GARF: Towards Self-optimised Random Forests

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Abstract. Ensemble learning is a machine learning approach that utilises a number of classifiers to contribute via voting to identifying the class label for any unlabelled instances. $Random\ Forests$ RF is an ensemble classification approach that has proved its high accuracy and superiority. However, most of the commonly used selection methods are static. Motivated by the idea of having self-optimised RF capable of dynamical changing the trees in the forest. This study uses a genetic algorithm GA approach to further enhance the accuracy of RF. The approach is termed as Genetic Algorithm based $RF\ (GARF)$. Our extensive experimental study has proved that RF performance is be boosted using the GA approach.

Keywords: Random Forest, Genetic Algorithms, Ensemble Classification

1 Introduction

Ensemble classification is an established machine learning approach to boost the performance of classification techniques. It is based on the process of building a number of classifiers, and then collectively using them to identify unlabelled instances. Two widely used ensemble approaches could be identified, namely, boosting and bagging. Boosting is an incremental process of building a sequence of classifiers, where each classifier works on the incorrectly classified instances of the previous one in the sequence. AdaBoost [8] is the representative of this class of techniques. However, AdaBoost is pruned to overfitting. The other class of ensemble approaches is the Bootstrap Aggregating (Bagging) [5]. Bagging involves building each classifier in the ensemble using a randomly drawn sample of the data, having each classifier giving an equal vote when labelling unlabelled instances. Bagging is known to be more robust than boosting against model overfitting. The main representative of bagging is Random Forests (RF) [6]. In RF, a number of trees are generated, having each tree built using randomly drawn instances from the data set. Randomisation is also applied when selecting the best node to split on for all the trees. Typically this is an input parameter which is equal to \sqrt{F} , where F is the number of features in the data set. More details about RF are presented in Section 2.

Genetic algorithm [10] is an optimisation approach that belongs to the family of stochastic optimisation. It has long been used successfully in many applications .The process of applying genetic algorithm goes through four main steps,

initialisation, selection, reproduction and termination. In the initialisation step, an initial population of individual solutions is generated. Using a fitness function, individuals of good performance are used to produce a new generation. This process is the selection step. The reproduction step uses mainly two techniques, crossover and mutation, to produce a new generation. The reproduction process continues until a termination condition is reached.

Motivated by the observation that a number of RFs could be drawn from a larger RF forming an initial population of individuals, genetic algorithms could be an ideal optimisation solution to build a more accurate ensemble. It is worth noting that this observation also applies to other ensemble approaches. Thus, our hypothesis in conducting this research could be stated as follows: genetic algorithm is able to further enhance the performance of ensemble classification.

In this paper, we have proposed, developed and empirically evaluated a novel approach to optimising RFs boosting their performance. Our approach is termed Genetic Algorithm based RFs (GARF). The GARF approach starts by generating a large RF of N decision trees, forming a vector \overrightarrow{RF} . Drawing randomly from \overrightarrow{RF} a number of vectors each denoted as $\overrightarrow{rf_i}$, where the number of trees in $\overrightarrow{rf_i}$ is denoted as $n_i \leq N$, i=1...S, and S is the number of RFs. In genetic algorithms terminology, S is the size of the population. This initial population is then evolved through a number of generations, with the fitness function for each individual being its classification accuracy.

The paper is organised as follows. Section 2 provides necessary background about RFs and genetic algorithm; the main constructs of our GARF technique. Our proposed approach GARF to boost the performance of RFs is detailed in Section 3. Extensive experimental study validating GARF is presented in Section 4. A discussion of related work is given in Section 2.3. Finally, the paper is concluded with a short summary and pointer to future developments in Section 5.

2 Background

As we build our GARF technique based on RFs and genetic algorithm, the following subsections provide necessary background on the two methods.

