Mechanics of Materials II:

Thin-Walled Pressure Vessels and Torsion

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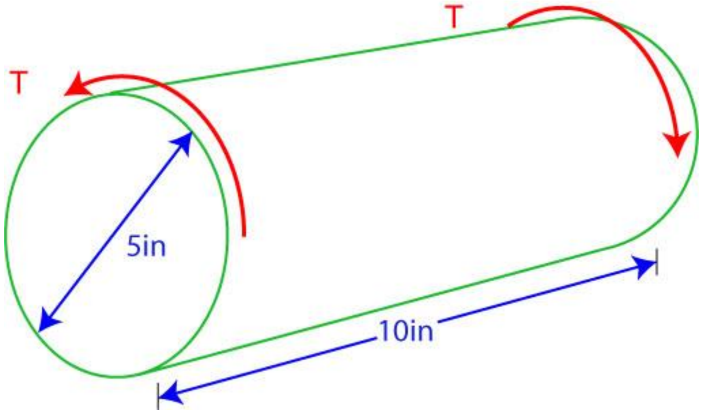
Module 21 Learning Outcome

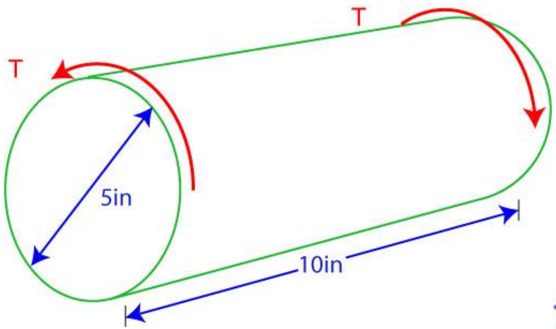
- Solve a problem for the inelastic torsion of straight cylindrical shafts

Inelastic Torsion of Straight Cylindrical Shafts

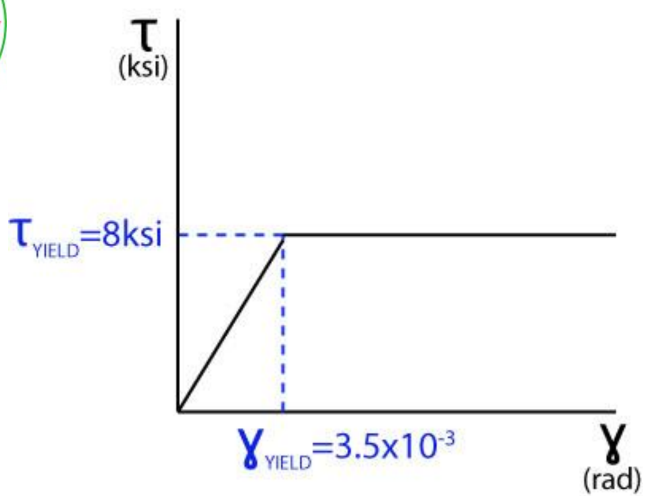
A scale-model prototype of a small portion of a turbine shaft is analyzed for inelastic behavior



Inelastic Torsion of Straight Cylindrical Shafts



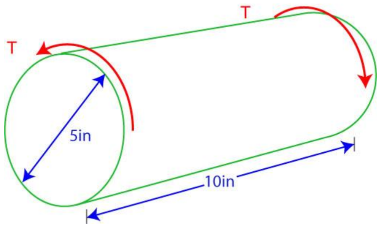
Idealized Magnesium
Shear Stress-Strain Diagram

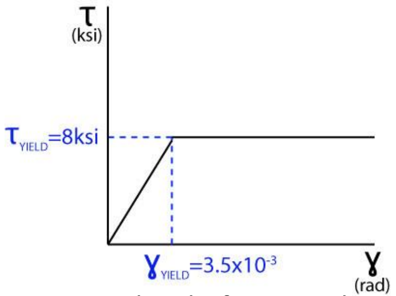


Worksheet:

a) Determine the maximum torque the shaft can withstand if the entire cross section remains elastic.
b) Determine the maximum torque the shaft can withstand if the maximum shear strain is 7 X 10^{-3} rad.
c) Determine the angle of twist experienced by the shaft for part b)

Inelastic Torsion of Straight Cylindrical Shafts





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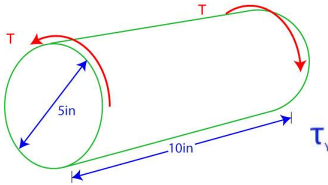
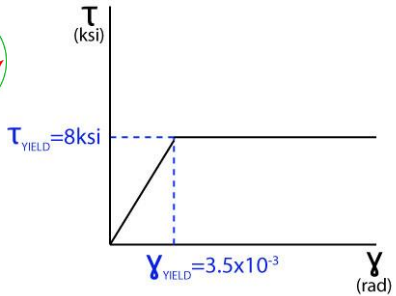
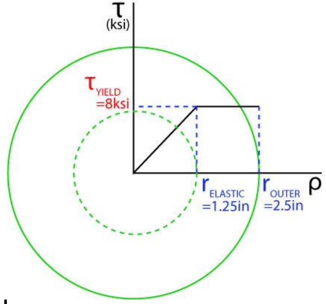
Inelastic Torsion of Straight Cylindrical Shafts







Worksheet:

b) Determine the maximum torque the shaft can withstand if the maximum shear strain is $7 \text{ X } 10^{-3}$ rad.

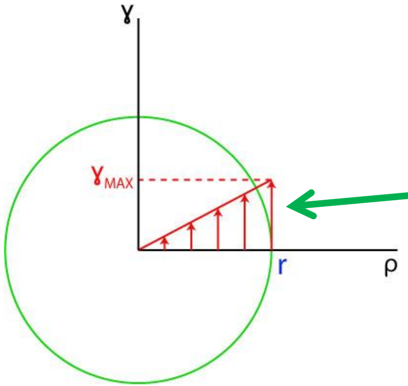
$$\frac{\gamma_{MAX}}{r_{OUTER}} = \frac{\gamma}{\rho} = \frac{\gamma_{YIELD}}{r_{ELASTIC}}$$

$$T_{PLASTIC} = \frac{2}{3}\pi\tau_{YIELD}\left(r_{OUTER}^3 - r_{ELASTIC}^3\right)$$

$$T_{ELASTIC} = \frac{\tau_{YIELD} J_{\substack{ELASTIC \\ REGION}}}{r_{ELASTIC}}$$

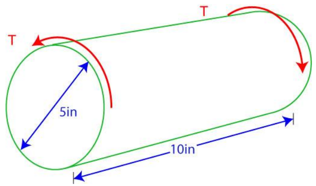
Inelastic Torsion of Straight Cylindrical Shafts

Shear Strains vary linearly with ρ



$$\gamma = \rho\theta = \frac{\rho}{r}\gamma_{MAX}$$

radial distance from center



Rate of Twist, θ
(angle of twist per unit length)

$$\theta = \frac{d\phi}{dx} = \frac{\phi}{L}$$

$$\gamma = \frac{\rho\phi}{L}$$

Note: So far we haven't specified any material properties:

material could be in elastic or inelastic region

material could homogeneous or heterogeneous

we have specified small angles: $\tan\gamma \approx \gamma = \frac{s}{L}$

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