Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

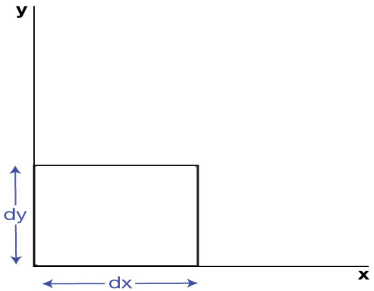
Dr. Wayne Whiteman
Senior Academic Professional and Director of the Office of Student Services
Woodruff School of Mechanical Engineering

Georgia Institute of Technology

Module 30 Learning Outcome

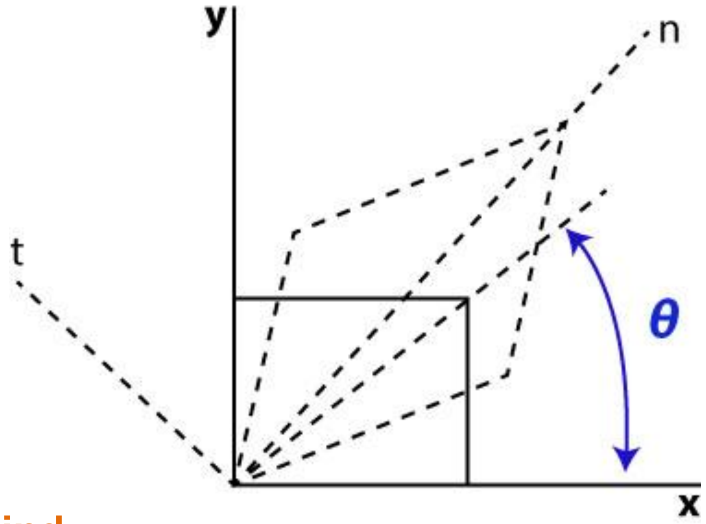
- Derive the strain transformation equations for the case of plane strain

Plane Strain



$$\varepsilon_z = \gamma_{xz} = \gamma_{zx} = \gamma_{yz} = \gamma_{zy} = 0$$

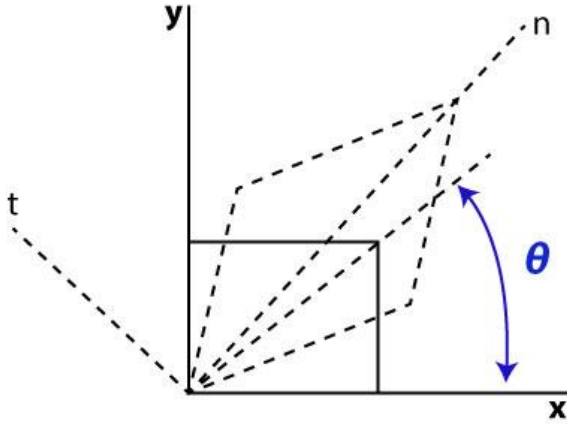
In general, ε_x , ε_y , *and* $\gamma_{xy} = \gamma_{yx}$
are known or can be found

