

第十一次作业

5-14

(1) 已知 $G(s) = \frac{K}{(Ts+1)(Ts+1)(Ts+1)}$ $\therefore p=0$

由图(1)可知 $N_- = 1$ $N_+ = 0$

$\therefore Z = P - R = P - 2(N_+ - N_-) = 2$

\therefore 该系统在右半平面闭环极点个数为2, 闭环系统不稳定

(2) 已知 $G(s) = \frac{K}{s(Ts+1)(Ts+1)}$ 易得 $p=0$

由图2可知 $N_+ = 0$ $N_- = 0$

$\therefore Z = P - R = P - 2(N_+ - N_-) = 0$

\therefore 由奈氏判据可知闭环系统稳定

(3) 已知 $G(s) = \frac{K}{s^2(Ts+1)}$ 易得 $p=0$

由图3可知 $N_+ = 0$ $N_- = 1$

$\therefore Z = P - R = P - 2(N_+ - N_-) = 2$

\therefore 由奈氏判据可知闭环系统不稳定

S右半面闭环极点数为2

(4) 已知 $G(s) = \frac{K(1s+1)}{s^2(Ts+1)}$ 易得 $p=0$

\therefore 由图4可知 $N_+ = 0$ $N_- = 0$

$\therefore Z = P - R = P - 2(N_+ - N_-) = 0$

\therefore 由奈氏判据可知闭环系统稳定

(5) 已知 $G(s) = \frac{K}{s^3}$ 易得 $p=0$

由图5可知 $N_+ = 0$ $N_- = 1$

$\therefore Z = P - R = P - 2(N_+ - N_-) = 2$

\therefore 由奈氏判据可知闭环系统不稳定

S右半平面有两个闭环极点

(6) 已知 $G(s) = \frac{K(Ts+1)(Ts+1)}{s^3}$ 易得 $p=0$

由图(6)可知 $N_+ = 1$ $N_- = 1$

$\therefore Z = P - R = P - 2(N_+ - N_-) = 0$

\therefore 由奈氏判据可知闭环系统稳定

(7) 已知 $G(s) = \frac{K(Ts+1)(Ts+1)}{s \prod_{i=1}^n (Ts+1)}$ 易得 $p=0$

由图(7)可知 $N_+ = 1$ $N_- = 1$

$\therefore Z = P - R = P - 2(N_+ - N_-) = 0$

\therefore 由奈氏判据可知闭环系统稳定

(8) 已知 $G(s) = \frac{K}{(Ts-1)}$ 易得 $p=1$

由图(8)得 $N_+ = \frac{1}{2}$ $N_- = 0$

$\therefore Z = P - R = P - 2(N_+ - N_-) = 0$

\therefore 由奈氏判据可知闭环系统稳定

(9) 已知 $G(s) = \frac{-K}{-Ts+1} = \frac{K}{Ts-1}$ 易得 $P=1$

由图9可知 $N_+ = N_- = 0$

$\therefore Z = P - 2(N_+ - N_-) = 1$

\therefore 奈氏判据可知闭环系统不稳定

s 右半平面闭环极点数为1

(10) 已知 $G(s) = \frac{K}{s(Ts-1)}$ 易得 $P=1$

由图10可得 $N_+ = 0$ $N_- = \frac{1}{2}$

$\therefore Z = P - 2(N_+ - N_-) = 2$

\therefore 奈氏稳定判据可知系统不稳定

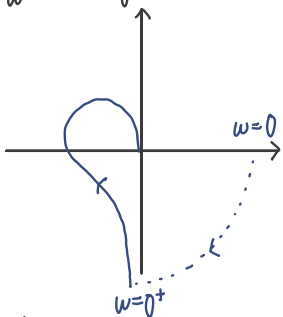
s 右半平面闭环极点数为2

5-16 (1) 已知 $G(s) = \frac{K}{s(2s+1)(s+1)}$ 易得 $P=0$

$|G(j\omega)| = \frac{K}{\sqrt{\omega^2+1} \sqrt{\omega^2+1} \cdot \omega}$

$G(s)$ 奈氏曲线如下

$\angle G(j\omega) = -\frac{\pi}{2} - \arctan \omega - \arctan \omega = -\frac{\pi}{2} - \arctan \frac{\frac{3}{2}\omega}{1 - \frac{1}{2}\omega^2}$



