Stat330: Homework #1

Due on January 22, 2014at 3:10pm

Mr. Lanker Section A

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Problem 1

Math and calculus review.

Part A

Evaluate $\sum_{k=1}^{5} k^2$ and $\sum_{k=1}^{5}(k-1)^2$.

Solution

$$\sum_{k=1}^{5} k^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$$

$$\sum_{k=1}^{5}(k-1)^2 = (1-1)^2 + (2-1)^2 + (3-1)^2 + (4-1)^2 + (5-1)^2 = 30$$

Part B

Find the derivative of $f(x) = x^4 + 3x^2 - 2$

Solution

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^4 + 3x^2 - 2) = 4x^3 + 6x$$

Part C

Find the derivative of $f(x) = 1 - \mathrm{e}^{-\lambda x}$.

Solution

$$\frac{\mathrm{d}}{\mathrm{d}x}(1 - \mathrm{e}^{-\lambda x}) = \lambda \cdot \mathrm{e}^{-\lambda x}$$

Part D

Evaluate the integrals $\int_0^1 (1-x^2)\mathrm{d}x$ and $\int_1^\infty \frac{1}{x^2}\mathrm{d}x$.

Solution

$$\int_0^1 (1-x^2)\mathrm{d}x = x - \frac{1}{3}x^3\Big|_0^1 = (1 - \frac{1}{3}) - (0 - 0) = \frac{2}{3}$$

$$\begin{aligned}
\int_1^\infty \frac{1}{x^2}\mathrm{d}x &= \lim_{b\to\infty}\int_1^b \frac{1}{x^2}\mathrm{d}x \\
&= \lim_{b\to\infty} -\frac{1}{x}\Big|_1^b \\
&= \lim_{b\to\infty} -\frac{1}{b} + \frac{1}{1} \\
&= 1
\end{aligned}$$

Problem 2

Examples of sample spaces.

Part A

Driving to work and passing through 3 intersections.

Solution

$$\Omega = \{ccc, ccs, csc, css, scc, scs, ssc, sss\}$$

Part B

What is the probability that she doesn't stop?

$$\Pr(\text{no stops}) = \frac{|\{ccc\}|}{|\Omega|} = \frac{1}{8}$$

Part C

Let A be the event that the commuter stops at the first light. Let B be the event that the commuter stops at the second light.

Solution

1. $A = \{scc, scs, ssc, sss\}$
2. $B = \{csc, css, ssc, sss\}$
3. $\overline{B} = \{ccc, ccs, scc, scs\}$
4. $A \cup B = \{scc, scs, csc, css, ssc, sss\}$
5. $A \cap B = \{ssc, sss\}$
6. $A \cap \overline{B} = \{scc, scs\}$

Problem 3

Let G and H be disjoint events in some sample space Ω.

Part A
Describe the event $G \cup H$.

Solution
A new event that includes outcomes in G **or** in H.

Part B
What is $P(G \cup H)$ in terms of $P(G)$ and $P(H)$?

Solution
Since G and H are disjoint, $P(G \cap H) = 0$:

$$\begin{aligned} P(G \cup H) &= P(G) + P(H) - P(G \cap H) \\ &= P(G) + P(H) - 0 \\ &= P(G) + P(H) \end{aligned}$$

Part C
Describe the event $G \cap H$.

Solution
A new event that includes the outcomes that happen in event G **and** in H.

Part D
What is the probability of event $G \cap H$?

Solution
The probability of $G \cap H = 0$ because the two events are disjoint.

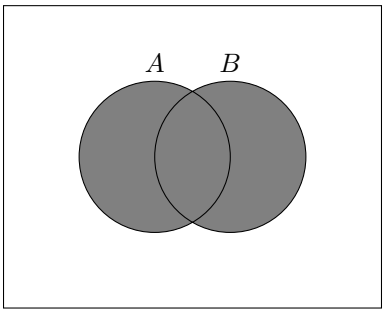
Problem 4

Venn diagrams. The grey areas indicate the solution.

Part A

$A \cup B$

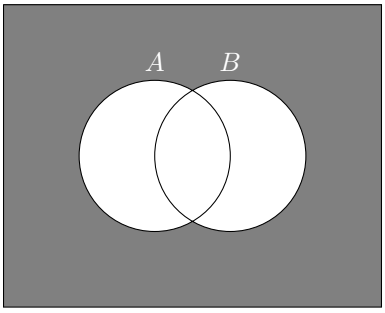
Solution



Part B

$\overline{A \cup B}$

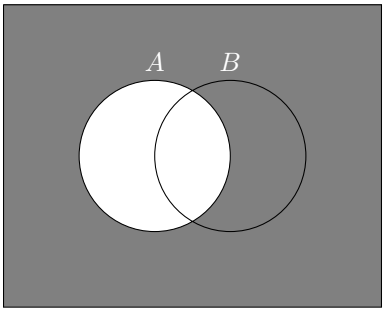
Solution



Part C

$\overline{A}$

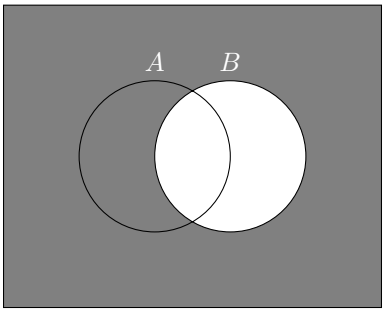
Solution



Problem 4 continued on next page...

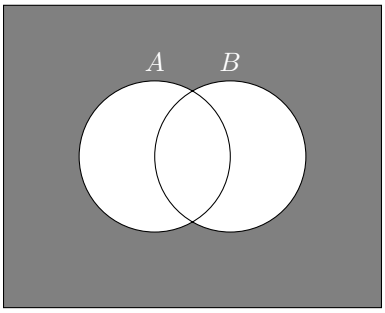
Part D
$\overline{B}$

Solution



Part E
$\overline{A} \cap \overline{B}$

Solution



Problem 5

Employees at a firm, 70% know C, 60% know Fortran and 50% know both.

Part A
Let A be the event that an employee knows C and B be the event that employee knows Fortran. Draw a Venn diagram.

Solution(Drawing)

Part B
What percentage of programmers do not know Fortran?

Solution
$$\Pr(\neg \mathrm{B}) = 1 - \Pr(\mathrm{B}) = 1 - .6 = .4$$

Part C
What percentage of programmers do not know Fortran and C?

Solution
$$\Pr(\neg(\mathrm{B} \wedge \mathrm{C})) = 1 - \Pr(\mathrm{B} \wedge \mathrm{C}) = 1 - \Pr(\mathrm{B} \cap \mathrm{C}) = 1 - (.5) = .5$$

Part D
What percentage of programmers know Fortran but not C?

Solution
$$\Pr(\mathrm{B} \wedge \neg \mathrm{C}) = \Pr(B) - \Pr(\text{both}) = .6 - .5 = .1$$

Problem 6

Total 60 students attending University. 9 were living off campus, 36 were undergrads, 3 were undergrads living off campus.

Let A be the event denoting undergraduates and B denote living off campus.

$$A = 36, \quad B = 9, \quad \overline{A} = 60 - 36 = 24, \quad \overline{B} = 60 - 9 = 51, \quad A \cap B = 3$$

Part A
Number of students who were undergrads living on campus.

Solution
$$A \cap \overline{B} = A \setminus B = 36 - 9 + 3 = 30$$

Part B
Number of students who were graduate students living on campus.

Solution
$$\overline{A} \cap \overline{B} = \overline{A} \setminus B = 24 - 9 + 3 = 18$$