Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

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Mechanics of Materials I:
Fundamentals of Stress & Strain and Axial Loading

- ✓ Internal Forces due to External Loads
- ✓ Axial Centric Loads
- ✓ Normal Stress and Shear Stress
- ✓ General State of Stress at a Point (3D)
- ✓ Plane Stress (2D)
- ✓ Normal Strain and Shear Strain
- ✓ Stress-Strain Diagrams
- ✓ Mechanical Properties of Materials
- ✓ Linear Elastic Behavior, Hooke’s Law, and Poisson’s Ratio
- ✓ Stresses on Inclined Planes
- ✓ Principal Stresses and Max Shear Stress
- ✓ Mohr’s Circle for Plane Stress
- ✓ Stress Concentrations
- ✓ Mohr’s Circle for Plane Strain
- ✓ Strain Transformation and Measuring Strains
- ✓ Factor of Safety and Allowable Stresses/Loads
- ✓ Nonlinear Behavior and Plasticity
- ✓ Statically Indeterminate Structures
- ❑ Thermal and Pre-strain Effects

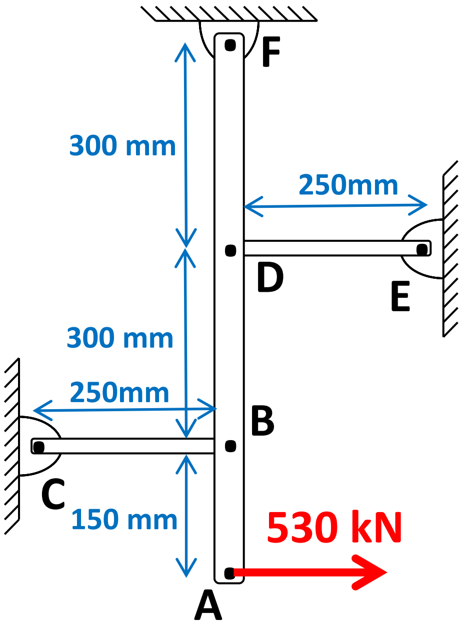
Module 43 Learning Outcomes

- Solve a statically indeterminate structure problem for axial loading

Example:

Bar DE is aluminum and has a cross sectional area of 5000 mm^2 and a modulus of elasticity of 70 GPa. $\sigma_{alum\ yield}$ = 280 MPa = 0.28 GPa
Bar BC is steel and has a cross sectional area of 1300 mm^2 and a modulus of elasticity of 200 GPa. $\sigma_{steel\ yield}$ = 250 MPa = 0.25 GPa
Bar ABDF can be considered rigid. Both the aluminum and steel bars are deformable. The weight of the bars can be assumed negligible in comparison to the forces they are supporting. Find:

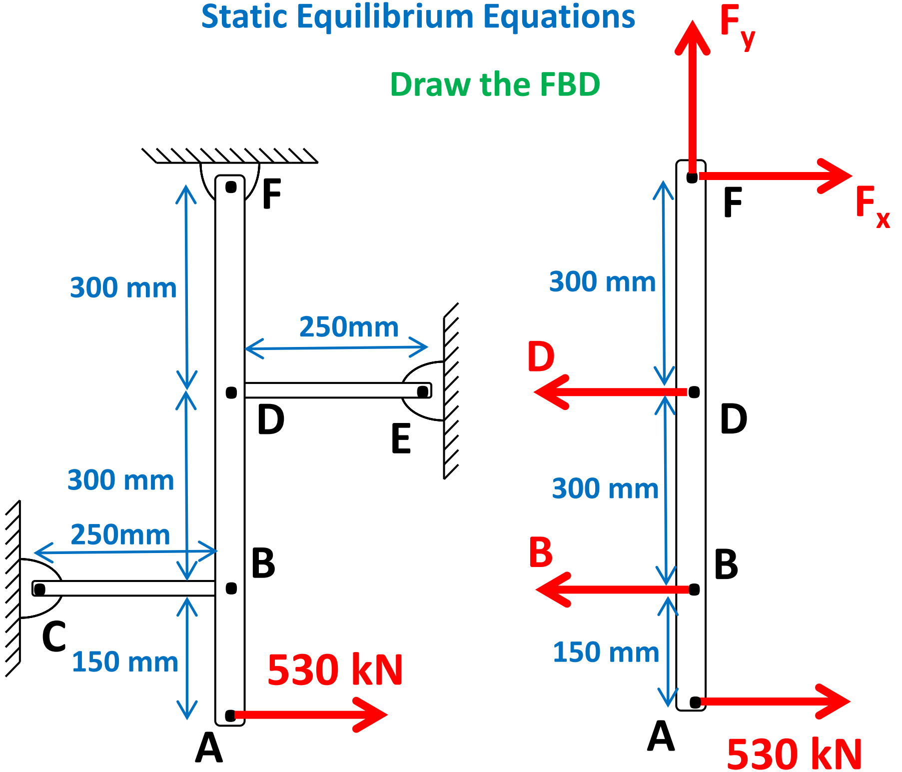
a) The axial stress in the aluminum and steel bars
b) The deflection at point A



