Write $k = (k - 1) + 1$ to obtain

$$E(X^2) = \lambda \sum_{k=1}^{\infty} \frac{(k-1)e^{-\lambda}\lambda^{k-1}}{(k-1)!} + \lambda \sum_{k=1}^{\infty} \frac{e^{-\lambda}\lambda^{k-1}}{(k-1)!}$$

The summation in the first term on the right-hand side of the previous equation is recognized to be the mean of X and this equals λ so that the first term is λ^2. The summation in the second term on the right-hand side is recognized to be the sum of the probabilities and this equals one. Therefore, the previous equation simplifies to $E(X^2) = \lambda^2 + \lambda$. Because the $V(X) = E(X^2) - (EX)^2$, we have

$$V(X) = \lambda^2 + \lambda - \lambda^2 = \lambda$$

and the variance is derived.

Mean and Variance

If X is a Poisson random variable with parameter λ, then

$$\mu = E(X) = \lambda \quad \text{and} \quad \sigma^2 = V(X) = \lambda \tag{3-17}$$

The mean and variance of a Poisson random variable are equal. For example, if particle counts follow a Poisson distribution with a mean of 25 particles per square centimeter, the variance is also 25 and the standard deviation of the counts is five per square centimeter. Consequently, information on the variability is very easily obtained. Conversely, if the variance of count data is much greater than the mean of the same data, the Poisson distribution is not a good model for the distribution of the random variable.

EXERCISES FOR SECTION 3-9

3-129. Suppose X has a Poisson distribution with a mean of 4. Determine the following probabilities:
(a) $P(X = 0)$ (b) $P(X \le 2)$
(c) $P(X = 4)$ (d) $P(X = 8)$

3-130. Suppose X has a Poisson distribution with a mean of 0.4. Determine the following probabilities:
(a) $P(X = 0)$ (b) $P(X \le 2)$
(c) $P(X = 4)$ (d) $P(X = 8)$

3-131. Suppose that the number of customers who enter a bank in an hour is a Poisson random variable, and suppose that $P(X = 0) = 0.05$. Determine the mean and variance of X.

3-132. The number of telephone calls that arrive at a phone exchange is often modeled as a Poisson random variable. Assume that on the average there are 10 calls per hour.
(a) What is the probability that there are exactly five calls in one hour?
(b) What is the probability that there are three or fewer calls in one hour?
(c) What is the probability that there are exactly 15 calls in two hours?
(d) What is the probability that there are exactly five calls in 30 minutes?

3-133. Astronomers treat the number of stars in a given volume of space as a Poisson random variable. The density in the Milky Way Galaxy in the vicinity of our solar system is one star per 16 cubic light-years.
(a) What is the probability of two or more stars in 16 cubic light-years?
(b) How many cubic light-years of space must be studied so that the probability of one or more stars exceeds 0.95?

3-134. Data from www.centralhudsonlabs determined the mean number of insect fragments in 225-gram chocolate bars was 14.4, but three brands had insect contamination more than twice the average. See the U.S. Food and Drug Administration–Center for Food Safety and Applied Nutrition for Defect Action Levels for food products. Assume the number of fragments (contaminants) follows a Poisson distribution.
(a) If you consume a 225-gram bar from a brand at the mean contamination level, what is the probability of no insect contaminants?

(b) Suppose you consume a bar that is one-fifth the size tested (45 grams) from a brand at the mean contamination level. What is the probability of no insect contaminants?
(c) If you consume seven 28.35-gram (one-ounce) bars this week from a brand at the mean contamination level, what is the probability that you consume one or more insect fragments in more than one bar?
(d) Is the probability of contamination more than twice the mean of 14.4 unusual, or can it be considered typical variation? Explain.

3-135. In 1898 L. J. Bortkiewicz published a book entitled *The Law of Small Numbers*. He used data collected over 20 years to show that the number of soldiers killed by horse kicks each year in each corps in the Prussian cavalry followed a Poisson distribution with a mean of 0.61.
(a) What is the probability of more than one death in a corps in a year?
(b) What is the probability of no deaths in a corps over five years?

3-136. The number of flaws in bolts of cloth in textile manufacturing is assumed to be Poisson distributed with a mean of 0.1 flaw per square meter.
(a) What is the probability that there are two flaws in 1 square meter of cloth?
(b) What is the probability that there is one flaw in 10 square meters of cloth?
(c) What is the probability that there are no flaws in 20 square meters of cloth?
(d) What is the probability that there are at least two flaws in 10 square meters of cloth?

3-137. When a computer disk manufacturer tests a disk, it writes to the disk and then tests it using a certifier. The certifier counts the number of missing pulses or errors. The number of errors on a test area on a disk has a Poisson distribution with $\lambda = 0.2$.
(a) What is the expected number of errors per test area?
(b) What percentage of test areas have two or fewer errors?

