5	4,0	0	1	2	2	0	0	1
4	0	0	1	2	1	0	0	1
0	6	0	1	2	1	0	0	0
6	3,1	0	0	1	0	0	0	0
3	2,1	0	0	0	0	0	0	0
1	2	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0

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I. 1.

$$\begin{cases} \dot{x} = x - 2xy \\ \dot{y} = \dfrac{x^2}{2} - y \end{cases} \Longleftrightarrow$$

a) $\begin{cases} x - 2xy = 0 \\ \dfrac{x^2}{2} - y = 0 \end{cases} \Leftrightarrow \cancel{x = 2} \Rightarrow \dfrac{x^2}{2} = y \Leftrightarrow x^2 = 2y \Bigg\} \Rightarrow x - x^3 = 0 \Leftrightarrow$

$\Leftrightarrow x(1 - x^2) = 0 \Leftrightarrow$ 1/7

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$\Rightarrow x(x-1)(x+1)=0 \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 1 \\ x_3 = -1 \end{cases}$

$y = \frac{x^2}{2} \xRightarrow{x_2, x_3} y_2 = \frac{1}{2}$

$y = \frac{x^2}{2} \xrightarrow{x_1} y_1 = 0$

$S = \{ (0,0), (1, \frac{1}{2}), (-1, \frac{1}{2}) \}$

b) $\dot{x} = Jf(\eta^*) \cdot x \quad \cancel{\eta^* = e_2} \quad \eta^* \in \mathbb{R}^2$

$Jf(x,y) = \begin{pmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} \\ \frac{df_2}{dx} & \frac{df_2}{dy} \end{pmatrix} = \begin{pmatrix} 1-2y & -2x \\ x & -1 \end{pmatrix}$

$\dot{x} = Jf(0,0) \cdot x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} x$

$\lambda_1 = 1, \lambda_2 = -1 \Rightarrow Re(\lambda_{1,2}) \neq 0 \Rightarrow \lambda_2 < 0 < \lambda_1 \Rightarrow 0 \in \mathbb{R}^2$ saddle point ~~fat~~

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$0 \in \mathbb{R}^2$ saddle point for $\dot{x} = J_f(0,0) \cdot X \Rightarrow \dot{x} = f(x)(0,0)$ unstable

$\dot{x} = J_f(1, \frac{1}{2})X$

$\dot{x} = \begin{pmatrix} 1 - 2\frac{1}{2} & -2 \\ 1 & -1 \end{pmatrix} X \Rightarrow \dot{x} = \begin{pmatrix} 0 & -2 \\ 1 & -1 \end{pmatrix} X$

$\left| \begin{pmatrix} 0 & -2 \\ 1 & -1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0 \Leftrightarrow \begin{vmatrix} -\lambda & -2 \\ 1 & -1-\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 + \lambda + 2 = 0$

$\Delta = 1 - 8$

$\Delta = -7 \Rightarrow \lambda_{1,2} = \frac{-1 \pm i\sqrt{7}}{2} = -\frac{1}{2} \pm i\frac{\sqrt{7}}{2}$

$0 \in \mathbb{R}^2$ is attracting focus for $\dot{x} = J_f(1, \frac{1}{2})X \Rightarrow (1, \frac{1}{2})$ hiperbolic $\Rightarrow$

$\Rightarrow (1, \frac{1}{2})$ is an attractor for $\dot{x} = J_f(1, \frac{1}{2})x$

stable

-2 ... X
-1

$+\lambda+2=0$

$(1, \frac{1}{2})x \Rightarrow$
...erbolic

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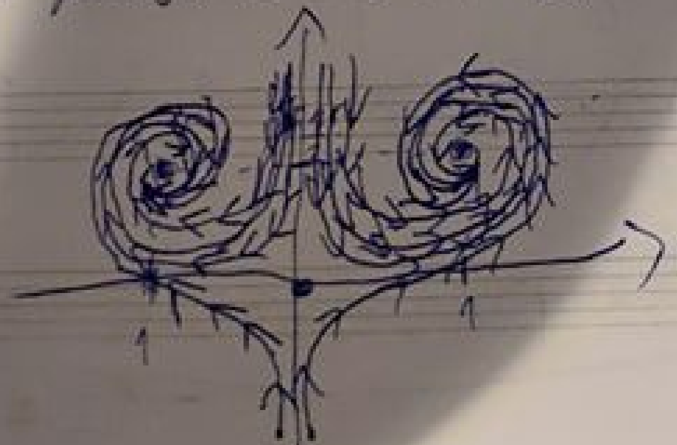
$\dot{x} = Jf(-1, \frac{1}{2})x$