2.1 Random Forests

Two broad categories of techniques could be identified in machine learning, supervised and unsupervised. Supervised techniques are also widely known as classification attempt to identify the value of an attribute, known as the *class attribute*, based on the values of the other attributes in the same instance or record of data. This identification is based on learning from historical data. The attributes other than the class are known as *predictors*. Thus, if the value of the class label is y, and the values of the predictors form the vector x, then y = f(x). Any classification technique attempts to find $\hat{f}(x)$ that approximates the function f(x).

The notion of using a set of classifiers to identify unlabelled instances is known as ensemble learning. Boosting and bagging are the two known successful approaches to ensemble learning. RFs belongs to the bagging approach. Bagging(Bootstrap Aggregating) has been proposed by Breiman in [5]. It is based on generating a number of replicas from the training data by uniformly sampling the instances with replacement. This sampling approach is known as Bootstrap. It allows duplicate instances to appear in the same replica, and also allows some instances to be left out. Statistically for a large replica that has the number of instances equal to the size of the data set, 63.2% of the instances do appear at least once in the replica. Having a number of replicas, each denoted as r out of the training data, a classifier c(r) is built using the sampled instances in r. The classification is done via voting among a vector of classifiers $\overline{c(r)}$ that have been built using the corresponding vector of replicas \overrightarrow{r} . A common performance evaluation approach in bagging is to use out of bag method. This is based on evaluating each instance using those classifiers in the ensemble that did not use that instance for training. This means that not all the classifiers are used together in testing.

Bagging has been applied successfully to an ensemble technique, termed RFs. In addition to the Bootstrap sampling, randomisation over the feature space is also used. The technique is based on building a number of decision tree classifiers, having each tree built from one replica out of the training data. However, when splitting the nodes of the decision tree, only a subset of all the features is used. Assuming that the number of features in the data set is F, the standard setting for the random features to be used at each split is $M=\sqrt{F}$. Breiman has used $Gini\ index$ as the goodness measure to split the attributes on. $Gini\ index$ has been introduced by Breiman et al [7] in building the Classification And Regression Trees CART technique. However, it has been first introduced by the Italian statistician $Corrado\ Gini\ in\ 1912$. The index is a function that could be used to measure the impurity of the data, i.e., how uncertain we are if an event will occur. In classification, this event would be the determination of the class label. The Gini impurity function in its original form is calculated as follows.

$$Gini(t) = 1 - \sum_{i=1}^{w} P(C_i|t)^2$$
 (1)

where t is a condition, w the number of classes in the data set, and C_i is the i^{th} class label in the data set.

By removing the condition t from the original form of the previous equation, we can calculate the level of impurity for any data set before splitting as follows.

$$Gini(Class) = 1 - \sum_{i=1}^{w} P(C_i)^2$$
(2)

The Gini index of any attribute A can then be calculated as follows.

$$GiniIndex(A) = Gini(Class) - \sum_{j=1}^{m} P(a_j).Gini(A = a_j)$$
 (3)

where m is the number of values for the attribute A.

The attribute with a higher $Gini\ index$ is the one chosen to be split on. It is worth noting that CART uses binary splits. Thus, for those attributes with m > 2, a preprocessing step is required to find the best was to combine the different values of the attribute to result in a binary split.

2.2 Genetic Algorithm

GA is a well established evolutionary approach. Basic details about GA can be found in [9] In an ordinary GA, the chromosome represents an encoded solution. For some problems, the direct encoding of a solution in a GA's chromosome results in complex and large chromosomes that may need complex repairs after the application of the GA's operators. In contrast, [12] introduced what could be called as indirect encoding or $Indirect\ GAs\ (IGAs)$, where each gene in the chromosome represents a heuristic – this could be seen as rule of thumb, an educated guess or small rules – instead of representing part of the solution. In an $indirect\ GA$, the chromosome which is known as $Heuristic\ Chromosome\ (HC)$ may be much more compact and robust, since it represents the heuristics that will be used in order to get a solution.

