

题号	一	二	三	四	五	六	七	八	九	十	总分	阅卷人
得分												

备注： 闭卷，可以带计算器

得分	阅卷人

一、 Fill the blanks(40%)

- Let there is a continuous time sequences $x(t) = x_1(t) + x_2(t) + x_3(t)$ with two continuous components, $x_1(t) = \cos(3\pi t)$, $x_2(t) = \cos(7\pi t)$, $x_3(t) = \cos(10\pi t)$, if the combined digital signal is $x[n] = x_1[n] + x_2[n] + x_3[n]$, then
 - if sampled by 10Hz, then the signal will get aliasing or not no ① and then sampled signal in time domain $x[n] = \underline{\cos(0.3\pi n) + \cos(0.7\pi n) + \cos(\pi n)}$ ②;
 - based on (1), to filter the $x_1[n]$ by passing $x[n]$ through a digital filter, choose one kind of digital filter LPF ③ and determine its cut-off frequency $0 \leq \omega \leq 0.7\pi$ ④;
 - based on (2), determine the period of $x_1[n]$, $N = \underline{20}$ ⑤.
- If $x[n] = \{1, 4, 5, -4, -1\}$, $-2 \leq n \leq 2$, then
 - $x[n]$ can be expressed in terms of the unit impulse signal $\delta[n]$ as $x[n] = \delta[n+2] + 4\delta[n+1] + 5\delta[n] - 4\delta[n-1] - \delta[n-2]$ ⑥;
 - If the impulse response of a LTI system is $h[n] = \{1, 4, 5\}$, $0 \leq n \leq 3$, then given by $x[n]$, the output sequence $y[1] = \underline{36}$ ⑦ and the range of $y[n]$ is between -2 ⑧ and 4 ⑨;
 - if calculate $y_c[n]$ by 6-points circulation convolution, the value of $y_c[1] = \underline{-8}$ ⑩;
 - without computing the DTFT, $X(e^{j0}) = \underline{5}$ ⑪; $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = \underline{84\pi}$ ⑫;
 - to make $h[n]$ to be a Type1 linear phrase FIR, then write out one of the possible new linear phase $h'[n] = \underline{[-1, 4, 5, -4, 1]}$ ⑬.
- Without computing the DTFT, determine what kind of $X(e^{j\omega})$ is real-valued ⑭; (real-valued or imaginary-valued or others types of DTFT).
- If $x[n]$ is a length-120 sequence and $h[n]$ is a FIR filter with $0 \leq n \leq 4$. If using 10-points overlap method to computer $y[n]$, then the length of each small segment of $x[n]$ is 6 ⑮ and their whole length output $y[n]$ is 1620 ⑯.
- Some samples of the 5-point DFT of a length-5 real sequence are given by $X[0] = -4.7$, $X[2] = 1.2 - j2$, $X[4] = -3.5 + j3$. The $X[1]$ should be $-3.5 - 3j$ ⑰.
- An IIR digital filter has the unit pulse response $h[n] = (0.5)^n \mu[-n] + (0.2)^n \mu[n]$, then

$$\frac{1}{1+0.5z} + \frac{1}{1+0.2z^{-1}} \quad 0.2 < |z| < 2$$
 - z-transform of $H(z)$ in closed form is ⑱ and its R.O.C is ⑲.
 - whether $h[n]$ is BIBO stable or not stable ⑳.

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二、 Comprehensive problems(60%)

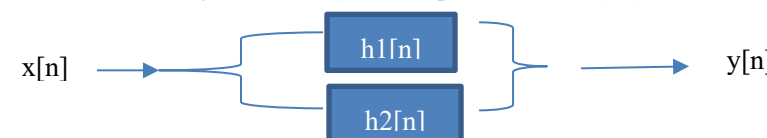
1. (40%) A causal LTI system is described by the recursive difference equation

$$y[n] = 2x[n] - x[n-1] + \frac{7}{12}y[n-1] - \frac{1}{12}y[n-2]$$

- (1) Find the transfer function of $H(z)$ and its R.O.C.(6%)

$$H(z) = \frac{2 - z^{-1}}{1 - \frac{7}{12}z^{-1} + \frac{1}{12}z^{-2}}, R.O.C |z| > \frac{1}{3}$$

