Homework 03

Submission Notices:

- *Conduct your homework by filling answers into the placeholders in this file (in Microsoft Word format). Questions are shown in black color, instructions/hints are shown in italics and blue color, and your content should use any color that is different from those.*
- *After completing your homework, prepare the file for submission by exporting the Word file (filled with answers) to a PDF file, whose filename follows the following format,*
  <StudentID-1>_<StudentID-2>_HW01.pdf (Student IDs are sorted in ascending order)
  E.g., ***2112001_2112002_HW02.pdf***
  and then submit the file to Moodle directly WITHOUT any kinds of compression (.zip, .rar, .tar, etc.).
- *Note that you will get zero credit for any careless mistake, including, but not limited to, the following things.*
  1. *Wrong file/filename format, e.g., not a pdf file, use "-" instead of "_" for separators, etc.*
  2. *Disorder format of problems and answers*
  3. *Conducted not in English*
  4. *Cheating, i.e., copying other students' works or letting other students copy your work.*

Problem 1. (2pts) Consider a first-order knowledge base that describes worlds containing people, songs, albums (e.g., "Meet the Beatles") and disks (i.e., particular physical instances of CDs). The vocabulary contains the following symbols:

- CopyOf(d, a): Disk d is a copy of album a
- Owns(p, d):Person p owns disk d
- Sings(p, s, a): Album a includes a recording of song s sung by person p
- Wrote(p, s): Person p wrote song s

Express the following statements in first-order logic using only the given vocabulary:

1. Either Gershwin or McCartney wrote "The Man I Love."
2. Joe owns a copy of *Revolver*.
3. Every song that McCartney sings on *Revolver* was written by McCartney.
4. Gershwin did not write any of the songs on *Revolver*.
5. Every song that Gershwin wrote has been recorded on some album. (Possibly different songs are recorded on different albums.)
6. There is a single album that contains every song that Joe has written
7. Joe owns a copy of an album that has Billie Holiday singing "The Man I Love."
8. Joe owns a copy of every album that has a song sung by McCartney. (Of course, each different album is instantiated in a different physical CD.)

Please write your answer to the following table.

No.	Score	Original statements and FOL sentences
1.	0.25pt	Either Gershwin or McCartney wrote "The Man I Love."
		$Wrote(Gershwin, \text{“The Man I Love”}) \lor Wrote(McCartney, \text{“The Man I Love”})$
2.	0.25pt	Joe owns a copy of *Revolver*.

		$\exists d(CopyOf(d, Revolver) \wedge Owns(Joe, d))$
3.	0.25pt	Every song that McCartney sings on *Revolver* was written by McCartney.
		$\forall s(Sings(McCartney, s, Revolver) \Rightarrow Wrote(McCartney, s))$
4.	0.25pt	Gershwin did not write any of the songs on *Revolver*.
		$\forall s \forall p\ (Sings(p, s, Revolver) \Rightarrow \neg Wrote(Gershwin, s))$
5.	0.25pt	Every song that Gershwin wrote has been recorded on some album. (Possibly different songs are recorded on different albums.)
		$\forall s(Wrote(Gershnwin, s) \Rightarrow \exists p \exists a(Sings(p, s, a))$
6.	0.25pt	There is a single album that contains every song that Joe has written
		$\exists a(\forall p \forall s(Sings(p, s, a) \Rightarrow Wrote(Joe, s)))$
7.	0.25pt	Joe owns a copy of an album that has Billie Holiday singing "The Man I Love."
		$\exists a \exists d(Sings(Billiie\ Holiday, The\ Man\ I\ Love, a) \wedge CopyOf(d, a) \wedge Owns(Joe, d))$
8.	0.25pt	Joe owns a copy of every album that has a song sung by McCartney. (Of course, each different album is instantiated in a different physical CD.)
		$\forall a(\exists s(Sings(McCartney, s, a)) \Rightarrow \exists d\ (CopyOf(d, a) \wedge Owns(Joe, d))$

Problem 2. (1pt) Consider a vocabulary with the following symbols:

Dog(x) *predicate "x is a dog"*   Cat(x) *predicate "x is a cat"*

Cute(x) *predicate "x is cute"*   Scary(x) *predicate "x is scary"*

Owns(x, y) *predicate "x owns y"*   Bites*(x,y) predicate "x bites y"*

Translate the following FOL sentences to English.

Please fill your answers in the table below.

