$$A = \begin{pmatrix} 4 & 8 & 1 \\ -2 & -9 & -2 \\ 0 & 10 & -4 \end{pmatrix}$$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{bmatrix}$$

$$\begin{vmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)\begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} - 8\begin{vmatrix} -2 & -2 \\ 0 & 5-\lambda \end{vmatrix} + (-1)\begin{vmatrix} -2 & -9-\lambda \\ 0 & 10 \end{vmatrix} = 0$$

$$(4-\lambda)(\lambda^2 - 4\lambda - 25) - 8(-10+2\lambda) + (-1)(-20)$$

$$-\lambda^3 + (4\lambda^2 - 4\lambda^2) + (16\lambda + 25\lambda) - 100$$

$$-\lambda^3 - 41\lambda - 100 = 0$$

By using the calculator,

$$\lambda_1 = 0, \quad \lambda_2 = 5, \quad \lambda_3 = -5$$

calculating for eigen vectors by peer 2

Eigenvector for λ=0

$$A - 0I = A = \begin{bmatrix} 4 & 8 & -1 \\ -2 & -9 & -2 \\ 0 & 10 & 5 \end{bmatrix}$$

$$Ax = 0 \text{ or } \begin{bmatrix} 4 & 8 & -1 \\ -2 & -9 & -2 \\ 0 & 10 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

After doing the Operation

$$V_1 = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

Eigenvector for λ= 5

$$A - 5\lambda = \begin{bmatrix} -1 & 8 & -1 \\ -2 & -14 & -2 \\ 0 & 10 & 0 \end{bmatrix}$$

solving (A - 5/)x = 0

Eigenvector: v2 $= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

Eigenvector for λ= -5

$$A + 5I = \begin{bmatrix} 9 & 8 & -1 \\ -2 & -4 & -2 \\ 0 & 10 & 10 \end{bmatrix}$$

After Solving $[A + 5I)x = 0$ we get

$$V_3 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

Importance of eigen value / PCA

Sum of the absolute Values

$$|X| = |-5| + |5| + |0| = 5 + 5 + 0 = 10$$

Percentage for each

$$\lambda_1 = |-5| \quad \left(\frac{5}{10}\right) \times 100 = 50\%$$

$$\lambda_2 = 5 \quad \left(\frac{5}{10}\right) \times 100 = 50\%$$

$$\lambda_3 = 0 \quad \left(\frac{0}{10}\right) \times 100 = 0\%$$

Final Answer:

- **Eigenvalue −5-:** 50% importance
- **Eigenvalue 5:** 50% importance
- **Eigenvalue 0:** 0% importance

This means the first two eigenvalues contribute equally to the system, while the third eigenvalue has no significance in this context