


# Mechanics of Materials I:

## Fundamentals of Stress & Strain and Axial Loading

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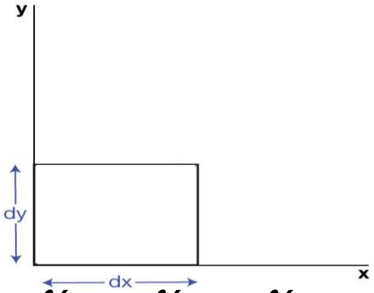
# Module 31 Learning Outcome

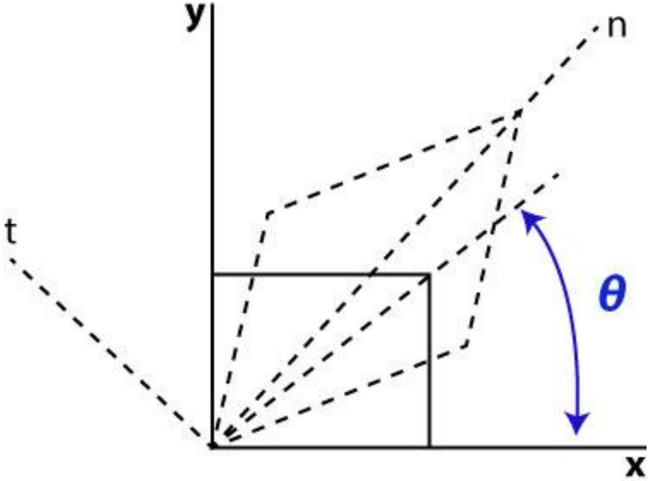
- Derive the strain transformation equations for the case of plane strain (continued)

# Plane Strain




$$\varepsilon_z = \gamma_{xz} = \gamma_{zx} = \gamma_{yz} = \gamma_{zy} = 0$$

**In general,** $\varepsilon_x\,,\ \ \varepsilon_y\ ,\ and\ \ \gamma_{xy} = \gamma_{yx}$ **are known or can be found**




**Find:** $\varepsilon_n, \gamma_{nt}\ \ for\ \ any\ \ angle\ \ \theta$

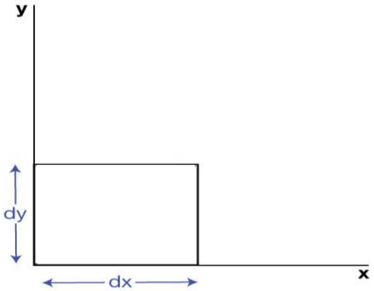
**Using**
**Law of Cosines,**
**Geometry,**
**and Trig Identities**

## Normal Strain Transformation Equation

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$$

$$\varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

# Plane Strain

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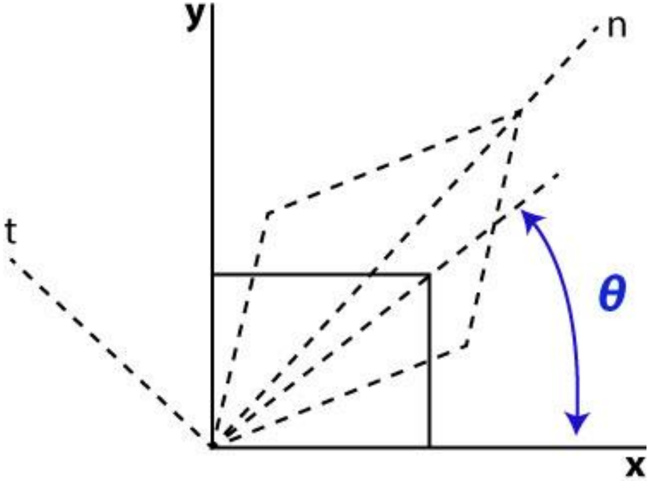
**In general,** $\varepsilon_x, \ \varepsilon_y, \ and \ \gamma_{xy} = \gamma_{yx}$ **are known or can be found**




**Find:** $\varepsilon_n, \gamma_{nt} \ \ for \ any \ angle \ \theta$

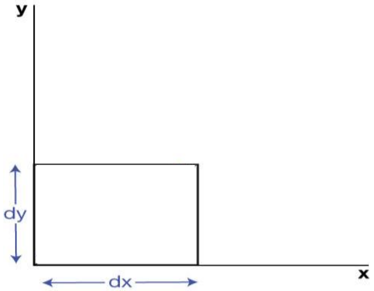
**Similarly using Law of Sines, Geometry, and Trig Identities the following shear strain transformation equations can be derived.**

## Shear Strain Transformation Equation

$$\gamma_{nt} = -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

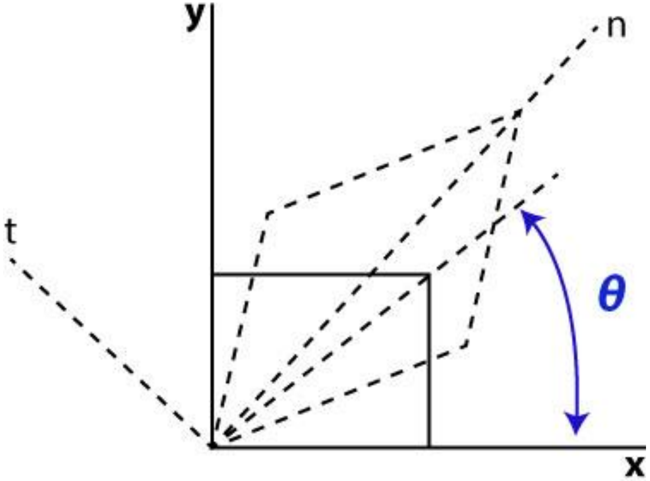
$$\frac{\gamma_{nt}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

# Plane Strain




$$\varepsilon_z = \gamma_{xz} = \gamma_{zx} = \gamma_{yz} = \gamma_{zy} = 0$$

**In general,** $\varepsilon_x$ , $\varepsilon_y$ , *and* $\gamma_{xy} = \gamma_{yx}$ **are known or can be found**




**Find:** $\varepsilon_n, \gamma_{nt}$ *for any angle* $\theta$

## Normal Strain Transformation Equation

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$\varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

## Shear Strain Transformation Equation

$$\gamma_{nt} = -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

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