Mechanics of Materials I:

Fundamentals of Stress & Strain and Axial Loading

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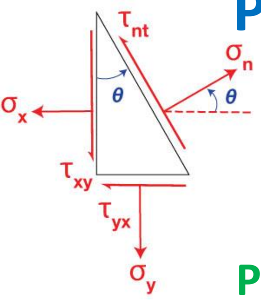
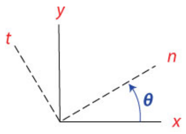
Module 22 Learning Outcome

- Show that the transformation equations for plane stress can be expressed in the form of the equation for a circle (Mohr's Circle)



Plane Stress



$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Maximum In-Plane Shear Stress

$$\tau_{MAX} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_S = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

The planes on which the maximum in-plane shear stresses occur are 45° from the Principal Planes

Stress Invariant

$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

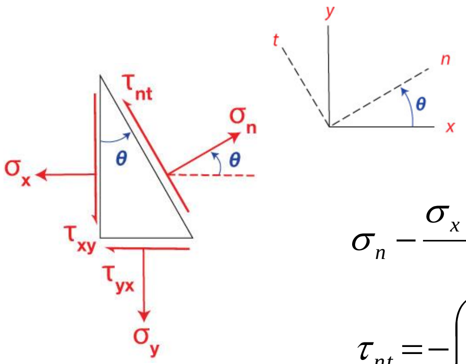
Also

$$\tau_{MAX} = \left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

Plane Stress

$$\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$



rearrange first equation

$$\sigma_n - \frac{\sigma_x+\sigma_y}{2} = \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

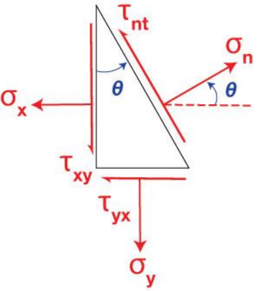
$$\tau_{nt} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

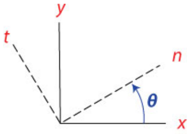
square equations and add

$$\left(\sigma_n - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau_{nt}^2 = \overbrace{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2\cos^2 2\theta + \left(\frac{\sigma_x-\sigma_y}{2}\right)^2\sin^2 2\theta}^{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2} + \overbrace{\tau_{xy}^2\sin^2 2\theta + \tau_{xy}^2\cos^2 2\theta}^{\tau_{xy}^2}$$

$$\left(\sigma_n - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau_{nt}^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

Plane Stress



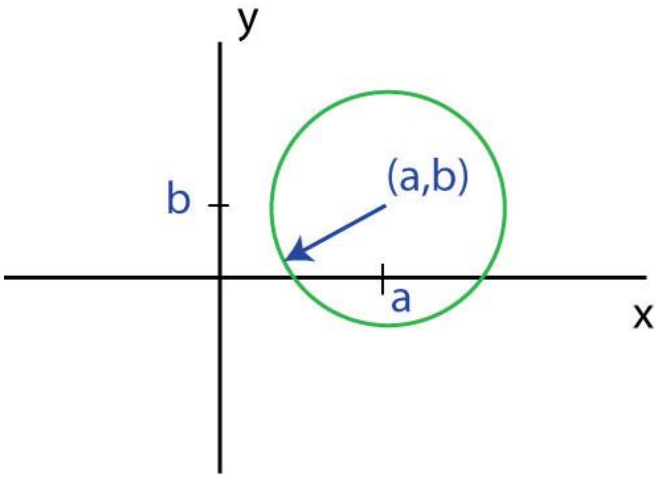


$$\left(\sigma_n - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{nt}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$\left(\sigma_n - \frac{\sigma_x + \sigma_y}{2}\right)^2 + (\tau_{nt} - 0)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

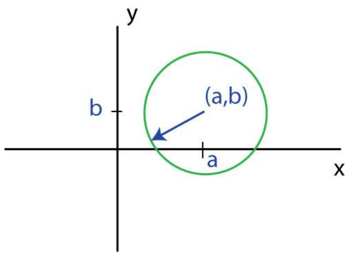
Recall the equation of a circle

$$(x - a)^2 + (y - b)^2 = R^2$$



Mohr's Circle

Otto Mohr (1835-1918)
German Engineer



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Graphical tool for the depiction of the transformation equations for plane stress

$$(x-a)^2+(y-b)^2=R^2$$

$$\left(\sigma_n-\frac{\sigma_x+\sigma_y}{2}\right)^2+(\tau_{nt}-0)^2=\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2$$

Radius = $\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

Center: $\left(\frac{\sigma_x+\sigma_y}{2},0\right)=(\sigma_{AVG},0)$

Where Average Normal Stress $=\sigma_{AVG}=\frac{\sigma_x+\sigma_y}{2}$

The stress transformation equation is based on an angle 2θ

Therefore the angle on Mohr's circle is 2 times the stress block angle