$\dot{x} = \begin{pmatrix} 1-2\frac{1}{2} & 2 \\ -1 & -1 \end{pmatrix} x \Rightarrow \dot{x} = \begin{pmatrix} 0 & -2 \\ 1 & -1 \end{pmatrix} x$

$\left| \begin{pmatrix} 0 & 2 \\ -1 & -1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0 \Leftrightarrow \begin{vmatrix} -\lambda & 2 \\ -1 & -1-\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 + \lambda + 2 = 0$

$\Delta = 1-8$

$\Delta = -7 \Rightarrow \lambda_{1,2} = \frac{-1 \pm i\sqrt{7}}{2} = -\frac{1}{2} \pm i\frac{\sqrt{7}}{2} \Rightarrow Re\left(-\frac{1}{2} \pm \frac{i\sqrt{7}}{2}\right) < 0 \Rightarrow 0 \in \mathbb{R}^2$ is attracting focus for $\dot{x} = Jf(-1, \frac{1}{2})x \Rightarrow (-1, \frac{1}{2})$ hiperbolic $\Rightarrow (-1, \frac{1}{2})$ is an attractor for $\dot{x} = Jf(-1, \frac{1}{2})x$



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$$\begin{cases}\dot{x} = x - xy \\ \dot{y} = -0{,}3y + 0{,}3xy\end{cases} \iff \begin{cases} x - xy = 0 \\ -0{,}3y + 0{,}3xy = 0 \Rightarrow -0{,}3y + 0{,}3xy = 0\end{cases}$$

a)

$$\Rightarrow 0{,}3xy = 0{,}3y \iff x = 1 \Rightarrow 1 - y = 0 \Rightarrow 1 = y \Rightarrow \begin{cases} x = 1 \\ y = 1\end{cases}$$

$S = \{(1,1)\} \Rightarrow$ ~~(1,1) is an equilibrium point~~

$\dot{X} = Jf(\eta^*) \cdot X$

$\cancel{Jf(x,y) =}$

$$Jf(x,y) = \begin{pmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} \\ \frac{df_2}{dx} & \frac{df_2}{dy} \end{pmatrix} = \begin{pmatrix} 1-y & -x \\ 0{,}3y & -0{,}3+0{,}3x \end{pmatrix}$$

$$\cancel{\dot{X} = Jf(1,1) \cdot X = \begin{pmatrix} 0 & -1 \\ 0{,}3 & -0{,}3 \end{pmatrix} \cdot X \Rightarrow \left| \begin{pmatrix} 0 & -1 \\ 0{,}3 & -0{,}3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0}$$

$$\cancel{\iff \begin{vmatrix} -\lambda & -1 \\ 0{,}3 & -0{,}3-\lambda \end{vmatrix} = 0 \iff \lambda^2 + 0{,}3\lambda + 0{,}3 = 0 \Rightarrow \Delta = 0{,}09 - 1{,}2}$$

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$0,3xy = 0 \Rightarrow$

$\Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}$

$\begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} = 0$

$0,e y -$

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$$\Delta = -1,11 \Rightarrow \lambda_1, \lambda_2 = \frac{-0,3 \pm i\sqrt{1,11}}{2} \Rightarrow \begin{cases} Re(\lambda_1) < 0 \\ Re(\lambda_2) < 0 \end{cases} \Rightarrow (1,1) \text{ is an}$$

equilibrium point

$$\dot{x} = Jf(1,1)\,x \Leftrightarrow \dot{x} = \begin{pmatrix} 0 & -1 \\ 0,3 & 0 \end{pmatrix} x \Rightarrow \left| \begin{pmatrix} 0 & -1 \\ 0,3 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$$

$$\Leftrightarrow \begin{vmatrix} -\lambda & -1 \\ 0,3 & -\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 + 0,3 = 0 \Leftrightarrow \lambda^2 = -0,3 \Rightarrow \lambda_{1,2} = \pm i\sqrt{0,3}$$

