Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

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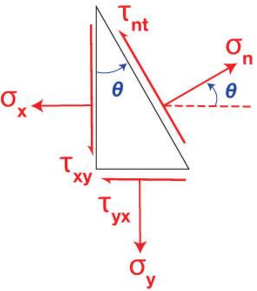
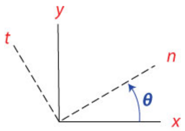
Module 20 Learning Outcome

- Find the Maximum and Minimum In-Plane Principal Stresses

Stresses on Inclined Planes for Plane Stress in general





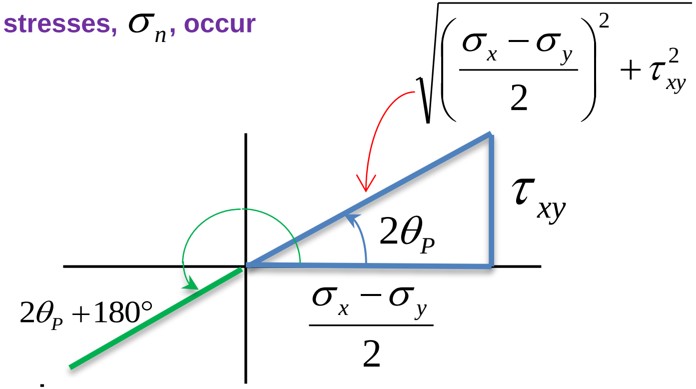
$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Angle(s) where the max/min normal stresses, σ_n, occur

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Where θ_P is the angle(s) to what are defined as the "Principal Planes"



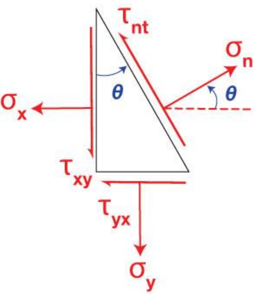
Consider $\frac{\sigma_x - \sigma_y}{2}$ and τ_{xy} the same sign

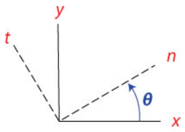
$\tan 2\theta_P$ is positive. Therefore $2\theta_P$ is between 0° and 90° and between 180° and 270°

There are 2 values of θ_P. One is between 0° and 45° and the other is 90° greater

The rotation is counterclockwise.

Stresses on Inclined Planes for Plane Stress in general



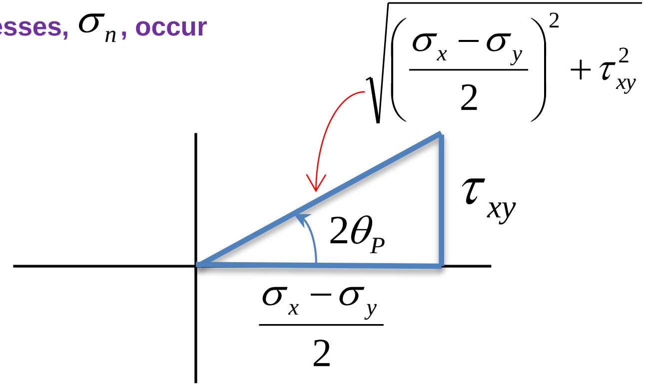


$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Angle(s) where the maximum normal stresses, σ_n, occur

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



Where θ_P is the angle(s) to what are defined as the "Principal Planes"

Consider $\frac{\sigma_x - \sigma_y}{2}$ **and** τ_{xy} **different signs**

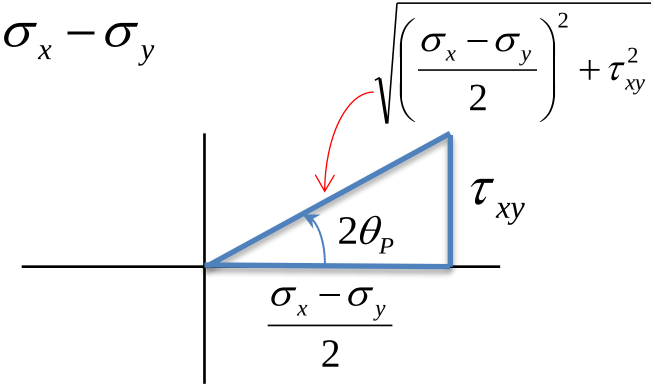
$\tan 2\theta_P$ **is negative.** **Therefore** $2\theta_P$ **is between 0° and -90°** **and between -180° and -270°**

There are 2 values of θ_P. **One is between 0° and -45° and the other is 90° less**

The rotation is clockwise.

Thus

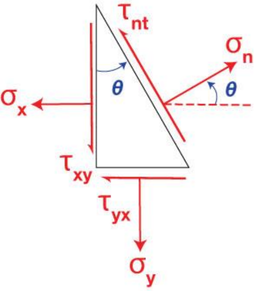
$$\tan 2\theta_P = \pm \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

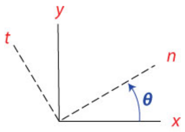


$$\cos 2\theta_P = \pm \frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\sin 2\theta_P = \pm \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

Find Maximum and Minimum In-Plane Principal Stresses





$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Sub

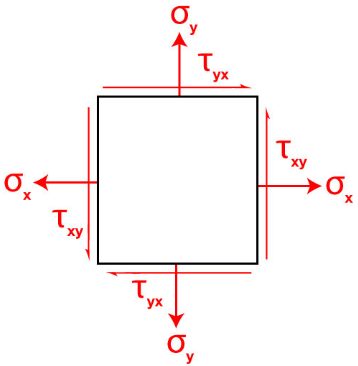
$$\cos 2\theta_P = \pm \frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad \sin 2\theta_P = \pm \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

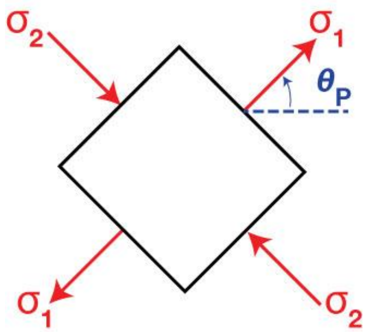
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\left(\pm \frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \right) + \tau_{xy}\left(\pm \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \right)$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} + \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]\left(\pm \frac{1}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \right)$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum and Minimum In-Plane Principal Stresses





$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Note that in this development we considered the maximum and minimum stresses as algebraic quantities. But the minimum algebraic stress may have a larger magnitude than that maximum stress.

For engineering problems, the term "maximum" will refer to the largest absolute value (largest magnitude)

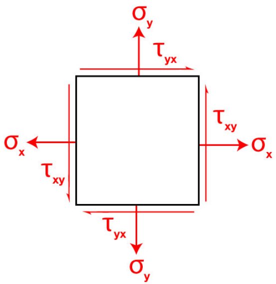
For example:

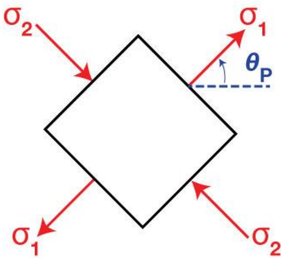
$$\sigma_1 = +700\ MPa$$

$$\sigma_2 = -1200\ MPa$$

Maximum Normal Stress

Maximum and Minimum In-Plane Principal Stresses





$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Stress Invariant

$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

The sum of the normal stresses on any two perpendicular (or orthogonal) planes is constant (or invariant).