GENERAL ASSEMBLY

# INTRO TO DATA SCIENCE
## LINEAR REGRESSION

# I. LINEAR REGRESSION
# II. POLYNOMIAL REGRESSION

# EXERCISES:
# III. LINEAR REGRESSION IN R
# IV. PREDICTING BASEBALL SALARIES

# I. LINEAR REGRESSION

| | *continuous* | *categorical* |
|---|---|---|
| *supervised* | ??? | ??? |
| *unsupervised* | ??? | ??? |

| | *continuous* | *categorical* |
| --- | --- | --- |
| ***supervised*** | regression | classification |
| ***unsupervised*** | dimension reduction | clustering |

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$\varepsilon$ = **residual** (the prediction error)

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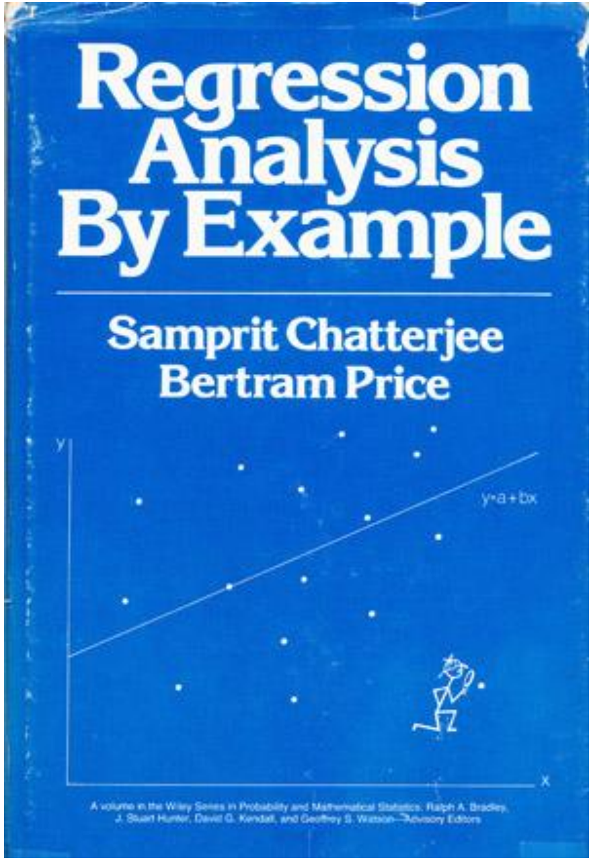
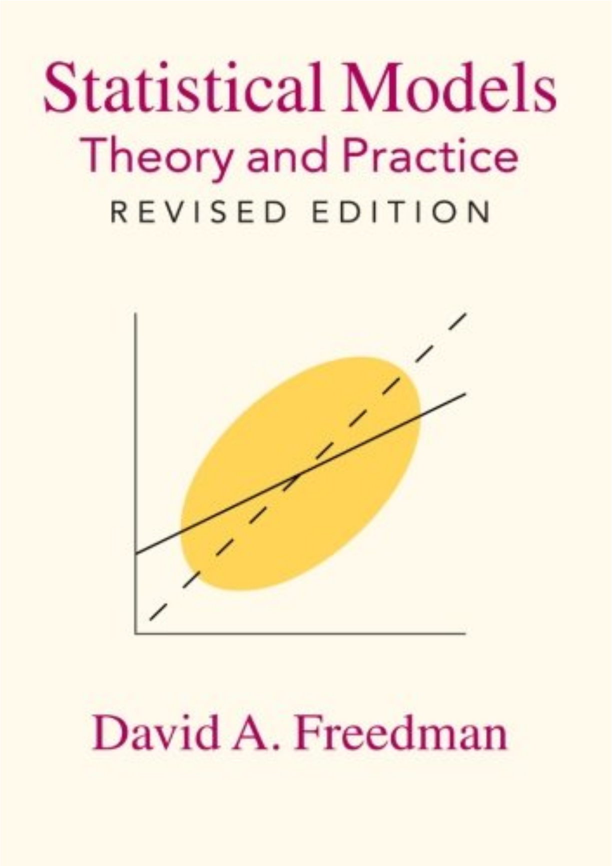
$$y = \alpha + \beta_1 x_1 + \ldots + \beta_n x_n + \varepsilon$$

Linear regression involves several technical assumptions and is often presented with lots of mathematical formality.

The details are not very important for our purposes, but you can check them out if you're interested.


Regression Analysis By Example
Samprit Chatterjee
Bertram Price
y
y=a+bx
x
A volume in the Wiley Series in Probability and Mathematical Statistics, Ralph A. Bradley,
J. Stuart Hunter, David G. Kendall, and Geoffrey S. Watson—Advisory Editors


Statistical Models
Theory and Practice
REVISED EDITION
David A. Freedman

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And there are other ways.

And software implements them as well.

# II: POLYNOMIAL REGRESSION

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Q: This represents a nonlinear relationship. Is it still a linear model?

A: Yes, because it's linear in the $\beta$'s!

"Although polynomial regression fits a *nonlinear* model to the data, as a statistical estimation problem it is *linear*, in the sense that the regression function E(y|x) is linear in the unknown parameters that are estimated from the data. For this reason, polynomial regression is considered to be a special case of multiple linear regression." -- Wikipedia

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But there is a problem with the model we've written down so far.

This model displays **collinearity**, which means the predictor variables are highly correlated with each other.

$$y = \alpha + \beta_1 x + \beta_2 x^2 + \dots + \beta_n x^n + \varepsilon$$

```
> x <- seq(1, 10, 0.1)
> cor(x^9, x^10)
[1] 0.9987608
```

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NOTE

For identical features, this results in a *singularity*. We will see an example of this in just a minute!

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OPTIONAL NOTE

These polynomial functions form an *orthogonal basis* of the function space.

# EXERCISES