Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

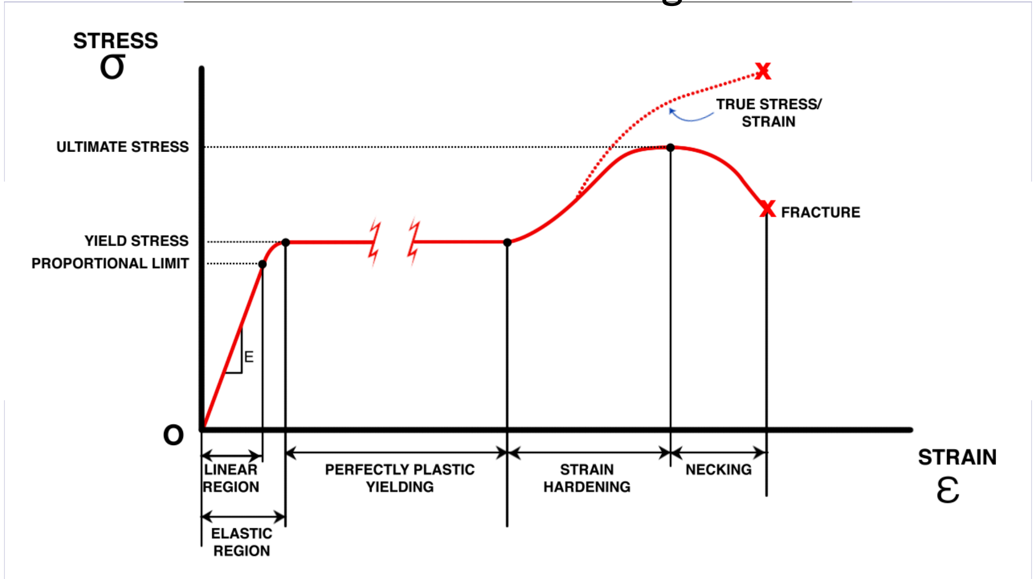
Dr. Wayne Whiteman
Senior Academic Professional and Director of the Office of Student Services
Woodruff School of Mechanical Engineering

Georgia Institute of Technology

Module 38 Learning Outcomes

- Define Isotropic materials
- Develop Generalized Hooke's Law for Isotropic Materials

Normal Stress-Strain Diagram



Material Properties

Stiffness: E = Modulus of Elasticity

= Young's Modulus

Hooke's Law (valid for linear elastic region):

$$\sigma = E\,\varepsilon$$

Hooke's Law
(Normal Stress-Strain; Uniaxial Loading)

$$\sigma = E\,\varepsilon$$

Can be extended for use in more engineering applications for situations involving biaxial and triaxial loading

Recall Hooke's Law assumed elastic behavior

Let's add another common assumption that the material is isotropic

Isotropic is defined as having the same material properties in all directions

Young's modulus, E , such that $E_x = E_y = E_z = E$

And Poisson's Ratio, υ , such that $\upsilon_x = \upsilon_y = \upsilon_z = \upsilon$

