Stat330: Homework #10

Due on April 23, 2014 at 3:10pm

Mr. Lanker Section A

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Problem 1

Let $\bar{X}$ denote the mean of a random sample of n i.i.d. observations from a distribution that is $Normal(\mu, \sigma^2)$, where $\sigma^2 > 0$, and σ is known but μ is unknown.

What is the probability that the confidence interval:

$$(\bar{X} - 2.2\frac{\sigma}{\sqrt{n}}, \bar{X} + 2.2\frac{\sigma}{\sqrt{n}})$$

contains the fixed point μ?

Solution

According to the standard Z table, when $z_{\alpha/2} = 2.2$, the probability is 0.9861. This represents the $z_{\alpha/2}$ probability however. This means that the α that we found is twice the value of ours. This gives us the value of $\frac{\alpha}{2} = 1 - 0.9861 = 0.0139$. Thus $\alpha = 0.0278$. We know that $1 - \alpha$ gives our probability, thus $1 - 0.0278 = 97.22\%$ is the probability that we are looking for.

Problem 2

Problem 9.7a from Baron on pg. 301. Answer in the back of the book.

Let there be a server that serves concurrent users. Let the average number of concurrent users at 100 randomly selected times be 37.7 with a standard deviation of $\sigma = 9.2$.

Construct a 90% confidence interval for the expectation of the number of concurrent users.

Solution

We use the standard equation for confidence interval when the mean when σ is known. We know that $\bar{X} = 37.7$, $n = 100$, and $\alpha = 0.1$. This gives us a $z_{\alpha/2} = z_{0.05} = 1.645$ by using a table. This gives us:

$$\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 37.7 \pm 1.645\frac{9.2}{10} = 37.7 \pm 1.513 = [36.19, 39.21]$$

Problem 3

Find a 95% confidence interval for μ, the true mean of a normal population which has a variance of $\sigma^2 = 100$. Consider a sample of size 25 that has a mean of 69.3.

Solution

We know that $\bar{X} = 69.3$, $n = 25$, and $\alpha = 0.05$. This gives us a $z_{\alpha/2} = z_{0.025} = 1.96$ by using a table. This gives us:

$$\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 69.3 \pm 1.96\frac{10}{5} = 69.3 \pm 3.92 = [65.38, 73.22]$$

Problem 4

A department store has 10,000 customers charge accounts. To establish the amount owed by all its customers, it selected 36 accounts at random and found a mean of $150 and a standard deviation of $60.

Part A

Establish a 95% confidence interval estimate of the mean amount owed by its customers.

Solution

We know that $\bar{X} = 150$, $n = 36$, and $\alpha = 0.05$. This gives us a $z_{\alpha/2} = z_{0.025} = 1.96$ by using a table. This gives us:

$$\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 150 \pm 1.96\frac{60}{6} = 150 \pm 19.6 = [130.4, 169.6]$$

Part B

Provide an interpretation for this confidence interval to someone with little statistical background. *Hint:* See pg. 248.

Solution

Say there is a piece of paper in a sealed envelope with a number from 1 to 200 on it. Then let's say that we don't know what this number is. By saying that our 95% confidence interval starts at 130.4 and goes to 169.6, we are saying that if we open the envelope and look at the number, we have a 95% chance that our range covers the number on the piece of paper.

Problem 5

Find a 90% confidence interval for $\mu_1 - \mu_2$ when $n_1 = 30$, $n_2 = 39$, $\bar{x}_1 = 4.2$, $\bar{x}_2 = 3.4$, $s_1^2 = 49$ and $s_2^2 = 32$.

Solution

This is a confidence interval for differene of means. This gives us $\alpha = 0.1$, so $z_{\alpha/2} = z_{0.05} = 1.645$ then:

$$\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 4.2 - 3.4 \pm 1.645\sqrt{\frac{49}{30} + \frac{32}{39}} = 0.8 \pm 2.58 = [-1.78, 3.38]$$

Problem 6

Problem 9.9a from Baron on pg. 301. Answer in the back of the book.

Salaries of entry level computer engineers have a Normal distribution with unknown mean and variance. Three randomly selected computer engineers have salaries: 30, 50, 70.

Construct a 90% confidence interval for the average salary of an entry-level computer engineer.

Solution

We have a small sample size, so we need to use the t-statistic. Using our sample, this gives us: $\bar{X} = 50$, $\alpha = 0.10$, $s = 20$. Using a t table, know our t-statistic has 2 degrees of freedom which gives $t_{0.05} = 2.353$. Thus:

$$\bar{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 50 \pm 2.920\frac{20}{\sqrt{3}} = 50 \pm 33.72 = [16.3, 83.7]$$

Problem 7

Cranston, Rhode Island, has the reputation for selling the most expensive bubble gum in the U.S. Ten candy stores were surveyed and it was found that the average price in the 10 stores was 40 cents with a standard deviation of 5 cents.

Part A

Find a 95% confidence interval for μ, the mean gum price.

Solution

With our sample, this gives us: $\bar{X} = 40$, $\alpha = 0.05$, $s = 5$. Using a t table, know our t-statistic has 9 degrees of freedom which gives $t_{0.025} = 2.262$. Thus:

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 40 \pm 2.262 \frac{5}{\sqrt{10}} = 40 \pm 3.58 = [36.4, 43.6]$$

Part B

Find a 99% confidence interval for μ, the mean gum price.

Solution

Our new value for α is 0.01. Using a t table, know our t-statistic has 9 degrees of freedom which gives $t_{0.005} = 3.250$. Thus:

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 40 \pm 3.250 \frac{5}{\sqrt{10}} = 40 \pm 5.14 = [34.9, 45.1]$$