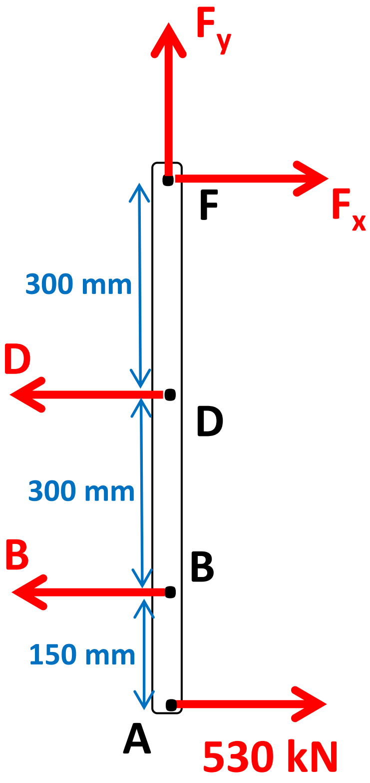
We will work in:
kN/mm^2 and GPa
where
1 kN/mm^2 = 1 GPa

Static Equilibrium Equations

Draw the FBD



Static Equilibrium Equations



3 Independent Equilibrium Equations:

4 Unknowns: F_x, F_y, B, D

Write the best equilibrium equation to start to solve the problem

$$+\circlearrowleft \sum M_D = 0$$

$$530(750) - D(300) - B(600) = 0$$

$$D + 2B = 1325 \qquad \textbf{EQN [1]}$$

1 Equation, 2 Unknowns: B, D

$$D + 2B = 1325$$ EQN [1]

1 Equation, 2 Unknowns: B, D

We need an additional equation

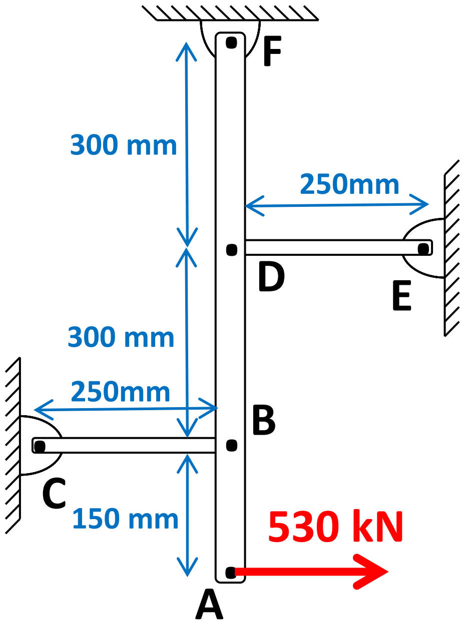
Deformation Equation

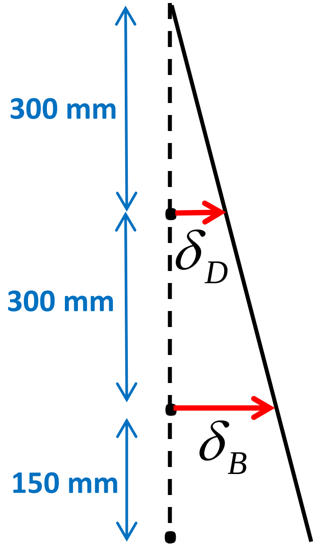
(geometry of the deformation of the members in the structure)

or Compatibility Equation

(compatibility between equilibrium and the deformation the structure undergoes)

assume small deformations and therefore small angles





Example:

