MOOC – "Mechanics of Materials II"
Section Two Quiz

Problem 1) A solid circular aluminum shaft is 50 mm in diameter. It is subjected to a pure torque of 600 N-m. The length of the bar is 2 meters. The modulus of rigidity for aluminum is 28 GPa.
Find the maximum shear stress in the shaft and the magnitude of the angle of twist over the 2 meter length.

MAX τ occurs at outer surface

$$\tau = \frac{T\rho}{J}$$

$$28\ GPa = 28{,}000\ MPa = 28{,}000\ N/mm^2$$

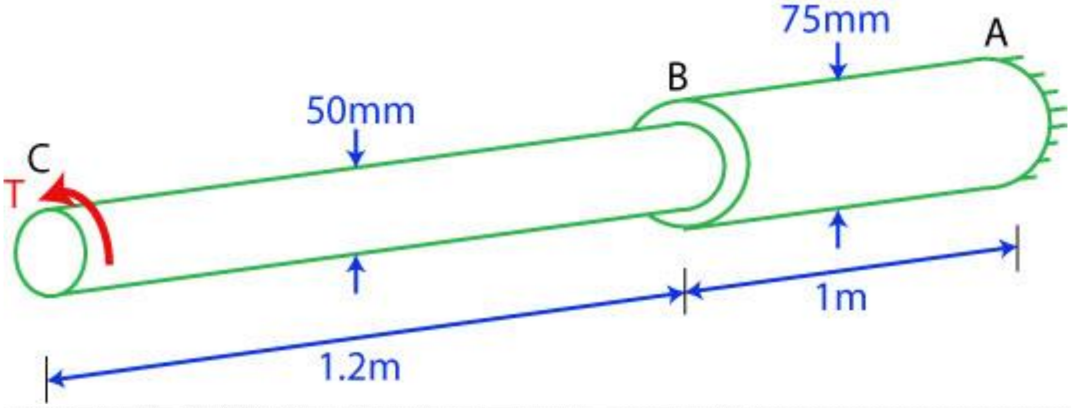
$$\tau_{MAX} = \frac{600\ N\text{-}m\left(\frac{1000\ mm}{m}\right)(25\ mm)}{\left[\frac{\pi}{2}(25\ mm)^4\right]} = \underline{\underline{24.4\ N/mm^2 = 24.4\ MPa}}$$

ANS.

$$\phi = \frac{TL}{GJ} = \frac{600\ N\text{-}m\left(\frac{1000\ mm}{m}\right) 2\ m\left(\frac{1000\ mm}{m}\right)}{(28{,}000\ N/mm^2)\left[\frac{\pi}{2}(25\ mm)^4\right]} = \underline{\underline{0.0698\ rad}}$$

ANS.

Problem 2) A solid circular steel shaft is composed of two sections as shown below. The modulus of rigidity for the steel is 80 GPa. The shaft is subjected to a torque, T, as shown. The allowable shear stress is 70 MPa. The maximum allowable angle of twist of point C with respect to point A is 0.05 radians.
Determine the maximum allowable value of T.



$$J_{AB} = \frac{\pi}{2} r^4 = \frac{\pi}{2}(37.5\,mm)^4 = 3.106 \times 10^6\,mm^4$$

$$J_{BC} = \frac{\pi}{2}(25\,mm)^4 = 6.136 \times 10^5\,mm^4$$

ELASTIC FORMULA: $\tau = \frac{T\rho}{J}$ NOTE: $\tau_{MAX} = 70\,MPa = 70\,N/mm^2$

LIMITING T BASED ON MAX SHEAR STRESS IN SECTION AB

$$T = \frac{(70\,N/mm^2)(3.106 \times 10^6\,mm^4)}{(37.5\,mm)} = 5.80 \times 10^6\,N\text{-}mm$$

LIMITING T BASED ON MAX SHEAR STRESS IN SECTION BC

$$T = \frac{(70\,N/mm^2)(6.136 \times 10^5\,mm^4)}{(25\,mm)} = 1.72 \times 10^6\,N\text{-}mm$$

