Mechanics of Materials II:

Thin-Walled Pressure Vessels and Torsion

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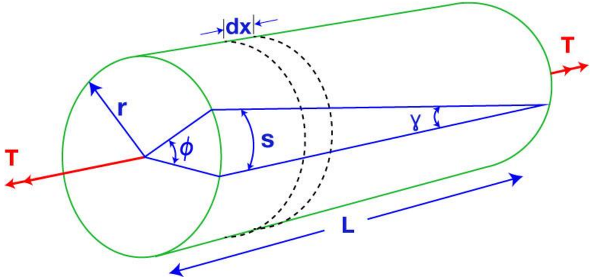
Mechanics of Materials II: Thin-Walled Pressure Vessels and Torsion

- ✓ Thin-Walled Pressure Vessels - Internal Pressure
- ✓ Torsional Shearing Stress and Strain
- ✓ Elastic Torsion Formula
- ✓ Elastic Torsion of Straight, Cylindrical Shafts
- ❑ Inelastic Torsion of Straight, Cylindrical Shafts
- ❑ Statically Indeterminate Torsion Members

Module 20 Learning Outcome

- Develop the relationships used in the analysis and design of inelastic torsion of straight cylindrical shafts

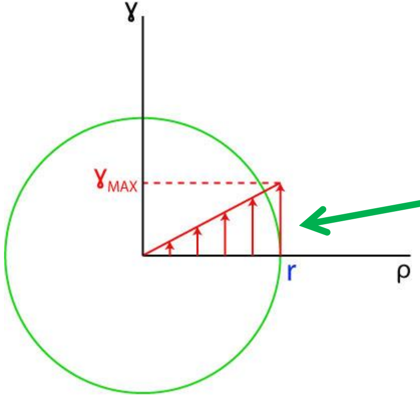
Review: Circular Bar Torsion



Torsional Shear Strain at Outer Surface

$$\gamma_{MAX} = \frac{r\phi}{L} = \frac{r\,d\phi}{dx} = r\theta$$

Shear Strains vary linearly with ρ



$$\gamma = \rho\theta = \frac{\rho}{r}\gamma_{MAX}$$

radial distance from center

Rate of Twist, θ
(angle of twist per unit length)

$$\theta = \frac{d\phi}{dx} = \frac{\phi}{L}$$

$$\gamma = \frac{\rho\phi}{L}$$

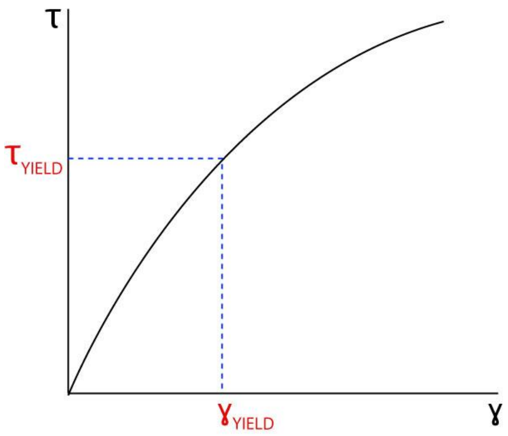
Note: So far we haven't specified any material properties:

material could be in elastic or inelastic region

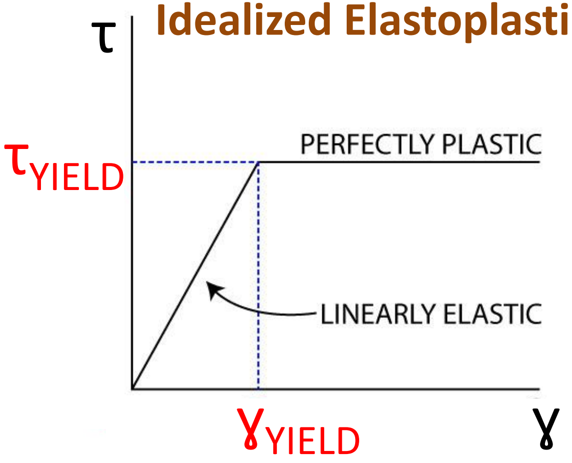
material could homogeneous or heterogeneous

