CSCA67 - Exercises #4

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For each of the following arguments, prove if they are valid or not.

1.1

Every insect has six legs. Charlotte has six legs. Therefore, Charlotte is an insect.
Let $I(x)$ be "x is an insect". $L(x)$ be "x has six legs. Universe of discourse is live beings.

The argument given is:

$$\forall x(I(x) \rightarrow L(x)) \qquad (1)$$
$$L(Charlotte) \qquad (2)$$
$$\overline{I(Charlotte)} \qquad \text{(Conclusion)}$$

1.2

Every insect has six legs. At least one insect flies. Therefore, at least one six-legged being flies.
Let $I(x)$ be "x is an insect". $L(x)$ be "x has six legs", $F(x)$ be "x flies." Universe of discourse is live beings.

The argument given is:

$$\forall x(I(x) \rightarrow L(x)) \qquad (1)$$
$$\exists x, (I(x) \wedge F(x)) \qquad (2)$$
$$\overline{\exists x, (L(x) \wedge F(x))} \qquad \text{(Conclusion)}$$

Proof.

Take an element c such that: $I(c) \wedge F(c)$ (3) (EI)

  $I(c)$ (4) (3, simp.)

  $F(c)$ (5) (3, simp.)

  $L(c)$ (6) (1, 4, Universal Modus Ponens)

  $L(c) \wedge F(c)$ (7) (5, 6, conj.)

$\exists x (L(x) \wedge F(x))$ (8) (7, EG)

$\therefore$ The argument is valid. ∎

1.3

Every insect has six legs. Only insects fly. Therefore, every flying being has six legs.
Let $I(x)$ be "x is an insect". $L(x)$ be "x has six legs, $F(x)$ be "x flies." Universe of discourse is live beings.

The argument given is:

$$\forall x(I(x) \rightarrow L(x)) \qquad (1)$$
$$\forall x, (F(x) \rightarrow I(x)) \qquad (2)$$
$$\overline{\forall x, (F(x) \rightarrow L(x))} \qquad \text{(Conclusion)}$$

Take an arbitrary c (3)

1.4

All birds eat at least one species of insect. At least one species of insect can fly. Therefore, all birds eat some flying being.

Let $I(x)$ be "x is a species of insect.", $B(x)$ be "x is a bird, $F(x)$ be "x flies.", and $E(x,y)$ be "x eats y". Universe of discourse is live beings.

The argument given is:

$$\forall x\ (B(x) \rightarrow \exists y(I(y) \wedge E(x,y))) \tag{1}$$

$$\exists x, (I(x) \wedge F(x)) \tag{2}$$

$$\overline{\forall x, (B(x) \rightarrow \exists y(F(y) \wedge E(x,y)))} \qquad \text{(Conclusion)}$$