0.0 Sets and Notation

We begin by defining the standard sets of numbers used throughout this book.

Definition 0.1.1 (Common Number Sets)**.** We use the following notation for common sets of numbers:

- $\mathbb{N}$: The set of natural numbers $\{1, 2, 3, \ldots\}$.
- $\mathbb{Z}$: The set of integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$.
- $\mathbb{Q}$: The set of rational numbers.
- $\mathbb{R}$: The set of real numbers.
- $\mathbb{C}$: The set of complex numbers.

Definition 0.1.2 (Cartesian Product)**.** Let A and B be sets. The **Cartesian product** of A and B, denoted $A \times B$, is the set of all ordered pairs:

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

More generally, for sets $A_1, A_2, \ldots, A_n$, the Cartesian product is:

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) : a_i \in A_i \text{ for } i = 1, 2, \ldots, n\}$$

Definition 0.1.3 (n-Tuple)**.** An **n-tuple** is an ordered list of n elements. For a set A, the set of all n-tuples with entries from A is denoted:

$$A^n = \underbrace{A \times A \times \cdots \times A}_{n \text{ times}} = \{(a_1, a_2, \ldots, a_n) : a_i \in A \text{ for } i = 1, 2, \ldots, n\}$$

Example (n-Tuples). Here are examples of n-tuples from various sets:

1. $\mathbb{R}^2$: The Cartesian plane. Elements include $(0, 0)$, $(1, 2)$, $(-3, \pi)$, $(\sqrt{2}, -5.7)$.
2. $\mathbb{R}^3$: 3-dimensional space. Elements include $(1, 0, 0)$, $(1, 2, 3)$, $(-1, \pi, e)$.
3. $\mathbb{Z}^2$: Pairs of integers. Elements include $(0, 0)$, $(1, -2)$, $(5, 7)$. Note that $(\frac{1}{2}, 3) \notin \mathbb{Z}^2$.
4. $\mathbb{C}^2$: Pairs of complex numbers. Elements include $(1 + i, 2 - 3i)$, $(i, 0)$, $(3, 4)$.
5. $\{0, 1\}^3$: Binary 3-tuples. This set has exactly 8 elements:

$$\{0, 1\}^3 = \{(0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1)\}$$

In general, if A is a finite set with $|A| = k$ elements, then A^n has k^n elements.