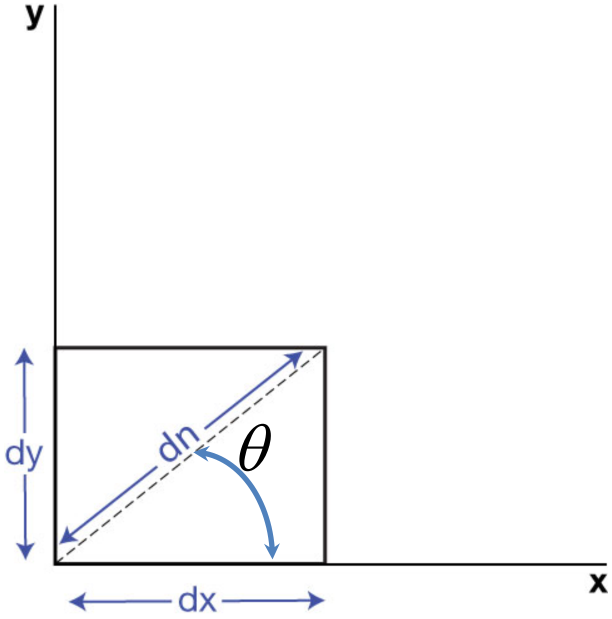


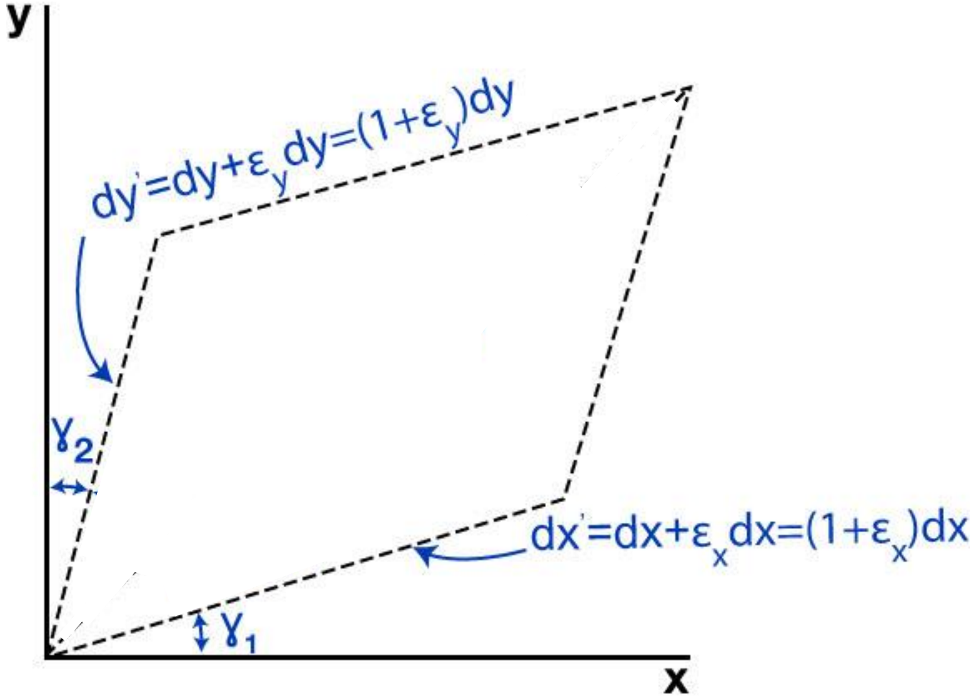
Find:

$$\varepsilon_n, \gamma_{nt} \ \ for\ \ any\ \ angle\ \ \theta$$

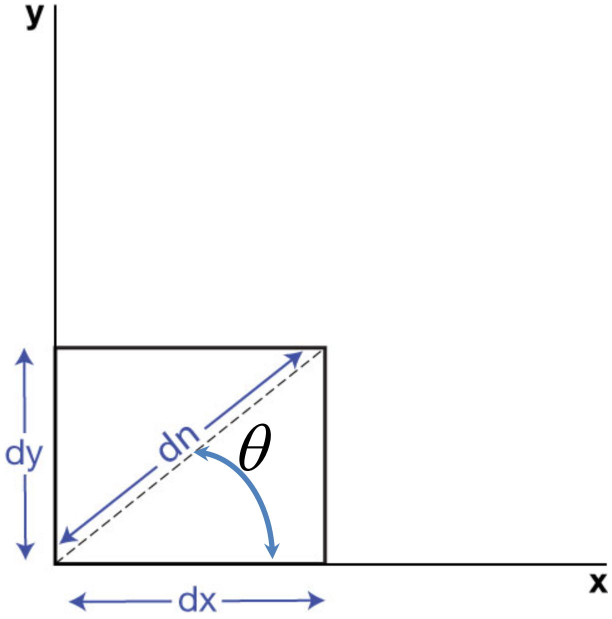
Relate strain in cartesian coordinates x and y to any other coordinates n and t

