Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

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Module 4 Learning Outcome

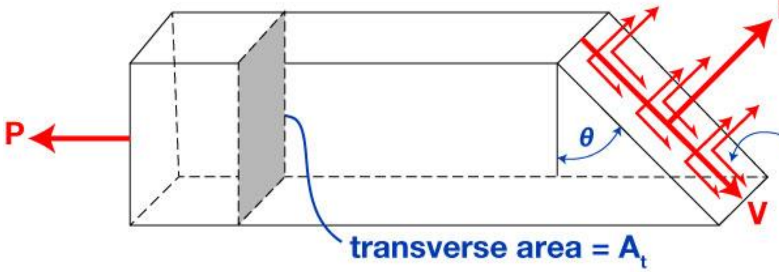
- Determine the maximum normal and shear stresses on inclined planes for the case of uniaxial loading

Maximum Normal and Shear Stresses on Inclined Planes for Uniaxial Loading

(For uniaxial loading, the structural member is subjected to simple tension or compression)

By Similar Triangles:

Maximum Normal and Shear Stresses on Inclined Planes for Uniaxial Loading



$$N = \frac{\sigma A_t}{\cos\theta}$$

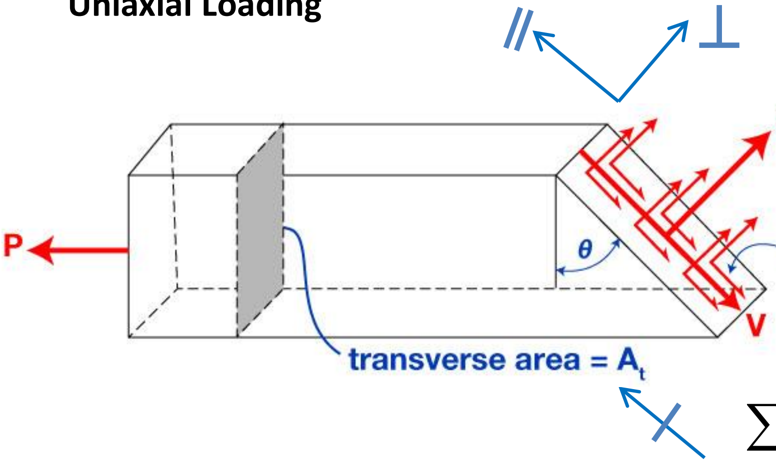
$$A = \frac{\tau A_t}{\cos\theta}$$

$$A = \frac{A_t}{\cos\theta}$$

$$\tau = \frac{V}{A}$$

$$V = \tau A = \frac{\tau A_t}{\cos\theta}$$

Maximum Normal and Shear Stresses on Inclined Planes for Uniaxial Loading



$$N = \frac{\sigma A_t}{\cos\theta}$$

$$A = \frac{\tau A_t}{\cos\theta}$$

$$\sum F = 0$$

$$P\sin\theta - V = 0$$

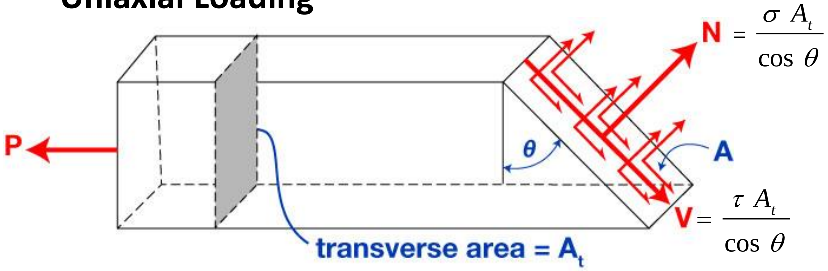
$$P\sin\theta = V = \frac{\tau A_t}{\cos\theta}$$

$$\frac{1}{2}\sin 2\theta$$

$$\tau = \frac{P}{A_t}(\sin\theta\cos\theta)$$

$$\tau = \frac{P}{2A_t}\sin 2\theta$$

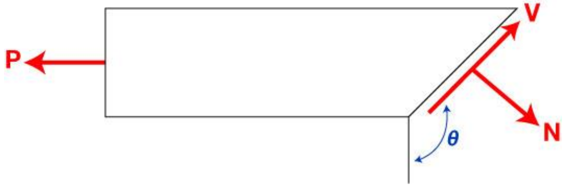
Maximum Normal and Shear Stresses on Inclined Planes for Uniaxial Loading



