Mechanics of Materials III:

Beam Bending

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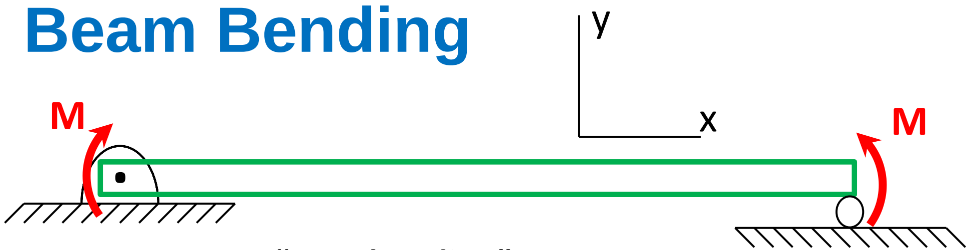
Module 7 Learning Outcome

- Derive the strain-curvature relationship for pure beam bending

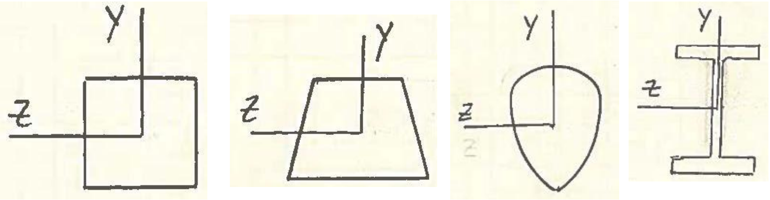
Beam Bending



"Pure bending"
Flexure under constant bending moment
No shear force

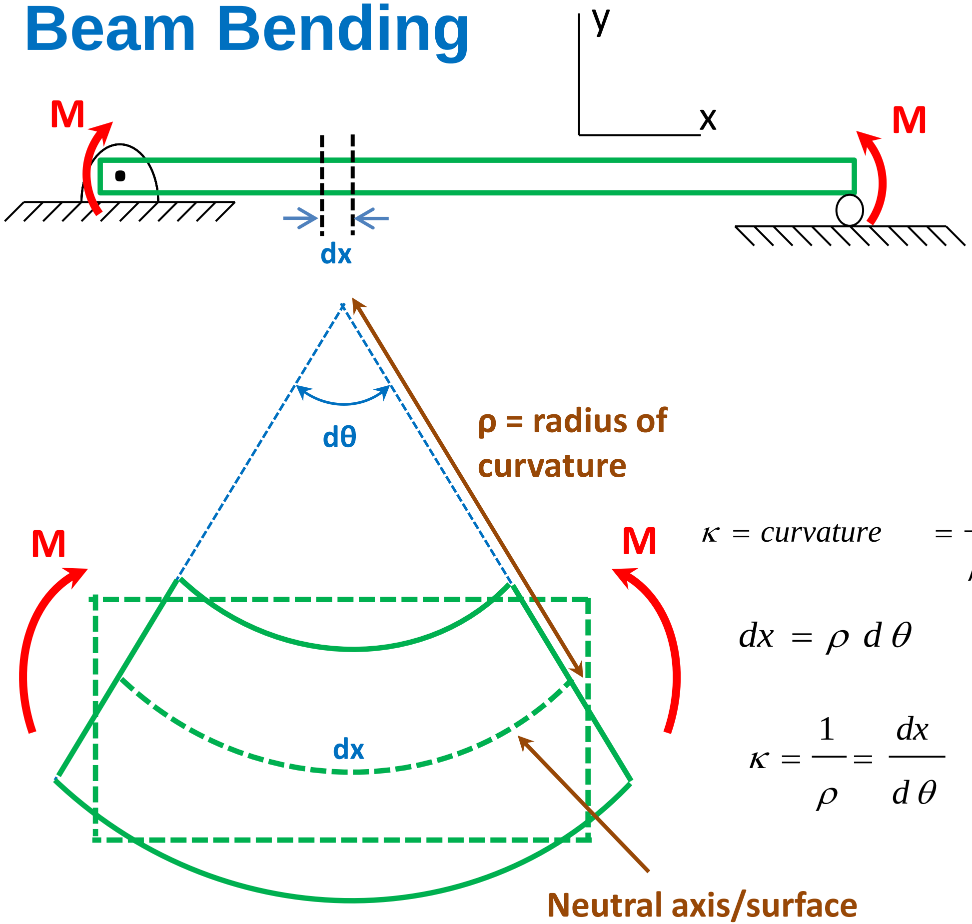
Assumptions:

- Symmetric about x-y plane (plane of bending)



- Plane sections remain plane
- No twisting
- No buckling
- Small deflections

Beam Bending

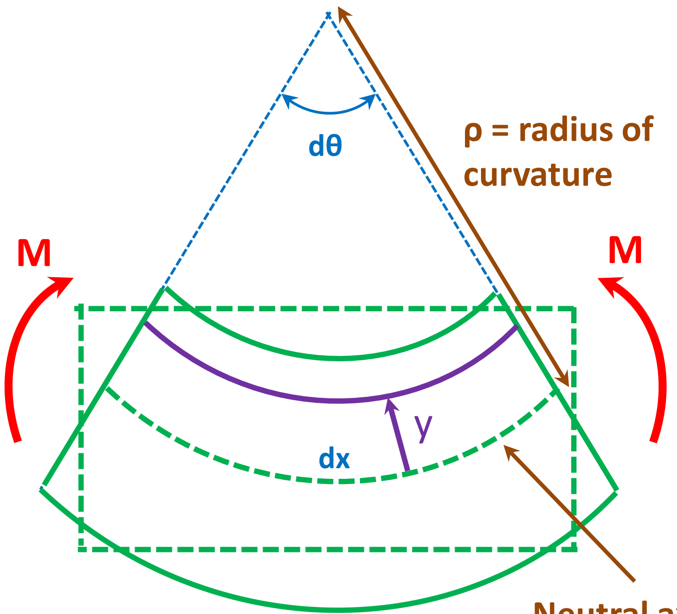


$$\kappa = curvature = \frac{1}{\rho}$$

$$dx = \rho \, d\theta$$

$$\kappa = \frac{1}{\rho} = \frac{dx}{d\theta}$$

Beam Bending



Neutral axis/surface

Recall: Normal Strain

Elongation per unit length

$$\varepsilon = \frac{\delta}{L}$$

$$\kappa = curvature \quad = \frac{1}{\rho}$$

$$dx = \rho \; d\,\theta$$

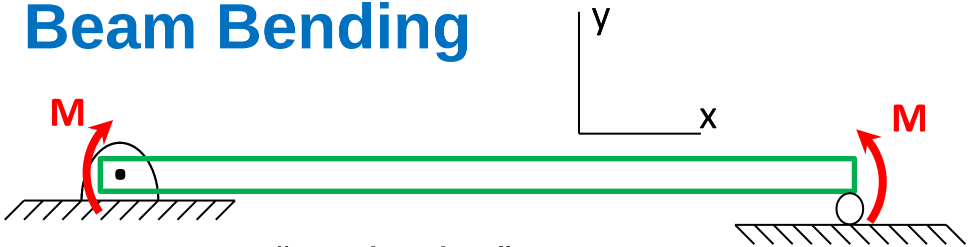
$$\left(\rho - y\right) d\theta = dx - \frac{y}{\rho} dx$$

$$d\theta = \frac{dx}{\rho}$$

Strain-Curvature Relationship

$$\varepsilon_x = \frac{-\frac{y}{\rho} d\cancel{x}}{d\cancel{x}} = -\frac{y}{\rho} = -\kappa\ y$$

Beam Bending



"Pure bending"
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Strain-Curvature Relationship

$$\varepsilon_x = -\frac{y}{\rho} = -\kappa y$$

Strain Sign Convention

(+) elongation
(-) shortening

Strain is proportional to curvature and varies linearly with distance, y, from the neutral axis.

Independent of material

Note: There are strains in the y and z direction due to Poisson's effect, but no stresses because the beam is free to deform laterally.

Therefore pure bending in beams produces uniaxial stress.

We'll start looking at the stresses next time!