The most common form of HC, is one where the HC consists of a number of genes and each of these genes represents the ID of a heuristic. In order to build a solution, the heuristics in an HC are called one after the other or in parallel based on the problem and what exactly the chromosome represents. One of the main differences between different HC approaches lies in structure of the HC and what exactly each gene represents.

The approach adopted in this paper is similar to the IGA approach, a single random tree could be considered as a heuristic. Each gene in the chromosome represents a pointer to a random tree classifier, and the chromosome as a whole represents an ensemble classifier (forest). In order to get a solution (classification) of a given instance, the genes in the chromosome are used to evaluate the instance as detailed in section 3.

2.3 Related Work

Genetic algorithm has been applied in machine learning and data mining extensively. The main application is the use of genetic algorithm in the feature selection problem. An early survey on this topic can be found in [11]. However, the relevant work to the research reported in this paper is detailed in the following.

Robnik-Sikonja [13] has proposed possible extensions to RFs that have proved to boost the accuracy of the original techniques presented in Section 2. The motivation behind these extensions is to decrease the correlation among the trees in the RF. As the original technique proposed by [6] uses $Gini\ index$ for finding the best split among the randomised vector of attributes $\overrightarrow{F_M}$. In an attempt to decrease the dependency among attributes, Robnik-Sikonja has used ReliefF

[14] as a measure of the quality of the attributes. This extension has not proved to have a good performance on real data sets. A combination of measures for the quality of attributes has been used to decide the split, having each fifth tree in the forest uses a different measure. This method has proved to boost the performance of the RF, but not significantly. The other approach proposed by Robnik-Sikonja was the use of weighted voting among the trees using similarity of the instances with regards to their performance on the individual trees. This method always has proven to boost the performance of the input forest, and very competitive with other state-of-the-art methods.

Sylvester and Chawla [15] have proposed the EVEN (EVolutionary ENsembles). They have also attempted to use weighted voting among a set of homogeneous or heterogeneous classifiers. The EVEN system uses each of the classifier's performance over a validation set of data to weight the tree. Experimental validation has proved that EVEN can outperform the unweighted ensemble. In a more recent work, Abdulsalam and Skillicorn [2] have used Hoeffding trees to build a window of RFs to tackle the concept drift problem when mining streaming data.

3 GARF

GARF uses variable size chromosomes. Each chromosome (individual) in the population represents a forest. Each of the genes in the chromosome represents a random tree. Traditional genetic operators are employed by the proposed GARF for the crossover; a standard single point crossover operator is adopted. Two modes of operation for the crossover operator have been developed and tested. In the first mode all the repeated genes that could occur because of the crossover in the new individuals are removed. This to make sure that the each evolved forest has no repeated trees. The second mode does not make this extra check and allows the repetition of the trees in the offspring. For the mutation, a standard uniform mutation operator is employed, where the operator replaces a randomly chosen tree/gene with another randomly selected tree from the input trees forest that does not already exist in the forest/individual.

Each dataset is divided into three sets, training, validation (for GA training) and testing. The training set is used for building the random tress (input RF). The accuracy of the trees during the training is very high, reaching in most cases above 99% accuracy. By the accuracy here we mean the ability of correctly classifying a given instance. This is because these instances have been seen before during the building stage and in random trees does not use burning. As a result, it is not possible to use these instances (training set) for training the GARF as well, and another indebtedness set of instances (Optimization set) is needed for training the GA. In this paper we may refer to the validation set as GA-training set.

3.1 Fitness

Before GRAF starts the evolution process, each tree in the input forest is used to classify each of the instances in the GA-training set, all the classification results of every tree for each instance is stored in a buffer. This is done to speed up the evolution process and especially the fitness evaluation. So in the fitness evaluation of each individual, instead of evaluating the performance of all the trees in the individual against all the instances in the GA-training set, the classification results are collected directly from the buffer.