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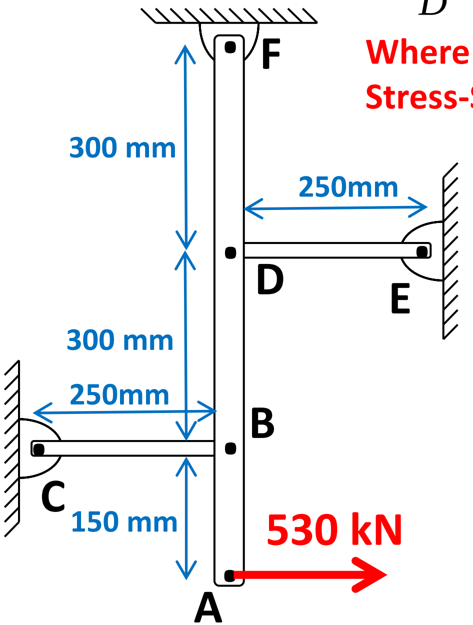
Equilibrium Equation

$$D + 2B = 1325$$

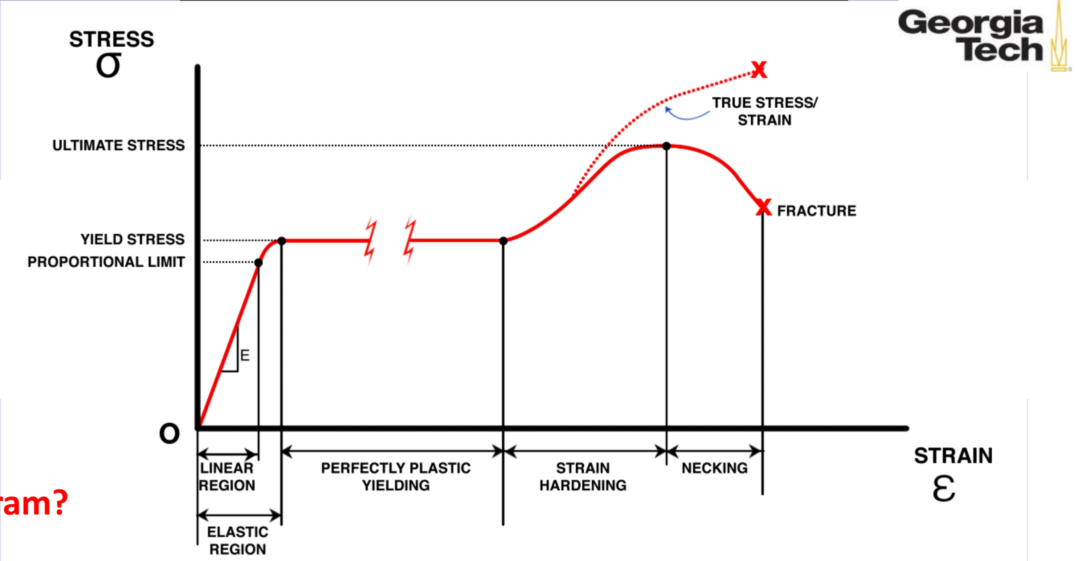
Deformation Equation

$$2\delta_D = \delta_B$$

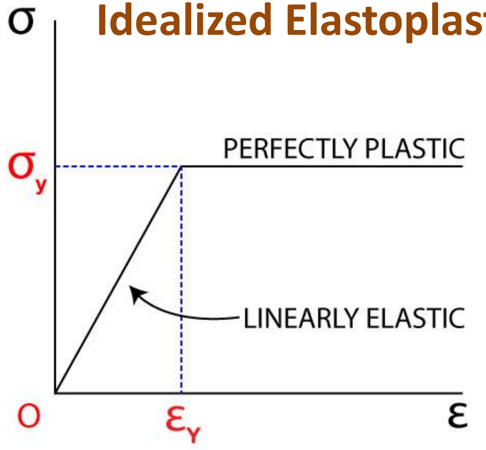


Where are we on Stress-Strain diagram?

Normal Stress-Strain Diagram

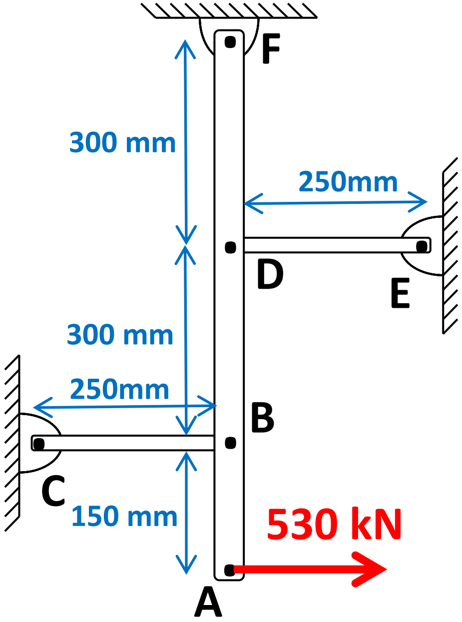


Idealized Elastoplastic Material



Yield Stress and Proportional Limit are assumed to be the same

Let's use the elastoplastic assumption and assume the steel and aluminum bars are on the linear elastic region



