PERTEMUAN 4

LANJUTAN BOOLEAN ALGEBRA, PENGENALAN LOGIKA KOMBINASIONAL

BOOLEAN THEOREMS (SINGLE-VARIABLE)

1. $x * 0 = 0$
2. $x * 1 = x$
3. $x * x = x$
4. $x * x' = 0$
5. $x + 0 = x$
6. $x + 1 = 1$
7. $x + x = x$
8. $x + x' = 1$
9. $\sim(\sim x) = x$

BOOLEAN THEOREMS (MULTIVARIABLE)

10.	$X + Y = Y + X$	11.	$XY = YX$	Commutative
12.	$X + (Y + Z) = (X + Y) + Z$	13.	$X(YZ) = (XY)Z$	Associative
14.	$X(Y + Z) = XY + XZ$	15.	$X + YZ = (X + Y)(X + Z)$	Distributive
16.	$\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17.	$\overline{X \cdot Y} = \overline{X} + \overline{Y}$	DeMorgan's

x+xy=x

x+x'y=x+y

x'+xy=x'+y

DEMORGAN'S THEOREMS

- (x+y)
- (xy)'

Truth Tables to Verify DeMorgan's Theorem

(a) X	Y	X + Y	$\overline{X + Y}$	(b) X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X} \cdot \overline{Y}$
0	0	0	1	0	0	1	1	1
0	1	1	0	0	1	1	0	0
1	0	1	0	1	0	0	1	0
1	1	1	0	1	1	0	0	0

2-2. *Prove the identity of each of the following Boolean equations, using algebraic manipulation:

(a) $\overline{X}\,\overline{Y} + \overline{X}Y + XY = \overline{X} + Y$

(b) $\overline{A}B + \overline{B}\,\overline{C} + AB + \overline{B}C = 1$

(c) $Y + \overline{X}Z + X\overline{Y} = X + Y + Z$

(d) $\overline{X}\,\overline{Y} + \overline{Y}Z + XZ + XY + Y\overline{Z} = \overline{X}\,\overline{Y} + XZ + Y\overline{Z}$

2-3. +Prove the identity of each of the following Boolean equations, using algebraic manipulation:

(a) $AB\overline{C} + B\overline{C}\,\overline{D} + BC + \overline{C}D = B + \overline{C}D$

(b) $WY + \overline{W}Y\overline{Z} + WXZ + \overline{W}X\overline{Y} = WY + \overline{W}X\overline{Z} + \overline{X}Y\overline{Z} + X\overline{Y}Z$

(c) $A\overline{D} + \overline{A}B + \overline{C}D + \overline{B}C = (\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + C + D)$

2-4. +Given that $A \cdot B = 0$ and $A + B = 1$, use algebraic manipulation to prove that

$$(A + C) \cdot (\overline{A} + B) \cdot (B + C) = B \cdot C$$

2-3. +Prove the identity of each of the following Boolean equations, using algebraic manipulation:

(a) $AB\overline{C} + B\overline{C}\,\overline{D} + BC + \overline{C}D = B + \overline{C}D$

(b) $WY + \overline{W}Y\overline{Z} + WXZ + \overline{W}X\overline{Y} = WY + \overline{W}X\overline{Z} + \overline{X}Y\overline{Z} + X\overline{Y}Z$

(c) $A\overline{D} + \overline{A}B + \overline{C}D + \overline{B}C = (\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + C + D)$

