CSCA67 - Exercises #7

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October 22, 2025

2.1

Prove that $\sqrt{2}$ is irrational.

Suppose $\sqrt{2}$ is rational	(1) for contradiction
$\exists a \exists b, \left(\sqrt{2} = \frac{a}{b}\right) \wedge (a, b$ have no common factors $\neq 1.)$	(2) (1, Definition of $\mathbb{Q}$)
$\left(\sqrt{2} = \frac{x}{y}\right) \wedge (x, y$ have no common factors $\neq 1.)$	(3) (2, E.I)
$x^2 = 2y^2$	(4) (3, simp. algebra)
y^2 is an integer	(5)
$\exists k, x^2 = 2k$	(6) (4, 5, E.G)
x^2 is even	(7) (6, Definition of even)
x is even	(8) (7, def., U.M.P)
$x = 2i$	(9) (8, def. E.I)
$y^2 = \frac{x^2}{2} = \frac{(2i)^2}{2} = 2i^2$	(10) (4, 9)
y^2 is even	(11) (10, def.)
y is even	(12) (11, def., U.M.P)
$y = 2j$	(13) 12, def. E.I
2 is a common factor ofx, y	(14) (9, 13)
$(a, b$ have no common factors $\neq 1)$	(15) (3, simp.)
a contradiction	(16) (14, 15)
$\sqrt{2}$ is irrational.	(17) (1, 16, contradiction.)

As required to prove. ■

2.2

Prove that the sum of a rational number and an irrational number is irrational.

$$WTS: \forall x \forall y, (x \in \mathbb{Q} \wedge y \notin \mathbb{Q}) \rightarrow ((x + y) \notin \mathbb{Q})$$

Let a be arbitrary	(1)
Let b be arbitrary	(2)
Assume $a \in \mathbb{Q}$	(3)
Assume $b \notin \mathbb{Q}$	(4)

As required to prove. ■