A synthetic magnetic field model for the island divertor stochasticity studies.

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ABSTRACT

The intolerable levels of stochasticity in the stellarator-reactor can violate the desirable island-like scrape-off layer flows. Given coil configuration constrains of Wendelstein 7-X, a more generic model for studying stochastisation of the island divertor is developed

To study stochasticity, the Hamiltonian system of equations for the field lines [1] can be employed. Initially, a single island chain is created atop circular magnetic surfaces. The Runge-Kutta scheme is tested against the classical pendulum solution [2] and diverges rapidly due to the numerical scheme's failure to preserve energy. A more advanced area-preserving numerical schemes, such as the Störmer-Verlet one is found successful to reproduce the pendulum solution.

Stochastisation of the magnetic field is achieved by overlapping two island chains using the Störmer-Verlet numerical scheme.

HAMILTONIAN SYSTEM FOR THE ONE ISLAND CHAIN ATOP CIRCULAR FLUX SURFACES

$$\frac{d\psi}{d\phi} = -\frac{\partial\chi}{\partial\theta} \qquad \frac{d\theta}{d\phi} = \frac{\partial\chi}{\partial\psi} \qquad r = \sqrt{\frac{\psi}{\pi B_0}}$$

$$\chi = \chi_0(\psi) + \chi_1(\psi)\cos[m(\theta - \iota(\psi_0)\phi)] \qquad \psi_0 = 1.0$$

$$\iota_a = 0.3$$

$$\frac{d\chi_0}{d\psi} = \iota(\psi) = \iota_a\psi + \iota_b \qquad \iota_b = 0.7$$

TIME INDEPENDENT (TI) HAMILTONIAN AND CLASSICAL PENDULUM SOLUTION

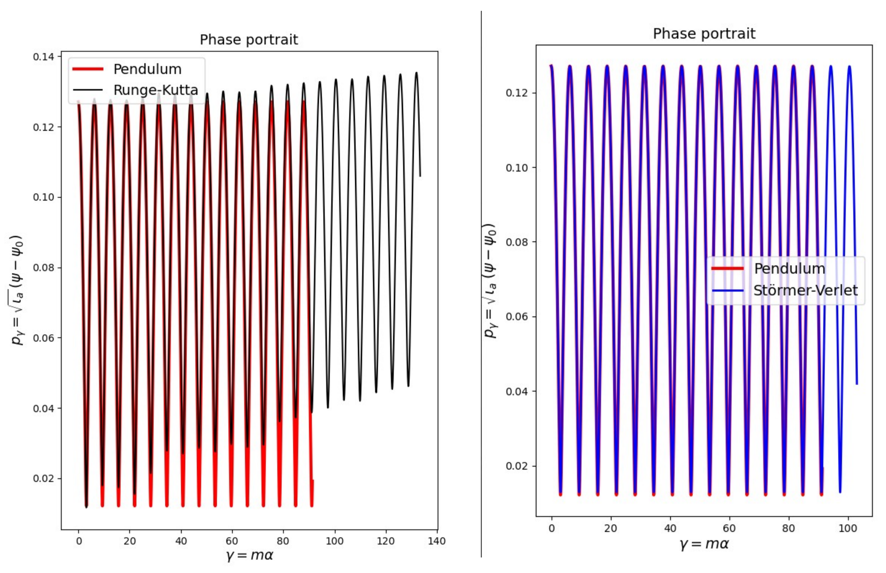
Use the frame, which rotates with the resonant transform: $\alpha = \theta - \iota(\psi_0)\phi$

$$\frac{d\psi}{d\phi} = -\frac{\partial\chi}{\partial\theta} = -\frac{\partial H}{\partial\alpha} \qquad \frac{d\alpha}{d\phi} = \frac{d\theta}{d\phi} - \iota(\psi_0) = \frac{\partial H}{\partial\psi}$$

