

自动控制原理第十四次作业

7-3

(1) 已知 $E(z) = \frac{10z}{(z-1)(z-2)}$

部分分式法: $\frac{E(z)}{z} = \frac{10}{(z-1)(z-2)} = \frac{10}{z-2} - \frac{10}{z-1}$

$\therefore E(z) = \frac{10z}{z-2} - \frac{10z}{z-1}$

$\therefore e(nT) = 10(2^n - 1)$

幂级数法: 长除法计算得

$E(z) = 10 + 10z^{-1} + 20z^{-2} + 70z^{-3} + \dots$

由数学归纳法可总结规律 $e(nT) = 10(2^n - 1)$

反演积分法 $\text{Res}_{z \rightarrow 1} E(z) = -10$

$\text{Res}_{z \rightarrow 2} E(z) = 10 \cdot 2^n$

$\therefore e(nT) = 10(2^n - 1)$

(2) 已知 $E(z) = \frac{-3z + z^{-1}}{1 - 2z^{-1} + z^{-2}} = \frac{-3z^2 + z}{z^2 - 2z + 1}$

部分分式法: $\frac{E(z)}{z} = \frac{1-3z}{(z-1)^2} = -\frac{2}{(z-1)^2} - \frac{3}{z-1}$

$\therefore E(z) = \frac{-2z}{(z-1)^2} - \frac{3z}{z-1}$

$\therefore e(nT) = -2n - 3$

幂级数法: 长除法计算得 $E(z) = -3 - 5z^{-1} - 7z^{-2} + \dots$

归纳总结得 $e(nT) = -2n - 3$

反演积分法:

$e(nT) = \text{Res}_{z \rightarrow 1} E(z) = \lim_{z \rightarrow 1} \frac{d}{dz} [(z-1)^2 E(z) \cdot z^{n-1}] = -2n - 3$

$$7-8 \quad (1) Z[c^*(t+2T)] = Z^2 C(z) - Z^2 c^*(0) - Z c^*(T) = Z^2 C(z)$$

$$Z[c^*(t+T)] = Z C(z) - Z c^*(0) = Z C(z)$$

$$Z[c^*(t)] = C(z) \quad Z[r^*(t)] = \frac{Z}{Z-1}$$

$$\therefore (Z^2 - 6Z + 8) C(z) = \frac{Z}{Z-1}$$

$$C(z) = \frac{Z}{(Z-1)(Z-2)(Z-4)}$$

$$\therefore c^*(nT) = \sum_{i=1}^3 \text{Res } C(z) = \lim_{Z \rightarrow 1} \frac{Z^n}{(Z-2)(Z-4)} + \lim_{Z \rightarrow 2} \frac{Z^n}{(Z-1)(Z-4)} + \lim_{Z \rightarrow 4} \frac{Z^n}{(Z-1)(Z-2)}$$

$$= \frac{1}{3} - 2^{n-1} - \frac{1}{6} 4^n$$

$$c^*(t) = \sum_{n=0}^{\infty} c^*(nT) \delta(t-nT)$$

$$(2) Z[c^*(t+2T)] = Z^2 C(z) - Z^2 c^*(0) - Z c^*(T) = Z^2 C(z) \quad Z[c^*(t)] = C(z)$$

$$Z[c^*(t+T)] = Z C(z) - Z c^*(0) = Z C(z) \quad Z[r^*(t)] = \frac{Z}{(Z-1)^2}$$

$$\therefore (Z+1)^2 C(z) = \frac{Z}{(Z-1)^2}$$

$$C(z) = \frac{Z}{(Z-1)^2 (Z+1)^2}$$

$$\therefore C(nT) = \text{Res } C(z) + \text{Res } C(z) = \lim_{Z \rightarrow 1} \frac{d}{dz} \left(\frac{Z^n}{(Z+1)^2} \right) + \lim_{Z \rightarrow -1} \frac{d}{dz} \left(\frac{Z^n}{(Z-1)^2} \right)$$

$$= \lim_{Z \rightarrow 1} \frac{n Z^{n-1} (Z+1)^2 - Z^n 2(Z+1)}{(Z+1)^4} + \lim_{Z \rightarrow -1} \frac{n Z^{n-1} (Z-1)^2 - Z^n 2(Z-1)}{(Z-1)^4}$$

$$= \frac{n-1}{4} + \frac{(-1)^{n-1} n - (-1)^{n-1}}{4} = \frac{1}{4} [1 + (-1)^{n-1}] (n-1)$$

$$c^*(t) = \sum_{n=0}^{\infty} C(nT) \delta(t-nT)$$

$$(3) Z[c(k+3)] = Z^3 C(z) - Z^3 c(0) - Z^2 c(1) - Z c(2) = Z^3 C(z) - Z^3 - Z^2$$

