MATA31 - Assignment #4

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A

Consider the linear function

$$f(x) = 3x + 1.$$

We know intuitively that

$$\lim_{x \to -1} f(x) = -2.$$

A. How close to -1 does x have to be such that $f(x)$ differs from -2 by less than 0.1?

B. How close to -1 does x have to be such that $f(x)$ differs from -2 by less than 0.01?

C. How close to -1 does x have to be such that $f(x)$ differs from -2 by less than 0.001?

B

Provide the formal definition of the limit

$$\lim_{x \to a} f(x) = L$$

in two ways: one using intervals and one using absolute value inequalities. Use this definition to prove that

$$\lim_{x \to 3} (2x + 4) = 10$$

C

Provide the formal definition of the limit

$$\lim_{x \to a^+} f(x) = \infty$$

in two ways: one using intervals and one using absolute value inequalities. Use this definition to prove that

$$\lim_{x \to 1^+} \frac{1}{x - 1} = \infty$$

D

Provide the formal definition of the limit

$$\lim_{x \to \infty} f(x) = L$$

in two ways: one using intervals and one using absolute value inequalities. Use this definition to prove that

$$\lim_{x \to \infty} \frac{2}{x + 1} = 0$$

E

Provide the formal definition of the limit

$$\lim_{x\to\infty} f(x) = \infty$$

in two ways: one using intervals and one using absolute value inequalities. Use this definition to prove that

$$\lim_{x\to\infty} \left(x^2 + 1\right) = \infty$$

F

Find the equation of a possible function f with $f(0) = 5$, $\lim_{x\to 1^+} f(x) = \infty$ and $\lim_{x\to 1^-} f(x) = \infty$

G

Does

$$\lim_{x\to 2} \frac{|x-2|}{x-2}$$

exist? Explain why or why not.

H

Find $\lim_{x\to 3} f(x)$ if it exists. Otherwise, explain by one-sided limits

$$f(x) = \begin{cases} x^2, & \text{if } x \le 3, \\ 3x + 2, & \text{if } x > 3. \end{cases}$$