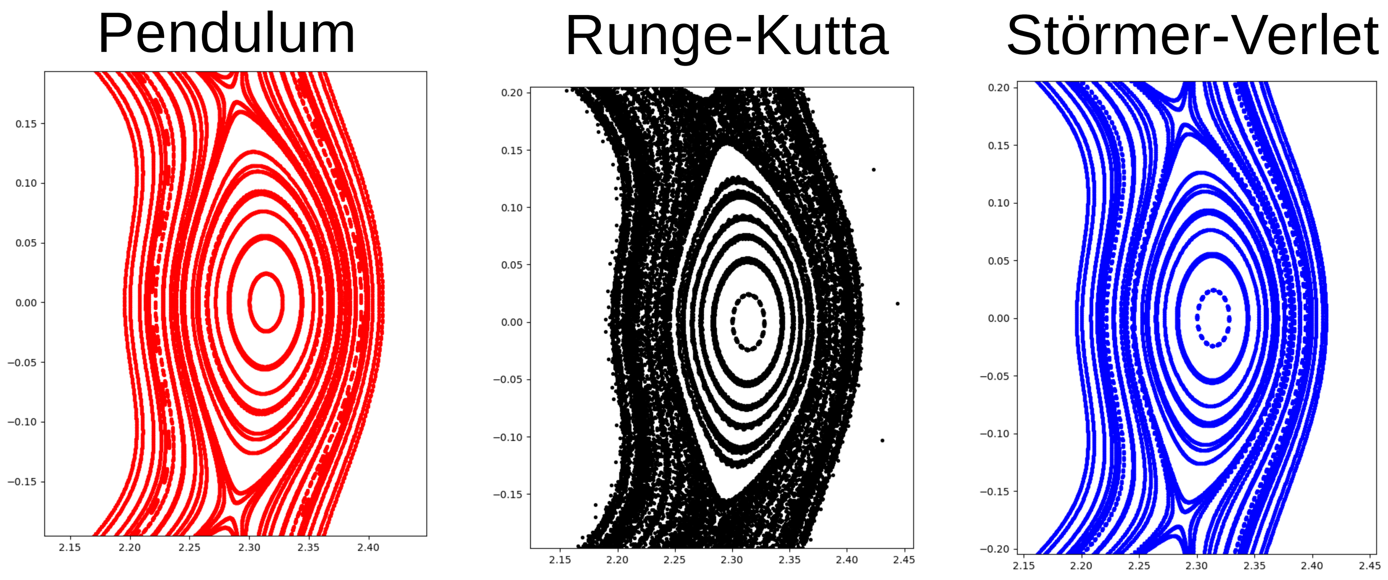
TI Hamiltonian (same as for the classical pendulum):

$$H = \frac{\iota_a(\psi-\psi_0)^2}{2} + A\cos(m\alpha) + \psi_0\left(\frac{\iota_a}{2}\psi_0 + \iota_b\right)$$

- The classical pendulum solution is expressed in terms of the **Jacobi elliptical functions**
- The Runge-Kutta numerical scheme **does not preserve energy** and diverges rapidly *(creates artificial stochasticity)*
- The energy preserving Störmer-Verlet scheme is more stable



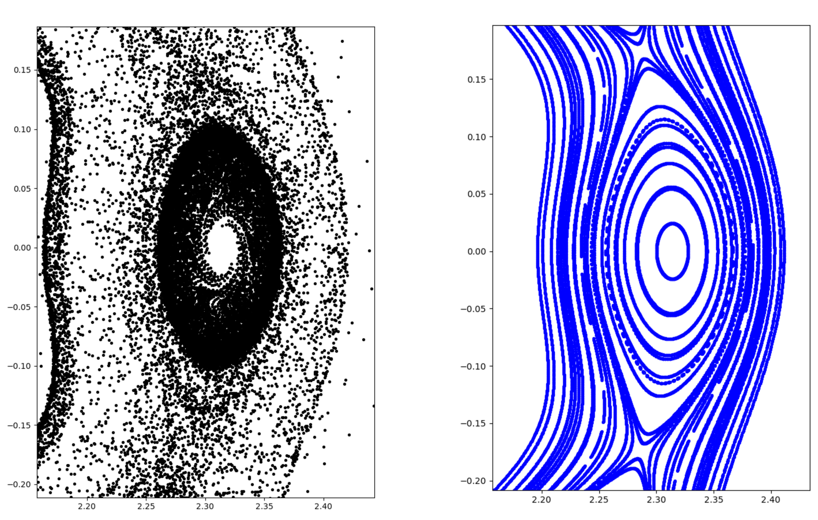
Pendulum Runge-Kutta Störmer-Verlet



TIME DEPENDENT (TD) HAMILTONIAN: NUMERICAL SCHEMES

- The Runge-Kutta performs even worse of TD Hamiltonian problem
- The energy preserving schemes (such is used in [1]) should be used for stochasticity studies

