Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

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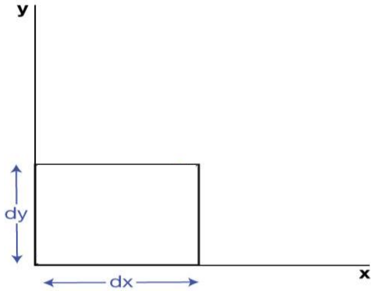
Module 33 Learning Outcome

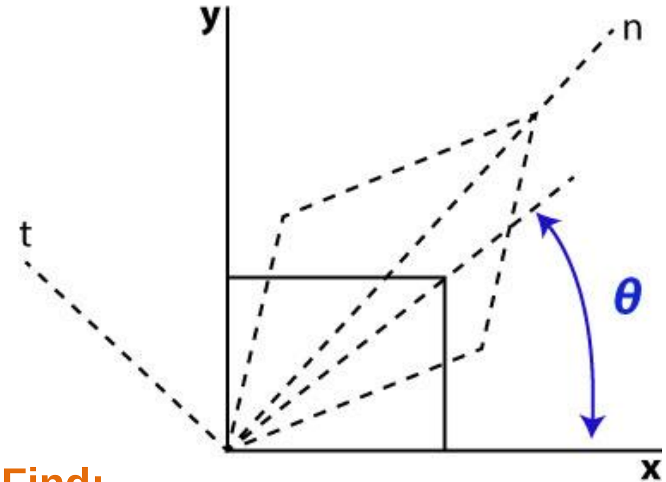
- For a given set of plane strain conditions at a point, determine the Principal Strains, Principle Planes, and Maximum Shear Strain using Mohr's Circle

Plane Strain



$$\varepsilon_z = \gamma_{xz} = \gamma_{zx} = \gamma_{yz} = \gamma_{zy} = 0$$

In general, $\varepsilon_x , \ \varepsilon_y \ , and \ \gamma_{xy} = \gamma_{yx}$ **are known or can be found**



Find: $\varepsilon_n , \gamma_{nt} \ \ for \ any \ angle \ \theta$

Plane Strain
Mohr's Circle

Graphical tool for the depiction of the transformation equations for plane stress

$$\left(\varepsilon_n - \frac{\varepsilon_x + \varepsilon_y}{2} \right)^2 + \left(\frac{\gamma_{xy}}{2} - 0 \right)^2 = \left(\frac{\varepsilon_x - \varepsilon_y}{2} \right)^2 + \left(\frac{\gamma_{xy}}{2} \right)^2$$

Radius = $\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2} \right)^2 + \left(\frac{\gamma_{xy}}{2} \right)^2}$

Center: $\left(\frac{\varepsilon_x + \varepsilon_y}{2} , 0 \right) = \left(\varepsilon_{AVG} , 0 \right)$

The angle on Mohr's circle is 2 times the stress block angle

Mohr's circle is a circle where each point represents the stress ε and $\gamma / 2$ on a particular plane through a single point

Plane Strain
Mohr's Circle

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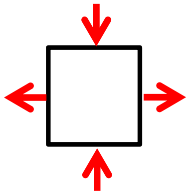
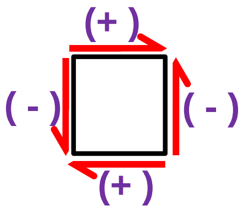
Mohr's Circle Sign Convention

Normal



(+) Tension
(-) Compression

Shear



Clockwise (+)
Counterclockwise (-)

Example

The measured strain components at a point in a body under a state of plane strain are shown. Using Mohr's circle, find:

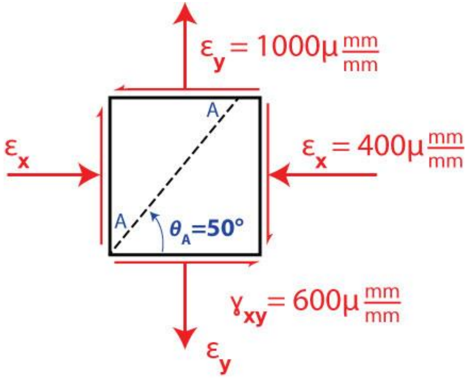
a) The principal strains and the maximum shear strain at that point, and find the orientation of the principal planes

