Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

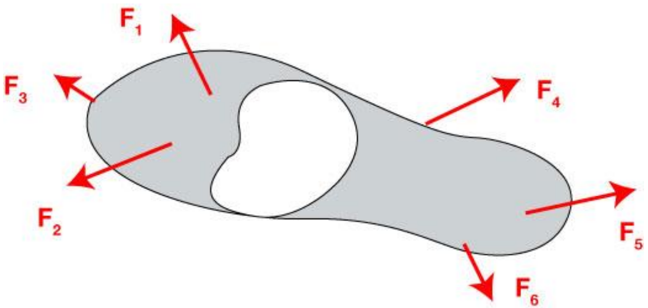
Dr. Wayne Whiteman
Senior Academic Professional and Director of the Office of Student Services
Woodruff School of Mechanical Engineering

Georgia Institute of Technology

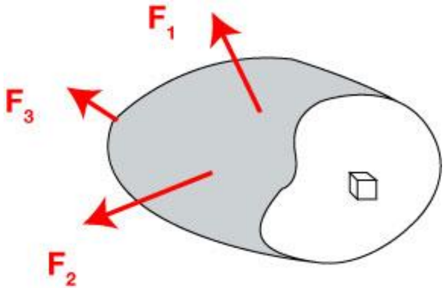
Module 26 Learning Outcome

- Describe the procedure for finding the principal stresses and principal planes for a general three-dimensional (3D) state of stress at a point by solving the eigenvalue problem

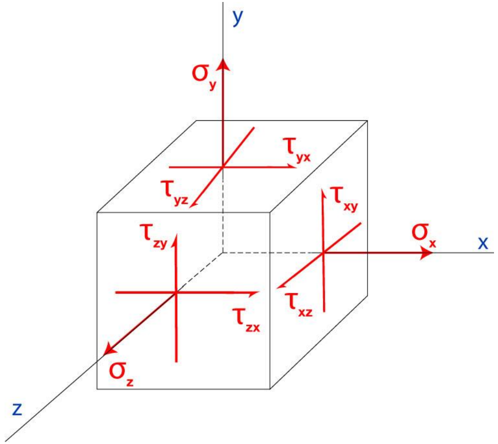
General 3D State of Stress at a Point
(Arbitrarily Loaded Member)



- For more complicated structural members , the stress distributions may not be uniform on arbitrary planes
- For an infinitesimally small point, the stress distribution approaches uniformity
- An infinite number or planes can be passed through each point.
- But, it can be shown that three mutually perpendicular planes is sufficient to completely describe the state of stress at any point for any orientation. (Hence we will use a cube to represent the state of stress at a point.)



3D State of Stress at a Point
(shown in positive sign convention)



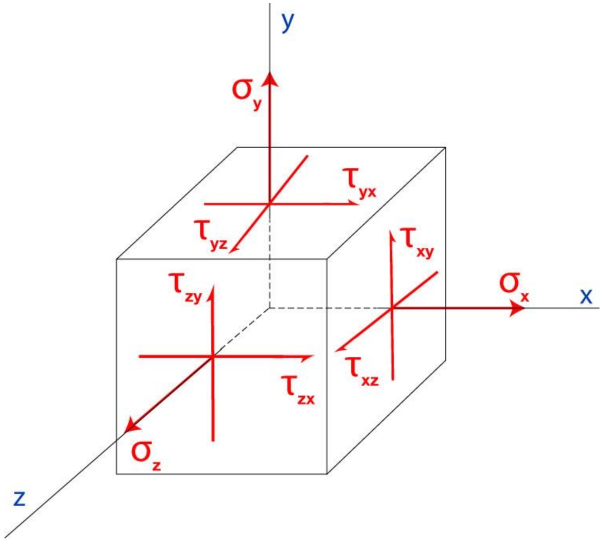
$$\tau_{xy} = \tau_{yx}$$

$$\tau_{yz} = \tau_{zy}$$

$$\tau_{xz} = \tau_{zx}$$

- **Stress is a tensor**
- **A tensor represents a physical/geometric property/quantity by a mathematical idealization of an array of numbers**
  (see Module 20 of my course "Advanced Engineering Systems in Motion: Dynamics of 3D Motion" for a more detailed discussion of tensors)

3D State of Stress at a Point
(shown in positive sign convention)



$$\tau_{xy} = \tau_{yx}$$

$$\tau_{yz} = \tau_{zy}$$

$$\tau_{xz} = \tau_{zx}$$

Matrix Notation:

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

3D State of Stress at a Point

See Modules 24, 25, 26 of my course "Advanced Engineering Systems in Motion: Dynamics of 3D Motion"

Matrix Notation:

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

Mathematically analogous to:

Inertia Matrix

$$\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

Given a general set of stresses if, for a particular coordinate orientation of the stress block, the shear stresses vanish, we arrive at principal normal stresses acting on principal planes

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

If for a particular coordinate orientation, the products of inertia vanish, we arrive at principal moments of inertia wrt principal axes

$$\begin{bmatrix} I_{xx}' & 0 & 0 \\ 0 & I_{yy}' & 0 \\ 0 & 0 & I_{zz}' \end{bmatrix}$$

3D State of Stress at a Point

Matrix Notation:

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

Given a general set of stresses if, for a particular coordinate orientation of the stress block, the shear stresses vanish, we arrive at principal normal stresses acting on principal planes

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

Solve via the Eigenvalue Problem

where the eigenvalues are the principal stresses (maximum, minimum, and one in-between)

the corresponding eigenvectors are three sets of direction cosines which define the normals to the three principal planes where the principal stresses occur