Mechanics of Materials III:

Beam Bending

Dr. Wayne Whiteman
Senior Academic Professional and Director of the Office of Student Services
Woodruff School of Mechanical Engineering

Georgia Institute of Technology

Module 14 Learning Outcome

- Solve an elastic beam bending problem.

Elastic Beam Bending

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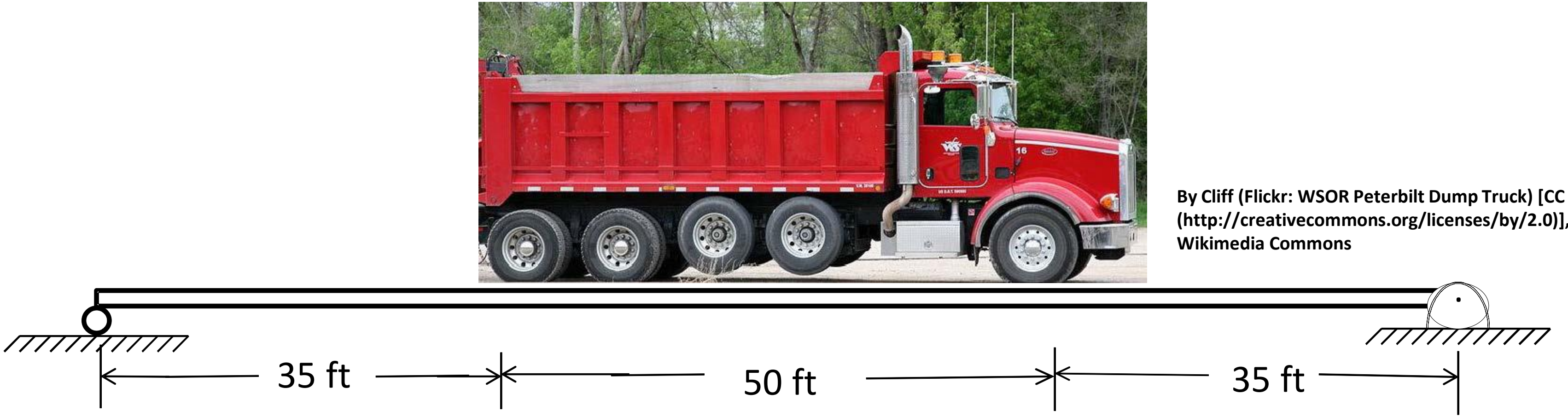

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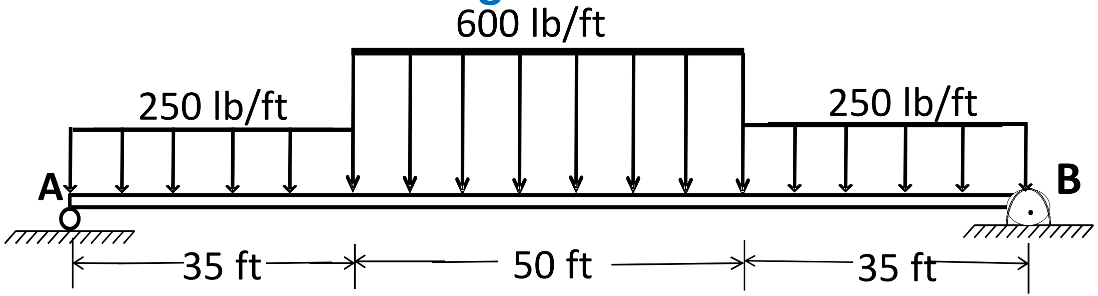
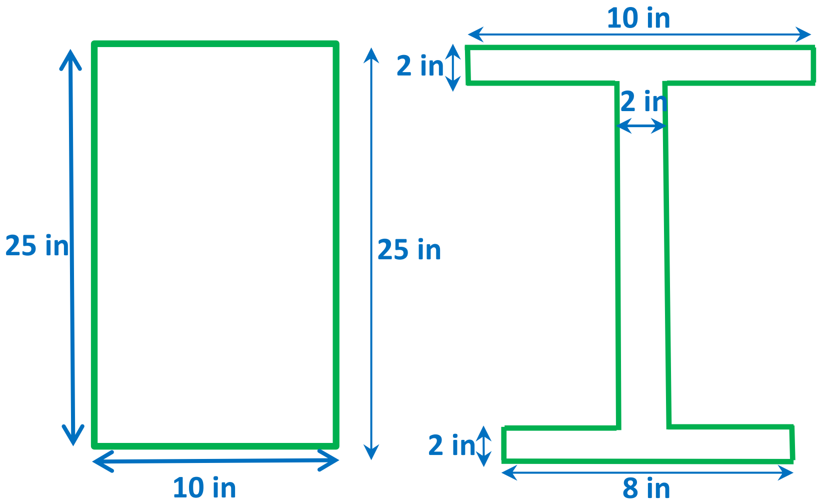
Elastic Beam Bending

