

自动控制原理第十四次作业

7-3

$$(1) \text{ 已知 } E(z) = \frac{10z}{(z-1)(z-2)}$$

$$\text{部分分式法: } \frac{E(z)}{z} = \frac{10}{(z-1)(z-2)} = \frac{10}{z-2} - \frac{10}{z-1}$$

$$\therefore E(z) = \frac{10z}{z-2} - \frac{10z}{z-1}$$

$$\therefore e(nT) = 10(2^n - 1)$$

幂级数法: 长除法计算得

$$E(z) = 10 + 10z^{-1} + 30z^{-2} + 70z^{-3} \dots$$

由数学归纳法可总结规律 $e(nT) = 10(2^n - 1)$

$$\text{反演积分法 } \underset{z \rightarrow 1}{\text{Res}} E(z) = -10$$

$$\underset{z \rightarrow 2}{\text{Res}} E(z) = 10 \cdot 2^n \quad \therefore e(nT) = 10(2^n - 1)$$

$$(2) \text{ 已知 } E(z) = \frac{-3z+2^{-1}}{1-2z^{-1}+z^{-2}} = \frac{-3z^2+z}{z^2-2z+1}$$

$$\text{部分分式法: } \frac{E(z)}{z} = \frac{1-3z}{(z-1)^2} = -\frac{2}{(z-1)^2} - \frac{3}{z-1}$$

$$\therefore E(z) = \frac{-2z}{(z-1)^2} - \frac{3z}{z-1}$$

$$\therefore e(nT) = -2n - 3$$

幂级数法: 长除法计算得 $E(z) = -3 - 5z^{-1} - 7z^{-2} \dots$

归纳总结得 $e(nT) = -2n - 3$

反演积分法:

$$e(nT) = \underset{z \rightarrow 1}{\text{Res}} E(z) = \lim_{z \rightarrow 1} \frac{d}{dz} [(z-1)^2 E(z)] \cdot z^{n-1} = -2n - 3$$

$$7-8 \quad (1) Z[C^*(t+2)] = Z^3(C(z)) - Z^2 C^*(0) - Z C^*(1) = Z^3(C(z))$$

$$Z[C^*(t+1)] = ZC(z) - ZC^*(0) = ZC(z)$$

$$Z[C^*(t)] = C(z) \quad Z[r^*(t)] = \frac{z}{z-1}$$

$$\therefore (z^2 - 6z + 8)C(z) = \frac{z}{z-1}$$

$$C(z) = \frac{z}{(z-1)(z-2)(z-4)}$$

$$\therefore C^*(nT) = \sum_{i=1}^3 \text{Res}(C(z)) = \lim_{z \rightarrow 1} \frac{z^n}{(z-2)(z-4)} + \lim_{z \rightarrow 2} \frac{z^n}{(z-1)(z-4)} + \lim_{z \rightarrow 4} \frac{z^n}{(z-1)(z-2)}$$

$$= \frac{1}{3} - 2^{n-1} - \frac{1}{6} 4^n$$

$$C^*(t) = \sum_{n=0}^{\infty} C^*(nT) \delta(t-nT)$$

$$(2) Z[C^*(t+2)] = Z^3(C(z)) - Z^2 C^*(0) - Z C^*(1) = Z^3(C(z)) \quad Z[C^*(t)] = C(z)$$

$$Z[C^*(t+1)] = ZC(z) - ZC^*(0) = ZC(z) \quad Z[r^*(t)] = \frac{z}{(z-1)^2}$$

$$\therefore (z+1)^2 C(z) = \frac{z}{(z-1)^2}$$

$$C(z) = \frac{z}{(z-1)^2 \cdot (z+1)^2}$$

$$\therefore C(nT) = \text{Res}(C(z)) + \text{Res}(C(z)) = \lim_{z \rightarrow 1} \frac{d}{dz} \left(\frac{z^n}{(z+1)^2} \right) + \lim_{z \rightarrow -1} \frac{d}{dz} \left(\frac{z^n}{(z-1)^2} \right)$$

$$= \lim_{z \rightarrow 1} \frac{n z^{n-1} (z+1)^2 - z^n 2(z+1)}{(z+1)^4} + \lim_{z \rightarrow -1} \frac{n z^{n-1} (z-1)^2 - z^n 2(z-1)}{(z-1)^4}$$

$$= \frac{n-1}{4} + \frac{(-1)^{n-1} - (-1)^{n-1}}{4} = \frac{1}{4} [1 + (-1)^{n-1}] (n-1)$$