2-4. +Given that $A \cdot B = 0$ and $A + B = 1$, use algebraic manipulation to prove that

$$(A + C) \cdot (\overline{A} + B) \cdot (B + C) = B \cdot C$$

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$$AD' + A'B + C'D + B'C$$

$$= \neg(\neg(AD' + A'B + C'D + B'C))$$

$$= \neg((A'+D)(A+B')(C+D')(B+C'))$$

$$= \neg(0 + A'B' + AD + DB')(CB + CC' + BD' + C'D')$$

$$= \neg(A'B' + AD + B'D)(BC + BD' + C'D')$$

$$= \neg(A'\cancel{BB'}C + A'\cancel{BB'}D' + A'B'C'D' + ABCD + A\cancel{BDD'} + A\cancel{C'D}D' + B\cancel{B'C}D + B\cancel{B'D}D' + B'\cancel{C'D}D')$$

$$= \neg(ABCD + A'B'C'D')$$

$$= (A'+B'+C'+D')(A+B+C+D)$$

$$\bar{C}(AB + B\bar{D} + D) + BC$$

$$\bar{C}(B(A+\bar{D})+D) + BC$$

$$\bar{C}((B+D)(\cancel{D+\bar{D}}+A)) + BC$$

$$\bar{C}((B+D)\cancel{(A+1)}) + BC$$

$$\bar{C}(B+D) + BC$$

$$\bar{C}B + \bar{C}D + BC$$

$$B(\cancel{C+\bar{C}}) + \bar{C}D$$

$$B + \bar{C}D$$

b. $\bar{A}B + \bar{B}\bar{C} + AB + \bar{B}C = 1$

$= B(\bar{A}+A) + \bar{B}(\bar{C}+C) = 1$

$= B + \bar{B} = 1$

$= 1 = 1$

Pro

c. $y + \bar{x}z + x\bar{y} = x + y + z$

$= y + \bar{y}x + \bar{x}z = x + y + z$

$= y + x + \bar{x}z = x + y + z$

$= y + x + z = x + y + z$

$= x + y + z = x + y + z$

d. $\bar{x}\bar{y} + \bar{y}z + xz + xy + y\bar{z} = \bar{x}\bar{y} + xz$

$= \bar{x}\bar{y} + \bar{y}z(x+\bar{x}) + xz + xy + y\bar{z} = \bar{x}\bar{y} + xz$

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2-3

a. $AB\bar{C} + B\bar{C}\bar{D} + BC + \bar{C}D = B + \bar{C}D$

$B(A\bar{C} + C) + \bar{C}(B\bar{D} + D) = B + \bar{C}D$

$B((A+C)(\bar{C}+C)) + \bar{C}((B+D)(\bar{D}+D)) = B + \bar{C}D$

$B(A+C) + \bar{C}(B+D) = B + \bar{C}D$

$AB + BC + B\bar{C} + \bar{C}D = B + \bar{C}D$

$B(C+\bar{C}) + AB + \bar{C}D = B + \bar{C}D$

$B + AB + \bar{C}D = B + \bar{C}D$

$B(1+A) + \bar{C}D = B + \bar{C}D$

$B \cdot 1 + \bar{C}D = B + \bar{C}D$

b. $wy + \bar{w}y\bar{z} + wxz + \bar{w}x\bar{y} = wy + \bar{w}x\bar{z} + \bar{x}y\bar{z} + x\bar{y}z$

(b) $wY + \bar{w}Y\bar{z} + wxz + \bar{w}x\bar{Y} = wY + \bar{w}x\bar{z} + \bar{x}Y\bar{z} + x\bar{Y}z$

$wY + \bar{w}Y\bar{z}(x+\bar{x}) + wxz(Y+\bar{Y}) + \bar{w}x\bar{Y}(z+\bar{z})$

$wY + \bar{w}Y\bar{z}x + \bar{w}Y\bar{z}\bar{x} + wxzY + wxz\bar{Y} + \bar{w}x\bar{Y}z + \bar{w}x\bar{Y}\bar{z}$

$wY + (\bar{w}Y\bar{z}x + \bar{w}\bar{Y}\bar{z}x) + (wxz\bar{Y} + \bar{w}x\bar{Y}z) + \bar{w}Y\bar{z}\bar{x} + wxYz$