3-138. The number of cracks in a section of interstate highway that are significant enough to require repair is assumed to follow a Poisson distribution with a mean of two cracks per mile.
(a) What is the probability that there are no cracks that require repair in 5 miles of highway?
(b) What is the probability that at least one crack requires repair in 1/2 mile of highway?
(c) If the number of cracks is related to the vehicle load on the highway and some sections of the highway have a heavy load of vehicles whereas other sections carry a light load, how do you feel about the assumption of a Poisson distribution for the number of cracks that require repair?

3-139. The number of surface flaws in plastic panels used in the interior of automobiles has a Poisson distribution with a mean of 0.05 flaw per square foot of plastic panel. Assume an automobile interior contains 10 square feet of plastic panel.
(a) What is the probability that there are no surface flaws in an auto's interior?
(b) If 10 cars are sold to a rental company, what is the probability that none of the 10 cars has any surface flaws?
(c) If 10 cars are sold to a rental company, what is the probability that at most one car has any surface flaws?

3-140. The number of failures of a testing instrument from contamination particles on the product is a Poisson random variable with a mean of 0.02 failure per hour.
(a) What is the probability that the instrument does not fail in an eight-hour shift?
(b) What is the probability of at least one failure in a 24-hour day?

3-141. The number of content changes to a Web site follows a Poisson distribution with a mean of 0.25 per day.
(a) What is the probability of two or more changes in a day?
(b) What is the probability of no content changes in five days?
(c) What is the probability of two or fewer changes in five days?

3-142. The number of views of a page on a Web site follows a Poisson distribution with a mean of 1.5 per minute.
(a) What is the probability of no views in a minute?
(b) What is the probability of two or fewer views in 10 minutes?
(c) Does the answer to the previous part depend on whether the 10-minute period is an uninterrupted interval? Explain.

Supplemental Exercises

3-143. Let the random variable X be equally likely to assume any of the values $1/8$, $1/4$, or $3/8$. Determine the mean and variance of X.

3-144. Let X denote the number of bits received in error in a digital communication channel, and assume that X is a binomial random variable with $p = 0.001$. If 1000 bits are transmitted, determine the following:
(a) $P(X = 1)$ (b) $P(X \geq 1)$
(c) $P(X \leq 2)$ (d) mean and variance of X

3-145. Batches that consist of 50 coil springs from a production process are checked for conformance to customer requirements. The mean number of nonconforming coil springs in a batch is five. Assume that the number of nonconforming springs in a batch, denoted as X, is a binomial random variable.
(a) What are n and p?
(b) What is $P(X \leq 2)$?
(c) What is $P(X \geq 49)$?

3-146. An automated egg carton loader has a 1% probability of cracking an egg, and a customer will complain if more than one egg per dozen is cracked. Assume each egg load is an independent event.
(a) What is the distribution of cracked eggs per dozen? Include parameter values.
(b) What are the probability that a carton of a dozen eggs results in a complaint?

(c) What are the mean and standard deviation of the number of cracked eggs in a carton of one dozen?

3-147. A total of 12 cells are replicated. Freshly synthesized DNA cannot be replicated again until mitosis is completed. Two control mechanisms have been identified—one positive and one negative—that are used with equal probability. Assume that each cell independently uses a control mechanism. Determine the following probabilities.
(a) All cells use a positive control mechanism.
(b) Exactly half the cells use a positive control mechanism.
(c) More than four, but less than seven, cells use a positive control mechanism.

3-148. A congested computer network has a 1% chance of losing a data packet and packet losses are independent events. An e-mail message requires 100 packets.
(a) What is the distribution of data packets that must be re-sent? Include the parameter values.
(b) What is the probability that at least one packet must be re-sent?
(c) What is the probability that two or more packets must be re-sent?
(d) What are the mean and standard deviation of the number of packets that must be re-sent?
(e) If there are 10 messages and each contains 100 packets, what is the probability that at least one message requires that two or more packets be re-sent?

3-149. A particularly long traffic light on your morning commute is green 20% of the time that you approach it. Assume that each morning represents an independent trial.
(a) What is the probability that the first morning that the light is green is the fourth morning that you approach it?
(b) What is the probability that the light is not green for 10 consecutive mornings?

3-150. The probability is 0.6 that a calibration of a transducer in an electronic instrument conforms to specifications for the measurement system. Assume the calibration attempts are independent. What is the probability that at most three calibration attempts are required to meet the specifications for the measurement system?

3-151. An electronic scale in an automated filling operation stops the manufacturing line after three underweight packages are detected. Suppose that the probability of an underweight package is 0.001 and each fill is independent.
(a) What is the mean number of fills before the line is stopped?
(b) What is the standard deviation of the number of fills before the line is stopped?

