《数字信号处理原理》2022-2023 年第一学期 期中考试

一、 Fill the blanks(40%)

1. If $x[n]=\{-2，0，0，3\},-1\le n\le 2$，

(1) then x[n] can be expressed in terms of the unit impulse signal $\delta[n]$ as ① $-2\delta[n+1]+3\delta[n-2]$ and the unit step signal $\mu[n]$ as $-2(\mu[n+1]-\mu[n])+3(\mu[n-2]-\mu[n-3])$ ② ;

(2) if the impulse response of a LTI system is h[n]= $\{-1，0,\ 0,\ 1.5,\ 0,\ 3\}$, $-1\le n\le 4$, then given by x[n], the output sequence y[n]= ③ $(-2\delta[n+1]+3\delta[n-2])(-\delta[n+1]+1.5\delta[n-2]+3\delta[n-4])$ ={2,0,0,-6,0,-6,4.5,0,9}.

(3) the even part of x[n] is ④ . $x[n]=\{0,-2，0，0，3\},-2\le n\le 2$，$x[-n]=\{3,0，0，-2，0\},-2\le n\le 2$;

$x_{ev}[n]=\frac{1}{2}\{3,-2，0，-2，3\}=\{1.5,-1，0，-1，1.5\}$, $-2\le n\le 2$

2. determine whether the following system is linear, causal , stable and shift-invariant: ⑤ linear,causal, not stable, shift-variant .

$$y[n]=n^3x[n]+x[n-4]$$

3. if $y[n]=x[n+1]-2x[n]+x[n-1]$, is it a LTI system? yes ⑥. If so , write out the impulse response of system h[n]: ⑦ $h[n]=\delta[n+1]-2\delta[n]+\delta[n-1]$.

4. determine the DTFT of the following sequences:

(1) $x[n]=n\alpha^n\mu[n],|\alpha|<1$: ⑧ ; $\frac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^2}$

(2) $x[n]=\begin{cases}N+1-|n|,-N\le n\le N,\\ 0.otherwise\end{cases}$: ⑨ solution 3.17 .

$$=\frac{\sin\left(\omega[N+\frac{1}{2}]\right)}{\sin(\omega/2)}+\frac{\sin^2(\omega N/2)}{\sin^2(\omega/2)}.$$

5. determine the IDTFT of the following sequences:

(1) $H_1(e^{j\omega})=1+2cos\omega+3cos2\omega$: ⑩ .solution 3.23 h[n]= $\{1.5，1,\ 1,\ 1,\ 1.5\}$

6. If $Y(e^{j\omega})=X(e^{j4\omega})$, then $y[n]$ can be expressed in terms of $x[n]$ as ⑪ solution 3.26

$y[n]=\begin{cases}x[n], & n=0,\pm4,\pm8,\pm16,\dots\\ 0, & \text{otherwise.}\end{cases}$ (注意和 Time-shifting $g[n-n_o]$ $e^{-j\omega n_o}G(e^{j\omega})$ 的区别)，应该是.

7. If $y[n]=x[n]e^{-j\pi n/3}$, then $Y(e^{j\omega})$ can be expressed in terms of $X(e^{j\omega})$ as ⑫ $Y(e^{j\omega})=X(e^{j(\omega+\pi/3)})$.

8. $H_1(e^{j\omega})=\begin{cases}|\omega|,0\le|\omega|\le\omega_c\\ 0,\omega_c\le|\omega|\le\pi\end{cases}$, determine it has IDTFT which is odd sequence or even sequence ⑬ .even solution 3.32；只要 DTFT 纯实数一定 IDTFT 偶序列，DTFT 纯虚数一定 IDTFT 奇序列，只要 DTFT 共轭一定 IDTFT 实数序列

9. if a continuous-time signal $g_a(t)$ is Ω_m. Determine the Nyquist frequency of

(1) $y_1(t)=g_a(t)g_a(t)$: ⑭ $2\Omega_m$ .solution 4.3

(2) $y_2(t) = \int_{-\infty}^{\infty} g_a(t-\tau)g_a(t)d\tau$: ⑮ Ω_m.

10. if $x[n]$ and $h[n]$ are two length-51 sequence defined for , $0 \le n \le 50$, denote the range of $y_L[n]$ ⑯ $0 \le n \le 100$, d__ and for which range $y_L[n] = y_c[n]$ ⑰ n=50 if circle convolution is 51 length.

11. determine the 5-points periodic convolution of the following sequences:

    (1) $x[n] = \{1, 2, -2, -1, 3\}$, $h[n] = \{2, 0, 1, 3, -4\}$, $0 \le n \le 4$: ⑱ y[n]={-13,-13,-13,-13};solution 5.2 ;

    (2) $x[n] = \{-1, 5, 3, 0, 3\}$, $h[n] = \{-2, 0, 5, 3, -2\}$, $0 \le n \le 4$: ⑲ y[n]={1,1,1,1,1} .