$wY + (\bar{w}x\bar{z}) + (x\bar{Y}z) + \bar{w}Y\bar{z}\bar{x} + wxYz$

$wY(xz + \overline{xz}) + \bar{w}Y\bar{x}\bar{z} + wxYz + (\bar{w}x\bar{z}) + (x\bar{Y}z)$

$\underline{wxYz} + wY(\overline{xz}) + \bar{w}Y\bar{x}\bar{z} + \underline{wxYz} + (\bar{w}x\bar{z}) + (x\bar{Y}z)$

$(wxYz + wxYz) + wY(\bar{x}+\bar{z}) + \bar{w}Y\bar{x}\bar{z} + \bar{w}x\bar{z} + x\bar{Y}z$

$wxYz + \underline{w\bar{x}Y} + w\bar{z}Y + \bar{w}Y\bar{x}\bar{z} + \bar{w}x\bar{z} + x\bar{Y}z$

$wxYz + (w\bar{x}Yz + w\bar{x}Y\bar{z}) + (w\bar{z}Yx + w\bar{z}Y\bar{x} + \bar{w}Y\bar{x}\bar{z} + \bar{w}x\bar{z} + x\bar{Y}z$

$(wxYz + w\bar{x}Yz) + \quad wY\bar{z} + \bar{x}Y\bar{z} + \bar{w}x\bar{z} + x\bar{Y}z$

$(wYz + wY\bar{z}) + \bar{x}Y\bar{z} + \bar{w}x\bar{z} + x\bar{Y}z$

$= wY + \bar{x}Y\bar{z} + \bar{w}x\bar{z} + x\bar{Y}z$

$= wY + \bar{w}x\bar{z} + \bar{x}Y\bar{z} + x\bar{Y}z$

(c) $A\bar{D}+\bar{A}B+\bar{C}D+\bar{B}C = (\bar{A}+\bar{B}+\bar{C}+\bar{D})(A+B+C+D)$

$(\bar{A}+\bar{B}+\bar{C}+\bar{D})(A+B+C+D)$

$=\bar{A}A+\bar{A}B+\bar{A}C+\bar{A}D+\bar{B}A+\bar{B}B+\bar{B}C+\bar{B}D+\bar{C}A+\bar{C}B+\bar{C}C+\bar{C}D+\bar{D}A+\bar{D}B+\bar{D}C+\bar{D}D$

$=\bar{A}B+\bar{A}C+\bar{A}D+\bar{B}A+\bar{B}C+\bar{B}D+\bar{C}A+\bar{C}B+\bar{C}D+\bar{D}A+\bar{D}B+\bar{D}C$

$=(B\bar{A}+\bar{B}D+\bar{A}D)+(B\bar{D}+\bar{B}C+\bar{D}C)+\bar{A}C+\bar{B}A+\bar{C}A+\bar{C}B+\bar{C}D+\bar{D}A$

$=\bar{A}B+\bar{B}D+\bar{D}B+\bar{B}C+\bar{A}C+\bar{B}A+\bar{C}A+\bar{C}B+\bar{C}D+\bar{D}A$

$=(D\bar{B}+\bar{D}A+\bar{B}A)+(A\bar{C}+\bar{A}B+\bar{C}B)+\bar{D}B+\bar{B}C+\bar{A}C+\bar{C}D$

$=\bar{B}D+\bar{D}A+\bar{C}A+\bar{A}B+\bar{D}B+\bar{B}C+\bar{A}C+\bar{C}D$

$=(B\bar{A}+\bar{B}C+\bar{A}C)+(\bar{C}D+\bar{D}A+\bar{C}A)+\bar{B}D+\bar{D}B$

$=\bar{A}B+\bar{B}C+\bar{C}D+\bar{D}A+\bar{B}D+\bar{D}B$

$=(C\bar{B}+\bar{C}D+\bar{B}D)+(A\bar{D}+\bar{A}B+\bar{D}B)$

$=C\bar{B}+\bar{C}D+A\bar{D}+\bar{A}B = A\bar{D}+\bar{A}B+\bar{C}D+\bar{B}C$ (proven)