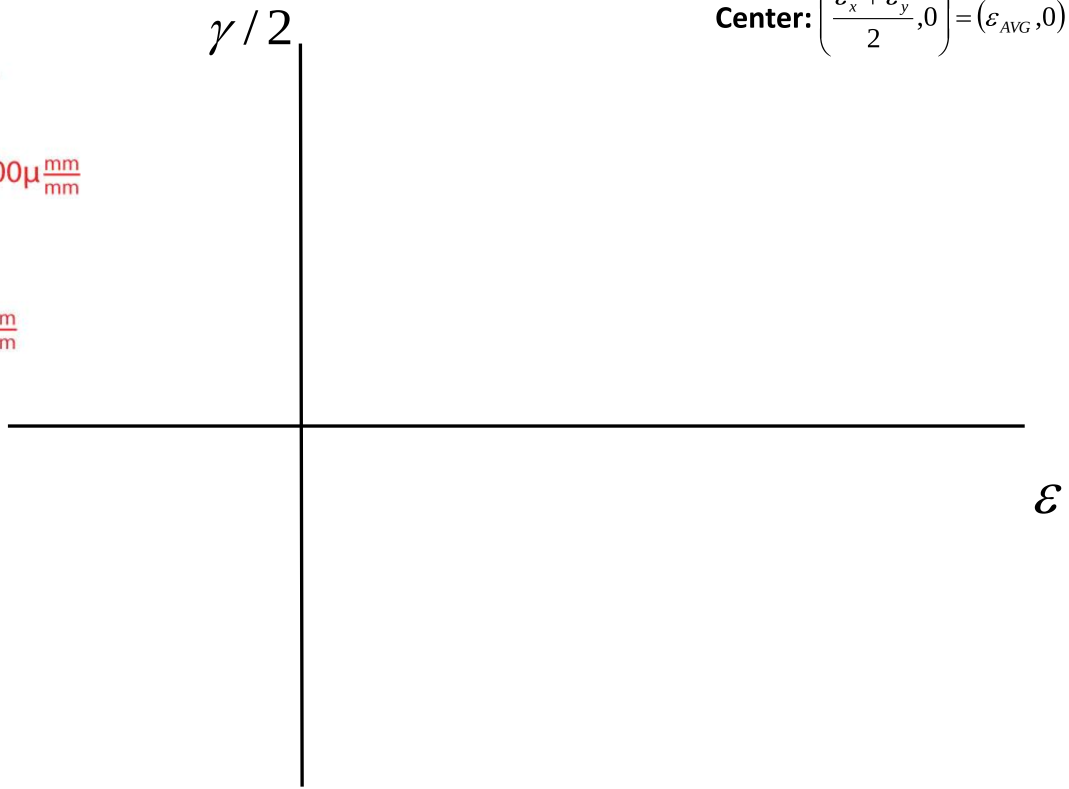
b) The normal and shear strains on plane AA oriented as shown.

$$\left(\varepsilon_n - \frac{\varepsilon_x + \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2} - 0\right)^2 = \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2$$

Radius = $\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$

Center: $\left(\frac{\varepsilon_x + \varepsilon_y}{2}, 0\right) = \left(\varepsilon_{AVG}, 0\right)$





The
bod
Usin
a)
b)

Example

The measured strain components at a point in a body under a state of plane strain are shown. Using Mohr's circle, find:

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Do by hand on previous page

$$\left(\varepsilon_n - \frac{\varepsilon_x + \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2} - 0\right)^2 = \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2$$

