et al, 2014)

Source

Abby

gosta

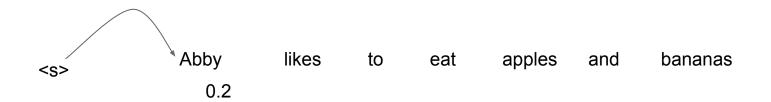
de

Translation 1	John	does	to	eat	coconuts	and	bananas
Translation 2	Abby	likes	to	eat	apples an	d ba	inanas
Translation 3	Abby	dislikes	to	drink	apples	and	bananas

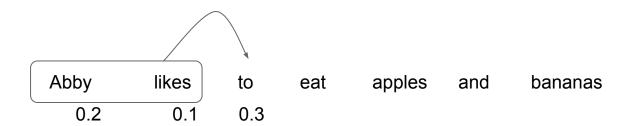
comer

macas

bananas



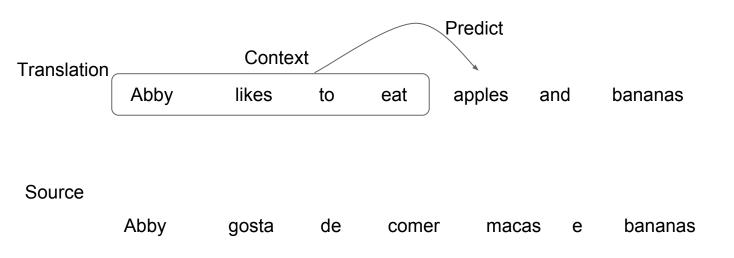


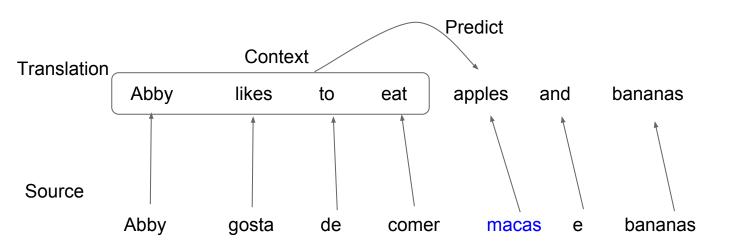


Abby	likes	to	eat	apples	and	bananas	0.000378
0.2	0.1	0.3	0.5	0.7	0.4	0.2	

John	does	to	eat	coconuts and	bananas	0.00003
Abby	likes	to	eat	apples and	bananas	0.000378
Abby	dislikes	to	drink	apples and	l bananas	0.00012

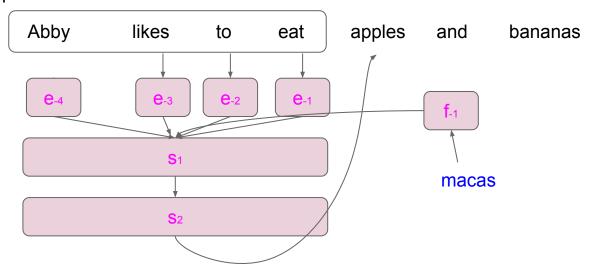
John	does	to	eat	coconuts	and	bananas	0.00003
Abby	likes	to	eat	apples and	d	bananas	0.000378
Abby	dislikes	to	drink	apples	and	bananas	0.00012





Translation Rescoring (Devlin et al, 2014)

Translation



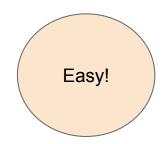
Translation Score (BLEU)	Arabic - English	Chinese - English
Best Rescored System	52.8	34.7
1st OpenMT12	49.5	32.6
Hierarchical	43.4	30.1

$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$
 $y = wx + b$

$$y = wx + b$$

$$\frac{\partial C}{\partial w} = \frac{\partial \sum_{n} (\hat{y}_{n} - y_{n})^{2}}{\partial w} = \sum_{n} -2(\hat{y}_{n} - y_{n}) x_{n}$$

$$\frac{\partial C}{\partial h} = \frac{\partial \sum_{n} (\hat{y}_{n} - y_{n})^{2}}{\partial h} = \sum_{n} -2(\hat{y}_{n} - y_{n})$$



$$y = wx + b + tanh(yx + b)^2$$



$$y = w_1x + b_1 + tanh(w_2x + b_2^2)$$

Computation
Graphs can
compute
gradients for you!

$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$
 $y = wx + b$

$$\frac{\partial C}{\partial w} = \frac{\partial \sum_{n} (\hat{y}_{n} - y_{n})^{2}}{\partial w} = \sum_{n} -2(\hat{y}_{n} - y_{n}) x_{n}$$

$$\frac{\partial C}{\partial b} = \frac{\partial \sum_{n} (\hat{y}_{n} - y_{n})^{2}}{\partial b} = \sum_{n} -2(\hat{y}_{n} - y_{n})$$

$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$
 $y = wx + b$

$$\frac{\partial C}{\partial w} = \sum_{n} \frac{\partial (\hat{y}_{n} - y_{n})^{2}}{\partial y_{n}} \frac{\partial y_{n}}{\partial w} = \sum_{n} -2(\hat{y}_{n} - y_{n}) x_{n}$$

$$\frac{\partial \mathbf{C}}{\partial \mathbf{b}} = \sum_{n} \frac{\partial (\hat{\mathbf{y}}_{n} - \mathbf{y}_{n})^{2}}{\partial \mathbf{y}_{n}} \frac{\partial \mathbf{y}_{n}}{\partial \mathbf{b}} = \sum_{n} -2(\hat{\mathbf{y}}_{n} - \mathbf{y}_{n})$$

$$C(w,b) = \sum_{n \in \{0,1,2\}} (y_n - \hat{y}_n)^2$$
 $y = wx + b$

$$\frac{\partial \mathbf{C}}{\partial \mathbf{w}} = \sum_{n} \frac{\partial (\hat{\mathbf{y}}_{n} - \mathbf{y}_{n})^{2}}{\partial \mathbf{y}_{n}} \frac{\partial \mathbf{y}_{n}}{\partial \mathbf{w}}$$

$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial (\hat{y}_{n} - y_{n})^{2}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$

$$C(\mathbf{w},\mathbf{b}) = \sum_{\mathbf{n} \in \{0,1,2\}} (\mathbf{y}_{\mathbf{n}} - \hat{\mathbf{y}}_{\mathbf{n}})^2$$

$$y = o + b$$

 $o = wx$

$$\frac{\partial C}{\partial w} = \sum_{n} \frac{\partial (\hat{y}_{n} - y_{n})^{2}}{\partial y_{n}} \frac{\partial y_{n}}{\partial w}$$

$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial (\hat{y}_{n} - y_{n})^{2}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$

$$C(w,b) = \sum_{n \in \{0,1,2\}} (d_n)^2$$

$$\frac{\partial \mathbf{C}}{\partial \mathbf{w}} = \sum_{n} \frac{\partial (\hat{\mathbf{y}}_{n} - \mathbf{y}_{n})^{2}}{\partial \mathbf{y}_{n}} \frac{\partial \mathbf{y}_{n}}{\partial \mathbf{w}}$$

$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial (\hat{y}_{n} - y_{n})^{2}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$

$$d = y - \hat{y}$$
$$y = o + b$$
$$o = wx$$

$$\frac{C(w,b)}{\sum_{n\in\{0,1,2\}}}$$

$$\frac{\partial \mathbf{C}}{\partial \mathbf{w}} = \sum_{n} \frac{\partial (\hat{\mathbf{y}}_{n} - \mathbf{y}_{n})^{2}}{\partial \mathbf{y}_{n}} \frac{\partial \mathbf{y}_{n}}{\partial \mathbf{w}}$$

$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial (\hat{y}_{n} - y_{n})^{2}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$

$$c = d2$$

$$d = y - \hat{y}$$

$$y = o + b$$

$$o = wx$$

$$\frac{C(w,b)}{\sum_{n\in\{0,1,2\}}}$$

$$\frac{\partial C}{\partial w} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial c_{n}} \frac{\partial c_{n}}{\partial w}$$

$$\frac{\partial \mathbf{C}}{\partial \mathbf{b}} = \sum_{n} \frac{\partial (\hat{\mathbf{y}}_{n} - \mathbf{y}_{n})^{2}}{\partial \mathbf{y}_{n}} \frac{\partial \mathbf{y}_{n}}{\partial \mathbf{b}}$$

$$c = d2$$

$$d = y - \hat{y}$$

$$y = o + b$$

$$o = wx$$

$$\frac{C(w,b)}{\sum_{n\in\{0,1,2\}}}$$

$$\frac{\partial C}{\partial w} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial O_{n}} \frac{\partial O_{n}}{\partial w}$$

