Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

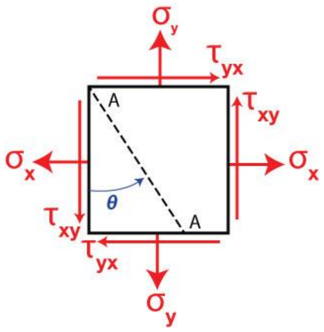
Dr. Wayne Whiteman
Senior Academic Professional and Director of the Office of Student Services
Woodruff School of Mechanical Engineering

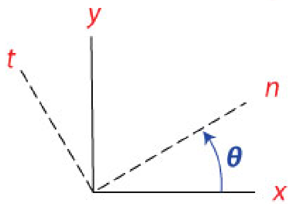
Georgia Institute of Technology

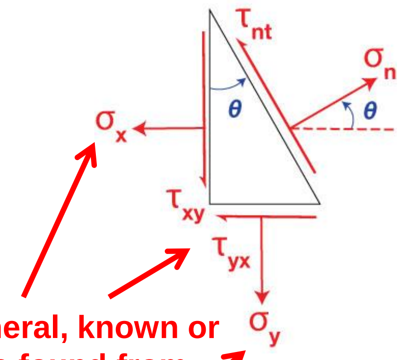
Module 17 Learning Outcomes

- Derive equations for stresses on inclined planes for the case of plane stress in general

Stresses on Inclined Planes for Plane Stress in general







In general, known or can be found from loading condition

Sign Convention

Normal Stress

(+) Tension

(-) Compression

Shear Stress

Shear stress is **(+)** if it is in the **(+)** direction of the coordinate axis

Angle

Counterclockwise angle θ is **(+)** as measured from positive x axis as reference

Find:

$$\sigma_n, \tau_{nt}$$

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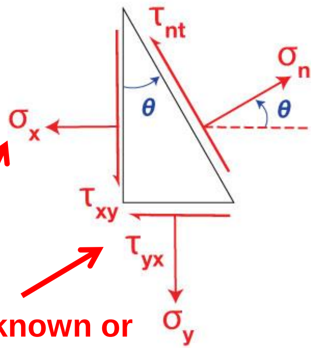


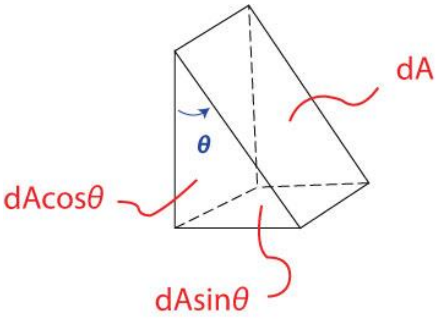
In general, known or can be found from loading condition

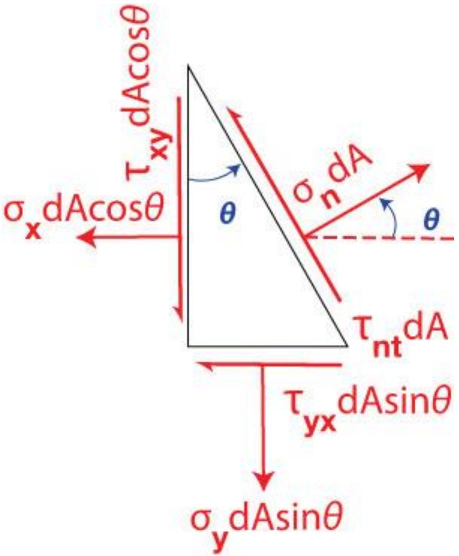
Find:

$$\sigma_n, \tau_{nt}$$

Apply Equilibrium

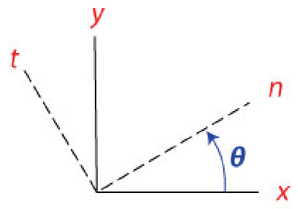
Convert stresses to forces

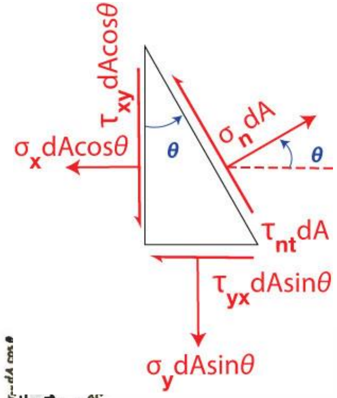




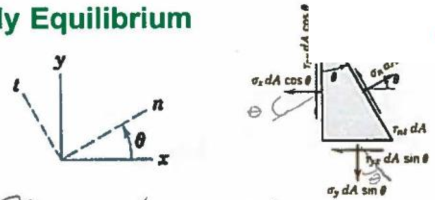
Stresses on Inclined Planes for Plane Stress in general

Apply Equilibrium





Apply Equilibrium



$$\sigma_n \cancel{dA} - (\sigma_x \cancel{dA} \cos\theta)\cos\theta - (\sigma_y \cancel{dA} \sin\theta)\sin\theta$$

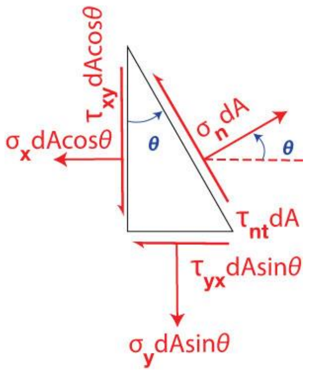
$$- (\tau_{xy} \cancel{dA} \cos\theta)\sin\theta - (\tau_{yx} \cancel{dA} \sin\theta)\cos\theta = 0$$

