



# Mechanics of Materials II:

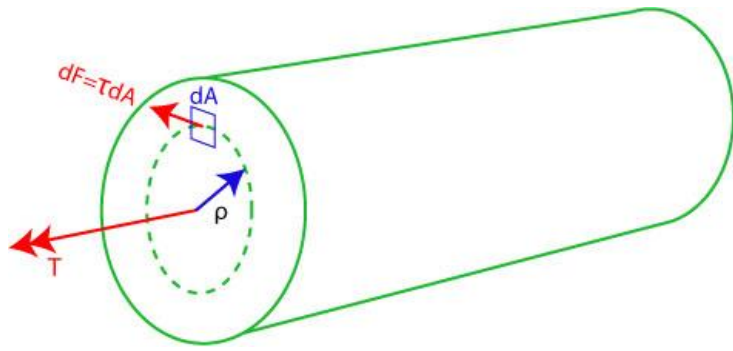
## Thin-Walled Pressure Vessels and Torsion

Dr. Wayne Whiteman

Senior Academic Professional and Director of the Office of Student Services  
Woodruff School of Mechanical Engineering

## Module 13 Learning Outcome

- Calculate the Polar Moment of Inertia for circular cross-sections



$$T = \int_A dT = \frac{\tau_{MAX}}{r} \underbrace{\int_A \rho^2 dA}$$

$J \equiv$  Polar Moment of Inertia

$$J = \int_A \rho^2 dA$$

**Elastic Torsion Formula**

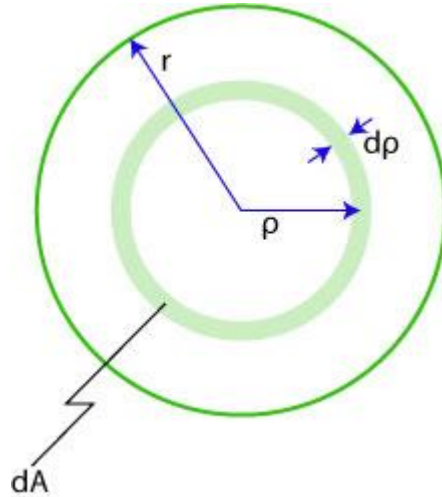
$$\tau = \frac{T \rho}{J}$$

# Polar Moment of Inertia, J

$J \equiv \text{Polar Moment of Inertia}$

$$J = \int_A \rho^2 dA$$

$$dA = 2\pi \rho d\rho$$

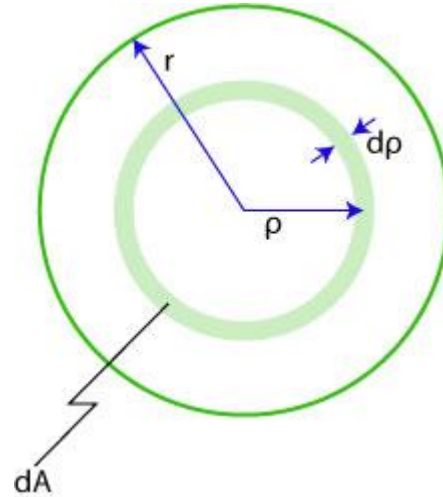


$$J = \int_0^r \rho^2 2\pi \rho d\rho = 2\pi \int_0^r \rho^3 d\rho = \frac{\pi r^4}{2}$$

# Polar Moment of Inertia, J

$J \equiv \text{Polar Moment of Inertia}$

$$J = \int_A \rho^2 dA$$



**Solid Circular Cross Section**

$$J = \frac{\pi r^4}{2} = \frac{\pi \left(\frac{D}{2}\right)^4}{2} = \frac{\pi D^4}{32}$$

**Hollow Circular Cross Section**

Often more efficient  
for torsion

$$J = \frac{\pi}{2} (r_{\text{outside}}^4 - r_{\text{inside}}^4) = \frac{\pi}{32} (D_{\text{outside}}^4 - D_{\text{inside}}^4)$$