$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$

$$c = d2$$

$$d = y - \hat{y}$$

$$y = o + b$$

$$o = wx$$

Sub

$$\frac{C(w,b)}{\sum_{n\in\{0,1,2\}}}$$

$$\frac{\partial C}{\partial w} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial c_{n}} \frac{\partial c_{n}}{\partial w}$$

$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$

$$C = d^{2}$$

$$d = y - \hat{y}$$

$$y = 0 + b$$

$$O = WX$$
Product

$$\frac{C(w,b)}{\sum_{n\in\{0,1,2\}}}$$

$$\frac{\partial C}{\partial w} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial c_{n}} \frac{\partial c_{n}}{\partial w}$$

$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$

$$C = d^{2}$$

$$Delta = y - \hat{y}$$

$$y = 0 + b$$

$$Delta = y - \hat{y}$$

$$Add$$

$$Delta = y - \hat{y}$$

$$Add$$

$$Delta = y - \hat{y}$$

$$Add$$

$$Delta = y - \hat{y}$$

$$Product$$

forward(x,y) \rightarrow z backward(x,y,dz) \rightarrow dx,dy

$$\frac{C(w,b)}{\sum_{n\in\{0,1,2\}}}$$

$$\frac{\partial C}{\partial w} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial c_{n}} \frac{\partial c_{n}}{\partial w}$$

$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$

$$C = d^{2}$$

$$d = y - \hat{y}$$

$$y = 0 + b$$

$$O = WX$$
Product

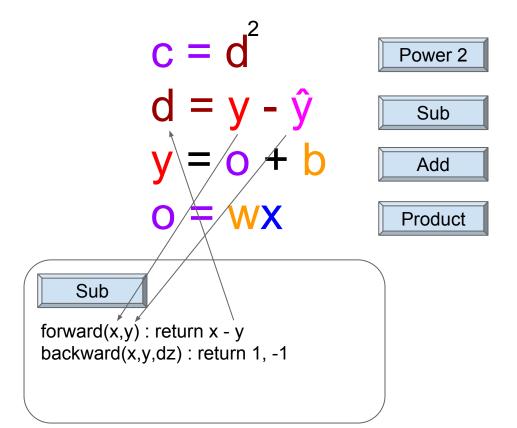
Sub

forward(x,y): return x - y backward(x,y,dz): return 1, -1

$$\frac{C(w,b)}{\sum_{n\in\{0,1,2\}}}$$

$$\frac{\partial C}{\partial w} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial c_{n}} \frac{\partial c_{n}}{\partial w}$$

$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$



$$C(w,b) = \sum_{n \in \{0,1,2\}} C_n$$

$$\frac{\partial C}{\partial w} = \sum_{n}^{\infty} \frac{\partial C_n}{\partial d_n} \frac{\partial d_n}{\partial y_n} \frac{\partial y_n}{\partial o_n} \frac{\partial o_n}{\partial w}$$

$$\frac{\partial C}{\partial b} = \sum_{n}^{\infty} \frac{\partial C_n}{\partial d_n} \frac{\partial d_n}{\partial y_n} \frac{\partial y_n}{\partial b}$$

$$C = d^2$$

$$y = y - \hat{y}$$

$$y = 0 + b$$

$$O = WX$$
Product
$$O = WX$$

$$O = WX$$
Product
$$\frac{\partial C}{\partial v} = \sum_{n}^{\infty} \frac{\partial C_n}{\partial d_n} \frac{\partial d_n}{\partial y_n} \frac{\partial y_n}{\partial b}$$

$$O = WX$$

$$O = WX$$

$$\frac{C(w,b)}{\sum_{n\in\{0,1,2\}}}$$

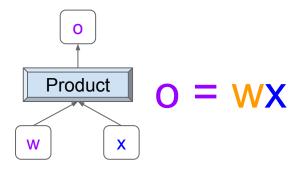
$$\frac{\partial C}{\partial w} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial o_{n}} \frac{\partial o_{n}}{\partial w}$$

$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$

$$c = d^{2}$$

$$d = y - \hat{y}$$

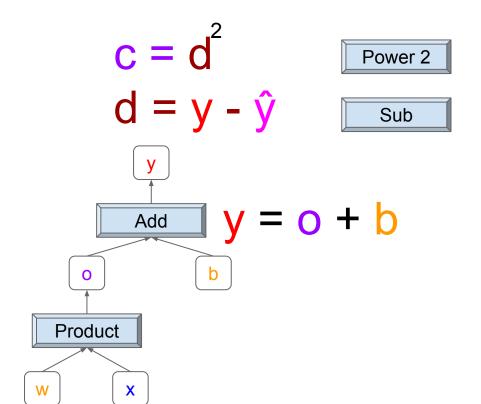
$$y = 0 + b$$
Add



$$\frac{C(w,b)}{\sum_{n\in\{0,1,2\}}}$$

$$\frac{\partial C}{\partial w} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial o_{n}} \frac{\partial o_{n}}{\partial w}$$

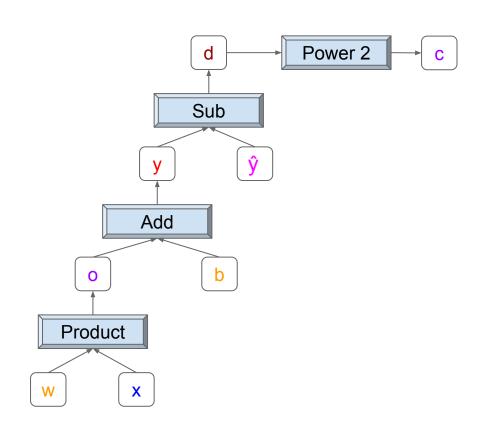
$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$



$$\frac{\mathbf{C}(\mathbf{w},\mathbf{b}) = \sum_{\mathbf{n} \in \{0,1,2\}} \mathbf{c}_{\mathbf{n}}$$

$$\frac{\partial C}{\partial w} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial o_{n}} \frac{\partial o_{n}}{\partial w}$$

$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$



$$\frac{\partial C}{\partial w} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial o_{n}} \frac{\partial o_{n}}{\partial w}$$

$$\frac{\partial C}{\partial b} = \sum_{n} \frac{\partial C_{n}}{\partial d_{n}} \frac{\partial d_{n}}{\partial y_{n}} \frac{\partial y_{n}}{\partial b}$$
Product