$$\Rightarrow \begin{cases} Re(\lambda_1) = 0 \\ Re(\lambda_2) = 0 \end{cases} \Rightarrow (1,1) \text{ is non-hyperbolic}$$

b) $\cancel{dx}$ $\dot{x}, \dot{y} \Rightarrow \frac{dx}{dt}, \frac{dy}{dt}$

$$\frac{\dot{x}}{\dot{y}} = \frac{\cancel{x - 2xy}}{\cancel{\frac{x^2}{2} - y}} \Leftrightarrow \frac{x - xy}{-0,3y + 0,3xy} \Leftrightarrow \frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{x(1-y)}{0,3y(-1+x)} \;\Big|\; \cdot \frac{dy}{\left(\frac{-1+x}{x}\right)} \Leftrightarrow$$

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$$\Leftrightarrow \frac{-1+x}{x}dx = \frac{1-y}{0,3y}dy \Leftrightarrow \int \frac{-1+x}{x}dx = \int \frac{1-y}{0,3y}dy$$

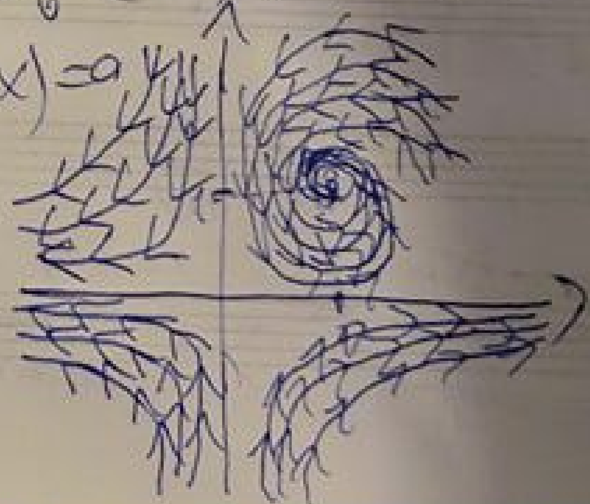
$$\int \frac{-1+x}{x}dx = \int \frac{x-1}{x}dx = \int\left(1-\frac{1}{x}\right)dx = x - \ln|x| + \mathcal{C}$$

$$\int \frac{1-y}{0,3y}dy = \int \frac{10(1-y)}{3y}dy = \int\left(\frac{10}{3y} - \frac{10}{3}\right)dy = \frac{10}{3}\ln|y| - \frac{10}{3}y + \mathcal{C}$$

$$= \frac{10(\ln|y|-y)}{3} + \mathcal{C} = \frac{\ln(y)-y}{0,3} + \mathcal{C}$$

$$x - \ln|x| + \mathcal{C} = \frac{\ln(y)-y}{0,3} + \mathcal{C} \overset{(*)}{\Leftrightarrow} x - \ln|x| = \frac{\ln|y|-y}{0,3}$$

$$(*) \Leftrightarrow \cancel{\ln x}\; y - \ln y + 0,3(x - \ln x) = a$$



> $solve\left(\left\{x - 2\cdot x\cdot y = 0, \frac{x^2}{2} - y = 0\right\}\right);$

$$\{x = 0, y = 0\}, \left\{x = 1, y = \frac{1}{2}\right\}, \left\{x = -1, y = \frac{1}{2}\right\} \quad \textbf{(1)}$$

