CSCA67 - Exercises #5

Satyajit Datta
1012033336

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For each of the following arguments, prove if they are valid or not.

1.1

Every insect has six legs. Charlotte has six legs. Therefore, Charlotte is an insect.
Let $I(x)$ be "x is an insect". $L(x)$ be "x has six legs. Universe of discourse is live beings.

The argument given is:

$$\begin{array}{ll} \forall x(I(x) \rightarrow L(x)) & (1) \\ L(Charlotte) & (2) \\ \hline I(Charlotte) & \text{(Conclusion)} \end{array}$$

Let W be $\{Charlotte\} \subseteq U$, such that $\neg I(Charlotte)$, and $L(Charlotte)$. (perhaps Charlotte is an octopus :0)

Premise (1)

$$\forall x(I(x) \rightarrow L(x))$$

Since Charlotte is the only element in our subset, we can rewrite the statement as such:

$$\begin{array}{ll} & I(Charlotte) \rightarrow L(Charlotte) \\ \textbf{eqv} & F \rightarrow T \\ \textbf{eqv} & T \end{array}$$

Premise (2)

$$\begin{array}{ll} & L(Charlotte) \\ \textbf{eqv} & T \end{array}$$

Conclusion

$$\begin{array}{ll} & I(Charlotte) \\ \textbf{eqv} & F \end{array}$$

Thus, we have proved that there is a case such that all the premises are true, while the conclusion is false, making this argument invalid. ■

1.2

Every insect has six legs. At least one insect flies. Therefore, at least one six-legged being flies.
Let $I(x)$ be "x is an insect". $L(x)$ be "x has six legs", $F(x)$ be "x flies." Universe of discourse is live beings.

The argument given is:

$$\begin{array}{ll} \forall x(I(x) \rightarrow L(x)) & (1) \\ \exists x,(I(x) \wedge F(x)) & (2) \\ \hline \exists x,(L(x) \wedge F(x)) & \text{(Conclusion)} \end{array}$$

Proof.

Choose c such that: $I(c) \wedge F(c)$	(3) (Existential Instantiation)
$I(c)$	(4) (3, Simp.)
$F(c)$	(5) (3, Simp.)
$L(c)$	(6) (1, 4, Universal Modus Ponens)
$L(c) \wedge F(c)$	(7) (5, 6, Conj.)
$\exists x\,(L(x) \wedge F(x))$	(8) (3, 7, Existential Generalisation)

$\therefore$ The argument is valid. ■

1.3

Every insect has six legs. Only insects fly. Therefore, every flying being has six legs.
Let $I(x)$ be "x is an insect". $L(x)$ be "x has six legs, $F(x)$ be "x flies." Universe of discourse is live beings.

The argument given is:

$$
\begin{array}{ll}
\forall x (I(x) \rightarrow L(x)) & (1) \\
\forall x, (F(x) \rightarrow I(x)) & (2) \\
\hline
\forall x, (F(x) \rightarrow L(x)) & \text{(Conclusion)}
\end{array}
$$

Take an arbitrary c	(3)
$I(c) \rightarrow L(c)$	(4) (1,3, Universal Instantiation)
$F(c) \rightarrow I(c)$	(5) (2,3, Universal Instantiation)
$\neg I(c) \vee L(c)$	(6) (4, Cond.)
$\neg F(c) \vee I(c)$	(7) (5, Cond.)
$\neg F(c) \vee L(c)$	(8) (6, 7, Res.)
$F(c) \rightarrow L(c)$	(9) (8, Cond.)
$\forall x (F(c) \rightarrow L(c))$	(10) (3, 9, Universal Generalisation)

$\therefore$ The argument is valid. ■

1.4

All birds eat at least one species of insect. At least one species of insect can fly. Therefore, all birds eat some flying being.
Let $I(x)$ be "x is a species of insect.", $B(x)$ be "x is a bird, $F(x)$ be "x flies.", and $E(x,y)$ be "x eats y". Universe of discourse is live beings.

The argument given is:

$$
\begin{array}{ll}
\forall x\,(B(x) \rightarrow \exists y (I(y) \wedge E(x,y))) & (1) \\
\exists x, (I(x) \wedge F(x)) & (2) \\
\hline
\forall x, (B(x) \rightarrow \exists y (F(y) \wedge E(x,y))) & \text{(Conclusion)}
\end{array}
$$

Let $W \subseteq U$ s.t. $W = \{i, j, b\}$

i	j	b
$F(i) = \text{False}$	$F(j) = \text{True}$	$F(b) = \text{True}$
$I(i) = \text{True}$	$I(j) = \text{True}$	$I(b) = \text{False}$
$B(i) = \text{False}$	$B(j) = \text{False}$	$B(b) = \text{True}$
$E(i,j) = \text{False}$	$E(j,i) = \text{False}$	$E(b,j) = \text{False}$
$E(i,b) = \text{False}$	$E(j,b) = \text{False}$	$E(b,i) = \text{True}$

Premise (1)

$$\forall x\,(B(x) \rightarrow \exists y\,(I(y) \wedge E(x,y)))$$

Since $\neg B(i)$ and $\neg B(j)$, the conditional is vacuously true for those cases. We only need to check for $x = b$:

$$\begin{aligned}
&B(b) \rightarrow \exists y\,(I(y) \wedge E(b,y)) \\
&\quad \text{Let } y = i \\
&\quad B(b) \rightarrow (I(i) \wedge E(b,i)) \\
&\quad T \rightarrow (T \wedge T) \\
&\quad T \rightarrow T \\
&\quad T
\end{aligned}$$

Premise (2)

$$\exists x\,(I(x) \wedge F(x))$$

$$\begin{aligned}
&\text{Let } x = j: \\
I(j) \wedge F(j)& \\
T \wedge T& \\
T&
\end{aligned}$$

Conclusion

$$\forall x\,(B(x) \rightarrow \exists y\,(F(y) \wedge E(x,y)))$$

Again, since $\neg B(i)$ and $\neg B(j)$, those are vacuously true. We check $x = b$:

$$\begin{aligned}
&B(b) \rightarrow \exists y\,(F(y) \wedge E(b,y)) \\
&\quad \text{Let } y = j \\
&\quad B(b) \rightarrow (F(j) \wedge E(b,j)) \\
&\quad T \rightarrow (T \wedge F) \\
&\quad T \rightarrow F \\
&\quad F
\end{aligned}$$

Thus, we have proved that there is a case such that all the premises are true, while the conclusion is false, making this argument invalid. ■