Power 2

C Id

C

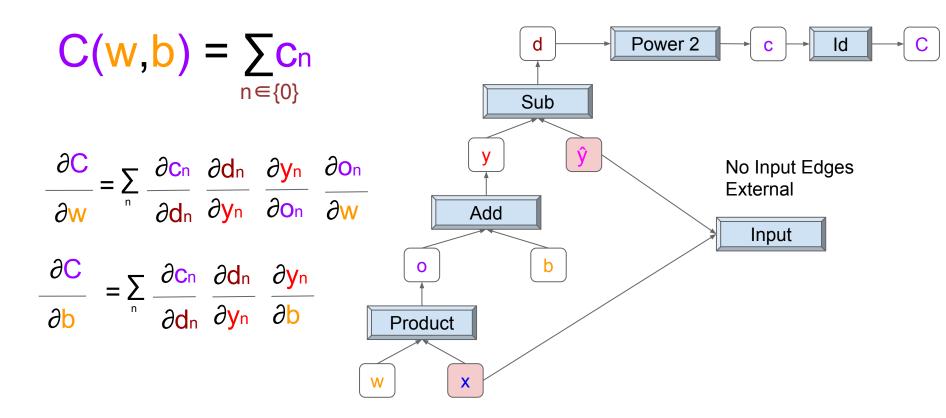
Power 2

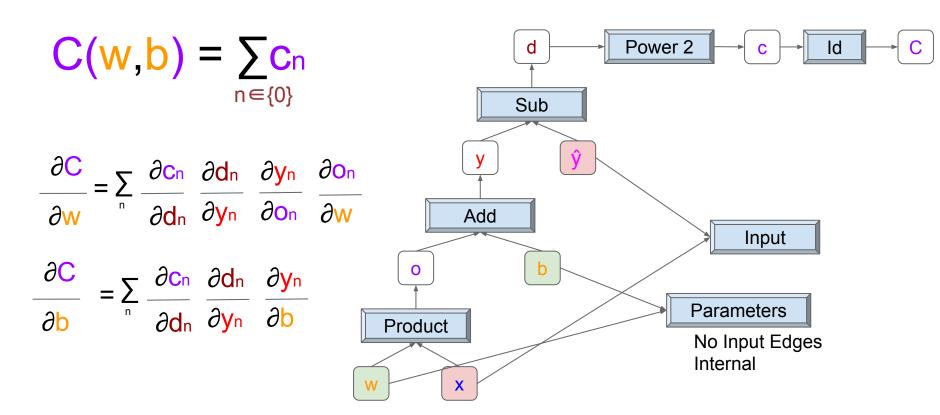
C

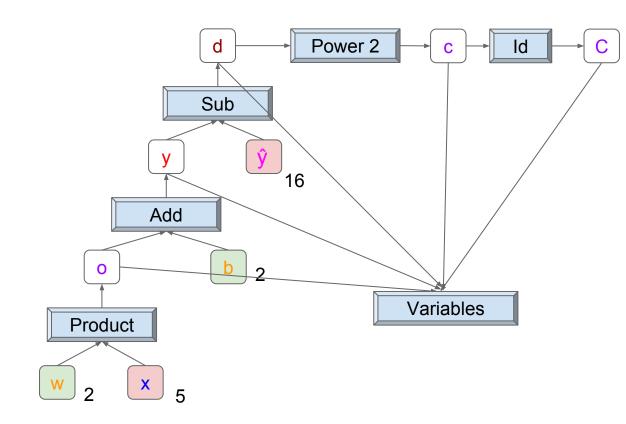
Id

C

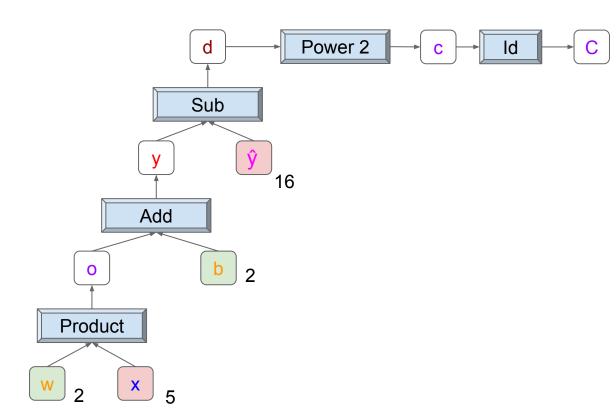
Product





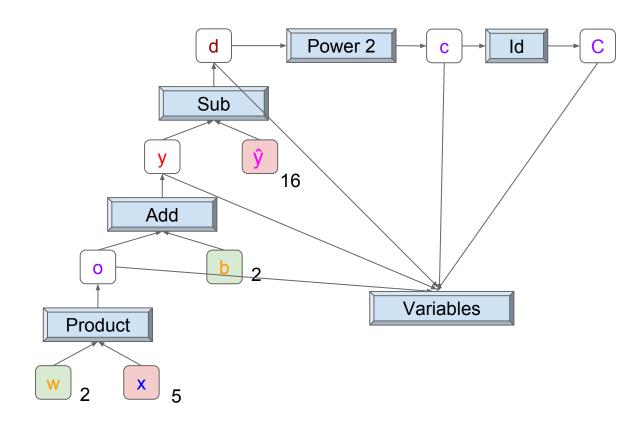


1-Initialize inputs



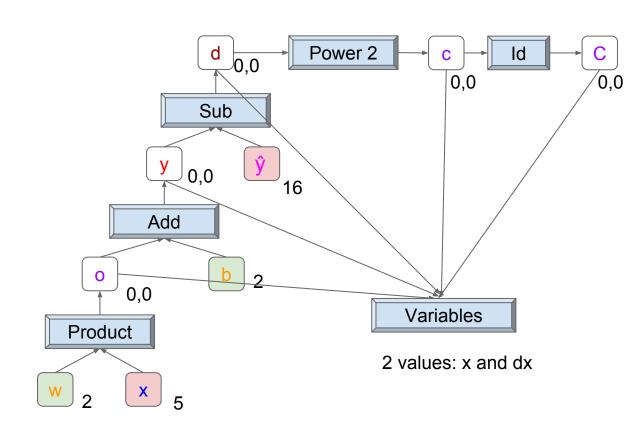
1-Initialize inputs

2-Initialize variables

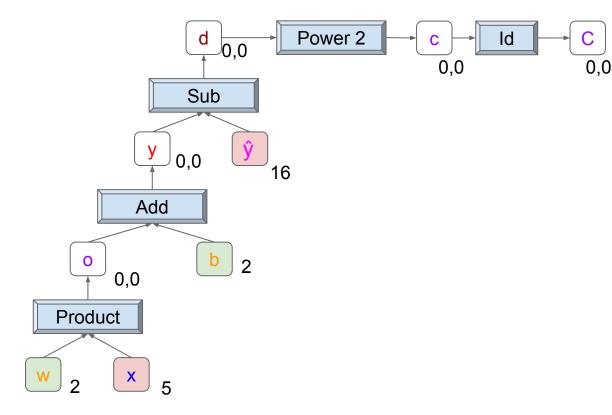


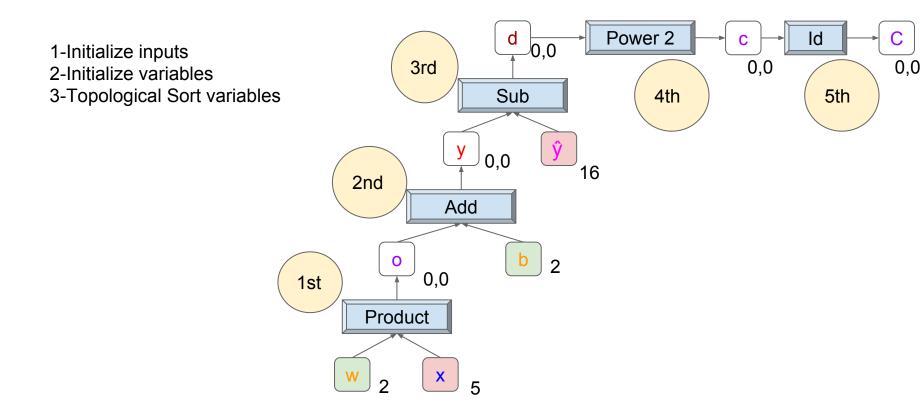
1-Initialize inputs

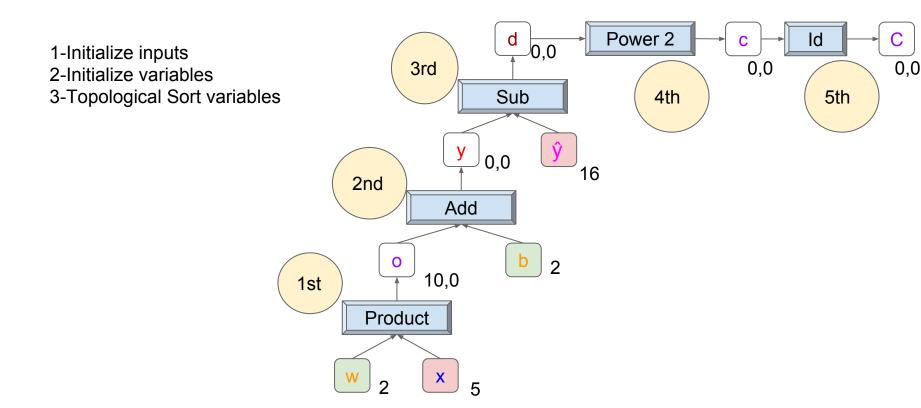
2-Initialize variables

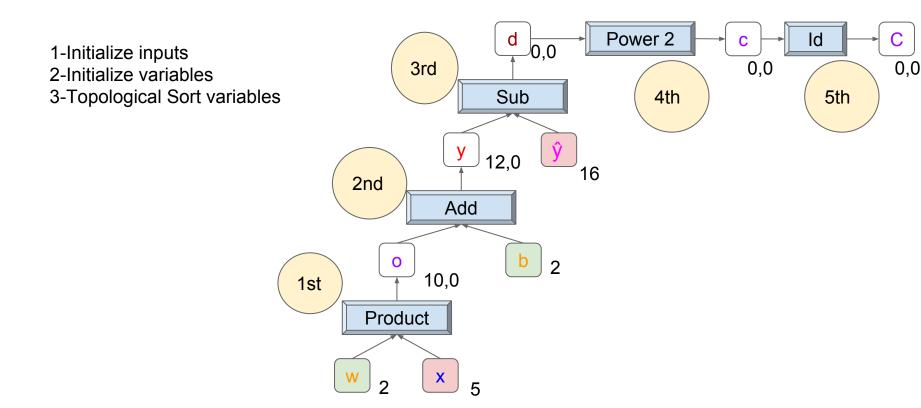


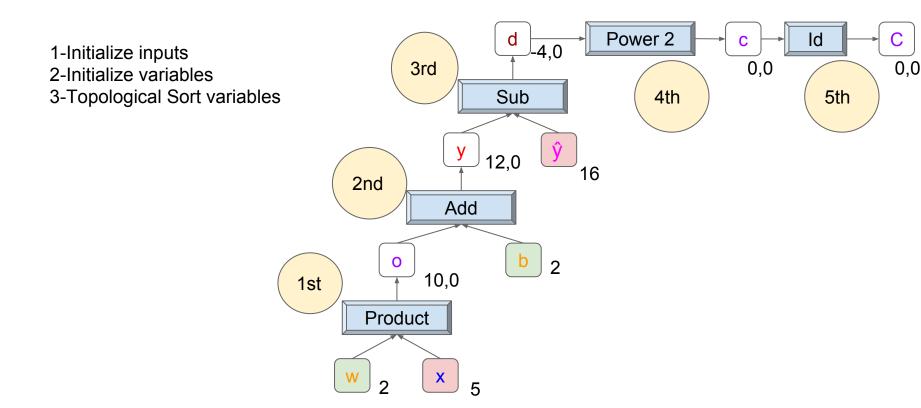
- 1-Initialize inputs
- 2-Initialize variables
- 3-Topological Sort variables