A given instance is considered as correctly classified, if the number of trees in the individual that has correctly classified it is greater than the number of trees that have given incorrect classification. In contrast, a given instance is considered as incorrectly classified, if the number of trees in the individual that has correctly classified it is less than or equal to the number of trees that have given incorrect classification. We call it a tie, if the number of trees that has correctly classified the instance is equal to the number of the instances that have been incorrectly classified.

The fitness of the individual is based on the number instances he has correctly classified.

$$f(v) = \sum_{i}^{K} c(v, i) + \frac{s(v, i)}{K}$$
 (4)

where K is the number of instances in the validation set. c(v, i) return 1 if individual v has correctly classified instance number i 0 otherwise. s(v, i) return 1 if it is a tie 0 otherwise.

If it is a tie we consider it as an incorrect classification. However, this could mean that the performance of the individual could be improved by a small change in the trees combination, and may benefit more from the genetic operators. Therefore, we slightly increase the fitness of the individual by 1/K for each tie.

3.2 GARF Algorithm

In this section, we provide details of our GARF method in an algorithmic format. The algorithm is depicted in Algorithm 1, where NG is the number of generations in GA, S denoting the size of the population (number of individual random forests) and n is the size of individual random forests in the initial population.

Having presented our proposed GARF technique in detail, the following section has validated the technique via extensive experimental study.

4 Experimental Study

We have conducted a series of experiments to evaluate the performance of GARF against the state of the art classification techniques. For our experiments, we used Waikato Environment for Knowledge Analysis (WEKA) [16]. We compared the performance of GARF against state of the art classification techniques; C4.5 decision tree, Support Vector Machines (SVM) and AdaBoost. We have also used WEKA to build the RF, we denote this as RFweka. The initial RF on which we used to build our initial population of RFs has been built using single calls of the random tree technique in WEKA. We denoted this in our experiments

