MIS ALGORITHM FOR INTERVAL GRAPH
&
t-RANDOM GRAPH

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INTRODUCTION

This project has been implemented based on the algorithm to find the Maximum Independent Set using interval graphs according to the research journal “”. The algorithm mentioned in the journal is an easy way to compute the Maximum Independent Set of size k for n intervals. The time complexity of this algorithm is O(nlogk) . This problem is analyzed by considering the problem of finding the maximum independent set on a real line, that is a subset of non-overlapping intervals of maximum size k. The alternative approach to solve this problem is to sort the intervals by the right end point, then applying the linear time algorithm on it to compute a maximum independent set in O(nlogk). But if we don’t sort the right end point of every set, then the time complexity of the algorithm is O(nlogn). In this algorithm the execution time is dependent on the size k of the maximum independent set.

Maximum independent set

If no two vertices of the graph have a direct edge between them then it is called the Maximum independent set.

Interval Graph

It is a graph showing intersecting intervals on a graph.

DIVIDE AND CONQUER

In divide and conquer algorithm the problem is divided into subproblems and the subproblems are solved by recursively calling the subproblems and after the calculations are computed the results of the subproblems are conquered to get the result of the problem.

Using the divide and conquer the efficiency of the algorithm can be increased and the difficult problems can be solved easily.

DIVIDE AND CONQUER IN MAXIMUM INDEPENDENT SET

The algorithm gives the maximum independent set as an output for the interval graph using the divide and conquer strategy . In this the input data in the form of csv(comma separated values) is fed to the algorithm and this data is represented as a set of subsets. The main set is divided into three categories called as Snegative, Spositive and Scrossing. This classification is done by finding the median of any interval chosen randomly from the given intervals. The intervals which span the median are considered as Scrossing(Sc), the intervals which are less than the median are taken into Snegative(S-) , the intervals which are greater than the median are taken into the Spositive(S+). After the given sub intervals of the main set are divided as mentioned, MIS(S-) is recursively called and all the segments/intervals from the Scrossing are deleted except the one containing the rightmost left end point. If S+ is empty and Scrossing contains only the single interval then that interval is the MIS output and the rightmost element of the interval is returned(x).If any interval contains the value x then those intervals are deleted .If the S+ is contains the intervals then those intervals are merged with the Scrossing and the MIS function is called again.

MIS FOR t-RANDOM

In the calculation of MIS using t- random set, it takes n intervals as the input for different values of t and n, where t values lies between 0 and 1($0 < t \leq 1$).

Generation of random Input

First the left endpoints of the intervals are generated randomly using the random function and using these left endpoints we will generate the right endpoints of the interval. For each of the left endpoints generated we will calculate the intermediate value vi where the vi values depend upon the t values. t - values for n interval input is taken as $\{1/n, 1/\sqrt{n}, 1/\log(n), 1/4\}$.Vi values in each interval lie between 0 and t . The right end point are now given by $\min(l_i+v_i, 1)$

Now the intervals are generated randomly $[l_i, r_i=\min(l_i+v_i, 1)]$.

Maximum independent set is calculated for the input generated.

GRAPH ANALYSIS

From the fig 1 below the graph is generated for different sizes of maximum independent sets output for various li and corresponding different sets ri values. The graph with the red color represents the number of MIS sets generated for the li and ri values. The ri value generated in this case is a minimum of li+vi and 1, where vi is randomly chosen between 0 and t. The t in this case is taken as 1/n where n is the number of intervals. This red colored graph has the maximum gradient among all other graphs which are generated for different values of t. The more gradient indicates that as more the number of intervals increases the more the number of maximum independent sets we are expected to get. We can see the asymptotic behavior here, that is when the number of intervals tends to increase to infinite, where we might have an infinite number of random intervals in a set.This gives a directly proportional relation between the number of maximum independent sets and the number of random intervals. At some point these two become equivalent to each other and the proportionality constant becomes 1. The next color graph which has very less gradient compared to this graph is the orange. This graph is generated by considering the t value equivalent to 1/square root of n. This graph also approaches the state where the number intervals are equal to the number of maximum independent sets but this takes more number of random sets to be generated compared to the above one. The other two graphs blue and green which are generated by considering the t value as 1/log(n) and ¼ respectively, their number of maximum independent sets grow gradually as the number of intervals are increased.