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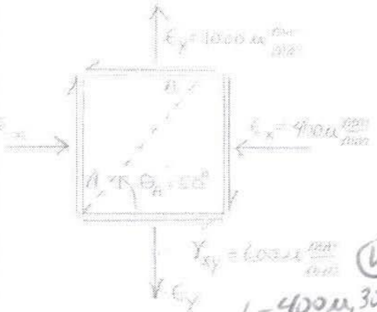
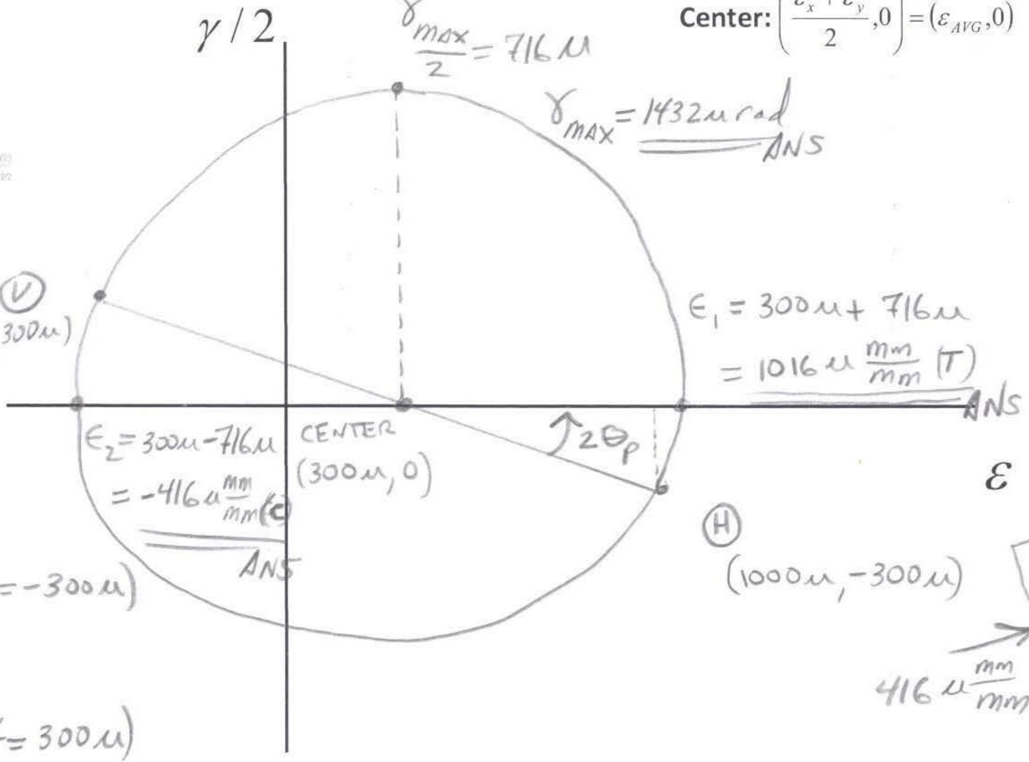
$$716\mu = \textbf{Radius} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{-400\mu - 1000\mu}{2}\right)^2 + \left(\frac{300}{2}\right)^2}$$

$$\textbf{Center: } \left(\frac{\varepsilon_x + \varepsilon_y}{2}, 0\right) = (\varepsilon_{AVG}, 0) = \left(\frac{-400\mu + 1000\mu}{2}, 0\right)$$

$$= (300\mu, 0)$$



$$\frac{\gamma_{xy}}{2} = 300\mu \frac{mm}{mm}$$

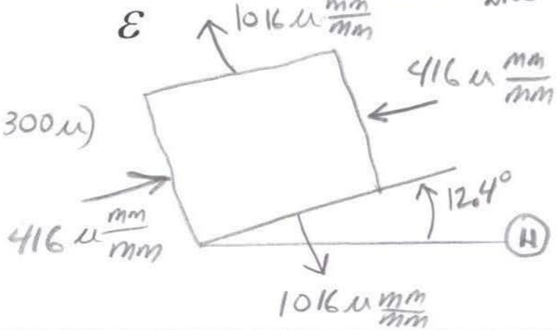


$$\sin 2\theta_p = \frac{300}{716}$$

$$2\theta_p = 24.8^\circ$$

$$\theta_p = 12.4^\circ$$

ANS



HORIZONTAL FACE

$$H = \left(+\sigma_y = 1000\mu, -\frac{\gamma_{xy}}{2} = -300\mu\right)$$

VERTICAL FACE

$$V = \left(-\sigma_x = -400\mu, \frac{\gamma_{xy}}{2} = 300\mu\right)$$