EXERCISES FOR SECTION 3-6

3-75. For each scenario described below, state whether or not the binomial distribution is a reasonable model for the random variable and why. State any assumptions you make.

(a) A production process produces thousands of temperature transducers. Let X denote the number of nonconforming transducers in a sample of size 30 selected at random from the process.

(b) From a batch of 50 temperature transducers, a sample of size 30 is selected without replacement. Let X denote the number of nonconforming transducers in the sample.

(c) Four identical electronic components are wired to a controller that can switch from a failed component to one of the remaining spares. Let X denote the number of components that have failed after a specified period of operation.

(d) Let X denote the number of accidents that occur along the federal highways in Arizona during a one-month period.

(e) Let X denote the number of correct answers by a student taking a multiple-choice exam in which a student can eliminate some of the choices as being incorrect in some questions and all of the incorrect choices in other questions.

(f) Defects occur randomly over the surface of a semiconductor chip. However, only 80% of defects can be found by testing. A sample of 40 chips with one defect each is tested. Let X denote the number of chips in which the test finds a defect.

(g) Reconsider the situation in part (f). Now, suppose the sample of 40 chips consists of chips with 1 and with 0 defects.

(h) A filling operation attempts to fill detergent packages to the advertised weight. Let X denote the number of detergent packages that are underfilled.

(i) Errors in a digital communication channel occur in bursts that affect several consecutive bits. Let X denote the number of bits in error in a transmission of 100,000 bits.

(j) Let X denote the number of surface flaws in a large coil of galvanized steel.

3-76. Let X be a binomial random variable with $p = 0.2$ and $n = 20$. Use the binomial table in Appendix A to determine the following probabilities.

(a) $P(X \le 3)$ (b) $P(X > 10)$
(c) $P(X = 6)$ (d) $P(6 \le X \le 11)$

3-77. Let X be a binomial random variable with $p = 0.1$ and $n = 10$. Calculate the following probabilities from the binomial probability mass function and also from the binomial table in Appendix A and compare results.

(a) $P(X \le 2)$ (b) $P(X > 8)$
(c) $P(X = 4)$ (d) $P(5 \le X \le 7)$

3-78. The random variable X has a binomial distribution with $n = 10$ and $p = 0.5$. Determine the following probabilities:

(a) $P(X = 5)$ (b) $P(X \le 2)$
(c) $P(X \ge 9)$ (d) $P(3 \le X < 5)$

3-79. The random variable X has a binomial distribution with $n = 10$ and $p = 0.01$. Determine the following probabilities.

(a) $P(X = 5)$ (b) $P(X \le 2)$
(c) $P(X \ge 9)$ (d) $P(3 \le X < 5)$

3-80. The random variable X has a binomial distribution with $n = 10$ and $p = 0.5$. Sketch the probability mass function of X.

(a) What value of X is most likely?
(b) What value(s) of X is(are) least likely?

3-81. Sketch the probability mass function of a binomial distribution with $n = 10$ and $p = 0.01$ and comment on the shape of the distribution.

(a) What value of X is most likely?
(b) What value of X is least likely?

3-82. Determine the cumulative distribution function of a binomial random variable with $n = 3$ and $p = 1/2$.

3-83. Determine the cumulative distribution function of a binomial random variable with $n = 3$ and $p = 1/4$.

3-84. An electronic product contains 40 integrated circuits. The probability that any integrated circuit is defective is 0.01, and the integrated circuits are independent. The product operates only if there are no defective integrated circuits. What is the probability that the product operates?

3-85. The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 10 calls are placed to the airline.

(a) What is the probability that for exactly three calls the lines are occupied?
(b) What is the probability that for at least one call the lines are not occupied?
(c) What is the expected number of calls in which the lines are all occupied?

3-86. A multiple-choice test contains 25 questions, each with four answers. Assume a student just guesses on each question.

