FUNGSI BETA

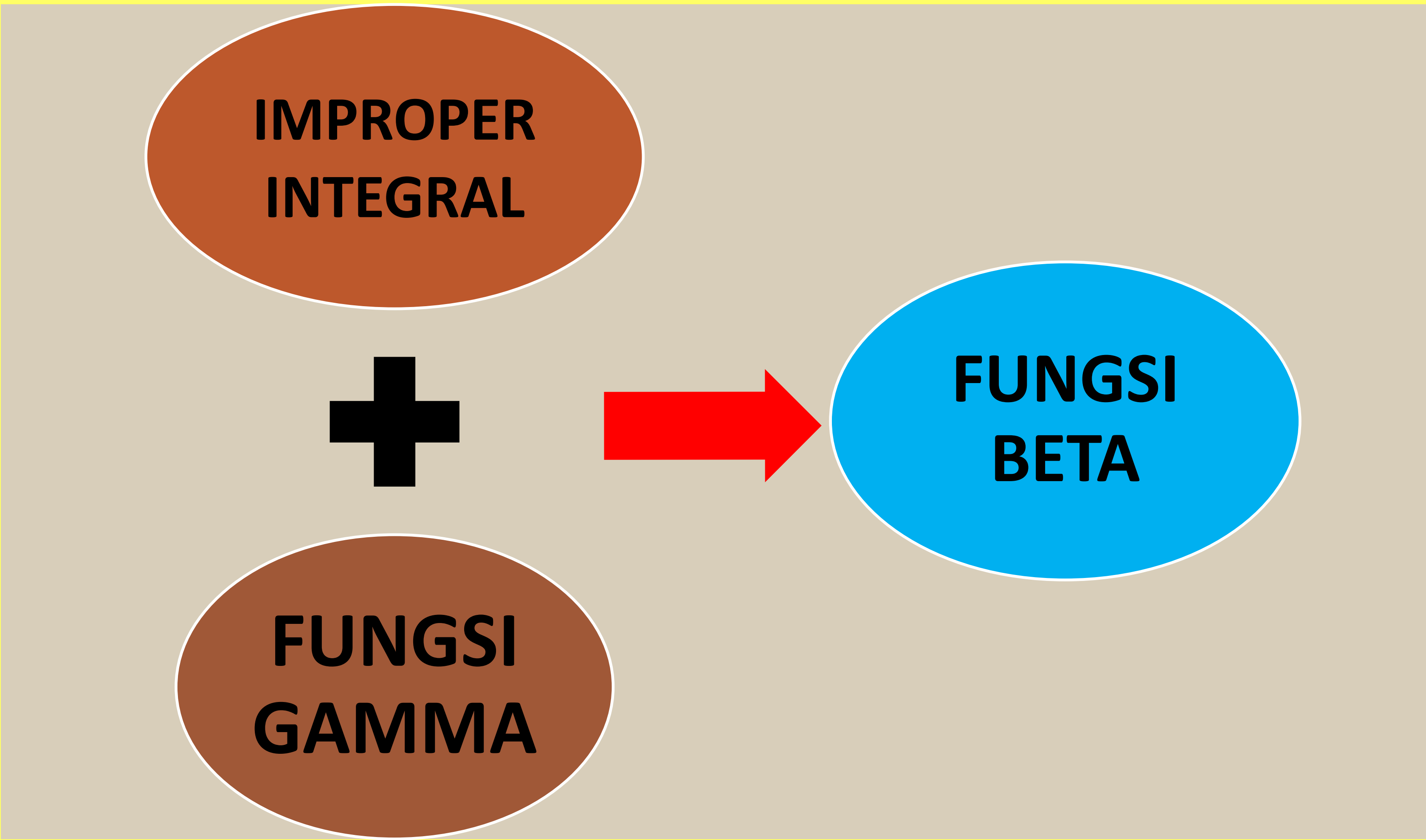
REFERENSI UTAMA :

WIDOWATI DKK, KALKULUS, UNDIP PRESS

TUJUAN PEMBELAJARAN

- Memahami bentuk standar fungsi Beta
- Menganalisis konvergensi fungsi Beta
- Menganalisis sifat-sifat fungsi Beta
- Mengaplikasikan sifat fungsi Beta untuk komputasi integral.

