

0.1 Sets and Notation

We begin by defining the standard sets of numbers used throughout this book.

Definition 0.1.1 (Common Number Sets). We use the following notation for common sets of numbers:

- \mathbb{N} : The set of natural numbers $\{1, 2, 3, \dots\}$.
- \mathbb{Z} : The set of integers $\{\dots, -2, -1, 0, 1, 2, \dots\}$.
- \mathbb{Q} : The set of rational numbers.
- \mathbb{R} : The set of real numbers.
- \mathbb{C} : The set of complex numbers.

Definition 0.1.2 (Cartesian Product). Let A and B be sets. The **Cartesian product** of A and B , denoted $A \times B$, is the set of all ordered pairs:

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

More generally, for sets A_1, A_2, \dots, A_n , the Cartesian product is:

$$A_1 \times A_2 \times \dots \times A_n = \{(a_1, a_2, \dots, a_n) : a_i \in A_i \text{ for } i = 1, 2, \dots, n\}$$

Definition 0.1.3 (n -Tuple). An **n -tuple** is an ordered list of n elements. For a set A , the set of all n -tuples with entries from A is denoted:

$$A^n = \underbrace{A \times A \times \dots \times A}_{n \text{ times}} = \{(a_1, a_2, \dots, a_n) : a_i \in A \text{ for } i = 1, 2, \dots, n\}$$

Example (n -Tuples). Here are examples of n -tuples from various sets:

1. \mathbb{R}^2 : The Cartesian plane. Elements include $(0, 0)$, $(1, 2)$, $(-3, \pi)$, $(\sqrt{2}, -5.7)$.
2. \mathbb{R}^3 : 3-dimensional space. Elements include $(1, 0, 0)$, $(1, 2, 3)$, $(-1, \pi, e)$.
3. \mathbb{Z}^2 : Pairs of integers. Elements include $(0, 0)$, $(1, -2)$, $(5, 7)$. Note that $(\frac{1}{2}, 3) \notin \mathbb{Z}^2$.
4. \mathbb{C}^2 : Pairs of complex numbers. Elements include $(1 + i, 2 - 3i)$, $(i, 0)$, $(3, 4)$.
5. $\{0, 1\}^3$: Binary 3-tuples. This set has exactly 8 elements:

$$\{0, 1\}^3 = \{(0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1)\}$$

In general, if A is a finite set with $|A| = k$ elements, then A^n has k^n elements.