Algorithm 1 GARF Algorithm

```
{User Settings}
input N, M, S, NG
{Process}
\overrightarrow{RF} = \text{Call RandomForest}(N, M)
for i = 1 \rightarrow S do
   for k = 1 \rightarrow n do
       x = Random(1 \rightarrow N)
       Add tree RF_x to forest i in the GA population \overrightarrow{P}_i
   end for
end for
Evaluate each forest in the initial population \overrightarrow{P}
for j = 1 \rightarrow NG do
    {Generate a new population by applying GA: operators mutation and crossover}
    \overrightarrow{PNew} = GAOperators(\overrightarrow{P})
   Evaluate each forest in \overrightarrow{P}
   \overrightarrow{bestForest} \leftarrow \text{copy of best } \overrightarrow{P}
   \overrightarrow{P} = \overrightarrow{PNew}
end for
{Output}
A vector of trees \overline{bestForest}
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as RFin. RFin was not created as one forest. Instead WEKA was used to create RFin as a set of independent random trees to enable us to evaluate each tree separately.

We have used 15 real standard data sets from UCI repository [1]. Description of the used data sets is given in Table 2. We have used a variety of data sets with diversity in the number of instances, number of classes and number of attributes.

As aforementioned, we have divided the data sets into three equal parts; one third for training, one third for optimisation (validation), and one third for testing. In GARF, we have used the validation part to evolve our RFs. To conduct fair experiments, we have combined the training and validation parts of the data sets to be used for training the other techniques. The same testing set has been used to calculate the performance of all the used classifiers.

The results of the experiments are shown in Table 2. The table shows our GARF technique has always been superior than the initial RF (RFin). The target in this paper is improving the performance of any given RF by changing the trees in the forest, which has been achieved. However, we compared the performance of GARF with state-of-the-art tree and bagging classifiers. Out of the 15 data sets, GARF has performed the best over all the other classification techniques in 8 data sets. For the letter data set, WEKA has not been able to scale to run such a large data set for the classifiers we have used. But as shown in the table, GARF has outperformed the initial RF. The GARF results shown in Table 2 are the 10 runs, parameters settings and statistics of these results are shown in Table 3 under Experiment 1.

Table 1: Data Sets Used

Name	Ins	Class.	Train.	Valid.	Test.	Name	Ins	Class.	Train.	Valid.	Test.
diabetes	768	2	256	256	256	glass	214	7	71	72	71
ionosphere	351	2	118	117	116	iris	150	3	50	50	50
labor	57	2	20	19	18	soybean	683	19	228	228	227
vote	435	2	145	145	145	credit-g	1000	2	333	334	333
ecoli	336	8	110	113	113	letter	20000	17	6666	6667	6667
liver-disorders	345	7	113	116	116	sonar	208	61	69	70	69
vehicle	846	19	282	282	282	vowel	990	14	330	330	330
waveform- 500	5000	41	1665	1668	1667						

Table 2: Performance of GARF against state-of-the-art techniques

Data Set Name	GARF RFin	RFweka AdaBoost C4.5 SVM
diabetes	78.5156 76.5625	75.3906 80.0781 76.5625 79.2969
glass	71.8310 67.6056	76.0563 33.8028 59.1549 47.8873
ionosphere	95.6896 93.1034	92.2414 91.3793 93.1034 91.3793
iris	96.0000 92.0000	94.0000 96.0000 96.0000 90.0000
labor	94.4444 88.8889	77.7778 83.3333 77.7778 83.3333
soybean	85.4626 81.4978	87.2247 32.5991 83.7004 N/A
vote	96.5517 95.8621	98.6207 99.3103 97.2414 7.2414
credit-g	73.8739 72.3724	72.6727 68.4685 69.6697 71.4715
ecoli	71.6814 69.9115	69.0265 24.7788 68.1416 61.0619
letter	84.0108 83.3508	N/A N/A N/A N/A
liver-disorders	69.8276 65.5172	68.9655 59.4828 61.2069 57.7586
sonar	88.4058 85.5072	81.1594 76.8116 76.8116 84.058
vehicle	73.7589 70.9220	74.8227 38.6525 65.9574 66.3121
vowel	74.5455 73.0303	80.0000 15.7576 65.1515 51.5152
waveform- 500	85.1830 84.5231	85.0030 73.0054 74.1452 86.0828

To assess the robustness of GARF with varying the experimental settings of genetic algorithm, we have conducted a set of experiments with different settings. The corresponding results are presented in Table 3. It can be noted that GARF has proved to be robust with the various setting of parameters, achieving consistently good accuracy over all the data sets used in our experimental study.