Baseline : Fungsi Beta



Bentuk standar/baku Fungsi Beta

Bentuk standar

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

atau

$$B(m,n) = 2\int_0^{\pi/2} sin^{2m-1}(x) cos^{2n-1}(x)\,dx$$

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

Kasus	Titik singular	Imp Integral
$m, n \geq 1$	Tidak ada	Konv.
$0 < m < 1$	$x = 0$	Konv.
$0 < n < 1$	$x = 1$	Konv.
$m, n \leq 0$	$x = 0, 1$	Divr.

Kesimpulan : Fungsi Beta well-defined untuk m, n > 0

Sifat-sifat

1. **Sifat 1. B(m,n) = B(n,m)**

Misalkan $x = 1 - y$ maka dx = -dy, sehingga

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

$$= \int_1^0 (1-y)^{m-1}y^{n-1}\,(-dy)$$

$$= \int_0^1 (1-y)^{m-1}y^{n-1}\,dy = B(n,m)$$

Sifat-sifat

2. **Sifat 2**. Hubungan fungsi Gamma dan fungsi Beta

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

Sifat-sifat

3. **Sifat ke-3**

$$\Gamma(n)\Gamma\left(n+\frac{1}{2}\right)=\frac{\Gamma(2n)\sqrt{\pi}}{2^{2n-1}}$$

Sifat-sifat

4. Sifat ke-4

$$B(m,n) = B(m+1,n) + B(m,n+1)$$

BENTUK LAIN FUNGSI BETA

Bentuk lain fungsi Beta pada interval $[0, \infty)$

$$B(m,n) = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}\,dy$$

Contoh 1

Hitung $B\left(\frac{1}{4},\frac{3}{4}\right)$

Tinjau $B\left(\frac{1}{4},\frac{3}{4}\right) = B\left(\frac{1}{4},1-\frac{1}{4}\right) = \frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(1-\frac{1}{4}\right)}{\Gamma(1)}$

Karena $\Gamma\left(\frac{1}{4}\right)\Gamma\left(1-\frac{1}{4}\right) = \frac{\pi}{sin\left(\frac{\pi}{4}\right)} = \pi\sqrt{2}$ maka

$$B\left(\frac{1}{4},\frac{3}{4}\right) = \pi\sqrt{2}$$

Contoh 2

Hitung $\int_0^\infty \frac{dx}{1+x^4} =$

Misalkan $y = x^4$ maka $dy = 4x^3 dx$

$$\int_0^\infty \frac{dx}{1+x^4} = \frac{1}{4}\int_0^\infty \frac{y^{\frac{1}{4}-1}}{(1+y)^{\frac{1}{4}+\frac{3}{4}}} dy$$

$$= \frac{1}{4} B\left(\frac{1}{4}, \frac{3}{4}\right)$$

$$= \frac{\pi\sqrt{2}}{4}$$

Contoh 3

Hitung $\int_0^{\pi/2} \sqrt{\tan(x)}dx =$

Perhatikan

$$\int_0^{\pi/2} \sqrt{\tan(x)}dx = \int_0^{\pi/2} sin^{1/2}(x)cos^{-1/2}(x)\, dx$$

Maka

$$\int_0^{\pi/2} \sqrt{\tan(x)}dx = \frac{1}{2}B\left(\frac{1}{4},\frac{3}{4}\right) = \frac{\pi\sqrt{2}}{2}$$

Latihan 2

BUKTIKAN

$$\int_0^\infty \frac{x^2}{1 + x^4}\, dx = \frac{\pi}{2\sqrt{2}}$$

Latihan 3

Dengan sifat fungsi beta, hitung

$$\int_0^1 x^4 \, (1 - \sqrt{x})^5 \, dx =$$

Latihan 4

Dengan sifat fungsi beta, hitung

$$\int_0^1 \frac{x^{a-1} + x^{b-1}}{(1+x)^{a+b}} dx =$$