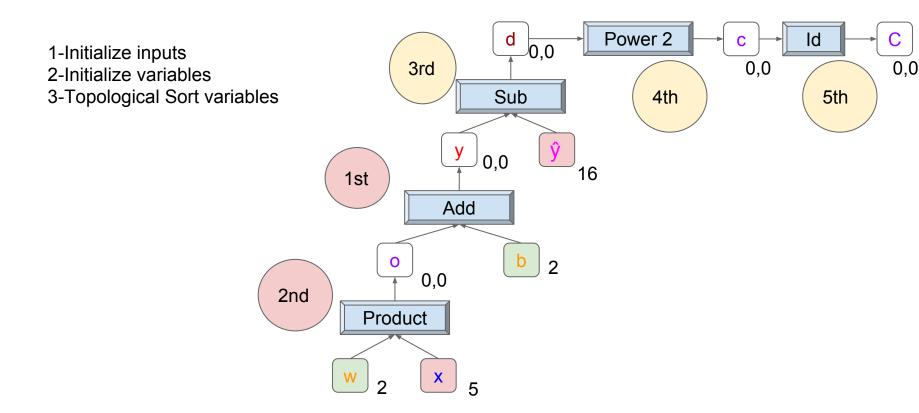


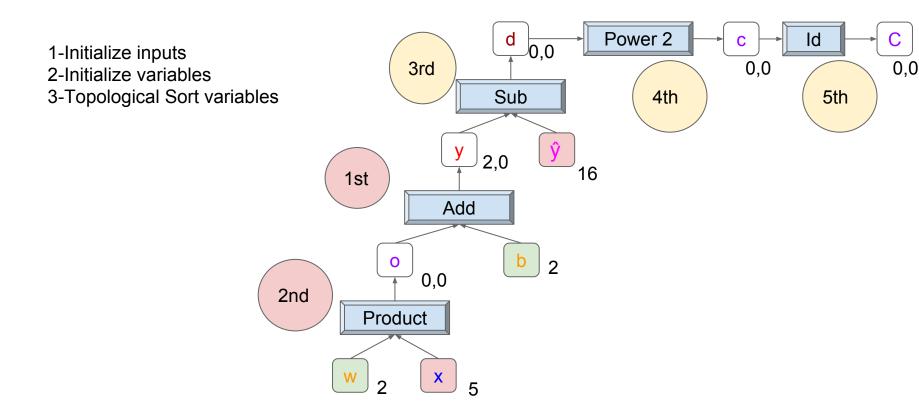


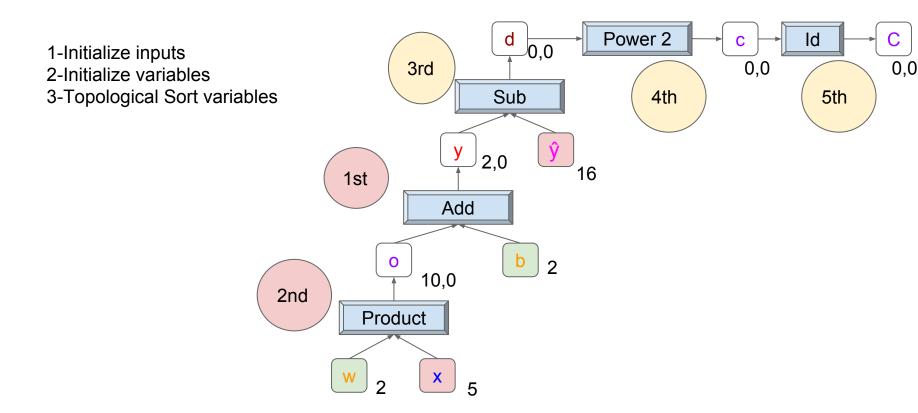


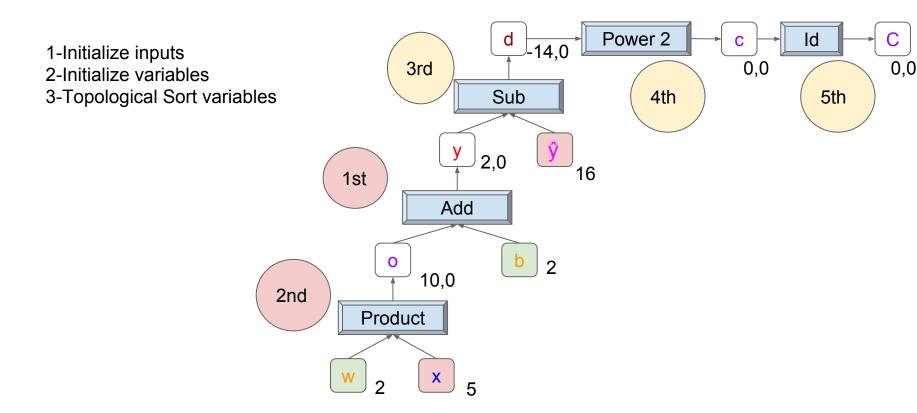




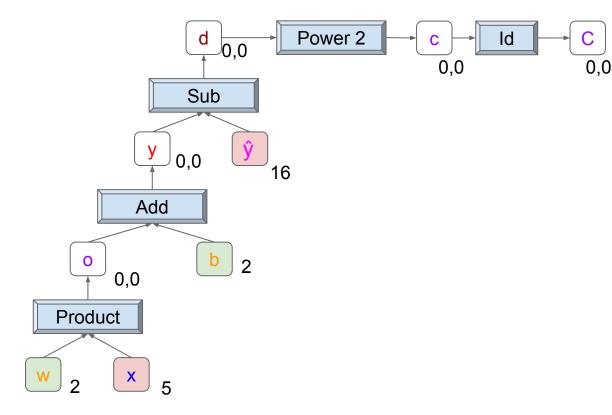




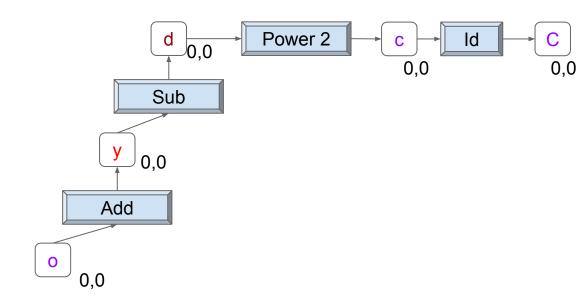




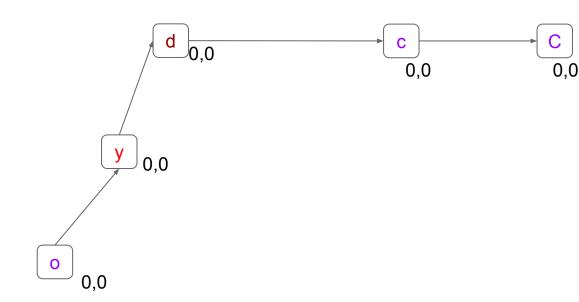
- 1-Initialize inputs
- 2-Initialize variables
- 3-Topological Sort variables



- 1-Initialize inputs
- 2-Initialize variables
- 3-Topological Sort variables



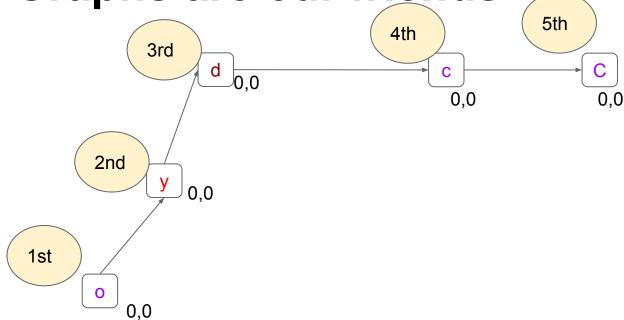
- 1-Initialize inputs
- 2-Initialize variables
- 3-Topological Sort variables



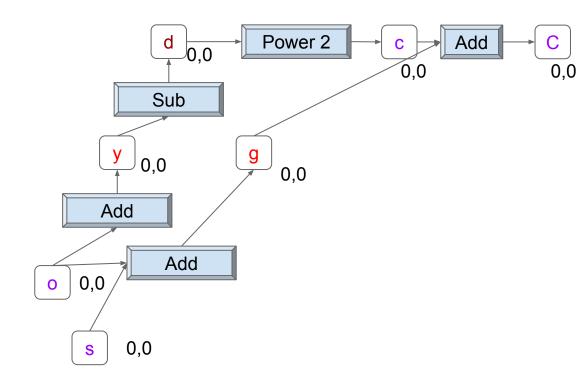
1-Initialize inputs

2-Initialize variables

3-Topological Sort variables



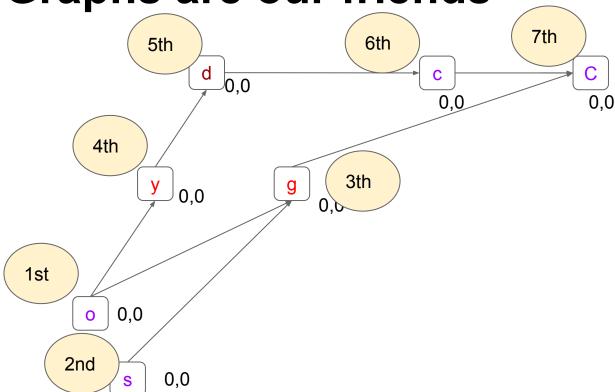
- 1-Initialize inputs
- 2-Initialize variables
- 3-Topological Sort variables