12. The even samples of the 12-point DFT of a length-12 real sequence $x[n]$ has the first 7 samples of are given by $X[k] = \{11, 8-2j, 1-12j, 6+3j, -3+2j, 2+j, 15\}, 0 \le n \le 6$, Determine the rest of 5 samples of $X[k]$: ⑳

$X[7] = X^*[\langle -7\rangle_{12}] = X^*[5] = 2 - j$, $X[8] = X^*[\langle -8\rangle_{12}] = X^*[4] = -3 - j2$,
$X[9] = X^*[\langle -9\rangle_{12}] = X^*[3] = 6 - j3$, $X[10] = X^*[\langle -10\rangle_{12}] = X^*[2] = 1 + j12$,
$X[11] = X^*[\langle -11\rangle_{12}] = X^*[1] = 8 + j2$.

二、Comprehensive problems(60%)

1. (40%)A causal LTI system is described by the recursive difference equation

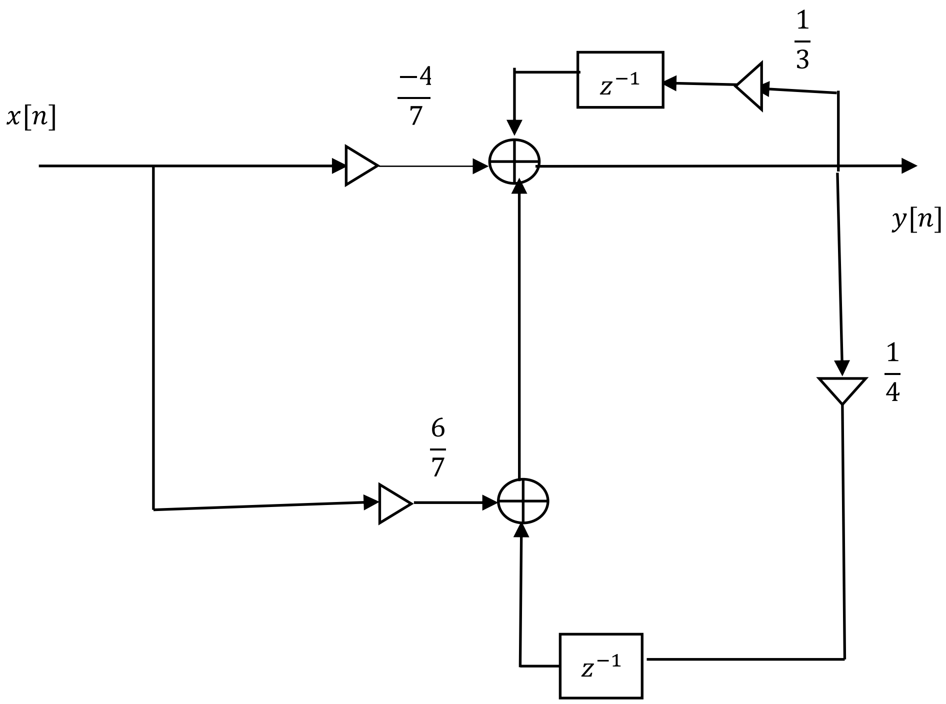
$$y[n] = 2x[n] - x[n-1] + \frac{7}{12}y[n-1] - \frac{1}{12}y[n-2]$$

(1) Draw the diagram of the system in parallel form. (10%)

Solution: $H(z) = \frac{2 - z^{-1}}{1 - \frac{7}{12}z^{-1} + \frac{1}{12}z^{-2}}, R.O.C\ |z| > \frac{1}{3},$

$$h1[n] = \frac{-4}{7}\left(\frac{1}{3}\right)^n u[n]$$

$$h2[n] = \frac{6}{7}\left(\frac{1}{4}\right)^n u[n]$$



(2) Find the impulse response h[n] by solving differential equations. (20%)

Solution: $y_h[n] = h[n] = \alpha_1 \left(\frac{1}{3}\right)^n + \alpha_2 \left(\frac{1}{4}\right)^n$, x[0]= $\delta[0]$, $y[0] = 2$; x[1]= $\delta[1]$, y[1]=-1,

$h[n] = -18\left(\frac{1}{3}\right)^n + 20\left(\frac{1}{4}\right)^n$

(3) Write out the magnitude function of the frequency response $H(e^{j\omega})$.(10%)

. $H(e^{j\omega}) = \sum_{n=1}^{N} h[n]e^{j\omega n}$

2. .(20%) For a continuous time signal x(t) with frequency spectrum of $X(e^{j\omega})$, which $-\pi/3 \le \omega \le \pi/3$ as figure shown. If there is a LPF $H(e^{j\omega})$ with cut-off frequency $-\pi/4 \le \omega_c \le \pi/4$, Plot the frequency spectrum of $H(e^{j\omega})$ and $Y(e^{j\omega})$and its 8-points DFT, H[k] and Y[k] .

Solution: