1. Let $\overline{X}$ denote the mean of a random sample of n i.i.d. observations from a distribution that is $Normal(\mu, \sigma^2)$, $\sigma^2 > 0$, where σ is known but μ is unknown.

   What is the probability that the confidence interval

   $$(\overline{X} - 2.2\frac{\sigma}{\sqrt{n}}, \overline{X} + 2.2\frac{\sigma}{\sqrt{n}})$$

   contains the (fixed) point μ? (Note: this is not a trick question. μ is fixed, but you can think of the endpoints $\bar{X} \pm z\frac{\sigma}{\sqrt{n}}$ as *random*, forming a random interval that either contains μ or doesn't.)

2. Baron 9.7(a), p. 301. (Answer in back of book.)

3. Find a 95% confidence interval for μ, the true mean of a normal population which has a variance $\sigma^2 = 100$. Consider a sample of size 25 that has a mean of 69.3.

4. A department store has 10,000 customers' charge accounts. To establish the total amount owed by all its customers, it selected 36 accounts at random and found a mean of \$150 and a standard deviation of \$60.

   (a) Establish a 95% confidence interval estimate of the mean amount owed by its customers. (They could then estimate the total owed on all charge accounts by multiplying by 10,000.)

   (b) Provide an interpretation for this confidence interval to someone with little statistical background. (Hint: see p. 248.)

5. Find a 90% confidence interval for $\mu_1 - \mu_2$ when $n_1 = 30$, $n_2 = 39$, $\bar{x}_1 = 4.2$, $\bar{x}_2 = 3.4$, $s_1^2 = 49$, and $s_2^2 = 32$, where $\bar{x}_1$ is the mean of a sample of size n_1 from the first population (mean μ_1) and has sample variance s_1^2, and $\bar{x}_2$ is the mean of a sample of size n_2 from the second population (mean μ_2) with sample variance s_2^2.

6. Baron 9.9(a), p. 301. (Answer in back of book.)

7. Cranston, Rhode Island, has the reputation for selling the most expensive bubble gum in the U.S. Ten candy stores were surveyed and it was found that the average price in the 10 stores was 40 cents with a standard deviation of 5 cents. Find (a) a 95% and (b) a 99% confidence interval for μ, the mean gum price in Cranston.

8. An experimenter tested for differences in attitudes toward smoking before and after a film on lung cancer was shown. The experimenter tested to see if there was a difference between attitudes that people held about smoking before and after viewing the film (either less or more favorable towards smoking). She found a difference which was significant between the .02 and .05 levels.

   (a) Let μ_1 and μ_2 represent the mean attitude towards smoking before viewing the film and after viewing the film, respectively. What are the assumed hypotheses (null and alternative)?

   (b) Which level of significance indicates the greater degree of significance, .05 or .02, i.e. for which level of significance will the experimenter be more confident in rejecting the null hypothesis in favor of the alternative hypothesis?

   (c) If her α level is .05, will she reject H_0 in favor of H_a?

   (d) Will she reject H_0 in favor of H_a if she employs the .01 level?

9. The mean yield of corn in the United States is about 120 bushels per acre. [from 1989] A survey of 50 farmers this year gives a sample mean yield of $\bar{x} = 123.6$ bushels per acre. We want to know whether this is good evidence that the national mean this year is not 120 bushels per acre. Assume that the sample is iid from the entire population and that the standard deviation of the yield in this population is $\sigma = 10$ bushels per acre.

   Give the p-value for the test of

   $$H_0 : \mu = 120 \qquad vs. \qquad H_a : \mu \neq 120$$

   Are you convinced that the population mean is not 120 bushels per acre? Use the .05 significance level in making your decision.

10. In the past, the mean score of the seniors at South High on the American College Testing (ACT) college entrance examination has been 20.0. This year a special preparation course is offered, and all 43 seniors planning to take the ACT test enroll in the course. The mean of their 43 ACT scores is 21.1. Assume that ACT scores vary normally with standard deviation 6.

    Is the outcome good evidence that this class's true mean score is not 20? State your hypotheses, compute the p-value, and assess the amount of evidence.

11. A computer has a random number generator designed to produce random numbers that are uniformly distributed within the interval from 0 to 1. If this is true, the numbers generated come from a population with $\mu = \frac{1}{2}$ and $\sigma^2 = \frac{1}{12}$.

    A command to generate one million random numbers results in a sample mean of 0.4992894. Assume that the population variance remains fixed. We want to use the results of this experiment to test if μ is in fact one-half.

    (a) State the hypotheses for this test.

    (b) Calculate the value of the z statistic.

    (c) Compute the p-value.

    (d) Is the result significant at the $\alpha = .05$ level?

    (e) Is the result significant at the $\alpha = .01$ level?

    (f) I performed this test in R, which has a decent random number generator. Are you surprised about the result of this experiment? Why or why not?

    (g) **[Extra credit.]** Now what if I told you that I generated 100 sets of one million random numbers and used the lowest sample mean of the 100 sets for this problem, now are you surprised about the result? Why or why not?