

Tugas Mandiri - 3

Pengantar Sistem Digital
2022/2023 - 1

Petunjuk pengerjaan:

- Kerjakan dengan tulisan tangan atau diketik.
 - Tuliskan Nama, Kelas, dan NPM pada setiap lembar jawaban.
 - Tuliskan penjelasan dari cara mendapatkan jawaban tersebut.
 - Apabila ditulis tangan, hasil pekerjaan di scan / foto dan dimasukkan ke dalam satu file berformat .pdf.
 - Format nama file (tanpa tanda kurung) : **[KodeAsdos]_TM3_[Nama]_[NPM].pdf**.
 - Tugas mandiri dikumpulkan **Jumat, 7 Oktober 2022 pukul 17.00** pada slot yang sudah disediakan di SCELE.
 - Jika **mengumpulkan telat sebelum pukul 23:59 pada hari yang sama**, akan dikenakan **penalti sebesar 50 poin**. Lebih dari waktu tersebut, tugas mandiri **tidak akan dinilai**.
 - Jika perlu pembulatan, bulat ke atas pada digit ke-n! (Dalam kasus pembulatan biner, digit "terakhir" akan selalu 1)
-

1) (27 poin) Sederhanakan fungsi berikut menggunakan aljabar boolean, tulis hukum yang digunakan. Dan hitung banyak L, G, GN sebelum dan setelah simplifikasi.

a) $F(A,B,C,D) = \overline{A}CD + A.(C.D + B) + \overline{B}C\overline{D}$

Sebelum Simplifikasi

Literal cost (L): 10

Gate input cost (G): 15

Gate input cost with NOTs (GN): 18

Simplifikasi

$$\begin{aligned}
 F(A,B,C,D) &= \overline{A}CD + A.(C.D + B) + \overline{B}C\overline{D} \\
 &= \overline{A}CD + ACD + AB + \overline{B}C\overline{D} && \text{Distributive Law} \\
 &= CD(\overline{A} + A) + AB + \overline{B}C\overline{D} && \text{Distributive Law} \\
 &= CD(1) + AB + \overline{B}C\overline{D} && \text{Inverse Law} \\
 &= CD + AB + \overline{B}C\overline{D} && \text{Identity Law} \\
 &= AB + C(D + \overline{B}\overline{D}) && \text{Distributive Law} \\
 &= AB + C(D + \overline{B}) && \text{Absorption Law}
 \end{aligned}$$

Sesudah Simplifikasi

Literal cost (L): 5

Gate input cost (G): 8

Gate input cost with NOTs (GN): 9

b) $F(A,B,C,D) = (\overline{B} + \overline{C}).(\overline{B} + D) + A.C.D$

Sebelum Simplifikasi

Literal cost (L): 7

Gate input cost (G): 11

Gate input cost with NOTs (GN): 14

Simplifikasi

$$\begin{aligned}
 F(A,B,C,D) &= (\overline{B} + \overline{C}).(\overline{B} + D) + A.C.D \\
 &= \overline{B}(\overline{B} + D) + \overline{C}(\overline{B} + D) + A.C.D && \text{Distributive Law} \\
 &= \overline{B}\overline{B} + \overline{B}D + \overline{B}\overline{C} + \overline{C}D + A.C.D && \text{Distributive Law} \\
 &= \overline{B} + \overline{B}D + \overline{B}\overline{C} + \overline{C}D + A.C.D && \text{Idempotent Law} \\
 &= \overline{B} + \overline{B}\overline{C} + \overline{C}D + A.C.D && \text{Absorption Law} \\
 &= \overline{B} + \overline{C}D + A.C.D && \text{Absorption Law} \\
 &= \overline{B} + AD + \overline{C}D && \text{Absorption Law} \\
 &= \overline{B} + D(A + \overline{C}) && \text{Distributive Law}
 \end{aligned}$$