we have specified small angles: $\tan\gamma \approx \gamma = \frac{s}{L}$

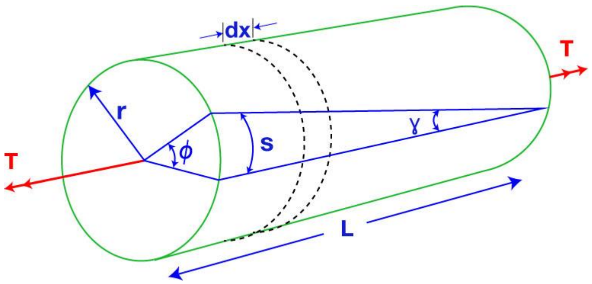
Typical Shear Stress-Strain Diagram

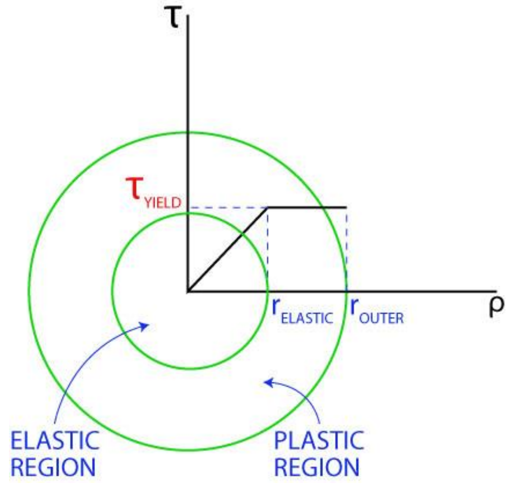


Idealized Elastoplastic Material

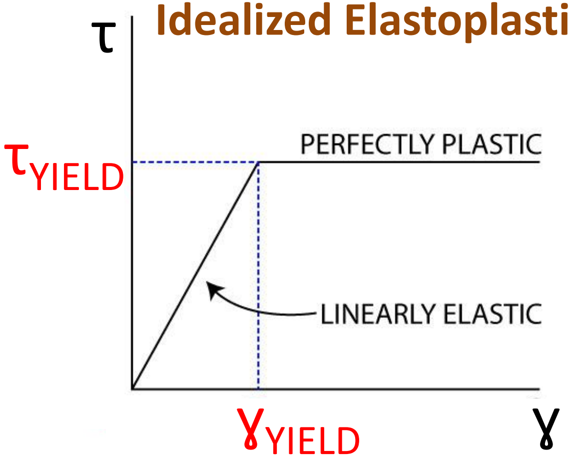


Inelastic Torsion of Straight Cylindrical Shafts





Idealized Elastoplastic Material



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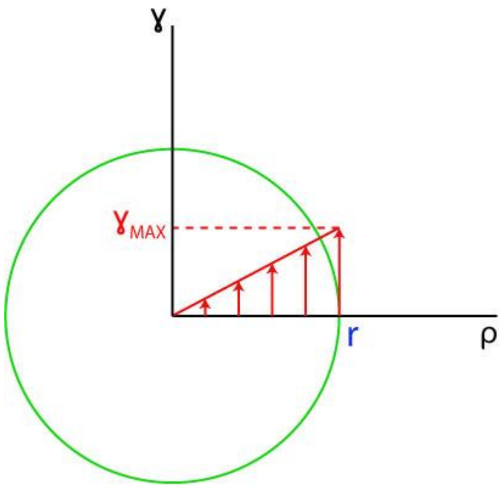

