Day 8: Unsupervised learning and dimensional reduction

Kenneth Benoit and Slava Mikhaylov

Introduction to Data Science and Big Data Analytics

26 August 2015

Day 8 Outline

Dimensional reduction methods

Parametric v. non-parametric methods

- Parametric methods model feature occurrence according to some stochastic distribution, typically in the form of a measurement model
  - for instance, model words as a multi-level Bernoulli distribution, or a Poisson distribution
  - feature effects and "positional" effects are unobserved parameters to be estimated

- Non-parametric methods typically based on the Singular Value Decomposition of a matrix
  - principal components analysis
  - correspondence analysis
  - other (multi)dimensional scaling methods

Non-parametric dimensional reduction methods

- Non-parametric methods are algorithmic, involving no "parameters" in the procedure that are estimated
- Hence there is no uncertainty accounting given distributional theory
- Advantage: don't have to make assumptions
- Disadvantages:
  - cannot leverage probability conclusions given distribtional assumptions and statistical theory
  - results highly fit to the data
  - not really assumption-free (if we are honest)

Principal Components Analysis

- For a set of features $X_1, X_2, \ldots, X_p$, typically centred (to have mean 0)
- the first principal component is the normalized linear combination of the features

$$Z_1 = \phi_{11}X_1 + \phi_{21}X_2 + \ldots + \phi_{p1}X_p$$

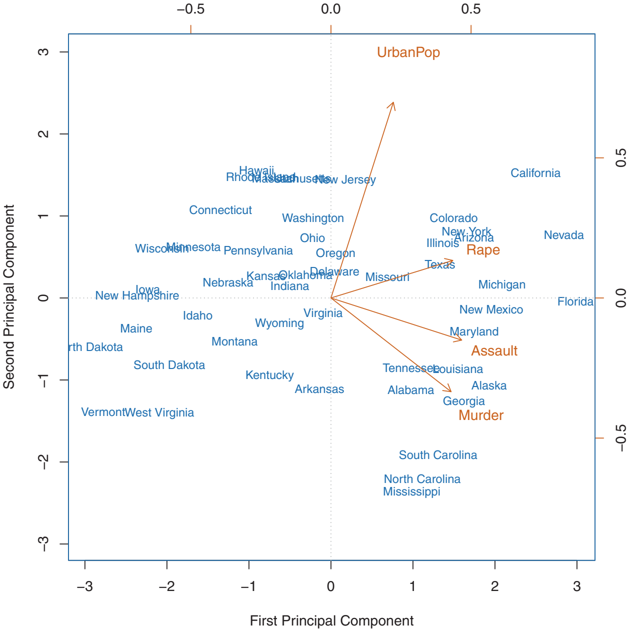
  that has the largest variance
- normalized means that $\sum_{j=1}^{p} \phi_{j1}^2 = 1$
- the elements $\phi_{11}, \ldots, \phi_{p1}$ are the loadings of the first principal component
- the second principal component is the linear combination Z_2 of $X_1, X_2, \ldots, X_p$ that has maximal variance out of all linear combinations that are *uncorrelated* with Z_1

PCA factor loadings example

	PC1	PC2
Murder	0.5358995	−0.4181809
Assault	0.5831836	−0.1879856
UrbanPop	0.2781909	0.8728062
Rape	0.5434321	0.1673186

TABLE 10.1. *The principal component loading vectors, ϕ_1 and ϕ_2, for the* `USArrests` *data. These are also displayed in Figure 10.1.*

PCA factor loadings biplot



PCA projection illustrated

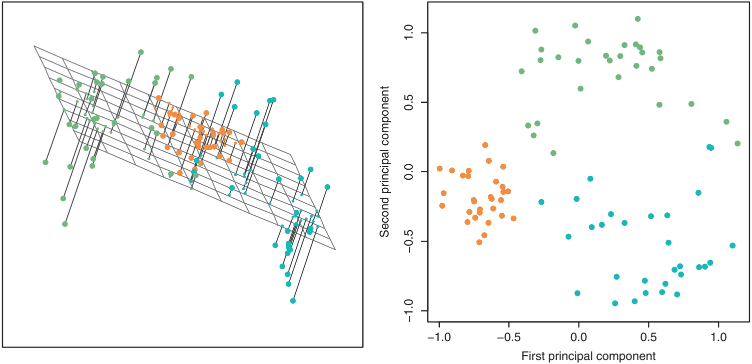


FIGURE 10.2. *Ninety observations simulated in three dimensions.* Left: *the first two principal component directions span the plane that best fits the data. It minimizes the sum of squared distances from each point to the plane.* Right: *the first two principal component score vectors give the coordinates of the projection of the 90 observations onto the plane. The variance in the plane is maximized.*

Correspondence Analysis

- CA is like factor analysis for categorical data
- Following normalization of the marginals, it uses Singular Value Decomposition to reduce the dimensionality of the word-by-text matrix
- This allows projection of the positioning of the words as well as the texts into multi-dimensional space
- The number of dimensions – as in factor analysis – can be decided based on the eigenvalues from the SVD