Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

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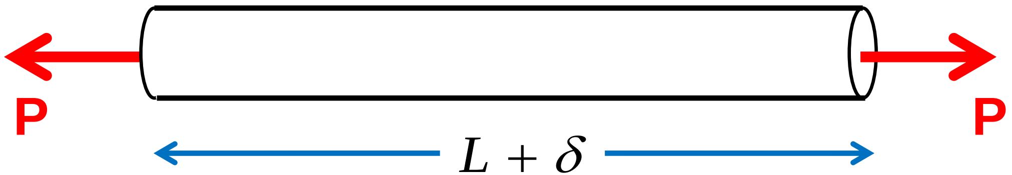
Mechanics of Materials I:
Fundamentals of Stress & Strain and Axial Loading

- ✓ Internal Forces due to External Loads
- ✓ Axial Centric Loads
- ✓ Normal Stress and Shear Stress
- ✓ General State of Stress at a Point (3D)
- ✓ Plane Stress (2D)
- ✓ Normal Strain and Shear Strain
- ✓ Stress-Strain Diagrams
- ✓ Mechanical Properties of Materials
- ✓ Linear Elastic Behavior, Hooke’s Law, and Poisson’s Ratio
- ✓ Stresses on Inclined Planes
- ✓ Principal Stresses and Max Shear Stress
- ✓ Mohr’s Circle for Plane Stress
- ✓ Stress Concentrations
- ❑ Mohr’s Circle for Plane Strain
- ❑ Strain Transformation and Measuring Strains
- ❑ Factor of Safety and Allowable Stresses/Loads
- ❑ Nonlinear Behavior and Plasticity
- ❑ Statically Indeterminate Structures
- ❑ Thermal and Pre-strain Effects

Module 29 Learning Outcome

- Define Two-Dimensional (2D) or Plane Strain

Recall



Normal Strain, ε

Elongation per unit length

$$\varepsilon = \frac{\delta}{L}$$

[dimensionless]

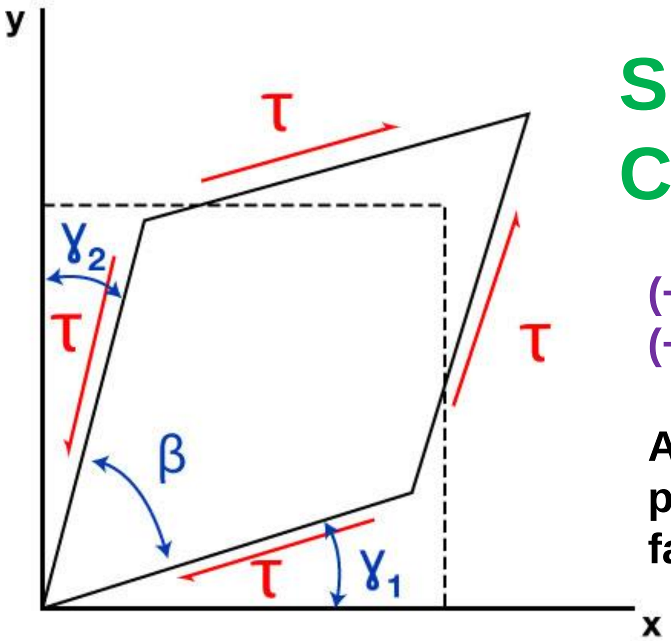
Sign Convention

(+) Tension causes **(+)** elongation
(-) Compression causes **(-)** shortening

Recall

Shear Strain, γ

Change in the angle between perpendicular reference axes; Angular Distortion (Shear Distortion)



Sign Convention

(+) Shear Stress causes
(+) Shear Strain

Angle reduced on 2 positive (or 2 negative) faces

$$\gamma \equiv Shear \quad Strain$$ [dimensionless]

$$\gamma = \gamma_1 + \gamma_2 = \frac{\pi}{2} - \beta$$

State of Strain at a Point

Strains are often easier to measure than stress

Therefore, we often use experimental analysis techniques to measure strains

And then we can use stress-strain equations (derived later) to calculate stresses