No.	Score	Original FOL sentences and English statements
1.	0.25pt	$\exists$x Owns(Joe, x) $\wedge$ Dog(x) $\wedge$ Cute(x)
		Joe owns some dogs that are cute
2.	0.25pt	$\neg$($\exists$x Owns(Joe, x) $\wedge$ Dog(x)) $\Rightarrow$ Scary(Joe)
		If Joe does not own a dog then Joe is Scary
3.	0.25pt	$\forall$x Dog(x) $\wedge$ (($\exists$y Dog(y) $\wedge$ (x $\neq$ y) $\wedge$ Bites(x,y)) $\vee$ (($\exists$z Cat(z) $\wedge$ Bites(x,z))) $\Rightarrow$ $\neg$ Cute(x)
		Every dog that bites another dog or cat is not cute
4.	0.25pt	$\exists$x Dog(x) $\wedge$ $\neg$(Scary(x) $\wedge$ Cute(x))
		There exists a dog that is not both scary and cute

Problem 3 (2pts) Minesweeper is a single-player computer game to clear a minefield without detonating a mine. Each square can be cleared, or uncovered, by clicking on it. If a square that contains a mine is clicked, the game is over. If the square does not contain a mine, one of two things can happen:

(1) A number between 1 and 8 appears indicating the number of adjacent squares containing mines, or

(2) no number appears; in which case there are no mines in the adjacent cells.

Given some the predicates to formalize the fact that:

1. A unary predicate **mine**, where mine(x) means that the cell x contains a mine
2. A binary predicate **adj**, where adj(x, y) means that the cell x is adjacent to the cell y
3. A binary predicate **contains**, where contains(x, n) means the cell x contains the number n mine.

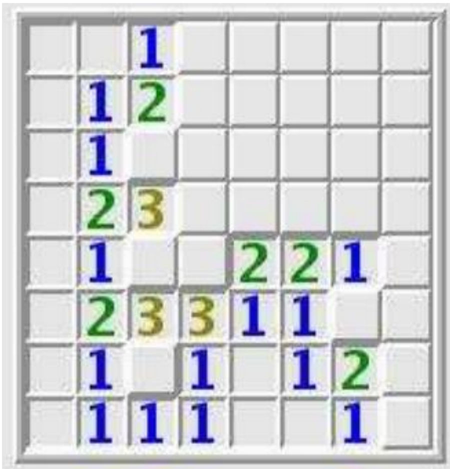
a) (1pt) In such a language you should be able to formalize the following knowledge:

- There are exactly n mines in the minefield.
- If a cell contains the number 1, then there is exactly one mine in the adjacent cells.

Please write your answer to the following table.

No.	Score	Axioms
1.	0.5pt	There are exactly n mines in the game.
2.	0.5pt	If a cell contains the number 1, then there is exactly one mine in the adjacent cells.
		$\forall x(contains(x,1) \Rightarrow \exists y\left(adj(x,y) \wedge mine(y) \wedge \forall z\big(adj(x,z)\big) \Rightarrow (z \neq y) \wedge \neg mine(z)\right))$

b) (1pt) Show by means of deduction that there must be a mine in the position (3,3) of the game state of below picture (8x8 size).



Please write your answer to the following table.

No.	CNF Sentences	Notes
1.	$contains((2,2),1)$	Cell (2,2) shows 1 mine
2.	$adj\big((2,2),(1,1)\big) \wedge adj\big((2,2),(1,2)\big) \wedge adj\big((2,2),(1,3)\big) \wedge adj\big((2,2),(2,1)\big) \wedge adj\big((2,2),(2,3)\big) \wedge adj\big((2,2),(3,1)\big) \wedge adj\big((2,2),(3,2)\big) \wedge adj\big((2,2),(3,3)\big)$	All adjacents of cell (2,2).
3.	$\neg mine\big((1,1)\big) \wedge \neg mine\big((1,2)\big) \wedge \neg mine\big((1,3)\big) \wedge \neg mine\big((2,1)\big) \wedge \neg mine\big((2,3)\big) \wedge \neg mine\big((3,1)\big) \wedge \neg mine\big((3,2)\big)$	All adjacents of cell (2,2) do not contain

		mine(excepted (3,3)
4.	$mine((3,3))$	From 1, 2, 3 we have a mine in (3,3)

Problem 4. (3pts) Consider the following statements. *Tony, Shi-Kuo and Ellen belong to the Hoofers Club. Every member of the Hoofers Club is either a skier or a mountain climber or both. No mountain climber likes rain, and all skiers like snow. Ellen dislikes whatever Tony likes and likes whatever Tony dislikes. Tony likes rain and snow.*

a) (1pt) Let S(x) mean "x is a skier", M(x) mean "x is a mountain climber", and L(x, y) mean "x likes y", where the domain of the first variable is Hoofers Club members, and the domain of the second variable is snow and rain. Build a FOL knowledge base from the above statements, using only given predicates.

