Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

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Module 45 Learning Outcome

- Solve a engineering problem when thermal effects are present

Example:

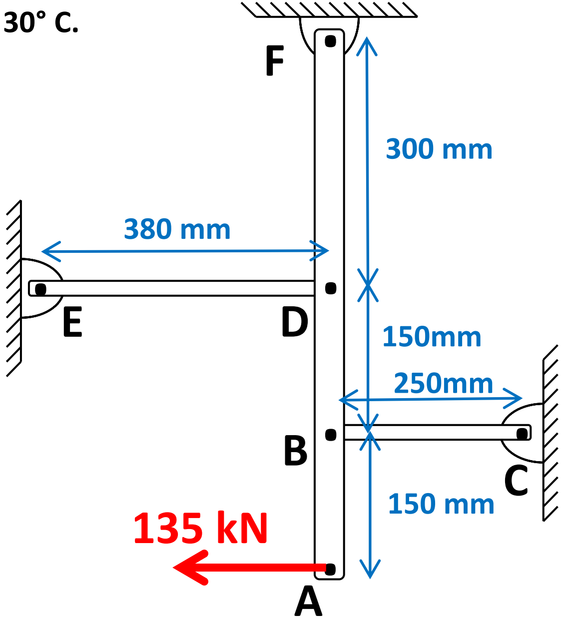
Bar BC is aluminum and has a cross sectional area of 2000 mm^2 and a modulus of elasticity of 70 GPa. $\sigma_{alum\ yield}$ = 280 MPa = 0.28 GPa.

The coefficient of thermal expansion of aluminum is α = 22.5 x 10-6/°C.

Bar DE is brass and has a cross sectional area of 1300 mm^2 and a modulus of elasticity of 100 GPa. $\sigma_{brass\ yield}$ = 100 MPa = 0.1 GPa.

The coefficient of thermal expansion of brass is α = 17.6 x 10-6/°C.

Bar ABDF can be considered rigid. Both the aluminum and brass bars are deformable. The weight of the bars can be assumed negligible in comparison to the forces they are supporting. Determine the axial stress in the aluminum and brass bars when the temperature decreases by 30° C.

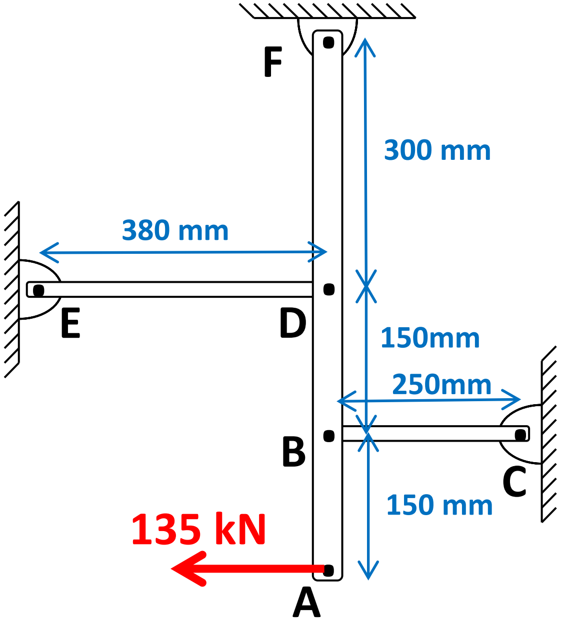


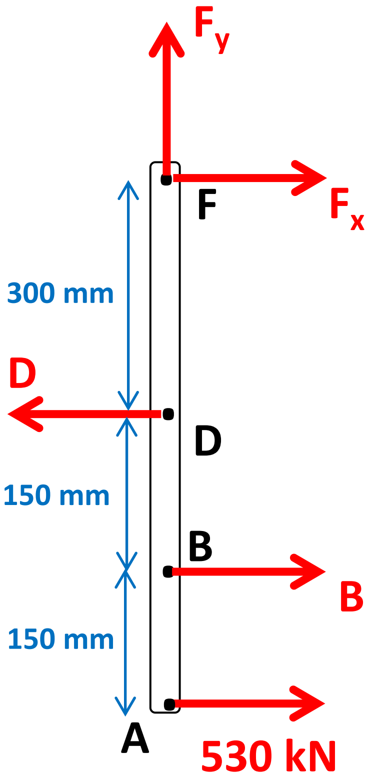
We will work in:
kN/mm^2 and GPa
where
1 kN/mm^2 = 1 GPa

