It is especially important that you show all of your work. Please staple. Please print off your computer simulation code and submit with your answers. You may combine all the code on one sheet, but clearly denote the problems related to the different parts of code.

1. Let X represent the outcome of a roll of a 20-sided die numbered 1 to 20. The values of X between $a = 1$ to $b = 20$ follows a discrete uniform probability distribution, with a expected value of $\frac{a+b}{2}$ and a variance of $\frac{b^2-1}{12}$.

   Suppose you roll this die 33 times. Let S be the sum of these 33 rolled values, and let random variable Y represent the mean of these 33 values minus the overall mean of 10.5:

   $$Y = \frac{S}{33} - 10.5.$$

   Determine the probability that

   (a) Y is less than -0.5
   (b) Y is less than -2.5

   hint: use the central limit theorem.

   Let Z be a standard normal random variable. Determine the probability that

   (c) Z is less than -0.5
   (d) Z is less than -2.5

   Simulate 10000 values for Y (by drawing 33 random values from an appropriate discrete uniform distribution) and calculate the proportion of these 10000 values that are

   (e) less than -0.5
   (f) less than -2.5

   Use the following command in R to help with your simulation: `ceiling(runif(33, 0, 20))` which will get 33 random rolls of this 20-sided die. Feel free to use code given to you on previous assignments.

   Submit your code that generates your 10000 random values for Y.

2. In Happyland, the weather each day is characterized by these three states: let state 1=a sunny day, state 2=a cloudy day, and state 3=a rainy day.

   The weather from one day to the next follows these transition probabilities:

   - If today is sunny, the probability it is sunny tomorrow is .4, and the probability it is rainy tomorrow is .2.
   - If today is cloudy, the probability it is sunny tomorrow is .5, and there is equal remaining probability that it is cloudy or rainy tomorrow.
   - If today is rainy, the probability that is rainy again tomorrow is .25, with no chance of it being sunny tomorrow.

   Use the concept of steady-state probabilities to get the probability of a rainy day this time next year in Happyland. Your answer should be in the form of a fraction.

   You may check your work via simulation, but this is not required.

3. Suppose that the average number of telephone calls arriving at the switchboard of a small corporation is 30 calls per hour. Assume that the arriving calls follows a Poisson process.

   (a) What is the probability that no calls will arrive in a 3-minute period?

   (b) What is the probability that four or more calls will arrive in a 5-minute interval?

   (c) What is the probability that one minute will pass before the next incoming call?

   (d) If you need to go refill your coffee, which will take two minutes to do, what is the probability you will have missed at most two calls during that trip to the coffeemaker? (Hint: find the probability that the third incoming call occurs after two minutes.)

4. *A university barbershop.* The campus barbershop has only one barber but unlimited chairs for waiting customers. No matter how many customers are in the shop, arriving customers always choose to wait.

   - The customers arrive according to a Poisson process. The average time between arrivals is 30 minutes.
   - The lone barber completes serving a customer at a rate of x per hour, where x is the number of customers in the barbershop.

   Assume that inter-arrival times and service times are independent exponential random variables.

   (a) Draw a state diagram with possible states and corresponding birth/death rates. Since there are an infinite number of states, show enough states to show the pattern or arrival and service rates.

   (b) What is the (large t) probability that the shop is empty? (Note: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$)

   (c) Let X be number of customers in the store. Determine the “large t” probability function for the different states of X. What probability distribution is this, and what is the value of the parameter in that distribution?

   (d) What is the (“large t”) probability that a customer arrives when the barbershop has more than two persons waiting?

   (e) What is the expected number of customers in the barbershop at any time (for “large t” time)? hint: Use the properties of the known probability distribution.

5. Queuing theory simulation problem. (To be added by March 25.)