2-4) $(A+C)\cdot(\bar{A}+B)\cdot(B+C) = BC$, dengan $A+B=1$ dan $AB=0$

$$(C+A)(C+B)(B+\bar{A}) = BC$$

$$(C+\underbrace{AB}_{0})\cdot\overset{1}{(A+B)}\cdot(B+\bar{A}) = BC$$

$$C(B+\bar{A})(B+\bar{B}) = BC$$

$$C(B+\bar{A}\bar{B}) = BC$$

$$C(B+\overline{A+B}) = BC$$

$$C(B+\bar{1}) = BC$$

$$C(B+0) = BC$$

$$BC = BC$$

2-3) a) $AB\bar{C} + B\bar{C}\bar{D} + BC + \bar{C}D = B + \bar{C}D$

$$= AB\bar{C} + BC + B\bar{C}\bar{D} + \bar{C}D$$

$$= B(A\bar{C} + C) + \bar{C}(B\bar{D} + D)$$

$$= B(A + C) + \bar{C}(B + D)$$

$$= BA + BC + \bar{C}B + \bar{C}D$$

$$= BC + \bar{C}B + BA + \bar{C}D$$

$$= B(C + \bar{C}) + BA + \bar{C}D$$

$$= B + BA + \bar{C}D$$

$$= B(1 + A) + \bar{C}D$$

$$= B + \bar{C}D \;//$$

c) $A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C = \overline{(\bar{A}+\bar{B}+\bar{C}+\bar{D})(A+B+C+D)}$

$\cancel{= (\bar{A}+D) + (A+\bar{B}) + (C+\bar{D}) +}$

$= \overline{(\bar{A}+D)(A+\bar{B})(C+\bar{D})(B+\bar{C})}$

$= \overline{(A\bar{A}+\bar{A}\bar{B}+AD+\bar{B}D)(BC+C\bar{C}+B\bar{D}+\bar{C}\bar{D})}$

$= \overline{\cancel{(A\bar{A}}\,(\bar{A}\bar{B}+AD+\bar{B}D)(BC+B\bar{D}+\bar{C}\bar{D})}$

$= \overline{(\bar{A}\bar{B}BC + \bar{A}\bar{B}BD + \bar{A}\bar{B}\bar{C}\bar{D}) + (\cancel{ADB}\,ABCD + ABD\bar{D} + A\bar{C}D\bar{D}) + (\bar{B}BCD + \bar{B}BD\bar{D} + \bar{B}C\bar{D}D)}$

$= \overline{\bar{A}\bar{B}\bar{C}\bar{D} + ABCD}$

$= (A+B+C+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})$ // (terbukti)

OVERVIEW

- latihan
- Penggambaran rangkaian
- Standard forms
- Minterm, maxterm
- Sum of minterm, product of maxterm
- Sum of product, product of sum

STANDARD FORMS

- Sebuah fungsi Boolean dapat dinyatakan dalam beberapa cara yang berbeda
  - A Boolean function expressed algebraically can be written in a variety of ways.
- There are, however, specific ways of writing algebraic equations that are considered to be standard forms.
  - Terdapat bentuk standar penulisan
- The standard forms facilitate the simplification procedures for Boolean expressions and, in some cases, may result in more desirable expressions for implementing logic circuits.
  - Bentuk standar memfasilitasi prosedur penyederhanaan untuk ekspresi boolean
- The standard forms contain product terms and sum terms
  - XYZ : product terms
  - X+Y+Z : sum terms

MINTERM

- A product term in which all the variables appear exactly once, either complemented or uncomplemented, is called a minterm
  - Perkalian dimana masing-masing variable akan muncul sekali, baik dalam bentuk komplemen maupun bukan komplemen
- There are 2^n distinct minterms for n variables
  - Untuk n variable terdapat 2 pangkat n minterm yang berbeda
- The four minterms for the two variables X and Y
- For each binary combination, there is a related minterm.
- Each minterm is a product term of exactly n literals, where n is the number of variables.
- **A literal** is a complemented variable if the corresponding bit of the related binary combination is 0 and is an uncomplemented variableif it is 1
- A symbol m_j for each minterm is also shown in the table, where the subscript j denotes the decimal equivalent of the binary combination corresponding to the minterm.

