# Stat330: Homework #1

Due on January 22, 2014at 3:10pm

*Mr. Lanker Section A*

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# Problem 1

Math and calculus review.

## Part A

Evaluate $\sum_{k=1}^{5} k^2$ and $\sum_{k=1}^{5} (k-1)^2$.

## Solution

$$\sum_{k=1}^{5} k^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$$

$$\sum_{k=1}^{5} (k-1)^2 = (1-1)^2 + (2-1)^2 + (3-1)^2 + (4-1)^2 + (5-1)^2 = 30$$

## Part B

Find the derivative of $f(x) = x^4 + 3x^2 - 2$

## Solution

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^4 + 3x^2 - 2) = 4x^3 + 6x$$

## Part C

Find the derivative of $f(x) = 1 - \mathrm{e}^{-\lambda x}$.

## Solution

$$\frac{\mathrm{d}}{\mathrm{d}x}(1 - \mathrm{e}^{-\lambda x}) = \lambda \cdot \mathrm{e}^{-\lambda x}$$

## Part D

Evaluate the integrals $\int_0^1 (1 - x^2)\mathrm{d}x$ and $\int_1^\infty \frac{1}{x^2}\mathrm{d}x$.

## Solution

$$\int_0^1 (1 - x^2)\mathrm{d}x = x - \frac{1}{3}x^3 \Big|_0^1 = (1 - \frac{1}{3}) - (0 - 0) = \frac{2}{3}$$

$$\begin{aligned}
\int_1^\infty \frac{1}{x^2}\mathrm{d}x &= \lim_{b\to\infty} \int_1^b \frac{1}{x^2}\mathrm{d}x \\
&= \lim_{b\to\infty} -\frac{1}{x}\Big|_1^b \\
&= \lim_{b\to\infty} -\frac{1}{b} + \frac{1}{1} \\
&= 1
\end{aligned}$$

# Problem 2

Examples of sample spaces.

## Part A

Driving to work and passing through 3 intersections.

## Solution

$$\Omega = \{ccc, ccs, csc, css, scc, scs, ssc, sss\}$$

## Part B

What is the probability that she doesn't stop?

$$\Pr(\text{no stops}) = \frac{|\{ccc\}|}{|\Omega|} = \frac{1}{8}$$

## Part C

Let $A$ be the event that the commuter stops at the first light. Let $B$ be the event that the commuter stops at the second light.

## Solution

1. $A = \{scc, scs, ssc, sss\}$
2. $B = \{csc, css, ssc, sss\}$
3. $\overline{B} = \{ccc, ccs, scc, scs\}$
4. $A \cup B = \{scc, scs, csc, css, ssc, sss\}$
5. $A \cap B = \{ssc, sss\}$
6. $A \cap \overline{B} = \{scc, scs\}$

# Problem 3

Let $G$ and $H$ be disjoint events in some sample space $\Omega$.

## Part A

Describe the event $G \cup H$.

## Solution

A new event that includes outcomes in $G$ **or** in $H$.

## Part B

What is $P(G \cup H)$ in terms of $P(G)$ and $P(H)$?

## Solution

Since $G$ and $H$ are disjoint, $P(G \cap H) = 0$:

$$\begin{aligned} P(G \cup H) &= P(G) + P(H) - P(G \cap H) \\ &= P(G) + P(H) - 0 \\ &= P(G) + P(H) \end{aligned}$$

## Part C

Describe the event $G \cap H$.

## Solution

A new event that includes the outcomes that happen in event $G$ **and** in $H$.

## Part D

What is the probability of event $G \cap H$?

## Solution

The probability of $G \cap H = 0$ because the two events are disjoint.

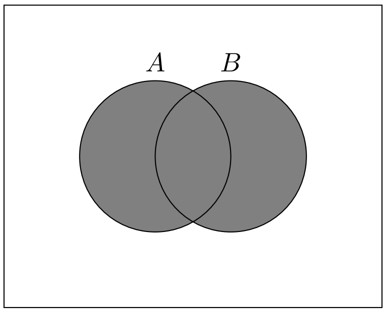
# Problem 4

Venn diagrams. The grey areas indicate the solution.

