Energy Institute Bengaluru, Centre of RGIPT

Assignment 3, Soft Computing Lab

Note: Use any programming language to implement the following.

Problem: Optimal Design of Life Testing Experiments under Progressive Censoring using Genetic Algorithms

Suppose n units are placed on a life test and the experiment is designed to continue until exactly m failures are observed ($m < n$). At the time of each observed failure X_i, a pre-specified number of surviving units are randomly removed (censored) from the test. The number of units removed at the time of the i-th failure is denoted by R_i, for $i = 1, 2, \ldots, m$. For fixed n and m, this leads to the censoring scheme $R = (R_1, R_2, \ldots, R_m)$ satisfying $\sum_{i=1}^{m} R_i = n - m$. This ensures that all n units are either observed to fail or are censored during the test. Let (X_i, R_i) denotes the observed data from this life testing experiment and assume the lifetimes follow an Weibull distribution with parameter α and λ. Then the expected Fisher information matrix representing the information about $\theta = (\alpha, \lambda)$ in the observed data can be obtained as given below:

$$I(\boldsymbol{\theta},; R_1, \ldots, R_m) = \begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix}$$

with individual elements defined as:

$$I_{11} = \frac{1}{\alpha^2} \sum_{i=1}^{m} \sum_{k=1}^{i} \frac{\sigma_{i-1} a_{k,i}}{\gamma_k} \int_0^\infty \left(1 + \ln\left(\frac{z}{\gamma_k}\right)\right)^2 e^{-z}\, dz$$

$$I_{22} = \left(\frac{\alpha}{\lambda}\right)^2 \sum_{i=1}^{m} \sum_{k=1}^{i} \frac{\sigma_{i-1} a_{k,i}}{\gamma_k}$$

$$I_{12} = I_{21} = \frac{1}{\lambda} \sum_{i=1}^{m} \sum_{k=1}^{i} \frac{\sigma_{i-1} a_{k,i}}{\gamma_k} \int_0^\infty \left(1 + \ln\left(\frac{z}{\gamma_k}\right)\right) e^{-z}\, dz$$

where

- $\gamma_r = m - r + 1 + \sum_{i=r}^{m} R_i, \quad$ for $r = 1, 2, \ldots, m$
- $\sigma_{r-1} = \prod_{i=1}^{r} \gamma_i$
- $a_{i,r} = \prod_{\substack{j=1 \\ j \neq i}}^{r} \frac{1}{\gamma_j - \gamma_i}$, with $a_{1,1} = 1$

Design Goal: Find the optimal values $R_1, R_2, \ldots, R_m$ such that the information obtained from the test is maximized. Mathematically, the aim is to

$$\max_{R_1, \ldots, R_m} det[I(\theta; R_1, \ldots, R_m)]$$

subject to

$$R_i \in \mathbb{Z}_{\geq 0}, \quad \text{for } i = 1, 2, \ldots, m \text{ and } \sum_{i=1}^{m} R_i = n - m$$

Input: $n = 15$, $m = 5$ and $(\alpha, \lambda) = (2, 1)$.