Total Time: 50min

Question 1:

This is a classical Bayesian condition problem

$$P(\,outside\ AFC\ East|\ win\,) = \frac{P(win\ outside\ AFC\ East)}{P(win)}$$

$$= \frac{\frac{4}{6+4+6}\times 50\% + \frac{6}{6+4+6}\times 60\%}{\frac{6}{6+4+6}\times 70\% + \frac{4}{6+4+6}\times 50\% + \frac{6}{6+4+6}\times 60\%} = 0.571$$

Question 2:

Assume the amount placed on the 4 horses are respectively a, b, c, d. (dollars)
They satisfy

$$\begin{cases} 2a - b - c - d > 0 \\ 3b - a - c - d > 0 \\ 4c - a - b - d > 0 \\ 5d - a - b - c > 0 \end{cases}$$

The answer can be infinite sets. For example, $a = 5, b = 4, c = 3, d = 2.5$ can always make money regardless who wins.

Question 3:

This problem can be divided into two situations, the score gap is larger than 2 and equal to 2.

Situation 1, there is only one score 4:1 or 1:4,

$$P(Situation\ 1\ win) = (60\%)^3 = 0.216$$

$$P(Situation\ 1\ lose) = (40\%)^3 = 0.064$$

Situation 2, one player must win the final two points to end game.

$$P(Situation\ 2\ win) = P(Former\ rounds)(60\%)^2$$

$$P(Situation\ 2\ lose) = P(Former\ rounds)(40\%)^2$$

So in situation 2, the probability of win and lose is in ratio $\frac{(60\%)^2}{(40\%)^2} = \frac{9}{4}$.

$$P(Situation\ 2\ win) = P(game\ ends\ in\ Situation\ 2) \times \frac{9}{9+4}$$
$$= [1 - P(game\ ends\ in\ Situation\ 1)] \times \frac{9}{9+4}$$
$$= \frac{9}{13}[1 - P(Situation\ 1\ win) - P(Situation\ 1\ lose)]$$
$$= \frac{9}{13} \times (1 - 0.216 - 0.064) = \frac{162}{325}$$

Thus,

$$P(win) = P(Situation\ 1\ win) + P(Situation\ 2\ win) = 0.216 + \frac{162}{325} = 0.714$$

Question 4:

All these teams can be seemed as Team A and Team B. Team A plays games 1, 2, 5, at home, Team B plays games 3, 4 at home.

$$P(Team\ A\ 3{:}0) = \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$$
$$P(Team\ A\ 3{:}1) = \left(\frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}\right) \times \frac{1}{3} = \frac{4}{27}$$
$$P(Team\ A\ 3{:}2) = \left(\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right) \times \frac{2}{3} = \frac{22}{81}$$

$$P(\ Team\ A\ win) = P(Team\ A\ 3{:}0) + P(Team\ A\ 3{:}1) + P(Team\ A\ 3{:}2) = \frac{4}{27} + \frac{4}{27} + \frac{22}{81} = \frac{46}{81}$$
$$P(\ Team\ B\ win) = 1 - P(\ Team\ A\ win) = 1 - \frac{46}{81} = \frac{35}{81}$$

Thus the question is to calculate $P(\ Team\ A\ win)\ P(\ Team\ B\ win)\ P(Team\ A\ 3{:}2) = \frac{46}{81} \times \frac{35}{81} \times \frac{22}{81} = 0.0666$