Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

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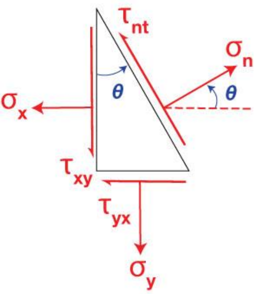
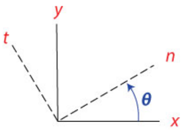
Module 19 Learning Outcome

- Show that Principal Planes are 90° apart, or on mutually perpendicular planes (normal to each other)

Stresses on Inclined Planes for Plane Stress in general





$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Angle(s) where the max/min normal stresses, σ_n, occur

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Where θ_P is the angle(s) to what are defined as the "Principal Planes"

These are the planes with the maximum and minimum normal stresses occur and these stresses are defined as "Principal Stresses"

Shear Stress is zero, $\tau_{nt} = 0$, on "Principal Planes"

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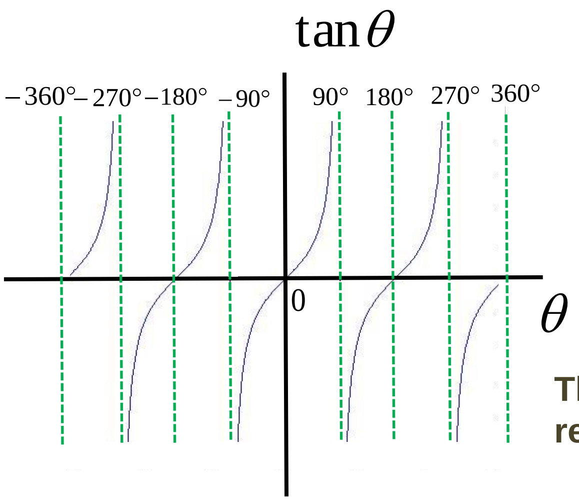
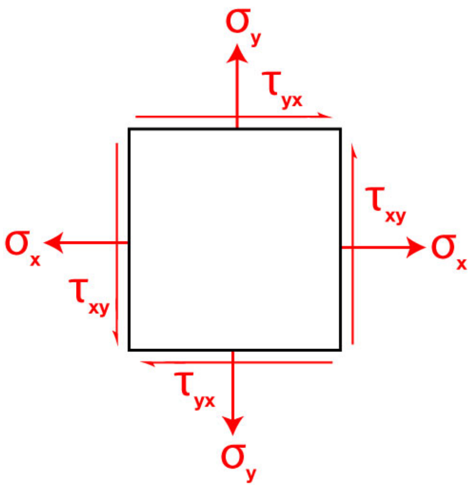
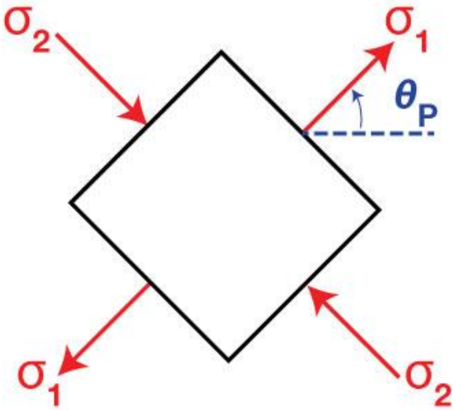


The tangent function repeats every 180°

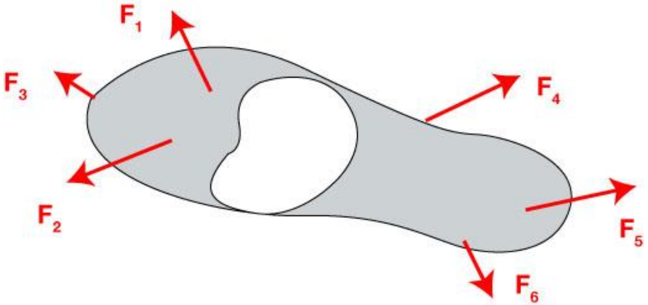




For a given set of values for σ_x, σ_y, *and* τ_{xy}, **the values of** $2\theta_P$ **differ by 180°**

Therefore "Principal Planes" are 90° apart
[mutually perpendicular planes (or normal to each other)]

General 3D State of Stress at a Point
(Arbitrarily Loaded Member)



- For more complicated structural members , the stress distributions may not be uniform on arbitrary planes
- For an infinitesimally small point, the stress distribution approaches uniformity
- An infinite number or planes can be passed through each point.
- But, it can be shown that three mutually perpendicular planes is sufficient to completely describe the state of stress at any point for any orientation. (Hence we will use a cube to represent the state of stress at a point.)