(a) What is the probability that the student answers more than 20 questions correctly?
(b) What is the probability the student answers less than five questions correctly?

3-87. A particularly long traffic light on your morning commute is green 20% of the time that you approach it. Assume that each morning represents an independent trial.

(a) Over five mornings, what is the probability that the light is green on exactly one day?
(b) Over 20 mornings, what is the probability that the light is green on exactly four days?
(c) Over 20 mornings, what is the probability that the light is green on more than four days?

3-88. Samples of rejuvenated mitochondria are mutated (defective) in 1% of cases. Suppose 15 samples are studied, and they can be considered to be independent for mutation. Determine the following probabilities. The binomial table in Appendix A can help.

(a) No samples are mutated.
(b) At most one sample is mutated.
(c) More than half the samples are mutated.

3-89. An article in *Information Security Technical Report* ["Malicious Software—Past, Present and Future" (2004, Vol. 9, pp. 6–18)] provided the following data on the top ten malicious software instances for 2002. The clear leader in the number of registered incidences for the year 2002 was the Internet worm "Klez," and it is still one of the most widespread threats. This virus was first detected on 26 October 2001, and it has held the top spot among malicious software for the longest period in the history of virology.

Place	Name	% Instances
1	I-Worm.Klez	61.22%
2	I-Worm.Lentin	20.52%
3	I-Worm.Tanatos	2.09%
4	I-Worm.BadtransII	1.31%
5	Macro.Word97.Thus	1.19%
6	I-Worm.Hybris	0.60%
7	I-Worm.Bridex	0.32%
8	I-Worm.Magistr	0.30%
9	Win95.CIH	0.27%
10	I-Worm.Sircam	0.24%

The 10 most widespread malicious programs for 2002 *(Source—Kaspersky Labs).*

Suppose that 20 malicious software instances are reported. Assume that the malicious sources can be assumed to be independent.

(a) What is the probability that at least one instance is "Klez"?
(b) What is the probability that three or more instances are "Klez"?
(c) What are the mean and standard deviation of the number of "Klez" instances among the 20 reported?

3-90. Heart failure is due to either natural occurrences (87%) or outside factors (13%). Outside factors are related to induced substances or foreign objects. Natural occurrences are caused by arterial blockage, disease, and infection. Suppose that 20 patients will visit an emergency room with heart failure. Assume that causes of heart failure between individuals are independent.

(a) What is the probability that three individuals have conditions caused by outside factors?
(b) What is the probability that three or more individuals have conditions caused by outside factors?
(c) What are the mean and standard deviation of the number of individuals with conditions caused by outside factors?

3-91. A computer system uses passwords that are exactly six characters and each character is one of the 26 letters (a–z) or 10 integers (0–9). Suppose there are 10,000 users of the system with unique passwords. A hacker randomly selects (with replacement) one billion passwords from the potential set, and a match to a user's password is called a **hit.**

(a) What is the distribution of the number of hits?
(b) What is the probability of no hits?
(c) What are the mean and variance of the number of hits?

3-92. **A statistical process control chart example.** Samples of 20 parts from a metal punching process are selected every hour. Typically, 1% of the parts require rework. Let X denote the number of parts in the sample of 20 that require rework. A process problem is suspected if X exceeds its mean by more than three standard deviations.

(a) If the percentage of parts that require rework remains at 1%, what is the probability that X exceeds its mean by more than three standard deviations?
(b) If the rework percentage increases to 4%, what is the probability that X exceeds 1?
(c) If the rework percentage increases to 4%, what is the probability that X exceeds 1 in at least one of the next five hours of samples?

3-93. Because not all airline passengers show up for their reserved seat, an airline sells 125 tickets for a flight that holds only 120 passengers. The probability that a passenger does not show up is 0.10, and the passengers behave independently.

(a) What is the probability that every passenger who shows up can take the flight?
(b) What is the probability that the flight departs with empty seats?

