Mechanics of Materials III:

Beam Bending

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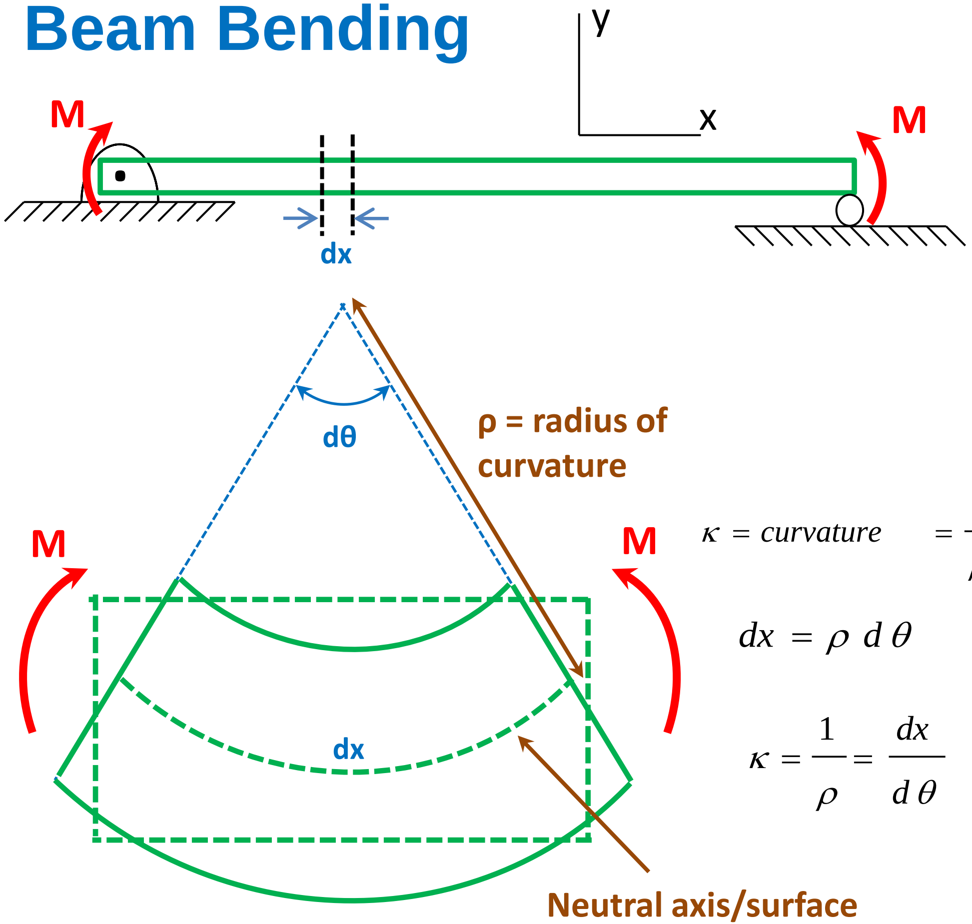
Module 8 Learning Outcome

- Locate the neutral axis/surface for a cross-section of a beam subject to pure bending

Beam Bending

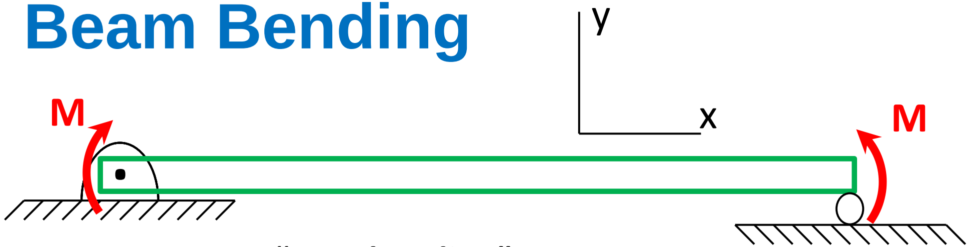


$$\kappa = curvature \quad = \frac{1}{\rho}$$

$$dx = \rho \, d\,\theta$$

$$\kappa = \frac{1}{\rho} = \frac{dx}{d\,\theta}$$

Beam Bending



"Pure bending"
Flexure under constant bending moment
No shear force

Strain-Curvature Relationship

$$\varepsilon_x = -\frac{y}{\rho} = -\kappa\, y$$

Strain Sign Convention

(+) elongation
(-) shortening

Strain is proportional to curvature and varies linearly with distance, y, from the neutral axis.

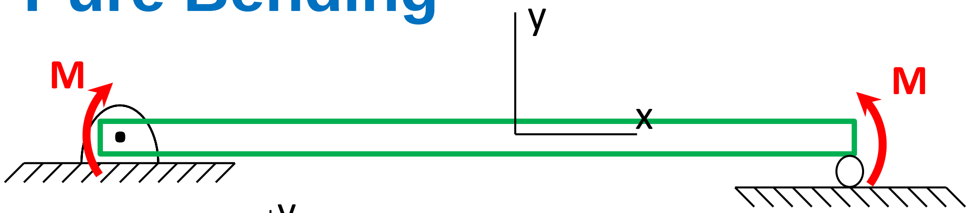
Independent of material

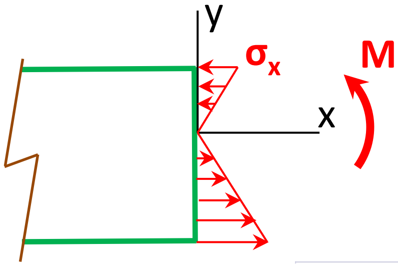
Note: There are strains in the y and z direction due to Poisson's effect, but no stresses because the beam is free to deform laterally.