The above results open the door for a range of possibilities to build on the success of GARF. One important success factor for any ensemble of classification is to increase the diversity among the classifiers. We note that throughout our experimental study we have used a fixed number of randomised features $M = \sqrt{F}$. However, having a variable M will lead to having a more diverse trees in the RF. Evolving this diversity of trees using genetic algorithm would have the potential to further improve GARF.

Moreover, given the empirically validated robustness of RFs against noise, it is suitable to address the problem of changing data, known as concept drift. GARF can address this issue, because of the natural evolution of genetic algorithm. However, an important issue needs to be addressed. One one hand,

Table 3: Performance of GARF with Varying Experimental Settings.

Experiment 1: PSize 100, NG 50, CR 0.9, MR 0.1 and variable InvSize of 100.

Experiment 2: PSize 100, NG 50, CR 0.9, MR 0.1 and fixed InvSize of 200.

Experiment 3: PSize 500, NG 50, CR 0.9, MR 0.1 and variable InvSize of 100.

Experiment 4: PSize 400, NG 50, CR 0.9, MR 0.1 and variable InvSize of 400.

where PSize = Population Size, NG = Number of Generations, CR = crossover rate,

MR = mutation rate, InvSize = GA string length (individual).

	Experiment 1				Experiment 2				
Data Set Name	Best	Size	Worst	Std.	Best	Size	Worst	Std.	
	Best	Size	Worst	Std.	Best	Size	Worst	Std.	
diabetes	78.5156	89	76.5625	3.124	77.7344	200	75.7813	3.1162	
glass	71.831	87	67.6056	2.8169	73.2394	200	69.0141	2.9001	
ionosphere	95.6896	87	93.1034	3.8204	94.8276	200	93.1034	3.7099	
iris	96	95	92	3.6878	96	200	92	3.9396	
labor	94.4444	95	88.8889	2.2103	94.4444	200	88.8889	2.7011	
soybean	85.4626	89	82.8194	3.4327	84.141	200	81.9383	3.3849	
vote	96.5517	91	95.1724	3.8448	96.5517	200	95.1724	3.8399	
credit-g	73.8739	97	71.7718	2.974	73.5736	200	71.4715	2.9558	
ecoli	71.6814	89	64.6018	3.0656	70.7965	200	68.1416	2.7534	
letter	84.0108	89	83.5458	3.3576	83.7708	200	83.3358	3.3423	
liver-disorders	69.8276	89	66.3793	2.8172	68.9655	200	65.5172	2.7801	
sonar	88.4058	95	84.058	3.4174	88.4058	200	84.058	3.5381	
vehicle	73.7589	89	70.922	2.9694	73.7589	200	71.6312	2.9846	
vowel	74.5455	89	72.7273	2.9536	74.2424	200	72.7273	2.9709	
waveform- 500	85.183	87	83.9232	3.3813	84.823	200	84.0432	3.3816	
		Expe	eriment 3		Experiment 4				
Data Set Name	Best	Size	Worst	Std.	Best	Size	Worst	Std.	
diabetes	78.5156	67	76.1719	3.123	78.125	154	76.1719	3.124	
glass	70.4225	71	64.7887	2.8727	70.4225	171	67.6056	2.6575	
ionosphere	96.5517	69	93.1034	3.8476	95.6897	131	93.1034	3.7853	
iris	96	69	92	3.7094	96	161	92	3.7524	
labor	94.4444	71	88.8889	2.3012	94.4444	161	88.8889	2.4125	
soybean	88.5463	67	85.022	3.5571	85.9031	169	83.7004	3.4316	
vote	96.5517	71	95.1724	3.8621	97.2414	167	95.1724	3.82	
credit-g	74.7748	72	71.1712	3.0263	73.8739	161	71.7718	3.0042	
ecoli	69.0265	73	63.7168	2.8263	70.7965	161	66.3717	2.9458	
letter	84.3558	65	83.6508	3.3602	83.9658	149	83.5908	3.3521	
liver-disorders	69.8276	71	66.3793	2.8799	69.8276	167	66.3793	2.7694	
sonar	88.4058	69	81.1594	3.6435	88.4058	131	84.058	3.3803	
vehicle	73.7589	67	70.922	2.9165	73.7589	169	71.6312	2.9559	
vowel	75.1515	69	72.4242	3.0454	74.5455	151	72.7273	2.9654	
waveform- 500	85.4229	71	83.8632	3.4368	85.243	161	83.9232	3.4025	

sudden and strong concept drift requires new trees to be added to the forest. On the other hand, gradual and weak concept drift can easily utilise the genetic algorithm to use existing trees in the RF. Extensions to GARF using more pow-

erful powerful methods such as Genetic Programming [4][3] to address this issue would also have a great potential in the data stream mining area, also with the use of more.

5 Conclusion

In this paper, we have empirically validated our novel approach to developing an optimised $Random\ Forest\ (RF)$ using genetic algorithms that we termed GARF. The approach is based on generating a large RF, which is decomposed into a number of smaller RF. The smaller forests are composed of trees drawn randomly with replacement from the initial large RF. Genetic algorithm is an optimisation technique which is then applied to evolve this initial population of individual RF with the fitness function being the classification of the forest.

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