3-152. The probability that an eagle kills a jackrabbit in a day of hunting is 10%. Assume that results are independent between days.
(a) What is the distribution of the number of days until a successful jackrabbit hunt?
(b) What is the probability that the eagle must wait five days for its first successful hunt?
(c) What is the expected number of days until a successful hunt?
(d) If the eagle can survive up to 10 days without food (it requires a successful hunt on the tenth day), what is the probability that the eagle is still alive 10 days from now?

3-153. Traffic flow is traditionally modeled as a Poisson distribution. A traffic engineer monitors the traffic flowing through an intersection with an average of six cars per minute. To set the timing of a traffic signal, the following probabilities are used.
(a) What is the probability of no cars through the intersection within 30 seconds?
(b) What is the probability of three or more cars through the intersection within 30 seconds?
(c) Calculate the minimum number of cars through the intersection so that the probability of this number or fewer cars in 30 seconds is at least 90%.
(d) If the variance of the number of cars through the intersection per minute is 20, is the Poisson distribution appropriate? Explain.

3-154. A shipment of chemicals arrives in 15 totes. Three of the totes are selected at random, without replacement, for an inspection of purity. If two of the totes do not conform to purity requirements, what is the probability that at least one of the nonconforming totes is selected in the sample?

3-155. The probability that your call to a service line is answered in less than 30 seconds is 0.75. Assume that your calls are independent.
(a) If you call 10 times, what is the probability that exactly nine of your calls are answered within 30 seconds?
(b) If you call 20 times, what is the probability that at least 16 calls are answered in less than 30 seconds?
(c) If you call 20 times, what is the mean number of calls that are answered in less than 30 seconds?

3-156. Continuation of Exercise 3-155.
(a) What is the probability that you must call four times to obtain the first answer in less than 30 seconds?
(b) What is the mean number of calls until you are answered in less than 30 seconds?

3-157. Continuation of Exercise 3-155.
(a) What is the probability that you must call six times in order for two of your calls to be answered in less than 30 seconds?
(b) What is the mean number of calls to obtain two answers in less than 30 seconds?

3-158. The number of messages sent to a computer bulletin board is a Poisson random variable with a mean of five messages per hour.
(a) What is the probability that five messages are received in 1 hour?
(b) What is the probability that 10 messages are received in 1.5 hours?
(c) What is the probability that less than two messages are received in one-half hour?

3-159. A Web site is operated by four identical computer servers. Only one is used to operate the site; the others are spares that can be activated in case the active server fails. The probability that a request to the Web site generates a failure in the active server is 0.0001. Assume that each request is an independent trial. What is the mean time until failure of all four computers?

3-160. The number of errors in a textbook follows a Poisson distribution with a mean of 0.01 error per page. What is the probability that there are three or less errors in 100 pages?

3-161. The probability that an individual recovers from an illness in a one-week time period without treatment is 0.1. Suppose that 20 independent individuals suffering from this illness are treated with a drug and four recover in a one-week time period. If the drug has no effect, what is the probability that four or more people recover in a one-week time period?

3-162. Patient response to a generic drug to control pain is scored on a 5-point scale, where a 5 indicates complete relief. Historically, the distribution of scores is

1	2	3	4	5
0.05	0.1	0.2	0.25	0.4

Two patients, assumed to be independent, are each scored.

(a) What is the probability mass function of the total score?

(b) What is the probability mass function of the average score?

3-163. In a manufacturing process that laminates several ceramic layers, 1% of the assemblies are defective. Assume that the assemblies are independent.

(a) What is the mean number of assemblies that need to be checked to obtain five defective assemblies?

(b) What is the standard deviation of the number of assemblies that need to be checked to obtain five defective assemblies?

3-164. Continuation of Exercise 3-163. Determine the minimum number of assemblies that need to be checked so that the probability of at least one defective assembly exceeds 0.95.

3-165. Determine the constant c so that the following function is a probability mass function: $f(x) = cx$ for $x = 1, 2, 3, 4$.

3-166. A manufacturer of a consumer electronics product expects 2% of units to fail during the warranty period. A sample of 500 independent units is tracked for warranty performance.

(a) What is the probability that none fails during the warranty period?

(b) What is the expected number of failures during the warranty period?

(c) What is the probability that more than two units fail during the warranty period?

3-167. Messages that arrive at a service center for an information systems manufacturer have been classified on the basis of the number of keywords (used to help route messages) and the type of message, either e-mail or voice. Also, 70% of the messages arrive via e-mail and the rest are voice.

number of keywords	0	1	2	3	4
e-mail	0.1	0.1	0.2	0.4	0.2
voice	0.3	0.4	0.2	0.1	0

Determine the probability mass function of the number of keywords in a message.

