

自动控制原理第10次作业

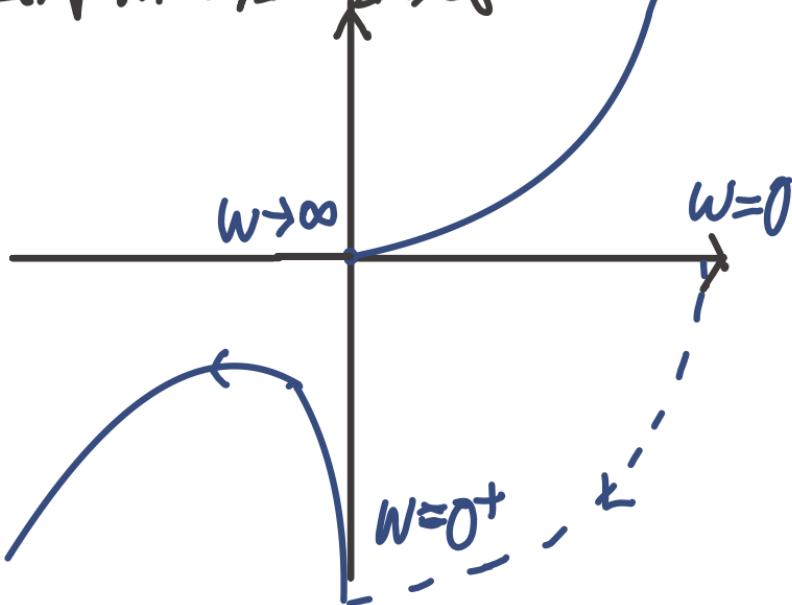
5-9 已知 $G(s)H(s) = \frac{10}{s(s+1)(\frac{s^2}{4} + 1)}$

易得 $|G(j\omega)H(j\omega)| = \frac{10}{\omega\sqrt{\omega^2 + 1}\sqrt{1 - \frac{\omega^4}{16}}}$

$\angle G(j\omega)H(j\omega) = -\frac{\pi}{2} - \arctan \omega \quad (\omega < 2)$

$\frac{\pi}{2} - \arctan \omega \quad (\omega > 2)$

试绘制开环幅相曲线



$$5-11 (1) G(s) = \frac{2}{(2s+1)(8s+1)}$$

分析各个环节

$$\frac{1}{2s+1} \text{ 转折点 } \omega = \frac{1}{2} \text{ 斜率 } -20\text{dB}$$

$$\frac{1}{8s+1} \text{ 转折点 } \omega = \frac{1}{8} \text{ 斜率 } -40\text{dB}$$

代入初值 $\omega=0$, 得 $L(0)=20\lg 2$

$$\therefore L\left(\frac{1}{8}\right) = L(0) = 20\lg 2$$

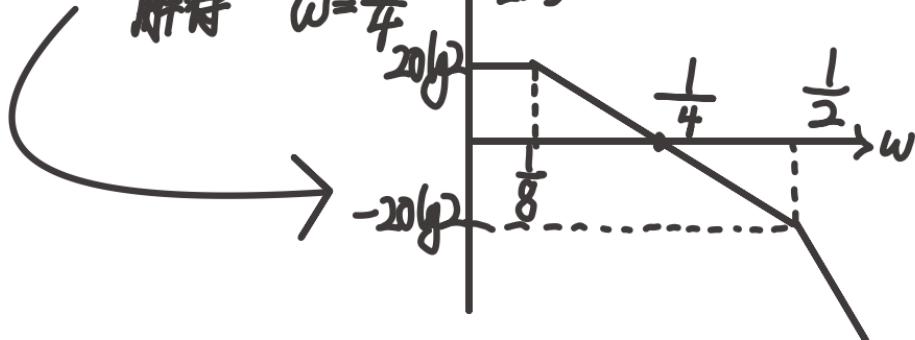
$$L\left(\frac{1}{2}\right) = L\left(\frac{1}{8}\right) - 20\left(\lg \frac{1}{2} - \lg \frac{1}{8}\right) = -20\lg 2$$

截止频率 ω_0 处于 $\frac{1}{8}$ 和 $\frac{1}{2}$ 之间

$$\therefore L(\omega_0) = L\left(\frac{1}{8}\right) - 20\left(\lg \omega_0 - \lg \frac{1}{8}\right) = 0$$