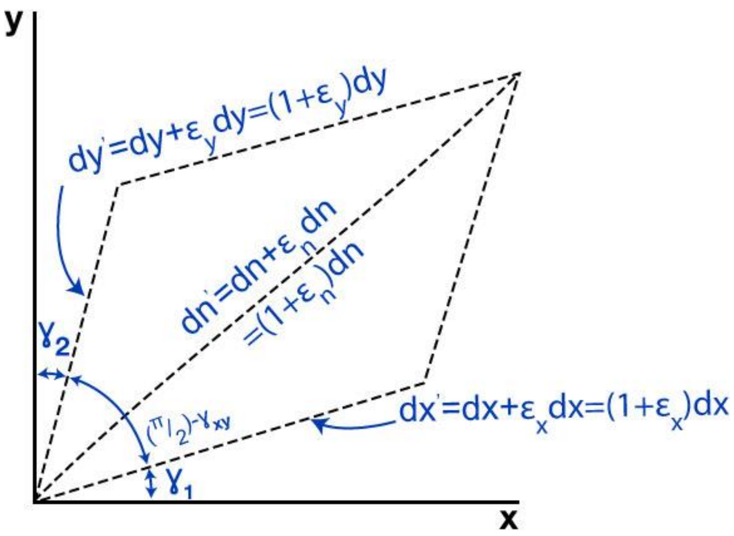






Recall: Rectangular Parallelpiped



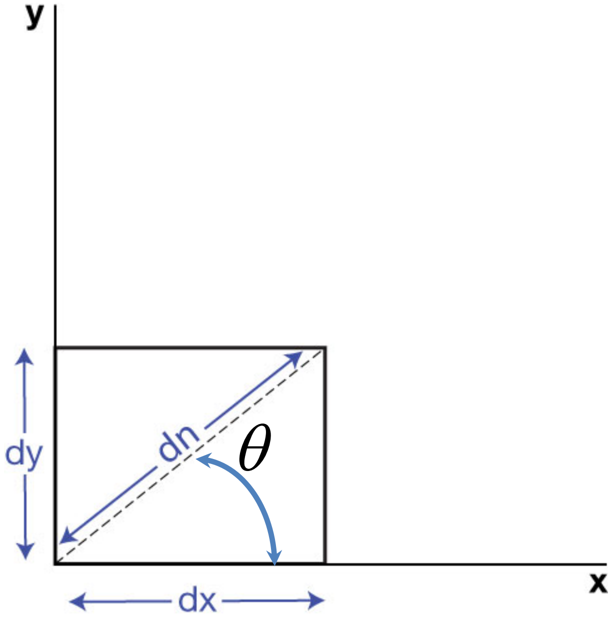


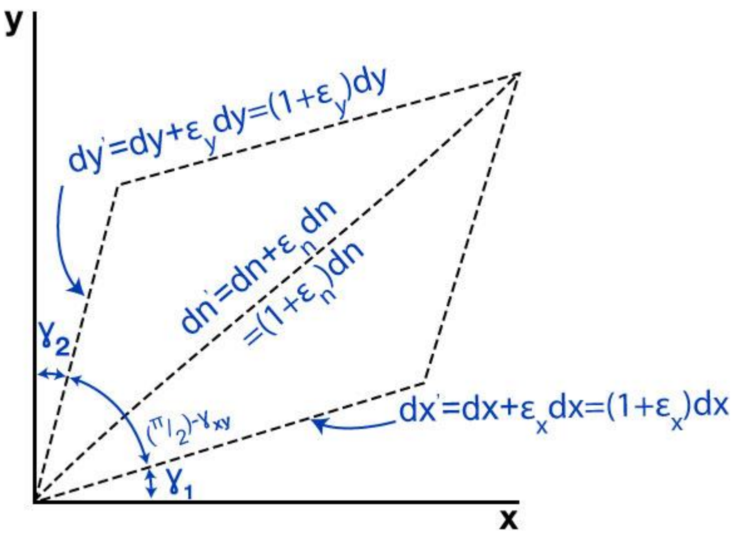
Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\phi$$

$$\left(\frac{\pi}{2} + \gamma_{xy}\right)$$

$$(1+\varepsilon_n)^2 dn^2 = (1+\varepsilon_x)^2 dx^2 + (1+\varepsilon_y)^2 dy^2 - 2(1+\varepsilon_x)dx(1+\varepsilon_y)dy\cos\phi$$





$$(1+\varepsilon_n)^2 dn^2 = (1+\varepsilon_x)^2 dx^2 + (1+\varepsilon_y)^2 dy^2 - 2(1+\varepsilon_x)dx(1+\varepsilon_y)dy\cos\left(\frac{\pi}{2}+\gamma_{xy}\right)$$

Also note

$$\frac{dx}{dn} = \cos\theta \rightarrow dx = dn\cos\theta$$

Trig Identity:

$$\cos\left(\frac{\pi}{2}+\gamma_{xy}\right) = -\sin\gamma_{xy}$$

$$\frac{dy}{dn} = \sin\theta \rightarrow dy = dn\sin\theta$$

$$(1+\varepsilon_n)^2 dn^2 = (1+\varepsilon_x)^2 dx^2 + (1+\varepsilon_y)^2 dy^2 + 2(1+\varepsilon_x)dx(1+\varepsilon_y)dy\sin\gamma_{xy}$$

$$(1+\varepsilon_n)^2 dn^2 = (1+\varepsilon_x)^2 dx^2 + (1+\varepsilon_y)^2 dy^2 + 2(1+\varepsilon_x)dx(1+\varepsilon_y)dy\sin\gamma_{xy}$$