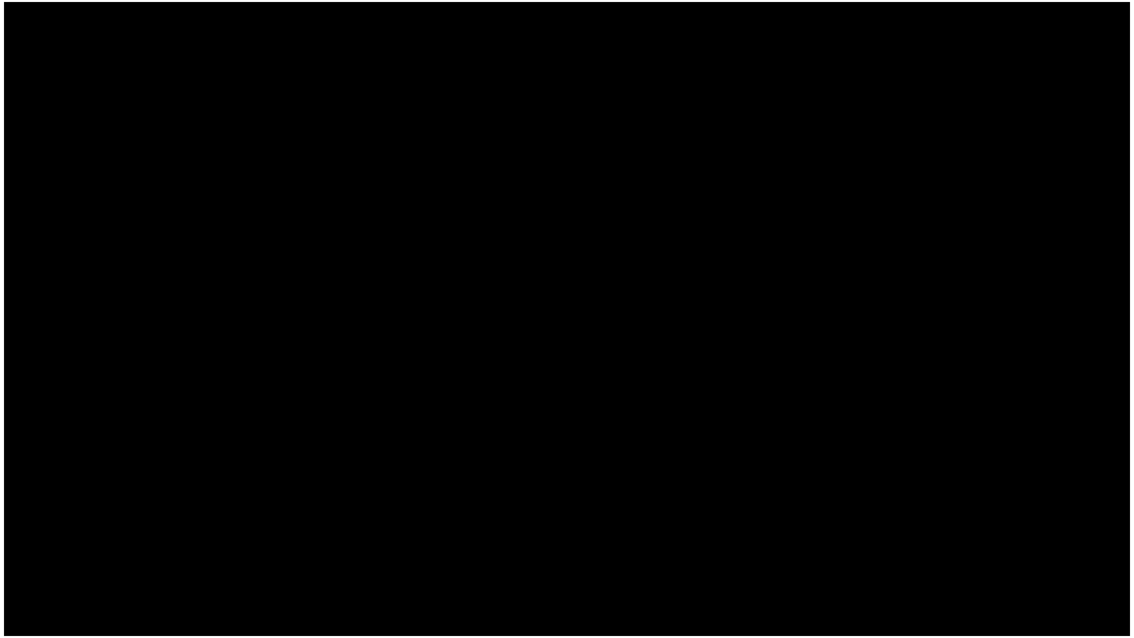
Isotropic material examples

- Rubber
- Steel
- Most metals

Anisotropic material examples

- Carbon fiber
- Wood

Recall Poisson's Ratio



Lateral Strain: $\varepsilon' = \dfrac{\delta_{Lateral}}{w_O}$

Longitudinal Strain: $\varepsilon = \dfrac{\delta_{Longitudinal}}{L_O}$

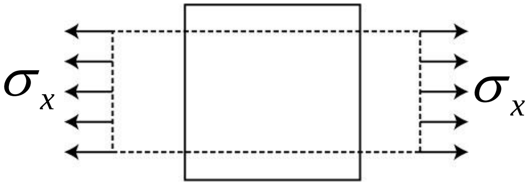
Poisson's ratio:

$$\upsilon = -\frac{\varepsilon'}{\varepsilon}$$

Georgia Tech

$$\upsilon = -\frac{\varepsilon'}{\varepsilon}\qquad \begin{matrix}\leftarrow \text{Lateral Strain}\\[1em] \leftarrow \text{Longitudinal Strain}\end{matrix}$$

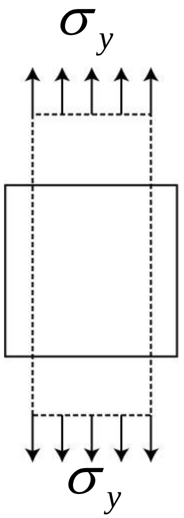
Consider Biaxial Principal Stresses



$$\upsilon = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$\varepsilon_x = \frac{\sigma_x}{E}$$

$$\varepsilon_y = -\upsilon\varepsilon_x = -\upsilon\frac{\sigma_x}{E}$$

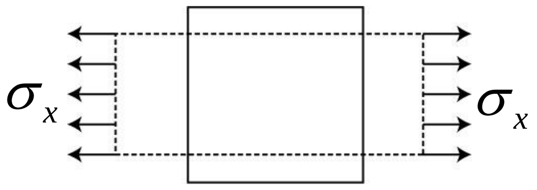


$$\upsilon = -\frac{\varepsilon_x}{\varepsilon_y}$$

$$\varepsilon_y = \frac{\sigma_y}{E}$$

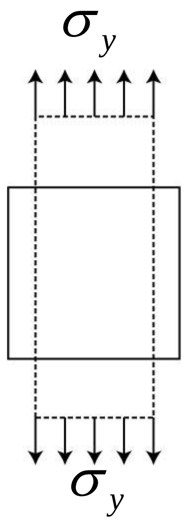
$$\varepsilon_x = -\upsilon\varepsilon_y = -\upsilon\frac{\sigma_y}{E}$$