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```
> f1:=(x,y)->x-2*x*y;
```

$$f1 := (x, y) \to x - 2\,x\,y \quad \textbf{(2)}$$

write the equation of x' as the function f1

> $f2 := (x, y) \to \frac{x^2}{2} - y;$

$$f2 := (x, y) \to \frac{1}{2}\,x^2 - y \quad \textbf{(3)}$$

write the equation of y' as the function f2

> $solve(\{f1(x, y) = 0, f2(x, y) = 0\});$

$$\{x = 0, y = 0\}, \left\{x = 1, y = \frac{1}{2}\right\}, \left\{x = -1, y = \frac{1}{2}\right\} \quad \textbf{(4)}$$

solving the equation in order to compute all its equilibria

> $with(linalg) : with(DEtools) : with(VectorCalculus) :$

Importing libraries

> $JM := Jacobian([f1(x, y), f2(x, y)], [x, y]);$

$$JM := \begin{bmatrix} 1 - 2\,y & -2\,x \\ x & -1 \end{bmatrix} \quad \textbf{(5)}$$

Computing the Jacobian Matrix

> $A1 := subs([x = 0, y = 0], JM);$

$$A1 := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \textbf{(6)}$$

replacing x with 0 and y with 0 in the Jacobian Matrix

> $eigenvalues(A1);$

$$1, -1 \quad \textbf{(7)}$$

Computing the eigenvalues of A1. Re(e1) > 0 & Re(e2) < 0

> $A2 := subs\left(\left[x = 1, y = \frac{1}{2}\right], JM\right);$

$$A2 := \begin{bmatrix} 0 & -2 \\ 1 & -1 \end{bmatrix} \quad \textbf{(8)}$$

replacing x with 1 and y with 1/2 in the Jacobian Matrix

> $eigenvalues(A2);$

$$-\frac{1}{2} + \frac{1}{2}\,\mathrm{I}\sqrt{7}, -\frac{1}{2} - \frac{1}{2}\,\mathrm{I}\sqrt{7} \quad \textbf{(9)}$$

Computing the eigenvalues of A2. Re(e1) < 0 & Re(e2) < 0

> $A3 := subs\left(\left[x =-1, y = \frac{1}{2}\right], JM\right);$

