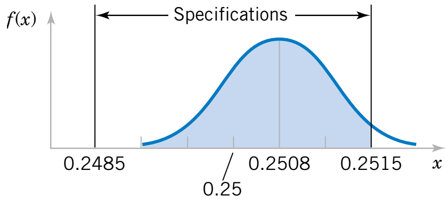

Figure 4-18 Distribution for Example 4-16.

EXERCISES FOR SECTION 4-6

4-49. Use Appendix Table III to determine the following probabilities for the standard normal random variable Z:
(a) $P(Z < 1.32)$ (b) $P(Z < 3.0)$
(c) $P(Z > 1.45)$ (d) $P(Z > -2.15)$
(e) $P(-2.34 < Z < 1.76)$

4-50. Use Appendix Table III to determine the following probabilities for the standard normal random variable Z:
(a) $P(-1 < Z < 1)$ (b) $P(-2 < Z < 2)$
(c) $P(-3 < Z < 3)$ (d) $P(Z > 3)$
(e) $P(0 < Z < 1)$

4-51. Assume Z has a standard normal distribution. Use Appendix Table III to determine the value for z that solves each of the following:
(a) $P(Z < z) = 0.9$ (b) $P(Z < z) = 0.5$
(c) $P(Z > z) = 0.1$ (d) $P(Z > z) = 0.9$
(e) $P(-1.24 < Z < z) = 0.8$

4-52. Assume Z has a standard normal distribution. Use Appendix Table III to determine the value for z that solves each of the following:
(a) $P(-z < Z < z) = 0.95$ (b) $P(-z < Z < z) = 0.99$
(c) $P(-z < Z < z) = 0.68$ (d) $P(-z < Z < z) = 0.9973$

4-53. Assume X is normally distributed with a mean of 10 and a standard deviation of 2. Determine the following:
(a) $P(X < 13)$ (b) $P(X > 9)$
(c) $P(6 < X < 14)$ (d) $P(2 < X < 4)$
(e) $P(-2 < X < 8)$

4-54. Assume X is normally distributed with a mean of 10 and a standard deviation of 2. Determine the value for x that solves each of the following:
(a) $P(X > x) = 0.5$
(b) $P(X > x) = 0.95$
(c) $P(x < X < 10) = 0.2$
(d) $P(-x < X - 10 < x) = 0.95$
(e) $P(-x < X - 10 < x) = 0.99$

4-55. Assume X is normally distributed with a mean of 5 and a standard deviation of 4. Determine the following:
(a) $P(X < 11)$ (b) $P(X > 0)$
(c) $P(3 < X < 7)$ (d) $P(-2 < X < 9)$
(e) $P(2 < X < 8)$

4-56. Assume X is normally distributed with a mean of 5 and a standard deviation of 4. Determine the value for x that solves each of the following:
(a) $P(X > x) = 0.5$ (b) $P(X > x) = 0.95$
(c) $P(x < X < 9) = 0.2$ (d) $P(3 < X < x) = 0.95$
(e) $P(-x < X - 5 < x) = 0.99$

4-57. The compressive strength of samples of cement can be modeled by a normal distribution with a mean of 6000 kilograms per square centimeter and a standard deviation of 100 kilograms per square centimeter.
(a) What is the probability that a sample's strength is less than 6250 Kg/cm^2?
(b) What is the probability that a sample's strength is between 5800 and 5900 Kg/cm^2?
(c) What strength is exceeded by 95% of the samples?

4-58. The time until recharge for a battery in a laptop computer under common conditions is normally distributed with a mean of 260 minutes and a standard deviation of 50 minutes.
(a) What is the probability that a battery lasts more than four hours?
(b) What are the quartiles (the 25% and 75% values) of battery life?
(c) What value of life in minutes is exceeded with 95% probability?

4-59. An article in *Knee Surgery Sports Traumatol Arthrosc* ["Effect of Provider Volume on Resource Utilization for Surgical Procedures" (2005, Vol. 13, pp. 273–279)] showed a mean time of 129 minutes and a standard deviation of 14 minutes for ACL reconstruction surgery at high-volume hospitals (with more than 300 such surgeries per year).
(a) What is the probability that your ACL surgery at a high-volume hospital requires a time more than two standard deviations above the mean?
(b) What is the probability that your ACL surgery at a high-volume hospital is completed in less than 100 minutes?
(c) The probability of a completed ACL surgery at a high-volume hospital is equal to 95% at what time?
(d) If your surgery requires 199 minutes, what do you conclude about the volume of such surgeries at your hospital? Explain.

