

《数字信号处理原理》2021-2-2022 年第一学期 期中考试

一、 Fill the blanks(40%)

1. If $x[n] = \{-2, 0, 0, 3\}, -1 \leq n \leq 2$,
 - (1) then $x[n]$ can be expressed in terms of the unit impulse signal $\delta[n]$ as ① and the unit step signal $\mu[n]$ as ②;
 - (2) if the impulse response of a LTI system is $h[n] = \{-1, 0, 0, 1.5, 0, 3\}, -1 \leq n \leq 4$, then given by $x[n]$, the output sequence $y[n] =$ ③.
 - (3) the even part of $x[n]$ is ④.
2. determine whether the following system is linear, causal, stable and shift-invariant: ⑤.

$$y[n] = n^3 x[n] + x[n-4]$$
3. if $y[n] = x[n+1] - 2x[n] + x[n-1]$, is it a LTI system? ⑥. If so, write out the impulse response of system $h[n]$: ⑦.
4. determine the DTFT of the following sequences:
 - (1) $x[n] = n\alpha^n \mu[n], |\alpha| < 1$: ⑧;
 - (2) $x[n] = \begin{cases} N+1-|n|, & -N \leq n \leq N, \\ 0, & \text{otherwise} \end{cases}$: ⑨.
5. determine the IDTFT of the following sequences:
 - (1) $H_1(e^{j\omega}) = 1 + 2\cos\omega + 3\cos 2\omega$: ⑩.
6. If $Y(e^{j\omega}) = X(e^{j4\omega})$, then $y[n]$ can be expressed in terms of $x[n]$ as ⑪.
7. If $y[n] = x[n]e^{-j\pi n/3}$, then $Y(e^{j\omega})$ can be expressed in terms of $X(e^{j\omega})$ as ⑫.
8. $H_1(e^{j\omega}) = \begin{cases} |\omega|, & 0 \leq |\omega| \leq \omega_c \\ 0, & \omega_c \leq |\omega| \leq \pi \end{cases}$, determine it has IDTFT which is odd sequence or even sequence ⑬.
9. if a continuous-time signal $g_a(t)$ is Ω_m . Determine the Nyquist frequency of
 - (1) $y_1(t) = g_a(t)g_a(t)$: ⑭.
 - (2) $y_2(t) = \int_{-\infty}^{\infty} g_a(t-\tau)g_a(t)d\tau$: ⑮.
10. if $x[n]$ and $h[n]$ are two length-51 sequence defined for $0 \leq n \leq 50$, denote the range of $y_L[n]$ ⑯ and for which range $y_L[n] = y_c[n]$ ⑰.
11. determine the 5-points periodic convolution of the following sequences:
 - (1) $x[n] = \{1, 2, -2, -1, 3\}$, $h[n] = \{2, 0, 1, 3, -4\}$, $0 \leq n \leq 4$: ⑱;
 - (2) $x[n] = \{-1, 5, 3, 0, 3\}$, $h[n] = \{-2, 0, 5, 3, -2\}$, $0 \leq n \leq 4$: ⑲.
12. The even samples of the 12-point DFT of a length-12 real sequence $x[n]$ has the first 7 samples of are given by $X[k] = \{11, 8-2j, 1-12j, 6+3j, -3+2j, 2+j, 15\}, 0 \leq n \leq 6$, Determine the rest of 5 samples of $X[k]$: ⑳.

二、 Comprehensive problems(60%)

1. (40%) A causal LTI system is described by the recursive difference equation

$$y[n] = 2x[n] - x[n-1] + \frac{7}{12}y[n-1] - \frac{1}{12}y[n-2]$$

- (1) Draw the diagram of the system in parallel form. (10%)

- (2) Find the impulse response $h[n]$ by solving differential equations. (20%)
- (3) Write out the magnitude function of the frequency response $H(e^{j\omega})$. (10%)
2. (20%) For a continuous time signal $x(t)$ with frequency spectrum of $X(e^{j\omega})$, which $-\pi/4 \leq \omega \leq \pi/4$ as figure shown. If there is a LPF $H(e^{j\omega})$ with cut-off frequency $-\pi/4 \leq \omega_c \leq \pi/4$, Plot the frequency spectrum of $H(e^{j\omega})$ and $Y(e^{j\omega})$ and its 8-points DFT, $H[k]$ and $Y[k]$.

