Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

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Mechanics of Materials I:
Fundamentals of Stress & Strain and Axial Loading

- ✓ Internal Forces due to External Loads
- ✓ Axial Centric Loads
- ✓ Normal Stress and Shear Stress
- ✓ General State of Stress at a Point (3D)
- ✓ Plane Stress (2D)
- ✓ Normal Strain and Shear Strain
- ✓ Stress-Strain Diagrams
- ✓ Mechanical Properties of Materials
- ✓ Linear Elastic Behavior, Hooke's Law, and Poisson's Ratio
- ✓ Stresses on Inclined Planes
- ✓ Principal Stresses and Max Shear Stress
- ✓ Mohr's Circle for Plane Stress
- ✓ Stress Concentrations
- ✓ Mohr's Circle for Plane Strain
- ✓ Strain Transformation and Measuring Strains
- ❑ Factor of Safety and Allowable Stresses/Loads
- ❑ Nonlinear Behavior and Plasticity
- ❑ Statically Indeterminate Structures
- ❑ Thermal and Pre-strain Effects

Module 35 Learning Outcome

- Calculate in-plane strains based on measurements using experimental analysis techniques

Experimental Strain Measurement

Strains are often easier to measure than stress

Therefore, we often use experimental analysis techniques to measure strains

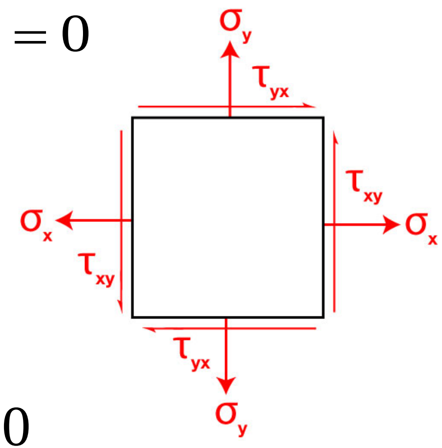
Strains are often measured on a free (unstressed) surface of a member

This is the condition of “Plane Stress”

Recall Two-Dimensional (2D) or Plane Stress

$$\sigma_z = \tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0$$

Even though there is no stress in the z-direction, there is a strain in the z-direction and it is a principal strain (There are no shear strains on the z-face) (It is a free surface)



$$\therefore \ \gamma_{xz} = \gamma_{yz} = 0 \qquad \varepsilon_z \neq 0$$

For small deformations, it can be shown that the in-plane normal strains are not affected by this out-of-plane displacement and the Plane Strain Equations and the Mohr’s Circle for Plane Strain representation are valid for the case of plane stress when measurements are made on a free surface.

Transformation equations of Plane Strain apply to strains in Plane Stress!

Similarly, transformation equations of Plane Stress can be used for stresses in Plane Strain!

Electrical resistance strain gages

Changes in length of strain gage/specimen

↓

Changes in electrical resistance are measured

↓

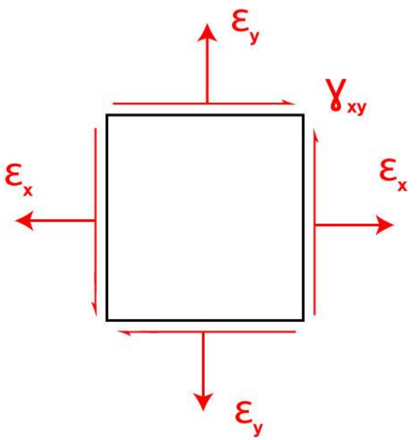
These electrical resistance measurements are calibrated to strain

↓

Then any reading can be converted to strain

Electrical resistance strain gages

Electrical resistance measurements are calibrated to measure strain



Strain gages measure normal strain.
Shear strain can then be calculated by measuring normal strains in 3 different directions and using the strain transformation equations to calculate shear strain.

Therefore strain gage rosettes are used.
Standard grades are 45 degrees and 60 degrees.
Positive gage orientation angles are measured counter-clockwise from the x-axis

