

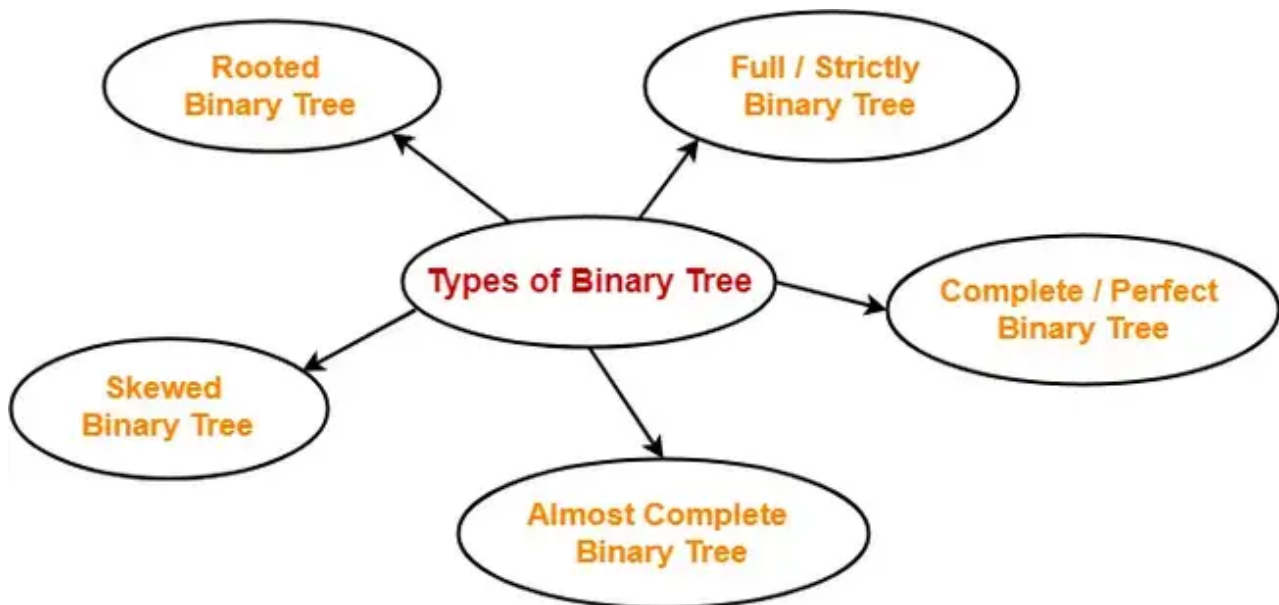
Binary Tree | Binary Tree Properties

Binary Tree-

Before you go through this article, make sure that you gone through the previous article on **Binary Trees**.

We have discussed-

- Binary tree is a special tree data structure.
- In a binary tree, each node can have at most 2 children.
- There are following types of binary trees-



In this article, we will discuss properties of binary trees.

Binary Tree Properties-

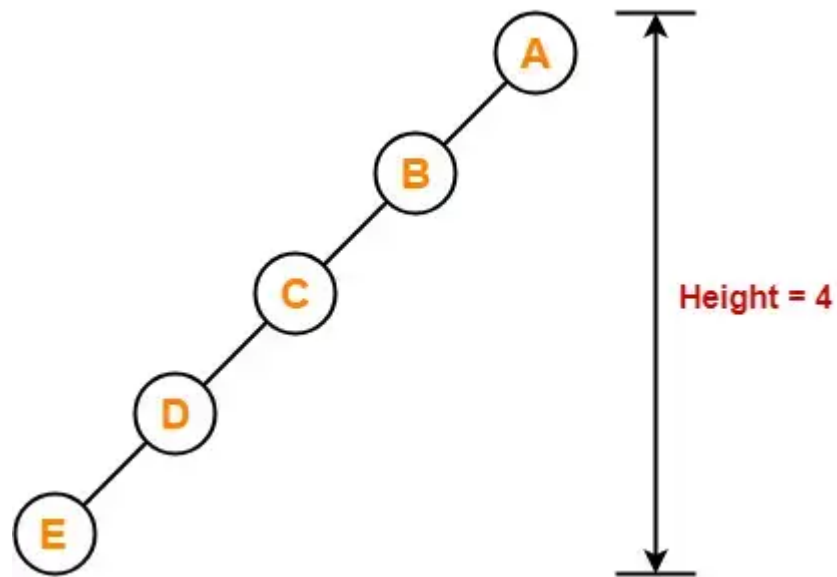
Important properties of binary trees are-

Property-01:

Minimum number of nodes in a binary tree of height H
 $= H + 1$

Example-

To construct a binary tree of height = 4, we need at least $4 + 1 = 5$ nodes.