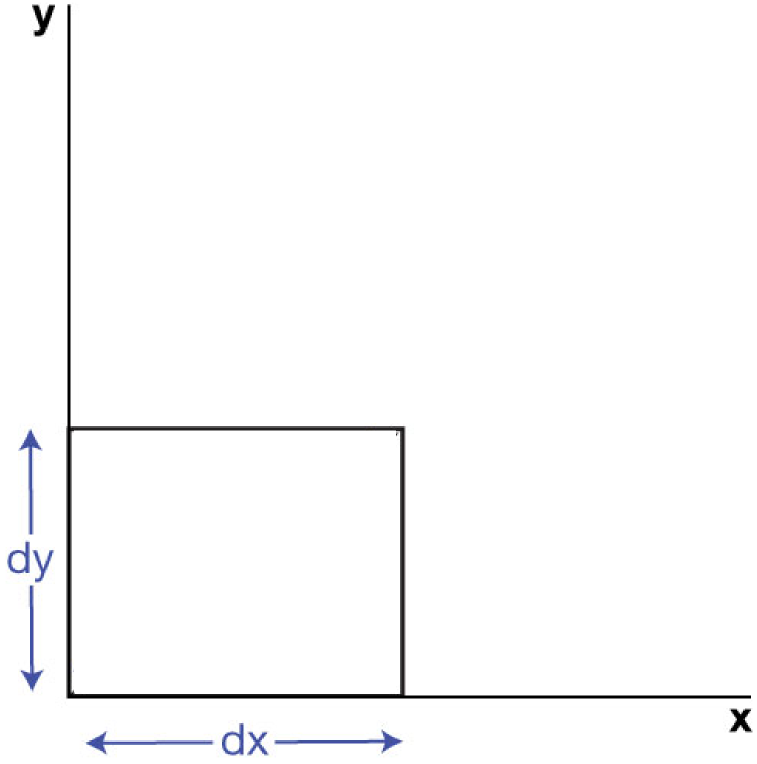
Again we will focus on biaxial or two-dimensional (2D) loading

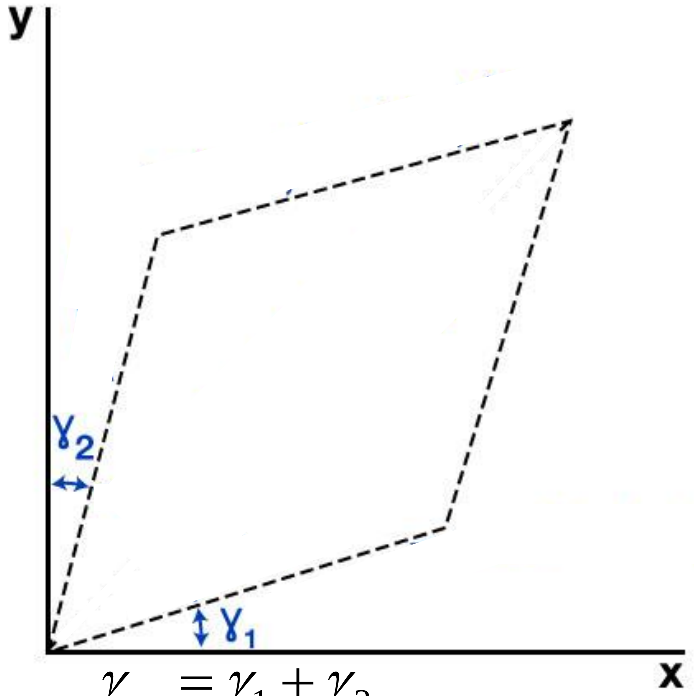
Therefore let's subject a small unrestrained rectangular parallelepiped to a system of two dimension loads (combination of both axial and shear loads)

