Show all of your work. Please staple. Problems denoted with (a.i.b.) mean the answers are in the back of the book: be sure to show your work for full credit. There are 10 problems on 2 pages.

1. Baron 3.15. (a.i.b.)

2. Based on Baron 3.13: Two random variables X and Y have the joint distribution defined by probabilities $P(0,0) = .4$, $P(1,0) = .2$, $P(1,1) = .2$, $P(2,0) = .1$, $P(2,2) = -.1$, and $P(x,y) = 0$ for all other pairs (x,y).

   (a) Find the probability mass function of a new random variable $U = 2X - Y$ by looking at the possible values for u based on the defined joint distribution. Hint: the value $u = 0$ has probability .4.

   (b) Find $E(U)$ using the probability mass function $p(U)$.

   (c) You can check that the expected value of the (marginal distributions) X and Y are $E(X) = 0.8$ and $E(Y) = 0.4$. This time using the properties of expected values (see p. 49), again calculate $E(U)$ to confirm your answer in (2b).

   (d) Calculate $P(X = 2|U \geq 2)$.

3. Based on Baron 4.2: The time in minutes for a certain system to reboot can be modeled with a continuous random variable T that has the following probability density function

$$f(t) = \begin{cases} C(2-t)^2 & 0 \leq t \leq 2 \\ 0 & \text{any other } t \end{cases}$$

   (a) Compute C.

   (b) Compute $E(T)$.

   (c) Compute $Var(T)$.

   (d) Computer the probability that it takes between 1 and 2 minutes to reboot.

4. *The Poisson process.* When you first learned the Poisson distribution, you found the parameter λ equal to a rate times the interval size. Now we will let λ represent the rate and make the interval size t an explicit variable.

   Say that jobs arrive at a print queue in a busy computer lab at a rate of 1 print jobs every two minutes (rate is $\lambda = 0.5$ jobs $\cdot$ min^{-1}). The number of jobs arriving in an time interval of size t is distributed Poisson$(\lambda\, t)$.

   (a) Calculate the probability that less than 2 jobs arrive at the print queue in 4 minutes.

   For a Poisson process, the time between incoming jobs is modeled with a Exponential(λ) probability distribution.

   (b) If a print job just arrived at the print queue, what is the probability that the queue will wait less than 1.5 minutes for the next print job to arrive?

5. *The memoryless property of the exponential.*

   (a) Using your answer to (4b), if a print job has just arrived, calculate the probability that the next print job arrives after 1.5 minutes.

   (b) Similarly if a print job has just arrived, calculate the probability that the next print job arrives after 3.0 minutes.

   (c) Calculate the conditional probability that the next print job arrives after 3.0 minutes given that the job does not arrive in the first 1.5 minutes. To start you on your answer, let I represent the time until the next arrival (for Interarrival time):

   $$P(I > 3.0|I > 1.5) = \frac{P(I > 3.0 \text{ and } I > 1.5)}{P(I > 1.5)} = \frac{P(I > 3.0)}{P(I > 1.5)} =$$

   with the simplification in the numerator due to the fact if $I > 3.0$ then it is automatically greater than 1.5 minutes, so the last part can be ignored. (In fancier notation, if $A \subset B$ then $P(A \cap B) = P(A)$.)

   (d) Why is the exponential probability distribution called memoryless? Compare your answers for (5a) and (5c).

6. Baron 4.16. (a.i.b.) I dislike his answers for (e) and (f): I prefer (e) $> .9999$, (f) $< .0001$. Saying that $P(Z > 6) = 0.0$ implies it cannot happen, but it has probability 9.9×10^{-10}, which is a lot larger than 0.

7. Baron 4.17.

8. If X is continuous random variable that follows a normal probability distribution with mean 100 points and standard deviation 15 points (variance of 225 points^2), i.e. $X \sim Normal(\mu = 100, \sigma = 15)$, then find the probability that X is

   (a) less than 106.

   (b) greater than 88.

   (c) between 85 and 115.

   In addition:

   (d) Find the point value that separates the lower half and upper half of values of X (i.e., the median, the value c where $P(X < c) = .50$).

   (e) Find the point value that separates the lower quarter from the upper three-quarters of values of X (i.e., the first quartile, the value c where $P(X < c) = .25$).

   (f) Find the point value that separates the lower three-quarters from the upper quarter of values of X (i.e., the third quartile, the value c where $P(X < c) = .75$).

9. Baron 4.22 (a.i.b.)

10. Baron 4.21. The answer for (b) should be a range of values.