3-94. This exercise illustrates that poor quality can affect schedules and costs. A manufacturing process has 100 customer orders to fill. Each order requires one component part that is purchased from a supplier. However, typically, 2% of the components are identified as defective, and the components can be assumed to be independent.

(a) If the manufacturer stocks 100 components, what is the probability that the 100 orders can be filled without reordering components?
(b) If the manufacturer stocks 102 components, what is the probability that the 100 orders can be filled without reordering components?
(c) If the manufacturer stocks 105 components, what is the probability that the 100 orders can be filled without reordering components?

3-95. Consider the lengths of stay at a hospital's emergency department in Exercise 3-29. Assume that five persons independently arrive for service.

(a) What is the probability that the length of stay of exactly one person is less than or equal to 4 hours?
(b) What is the probability that exactly two people wait more than 4 hours?

(c) What is the probability that at least one person waits more than 4 hours?

3-96. Consider the visits that result in leave without being seen (LWBS) at an emergency department in Example 2-8. Assume that four persons independently arrive for service at Hospital 1.

(a) What is the probability that exactly one person will LWBS?
(b) What is the probability, that two or more two people will LWBS?
(c) What is the probability that at least one person will LWBS?

3-97. Assume a Web site changes its content according to the distribution in Exercise 3-30. Assume 10 changes are made independently.

(a) What is the probability that the change is made in less than 4 days in seven of the 10 updates?
(b) What is the probability that the change is made in less than 4 days in two or fewer of the 10 updates?
(c) What is the probability that at least one change is made in less than 4 days?
(d) What is the expected number of the 10 updates that occur in less than 4 days?

3-98. Consider the endothermic reactions in Exercise 3-28. A total of 20 independent reactions are to be conducted.

(a) What is the probability that exactly 12 reactions result in a final temperature less than 272 K?
(b) What is the probability that at least 19 reactions result in a final temperature less than 272 K?
(c) What is the probability that at least 18 reactions result in a final temperature less than 272 K?
(d) What is the expected number of reactions that result in a final temperature of less than 272 K?

3-7 GEOMETRIC AND NEGATIVE BINOMIAL DISTRIBUTIONS

Geometric Distribution

Consider a random experiment that is closely related to the one used in the definition of a binomial distribution. Again, assume a series of Bernoulli trials (independent trials with constant probability p of a success on each trial). However, instead of a fixed number of trials, trials are conducted until a success is obtained. Let the random variable X denote the number of trials until the first success. In Example 3-5, successive wafers are analyzed until a large particle is detected. Then, X is the number of wafers analyzed. In the transmission of bits, X might be the number of bits transmitted until an error occurs.

EXAMPLE 3-20 Digital Channel

The probability that a bit transmitted through a digital transmission channel is received in error is 0.1. Assume the transmissions are independent events, and let the random variable X denote the number of bits transmitted *until* the first error.

Then, $P(X = 5)$ is the probability that the first four bits are transmitted correctly and the fifth bit is in error. This event can be denoted as $\{OOOOE\}$, where O denotes an okay bit. Because the trials are independent and the probability of a correct transmission is 0.9,

$$P(X = 5) = P(OOOOE) = 0.9^4 0.1 = 0.066$$

Note that there is some probability that X will equal any integer value. Also, if the first trial is a success, $X = 1$. Therefore, the range of X is $\{1, 2, 3, \dots\}$, that is, all positive integers.

Geometric Distribution

In a series of Bernoulli trials (independent trials with constant probability p of a success), let the random variable X denote the number of trials until the first success. Then X is a **geometric random variable** with parameter $0 < p < 1$ and

$$f(x) = (1 - p)^{x-1} p \qquad x = 1, 2, \dots \tag{3-9}$$

Examples of the probability mass functions for geometric random variables are shown in Fig. 3-9. Note that the height of the line at x is $(1 - p)$ times the height of the line at $x - 1$. That is, the probabilities decrease in a geometric progression. The distribution acquires its name from this result.