Equilibrium Equation

$$D + 2B = 1325$$

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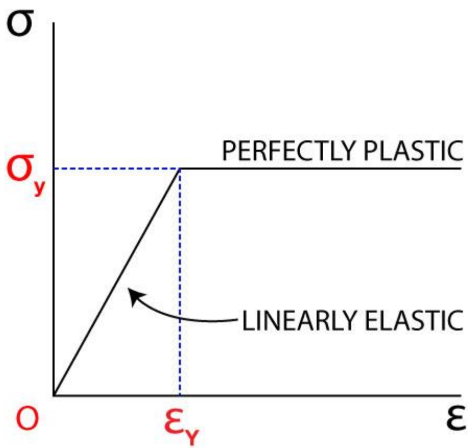


Axial Centric Loading

Normal Stress

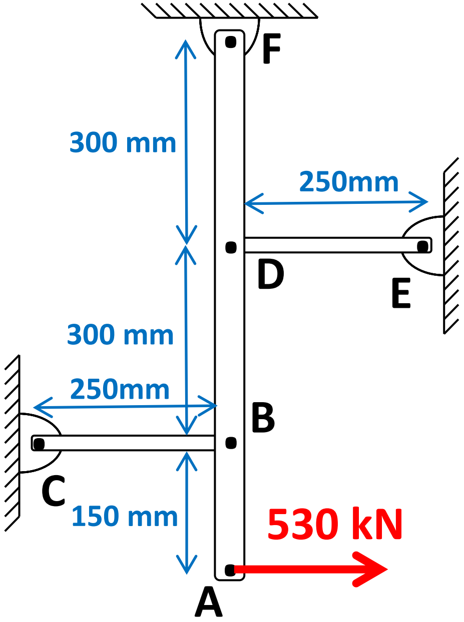
$$\sigma = \frac{N}{A}$$

Normal Strain

$$\varepsilon = \frac{\delta}{L}$$

$$\delta = \frac{PL}{AE}$$

Let’s use the elastoplastic assumption and assume the steel and aluminum bars are on the linear elastic region