3-168. The random variable X has the following probability distribution:

x	2	3	5	8
probability	0.2	0.4	0.3	0.1

Determine the following:

(a) $P(X \le 3)$ (b) $P(X > 2.5)$

(c) $P(2.7 < X < 5.1)$ (d) $E(X)$

(e) $V(X)$

3-169. Determine the probability mass function for the random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < 2 \\ 0.2 & 2 \le x < 5.7 \\ 0.5 & 5.7 \le x < 6.5 \\ 0.8 & 6.5 \le x < 8.5 \\ 1 & 8.5 \le x \end{cases}$$

3-170. Each main bearing cap in an engine contains four bolts. The bolts are selected at random, without replacement, from a parts bin that contains 30 bolts from one supplier and 70 bolts from another.

(a) What is the probability that a main bearing cap contains all bolts from the same supplier?

(b) What is the probability that exactly three bolts are from the same supplier?

3-171. Assume the number of errors along a magnetic recording surface is a Poisson random variable with a mean of one error every 10^5 bits. A sector of data consists of 4096 eight-bit bytes.

(a) What is the probability of more than one error in a sector?

(b) What is the mean number of sectors until an error is found?

3-172. An installation technician for a specialized communication system is dispatched to a city only when three or more orders have been placed. Suppose orders follow a Poisson distribution with a mean of 0.25 per week for a city with a population of 100,000, and suppose your city contains a population of 800,000.

(a) What is the probability that a technician is required after a one-week period?
(b) If you are the first one in the city to place an order, what is the probability that you have to wait more than two weeks from the time you place your order until a technician is dispatched?

3-173. From 500 customers, a major appliance manufacturer will randomly select a sample without replacement. The company estimates that 25% of the customers will provide useful data. If this estimate is correct, what is the probability mass function of the number of customers that will provide useful data?
(a) Assume that the company samples five customers.
(b) Assume that the company samples 10 customers.

3-174. It is suspected that some of the totes containing chemicals purchased from a supplier exceed the moisture content target. Samples from 30 totes are to be tested for moisture content. Assume that the totes are independent. Determine the proportion of totes from the supplier that must exceed the moisture content target so that the probability is 0.90 that at least one tote in the sample of 30 fails the test.

3-175. Messages arrive to a computer server according to a Poisson distribution with a mean rate of 10 per hour. Determine the length of an interval of time such that the probability that no messages arrive during this interval is 0.90.

3-176. Flaws occur in the interior of plastic used for automobiles according to a Poisson distribution with a mean of 0.02 flaw per panel.
(a) If 50 panels are inspected, what is the probability that there are no flaws?
(b) What is the expected number of panels that need to be inspected before a flaw is found?
(c) If 50 panels are inspected, what is the probability that the number of panels that have one or more flaws is less than or equal to 2?

MIND-EXPANDING EXERCISES

3-177. Derive the convergence results used to obtain a Poisson distribution as the limit of a binomial distribution.

3-178. Show that the function $f(x)$ in Example 3-5 satisfies the properties of a probability mass function by summing the infinite series.

3-179. Derive the formula for the mean and standard deviation of a discrete uniform random variable over the range of integers $a, a + 1, \ldots, b$.

3-180. Derive the expression for the variance of a geometric random variable with parameter p.

3-181. An air flight can carry 120 passengers. A passenger with a reserved seat arrives for the flight with probability 0.95 Assume the passengers behave independently. (Computer software is expected.)
(a) What is the minimum number of seats the airline should reserve for the probability of a full flight to be at least 0.90?
(b) What is the maximum number of seats the airline should reserve for the probability that more passengers arrive than the flight can seat to be less than 0.10?
(c) Discuss some reasonable policies the airline could use to reserve seats based on these probabilities.

3-182. A company performs inspection on shipments from suppliers in order to defect nonconforming products. Assume a lot contains 1000 items and 1% are nonconforming. What sample size is needed so that the probability of choosing at least one nonconforming item in the sample is at least 0.90? Assume the binomial approximation to the hypergeometric distribution is adequate.

3-183. A company performs inspection on shipments from suppliers in order to detect nonconforming products. The company's policy is to use a sample size that is always 10% of the lot size. Comment on the effectiveness of this policy as a general rule for all sizes of lots.

3-184. A manufacturer stocks components obtained from a supplier. Suppose that 2% of the components are defective and that the defective components occur independently. How many components must the manufacturer have in stock so that the probability that 100 orders can be completed without reordering components is at least 0.95?

3-185. A large bakery can produce rolls in lots of either 0, 1000, 2000, or 3000 per day. The production cost per item is \$0.10. The demand varies randomly according to the following distribution:

demand for rolls	0	1000	2000	3000
probability of demand	0.3	0.2	0.3	0.2

Every roll for which there is a demand is sold for \$0.30. Every roll for which there is no demand is sold in a secondary market for \$0.05. How many rolls should the bakery produce each day to maximize the mean profit?