$$A3 := \begin{bmatrix} 0 & 2 \\ -1 & -1 \end{bmatrix} \tag{10}$$

replacing x with -1 and y with 1/2 in the Jacobian Matrix

> *eigenvalues*(*A3*);

$$-\frac{1}{2}+\frac{1}{2}\,\mathrm{I}\sqrt{7},\ -\frac{1}{2}-\frac{1}{2}\,\mathrm{I}\sqrt{7} \tag{11}$$

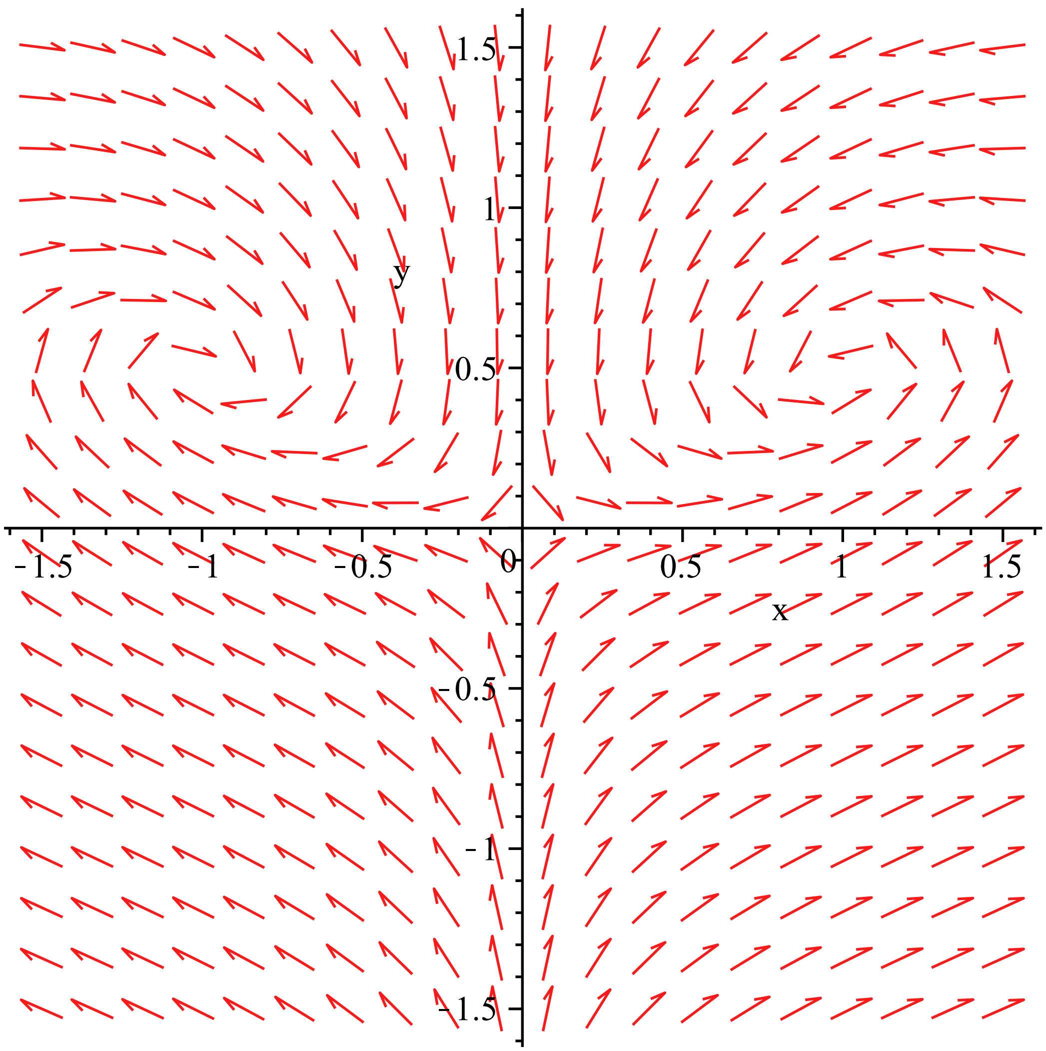
Computing the eigenvalues of A3. Re(e1) < 0 & Re(e2) < 0

> $plotField := (s, e) \rightarrow dfieldplot([diff(x(t), t) = f1(x(t), y(t)), diff(y(t), t) = f2(x(t), y(t))], [x(t), y(t)], t = 0..1, x = s, y = e);$

$$plotField := (s, e) \rightarrow dfieldplot\left(\left[\frac{d}{dt}\,x(t) = f1(x(t), y(t)),\ \frac{d}{dt}\,y(t) = f2(x(t), y(t))\right], [x(t), y(t)], t = 0..1, x = s, y = e\right) \tag{12}$$

defining a function for a easier use and with no agglomerated text for dfieldplot

