Stat330: Homework #3

Due on February 5, 2014 at 3:10pm

Mr. Lanker Section A

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Problem 1

Define a discrete random variable and the sample space.

Part A

The number of bowling games needed for you to have at least 100 points.

Solution

X = number of bowling games to score at least 100 points. $\Omega = \{1, 2, 3, 4, ...\}$.

Part B

Analyze how many accidents occur at the intersection of Lincoln Way & Welch Ave. during any week.

Solution

X = number of accidents at Lincoln Way & Welch during a week. $\Omega = \{0, 1, 2, 3, 4, ...\}$.

Part C

You play a game where you roll a 6-sided die and win a number of points equal to 3 divided by your roll.

Solution

X = number when dividing 3 by the number when rolling a 6-sided die. $\Omega = \{3, \frac{3}{2}, 1, \frac{3}{4}, \frac{3}{5}, \frac{3}{6}\}$.

Problem 2

Five balls numbered 1, 3, 5, 7 and 9 are placed in an urn. Two balls are randomly selected from the five (without replacement), and their numbers noted. Find the probability distribution for the following.

1st	2nd	Max	Avg	1st	2nd	Max	Avg	1st	2nd	Max	Avg	1st	2nd	Max	Avg
1	3	3	2	1	5	5	3	1	7	7	4	1	9	9	5
3	1	3	2	3	5	5	4	3	7	7	5	3	9	9	6
5	1	5	3	5	3	5	4	5	7	7	6	5	9	9	7
7	1	7	4	7	3	7	5	7	5	7	6	7	9	9	8
9	1	9	5	9	3	9	6	9	5	9	7	9	7	9	8

Part A

The *largest* of the two sampled numbers.

Solution

$$P(x) = \begin{cases} 0 & x = 1 \\ 2/20 = 0.1 & x = 3 \\ 4/20 = 0.2 & x = 5 \\ 6/20 = 0.3 & x = 7 \\ 8/20 = 0.4 & x = 9 \\ 0 & \text{any other } x \end{cases}$$

Part B

The *average* of the two sampled numbers.

Solution

$$P(x) = \begin{cases} 2/20 = 0.1 & x = 2 \\ 2/20 = 0.1 & x = 3 \\ 4/20 = 0.2 & x = 4 \\ 4/20 = 0.2 & x = 5 \\ 4/20 = 0.2 & x = 6 \\ 2/20 = 0.1 & x = 7 \\ 2/20 = 0.1 & x = 8 \\ 0 & \text{any other } x \end{cases}$$

Problem 3

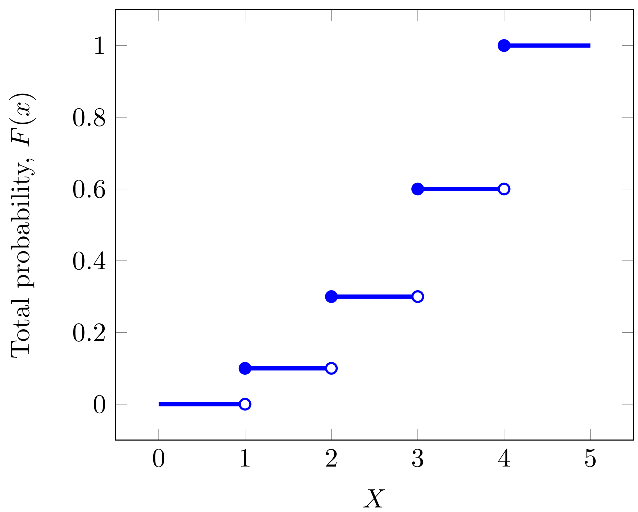
Calculate the cumulative probability function for the valid probability mass function:

$$P(x) = \begin{cases} x/10 & x = 1, 2, 3, 4 \\ 0 & \text{any other } x \end{cases}$$

Carefully plot the cumulative probability function for X, $F(x)$, labeling all axes.

Solution

$$F(x) = \begin{cases} 0 & x < 1 \\ 1/10 & 1 \le x < 2 \\ 3/10 & 2 \le x < 3 \\ 6/10 & 3 \le x < 4 \\ 1 & x = 4 \\ 1 & x > 4 \end{cases}$$



Problem 4

Every day, the number of network blackouts has a distribution (pmf):

x	0	1	2
$P(x)$	.7	.2	.1

A small internet trading company estimates that each network balckout results in a \$500 loss. Compute expectation and variance of this company's daily loss due to blackouts.

Solution

Expectation:

$$\begin{aligned} \mathrm{E}[X] &= \sum_{0}^{2} xP(x) \\ &= (0)(.7) + (1)(.2) + (2)(.1) \\ &= 0 + .2 + .2 \\ &= .4 \end{aligned}$$

Variance:

$$\begin{aligned} \mathrm{Var}[X] &= \mathrm{E}X^2 - (\mathrm{E}X)^2 \\ &= (0^2)(.7) + (1^2)(.2) + (2^2)(.1) - .4^2 \\ &= .4 + .2 - .16 \\ &= .44 \end{aligned}$$

Problem 5

Calculate the mean, variance, and standard deviation of the discrete probability distribution in question 3.

Solution

Using the probability distribution:

x	1	2	3	4
$P(x)$	1/10	2/10	3/10	4/10

Expectation:

$$\begin{aligned}
\mathrm{E}[X] &= \sum_{0}^{2} xP(x) \\
&= (1)(1/10) + (2)(2/10) + (3)(3/10) + (4)(4/10) \\
&= \frac{1}{10} + \frac{4}{10} + \frac{9}{10} + \frac{16}{10} \\
&= \frac{30}{10} \\
&= 3
\end{aligned}$$

Variance:

$$\begin{aligned}
\mathrm{Var}[X] &= \mathrm{E}X^2 - (\mathrm{E}X)^2 \\
&= (1^2)(1/10) + (2^2)(2/10) + (3^2)(3/10) + (4^2)(4/10) - (3)^2 \\
&= \frac{1}{10} + \frac{8}{10} + \frac{27}{10} + \frac{64}{10} - 9 \\
&= \frac{100}{10} - 9 \\
&= 1
\end{aligned}$$

Problem 6

A single fair die is tossed once. Let Y be the number facing up. Find the expected value and variance of Y.

Solution

Expectation:

$$\begin{aligned}
\mathrm{E}[X] &= \sum_{0}^{2} xP(x) \\
&= (1)(1/6) + (2)(1/6) + (3)(1/6) + (4)(1/6) + (5)(1/6) + (6)(1/6) \\
&= 3.5
\end{aligned}$$

Variance:

$$\begin{aligned}
\mathrm{Var}[X] &= \mathrm{E}X^2 - (\mathrm{E}X)^2 \\
&= (1^2)(1/6) + (2^2)(1/6) + (3^2)(1/6) + (4^2)(1/6) + (5^2)(1/6) + (6^2)(1/6) - (3.5)^2 \\
&= 2.92
\end{aligned}$$