Sesudah Simplifikasi

Literal cost (L): 4

Gate input cost (G): 6

Gate input cost with NOTs (GN): 8

c) $F(A,B,C,D) = C \cdot \bar{D} \cdot (A + B) + \bar{C} + D$

Sebelum Simplifikasi

Literal cost (L): 6

Gate input cost (G): 8

Gate input cost with NOTs (GN): 10

Simplifikasi

$$F(A,B,C,D) = C \cdot \bar{D} \cdot (A + B) + \bar{C} + D$$

$$= AC\bar{D} + BC\bar{D} + \bar{C} + D$$

$$= A\bar{D} + \bar{C} + BC\bar{D} + D$$

$$= A\bar{D} + \bar{C} + BC + D$$

$$= A\bar{D} + \bar{C} + B + D$$

$$= A + B + \bar{C} + D$$

Distributive Law

Absorption Law

Absorption Law

Absorption Law

Absorption Law

Sesudah Simplifikasi

Literal cost (L): 4

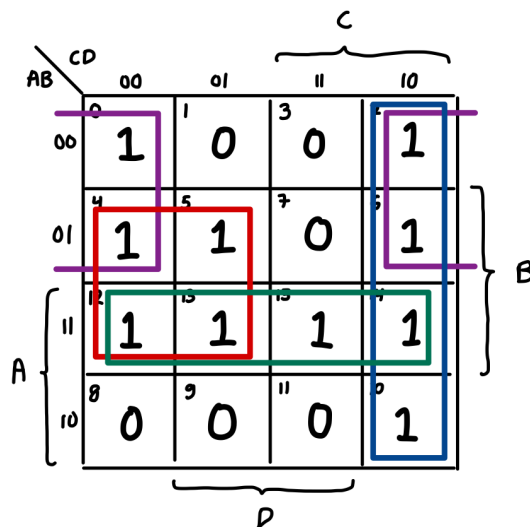
Gate input cost (G): 4

Gate input cost with NOTs (GN): 5

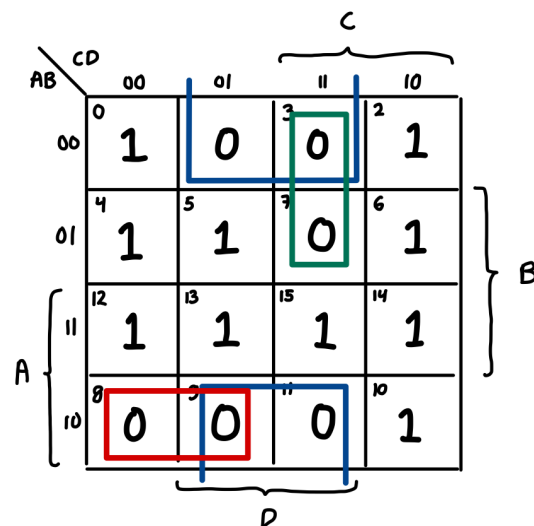
- 2) (27 poin) Optimalkan fungsi-fungsi berikut menggunakan K-Map dan cari bentuk paling sederhana dari SOP dan POS-nya.

a) $F(A,B,C,D) = \sum m(0,2,4,5,6,10,12,13,14,15)$

K-Map:



Minterm K-Map



Maxterm K-Map

SOP: $A'D' + BC' + AB + CD'$

POS: $(AB'C' + B'D + A'CD)' = (A'+B+C)(B+D')(A+C'+D')$

b) $F(A,B,C,D) = \prod M(1,2,3,5,6,7,8,9)$

K-Map:

		C			
		CD			
		00	01	11	10
AB	00	0	1	3	2
	01	4	5	7	6
	11	12	13	15	14
	10	8	9	11	10
		0	1	1	1
		0	0	1	1

Minterm K-Map

		C			
		CD			
		00	01	11	10
AB	00	0	1	3	2
	01	4	5	7	6
	11	12	13	15	14
	10	8	9	11	10
		1	0	0	0
		1	0	0	0
		1	1	1	1
		0	0	1	1

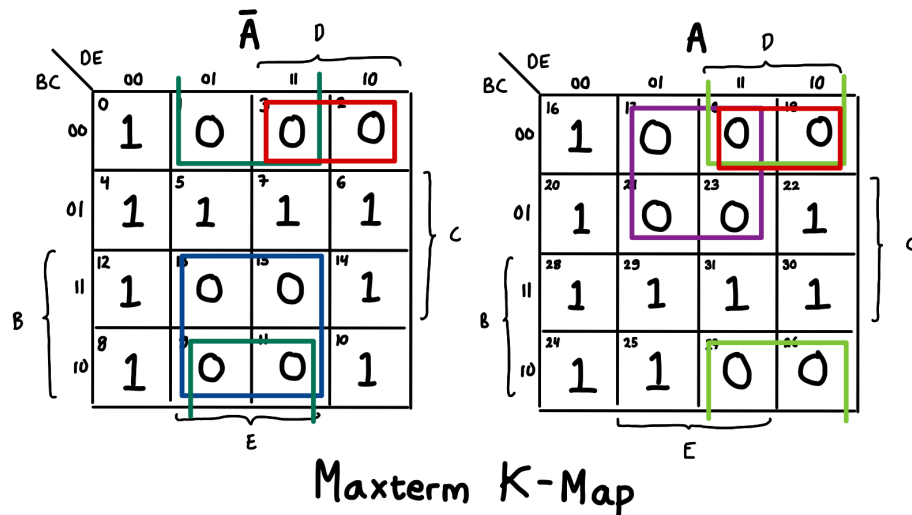
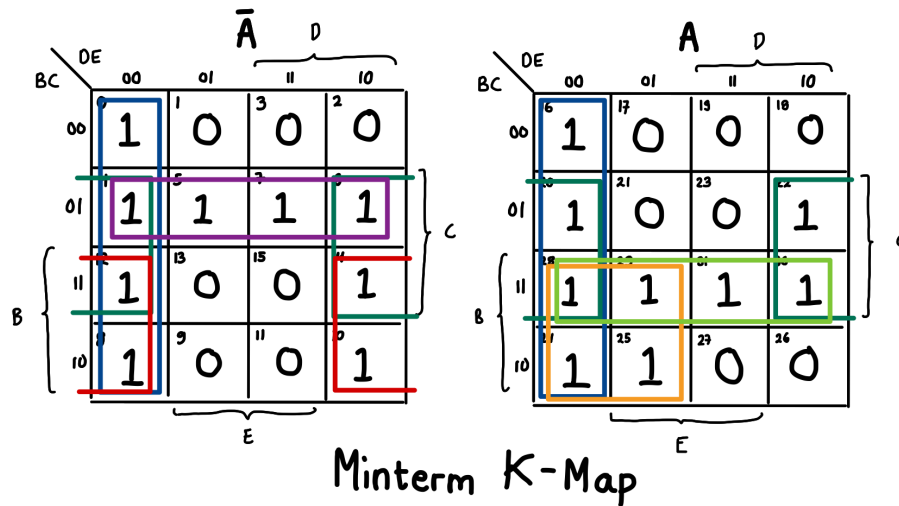
Maxterm K-Map

SOP: $A'C'D' + AB + AC$

POS: $(AB'C' + A'D + A'C)' = (A'+B+C)(A+D')(A+C')$

$$\begin{aligned}
 c) F(A,B,C,D,E) &= (\bar{E} + A.B).(C + \bar{D}) + \bar{A}.(\bar{B}.C + B.\bar{E}) \\
 &= (\bar{E} + A.B).(C + \bar{D}) + \bar{A}\bar{B}C + \bar{A}B\bar{E} && \text{Distributive Law} \\
 &= \bar{E}(C + \bar{D}) + AB(C + \bar{D}) + \bar{A}\bar{B}C + \bar{A}B\bar{E} && \text{Distributive Law} \\
 &= \bar{E}C + \bar{E}\bar{D} + ABC + AB\bar{D} + \bar{A}\bar{B}C + \bar{A}B\bar{E} && \text{Distributive Law}
 \end{aligned}$$

K-Map:



SOP: $D'E' + CE' + A'BE' + ABD' + ABC + A'B'C$

POS:

$$\begin{aligned}
 &= \overline{ABE} + \overline{ACE} + \overline{BCD} + \overline{ABE} + \overline{ACD} \\
 &= (A + \bar{B} + \bar{E})(A + C + \bar{E})(\bar{B} + C + \bar{D})(\bar{A} + B + \bar{E})(\bar{A} + C + \bar{D})
 \end{aligned}$$

3) (18 poin) Optimalkan fungsi-fungsi berikut menggunakan aljabar boolean dan buat tabel kebenarannya.

$$\begin{aligned}
 \text{a) } F(A,B,C,D) &= A.\bar{B} + \bar{C}.(D + A.B + \bar{A}.B) \\
 &= A.\bar{B} + \bar{C}.(D + B(A + \bar{A})) && \text{Distributive Law} \\
 &= A.\bar{B} + \bar{C}.(D + B(1)) && \text{Inverse Law} \\
 &= A.\bar{B} + \bar{C}.(D + B) && \text{Identity Law}
 \end{aligned}$$

Truth Table:

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

$$\begin{aligned}
 \text{b) } F(A,B,C,D) &= A.\bar{B} + D(B.\bar{C} + \bar{A}.\bar{B}) + AB\bar{D} \\
 &= A(\bar{B} + B\bar{D}) + D(B.\bar{C} + \bar{A}.\bar{B}) && \text{Distributive Law} \\
 &= A(\bar{B} + \bar{D}) + D(B.\bar{C} + \bar{A}.\bar{B}) && \text{Absorption Law} \\
 &= A\bar{B} + A\bar{D} + DBC + D\bar{A}\bar{B} && \text{Distributive Law} \\
 &= (A + D\bar{A})\bar{B} + A\bar{D} + DBC && \text{Distributive Law}
 \end{aligned}$$

$$\begin{aligned}
 &= (A + D) \bar{B} + A\bar{D} + D\bar{B}\bar{C} \\
 &= A\bar{B} + D\bar{B} + A\bar{D} + D\bar{B}\bar{C} \\
 &= A\bar{B} + A\bar{D} + D(\bar{B} + \bar{B}\bar{C}) \\
 &= A\bar{B} + A\bar{D} + D(\bar{B} + \bar{C}) \\
 &= A\bar{B} + A\bar{D} + \bar{B}D + \bar{C}D \\
 &= A\bar{D} + \bar{B}D + \bar{C}D \\
 &= A\bar{D} + D(\bar{B} + \bar{C})
 \end{aligned}$$

Absorption Law

Distributive Law

Distributive Law

Absorption Law

Distributive Law

Consensus Law

Distributive Law

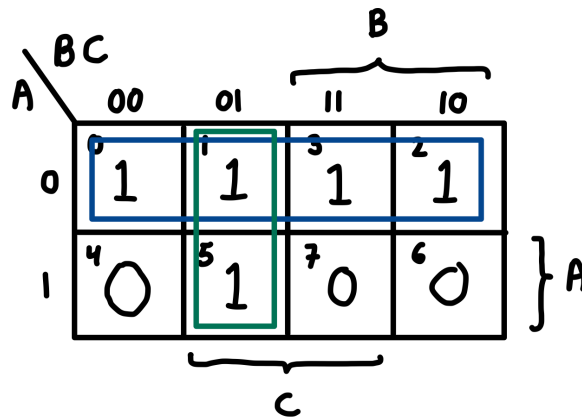
Truth Table:

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

- 4) (18 poin) Optimalkan fungsi-fungsi berikut menggunakan K-Map dan implementasikan dengan NOR atau NAND gate.

a) $F(A, B, C) = \bar{A}\bar{B} + \bar{A}B + A\bar{B}C$ (gunakan NAND gate)