## Part A

$A \cup B$

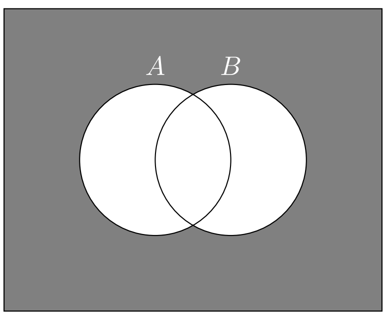
### Solution




## Part B

$\overline{A \cup B}$

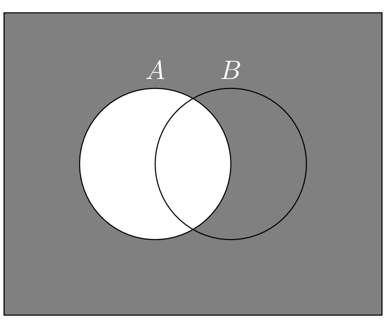
### Solution




## Part C

$\overline{A}$

### Solution

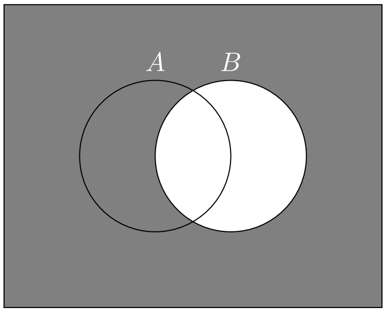



Problem 4 continued on next page...

## Part D

$\overline{B}$

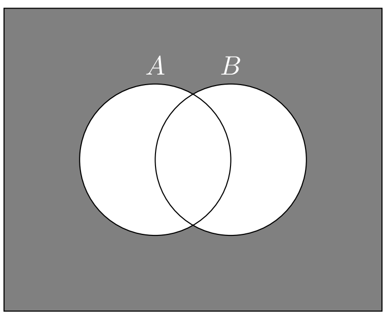
## Solution




## Part E

$\overline{A} \cap \overline{B}$

## Solution



# Problem 5

Employees at a firm, 70% know C, 60% know Fortran and 50% know both.

## Part A

Let $A$ be the event that an employee knows C and $B$ be the event that employee knows Fortran. Draw a Venn diagram.

## Solution

## Part B

What percentage of programmers do not know Fortran?

## Solution

$$\begin{aligned}
\Pr(\neg \mathrm{F}) &= 1 - \Pr(\mathrm{F}) \\
&= 1 - .6 \\
&= .4
\end{aligned}$$

## Part C

What percentage of programmers do not know Fortran and C?

## Solution

$$\begin{aligned}
\Pr(\neg \mathrm{F} \wedge \neg \mathrm{C}) &= 1 - \Pr(\mathrm{F} \vee \mathrm{C}) \\
&= 1 - (\Pr(\mathrm{F}) + \Pr(\mathrm{C}) - \Pr(\mathrm{F} \cap \mathrm{C})) \\
&= 1 - (.6 + .7 - .5) \\
&= .2
\end{aligned}$$

## Part D

What percentage of programmers know Fortran but not C?

## Solution

$$\begin{aligned}
\Pr(\mathrm{F} \wedge \neg \mathrm{C}) &= \Pr(F) - \Pr(\mathrm{both}) \\
&= .6 - .5 \\
&= .1
\end{aligned}$$

# Problem 6

Total 60 students attending University. 9 were living off campus, 36 were undergrads, 3 were undergrads living off campus.

Let $A$ be the event denoting undergraduates and $B$ denote living off campus.

$$\begin{aligned} A &= 36 \\ B &= 9 \\ \overline{A} &= 60 - 36 = 24 \\ \overline{B} &= 60 - 9 = 51 \\ A \cap B &= 3 \end{aligned}$$

## Part A

Number of students who were undergrads living on campus.

## Solution

$$\begin{aligned} A \cap \overline{B} &= A \setminus B \\ &= 36 - 9 + 3 \\ &= 30 \end{aligned}$$

## Part B

Number of students who were graduate students living on campus.

## Solution

$$\begin{aligned} \overline{A} \cap \overline{B} &= \overline{A} \setminus B \\ &= 24 - 9 + 3 \\ &= 18 \end{aligned}$$