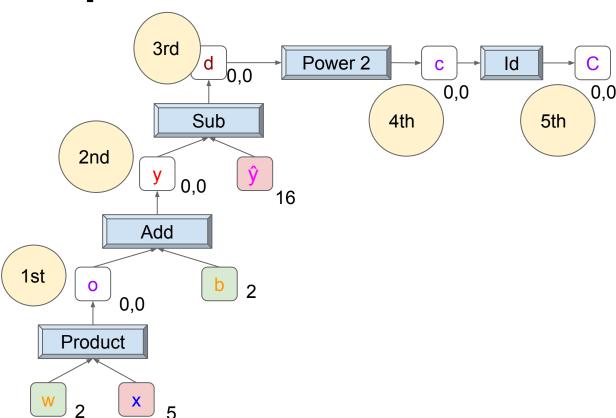
1-Initialize inputs

2-Initialize variables

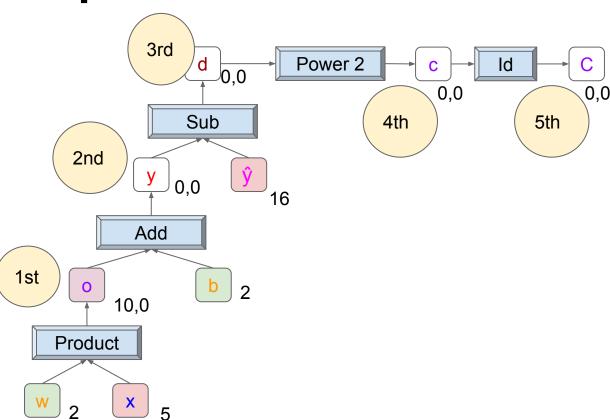
3-Topological Sort variables



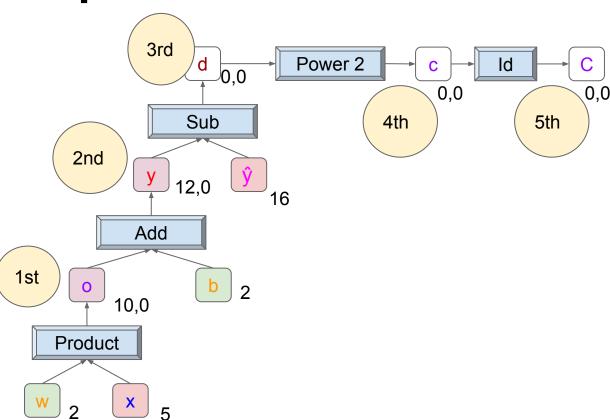
- 1-Initialize inputs
- 2-Initialize variables
- 3-Topological Sort variables
- 4-For each variable in topological order, run the forward method of all operations that link to them



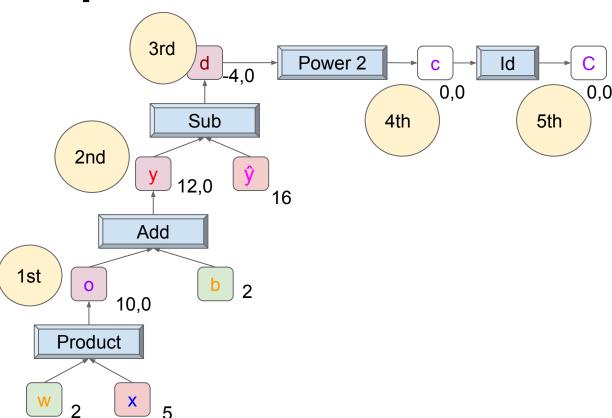
- 1-Initialize inputs
- 2-Initialize variables
- 3-Topological Sort variables
- 4-For each variable in topological order, run the forward method of all operations that link to them



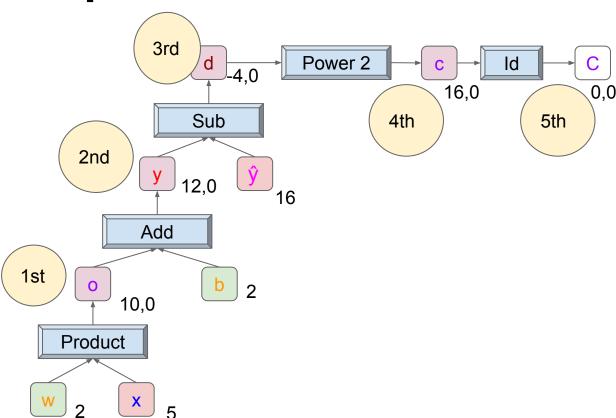
- 1-Initialize inputs
- 2-Initialize variables
- 3-Topological Sort variables
- 4-For each variable in topological order, run the forward method of all operations that link to them