Static Equilibrium Equations

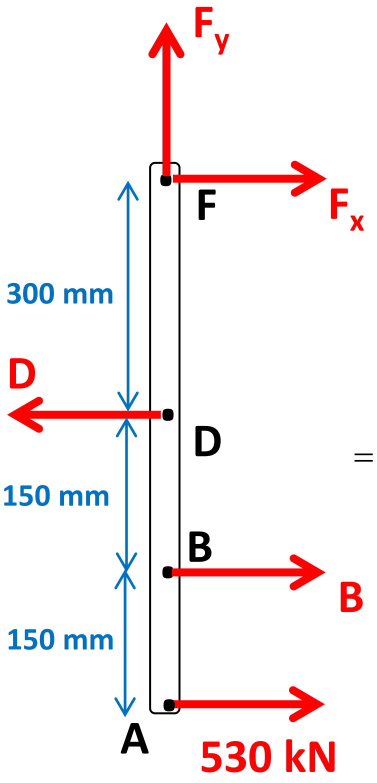
Draw the FBD

Assume tension in bars BC and DE





Static Equilibrium Equations



Write the best equilibrium equation to start to solve the problem

$$\circlearrowleft + \quad \sum M_D = 0$$

$$450B - D(300) - 135(600) = 0$$

$$1.5B - D = 270$$

$$B = \sigma_{BC} A_{BC} = 2000\,\sigma_{BC} \qquad D = \sigma_{DE} A_{DE} = 1300\,\sigma_{DE}$$

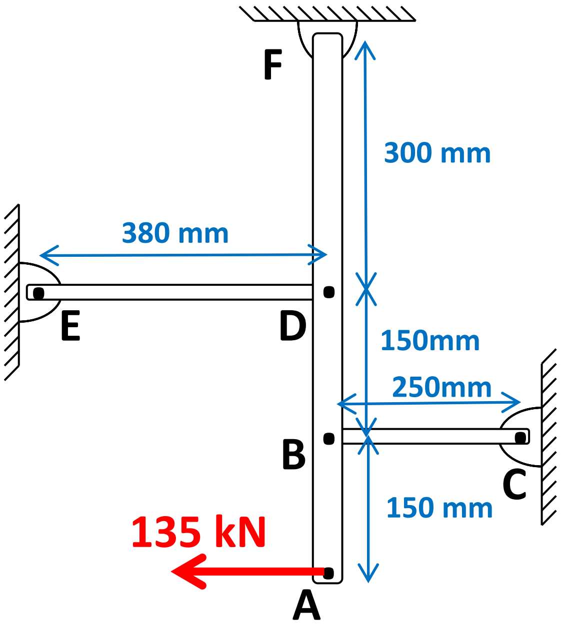
$$3000\,\sigma_{BC} - 1300\,\sigma_{DE} = 270$$ **EQN [*]**

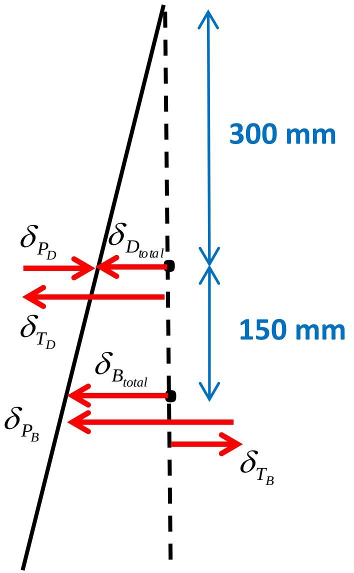
1 Equation, 2 Unknowns: σ_{BC}, σ_{DE}

We need an additional equation

Deformation Equation/ Compatibility Equation

assume small deformations
and therefore small angles





similar triangles

$$\frac{\delta_{D_{total}}}{300} = \frac{\delta_{B_{total}}}{450} \qquad 1.5\delta_{D_{total}} = \delta_{B_{total}}$$

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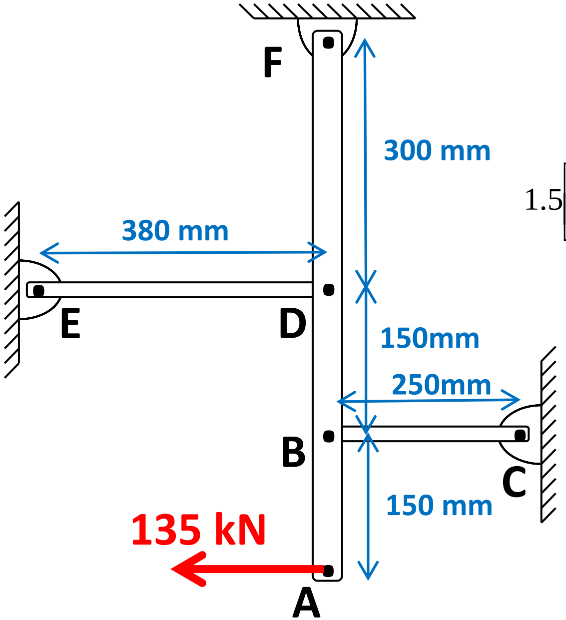