易得 $\omega = \frac{\sqrt{2}}{2}$ 时 $\angle G(j\omega) = -\pi$

此时 $|G(j\omega)| = \frac{K}{\sqrt{2} \sqrt{\frac{3}{2}} \frac{\sqrt{2}}{2}} = \frac{2}{3}K$

想让 $G(s)$ 闭环稳定 则 $\frac{2}{3}K < 1$ 即 $K < \frac{3}{2}$

$\therefore K$ 值范围为 $K \in (0, \frac{3}{2})$

(2) 已知 $G(s) = \frac{10}{s(Ts+1)(s+1)}$ 易得 $P=0$

$|G(j\omega)| = \frac{10}{\omega \sqrt{T^2\omega^2+1} \cdot \sqrt{\omega^2+1}}$ $\angle G(j\omega) = -\frac{\pi}{2} - \arctan T\omega - \arctan \omega = -\frac{\pi}{2} - \arctan \frac{(T+1)\omega}{1 - T\omega^2}$

奈氏曲线大致如上

易得 $\omega = \frac{1}{\sqrt{T}}$ 时 $\angle G(j\omega) = -\pi$, 此时 $|G(j\omega)| = \frac{10\sqrt{T}}{\sqrt{\frac{1}{T}+1} \cdot \sqrt{\frac{1}{T}+1}}$

想让 $G(s)$ 闭环稳定 则 $|G(j\omega)| < 1$

解得 $T < \frac{1}{9}$

$\therefore T$ 的取值范围为 $(0, \frac{1}{9})$

(3) 易得 $p=0$

同(2)可得 $w=\frac{1}{T}$ 时, $|G(jw)| = \frac{KT}{\sqrt{T+1}\sqrt{\frac{T}{T+1}+1}}$

欲使 $|G(jw)| < 1$, 可得 $0 < K < \frac{T+1}{T}$, $0 < T < \frac{1}{K-1}$

5-19 已知 $G(s) = \frac{Ke^{-0.8s}}{s+1}$ 易得 $p=0$

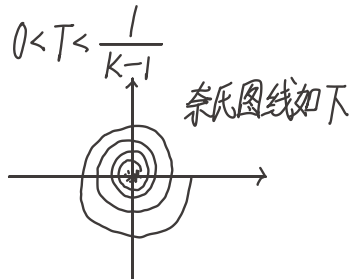
$\therefore |G(jw)| = \frac{K}{\sqrt{w^2+1}}$ $\angle G(jw) = -0.8w - \arctan w$

为了系统稳定, 要求 $N_- = 0$

$\therefore \begin{cases} -0.8w - \arctan w = -\pi \\ |G(jw)| < 1 \end{cases}$

解得 $K < 2.65$ $w \approx 2.45$

$\therefore K \in (0, 2.65)$



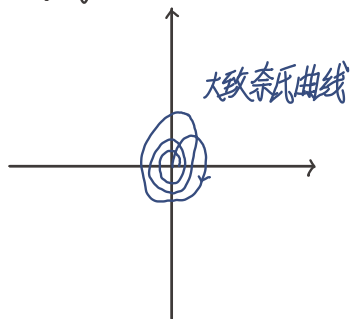
5-20 已知 $G(s) = \frac{5s^2 e^{-\tau s}}{(s+1)^4}$ 易得 $p=0$

$\therefore |G(jw)| = \frac{5w^2}{(1+w^2)^2}$ $\angle G(jw) = \pi - \tau w - 4\arctan w$

为了系统稳定, 要求 $N_- = 0$

$\therefore \begin{cases} \pi - \tau w - 4\arctan w = -\pi \\ \frac{5w^2}{(1+w^2)^2} < 1 \end{cases}$ 解得 $w \approx 1.618$
 $\tau < 1.37$

$\therefore \tau$ 的取值范围为 $(0, 1.37)$



5-21 已知 $G(s) = \frac{as+1}{s^2}$

$|G(jw)| = \frac{\sqrt{a^2 w^2 + 1}}{w^2} = 1$

$\angle G(jw) = -\pi + \arctan aw = \frac{\pi}{4} - \pi$

解得 $a = \frac{1}{\sqrt{2}}$ $w = \sqrt{2}$

\therefore 相角裕度为 45° 时 a 的值为 $\frac{1}{\sqrt{2}}$