One assignment submitted per group. Show all of your work. Please staple.

1. The *discrete uniform* distribution defined on $[a, b]$, where a and b are integers, is the probability distribution where any number between a and b (inclusive) is equally likely to occur. Let X be a discrete uniform variable on $[a, b]$. The probability mass function of X is

$$p_X(x) = \begin{cases} \dfrac{1}{b-a+1} & , \quad x \in \{a, a+1, \ldots, b-1, b\} \\ 0 & , \quad \text{otherwise} \end{cases}$$

   (a) Find $E[X]$.

   (b) Suppose c is an integer between a and b. How does $p_X(c)$ change as a and b move further away from each other?

2. In the board game *Monopoly*, a player in jail gets out by rolling "doubles" on their turn, i.e., by rolling the same number on the two dice thrown. Since the only outcome that allows the player to move his piece is doubles, the player will be less interested in the sum of the two dice and more interested in whether or not the he or she rolled doubles.

   Let X be the outcome of a single roll of the dice, with "success" considered to be rolling doubles and "failure" rolling anything else. Then $X \sim \text{Bernoulli}(p)$.

   (a) What is p?

   (b) What is E(X) and Var(X)?

   (c) Construct a graph of the cumulative distribution function $F_X(t)$.

   Now let's look at the number of turns that are needed until doubles are rolled (and a player "gets out of jail"). Let Y be a random variable representing the number of rolls of the dice until doubles comes up. Then $Y \sim \text{Geometric}(p)$.

   (d) Using your answer to part (2a) for p, what is the expected number of turns a player will need to get out of jail? (Hint: this is $E[Y]$)

   (e) What is the probability that a player will need four or more rolls to get out of jail?

3. A student is taking a 10 question multiple choice exam, where each question has four possible answers and the correct answer for each question is randomly chosen and independent of the other answers. If the student guesses "C" on every question, then X, the number of questions that the student answers correctly, follows a Binomial$(10, 0.25)$ distribution.

   (a) Find $E[X]$ and $Var(X)$.

   (b) Does $E[X]$ represent a possible outcome? Does this matter? Why or why not?

   (c) What is the probability that the student answers no questions correctly?

   (d) The student will pass the exam if he answers 6 or more questions correctly. What is the probability that the student passes the exam?

4. In some city, the probability of rain on any day is 0.60, determined using historical climate records. It is known that on rainy days the number of traffic accidents has a Poisson(10) distribution; otherwise, the number of traffic accidents has a Poisson(4) distribution.

   An accident occurred years ago and there are no precise weather records available, but we do know that on the day of the accident in question there were 8 accidents in this city. Use Bayes Rule to determine the probability that it was in fact a rainy day.

5. In his NBA career, Coach Fred Hoiberg had a free throw shooting percentage of 85.4%. Suppose that in practice one day he shoots free throws until his first miss. Let X bet the number of shots he takes.
   (a) Define the probability distribution of X, carefully determining the appropriate parameter(s).
   (b) What is the probability that Coach Hoiberg makes his first two shots but misses his third shot?
   (c) What is the probability that Coach Hoiberg takes more than fifteen shots? (Hints: this is strictly greater than fifteen, and it would help to use a cumulative probability function.)
   (d) What is the probability that Coach Hoiberg takes between two and four (inclusive) shots?
6. On average, A certain region in Alaska experiences on average 12 severe earthquakes every 10 years. Assume that the timings of the severe earthquakes do not depend on the others (and this is not a good assumption because physically stress builds in the plates to cause severe earthquakes). If you have a three year assignment in this region of Alaska:
   (a) what is the probability that there be no severe earthquakes during your assignment?
   (b) what is the probability that there are three or more severe earthquakes during your assignment?
7. A type of photo paper has on average 1.5 blemishes per square foot. You can reasonably assume that blemishes are randomly distributed. If the photograph you are printing is 0.4 square feet, what is the probability that the randomly selected photo paper for your photograph will have at least one blemish?
8. As an exemplary student, the probability you attend lecture for a certain class is .95. If there are 30 lectures during the semester, what is the probability that you will miss 2 or more lectures throughout the semester? There is no attendance requirement, and the probability of missing any lecture is independent of attendance in all other lectures.