Strain gage rosettes

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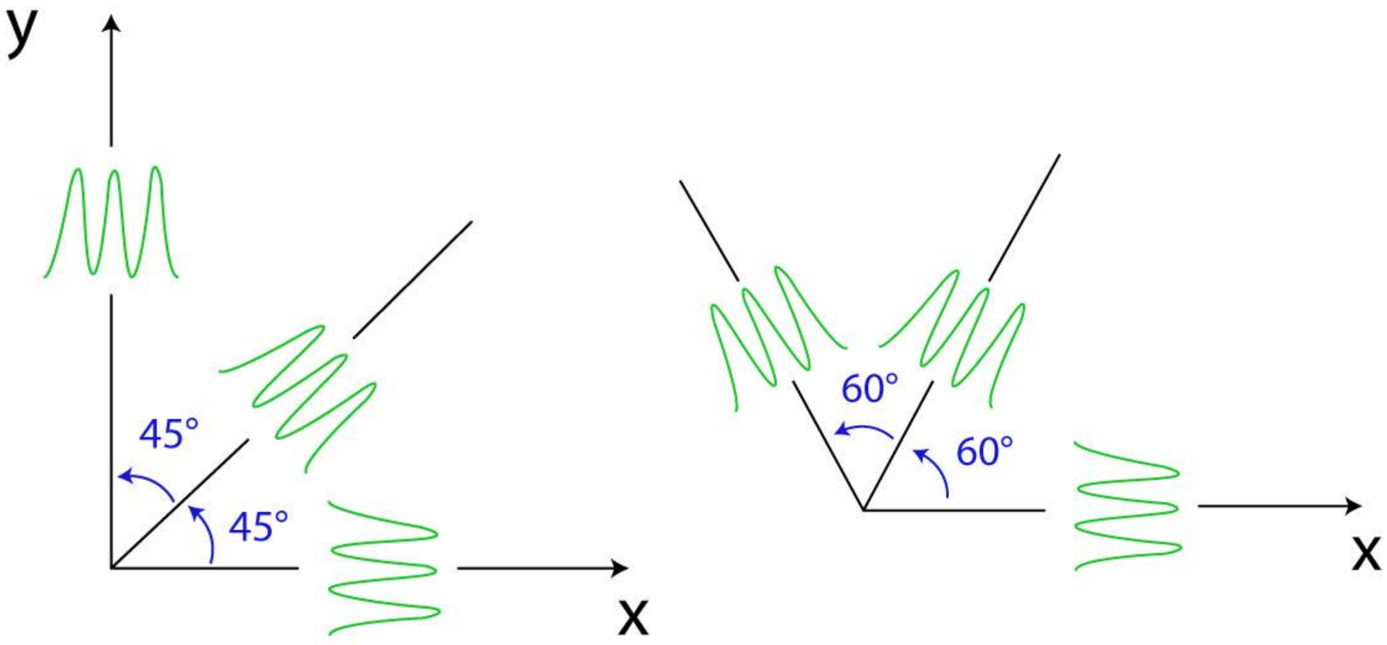
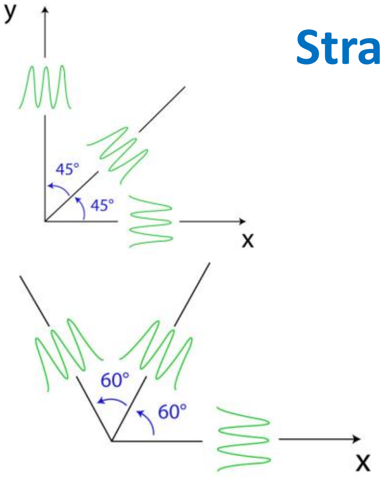
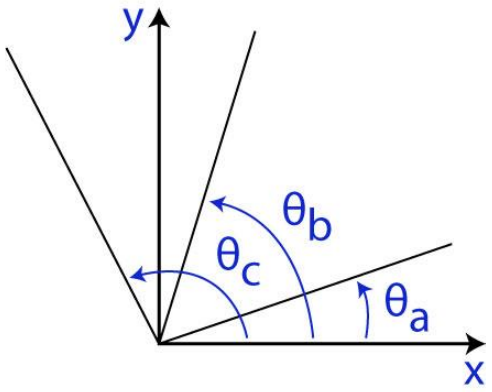
Positive gage orientation angles are measured counter-clockwise from the x-axis



Strain gage rosettes





Recall Normal Strain Transformation Equation

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

measured

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a\cos\theta_a$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b\cos\theta_b$$

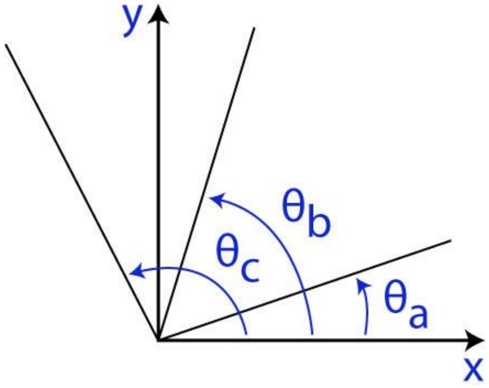
$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c\cos\theta_c$$

Solve 3 equations

For 3 unknowns

$$\varepsilon_x,\ \varepsilon_y,\ \gamma_{xy}$$

Strain gage rosettes



$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$

measured

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$$

$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy}\sin\theta_c\cos\theta_c$$

Solve 3 equations
For 3 unknowns

$$\varepsilon_x,\ \varepsilon_y,\ \gamma_{xy}$$

Now we can solve for in-plane principal strains/planes and max in-plane shear stress