解得

$$\omega = \frac{1}{4}$$



$$(2) G(s) = \frac{200}{s^2(s+1)(10s+1)}$$

分析幅频特性

取初值

ω 取值 斜率

$(0, \frac{1}{10})$ -40dB

$(\frac{1}{10}, 1)$ -60dB

$(1, +\infty)$ -80dB

代入 $\omega=1$, 得

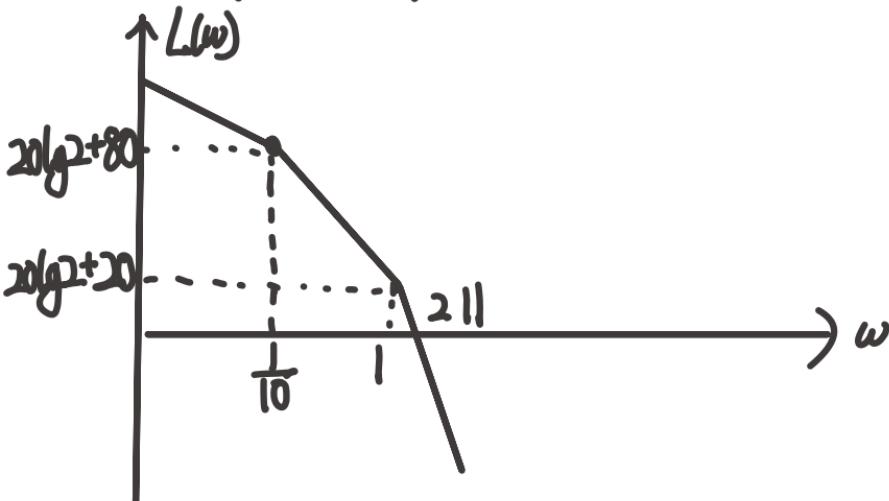
$$L(1) \approx 20 \lg 20 = 20 \lg 2 + 20$$

$$\therefore L(\frac{1}{10}) = L(1) + 60(\lg 1 - \lg \frac{1}{10}) = 20 \lg 2 + 80$$

可知 ω_0 在 $(1, +\infty)$ 段

$$L(\omega_0) = L(1) - 80(\lg \omega_0 - \lg 1) = 0$$

$$\text{解得 } \omega_c = \sqrt[4]{20} \approx 2.11$$



$$(3) \text{ 已知 } G(s) = \frac{8(10s+1)}{s(s^2+s+1)(\frac{s}{2}+1)}$$

分析幅频特性曲线

ω 取值

直线斜率

求极值, 代入 $\omega=1$

$$(0, \frac{1}{10})$$

$$-20dB$$

$$\begin{aligned} \text{得 } L(1) &= 20 \lg \frac{8 \times 10}{1 \times 1 \times 1} \\ &= 60 \lg 2 + 20 \end{aligned}$$

$$(\frac{1}{10}, 1)$$

$$0dB$$

$$(1, 2)$$

$$-40dB$$

$$(2, +\infty)$$

$$-60dB$$

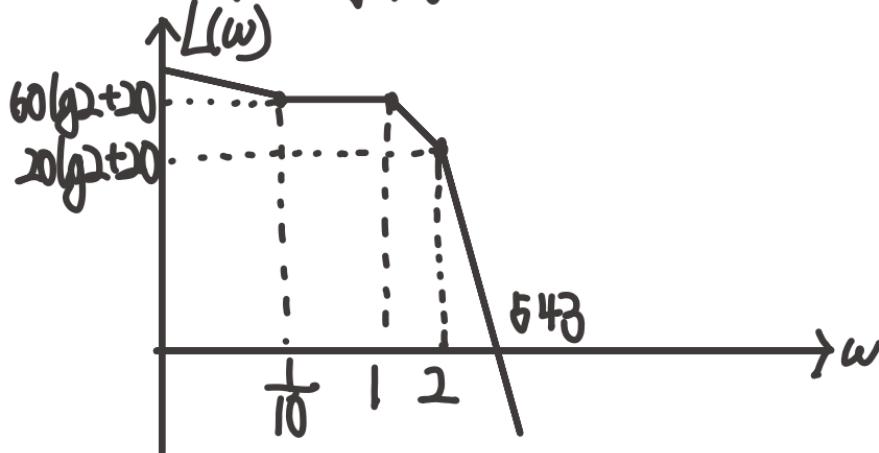
$$\therefore L(\frac{1}{10}) = L(1) = 60 \lg 2 + 20$$

$$L(2) = L(1) - 40(\lg 2 - \lg 1) = 20 \lg 2 + 20$$

可知截止频率 ω_c 位于 $(2, +\infty)$ 之间

$$\therefore L(\omega_c) = L(2) - 60(\lg \omega_c - \lg 2) = 0$$

$$\text{解得 } \omega_c = \sqrt[3]{160} \approx 5.43$$



$$(4) G(s) = \frac{10(\frac{s^2}{400} + \frac{s}{10} + 1)}{s(s+1)(\frac{s}{10} + 1)}$$

分析幅频特性曲线

ω 取值 直线斜率

(0, 0)	-20dB
(0.1, 1)	-40dB
(1, 20)	-60dB
(20, 0)	-20dB

取初值 代入 $\omega = 1$

$$\text{得 } L(1) = 20 \lg \frac{10 \times 1}{1 \times 1 \times 10} = 0$$

$$L(0.1) = L(1) + 40(\lg 1 - \lg 0.1) = 40$$

$$L(20) = L(1) - 60(\lg 20 - \lg 1) = -60(\lg 2 - 60)$$

