Mechanics of Materials III:

Beam Bending

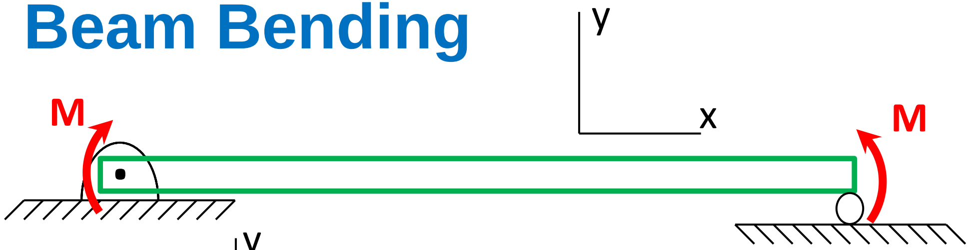
Dr. Wayne Whiteman
Senior Academic Professional and Director of the Office of Student Services
Woodruff School of Mechanical Engineering

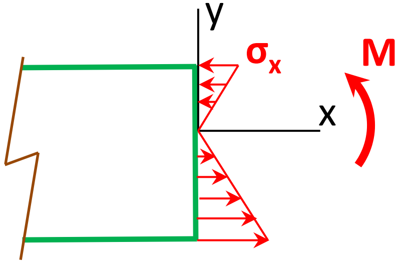
Georgia Institute of Technology

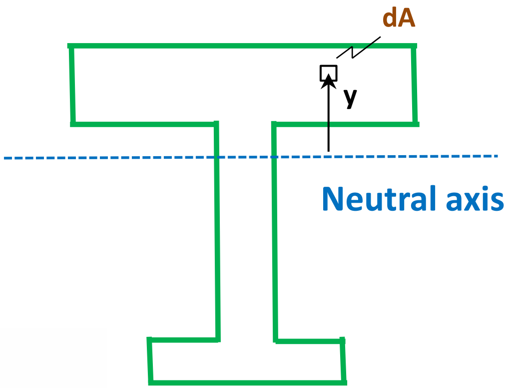
Module 9 Learning Outcome

- Derive the moment-curvature relationship

Beam Bending





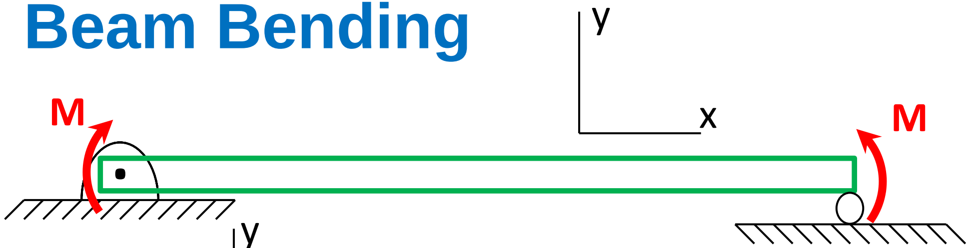


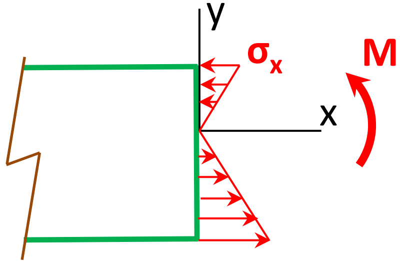
$$(+\circlearrowleft)\ \Sigma M_z = 0$$

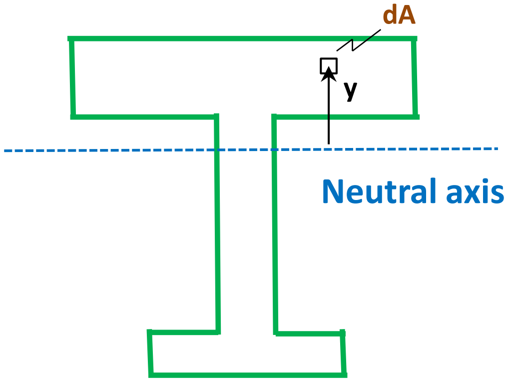
$$dM + \sigma_x\, y\, dA = 0$$

$$dM = -\sigma_x\, y\, dA = 0$$

Beam Bending







$$dM = -\sigma_x \, y \, dA$$

$$M = -\int_A \overset{-E\kappa y}{\overbrace{\sigma_x}} \, y \, dA$$

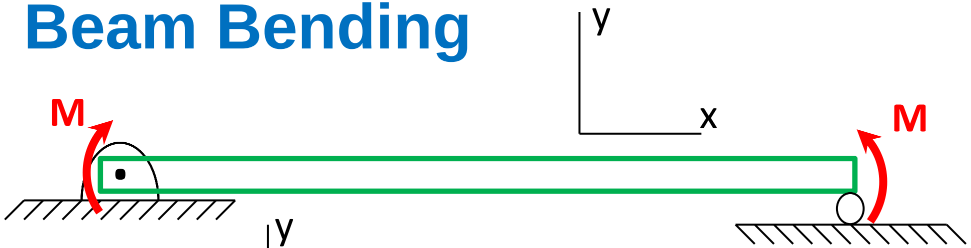
$$M = \int_A E \kappa y^2 \, dA$$

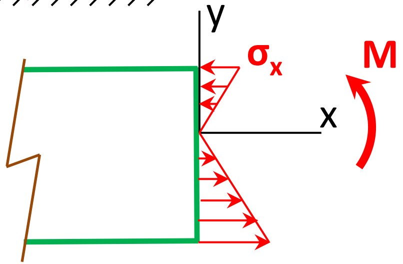
From last module:

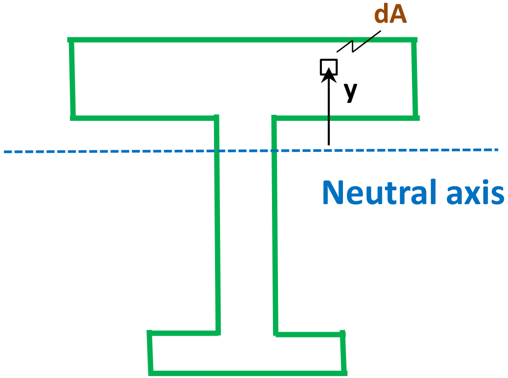
$$\sigma_x = -\frac{E\,y}{\rho} = -E\,\kappa\,y$$

For linear elastic material, stress is also proportional to curvature and varies linearly with distance, y, from the neutral axis.

Beam Bending







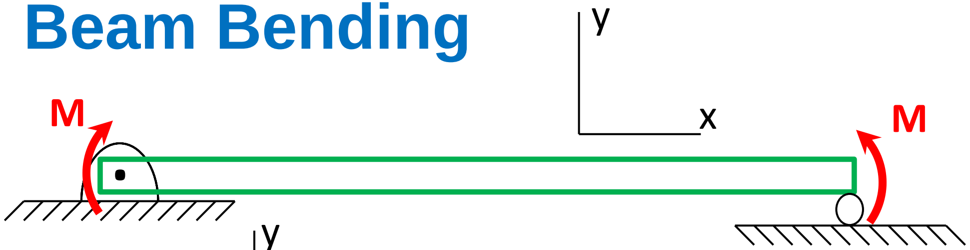
$$M = \int_A E \,\kappa\, y^2 \, dA$$

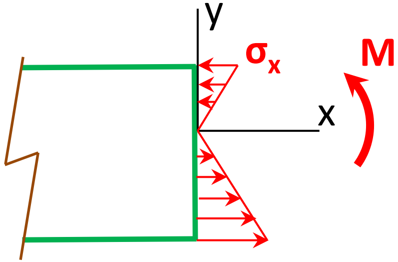
$$M = E\kappa \underbrace{\int_A y^2 \, dA}_{I \equiv}$$

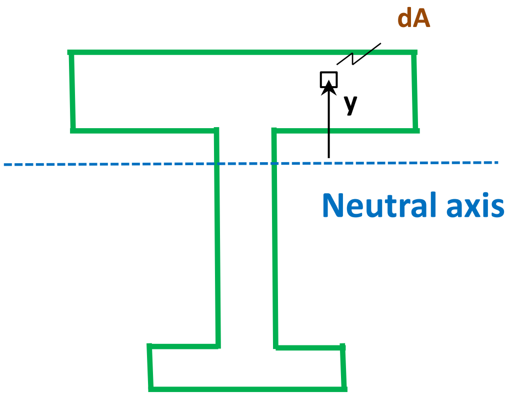
$I \equiv$ Area moment of inertia (Second moment of area)

$$M = E\kappa I$$

Beam Bending





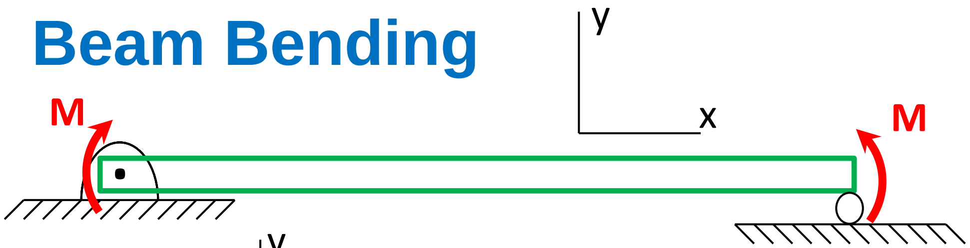


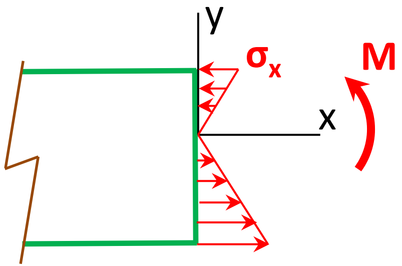
$$M = E\,\kappa\,I$$

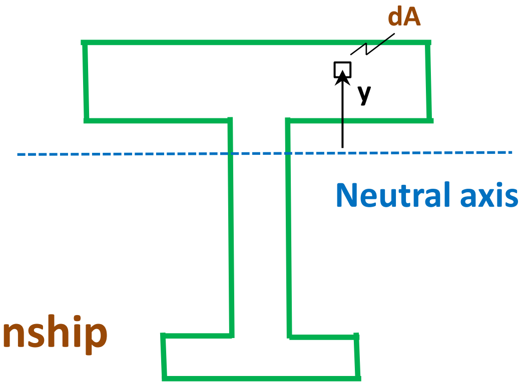
Moment-curvature relationship

$$\kappa = \frac{1}{\rho} = \frac{M}{EI}$$

Beam Bending







Moment-curvature relationship

$$\kappa = \frac{1}{\rho} = \frac{M}{EI}$$

EI ≡ Flexural Rigidity

Resistance of the beam to bending for a given curvature

Curvature is proportional to moment

$\uparrow EI$ $\uparrow M$