GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

REGULARIZATION FOR REGRESSION

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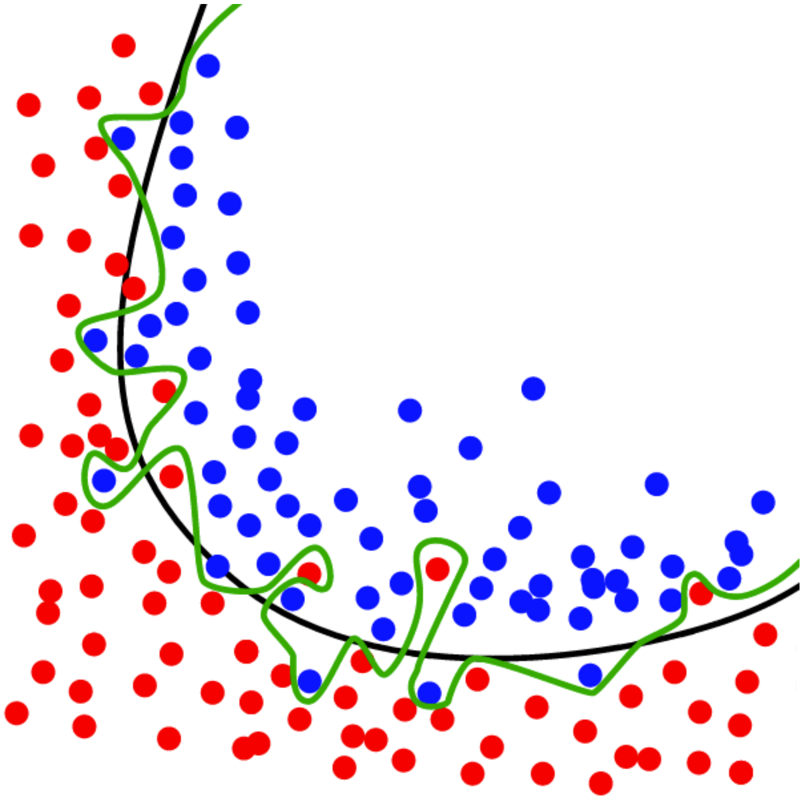
In other words, a model that is overfit has learned from the **noise** in the dataset instead of just the **signal**.

OVERFITTING EXAMPLE (CLASSIFICATION)



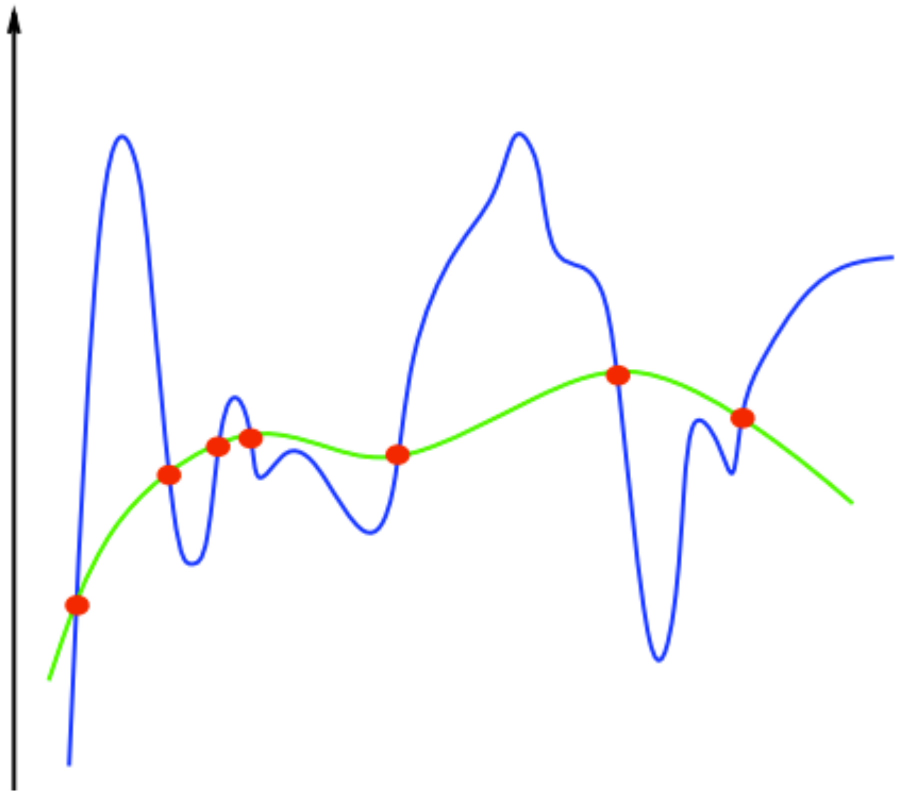
source: http://upload.wikimedia.org/wikipedia/commons/1/19/Overfitting.svg

The same thing can happen in regression.

It's possible to design a regression model that matches the noise in the data instead of the signal.

This happens when our model becomes *too complex* for the data to support.

OVERFITTING EXAMPLE (REGRESSION)



source: http://www.mit.edu/~9.520/spring12/slides/class02/class02.pdf

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Ex 1: $\Sigma |\beta_i|$

Ex 2: $\Sigma \beta_i^2$

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A: One method is to define complexity as a function of the size of the coefficients.

Ex 1: $\Sigma\,|\beta_i|$ this is called the **L1-norm**

Ex 2: $\Sigma\,\beta_i{}^2$ this is called the **L2-norm**

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Regularization refers to the method of preventing **overfitting** by explicitly controlling model **complexity**.