- (2) Draw the diagram of the system in parallel form. (8%)



$$h_1[n] = \frac{-4}{7} \left(\frac{1}{3}\right)^n u[n]$$

$$h_2[n] = \frac{6}{7} \left(\frac{1}{4}\right)^n u[n]$$

- (3) If using bilinear form to transfer the digital filter to analog filter, write out the transform equation. (4%)

$$z = \frac{2 + T \cdot s}{2 - T \cdot s}$$

- (4) Find the impulse response $h[n]$ by solving differential equations. (8%)

$$y_h[n] = h[n] = \alpha_1 \left(\frac{1}{3}\right)^n + \alpha_2 \left(\frac{1}{4}\right)^n, x[0] = \delta[0], y[0] = 2; x[1] = \delta[1], y[1] = -1,$$

$$h[n] = -18 \left(\frac{1}{3}\right)^n + 20 \left(\frac{1}{4}\right)^n$$

- (5) Write out the magnitude function of the frequency response $H(e^{j\omega})$. (6%) $z = e^{j\omega}$ 代入

- (6) If using FIR filter to approximate $h[n]$ and $N=10$, then what its magnitude function of the frequency response $H(e^{j\omega})$ will be. (6%) $H(e^{j\omega}) = \sum_{n=1}^N h[n] e^{j\omega n}$

- (7) How to make (6) to be linear phase. (2%) $h'[n] = h[N-n]$

2. (20%) For a continuous time signal $x(t)$ with frequency spectrum $X(j\Omega)$, which $-\pi \times 10^4 \text{ r/s} \leq \Omega \leq \pi \times 10^4 \text{ r/s}$ as figure 1 shown.

- (1) Plot corresponding frequency spectrum of $X(e^{j\omega})$ and $X_s(j\Omega)$ with a proper sampling period

$$T = 0.5 \times 10^{-4} \text{ s} \quad (5\%). \quad \Omega_T = \frac{2\pi}{T} = 4\pi \times 10^4 \text{ r/s}, \quad \omega_{\max} = \Omega_{\max} T = \pi \times 10^4 \times 0.5 \times 10^{-4} = 0.5\pi$$

- (2) If there is a LPF $H(e^{j\omega})$ with cut-off frequency $-\pi/4 \leq \omega_c \leq \pi/4$, Plot the frequency spectrum of $Y(e^{j\omega})$ and its 10-points DFT $Y[k]$. (5%) k 为 1 或 -1

- (3) Determine the range of sampling rate Ω_s if $Y_s(j\Omega)$ is reconstructed by filter in (2) without aliasing.

$$\text{and Draw the } Y_s(j\Omega) \text{ based on (3). (5\%)} \quad \begin{cases} \frac{\omega_{cc}}{T} \leq \frac{2\pi}{T} - \Omega_{\max} \rightarrow \frac{1}{4} \times 10^{-4} \text{ s} \leq T \leq \frac{7}{4} \times 10^{-4} \text{ s} \\ \frac{\omega_{cc}}{T} \leq \Omega_{\max} \end{cases}$$

$$\text{if chose } T = 0.5 \times 10^{-4} \text{ s, then } \Omega_{y_{\max}} = \frac{\omega_{y_{\max}}}{T} = \frac{\frac{\pi}{4}}{0.5 \times 10^{-4}} = 0.5\pi \times 10^4 \text{ r/s}$$

- (4) Design an antialiasing filter if $T = 0.2 \times 10^{-4} \text{ s}$ and plot the frequency spectrum of $X_s(j\Omega)$ after using anti-

aliasing filter . (5%) $H_a(j\Omega) = \begin{cases} 1, & |\Omega| < \frac{\Omega_T}{2} = \frac{\frac{2\pi}{0.2 \times 10^{-4}}}{2} = 5 \times 10^{-4} \pi, \\ 0, & |\Omega| \geq \frac{\Omega_T}{2}. \end{cases}$

$$X_{anti}(j\Omega) = H_a(j\Omega)X(j\Omega)$$

