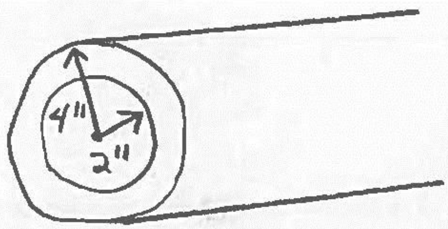
MOOC – "Mechanics of Materials II"
Section Three Quiz solution

Problem 1) A hollow circular shaft is made of steel that may be treated as perfectly elastoplastic. The outer diameter of the shaft is 4 inches and the inner diameter of the shaft is 2 inches. The shear stress yielding point for the steel is 20 ksi. The modulus of rigidity for the steel is 11,600 ksi.
Determine the maximum torque that the shaft can transmit if the allowable shear stress is 18 ksi and the entire cross section remains elastic.
Also determine the maximum torque that the shaft can transmit if the shear stress at the inside surface is allowed to reach the yield point and the entire cross section is on the verge of becoming totally plastic.



FOR TOTALLY ELASTIC

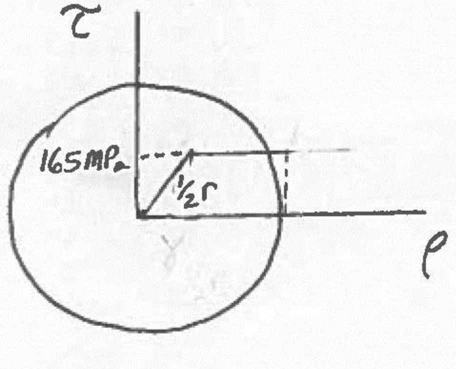
$$T_{ELASTIC} = \frac{\tau J}{r} = \frac{(18\ \text{Ksi})\,\frac{\pi}{2}\left((2)^4 - (1)^4\right)}{2\ \text{in}} = \underline{\underline{212\ \text{in-Kip}}}\ \text{ANS}$$

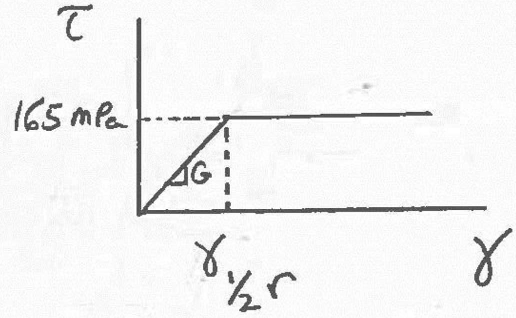
FOR TOTALLY PLASTIC

$$T_{PLASTIC} = \frac{2}{3}\pi\,\tau_{YIELD}\left(r_{OUTER}^{3} - r_{ELASTIC}^{3}\right)$$

$$T_{PLASTIC} = \frac{2}{3}\pi\,(20\ \text{Ksi})\left[(2\ \text{in})^3 - (1\ \text{in})^3\right] = \underline{\underline{293\ \text{in-Kip}}}\ \text{ANS.}$$

Problem 2) A solid circular shaft is made of steel that may be treated as perfectly elastoplastic. The shear stress yielding point for the steel is 165 MPa. The modulus of rigidity for the steel is 80 GPa. A torque is applied such that the shear stress reaches its yield point at 1/2 the radius of the shaft.
Find the shear strain at the outer surface of the shaft.





$$\tau = G\gamma$$

$$\gamma_{\frac{1}{2}r} = \frac{\tau}{G} = \frac{165\ \text{MPa}}{80{,}000\ \text{MPa}} = \underline{0.00206}$$

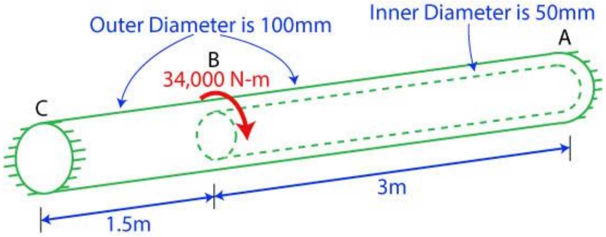
SHEAR STRAIN VARIES LINEARLY WITH ρ

$$\frac{\gamma_{MAX}}{r_{OUTER}} = \frac{\gamma_{YIELD}}{r_{ELASTIC}} \Rightarrow \gamma_{MAX} = 0.00206\left(\frac{r}{\frac{1}{2}r}\right)$$

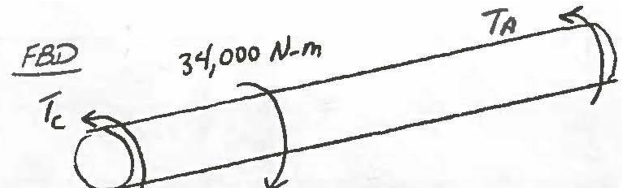
$$\gamma_{MAX} = 0.00413 \text{ radians}$$

ANS

Problem 3) A circular steel shaft is rigidly secured at both ends. The right part of the shaft is hollow as shown below. The modulus of rigidity for the steel is 80 GPa. Determine the maximum shearing stress in the shaft.



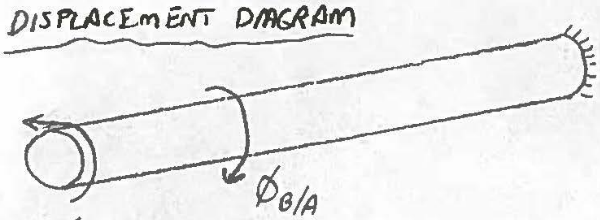
FBD



$\Sigma M = 0$

EQUILIBRIUM EQUATION $\Longrightarrow$ $T_A + T_B = 34{,}000\,N\text{-}m = 34{,}000{,}000\ N\text{-}mm$ [EQN 1]

DISPLACEMENT DIAGRAM



$\phi_{C/B} - \phi_{B/A} = 0 \Longrightarrow \phi_{C/B} = \phi_{B/A}$

ASSUME ELASTIC

$\leftarrow \phi = \frac{TL}{GJ}$

$$\frac{T_C\,(1500\,mm)}{(80\,GPa)\left[\frac{\pi}{2}(50\,mm)^4\right]} = \frac{T_A\,(3000\,mm)}{(80\,GPa)\left[\frac{\pi}{2}(50\,mm)^4 - (25\,mm)^4\right]}$$

$T_C = 2.133\,T_A$ [EQN 2]

SOLVING EQUATION [1] AND [2] SIMULTANEOUSLY

$T_A = 10.9 \times 10^6\ N\text{-}mm$

$T_C = 23.1 \times 10^6\ N\text{-}mm$

SOLVE SHEAR STRESSES

$$\tau_A = \frac{T\rho}{J} = \frac{10.9\times10^6\,N\text{-}mm\,(50\,mm)}{\frac{\pi}{2}\left[(50\,mm)^4 - (25\,mm)^4\right]} = 58.9\,\frac{N}{mm^2} = 58.9\,MPa$$

$$\tau_C = \frac{23.1\times10^6\,N\text{-}mm\,(50\,mm)}{\frac{\pi}{2}\left[50\,mm\right]^4} = \underline{\underline{118\,MPa}}\ ANS < 165\,MPa = \tau_{YIELD\ STEEL}$$

$\therefore$ ELASTIC ASSUMPTION HOLDS