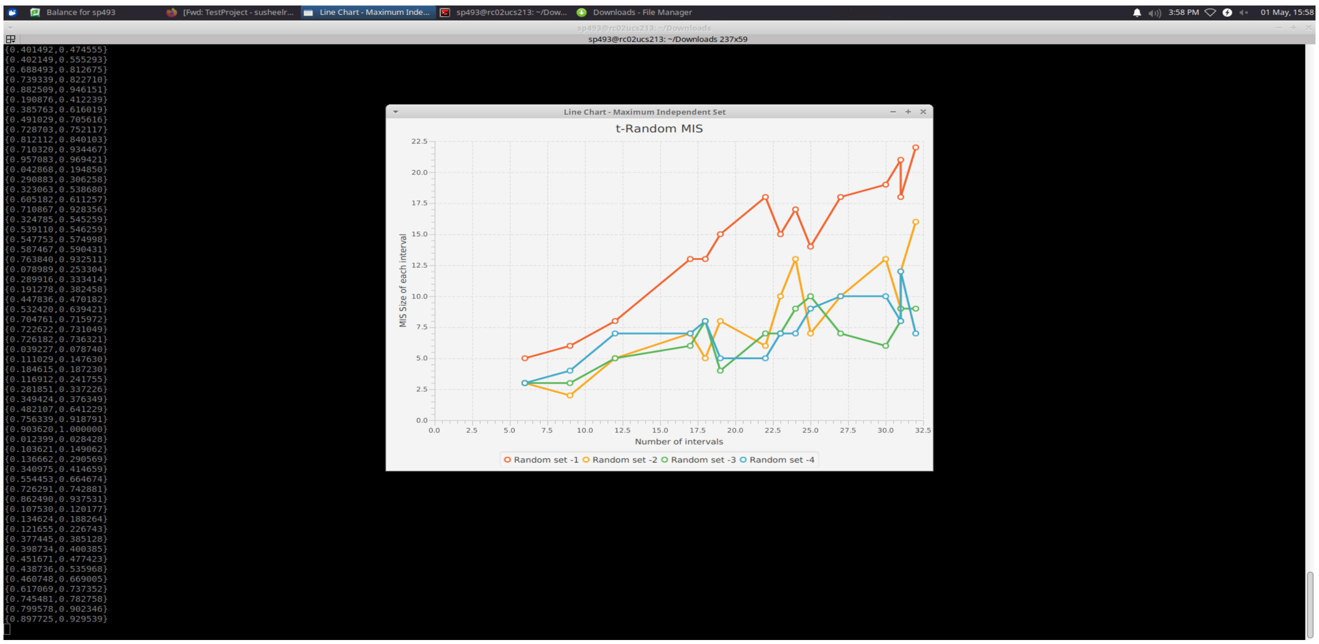
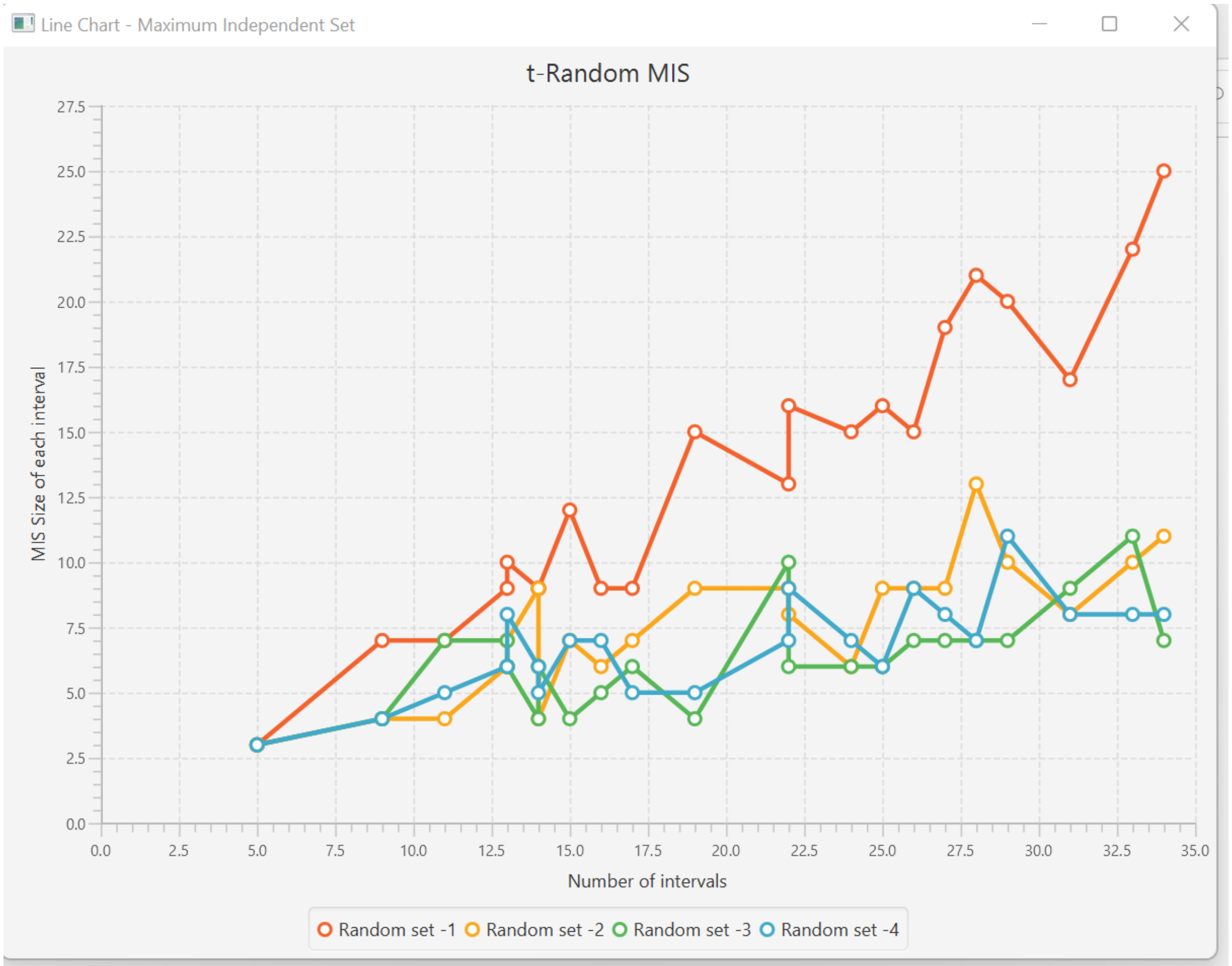


Figure 1 Graph for t- Random MIS


Line Chart - Maximum Independent Set
t-Random MIS
MIS Size of each interval
27.5
25.0
22.5
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
22.5
25.0
27.5
30.0
32.5
35.0
Number of intervals
Random set -1
Random set -2
Random set -3
Random set -4