Equilibrium Equation

$$D + 2B = 1325 \quad \textbf{EQN [1]}$$

Assuming linear elastic region

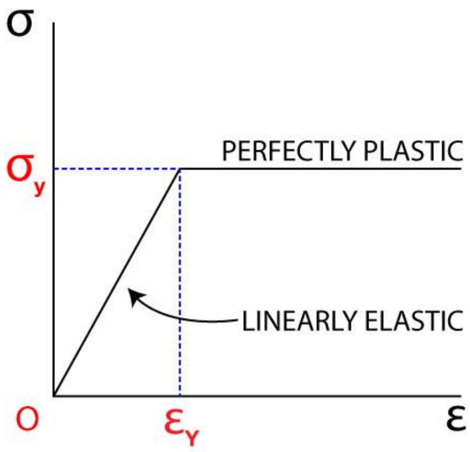
$$B = 1.49D \quad \textbf{EQN [2]}$$

Solving simultaneously

$$D = 446 \ kN$$
$$B = 664 \ kN$$

Check Stresses for linearly elastic assumption

Idealized Elastoplastic Material



$\sigma_{alum\ yield}$ = 280 MPa = 0.28 GPa

$\sigma_{steel\ yield}$ = 250 MPa = 0.25 GPa

$$\sigma_{BC} = \frac{664 \ kN}{1300 \ mm^2} = 0.511 \ GPa$$

$$0.511 \ GPa > 0.25 \ GPA$$

Not ok
steel bar has yielded

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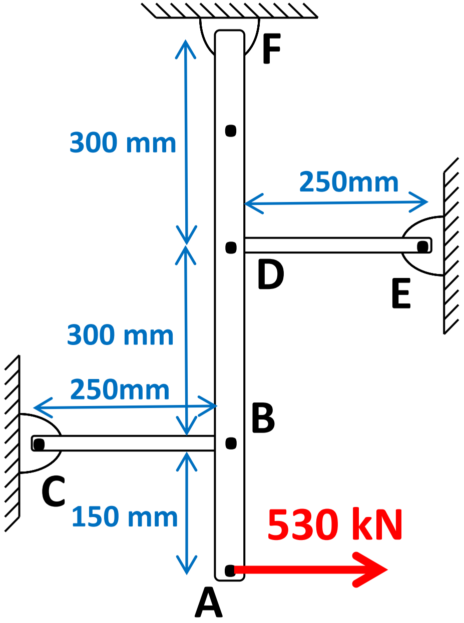
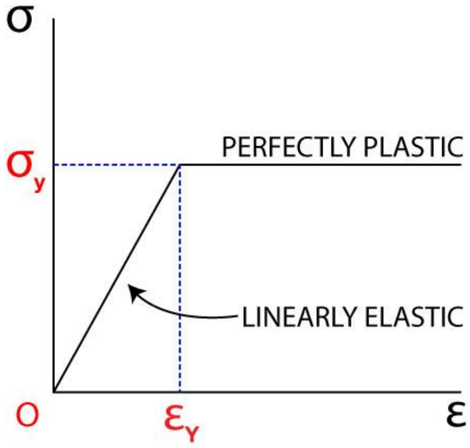


Idealized Elastoplastic Material



$\sigma_{alum\ yield}$ = 280 MPa = 0.28 GPa
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Equilibrium Equation

$$D + 2B = 1325$$ EQN [1]

Aluminum is in linear elastic region

Steel has yielded and is in perfectly plastic region for elastoplastic assumption

a) The axial stress in the steel bar

$\sigma_{BC} = \sigma_{steel\ yield}$ = 250 Mpa (T) = 0.25 Gpa (T)

ANS

Example:

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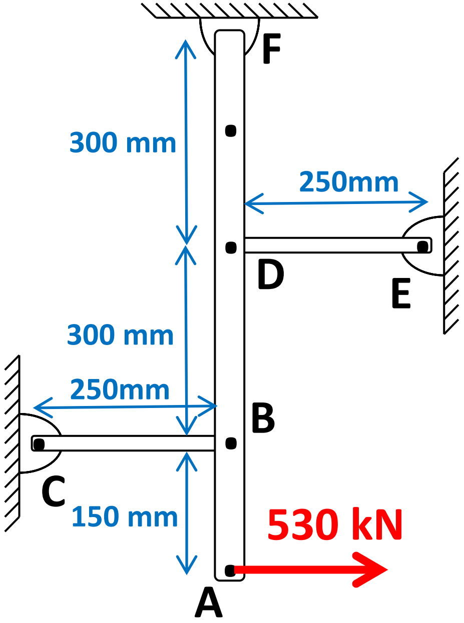
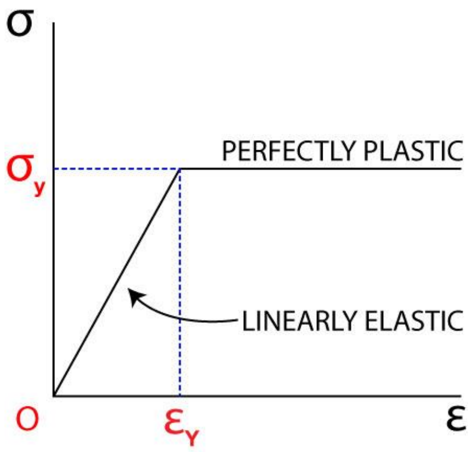


Idealized Elastoplastic Material



Equilibrium Equation

325

$$D + 2\cancel{B} = 1325$$ **EQN [1]**

$$D = 675\ kN$$

$\sigma_{BC} = \sigma_{steel\ yield}$ **= 250 Mpa (T) = 0.25 Gpa (T)**

ANS

$$\sigma = \frac{N}{A}$$

$$B = \sigma_{BC} A_{BC} = \left(0.25\ \frac{kN}{mm^2}\right)\left(1300\ mm^2\right) = 325\ kN$$

$$\sigma_{DE} = \frac{D}{A_{DE}} = \frac{675\ kN}{5000\ mm^2} = 0.135\ GPa\ (C) = 135\ MPa\ (C)$$