Recall $$\varepsilon = \frac{\delta}{L}$$

$$\varepsilon_x = \frac{dx' - dx}{dx}$$

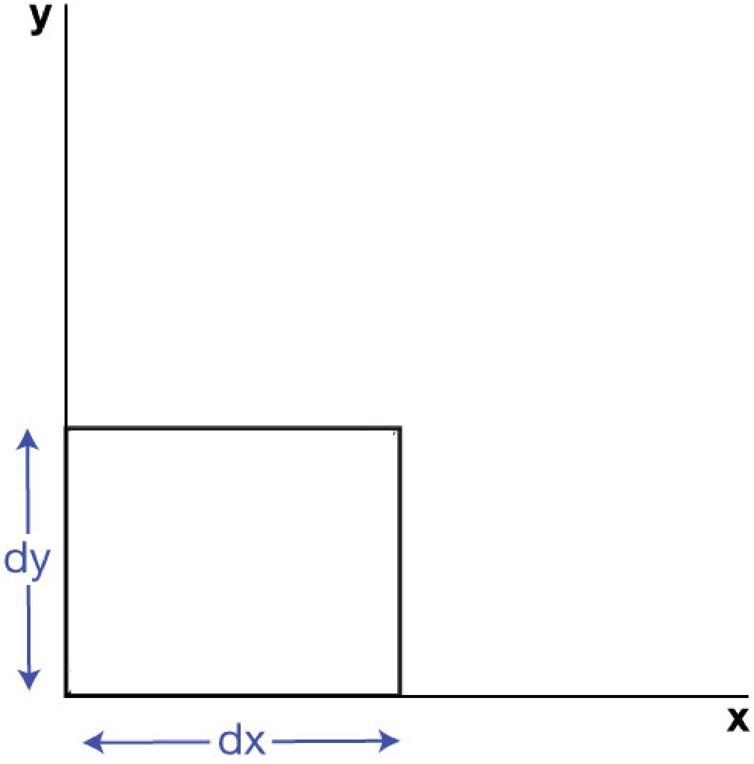
Similarly for dy'

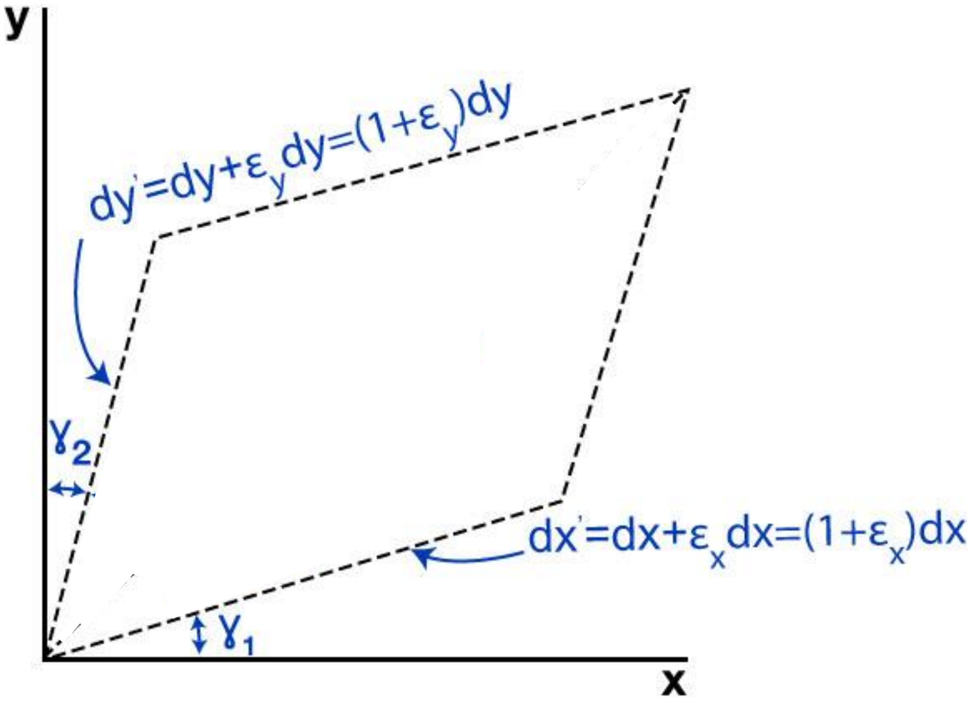




$$\gamma_{xy} = \gamma_1 + \gamma_2$$

Plane Strain



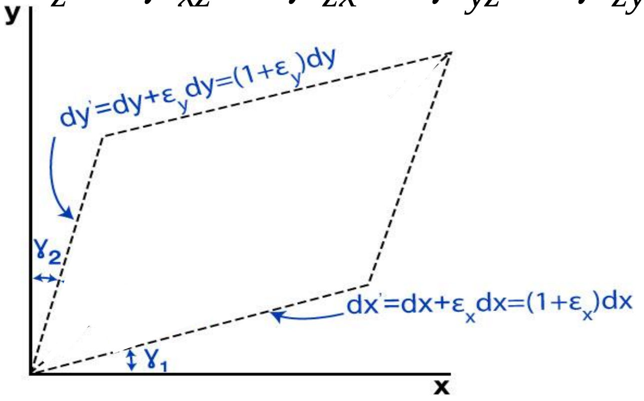


$$\varepsilon_z = \gamma_{xz} = \gamma_{zx} = \gamma_{yz} = \gamma_{zy} = 0$$

$$\gamma_{xy} = \gamma_1 + \gamma_2$$

Plane Strain

$$\varepsilon_z = \gamma_{xz} = \gamma_{zx} = \gamma_{yz} = \gamma_{zy} = 0$$



Plane Strain

No strains in z-direction.
But there can be stresses in the z-direction

Large relative dimension in z-direction with restraints to prevent strain in z-direction