These regularization problems can also be expressed as:

L1 regularization: $min(\|y - x\beta\|^2 + \lambda\|x\|)$

L2 regularization: $min(\|y - x\beta\|^2 + \lambda\|x\|^2)$

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Lasso tends to eliminate coefficients, so it's useful for reducing the number of features. L2 tends to make coefficients small but not necessarily zero.

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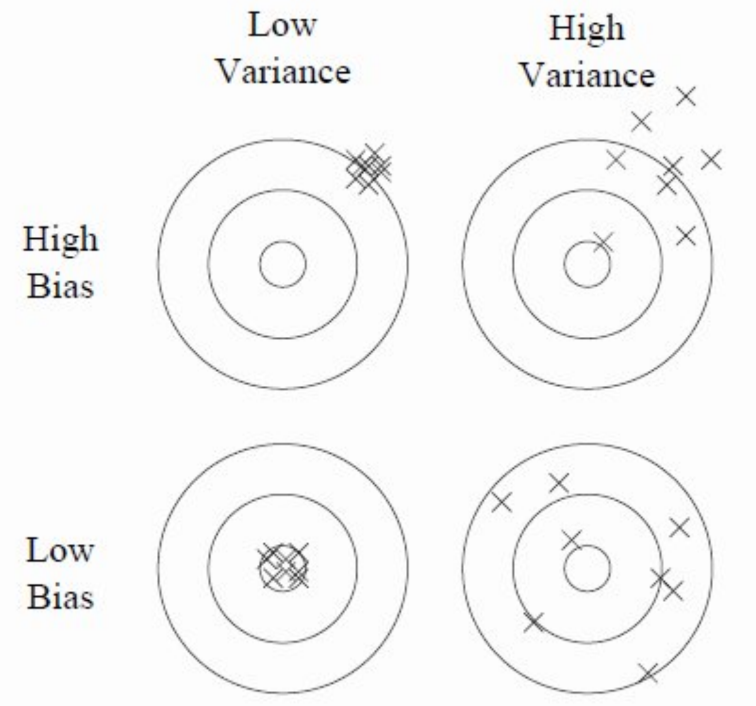

Figure 1: Bias and variance in dart-throwing.

source: http://homes.cs.washington.edu/~pedrod/papers/cacm12.pdf

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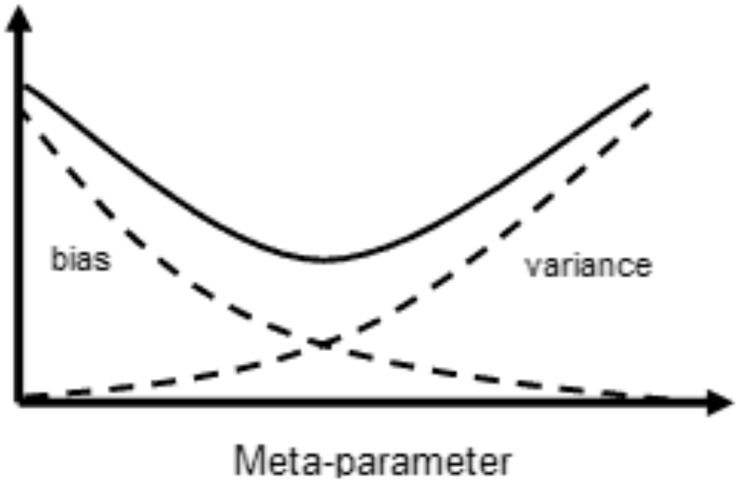
Generalization error can be decomposed into a bias component and variance component.

This is another example of the **bias-variance tradeoff**.



source: http://www.isu.edu/chem/images/kalivasmeta.gif

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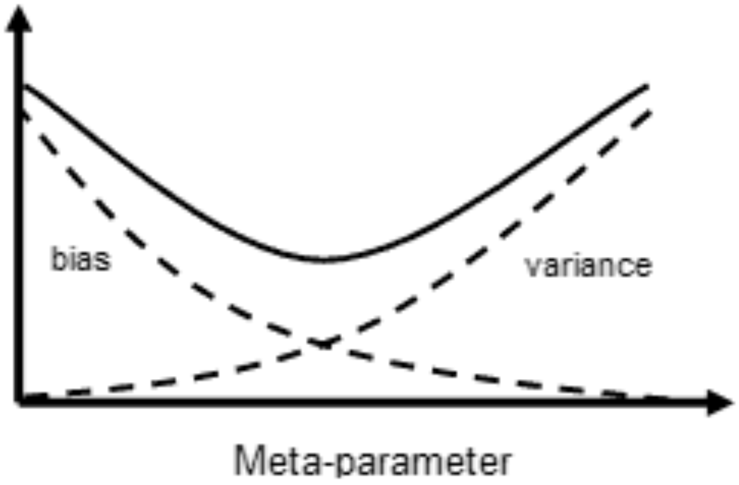

NOTE

The “meta-parameter” (or “hyperparameter”) here is the lambda we saw above.

source: *http://www.isu.edu/chem/images/kalivasmeta.gif*

This tradeoff is regulated by a **hyperparameter** λ, which we've already seen.

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So regularization represents a method to trade away some variance for a little bias in our model, thus achieving a better overall fit.

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NOTE

Combining the regularization terms (with a balancing parameter) we have *elastic net* regularization.

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