Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

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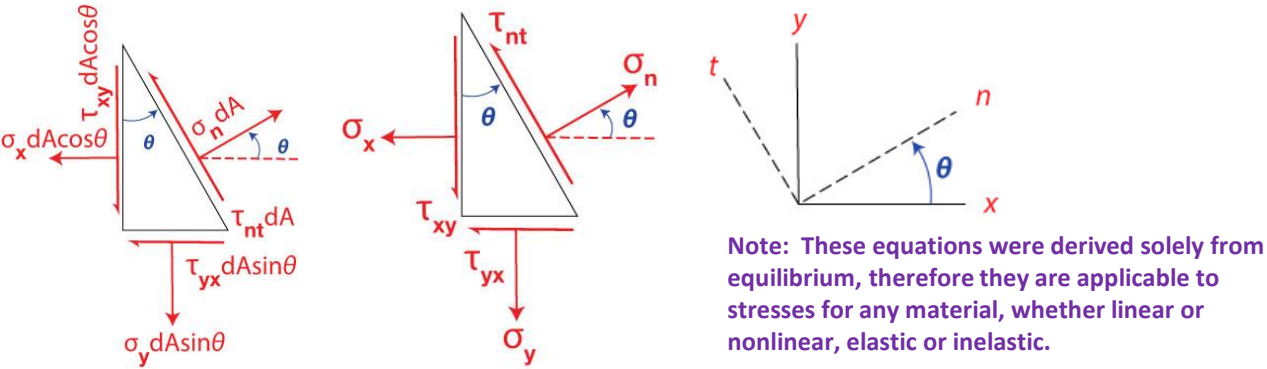
Module 18 Learning Outcomes

- Derive the angles to the “Principal Planes” where maximum and minimum normal stresses occur, which are defined as “Principal Stresses”
- Show that the shear stress is zero on “Principal Planes”

Stresses on Inclined Planes for Plane Stress in general



Note: These equations were derived solely from equilibrium, therefore they are applicable to stresses for any material, whether linear or nonlinear, elastic or inelastic.

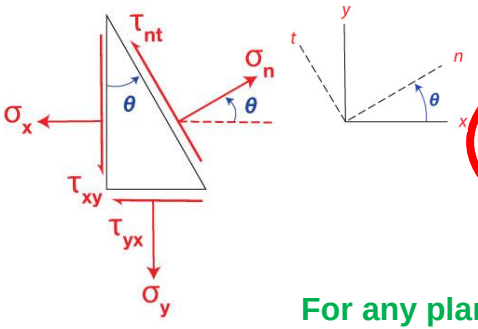
$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Transformation equations for plane stress

For any plane at an angle θ,
We can find σ_n and τ_{nt}

Stresses on Inclined Planes for Plane Stress in general



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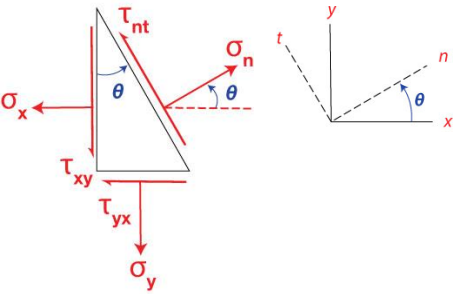
For any plane at an angle θ, we can find σ_n and τ_{nt}

For all structural and machine design, it is necessary to know at what planes and angles θ that the maximum values for σ_n and τ_{nt} occur

First, let's find the angle(s) where the max/min normal stresses, σ_n , occur

How should we proceed?

Stresses on Inclined Planes for Plane Stress in general



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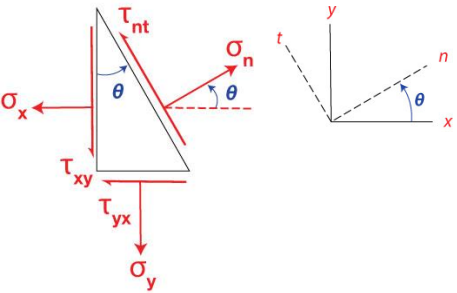
First, let's find the angle(s) where the max/min normal stresses, σ_n , occur

θ_P is the angle(s) to what are defined as the

"Principal Planes"

These are the planes where the maximum/minimum normal stresses occur and these stresses are defined as **"Principal Stresses"**

Stresses on Inclined Planes for Plane Stress in general



$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

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Angle(s) where the maximum normal stresses, σ_n , occur

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Where θ_P is the angle(s) to what are defined as the "Principal Planes"

These are the planes with the maximum and minimum normal stresses occur and these stresses are defined as "Principal Stresses"

Note for "Principal Planes" that we found:

$$\frac{d\sigma_n}{d\theta} = 0 = -(\sigma_x - \sigma_y)\sin 2\theta + 2\tau_{xy}\cos 2\theta$$

Therefore, Shear Stress is zero, $\tau_{nt} = 0$, on "Principal Planes"