Consider Biaxial Principal Stresses



$$\varepsilon_y = -\upsilon\varepsilon_x = -\upsilon\frac{\sigma_x}{E}$$

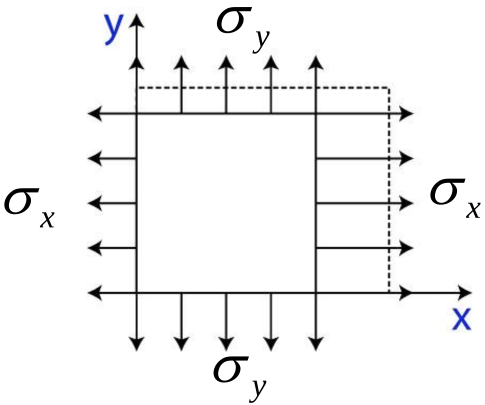
due to poisson effect



$$\varepsilon_x = -\upsilon\varepsilon_y = -\upsilon\frac{\sigma_y}{E}$$

due to poisson effect

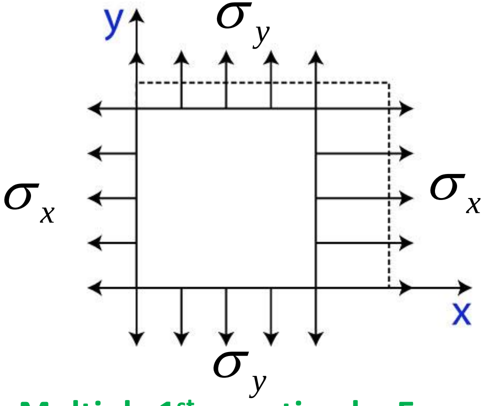
Combine



$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E}$$

Consider Biaxial Principal Stresses



$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon \frac{\sigma_y}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon \frac{\sigma_x}{E}$$

Multiply 1st equation by E

$$\varepsilon_x E = \sigma_x - \upsilon \sigma_y$$

Multiply 2nd equation by E and γ

$$\upsilon \varepsilon_y E = \upsilon \sigma_y - \upsilon^2 \sigma_x$$

Add

$$\varepsilon_x E + \upsilon \varepsilon_y E = \sigma_x - \upsilon^2 \sigma_x$$

$$\sigma_x \left(1 - \upsilon^2\right) = E\left(\varepsilon_x + \upsilon \varepsilon_y\right)$$

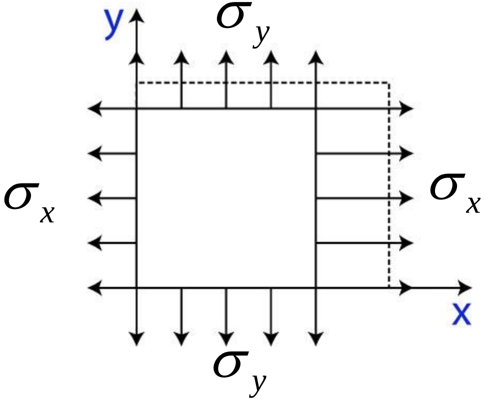
Generalized Hooke's Law for Biaxial Stress-Strain for Isotropic Materials

$$\sigma_x = \frac{E}{1-\upsilon^2}\left(\varepsilon_x + \upsilon \varepsilon_y\right)$$

Similarly

$$\sigma_y = \frac{E}{1-\upsilon^2}\left(\varepsilon_y + \upsilon \varepsilon_x\right)$$

Generalized Hooke's Law for Biaxial Stress-Strain for Isotropic Materials



$$\sigma_x = \frac{E}{1-\upsilon^2}\left(\varepsilon_x + \upsilon\varepsilon_y\right)$$

$$\sigma_y = \frac{E}{1-\upsilon^2}\left(\varepsilon_y + \upsilon\varepsilon_x\right)$$

Note: This development was for principal stress loading. However, for small deformations, recall that normal strains are unaffected by displacements perpendicular to the normal strain direction (such as produced by shear strains). Therefore these equations are valid even when shear stresses exist.

This development can be extended to triaxial states of stress