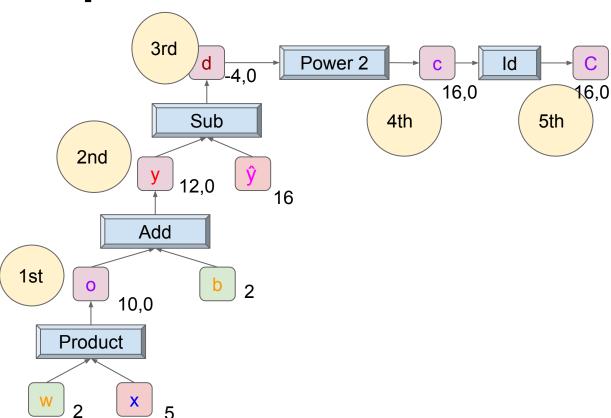
- 1-Initialize inputs
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- 3-Topological Sort variables
- 4-For each variable in topological order, run the forward method of all operations that link to them



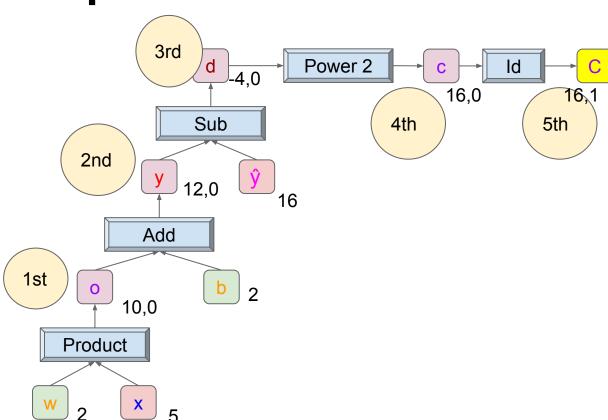
- 1-Initialize inputs
- 2-Initialize variables
- 3-Topological Sort variables
- 4-For each variable in topological order, run the forward method of all operations that link to them

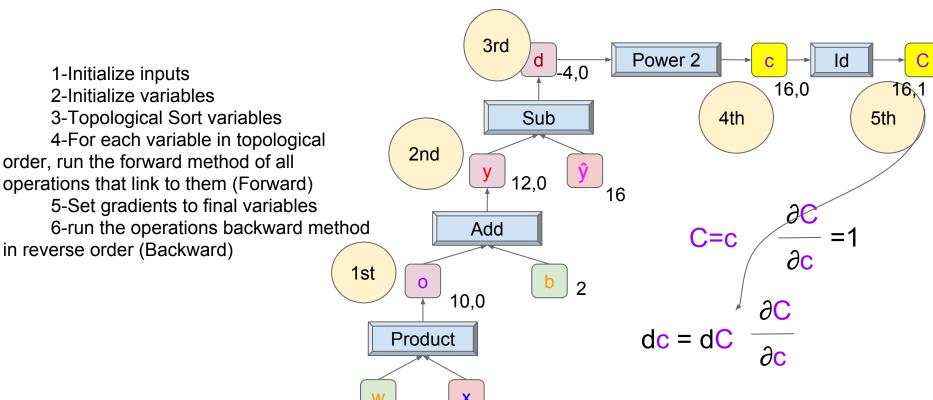


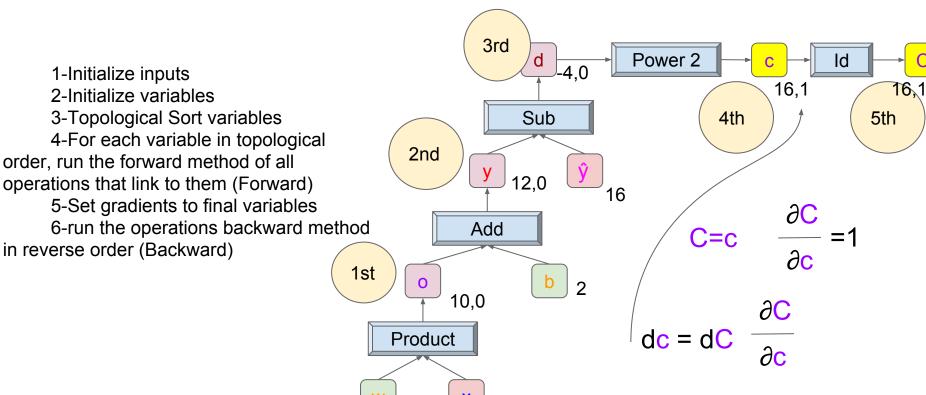
- 1-Initialize inputs
- 2-Initialize variables
- 3-Topological Sort variables
- 4-For each variable in topological order, run the forward method of all operations that link to them



- 1-Initialize inputs
- 2-Initialize variables
- 3-Topological Sort variables
- 4-For each variable in topological order, run the forward method of all operations that link to them
 - 5-Set gradients to final variables