4-60. Cholesterol is a fatty substance that is an important part of the outer lining (membrane) of cells in the body of animals. Its normal range for an adult is 120–240 mg/dl. The Food and Nutrition Institute of the Philippines found that the total cholesterol level for Filipino adults has a mean of 159.2 mg/dl and 84.1% of adults have a cholesterol level below 200 mg/dl (http://www.fnri.dost.gov.ph/). Suppose that the total cholesterol level is normally distributed.

(a) Determine the standard deviation of this distribution.
(b) What are the quartiles (the 25% and 75% values) of this distribution?
(c) What is the value of the cholesterol level that exceeds 90% of the population?
(d) An adult is at moderate risk if cholesterol level is more than one but less than two standard deviations above the mean. What percentage of the population is at moderate risk according to this criterion?
(e) An adult is thought to be at high risk if his cholesterol level is more than two standard deviations above the mean. What percentage of the population is at high risk?
(f) An adult has low risk if cholesterol level is one standard deviation or more below the mean. What percentage of the population is at low risk?

4-61. The line width for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.

(a) What is the probability that a line width is greater than 0.62 micrometer?
(b) What is the probability that a line width is between 0.47 and 0.63 micrometer?
(c) The line width of 90% of samples is below what value?

4-62. The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation of 0.1 fluid ounce.

(a) What is the probability that a fill volume is less than 12 fluid ounces?
(b) If all cans less than 12.1 or greater than 12.6 ounces are scrapped, what proportion of cans is scrapped?
(c) Determine specifications that are symmetric about the mean that include 99% of all cans.

4-63. In the previous exercise, suppose that the mean of the filling operation can be adjusted easily, but the standard deviation remains at 0.1 ounce.

(a) At what value should the mean be set so that 99.9% of all cans exceed 12 ounces?
(b) At what value should the mean be set so that 99.9% of all cans exceed 12 ounces if the standard deviation can be reduced to 0.05 fluid ounce?

4-64. The reaction time of a driver to visual stimulus is normally distributed with a mean of 0.4 seconds and a standard deviation of 0.05 seconds.

(a) What is the probability that a reaction requires more than 0.5 seconds?
(b) What is the probability that a reaction requires between 0.4 and 0.5 seconds?
(c) What is the reaction time that is exceeded 90% of the time?

4-65. The speed of a file transfer from a server on campus to a personal computer at a student's home on a weekday evening is normally distributed with a mean of 60 kilobits per second and a standard deviation of 4 kilobits per second.

(a) What is the probability that the file will transfer at a speed of 70 kilobits per second or more?
(b) What is the probability that the file will transfer at a speed of less than 58 kilobits per second?
(c) If the file is 1 megabyte, what is the average time it will take to transfer the file? (Assume eight bits per byte.)

4-66. The average height of a woman aged 20–74 years is 64 inches in 2002, with an increase of approximately one inch from 1960 (http://usgovinfo.about.com/od/healthcare). Suppose the height of a woman is normally distributed with a standard deviation of 2 inches.

(a) What is the probability that a randomly selected woman in this population is between 58 inches and 70 inches?
(b) What are the quartiles of this distribution?
(c) Determine the height that is symmetric about the mean that includes 90% of this population.
(d) What is the probability that five women selected at random from this population all exceed 68 inches?

4-67. In an accelerator center, an experiment needs a 1.41-cm-thick aluminum cylinder (http://puhep1.princeton.edu/mumu/target/Solenoid_Coil.pdf). Suppose that the thickness of a cylinder has a normal distribution with a mean of 1.41 cm and a standard deviation of 0.01 cm.

(a) What is the probability that a thickness is greater than 1.42 cm?
(b) What thickness is exceeded by 95% of the samples?
(c) If the specifications require that the thickness is between 1.39 cm and 1.43 cm, what proportion of the samples meet specifications?

4-68. The demand for water use in Phoenix in 2003 hit a high of about 442 million gallons per day on June 27, 2003 (http://phoenix.gov/WATER/wtrfacts.html). Water use in the summer is normally distributed with a mean of 310 million gallons per day and a standard deviation of 45 million gallons per day. City reservoirs have a combined storage capacity of nearly 350 million gallons.

(a) What is the probability that a day requires more water than is stored in city reservoirs?
(b) What reservoir capacity is needed so that the probability that it is exceeded is 1%?
(c) What amount of water use is exceeded with 95% probability?
(d) Water is provided to approximately 1.4 million people. What is the mean daily consumption per person at which the probability that the demand exceeds the current reservoir capacity is 1%? Assume that the standard deviation of demand remains the same.

4-69. The life of a semiconductor laser at a constant power is normally distributed with a mean of 7000 hours and a standard deviation of 600 hours.