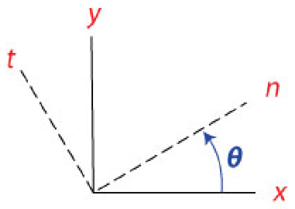
$$\sigma_n = \sigma_x \overset{\frac{1+\cos 2\theta}{2}}{\cos^2\theta} + \sigma_y \overset{\frac{1-\cos 2\theta}{2}}{\sin^2\theta} + 2\tau_{xy} \overset{\frac{\sin 2\theta}{2}}{\sin\theta\cos\theta}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Stresses on Inclined Planes for Plane Stress in general

Apply Equilibrium





$$+\nwarrow \quad \sum F_t = 0$$

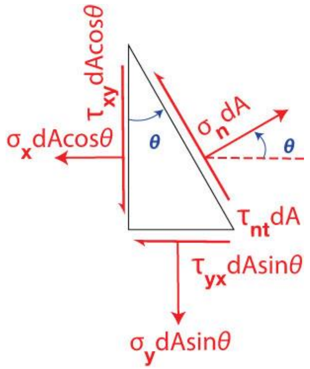
$$\tau_{nt}\, dA - \left(\tau_{xy}\, dA\cos\theta\right)\cos\theta + \left(\tau_{yx}\, dA\sin\theta\right)\sin\theta$$
$$+\left(\sigma_x\, dA\cos\theta\right)\sin\theta - \left(\sigma_y\, dA\sin\theta\right)\cos\theta = 0$$

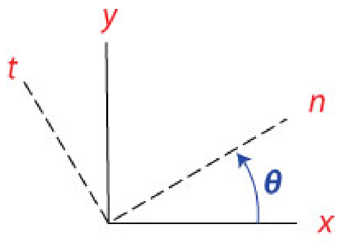
$$\left(\frac{\sin 2\theta}{2}\right) \qquad \left(\frac{1+\cos 2\theta}{2}\right) \qquad \left(\frac{1-\cos 2\theta}{2}\right)$$

$$\tau_{nt} = -\left(\sigma_x - \sigma_y\right)\sin\theta\cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Stresses on Inclined Planes for Plane Stress in general





Say transform the stress components from one set of axes to another

They were derived solely from equilibrium, therefore they are applicable to stresses for any material, whether linear or nonlinear, elastic or inelastic

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Transformation equations for plane stress

For any plane at an angle θ,
We can find σ_n and τ_{nt}

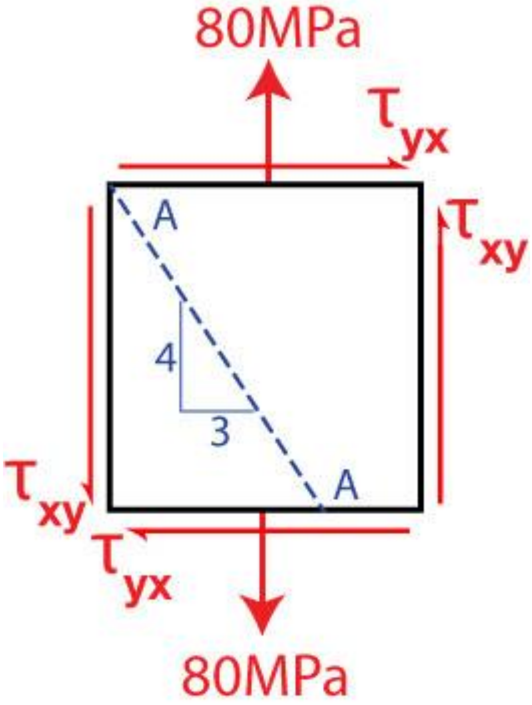
Worksheet:

The stresses on the horizontal and vertical planes at a point are shown in the figure below. The normal stress on a plane A-A at this point is found to be 80 MPa in tension.

Find:

a) The magnitude and direction of the shear stresses, $\tau_{xy} = \tau_{yx}$

b) The magnitude and direction of the shear stress on the inclined plane A-A

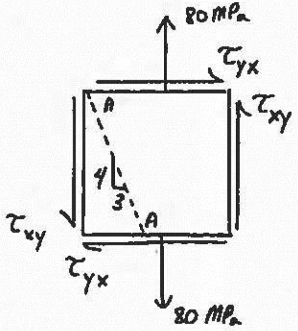


Worksheet: The stresses on the horizontal and vertical planes at a point are shown in the figure to the right. The normal stress on a plane A-A at this point is found to be 80 MPa in tension.

Find:

a) The magnitude and direction of the shear stresses, $\tau_{xy} = \tau_{yx}$

b) The magnitude and direction of the shear stress on the inclined plane A-A



FOR THIS EXAMPLE, $\sigma_x = 0$ $\quad \theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^{\circ}$

$\sigma_y = 80$ MPa

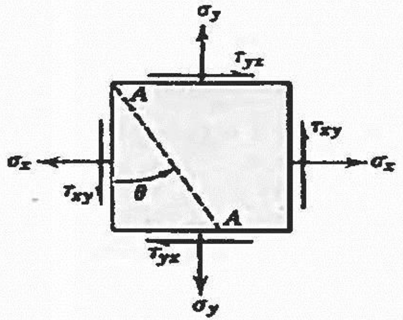
$\sigma_n = 80$ MPa

a)

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$80 = \left(\frac{0+80}{2}\right) + \left(\frac{0-80}{2}\right)\cos[2(36.87^{\circ})] + \tau_{xy}\sin[2(36.87^{\circ})]$$

$\tau_{xy} = 53.3$ MPa in the positive directions shown ANS.



$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{nt} = -\left(\frac{0-80}{2}\right)\sin[2(36.87^{\circ})] + 53.3\cos[2(36.87^{\circ})]$$

$\tau_{nt} = 53.3$ MPa in the positive direction as shown ANS