Equilibrium Equation

$$3000\sigma_{BC} - 1300\sigma_{DE} = 270 \quad \textbf{EQN [*]}$$

Deformation/Compatibility Equation

$$1.5\delta_{D_{total}} = \delta_{B_{total}}$$

Force-Thermal Displacement Relation

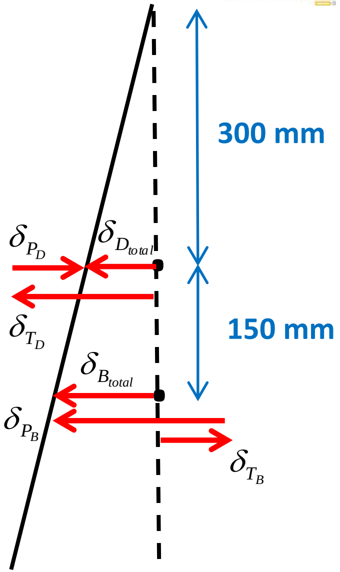
$$1.5\left(\delta_{T_D} - \delta_{P_D}\right) = \left(-\delta_{T_B} + \delta_{P_B}\right)$$

$$\sigma_{DE} \qquad\qquad\qquad\qquad \sigma_{BC}$$

$$1.5\left[\alpha_{DE}(\Delta T)L_{DE} - \frac{P_{DE}L_{DE}}{A_{DE}E_{DE}}\right] = \left[-\alpha_{BC}(\Delta T)L_{BC} - \frac{P_{BC}L_{BC}}{A_{BC}E_{BC}}\right]$$

$$1.5\left[\left(17.6\,x10^{-6}\right)30(380) - \frac{\sigma_{DE}(380)}{100}\right] = \left[\left(-22.5\,x10^{-6}\right)30(250) - \frac{\sigma_{DE}(250)}{70}\right]$$

$$3.57\sigma_{BC} + 5.70\sigma_{DE} = 0.470 \quad \textbf{EQN [**]}$$



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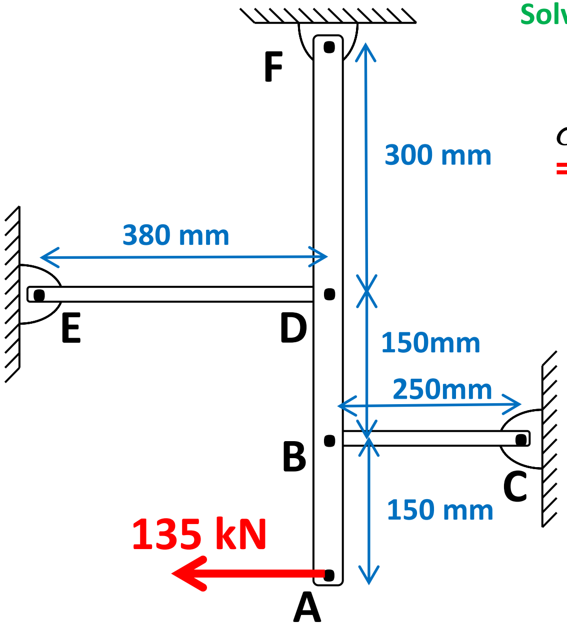

Equilibrium Equation

$$3000\sigma_{BC} - 1300\sigma_{DE} = 270$$ **EQN [*]**

Force-Thermal Displacement Relation

$$3.57\sigma_{BC} + 5.70\sigma_{DE} = 0.470$$ **EQN [**]**

Solving Simultaneously

ok with assumptions

$\sigma_{alum\ yield}$ = 280 MPa = 0.28 GPa

$$\sigma_{BC} = 0.0989\ GPa\,(T) < 0.28\ GPA$$

ANS

ok with assumptions

$\sigma_{brass\ yield}$ = 100 MPa = 0.1 GPa

$$\sigma_{DE} = 0.0205\ GPa\,(T) < 0.1\ GPA$$

ANS