> *plotField*(−1.5 ..1.5, −1.5 ..1.5);



Using the plotField function in order to use dfieldplot to draw the direction field in the box [-1.5,1.5]x [-1.5,1.5]. The effects of the equilibrium points (0,0), (-1, 1/2) and (1,1/2) can be seen.

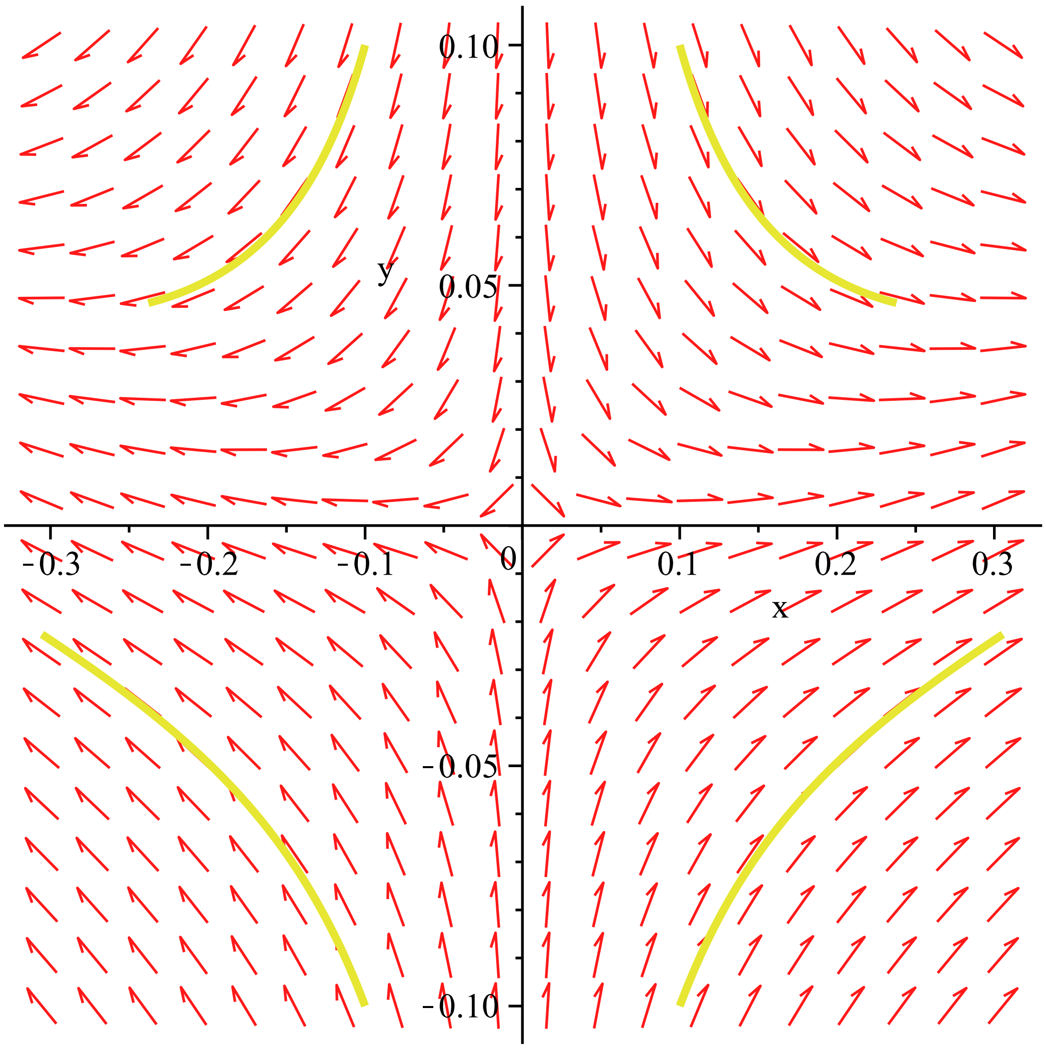
> $plotFieldDE := (interval, orbits) \rightarrow DEplot([diff(x(t), t) = f1(x(t), y(t)), diff(y(t), t)$

$= f2(x(t), y(t))], [x(t), y(t)], t = interval, orbits);$

$$plotFieldDE := (interval, orbits) \rightarrow DEplot\left(\left[\frac{d}{dt}\, x(t) = f1(x(t), y(t)),\ \frac{d}{dt}\, y(t) = f2(x(t), y(t))\right], [x(t), y(t)], t = interval, orbits\right) \quad (13)$$

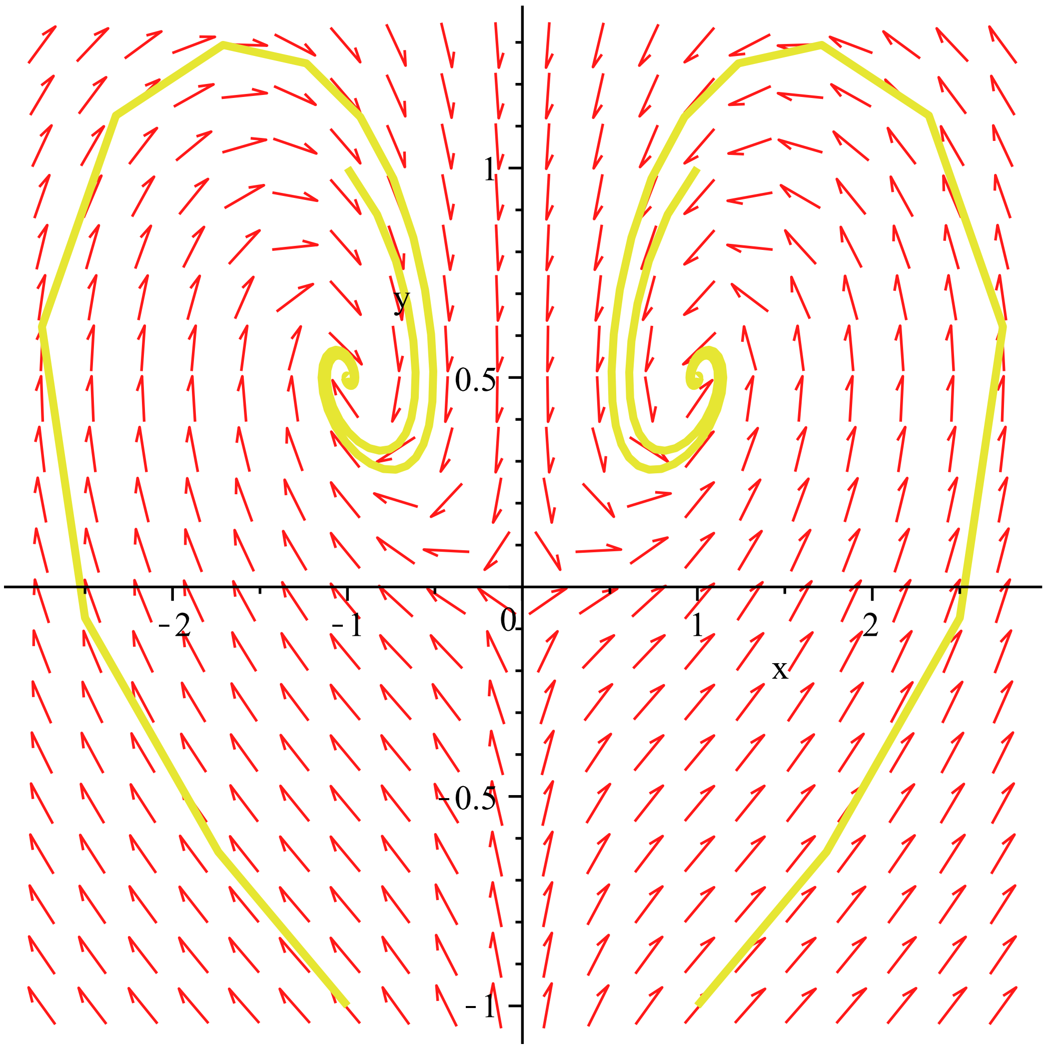
defining a function for a easier use and with no agglomerated text for DEplot

