Show all of your work, and *please* staple your assignment if you use more than one sheet.

1. A calculus and other math review:
   (a) Evaluate the sums $\sum_{k=1}^{5} k^2$ and $\sum_{k=1}^{5} (k-1)^2$
   (b) Find the derivative of $F(x)$ when $F(x) = x^4 + 3x^2 - 2$
   (c) Find the derivative of $F(x)$ (with respect to x) when $F(x) = 1 - e^{-\lambda x}$, where λ is a constant.
   (d) Evaluate the definite integrals $\int_0^1 (1 - x^2)\, dx$ and $\int_1^\infty \frac{1}{x^2}\, dx$
2. Examples of sample spaces
   (a) Driving to work, a commuter passes through a sequence of three intersections with traffic lights. At each light, she either stops, s, or continues, c. Determine the set of possible outcomes in the sample space. *Notation: use 'csc' to denote the outcome where she must stop only at the second traffic light.*
   (b) If each of the outcomes is equally likely in (2a), then what is the probability (or chance) that she doesn't have to stop at any of these lights? Comment if this value seems reasonable to you.
   (c) Let A be an event in the sample space defined in problem (2a), and have event A be the commuter stops at the first light. Let B be the event in (2a) that the commuter stops at the second light. Following the above notation, list the outcomes in event:
      i. A
      ii. B
      iii. B^C
      iv. $A \cup B$
      v. $A \cap B$
      vi. $A \cap B^C$
3. Disjoint events

   Let G and H be disjoint events in some sample space Ω. The probability that G occurs is $P(G)$ and H occurs is $P(H)$.
   (a) Describe the event $G \cup H$.
   (b) What is $P(G \cup H)$ in terms of $P(G)$ and $P(H)$?
   (c) Describe the event $G \cap H$.
   (d) What is the probability of event $G \cap H$?
4. Venn diagrams Let A and B be events in the same sample space Ω. Also, let $A \cap B \neq \emptyset$, i.e. A and B are not disjoint events (there are some outcomes in Ω common to both events A and B).

   Draw separate Venn diagrams for these events:
   (a) $A \cup B$
   (b) $\overline{A \cup B}$

   and draw separate Venn diagrams for these events:
   (c) $\overline{A}$
   (d) $\overline{B}$
   (e) $\overline{A} \cap \overline{B}$

Now you should be convinced that $\overline{A \cup B}$ equals $\overline{A} \cap \overline{B}$ from your Venn diagrams. This is one of DeMorgan's Laws, which are formally:

$$\overline{\left(\bigcup_{i=1}^{k} A_i\right)} = \bigcap_{i=1}^{k} \overline{A_i} \qquad \text{and} \qquad \overline{\left(\bigcap_{i=1}^{k} A_i\right)} = \bigcup_{i=1}^{k} \overline{A_i}$$

for any events $A_1, A_2, \ldots, A_k$.

5. based on Baron 2.4

   Among employees of a certain firm, 70% know C/C++, 60% know Fortran, and 50% know both languages.

   (a) Let A is the event that an employee knows C/C++ and B is the event that an employee knows Fortran and draw a Venn diagram denoting these percentages.

   (b) What percentage of programmers does not know Fortran?

   (c) What percentage of programmers does not know Fortran and C/C++?

   (d) What percentage of programmers knows Fortran but not C/C++?

6. From a survey of 60 students attending a university, it was found that 9 were living off campus, 36 were undergraduates, and 3 were undergraduates living off campus. Use set notation to solve this problem by defining event A to denote undergraduates and event B to denote living off campus.

   (a) Find the number of these students who were undergraduates living on campus.

   (b) Find the number of these students who were graduate students living on campus.