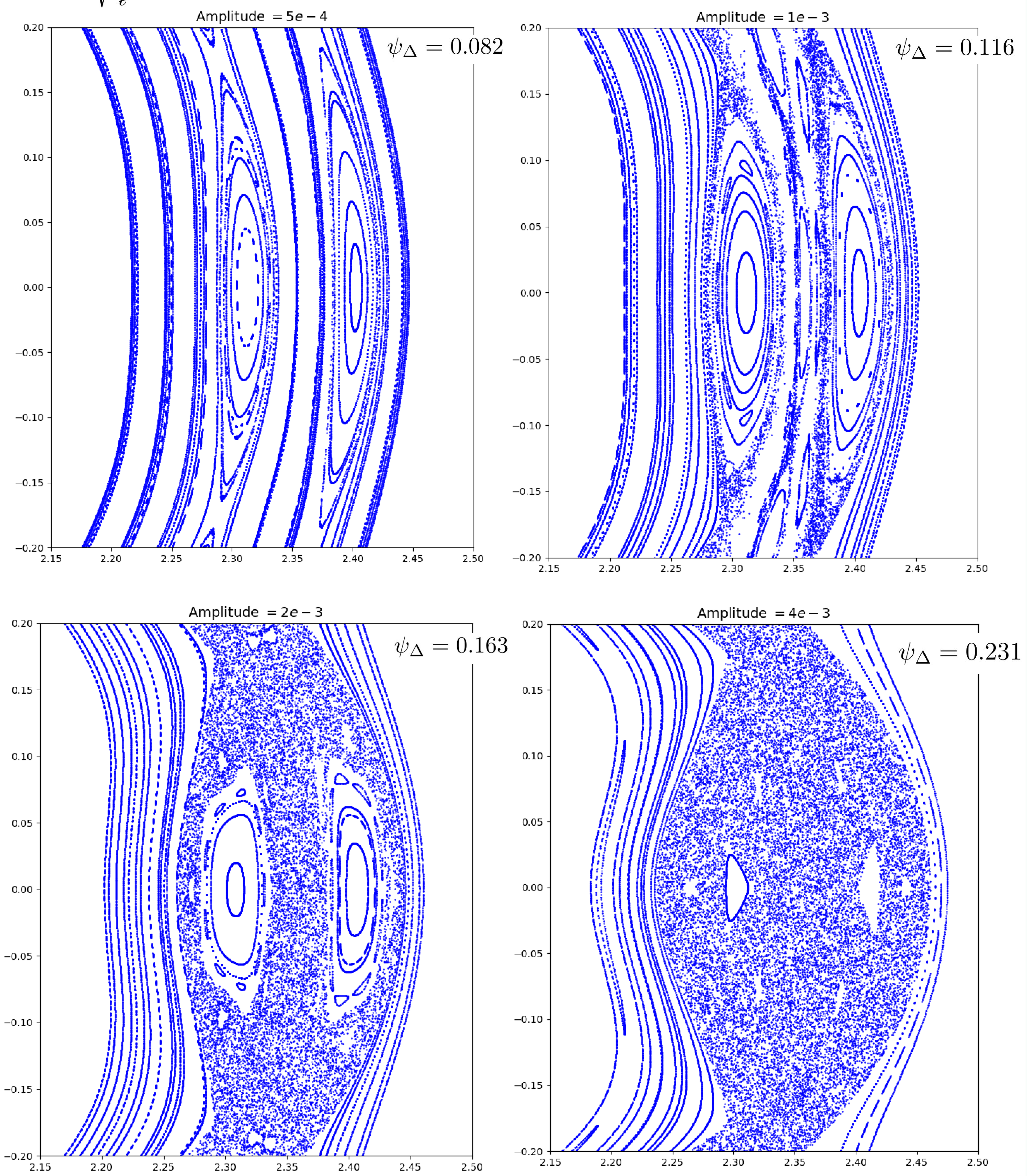
Runge-Kutta Störmer-Verlet



MAIN RESULTS: TWO ISLAND CHAIN OVERLAPING (STÖRMER-VERLET)

$$\chi = \psi\left(\frac{\iota_a}{2}\psi + \iota_b\right) + A\cos(m_1\theta - n_1\phi) + A\cos(m_2\theta - n_2\phi)$$

$$\psi_\Delta = 2\sqrt{\frac{A}{\iota'}} \qquad \psi_0^{(2)} - \psi_0^{(1)} = 0.33.. \qquad A^{overlap} = -2.08e-3 \qquad \begin{matrix} m_1 = 10 & m_2 = 10 \\ n_1 = 10 & n_2 = 11 \end{matrix}$$



- For A = 5e-4 the islands do not overlap and stochastic do not appear
- For A ≥ 1e-3 the islands start overlapping and creating stochastic regions, which can be investigated with the quadratic-flux-minimizing (QFMin) method and the EMC3-EIRENE code

FUTURE STEPS: QUADRATIC-FLUX-MINIMIZING (QFMIN) METHOD AND THE EMC3-EIRENE MODELING

- Future studies will concentrate on exploring the boundaries of the stochasticity level, where island divertor flows follow “ghost” surface structures.
- The QFMin [3] method will be employed to quantify the stochasticity level and form the “ghost” surfaces
- The EMC3-EIRENE code [4] will be utilized for scrape-off layer flows exploration.

OUTLOOK

- The 4-5 order explicit Runge-Kutta scheme is not suitable for stochasticity studies with Hamiltonian systems
- The 1 order area-preserving Störmer-Verlet scheme does not create substantial artificial stochasticity and can be used for the stochasticity studies
- The overlapping two island chains crate the stochasticity zones, which can further be investigated with the QFMin method and the EMC3-EIRENE code

[1] Alkesh Punjabi, Allen H. Boozer; Simulation of non-resonant stellarator divertor. Phys. Plasmas; 27 (1): 012503, (2020); https://doi.org/10.1063/1.5113907

[2] A. Baillod; Equilibrium pressure limits in stellarators. PhD Thesis, EPFL, p 221., (2023). https://doi.org/10.5075/epfl-thesis-10070

[3] S.R. Hudson, R.L. Dewar,, Are ghost surfaces quadratic-flux-minimizing?, Physics Letters A, Vol 373, 48, 2009, https://doi.org/10.1016/j.physleta.2009.10.005.

[4] Feng Y. et al, 3D fluid modelling of the edge plasma by means of a Monte Carlo technique, J. Nucl. Mater. 266–269 812–8, (1999), https://doi.org/10.1016/S0022-3115(98)00844-7