TABLE 2-9
Minterms for Three Variables

X	Y	Z	Product Term	Symbol	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7
0	0	0	$\overline{X}\,\overline{Y}\,\overline{Z}$	m_0	1	0	0	0	0	0	0	0
0	0	1	$\overline{X}\,\overline{Y}Z$	m_1	0	1	0	0	0	0	0	0
0	1	0	$\overline{X}Y\overline{Z}$	m_2	0	0	1	0	0	0	0	0
0	1	1	$\overline{X}YZ$	m_3	0	0	0	1	0	0	0	0
1	0	0	$X\overline{Y}\,\overline{Z}$	m_4	0	0	0	0	1	0	0	0
1	0	1	$X\overline{Y}Z$	m_5	0	0	0	0	0	1	0	0
1	1	0	$XY\overline{Z}$	m_6	0	0	0	0	0	0	1	0
1	1	1	XYZ	m_7	0	0	0	0	0	0	0	1

MAXTERM

- A sum term that contains all the variables in complemented or uncomplemented form is called a maxterm
- There are 2^n distinct maxterms for n variables
- A symbol M_j for each minterm is also shown in the table, where the subscript j denotes the decimal equivalent of the binary combination corresponding to the minterm.

TABLE 2-10
Maxterms for Three Variables

X	Y	Z	Sum Term	Symbol	M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7
0	0	0	$X + Y + Z$	M_0	0	1	1	1	1	1	1	1
0	0	1	$X + Y + \overline{Z}$	M_1	1	0	1	1	1	1	1	1
0	1	0	$X + \overline{Y} + Z$	M_2	1	1	0	1	1	1	1	1
0	1	1	$X + \overline{Y} + \overline{Z}$	M_3	1	1	1	0	1	1	1	1
1	0	0	$\overline{X} + Y + Z$	M_4	1	1	1	1	0	1	1	1
1	0	1	$\overline{X} + Y + \overline{Z}$	M_5	1	1	1	1	1	0	1	1
1	1	0	$\overline{X} + \overline{Y} + Z$	M_6	1	1	1	1	1	1	0	1
1	1	1	$\overline{X} + \overline{Y} + \overline{Z}$	M_7	1	1	1	1	1	1	1	0

SUM OF MINTERM

$$M_3 = X + \overline{Y} + \overline{Z} + = \overline{\overline{X}YZ} = \overline{m}_3$$

- minterm and maxterm with the same subscript are the complements of each other
- A Boolean function can be represented algebraically from a given truth table by forming the logical sum of all the minterms that produce a 1 in the function
- This expression is called a sum of minterms

$$F = \overline{X}\,\overline{Y}\,\overline{Z} + \overline{X}Y\overline{Z} + X\overline{Y}Z + XYZ = m_0 + m_2 + m_5 + m_7$$

$$F(X, Y, Z) = \Sigma m(0, 2, 5, 7)$$

TABLE 2-11
Boolean Functions of Three Variables

(a) X	Y	Z	F	$\overline{F}$	(b) X	Y	Z	E
0	0	0	1	0	0	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	1	0	0	1	0	1
0	1	1	0	1	0	1	1	0
1	0	0	0	1	1	0	0	1
1	0	1	1	0	1	0	1	1
1	1	0	0	1	1	1	0	0
1	1	1	1	0	1	1	1	0