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Also note $dx = dn\cos\theta$

$$dy = dn\sin\theta$$

Substitute: $\cancel{d}n^2\cos^2\theta$ $\quad \cancel{d}n^2\sin^2\theta$ $\quad \cancel{d}n^2\sin\theta\cos\theta$

$$(1+\varepsilon_n)^2 \cancel{d}n^2 = (1+\varepsilon_x)^2 \cancel{dx^2} + (1+\varepsilon_y)^2 \cancel{dy^2} + 2\cancel{(dx\,dy)}(1+\varepsilon_x)(1+\varepsilon_y)\sin\gamma_{xy}$$

$$1+2\varepsilon_n + \cancel{\varepsilon_n^2} = (1+2\varepsilon_x+\cancel{\varepsilon_x^2})\cos^2\theta + (1+2\varepsilon_y+\cancel{\varepsilon_y^2})\sin^2\theta + 2(1+\cancel{\varepsilon_x}+\cancel{\varepsilon_y}+\cancel{\varepsilon_x\varepsilon_y})\cancel{\sin\gamma_{xy}}\sin\theta\cos\theta$$

γ_{xy}

Note that strains are very small

$$\varepsilon^2 << \varepsilon$$

$$\sin\gamma \approx \gamma$$

$\varepsilon\gamma << \varepsilon$ or γ

$$1+2\varepsilon_n = (1+2\varepsilon_x)\cos^2\theta + (1+2\varepsilon_y)\sin^2\theta + 2\gamma_{xy}\sin\theta\cos\theta$$

$$1+2\varepsilon_n=(1+2\varepsilon_x)\cos^2\theta+(1+2\varepsilon_y)\sin^2\theta+2\gamma_{xy}\sin\theta\cos\theta$$

$$\cancel{1}+\cancel{2}\varepsilon_n=1\overbrace{(\sin^2\theta+\cos^2\theta)}^{1}+\cancel{2}\varepsilon_x\cos^2\theta+\cancel{2}\varepsilon_y\sin^2\theta+\cancel{2}\gamma_{xy}\sin\theta\cos\theta$$

Normal Strain Transformation Equation

$$\varepsilon_n=\varepsilon_x\cos^2\theta+\varepsilon_y\sin^2\theta+\gamma_{xy}\sin\theta\cos\theta$$

Normal Strain Transformation Equation

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$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

Trig Identities:

$$\cos 2\theta = 2\cos^2\theta - 1$$

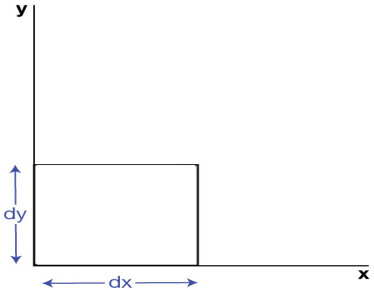
$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\varepsilon_n = \varepsilon_x \underbrace{\cancel{\cos^2}}_{\left(\frac{1+\cos 2\theta}{2}\right)}\theta + \varepsilon_y \underbrace{\cancel{\sin^2}}_{\left(\frac{1-\cos 2\theta}{2}\right)}\theta + \gamma_{xy} \underbrace{\cancel{\sin\theta\cos\theta}}_{\left(\frac{\sin 2\theta}{2}\right)}$$

$$\varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

Plane Strain



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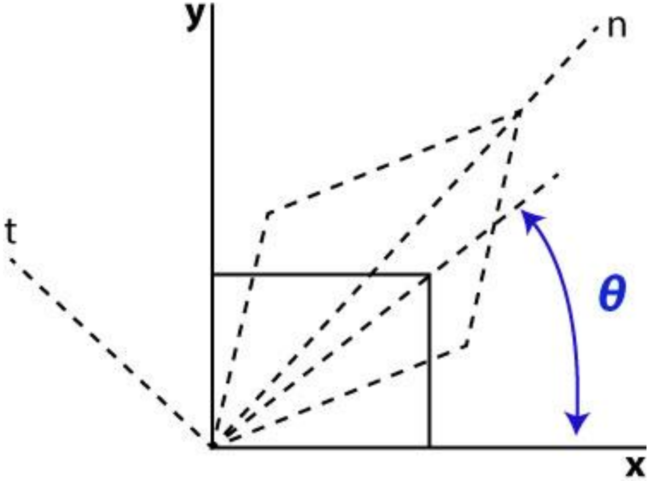

Find: $\varepsilon_n, \gamma_{nt}$ *for any angle* θ

Normal Strain Transformation Equation

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$$

$$\varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$