$$Z[c(k+2)] = Z^2 C(z) - Z^2 c(0) - Z c(1) = Z^2 C(z) - Z^2 - Z$$

$$Z[c(k+1)] = Z C(z) - Z c(0) = Z C(z) - Z$$

$$\therefore (z^3 + 6z^2 + 11z + 6)C(z) - z^3 - 7z^2 - 17z = 0$$

$$\therefore C(z) = \frac{z^3 + 7z^2 + 17z}{(z+1)(z+2)(z+3)}$$

$$\therefore C^*(n) = \underset{z \rightarrow -1}{\text{Res}} C(z) + \underset{z \rightarrow -2}{\text{Res}} C(z) + \underset{z \rightarrow -3}{\text{Res}} C(z)$$

$$= \frac{11}{2}(-1)^n - 7(-2)^n + \frac{5}{2}(-3)^n$$

$$C^*(t) = \sum_{n=0}^{\infty} C^*(n) \delta(t-n)$$

7-10

(a) 列出信号方程

$$E_1(s) = R(s) - G_3(s) C^*(s) \quad ①$$

$$C(s) = (E_1^*(s) - E_2^*(s)) G_1(s) \quad ② \quad \text{把②代入①③得}$$

$$E_2(s) = G_2(s) C(s) \quad ③$$

$$E_1(s) = R(s) - G_3(s) E_1^*(s) G_1^*(s) + G_3(s) E_2^*(s) G_1^*(s) \quad ④$$

$$E_2(s) = G_2(s) G_1(s) E_1^*(s) - G_2(s) G_1(s) E_2^*(s) \quad ⑤$$

$$\downarrow \quad E_1^*(s) = \frac{G_1 G_2^*(s) \cdot E_1^*(s)}{1 + G_1 G_2^*(s)} \quad E_2^*(s) = \frac{(1 + G_1^*(s) G_3^*(s)) E_1^*(s) - R^*(s)}{G_1^*(s) G_2^*(s)}$$

$$E_1^*(s) = \frac{1 + G_1 G_2^*(s)}{1 + G_1 G_2^*(s) + G_1^*(s) G_3^*(s)} R^*(s)$$

$$\therefore C^*(s) = (E_1^*(s) - E_2^*(s)) G_1^*(s) = \frac{G_1^*(s) R^*(s)}{1 + G_1 G_2^*(s) + G_1^*(s) G_3^*(s)}$$

$$\therefore G(z) = \frac{C(z)}{R(z)} = \frac{G_1(z)}{1 + G_1 G_2(z) + G_1(z) G_3(z)}$$

(b) 列出信号方程

$$E(s) = G(s) R(s) - C(s) \quad ①$$

$$C(s) = G_4(s) [G_2(s) R(s) + E^*(s) G_1(s) G_3(s)] \quad (2)$$

把①代入②得

$$C^*(s) = G_2 G_4 R^*(s) + G_4 G_3^*(s) G_1 R^*(s) - G_4 G_3^*(s) C^*(s)$$

$$\therefore C(z) = \frac{G_2 G_4 R(z) + G_4 G_3^*(z) G_1 R(z)}{1 + G_4 G_3 G_4(z)}$$

(c) 列出信号方程

① 先令 $R(s) = 0$, 求 $C(z)$ 的值

$$C(s) = G_2(s) [N(s) - G_1(s) G_3(s) D(z) C^*(s)]$$

$$\therefore C^*(s) = \frac{G_2 N^*(s)}{1 + G_1 G_3^*(s) D(z)}$$

$$C(z) = \frac{G_2 N(z)}{1 + G_1 G_3(z) D(z)}$$

② 再令 $N(s) = 0$, 求 $C(z)$ 的值

$$C(s) = G_1(s) G_1(s) G_2(s) B_2^*(s) \quad (1)$$

$$B_2^*(s) = D(z) E^*(s) + D_2(z) R^*(s) \quad (2)$$

$$E^*(s) = R^*(s) - C^*(s) \quad (3) \quad (R^*(s) - C^*(s))$$

$$\therefore \text{把③代入①得 } C^*(s) = G_1 G_1 G_2^*(s) [D(z) + D_2(z) R^*(s)]$$

$$\therefore [1 + G_1 G_1 G_2^*(s) D(z)] C^*(s) = [G_1 G_1 G_2^*(s) D(z) + G_1 G_1 G_2^*(s) D_2(z)] R^*(s)$$

$$\therefore C(z) = \frac{G_1 G_1 G_2(z) [D(z) + D_2(z)] R(z)}{1 + G_1 G_1 G_2(z)}$$

$$\therefore C(z) = \frac{G_2 N(z)}{1 + G_1 G_3(z) D(z)} + \frac{G_1 G_1 G_2(z) [D(z) + D_2(z)] R(z)}{1 + G_1 G_1 G_2(z)}$$