ANS

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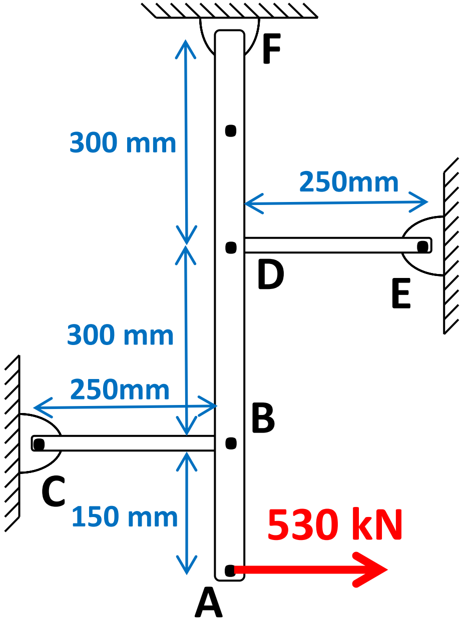
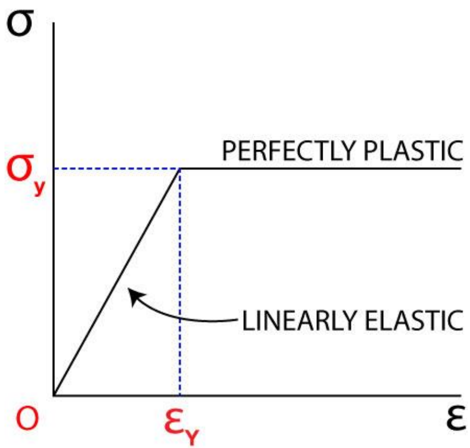


Equilibrium Equation

$$D + 2\cancel{B}^{\,325} = 1325 \quad \text{EQN [1]}$$

$$D = 675\ kN$$

Idealized Elastoplastic Material



Aluminum is in linear elastic region
Therefore, for deflection calculation, we must use the aluminum bar where the following relationship holds:

$$\delta = \frac{PL}{AE}$$

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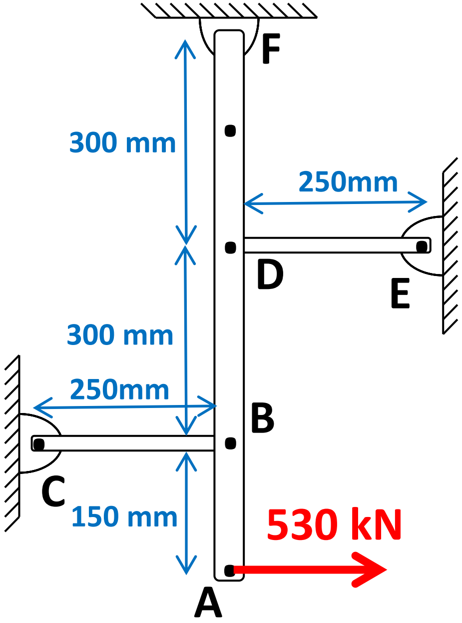
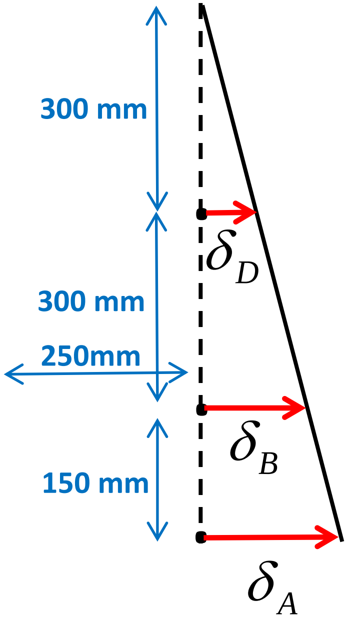
$$\delta_D = \frac{675(250)}{5000(70)} = 0.482\ mm$$





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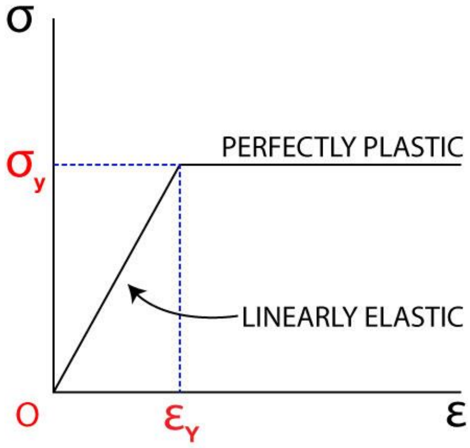
Idealized Elastoplastic Material



b) The deflection at point A