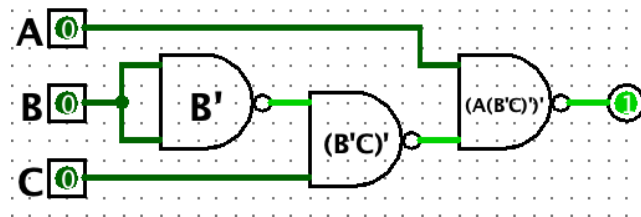
Minterm K-Map:



$$F(A, B, C) = \bar{A} + \bar{B}C$$

Dengan mengimplementasikan NAND gate:

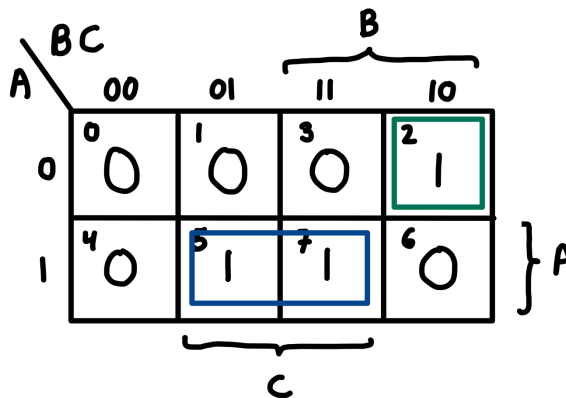
$$F(A, B, C) = ((A' + B'C))' = (A (B'C))' = (A ((B B')' C))'$$



- b) $F(A, B, C) = (A + B)(A + \bar{B} + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + C)$ (gunakan NOR gate)

$$F(A, B, C) = (A'B')' + (A'BC)' + (AB'C)' + (ABC)'$$

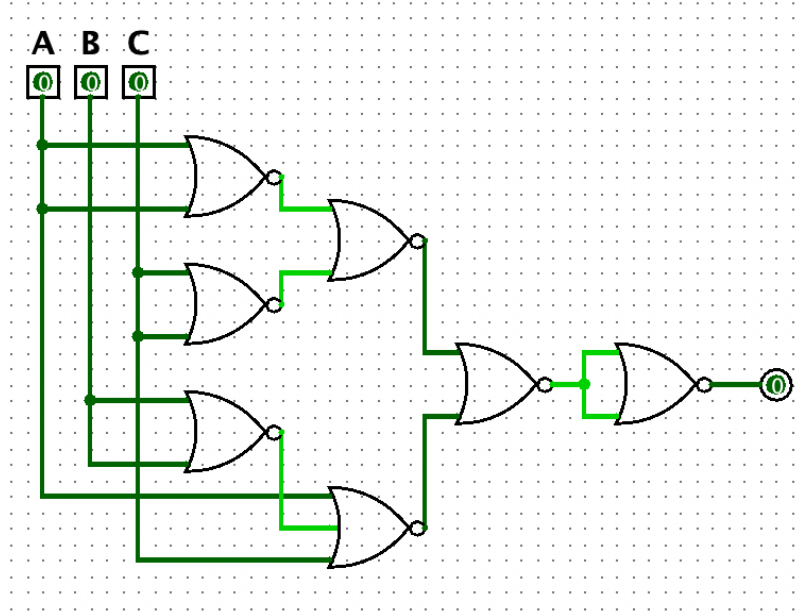
Minterm K-Map:



$$F(A, B, C) = AC + \overline{A}B\overline{C} = \overline{(\overline{A} + \overline{C})(A + \overline{B} + C)}$$

