Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

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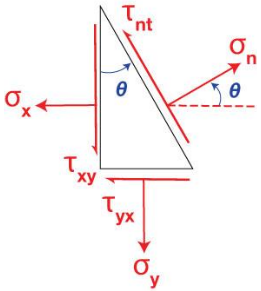
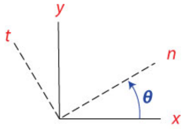
Module 21 Learning Outcome

- Find the Maximum In-Plane Shear Stress

Stresses on Inclined Planes for Plane Stress in general





$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

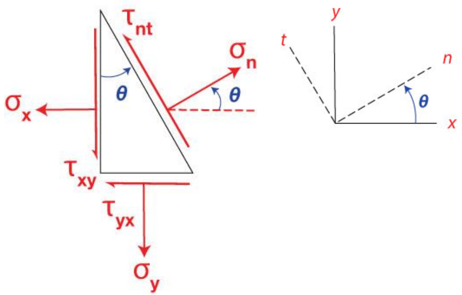
For any plane at an angle θ, we can find σ_n and τ_{nt}

For all structural and machine design, it is necessary to know at what planes and angles θ that the maximum values for σ_n and τ_{nt} occur

First, let's find the angle(s) where the maximum shear stress, τ_{nt} , occurs

How should we proceed?

Stresses on Inclined Planes for Plane Stress in general



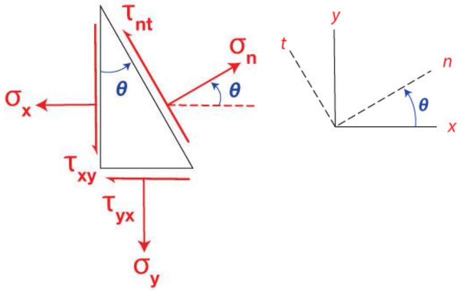
$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\frac{d\tau_{nt}}{d\theta} = 0 = -(\sigma_x - \sigma_y)\cos 2\theta - 2\tau_{xy}\sin 2\theta$$

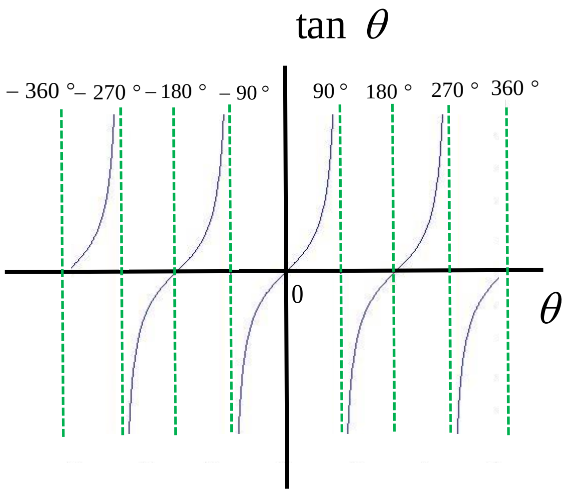
$$\tan 2\theta_S = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Where θ_S is the angle(s) where maximum stress occurs

Stresses on Inclined Planes for Plane Stress in general



$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$



$$\tan 2\theta_S = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Where θ_S is the angle(s) where maximum stress occurs

Recall

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Where θ_P is the angle(s) to what are defined as the **"Principal Planes"**

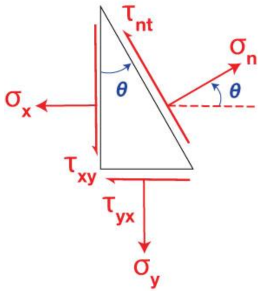
$\tan 2\theta_S$ is the negative reciprical of $\tan 2\theta_P$

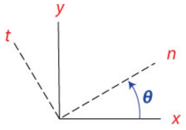
Therefore $2\theta_S$ and $2\theta_P$ are 90° apart

Therefore θ_S and θ_P are 45° apart

The planes on which the maximum in-plane shear stresses occur are 45° from the Principal Planes

Find Maximum In-Plane Shear Stress





$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Similar to before, substituting the angle functions for:

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Yields

$$\tau_{nt} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{MAX} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum In-Plane Shear Stress

$$\tau_{MAX} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Recall Principal Stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{MAX}$$

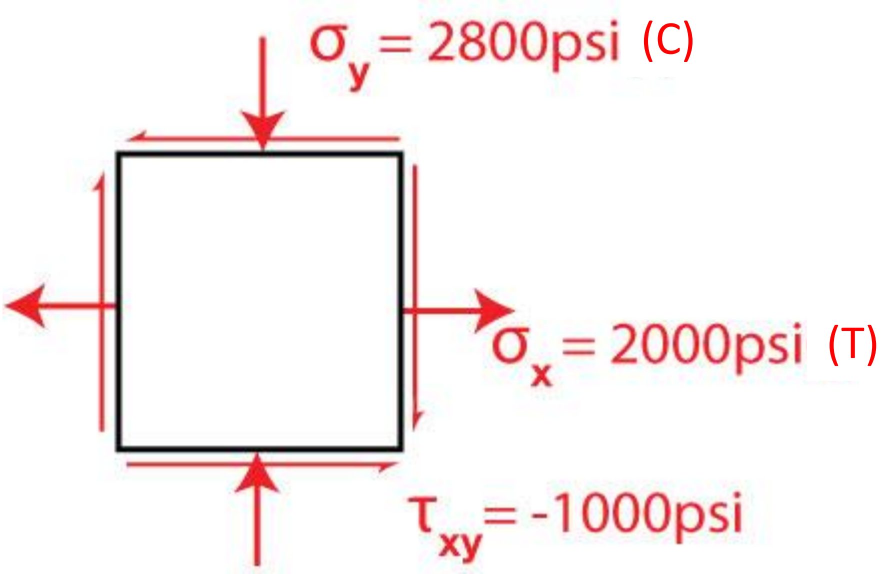
$$\sigma_1 - \sigma_2 = 2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{MAX} = \left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

The maximum in-plane shear stress equals ½ the difference of the two in-plane principal stresses

Worksheet:

For the stress block shown:
Find:
a) The principal stresses
b) The maximum in-plane shear stress

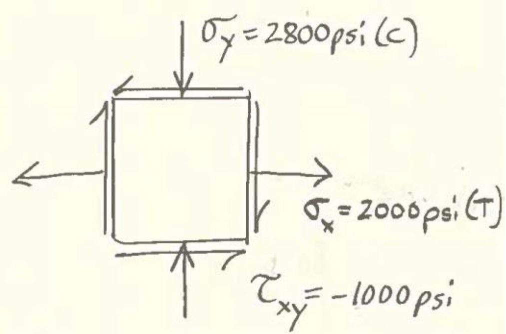


Worksheet Solution:

For the stress block shown:

Find:

a) The principal stresses
b) The maximum in-plane shear stress



$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}}$$

$$\sigma_1, \sigma_2 = \frac{2000 - 2800}{2} \pm \sqrt{\left[\frac{2000 - (-2800)}{2}\right]^2 + (-1000)^2}$$

$$\sigma_1, \sigma_2 = -400 \pm 2600$$

$$\underline{\underline{\sigma_1 = 2200 \quad psi\ (T)}}\ \text{ANS} \qquad \underline{\underline{\sigma_2 = 3000 \quad psi\ (C)}}\ \text{ANS}$$

$$\tau_{MAX} = \left(\frac{\sigma_1 - \sigma_2}{2}\right) = \left(\frac{2200 - (-3000)}{2}\right) = \underline{\underline{2600 \quad psi}}\ \text{ANS}$$