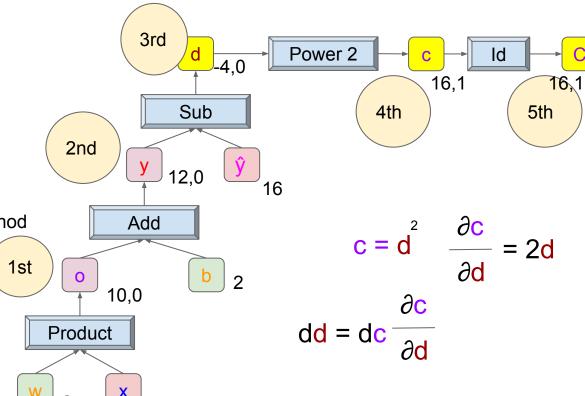


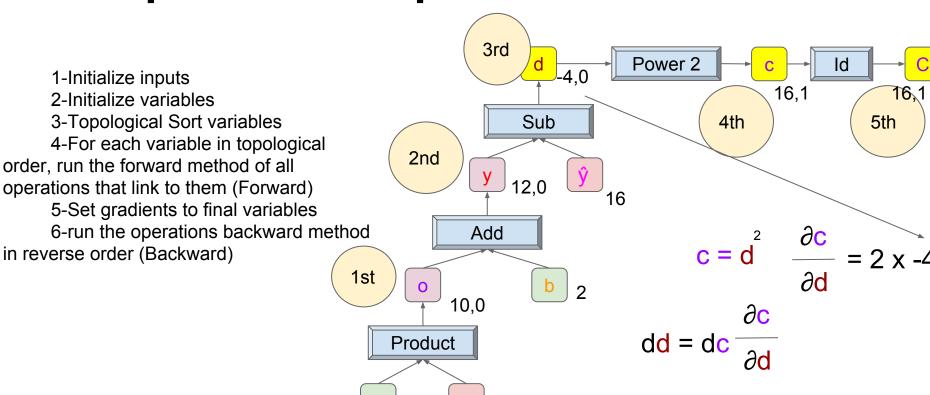


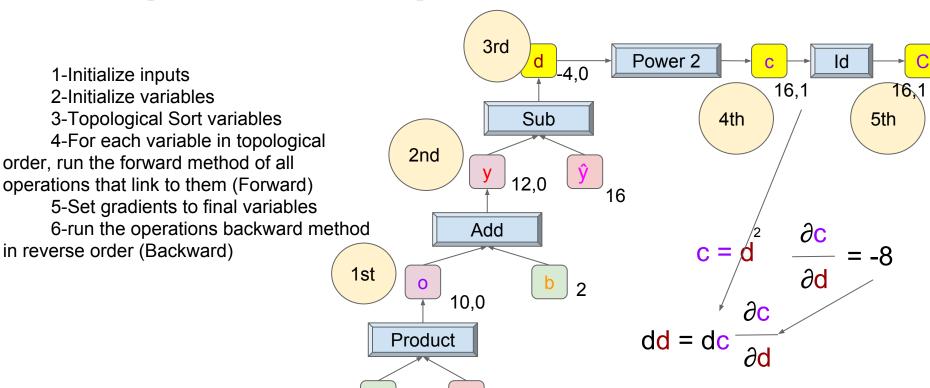


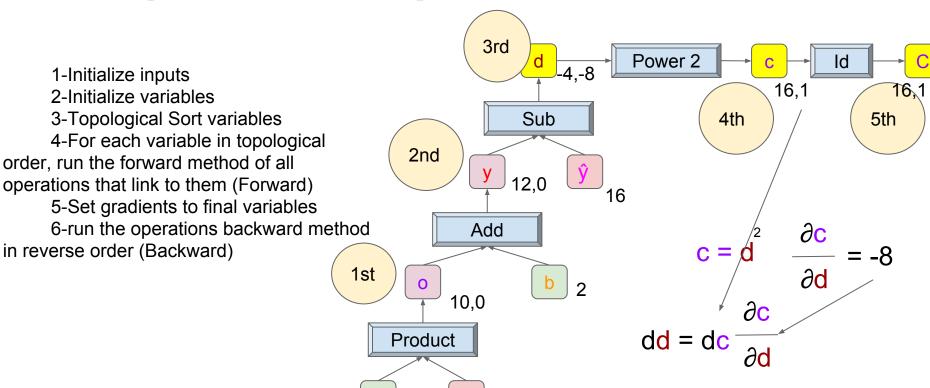
- 2-Initialize variables
- 3-Topological Sort variables
- 4-For each variable in topological order, run the forward method of all operations that link to them (Forward)
 - 5-Set gradients to final variables
- 6-run the operations backward method

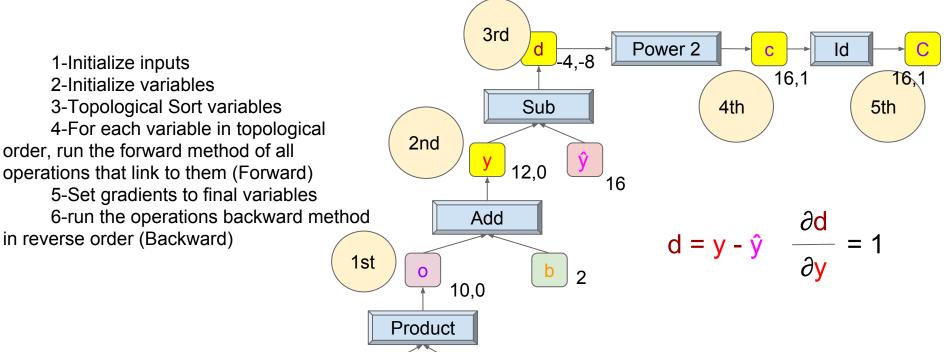
in reverse order (Backward)







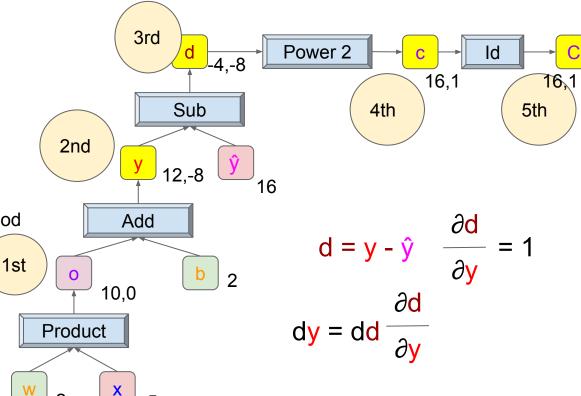


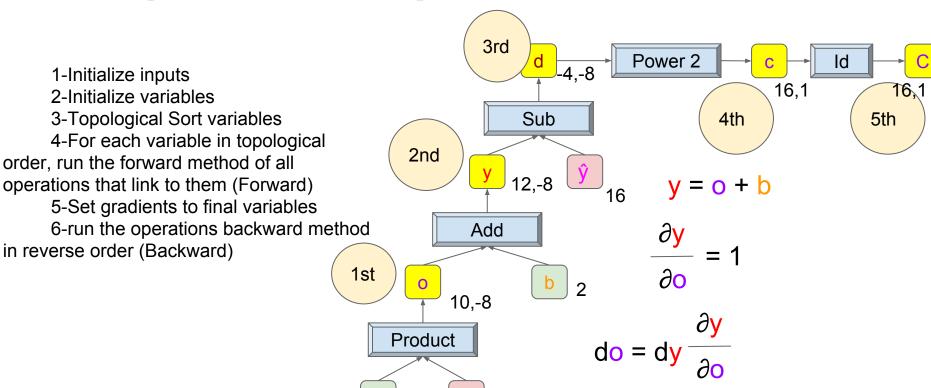


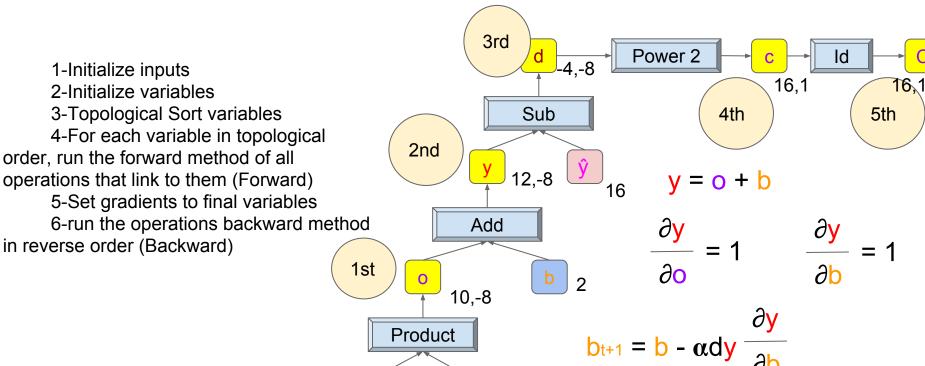


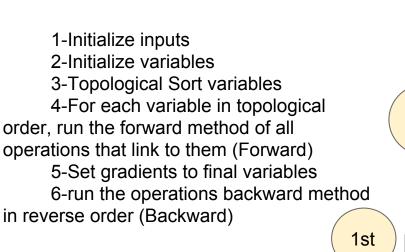
- 2-Initialize variables
- 3-Topological Sort variables
- 4-For each variable in topological order, run the forward method of all operations that link to them (Forward)
 - 5-Set gradients to final variables
- 6-run the operations backward method

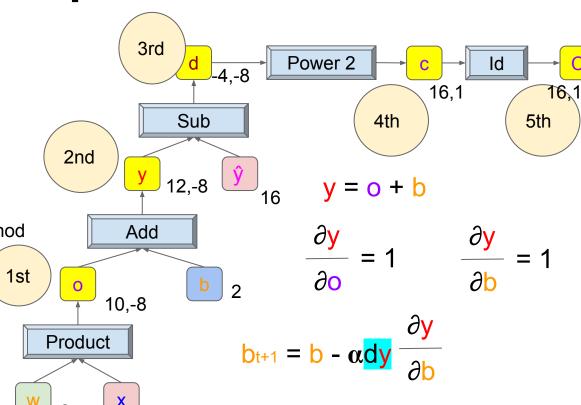
in reverse order (Backward)

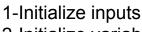










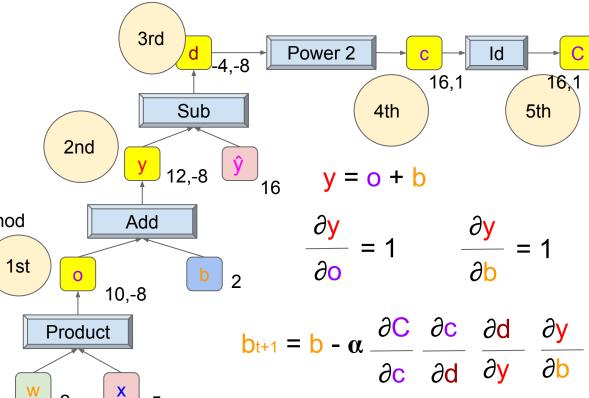


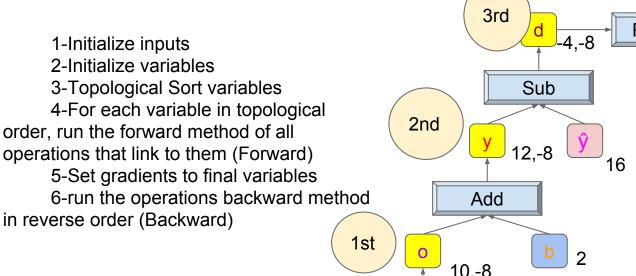
- 2-Initialize variables
- 3-Topological Sort variables
- 4-For each variable in topological order, run the forward method of all operations that link to them (Forward)