$$F(A, B, C) = (((A' + C')(A + B' + C)))' = (((A' + C)' + (A + B' + C))')'$$

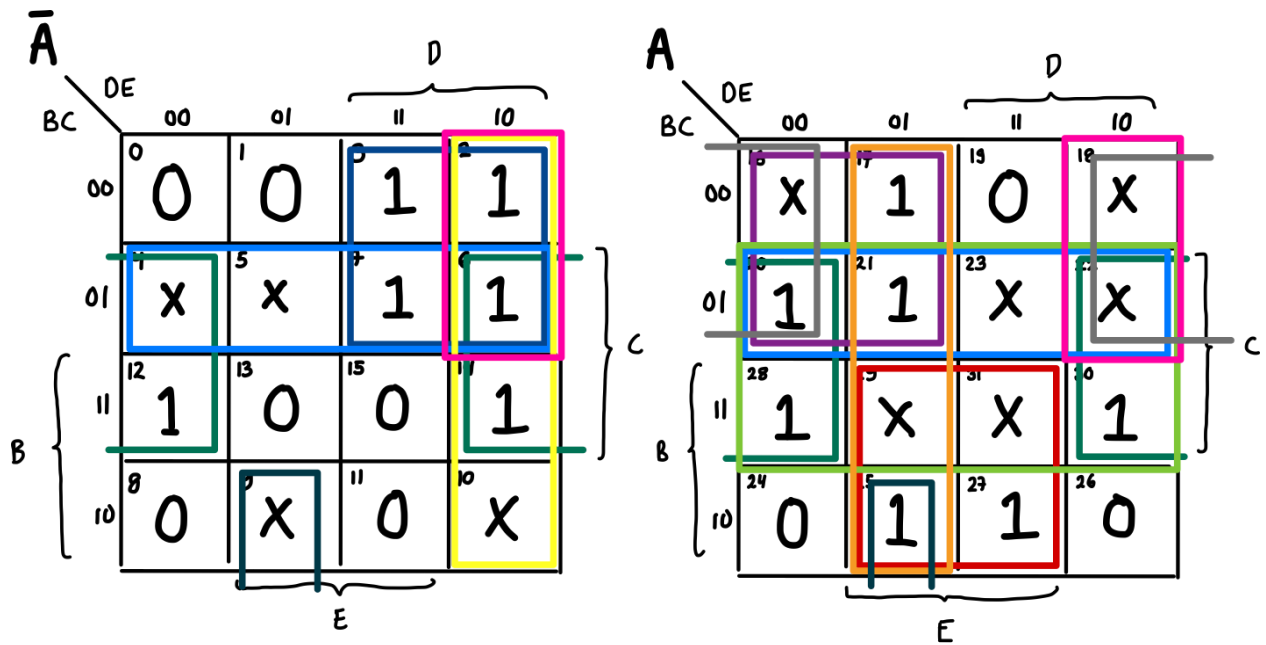
$$= (((A+A)' + (C+C)')' + (A + (B+B)' + C')')'$$



- 5) (10 poin) Buat K-Map dari tabel kebenaran dibawah (fungsi $F(A, B, C, D, E)$) dan tentukan semua *prime implicant* dan *essential prime implicant* nya!

A	B	C	D	E	F
0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	1	0	1
0	0	0	1	1	1
0	0	1	0	0	X
0	0	1	0	1	X
0	0	1	1	0	1
0	0	1	1	1	1
0	1	0	0	0	0

0	1	0	0	1	X
0	1	0	1	0	X
0	1	0	1	1	0
0	1	1	0	0	1
0	1	1	0	1	0
0	1	1	1	0	1
0	1	1	1	1	0
1	0	0	0	0	X
1	0	0	0	1	1
1	0	0	1	0	X
1	0	0	1	1	0
1	0	1	0	0	1
1	0	1	0	1	1
1	0	1	1	0	X
1	0	1	1	1	X
1	1	0	0	0	0
1	1	0	0	1	1
1	1	0	1	0	0
1	1	0	1	1	1
1	1	1	0	0	1
1	1	1	0	1	X
1	1	1	1	0	1
1	1	1	1	1	X



Prime Implicant :

- 1) $C\bar{E}$
- 2) $\bar{A}\bar{B}D$
- 3) $\bar{A}\bar{B}\bar{D}$
- 4) ABE
- 5) $\bar{B}C$
- 6) $\bar{A}D\bar{E}$
- 7) AC
- 8) $A\bar{D}E$
- 9) $\bar{B}D\bar{E}$
- 10) $A\bar{B}\bar{E}$
- 11) $B\bar{C}\bar{D}\bar{E}$

Essential Prime Implicant :

- 1) $C\bar{E}$
- 2) $\bar{A}\bar{B}D$
- 3) ABE