Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

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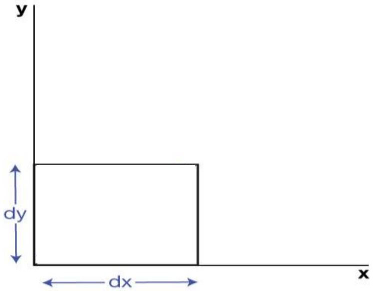
Module 32 Learning Outcome

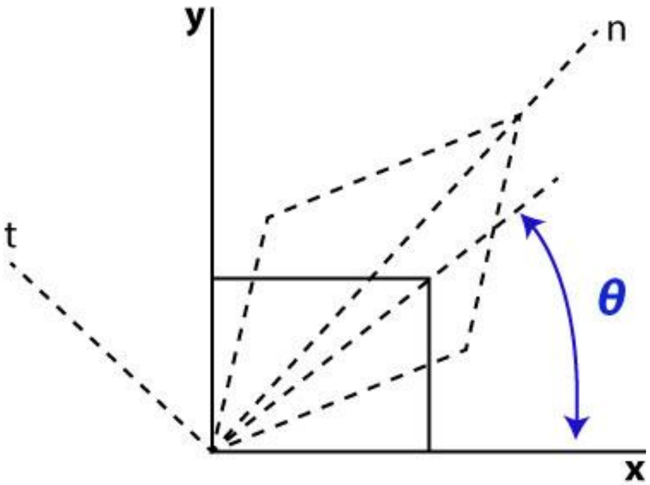
- Develop Mohr’s Circle for Plane Strain

Plane Strain



$$\varepsilon_z = \gamma_{xz} = \gamma_{zx} = \gamma_{yz} = \gamma_{zy} = 0$$

In general, ε_x , ε_y , *and* $\gamma_{xy} = \gamma_{yx}$
are known or can be found



Find: $\varepsilon_n, \gamma_{nt}$ *for any angle* θ

Normal Strain Transformation Equation

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$\varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

Shear Strain Transformation Equation

$$\gamma_{nt} = -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\frac{\gamma_{nt}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

Strain Transformation Equations for Plane Strain

$$\varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{nt}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

Recall Stress Transformation Equations for Plane Stress

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Recall Stress Transformation Equations for Plane Stress

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Strain Transformation Equations for Plane Strain

$$\varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{nt}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

Max/Min-Plane Principal Stresses/Principal Planes

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Max/Min-Plane Principal Strains/Principal Planes

$$\varepsilon_1, \varepsilon_2 = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

Maximum In-Plane Shear Stress

$$\tau_{MAX} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

These are the exact same form of the equations for Mohr's Circle for Plane Stress

Maximum In-Plane Shear Strain

$$\frac{\gamma_{MAX}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Therefore we can similarly graphically display Mohr's Circle for Plane Strains

Plane Stress Mohr's Circle

Graphical tool for the depiction of the transformation equations for plane stress

$$\left(\sigma_n - \frac{\sigma_x + \sigma_y}{2}\right)^2 + (\tau_{nt} - 0)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

Radius = $\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Center: $\left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = (\sigma_{AVG}, 0)$

The angle on Mohr's circle is 2 times the stress block angle

Mohr's circle is a circle where each point represents the stress σ and τ on a particular plane through a single point

Plane Strain Mohr's Circle

Graphical tool for the depiction of the transformation equations for plane stress

$$\left(\varepsilon_n - \frac{\varepsilon_x + \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2} - 0\right)^2 = \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2$$

Radius = $\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$

Center: $\left(\frac{\varepsilon_x + \varepsilon_y}{2}, 0\right) = (\varepsilon_{AVG}, 0)$

The angle on Mohr's circle is 2 times the stress block angle

Mohr's circle is a circle where each point represents the stress ε and $\gamma / 2$ on a particular plane through a single point