Chapter 3

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Excercise 9.

1. $\boldsymbol{5n^2 - 6n = \Theta(n^2)}$

   $(n \geq 1)\ \ 5n^2 - 6n < 5n^2 \ \ \Rightarrow \ \ f(n) = O(n^2).$

   $(n \geq 6)\ \ 5n^2 - 6n \geq 5n^2 - n^2 = 4n^2 \ \ \Rightarrow \ \ f(n) = \Omega(n^2).$

   i. e. $5n^2 - 6n = \Theta(n^2)$

2. $\boldsymbol{n! = O(n^n)}$

   $(\text{for } i = 1 \text{ to } n)\ \ i \leq n \ \ \Rightarrow \ \ \prod_{i=1}^{n} i \leq n^n.$

   i. e. $n! = O(n^n)$

3. $\boldsymbol{2n^2 2^n + n\log n = \Theta(n^2 2^n)}$

   $(n \geq 1)\ \ 2n^2 2^n + n\log n \leq 2n^2 2^n + n^2 \leq 3n^2 2^n \ \ \Rightarrow \ \ f(n) = O(n^2 2^n).$

   $(n \geq 1)\ \ 2n^2 2^n + n\log n > 2n^2 2^n \ \ \Rightarrow \ \ f(n) = \Omega(n^2 2^n).$

   i. e. $2n^2 2^n + n\log n = \Theta(n^2 2^n)$

4. $\boldsymbol{\sum_{i=0}^{n} i^2 = \Theta(n^3)}$

   $$(n \geq 4)\ \ \sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \leq \frac{4}{3}(n+1)^3 = \frac{4}{3}(n^3 + 3n^2 + 3n + 1) \leq 2n^3 + n^2 \leq 3n^3$$

   $$\Rightarrow \ \ f(n) = O(n^3)$$

   $$(n \geq 1)\ \ \sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} > \frac{1}{3}n^3 \ \ \Rightarrow \ \ f(n) = \Omega(n^3)$$

   i. e. $\sum_{i=0}^{n} i^2 = \Theta(n^3)$

5. $\boldsymbol{\sum_{i=0}^{n} i^3 = \Theta(n^4)}$

   $$(n \geq 4)\ \ \sum_{i=0}^{n} i^3 = \frac{n^2(n+1)^2}{4} \leq \frac{1}{4}(n+1)^4 \leq \frac{3}{4}n^3 \cdot 2n \leq \frac{3}{2}n^4 \ \ \Rightarrow \ \ f(n) = O(n^4)$$

   $$(n \geq 1)\ \ \sum_{i=0}^{n} i^3 = \frac{n^2(n+1)^2}{4} > \frac{1}{4}n^4 \ \ \Rightarrow \ \ f(n) = \Omega(n^4)$$

   i. e. $\sum_{i=0}^{n} i^3 = \Theta(n^4)$

6. $\boldsymbol{n^{2^n} + 6 \cdot 2^n = \Theta(n^{2^n})}$

$(n > 1)\ \ n^{2^n} + 6 \cdot 2^n < n^{2^n} + 6 \cdot n^{2^n} = 7n^{2^n} \quad \Rightarrow \quad f(n) = O(n^{2^n})$

$(n \geq 1) n^{2^n} + 6 \cdot 2^n > n^{2^n} \quad \Rightarrow \quad f(n) = \Omega(n^{2^n})$

i. e. $n^{2^n} + 6 \cdot 2^n = \Theta(n^{2^n})$

7. $\boldsymbol{n^3 + 10^6 n^2 = \Theta(n^3)}$

$(n \geq 10^6)\ \ n^3 + 10^6 n^2 \leq n^3 + n^3 = 2n^3 \quad \Rightarrow \quad f(n) = O(n^3)$

$(n \geq 1)\ \ n^3 + 10^6 n^2 > n^3 \quad \Rightarrow \quad f(n) = \Omega(n^3)$

i. e. $n^3 + 10^6 n^2 = \Theta(n^3)$

8. $\boldsymbol{\dfrac{6n^3}{\log n + 1} = O(n^3)}$

$(n \geq 1)\ \ \dfrac{6n^3}{\log n + 1} \leq 6n^3$

i. e. $\dfrac{6n^3}{\log n + 1} = O(n^3)$

9. $\boldsymbol{n^{1.001} + n \log n = \Theta(n^{1.001})}$

$(n \geq 3)\ \ n^{1.001} + n \log n < 2n^{1.001} \quad \Rightarrow \quad f(n) = O(n^{1.001})$

$(n \geq 1)\ \ n^{1.001} + n \log n > n^{1.001} \quad \Rightarrow \quad f(n) = \Omega(n^{1.001})$

i. e. $n^{1.001} + n \log n = \Theta(n^{1.001})$

0. $\boldsymbol{n^{k+\epsilon} + n^k \log n = \Theta(n^{k+\epsilon}),\ k \geq 0 \text{ and } \epsilon > 0}$

$(n > \epsilon^{-\epsilon})\ \ \log n < n^\epsilon \ \rightarrow\ n^{k+\epsilon} + n^k \log n < 2n^{k+\epsilon} \quad \Rightarrow \quad f(n) = O(n^{k+\epsilon})$

$(n \geq 1)\ \ n^{k+\epsilon} + n^k \log n > n^{k+\epsilon} \quad \Rightarrow \quad f(n) = \Omega(n^{k+\epsilon})$

i. e. $n^{k+\epsilon} + n^k \log n = \Theta(n^{k+\epsilon})$