No.	FOL sentences
1	$\forall x$(S(x) ∨ M(x))
2	$\forall x$(M(x) ⇒ ¬L(x, rain))
3	$\forall x$(S(x) ⇒ L(x, snow))
4	$\forall y$(L(Tony, y) ⇒ ¬L(Ellen, y))
5	$\forall y$(¬L(Tony, y) ⇒ L(Ellen, y))
6	L(Tony, rain) ∧ L(Tony, snow)
7	

b) (1pt) Convert each FOL sentence in the knowledge base into CNF.

No.	FOL sentences in CNF
1	S(x) ∨ M(x)
2	¬M(x) ∨ ¬L(x, rain)
3	¬S(x) ∨ L(x, snow)
4	¬L(Tony, y) ∨ ¬L(Ellen, y)
5	L(Tony, y) ∨ L(Ellen, y)
6	L(Tony, snow)
7	L(Tony, rain)

c) (1pt) Use resolution to prove that *"There is a member of the Hoofers Club who is a mountain climber but not a skier."*

Prove: $(M(x) \land \neg S(x))$

No.	FOL sentences in CNF
8	S(Ellen) ∨ M(Ellen) (1) {x/Ellen}
9	¬L(Tony, rain) ∨ ¬L(Ellen, rain) (4) {y/rain}
10	¬L(Ellen, rain) (7 and 9) {y/rain}
11	¬L(Tony, snow) ∨ ¬L(Ellen, snow) (4) {y/snow}
12	¬L(Ellen, snow) (6 and 11) {y/snow}
13	¬S(Ellen) ∨ L(Ellen, snow) (3) {x/Ellen}
14	¬S(Ellen) (13 and 12) {x/Ellen}
15	¬M(Ellen) ∨ ¬L(Ellen, rain) (2) {x/Ellen}
16	M(Ellen) (8 and 14) {x/Ellen}

17	M(Ellen) ∧ ¬S(Ellen) (14 and 16) (Proved)

Proved: There exists one member of the Hoofers Club who is a mountain climber but not a skier.

* The numbering in Table 2 continues sequentially from Table 1.

Problem 5. (2pts) Consider the following knowledge base.

1. $\forall x$ Milk(x) → Mammal(x)
2. $\forall y$ Feathers(y) ∧ Fly(y) ∧ Eggs(y) → Bird(y)
3. $\forall z$ Meat(z) ∧ Mammal(z) → Carnivore(z)
4. $\forall t$ Stripes(t) ∧ Carnivore(t) → Tiger(t)
5. Stripes(Hobbes)
6. Meat(Hobbes)
7. Feathers(Tweety)
8. Milk(Hobbes)
9. Fly(Tweety)

The list of predicates used to represent clauses in the knowledge base includes

- Milk(x): x gives milk
- Mammal(x): x is a mammal
- Feathers(x): x has feathers
- Fly(x): x flies
- Eggs(x): x lays eggs
- Bird(x): x is a bird
- Meat(x): x eats meat
- Stripes(x): x has black stripes
- Carnivore(x): x is a carnivore

a) (1pt) Use backward chaining to determine whether or not Hobbes is a tiger. Explain each reasoning step carefully and show the corresponding set of substitutions.

Tiger(t) requires Stripes(t) and Carnivore(t) from rule 4
- Substitutions: {t/Hobbes}.
- Stripes(Hobbes) is given in fact 5 (Proved).
- Carnivore(t) requires Meat(z) and Mammal(z) from rule 3.
  - Substitutions: {z/Hobbes}
  - Meat(Hobbes) is given in fact 6 (Proved).
  - Mammal(z) requires Milk(x) in rule 1
    - Substitutions: {x/Hobbes}
    - Milk(Hobbes) is given in fact 8.
    - Mammal(Hobbes) is proved
  - Carnival(Hobbes) is proved

⇨ Tiger(Hobbes) is proved.

b) (1pt) Use forward chaining to determine whether or not Hobbes is a tiger. Explain each reasoning step carefully and show the corresponding set of substitutions.

- From rule 1, we have Mammal(Hobbes) {x/Hobbes} (10)
- From rule 10 and rule 6, we have Carnivore(Hobbes) {z/Hobbes} (11)
- From rule 11 and rule 5, we have Tiger(Hobbes) {t/Hobbes} (12)
- Conclusion: Tiger(Hobbes) is proved.