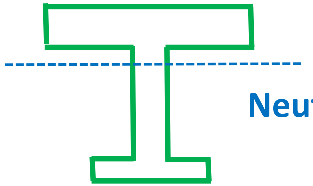
Therefore pure bending in beams produces uniaxial stress.

We'll start looking at the stresses next time!

Pure Bending

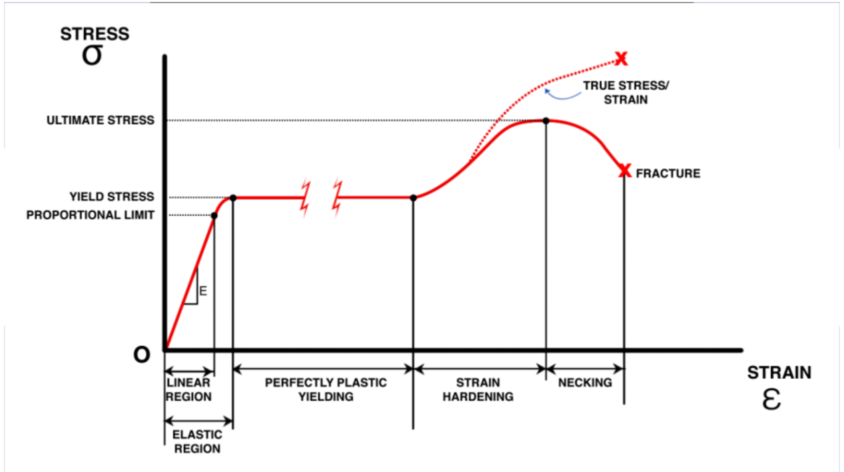






Neutral axis

Normal Stress-Strain Diagram



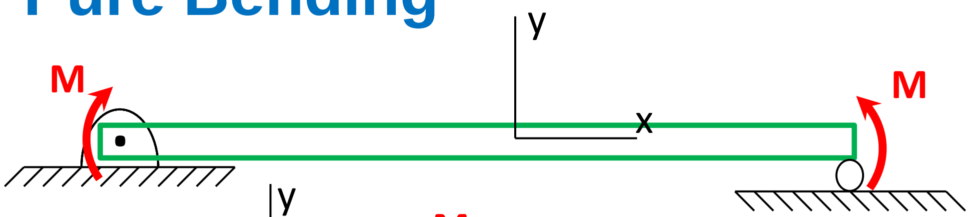
Stiffness:

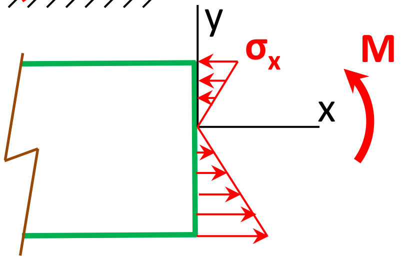
E = Modulus of Elasticity
= Young's Modulus

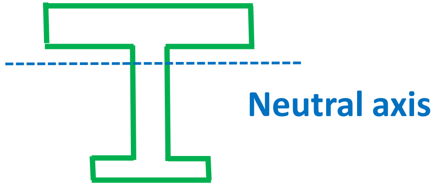
Hooke's Law (valid for linear elastic region):

$$\sigma = E\ \varepsilon$$

Pure Bending







Hooke's Law
(valid for linear elastic region)

$$\sigma = E\,\varepsilon$$

Note for most materials in the elastic range that it is reasonable to assume that the tension and compression stress-strain curves are the same. Therefore Hooke's Law applies the same for both tension and compression.

Strain-Curvature Relationship

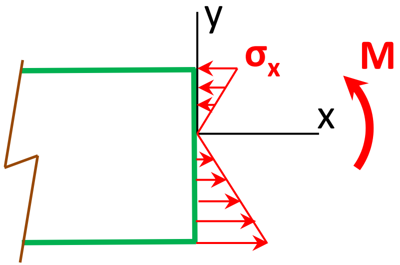
$$\varepsilon_x = -\frac{y}{\rho} = -\kappa\, y$$

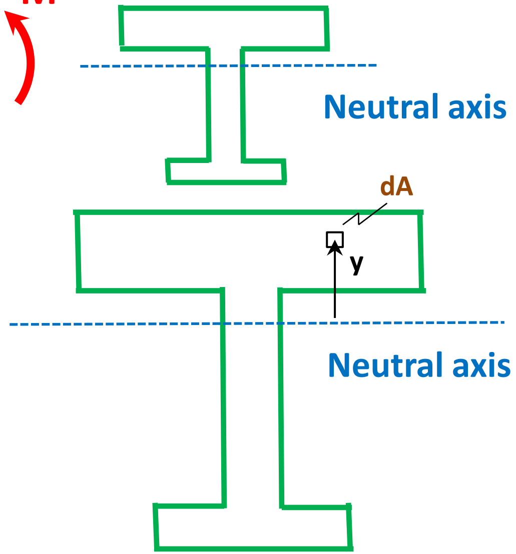
$$\sigma_x = -\frac{E\,y}{\rho} = -E\,\kappa\, y$$

For linear elastic material, stress is also proportional to curvature and varies linearly with distance, y, from the neutral axis.