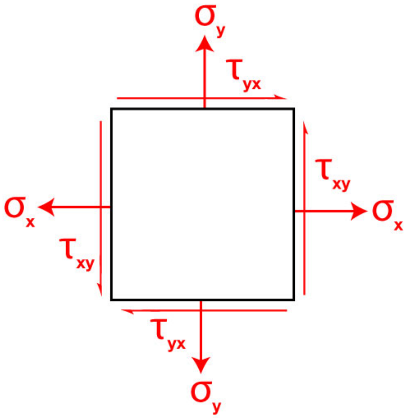
Examples: thick plates

Recall Plane Stress

For Two-Dimensional (2D) or Plane Stress, all out of plane stresses are zero

$$\sigma_z = \tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0$$

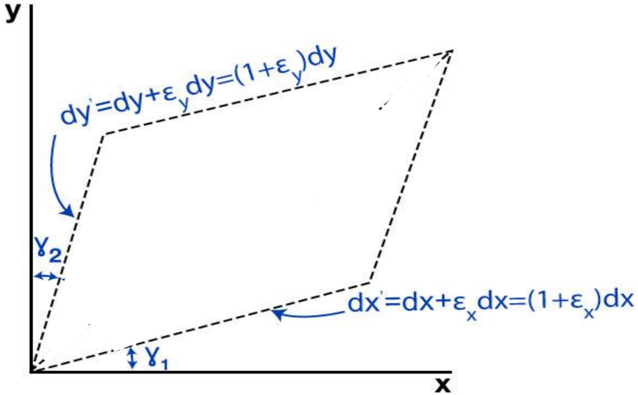
Small relative dimension in z-direction with no surface stress in z-direction
No stresses in z-direction.
But there can be strains in the z-direction



Examples: thin plates

Plane Strain

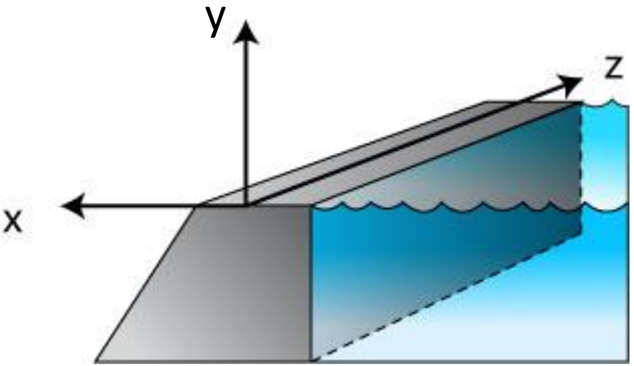
$$\varepsilon_z = \gamma_{xz} = \gamma_{zx} = \gamma_{yz} = \gamma_{zy} = 0$$



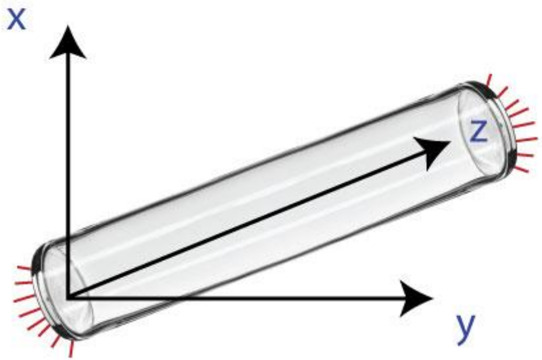
No strains in z-direction.
But there can be stresses in the z-direction

Large relative dimension in z-direction with restraints to prevent strain in z-direction

Dams, Retaining Walls, Tunnels



Bars, tubes, etc. compressed by forces normal to their cross-section



Should I use Plane Stress or Plane Strain?

You must use engineering judgment in modeling and be aware of the assumptions you are making!