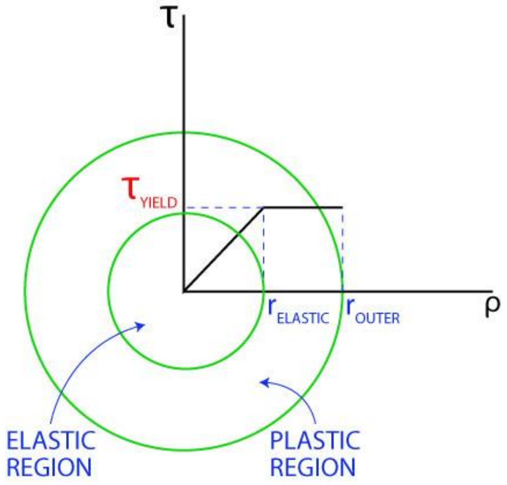

$$\gamma_{MAX} = r_{OUTER}\,\theta$$

$$\tau = G\gamma$$

$$\gamma = \rho\theta$$

$$\tau_{\substack{ELASTIC\\REGION}} = G\gamma_{\substack{ELASTIC\\REGION}}$$

$$\therefore \quad \frac{\gamma_{MAX}}{r_{OUTER}} = \frac{\gamma}{\rho} = \frac{\gamma_{YIELD}}{r_{ELASTIC}}$$

$$\tau_{PLASTIC} = \tau_{YIELD}$$

Inelastic Torsion of Straight Cylindrical Shafts

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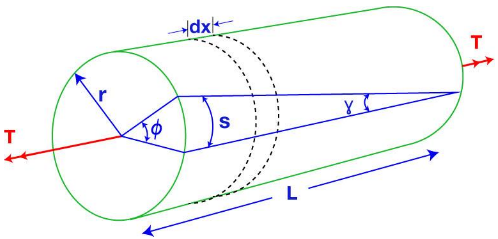
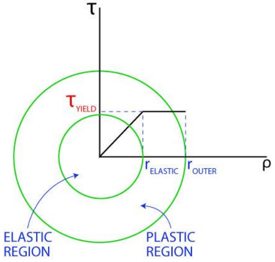
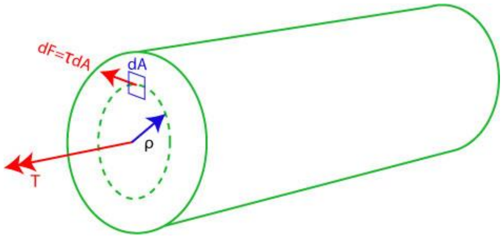
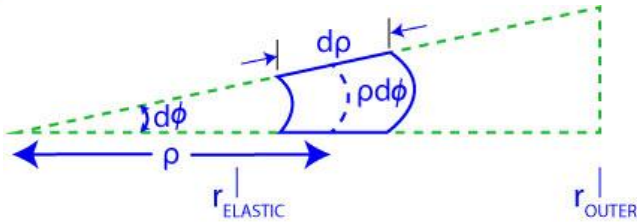


Relate Applied Torque to Stresses and Geometry

$$T_{TOTAL} = T_{ELASTIC} + T_{PLASTIC}$$







Elastic Torsion Formula

$$\tau = \frac{T\,\rho}{J}$$

$$T_{ELASTIC} = \frac{\tau_{YIELD}\, J_{\substack{ELASTIC\\REGION}}}{r_{ELASTIC}}$$

$$T_{PLASTIC} = \int_{\substack{PLASTIC\\AREA}} \rho\,\tau_{YIELD}\, dA$$

$$T_{PLASTIC} = \int_0^{2\pi} \int_{r_{ELASTIC}}^{r_{OUTER}} \rho\,\tau_{YIELD}\,\rho\, d\rho\, d\phi$$

$$T_{PLASTIC} = 2\pi\tau_{YIELD} \int_{r_{ELASTIC}}^{r_{OUTER}} \rho^2 d\rho = 2\pi\tau_{YIELD}\left[\frac{\rho^3}{3}\right]_{r_{ELASTIC}}^{r_{OUTER}}$$

$$T_{PLASTIC} = \frac{2}{3}\pi\tau_{YIELD}\left(r_{OUTER}^3 - r_{ELASTIC}^3\right)$$