```
> plotFieldDE(0..1, [[x(0) = 0.1, y(0) = 0.1], [x(0) =-0.1, y(0) = 0.1], [x(0) = 0.1, y(0) =
    -0.1], [x(0) =-0.1, y(0) =-0.1]]);
```



Using the plotFieldDE function in order to use DEplot to

```
> plotFieldDE(0..10, [[x(0) = 1, y(0) = 1], [x(0) =-1, y(0) = 1], [x(0) = 1, y(0) =-1], [x(0) =
    -1, y(0) =-1]]);
```



> g1 := (x, y) → x − x·y;

$$g1 := (x, y) \rightarrow x + VectorCalculus:\text{-}`\text{-}`(x\, y) \quad \textbf{(14)}$$

> g2 := (x, y) → -0.3·y + 0.3·x·y;

$$g2 := (x, y) \rightarrow VectorCalculus:\text{-}`\text{-}`(1) \cdot 0.3\, y + 0.3\, x\, y \quad \textbf{(15)}$$

> subs([x = 1, y = 1], [g1(x, y), g2(x, y)]);

$$[0, 0.] \quad \textbf{(16)}$$

> JM := Jacobian([g1(x, y), g2(x, y)], [x, y]);

$$JM := \begin{bmatrix} 1 - y & -x \\ 0.3\, y & -0.3 + 0.3\, x \end{bmatrix} \quad \textbf{(17)}$$

> A := subs([x = 1, y = 1], JM);

$$A := \begin{bmatrix} 0 & -1 \\ 0.3 & 0. \end{bmatrix} \quad \textbf{(18)}$$

> eigenvalues(A);

$$0. + 0.547722557505166\, I,\ 0. - 0.547722557505166\, I \quad \textbf{(19)}$$

> dsolve$\left(diff(y(x), x) = \dfrac{g2(x, y(x))}{g1(x, y(x))} \right)$;

$$y(x) = e^{-\mathrm{LambertW}\left(-\frac{e^{\frac{3}{10}x+\frac{3}{10}\_C1}}{x^{3/10}}\right)+\frac{3}{10}x-\frac{3}{10}\ln(x)+\frac{3}{10}\_C1} \tag{20}$$

> $H := (x, y) \rightarrow y - \ln(y) + 0.3 \cdot (x - \ln(x));$

$$H := (x, y) \rightarrow y + \mathit{VectorCalculus}:\text{-}\grave{}\text{-}\grave{}(\ln(y)) + 0.3\,(x + \mathit{VectorCalculus}:\text{-}\grave{}\text{-}\grave{}(\ln(x))) \tag{21}$$

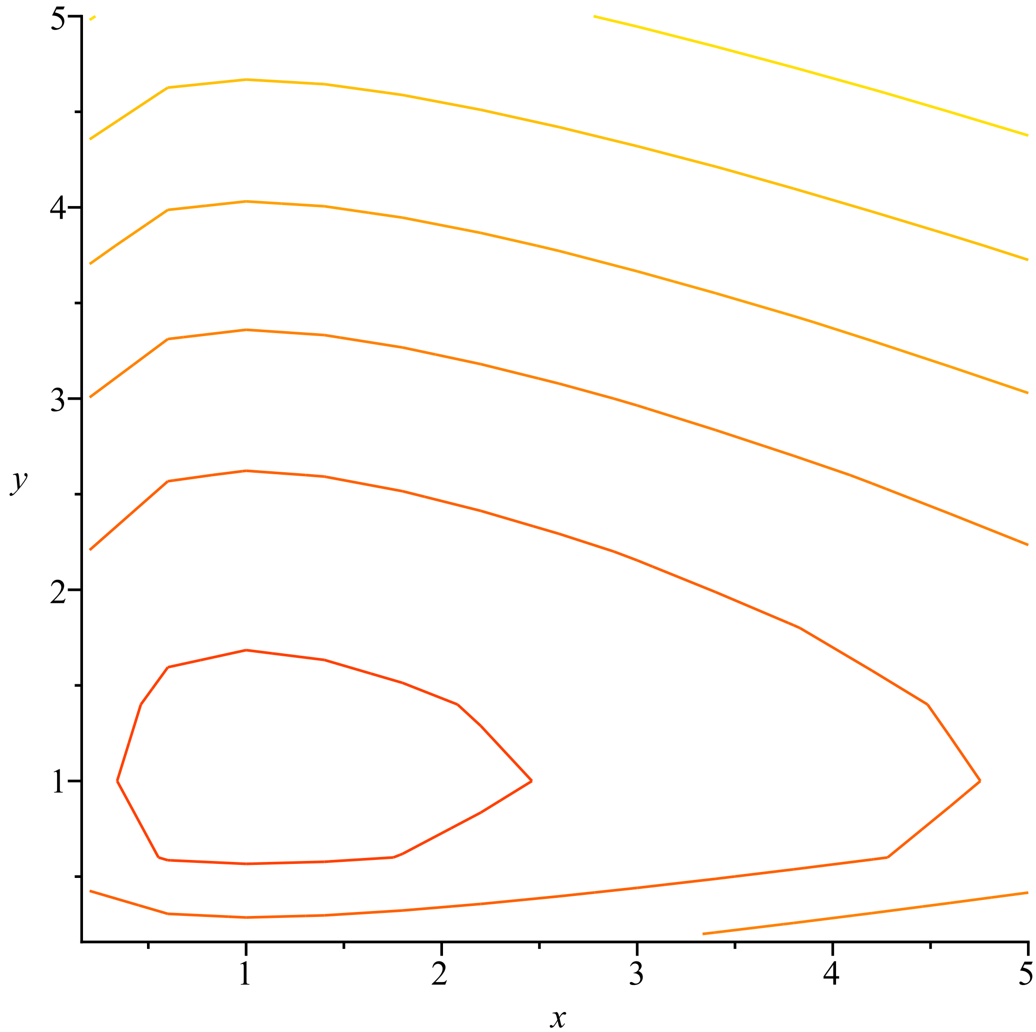
> $g1(x, y) \cdot \mathit{diff}(H(x, y), x) + g2(x, y) \cdot \mathit{diff}(H(x, y), y);$

$$(x - x\,y)\left(0.3 - \frac{0.3}{x}\right) + (-0.3\,y + 0.3\,x\,y)\left(1 - \frac{1}{y}\right) \tag{22}$$

> *with*(*plots*);

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(23)**

> $\mathit{contourplot}(H(x, y), x = -5..5, y = -5..5);$



5
4
3
2
1
y
1
2
3
4
5
x