

Answer the following problems completely given the context. Show all of your work. Please staple.

- If we wish to expand $(x + y)^8$, what is the coefficient of x^5y^3 ? What is the coefficient of x^3y^5 ?
(Hint: search for "binomial theorem" on <http://mathworld.wolfram.com>.)
- Simpson's Paradox
 - A black urn contains 5 red and 6 green balls and a white urn contains 3 red and 4 green balls. You are allowed to choose a urn and then choose a ball at random from the urn. Assume that choosing any ball in the urn is equally likely. If you choose a red ball you get a prize.
Which urn should you choose to draw from?
 - A second black urn contains 6 red and 3 green balls and a second white urn contains 9 red and 5 green balls. Now which of these urns should you choose to draw from?
 - Now the contents of the two black urns are combined into a large black urn and the contents of the two white urns are combined into a large white urn. Which of these large urns should you choose to draw from?
- A group of four undergraduate and five graduate students are available to fill four student government posts.
 - Find the probability that at least three undergraduates will be among the four chosen *assuming the selection was random*?
 - What is the probability of no undergraduates being selected again assuming the selection was random?
 - Later we will talk about a p -value, which represents the probability of something happening under certain model assumptions. Here our model assumptions are that the selection process was purely random. It is common that if the p -value is very low, say less than .05, we reject that our model assumptions are true. Our p -value here is the result you determined in (3b). What does this value say about the model statement that the selections were made at random?
- Suppose there are two events A and B in Ω such that $P(A) > 0$ and $P(B) > 0$. Further suppose that the two events are mutually exclusive. Can they also be independent? Explain.
- A computer has a dual-core processor. At any time, the probability that each of the processors are active is

Processor 1		In Use	Not In Use	
	In Use	0.70	0.05	0.75
	Not In Use	0.10	0.15	0.25
		0.8	0.2	

Processor 2

Let A be the event that processor 1 is in use and B be the event that processor 2 is in use.

- Calculate $P(A|B)$
- What is $P(A)$?
- Using your answers to part (a) and part (b), are the events A and B independent?
- Calculate $P(B|A)$
- Show that $P(A|B)P(B) = P(B|A)P(A)$

6. Suppose that A and B are independent events with $P(A) > 0$ and $P(B) > 0$. Show that \bar{A} and B are also independent.

Hints:

- Kolmogorov's axioms and their consequences still apply to conditional probabilities
- Note that event B is equivalent to

$$(B \cap A) \cup (B \cap \bar{A})$$

and that those two parts are disjoint

- $P(A) + P(\bar{A}) = ?$
7. Suppose that an office fires an employee who tests positive for drug use. The employee declares that she was fired unjustly. From previous studies, the office knows that the probability of any individual being a drug user is
- $$P(\text{Drug User}) = 0.02$$
- $$P(\text{Not Drug User}) = 0.98$$
- Furthermore, the office used a drug test that was 99% accurate, that is
- $$P(\text{Test Positive} \mid \text{Drug User}) = 0.99$$
- $$P(\text{Test Positive} \mid \text{Not Drug User}) = 0.01$$
- Does the employee have a legitimate claim that she was unjustly fired? (*Hint: Use Bayes rule to turn around one of the conditional probabilities given.*)
8. In a bag there are two standard 6-sided dice with sides labeled 1 to 6. One of the dice is fair, and the other is loaded so that the die rolls a 6 exactly half of the time. You reach into the bag, randomly select a dice, and roll a 6. What is the probability that you selected the loaded die?
9. For each of the following situations to model, (i) define a discrete random variable and (ii) declare the range of values the random variable can take.
- (a) analyze the number of points allowed by the ISU football team during their games this season.
 - (b) analyze the attendance at ISU home hockey games this season.
 - (c) analyze the accident rate at the intersection of Lincoln Way & Welch Ave.