Mechanics of Materials II:

Thin-Walled Pressure Vessels and Torsion

Dr. Wayne Whiteman
Senior Academic Professional and Director of the Office of Student Services
Woodruff School of Mechanical Engineering

Georgia Institute of Technology

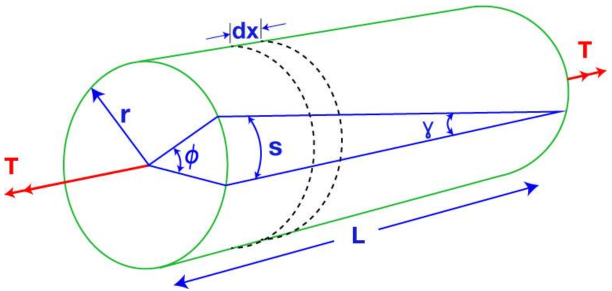
Mechanics of Materials II: Thin-Walled Pressure Vessels and Torsion

- ✓ Thin-Walled Pressure Vessels - Internal Pressure
- ✓ Torsional Shearing Stress and Strain
- ❑ Elastic Torsion Formula
- ❑ Elastic Torsion of Straight, Cylindrical Shafts
- ❑ Inelastic Torsion of Straight, Cylindrical Shafts
- ❑ Statically Indeterminate Torsion Members

Module 12 Learning Outcome

- Derive the Elastic Torsion Formula

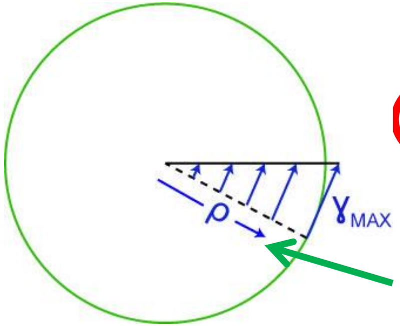
Circular Bar Torsion



Torsional Shear Strain at Outer Surface

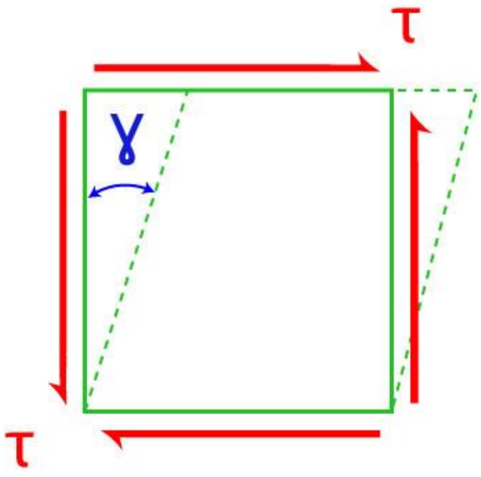
$$\gamma_{MAX} = \frac{r\phi}{L} = \frac{r\,d\phi}{dx} = r\theta$$

Shear Strains vary linearly with ρ



$$\gamma = \rho\theta = \frac{\rho}{r}\gamma_{MAX}$$

radial distance from center

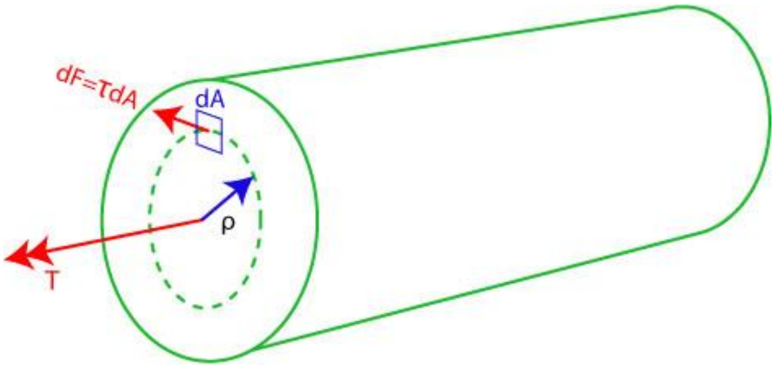


Shear Stresses also vary linearly with ρ

$$\tau_{MAX} = Gr\theta$$

$$\tau = \frac{\rho}{r}\tau_{MAX}$$

Now let's relate Shear Stress to the Applied Torque



$$\tau = \frac{\rho}{r}\tau_{MAX}$$

$$dT = (dF)\rho = \tau\rho\, dA = \frac{\tau_{MAX}}{r}\rho^2\, dA$$

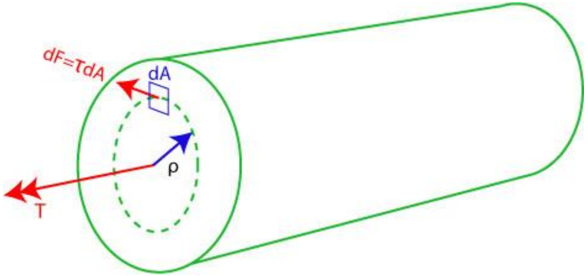
$$T = \int_A dT = \frac{\tau_{MAX}}{r}\underbrace{\int_A \rho^2\, dA}$$

$$J \equiv Polar\ \ Moment\ \ of\ \ Inertia$$

$$J = \int_A \rho^2\, dA$$

Now let's relate Shear Stress to the Applied Torque

Georgia Tech



$$T = \int_A dT = \frac{\tau_{MAX}}{r} \underbrace{\int_A \rho^2 \, dA}$$

$$J \equiv Polar\ \ Moment\ \ of\ \ Inertia$$

$$J = \int_A \rho^2 \, dA$$

$$T = \frac{\tau_{MAX}}{r} J$$

$$\tau_{MAX} = \frac{T\,r}{J}$$

recall $\tau = \frac{\rho}{r} \tau_{MAX}$

$$\tau_{MAX} = \frac{r}{\rho} \tau = \frac{T\,r}{J}$$

Elastic Torsion Formula

$$\tau = \frac{T\,\rho}{J}$$

τ is the shearing stress on a transverse plane at a distance ρ from the center axis

T is the resisting torque (generally obtained from the FBD and equilibrium equation)

Note that T (resisting Torque) is greater for larger J, polar moment of inertia.

J, the polar moment of inertia, is larger when we have more area further from the axis of rotation

Therefore J, the polar moment of inertia, is a measure of the resistance to twisting/torsion