PRODUCT OF MAXTERM)

$$\overline{F}(X,Y,Z) = \overline{X}\,\overline{Y}Z + \overline{X}YZ + X\overline{Y}\,\overline{Z} + XY\overline{Z} = m_1 + m_3 + m_4 + m_6$$

$$\overline{F}(X, Y, Z) = \Sigma m(1, 3, 4, 6)$$

$$F = \overline{m_1 + m_3 + m_4 + m_6} = \overline{m_1} \cdot \overline{m_3} \cdot \overline{m_4} \cdot \overline{m_6}$$

$$= M_1 \cdot M_3 \cdot M_4 \cdot M_6 \text{ (since } \overline{m_j} = M_j)$$

$$= (X + Y + \overline{Z})(X + \overline{Y} + \overline{Z})(\overline{X} + Y + Z)(\overline{X} + \overline{Y} + Z)$$

$$F(X, Y, Z) = \Pi M(1, 3, 4, 6)$$

MINTERM AND MAXTERM SUMMARIZED

1. There are 2^n minterms for n Boolean variables. These minterms can be generated from the binary numbers from 0 to $2^n - 1$.
2. Any Boolean function can be expressed as a logical sum of minterms.
3. The complement of a function contains those minterms not included in the original function.
4. A function that includes all the 2^n minterms is equal to logic 1.

CONVERT FUNCTION TO MINTERMS

$$E = \overline{Y} + \overline{X}\,\overline{Z}$$

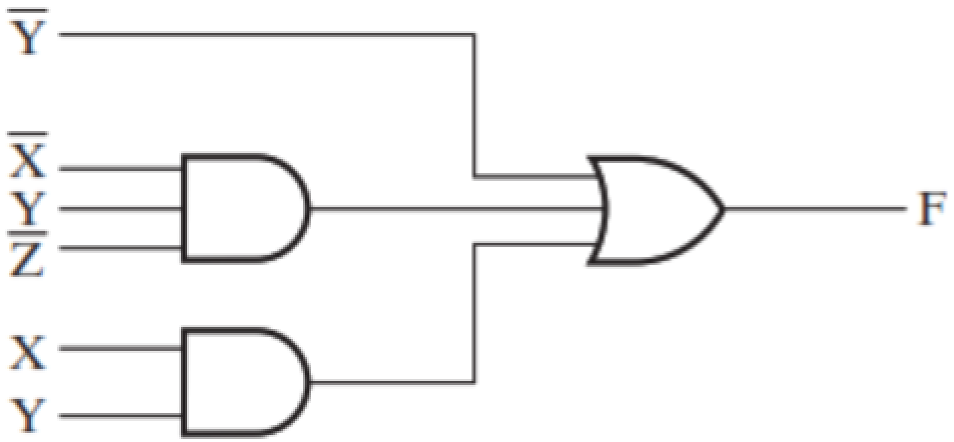
- A function that is not in the sum- of- minterms form can be converted to that form by means of a truth table, since the truth table always specifies the minterms of the function

$$E(X, Y, Z) = \Sigma m(0, 1, 2, 4, 5)$$

SUM OF PRODUCT

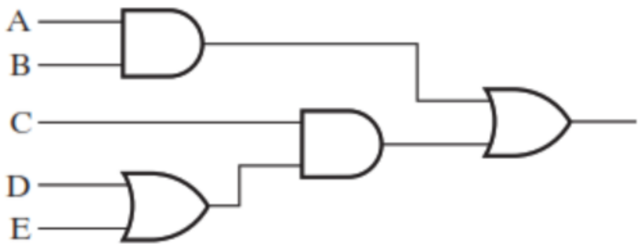
- Once the sum of minterms is obtained from the truth table, the next step is to try to simplify the expression to see whether it is possible to reduce the number of product terms and the number of literals in the terms.
- The result is a simplified expression in **<u>sum- of- products form</u>**

