GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

ORTHOGONALIZATION FOR REGRESSION

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"Although polynomial regression fits a *nonlinear* model to the data, as a statistical estimation problem it is *linear*, in the sense that the regression function E(y|x) is linear in the unknown parameters that are estimated from the data. For this reason, polynomial regression is considered to be a special case of multiple linear regression." -- Wikipedia

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But there is a problem with the model we've written down so far.

This model displays **collinearity**, which means the predictor variables are highly correlated with each other.

$$y = \alpha + \beta_1 x + \beta_2 x^2 + \ldots + \beta_n x^n + \varepsilon$$

```
> x <- seq(1, 10, 0.1)
> cor(x^9, x^10)
[1] 0.9987608
```

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NOTE

For identical features, this results in a *singularity*.

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OPTIONAL NOTE

These polynomial functions form an *orthogonal basis* of the function space.