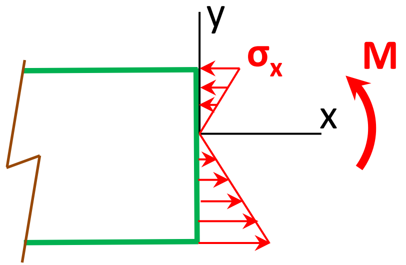


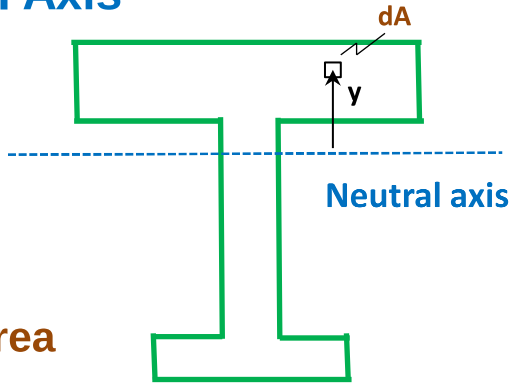


Worksheet: You decide as an engineer to change the rectangular cross section to an I-beam shape to reduce the weight of the beam, and thereby reduce the amount of material/cost. (more efficient design)

If the beam is made of steel with an elastic yield strength of 36 ksi, what is the maximum moment the beam can support and remain in the elastic range?

Location of Neutral Axis



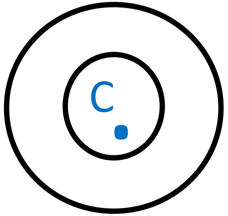


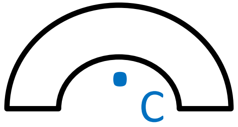
Say see modules 19 and 22 from "Introduction of Engineering Mechanics" for more details regarding the centroid.

First Moment of Area wrt z-axis $\int_A y \; dA = 0$

Centroids of areas and volumes (geometric center)

- **Does not necessarily have to lie on the body**

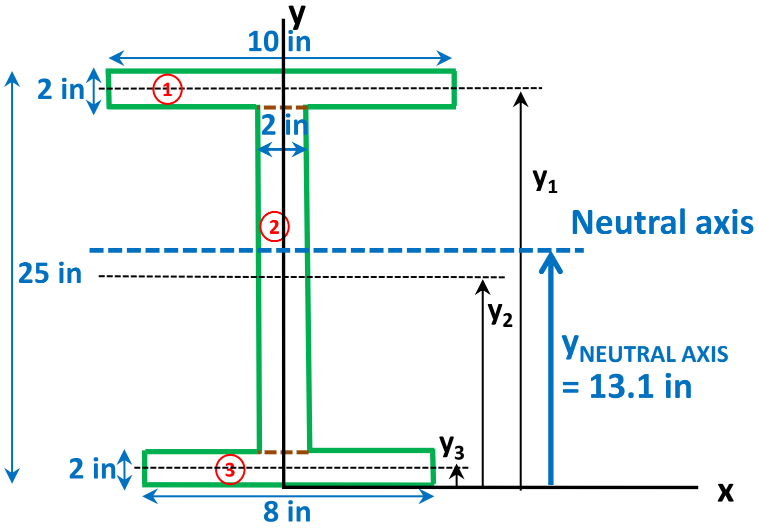




- **Will lie on an axis of symmetry**

Therefore the neutral axis coincides with the centroidal axis of the cross section (for flexural loading and elastic action)

Locate the neutral axis



$$y_{\substack{NEUTRAL\ AXIS \\ (CENTROID)}} = \frac{\sum y_i A_i}{A_{TOTAL}}$$

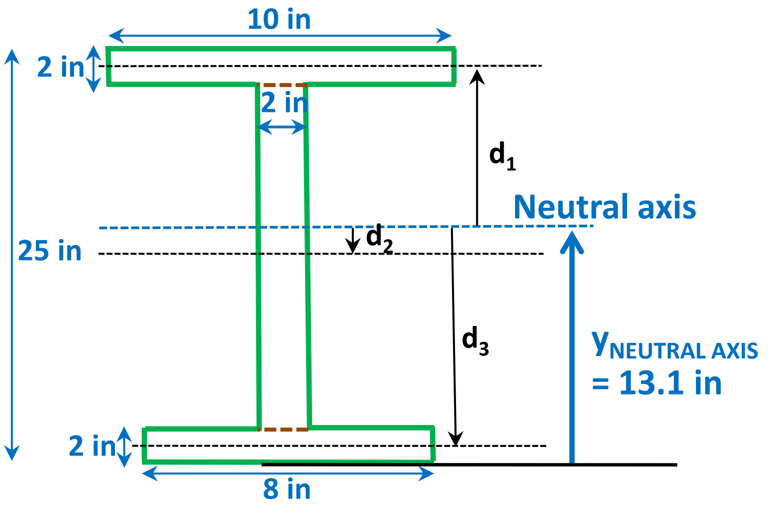
$$A_{TOTAL} = 2(10) + 21(2) + 2(8) = 78 \text{ in}^2$$

$$Y_{NA} = \frac{24(10)2 + 12.5(21)2 + 1(2)8}{78} = \underline{13.1 \text{ in}}$$