$$F = \overline{Y} + \overline{X}Y\overline{Z} + XY$$

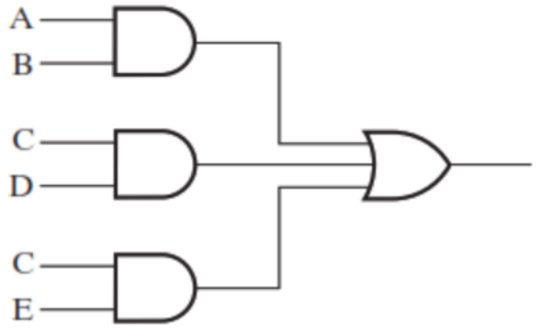


SUM OF PRODUCT

- If an expression is not in sum- of- products- form, it can be converted to the standard form by means of the distributive laws



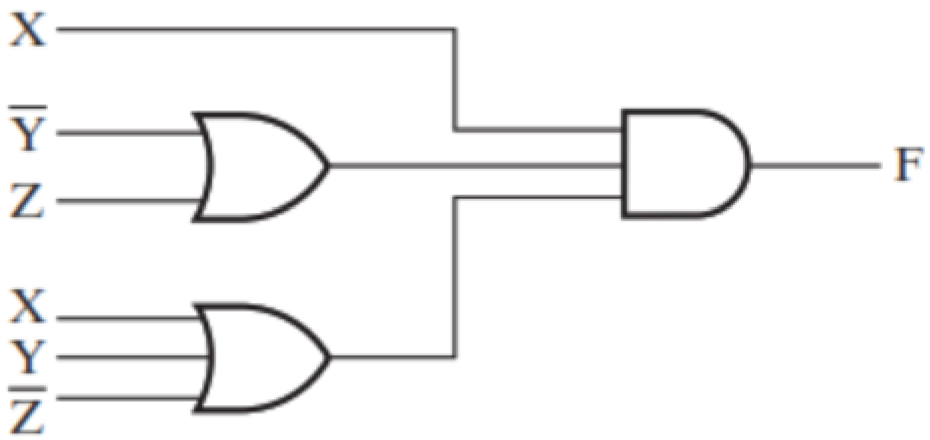
(a) $AB + C(D + E)$



(b) $AB + CD + CE$

PRODUCT OF SUM

$$F = X(\overline{Y} + Z)(X + Y + \overline{Z})$$



- obtained by forming a logical product of sum terms

2-10. *Obtain the truth table of the following functions, and express each function in sum-of-minterms and product-of-maxterms form:

(a) $(XY + Z)(Y + XZ)$

(b) $(\overline{A} + B)(\overline{B} + C)$

(c) $WX\overline{Y} + WX\overline{Z} + WXZ + Y\overline{Z}$

- Latihan 2.11

X	Y	Z	E	F
0	0	0	0	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	0
1	1	1	0	1

(a) List the minterms and maxterms of each function.

(b) List the minterms of $\overline{E}$ and $\overline{F}$

(c) List the minterms of $E + F$ and $E \cdot F$.

(d) Express E and F in sum-of-minterms algebraic form.

(e) Simplify E and F to expressions with a minimum of literals.

2-12. *Convert the following expressions into sum-of-products and product-of-sums forms:

(a) $(AB + C)(B + \overline{C}D)$

(b) $\overline{X} + X(X + \overline{Y})(Y + \overline{Z})$

(c) $(A + B\overline{C} + CD)(\overline{B} + EF)$

2-13. Draw the logic diagram for the following Boolean expressions. The diagram should correspond exactly to the equation. Assume that the complements of the inputs are not available.

(a) $\overline{A}\,\overline{B}\,\overline{C} + AB + AC$

(b) $X(Y\overline{Z} + \overline{Y}Z) + \overline{W}(\overline{Y} + \overline{X}Z)$

(c) $AC(\overline{B} + D) + \overline{A}C(\overline{B} + \overline{D}) + BC(\overline{A} + \overline{D})$

KARNAUGH MAP /K-MAP