

INTRO to DATA SCIENCE

NAIVE BAYES CLASSIFICATION

I. INTRO TO PROBABILITY

II. NAÏVE BAYES CLASSIFICATION

LAB:

III. A SPAM FILTER

I. INTRO TO PROBABILITY

Q: What is a **probability**?

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The probability of event A is denoted $P(A)$.

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NOTE

The symbol \cap is often used for intersection. For example, " $P(A \cap B)$ ".

Q: Is $P(AB)$ equal to $P(A)P(B)$?

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A: Maybe, maybe not. More later...

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This information about B *transforms* the sample space.

Take a moment to convince yourself of this!

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This is called the **conditional probability** of A given B , written $P(A|B) = P(AB) / P(B)$.

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Notice, with this we can also write $P(AB) = P(A|B) * P(B)$.

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Using the definition of the conditional probability, we can also write:

$$P(A|B) = P(AB) / P(B) = P(A) \rightarrow P(AB) = P(A) * P(B)$$

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$$\rightarrow P(A|B) = P(B|A) * P(A) / P(B) \text{ by rearranging last step}$$

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Some facts:

- This is a simple algebraic relationship using elementary definitions.
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- It's a very powerful computational tool.

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The *frequentist interpretation* regards an event's probability as its limiting frequency across a very large number of trials.

The *Bayesian interpretation* regards an event's probability as a “degree of belief,” which can apply even to events that have not yet occurred.

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This a good direction to head if you like math and/or if you're interested in learning about cutting-edge data science techniques.

II. NAÏVE BAYES CLASSIFICATION

Suppose we have a dataset with features x_1, \dots, x_n and a class label c .
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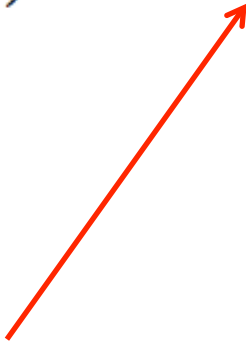
$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Bayes' theorem can help us to determine the probability of a record belonging to a class, *given* the data we observe.

Each term in this relationship has a name, and each plays a distinct role in any Bayesian calculation (including ours).

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **likelihood function**. It represents the joint probability of observing features $\{x_i\}$ given that that record belongs to class C .

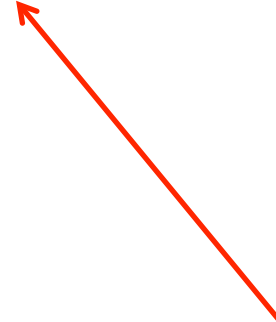
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We can approximate the value of the likelihood function from the training data.

This term is the **prior probability** of c . It represents the probability of a record belonging to class c before the data is taken into account.

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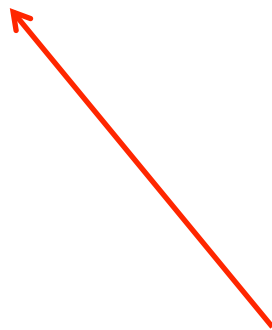
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The value of the prior is also observed from the data.

This term is the **normalization constant**. It doesn't depend on C , and is generally ignored until the end of the computation.

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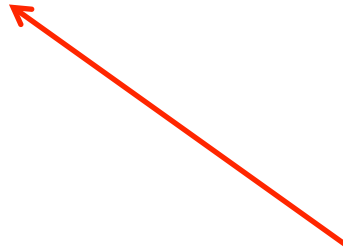
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The normalization constant doesn't tell us much.

This term is the **posterior probability** of c . It represents the probability of a record belonging to class c after the data is taken into account.

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The goal of any Bayesian computation is to find (“learn”) the posterior distribution of a particular variable.

The idea of Bayesian inference, then, is to **update** our beliefs about the distribution of C using the data (“evidence”) at our disposal.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Then we can use the posterior for prediction.

Methods

Predictions

“classical” (frequentist)

point estimates

Bayesian

distributions

Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

Remember the likelihood function?

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Observing this exactly would require us to have enough data for every possible combination of features to make a reasonable estimate.

Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

A: Estimating the full likelihood function.

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This “naïve” assumption simplifies the likelihood function to make it tractable.

III. SPAM FILTER

KEY OBJECTIVES

- preprocess data
- perform naïve Bayes classification

TOOLS

- tm, R