Location of Neutral Axis

$$\sigma_x = -\frac{E\,y}{\rho} = -E\,\kappa\,y$$





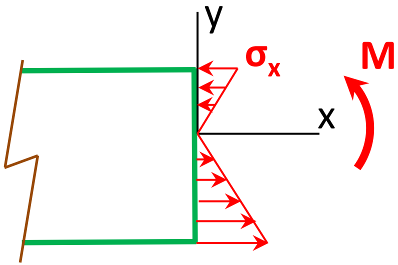
$$\sum F_x = 0$$

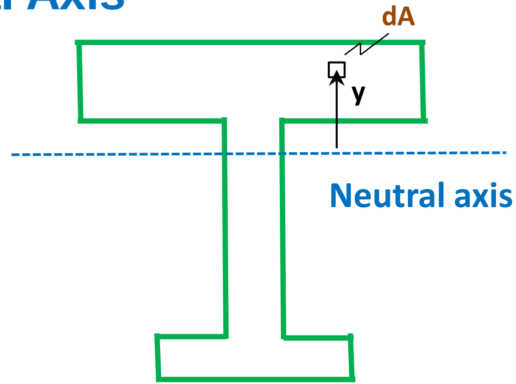
$$\int_A \sigma_x \, dA = 0$$

$$-\int_A E\,\kappa\,y\,dA = 0$$

$$\int_A y\,dA = 0$$

Location of Neutral Axis



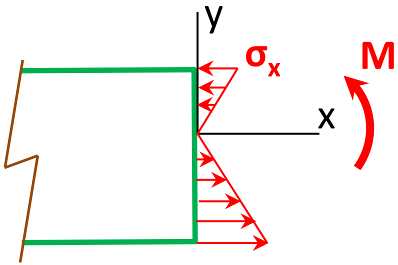


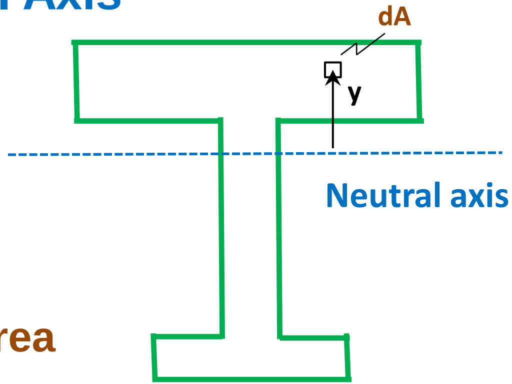
$$\sum F_x = 0$$

$$\int_A y \; dA = 0$$

First Moment of Area wrt z-axis

Location of Neutral Axis

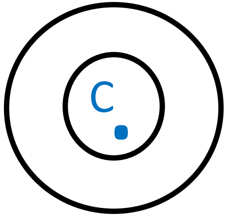


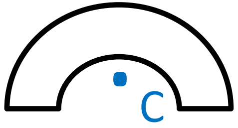


First Moment of Area wrt z-axis $\int_A y \; dA = 0$

Centroids of areas and volumes (geometric center)

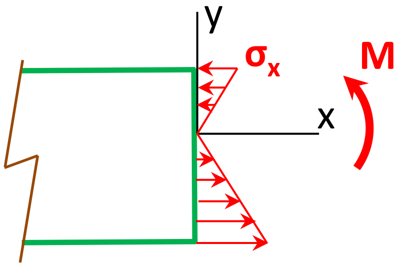
- Does not necessarily have to lie on the body

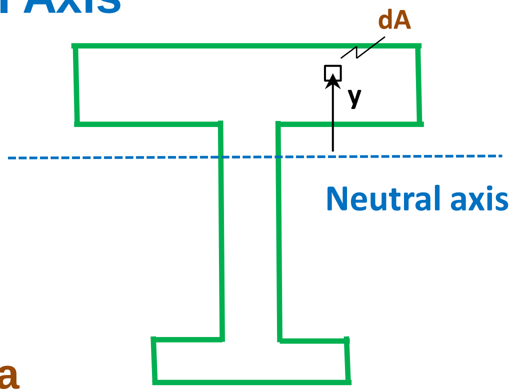




- Will lie on an axis of symmetry

Location of Neutral Axis





First Moment of Area

$$\int_A y \ dA = 0$$

Therefore the neutral axis coincides with the centroidal axis of the cross section (for flexural loading and elastic action)