(a) What is the probability that a laser fails before 5000 hours?
(b) What is the life in hours that 95% of the lasers exceed?
(c) If three lasers are used in a product and they are assumed to fail independently, what is the probability that all three are still operating after 7000 hours?

4-70. The diameter of the dot produced by a printer is normally distributed with a mean diameter of 0.002 inch and a standard deviation of 0.0004 inch.
(a) What is the probability that the diameter of a dot exceeds 0.0026 inch?
(b) What is the probability that a diameter is between 0.0014 and 0.0026 inch?
(c) What standard deviation of diameters is needed so that the probability in part (b) is 0.995?

4-71. The weight of a sophisticated running shoe is normally distributed with a mean of 12 ounces and a standard deviation of 0.5 ounce.
(a) What is the probability that a shoe weighs more than 13 ounces?
(b) What must the standard deviation of weight be in order for the company to state that 99.9% of its shoes are less than 13 ounces?
(c) If the standard deviation remains at 0.5 ounce, what must the mean weight be in order for the company to state that 99.9% of its shoes are less than 13 ounces?

4-72. Measurement error that is normally distributed with a mean of zero and a standard deviation of 0.5 gram is added to the true weight of a sample. Then the measurement is rounded to the nearest gram. Suppose that the true weight of a sample is 165.5 grams.
(a) What is the probability that the rounded result is 167 grams?
(b) What is the probability that the rounded result is 167 grams or greater?

4-73. Assume that a random variable is normally distributed with a mean of 24 and a standard deviation of 2. Consider an interval of length one unit that starts at the value a so that the interval is $[a, a + 1]$. For what value of a is the probability of the interval greatest? Does the standard deviation affect that choice of interval?

4-74. A study by Bechtel, et al., 2009, in the *Archives of Environmental & Occupational Health* considered polycyclic aromatic hydrocarbons and immune system function in beef cattle. Some cattle were near major oil- and gas-producing areas of western Canada. The mean monthly exposure to PM1.0 (particulate matter that is < 1 μm in diameter) was approximately 7.1 $\mu g/m^3$ with standard deviation 1.5. Assume the monthly exposure is normally distributed.
(a) What is the probability of a monthly exposure greater than 9 $\mu g/m^3$?
(b) What is the probability of a monthly exposure between 3 and 8 $\mu g/m^3$?
(c) What is the monthly exposure level that is exceeded with probability 0.05?
(d) What value of mean monthly exposure is needed so that the probability of a monthly exposure greater than 9 $\mu g/m^3$ is 0.01?

4-75. An article under review for *Air Quality, Atmosphere & Health* titled "Linking Particulate Matter (PM10) and Childhood Asthma in Central Phoenix" used PM10 (particulate matter < 10 μm in diameter) air quality data measured hourly from sensors in Phoenix, Arizona. The 24-hour (daily) mean PM10 for a centrally located sensor was 50.9 $\mu g/m^3$ with a standard deviation of 25.0. Assume that the daily mean of PM10 is normally distributed.
(a) What is the probability of a daily mean of PM10 greater than 100 $\mu g/m^3$?
(b) What is the probability of a daily mean of PM10 less than 25 $\mu g/m^3$?
(c) What daily mean of PM10 value is exceeded with probability 5%?

4-76. The length of stay at a specific emergency department in Phoenix, Arizona, in 2009 had a mean of 4.6 hours with a standard deviation of 2.9. Assume that the length of stay is normally distributed.
(a) What is the probability of a length of stay greater than 10 hours?
(b) What length of stay is exceeded by 25% of the visits?
(c) From the normally distributed model, what is the probability of a length of stay less than zero hours? Comment on the normally distributed assumption in this example.

4-7 NORMAL APPROXIMATION TO THE BINOMIAL AND POISSON DISTRIBUTIONS

We began our section on the normal distribution with the central limit theorem and the normal distribution as an approximation to a random variable with a large number of trials. Consequently, it should not be a surprise to learn that the normal distribution can be used to approximate binomial probabilities for cases in which n is large. The following example illustrates that for many physical systems the binomial model is appropriate with an extremely large value for n. In these cases, it is difficult to calculate probabilities by using the binomial distribution. Fortunately, the normal approximation is most effective in these cases. An illustration is provided in Fig. 4-19. The area of each bar equals the binomial probability of x. Notice that the area of bars can be approximated by areas under the normal density function.

From Fig. 4-19 it can be seen that a probability such as $P(3 \le X \le 7)$ is better approximated by the area under the normal curve from 2.5 to 7.5. This observation provides a method