Find Area Moment of Inertia, I

For composite shapes (parallel axis theorem):

$$I_{\substack{NEUTRAL\\AXIS}} = I_{\substack{STANDARD\ SHAPE'S\\NEUTRAL\ AXIS}} + Ad^2$$



$$I_{NA} = \frac{1}{12}10(2)^3 + 10(2)(24-13.1)^2$$

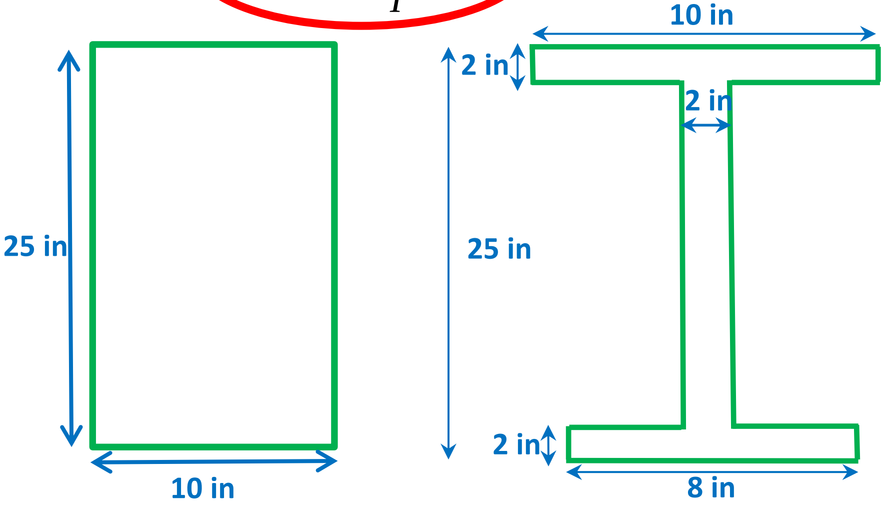
$$+\frac{1}{12}2(21)^3 + (2)21(13.1-12.5)^2 + \frac{1}{12}8(2)^3 + (2)8(13.1-1)^2$$

$$I = 6290\ in^4$$

Elastic Beam Bending

Elastic Flexural Formula

$$\sigma_x = -\frac{M\,y}{I}$$



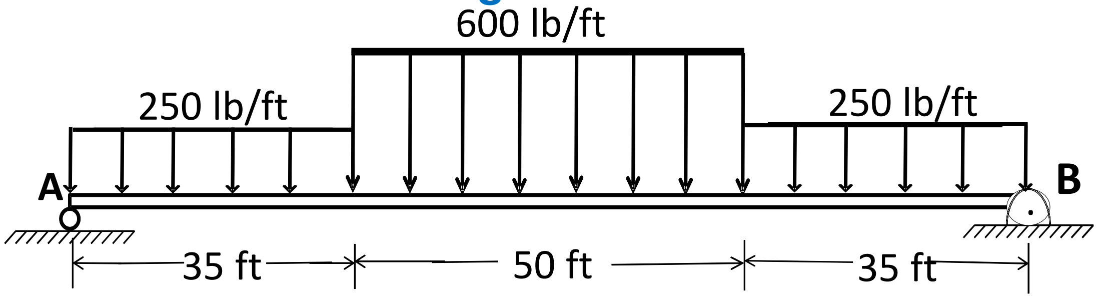
$A = 250 \quad in^2$ $\qquad$ $A = 78 \quad in^2$

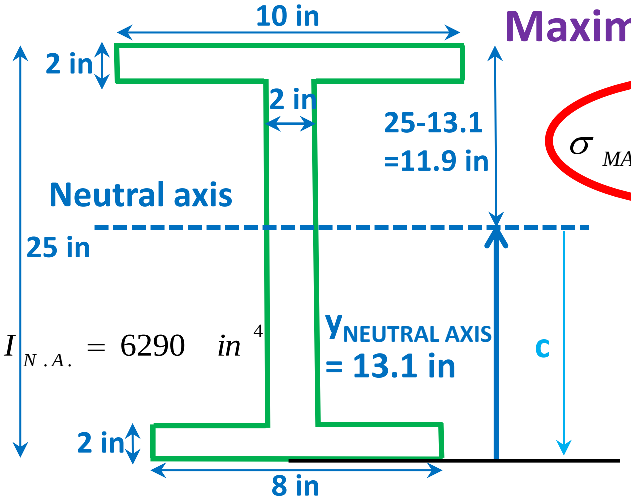
31.2 % of the original area

$I_{N.A.} = 13020 \quad in^4$ $\qquad$ $I_{N.A.} = 6290 \quad in^4$

48.3 % of the original area moment of inertia

Elastic Beam Bending





$$I_{N.A.} = 6290 \quad in^4$$

Maximum Stress

$$\sigma_{MAX} = \frac{M\ c}{I}$$

c is the furthest distance on the cross section from the neutral axis

$$M_{MAX} = \frac{\sigma_{MAX}\ I}{c}$$

$$M_{MAX} = \frac{36\,ksi\ (6290\,in^4)}{13.1\,in} = \underline{\underline{17{,}300\ in\text{-}k}}$$

ANS

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