Stat330: Homework #11

Due on April 30, 2014 at 3:10pm

Mr. Lanker Section A

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Problem 8

An experimenter tested for differences in attitudes toward smoking before and after a film on lung cancer was shown. The experimenter tested to see if there was a difference between attitudes that people held about smoking before and after viewing the film (either less or more favorable). She found a difference which was significant between the 0.02 and 0.05 levels.

Part A

Let μ_1 and μ_2 represent the mean attitude towards smoking before viewing the film and after viewing the film, respectively. What are the assumed hypotheses (null and alternative)?

Solution

The hypotheses are as follows:

$$H_0 : \mu_1 = \mu_2 \quad vs. \quad H_a : \mu_1 \neq \mu_2$$

Part B

What level of significance indicates the greater degree of significance, 0.05 or 0.02, i.e. for which level of significance will the experimenter be more confidence in rejecting null hypothesis in favor of the alternative?

Solution

A lower value is better as it indicates a smaller possibility of the value being that extreme.

Part C

If her α level is 0.05, will she reject H_0 in favor of H_a?

Solution

Yes, if her significance is between 0.05 and 0.02 and her α level is 0.05, then that means that the level of significance is less than α which is when we reject the H_0.

Part D

Will she reject H_0 in favor of H_a if she employs the 0.01 level?

Solution

No, since our value is between 0.05 and 0.02, an α value of 0.01 is too small and we can't reject H_0.

Problem 9

The mean yield of corn in the US is about 120 bushels per acre (from 1989). A survey of 50 farmers this year gives a sample mean yield of $\bar{x} = 123.6$ bushels per acre. We want to know whether this is good evidence that the national mean this year is not 120 bushels per acre. Assume that the sample is i.i.d. from the entire population and that the standard deviation of the yield in this population is $\sigma = 10$ bushels per acre.

Give the p-value for the test of

$$H_0 : \mu = 120 \quad vs. \quad H_a : \mu \neq 120$$

Are you convinced that the population mean is not 120 bushels per acre? Use the 0.05 significance level in making your decision.

Solution

First let's calculate the z-statistic, which gives us:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{123.6 - 120}{10/\sqrt{50}} = \frac{3.6}{1.414} = 2.55$$

This gives us a p-value of 0.015. Since this value is less than 0.05, we have moderate evidence that we can reject the null hypothesis and thus we accept that we have evidence that $\mu \neq 120$.

Problem 10

In the past, the mean score of the seniors at South High on the ACT exam has been 20.0. This year a special preparation course is offered, and all 43 seniors planning to take the ACT enroll in the course. The mean of their ACT scores is 21.1. Assume that the ACT scores vary normally with $\sigma = 6$.

Is the outcome good evidence that this class's true mean is not 20? State your hypotheses, compute the p-value, and assess the amount of evidence.

Solution

Our hypotheses are:

$$H_0 : \mu = 20 \quad vs. \quad H_a : \mu \neq 20$$

Let's calculate the z-statistic, which gives us:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{21.1 - 20}{6/\sqrt{43}} = \frac{1.1}{0.915} = 1.202$$

This gives us a p-value of 0.194. Since this value is not less than 0.05 (and not anywhere near it), we have no evidence that we can reject the null hypothesis and thus we fail to reject H_0 and accept it. This means we accept that $\mu = 20$.

Problem 11

A computer has a random number generator designed to produce random numbers that are uniformly distributed within the interval from 0 to 1. If this is true, the numbers come from a population with $\mu = \frac{1}{2}$ and $\sigma^2 = \frac{1}{12}$ and $\sigma = 0.289$.

A command to generate one million random numbers results in a sample mean of 0.4992894. Assume that the population variance remains fixed. We want to use the results of this experiment to test if μ is in fact one-half.

Part A

State the hypotheses for this test.

Solution

The hypotheses are:

$$H_0 : \mu = 0.5 \quad vs. \quad H_a : \mu \neq 0.5$$

Part B

Calculate the value of the z-statistic.

Solution

The value of the z-statistic is:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{0.4992894 - 0.5}{0.289/\sqrt{1000000}} = \frac{-0.0007106}{0.000289} = -2.459$$

Part C

Compute the p-value.

Solution

Using a standard Z table, this gives us the p-value $= 0.0139$.

Part D

Is the result significant at the $\alpha = 0.05$ level?

Solution

Yes, because the p-value is less than our α of 0.05.

Part E

Is the result significant at the $\alpha = 0.01$ level?

Solution

No, because the p-value is greater than our α of 0.01.

Part F

I performed this test in R, which has a decent random number generator. Are you surprised about the result of this experiment? Why or why not?

Solution

Yes, you'd think that 1 million random numbers would be enough to say that our μ value is 0.5. But using our p-value we have significant evidence to reject H_0 and thus accept H_a that $\mu \neq 0.5$.

Problem 11 continued on next page. . .

Part G

Extra Credit: Now what if I told you that I generated 100 sets of one million random numbers and used the lowest sample mean of the 100 sets for this problem, now are you surprised about the result? Why or why not?

Solution

By calculating the mean 100 times, it is likely that we will get one set of sample values that allows us to reject H_0. By taking the minimum of all of these means, we are basically just selecting the sample that gave us the most extreme value. Thus I'm not surprised anymore because we picked the mean that was most extreme thus making the p-value as low as possible.