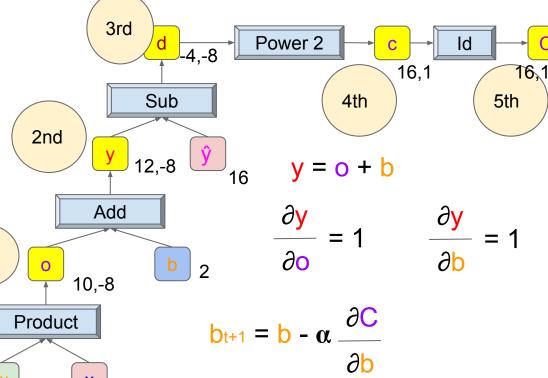
5-Set gradients to final variables

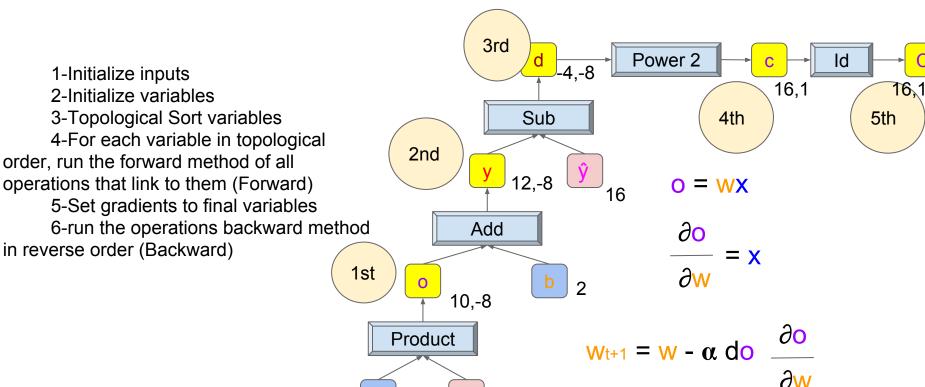
6-run the operations backward method

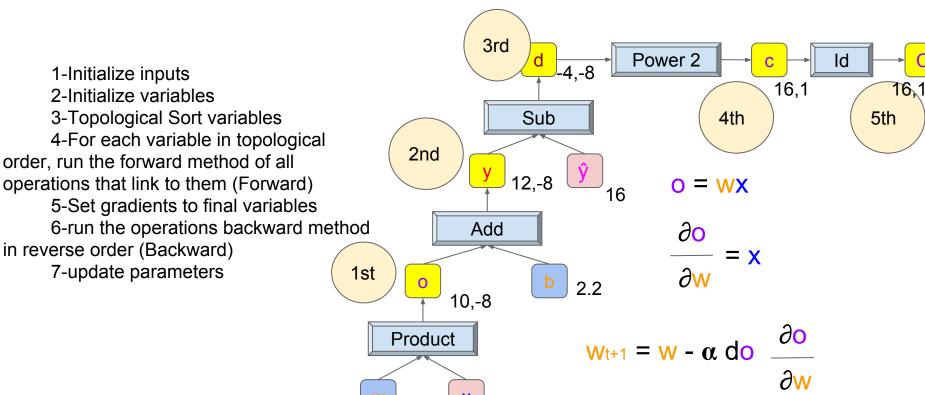
in reverse order (Backward)

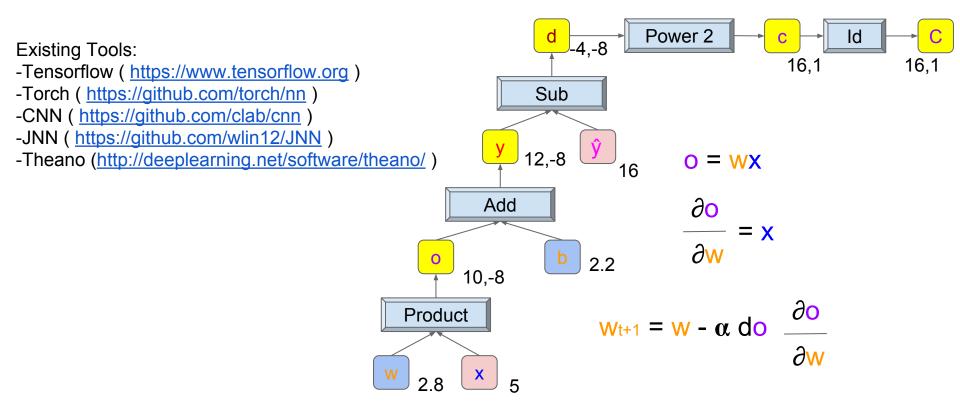


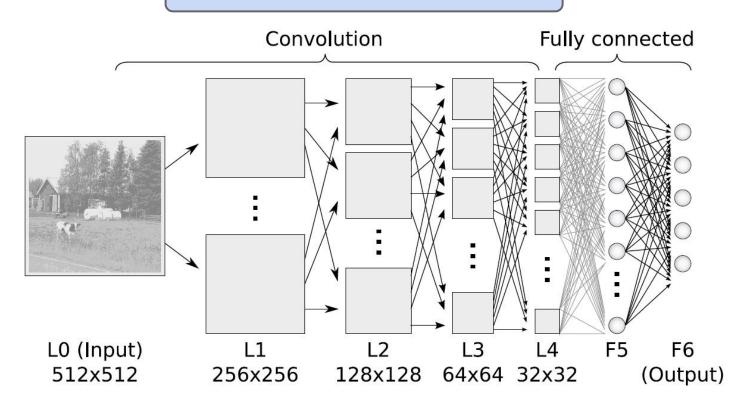






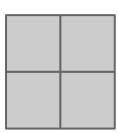




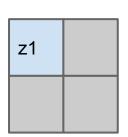


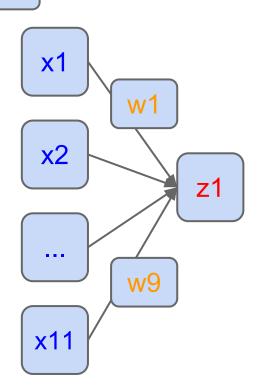
x1	x2	x3	x4
x5	x6	x7	x8
x9	x10	x11	x12
x13	x14	x15	x16

x1	x2	х3	x4
x5	x6	x7	x8
x9	x10	x11	x12
x13	x14	x15	x16



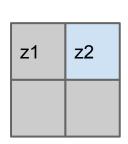
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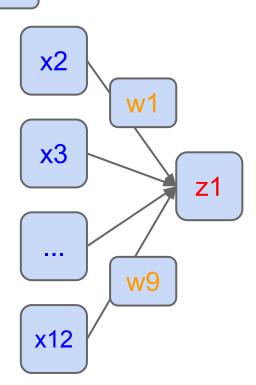




4x4 image

x1	x2	x3	x4
x5	x6	x7	x8
x9	x10	x11	x12
x13	x14	x15	x16





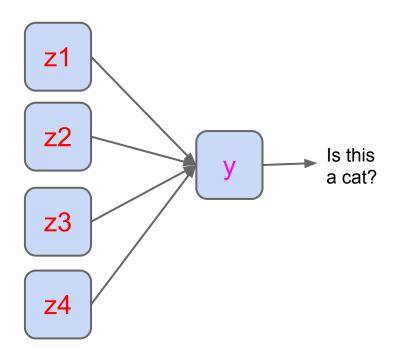
4x4 image

x1	x2	x3	x4
x5	x6	x7	x8
x9	x10	x11	x12
x13	x14	x15	x16

z1	z2
z3	z4

x1	x2	x3	x4
x5	x6	x7	x8
x9	x10	x11	x12
x13	x14	x15	x16

z1	z2
z3	z4



4x4 image