$$C^*(t) = \sum_{n=0}^{\infty} C(nT) \delta(t-nT)$$

$$(3) Z[C(k+3)] = Z^3(C(z)) - Z^2 C(0) - Z^1 C(1) - Z C(2) = Z^3(C(z)) - Z^3 - Z^2$$

$$Z[C(k+2)] = Z^2(C(z)) - Z^1 C(0) - Z C(1) = Z^2(C(z)) - Z^2 - Z$$

$$Z[C(k+1)] = Z(C(z)) - Z C(0) = Z(C(z)) - Z$$

$$(z^3 + 6z^2 + 11z + 6)(z) - z^3 - 7z^2 - 17z = 0$$

$$\therefore C(z) = \frac{z^3 + 7z^2 + 17z}{(z+1)(z+2)(z+3)}$$

$$\therefore C(n) = \underset{z \rightarrow -1}{\text{Res}}(C(z)) + \underset{z \rightarrow -2}{\text{Res}}(C(z)) + \underset{z \rightarrow -3}{\text{Res}}(C(z))$$

$$= \frac{1}{2}(-1)^n - 7(-2)^n + \frac{5}{2}(-3)^n$$

$$C^*(t) = \sum_{n=0}^{\infty} C^*(n) \delta(t-n)$$

7-10

(a) 列出信号方程

$$E(s) = R(s) - G_3(s) C^*(s) \quad ①$$

$$C(s) = (E_1^*(s) - E_2^*(s)) G_1(s) \quad ② \quad \text{把} ② \text{代入} ① ③ \text{得}$$

$$E_2(s) = G_2(s) C(s) \quad ③$$

$$E_1(s) = R(s) - G_3(s) E_1^*(s) G_1^*(s) + G_3(s) E_2^*(s) G_1^*(s) \quad ④$$

$$E_1(s) = G_3(s) G_1(s) E_1^*(s) - G_3(s) G_1(s) E_2^*(s) \quad ⑤ \quad \downarrow$$

$$E_2^*(s) = \frac{G_3 G_1^*(s) \cdot E_1^*(s)}{1 + G_3 G_1^*(s)} \quad \downarrow \quad E_2^*(s) = \frac{(1 + G_1^*(s) G_3^*(s)) E_1^*(s) - R(s)}{G_1^*(s) G_3^*(s)}$$

$$E_1^*(s) = \frac{1 + G_1 G_3^*(s)}{1 + G_1 G_3^*(s) + G_1^*(s) G_3^*(s)} R^*(s)$$

$$\therefore C^*(s) = (E_1^*(s) - E_2^*(s)) G_1^*(s) = \frac{G_1^*(s) R^*(s)}{1 + G_1 G_3^*(s) + G_1^*(s) G_3^*(s)}$$

$$\therefore G(z) = \frac{C(z)}{R(z)} = \frac{G_1(z)}{1 + G_1 G_3(z) + G_1(z) G_3(z)}$$

(b) 列出信号方程

$$E(s) = G(s) R(s) - C(s) \quad ①$$

$$C(s) = G_4(s) \left[G_1(s) R(s) + E^*(s) G_1(s) G_3(s) \right] \quad ②$$

把①代入②得

$$\begin{aligned} C^*(s) &= G_2 G_4 R^*(s) + G_h G_2^*(s) G_1 R^*(s) - G_h G_2^*(s) \frac{G_1}{G_4} C^*(s) \\ \therefore C(z) &= \frac{G_2 G_4 R(z) + G_h G_2^*(z) G_1 R(z)}{1 + G_h G_2 G_4(z)} \end{aligned}$$

(c) 列出信号方程

① 先令 $R(s)=0$, 求 $C(z)$ 的值

$$C(s) = G_2(s) \left[N(s) - G_h(s) G_1(s) D(z) C^*(s) \right]$$

$$\therefore C^*(s) = \frac{G_2 N^*(s)}{1 + G_1 G_2 \frac{G_h^*(s)}{G_h} D(z)}$$

$$C(z) = \frac{G_2 N(z)}{1 + G_1 G_2 D(z) D(z)}$$

② 再令 $N(s)=0$, 求 $C(z)$ 的值

$$C(s) = G_h(s) G_1(s) G_2(s) B_2^*(s) \quad ①$$

$$B_2^*(s) = D(z) E^*(s) + D_b(z) R^*(s) \quad ②$$

$$E^*(s) = R^*(s) - C^*(s) \quad ③ \qquad (R^*(s) - C^*(s))$$

$$\therefore \text{把 } ③ \text{ 代入 } ① \text{ 得 } C(z) = G_h G_1 G_2^*(z) \left[D(z) + D_b(z) R^*(z) \right]$$

$$\therefore [1 + G_h G_1 G_2^*(z) D(z)] C^*(z) = [G_h G_1 G_2^*(z) D(z) + G_h G_1 G_2^*(z) D_b(z)] R^*(z)$$

$$\therefore C(z) = \frac{G_h G_1 G_2(z) [D(z) + D_b(z)] R(z)}{1 + G_h G_1 G_2(z)}$$

$$\therefore C(z) = \frac{G_2 N(z)}{1 + G_1 G_2 D(z)} + \frac{G_h G_1 G_2(z) [D(z) + D_b(z)] R(z)}{1 + G_h G_1 G_2(z)}$$