$$\sigma = \frac{P}{A_t}\cos^2\theta$$

$$\tau = \frac{P}{2A_t}\sin 2\theta$$

$$\tau_{MAX} \quad occurs \quad if \quad \theta = 45°, 135°$$

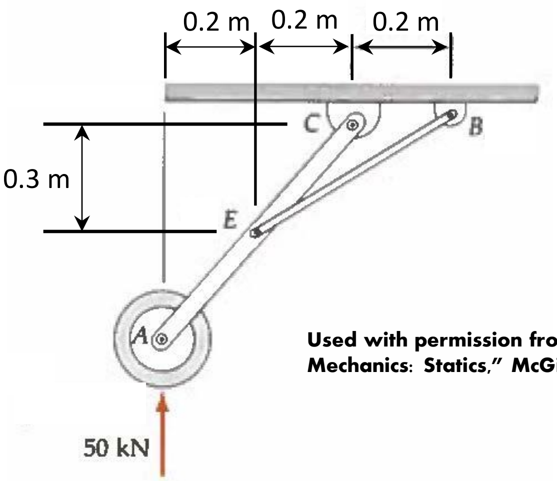


(Note: The sign of Shear Stress changes
for θ > 90 degrees
and the Shear Force vector changes
direction.)

Worksheet:

For the simple model of the engineering structure shown, the bar BE is a 50 mm diameter round steel bar. You may neglect the weight of the individual members and the wheel in your analysis.

a) For a transverse cut of the bar, find the normal stress in member BE
b) For a non-transverse cut of the bar at an angle of 30 degrees, find the normal stress and the shear stress in member BE



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Worksheet: SOLUTION

For the simple model of the engineering structure shown, the bar BE is a 50 mm diameter round steel bar. You may neglect the weight of the individual members and the wheel in your analysis.

a) For a transverse cut of the bar, find the normal stress in member BE

b) For a non-transverse cut of the bar at an angle of 30 degrees, find the normal stress and the shear stress in member BE

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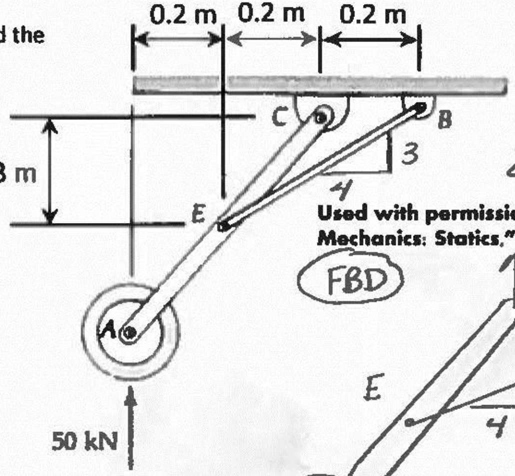

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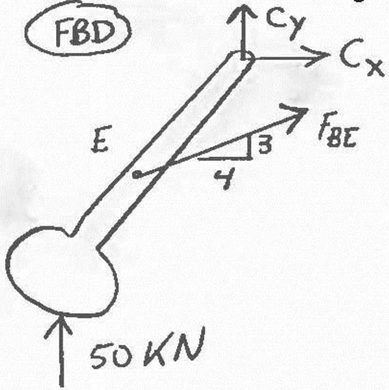
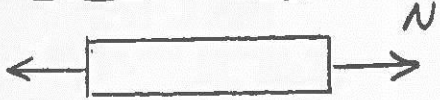
MEMBER BE IS A 2 FORCE MEMBER

FBD



TRANSVERSE CUT



$$\sigma = \frac{N}{A} = \frac{167}{1963} = 0.085 \frac{kN}{mm^2} \ (T)$$

$$= \underline{\underline{85 \ MPa \ (T)}} \quad ANS$$

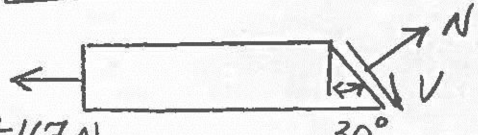
TRANSVERSE AREA

$$A_t = \pi r^2$$

$$= \pi (25)^2$$

$$A_t \doteq 1963 \ mm^2$$

NON-TRANSVERSE CUT



$$\sigma = \frac{P}{A_t} \cos^2\theta = \frac{167}{1963} \cos^2 30^\circ$$

$$\underline{\underline{\sigma = 0.0637 \frac{kN}{mm^2} (T) = 63.7 \ MPa (T)}} \ ANS$$

$$\tau = \frac{P}{2A_t} \sin 2\theta$$

$$\tau = \frac{167}{2(1963)} \sin[2(30^\circ)]$$

$$\tau = 0.0368 \ KN/mm^2$$

$$\underline{\underline{= 36.8 \ MPa}} \ ANS.$$

$$\circlearrowleft + \ \Sigma M_c = 0$$

$$-50(0.4) + \left(\tfrac{4}{5}\right) F_{BE}(0.3) - \left(\tfrac{3}{5}\right) F_{BE}(0.2) = 0$$

$$F_{BE} = 167$$

$$\underline{\vec{F}_{BE} = 167 \ \text{kN (T)}}$$