COMPARE

$T_{ALLOWABLE} = 1.72 \times 10^6\,N\text{-}mm$

ANS

LIMITING T BASED ON MAX ALLOWABLE ANGLE OF TWIST

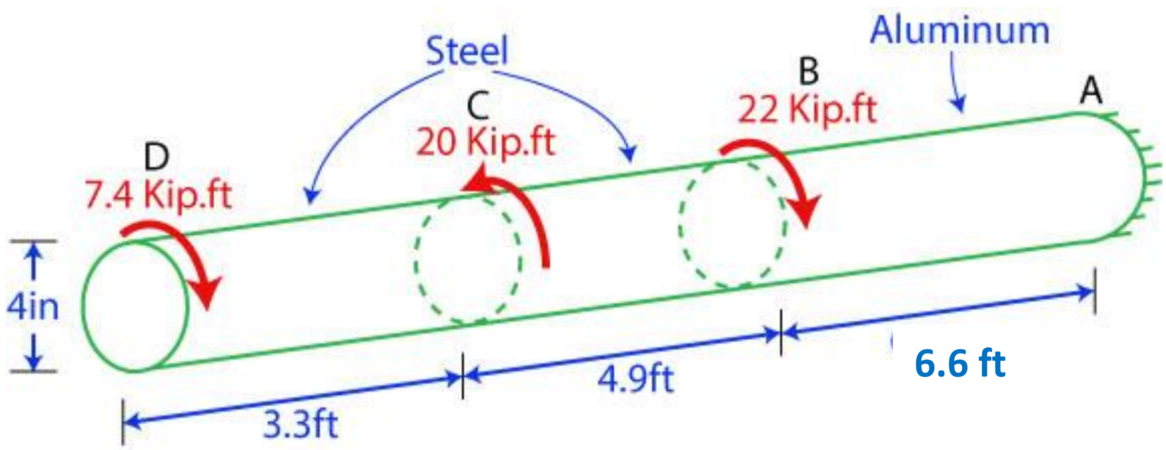
$$\phi_{AB} = \frac{TL}{GJ} = \frac{T(1000\,mm)}{(80\,GPa)\left(\frac{1000\,MPa}{GPa}\right)(3.106 \times 10^6\,mm^4)} = 4.024 \times 10^{-9}\,T$$

$$\phi_{BC} = \frac{T(1200\,mm)}{(80\,GPa)\left(\frac{1000\,MPa}{GPa}\right)(6.136 \times 10^5\,mm^4)} = 2.44 \times 10^{-8}\,T$$

$$\phi_{TOTAL} = \phi_{AB} + \phi_{CD} = 2.85 \times 10^{-8}\,T = 0.05 \Rightarrow$$

$T = 1.76 \times 10^6$ N-mm

Problem 3) A 4 inch diameter composite shaft is subjected to the torques shown below. The sections are perfectly bonded rigidly together. Use G_{STEEL} = 11,600 ksi and $G_{ALUMINUM}$ =4000 ksi. Determine the maximum shearing stress in the shaft and the magnitude of the angle of twist of point D with respect to point A



FBD SECTION CD

7.4 k-ft

$T_{CD} = 7.4$ Kip-ft

FBD SECTION BC

7.4 Kip ft   20 ft-Kip

$T_{BC} = 12.6$ Kip-ft

FBD SECTION AB

20 Kip ft

7.4 Kip ft   22 Kip ft T_{AB}

$T_{AB} = 9.4$ Kip-ft

MAX SHEAR STRESS OCCURS IN SECTION WITH HIGHEST TORQUE $\Rightarrow T_{BC} = 12.6$ Kip-ft

ELASTIC FORMULA

$$\tau_{MAX} = \frac{T\rho}{J} = \frac{(12.6\ \text{Kip-ft})(12\ \text{in/ft})(2\ \text{in})}{\frac{\pi}{2}(2\ \text{in})^4} = \underline{\underline{12.03\ \text{Ksi}}}$$

ANS

$$\phi_{AB} = \left(\frac{TL}{GJ}\right)_{Alum} = \frac{(9.4\ \text{Kip-ft})(12\ \text{in/ft})(6.6\ \text{ft})(12\ \text{in/ft})}{(4000\ \text{ksi})\left[\frac{\pi}{2}(2\ \text{in})^4\right]} = 0.08887$$

$$\phi_{BC} = \frac{TL}{GJ} = \frac{(12.6\ \text{Kip-ft})(12\ \text{in/ft})(4.9\ \text{ft})(12\ \text{in/ft})}{(11{,}600\ \text{Ksi})\left[\pi/2\ (2\ \text{in})^4\right]} = 0.0305$$

$$\phi_{CD} = \frac{TL}{GJ} = \frac{(7.4\ \text{Kip-ft})(12\ \text{in/ft})(3.3\ \text{ft})(12\ \text{in/ft})}{(11{,}600\ \text{Ksi})\left[\pi/2\ (2\ \text{in})^4\right]} = 0.01206$$

$$\underline{\underline{\phi_{TOTAL} = 0.0704\ \text{rad}}}$$

ANS.