Property-02:

Maximum number of nodes in a binary tree of height H
 $= 2^{H+1} - 1$

Example-

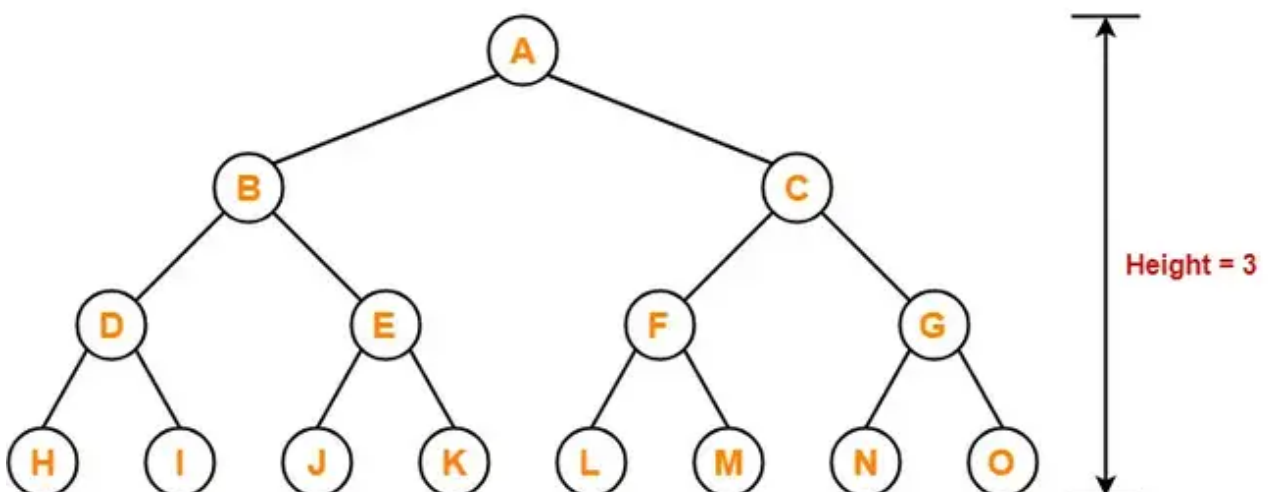
Maximum number of nodes in a binary tree of height 3

$$= 2^{3+1} - 1$$

$$= 16 - 1$$

$$= 15 \text{ nodes}$$

Thus, in a binary tree of height = 3, maximum number of nodes that can be inserted = 15.



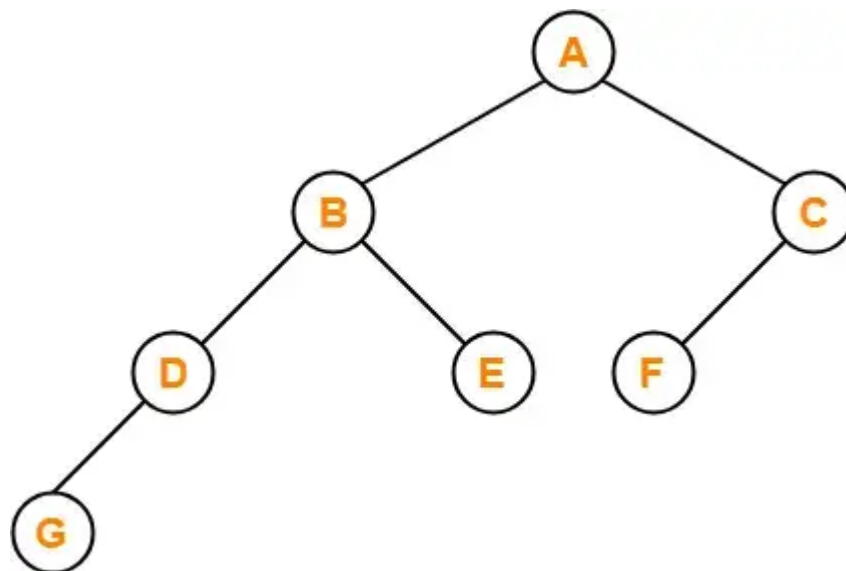
We can not insert more number of nodes in this binary tree.

Property-03:

**Total Number of leaf nodes in a Binary Tree
= Total Number of nodes with 2 children + 1**

Example-

Consider the following binary tree-



Here,

- Number of leaf nodes = 3
- Number of nodes with 2 children = 2

Clearly, number of leaf nodes is one greater than number of nodes with 2 children.

This verifies the above relation.

NOTE

It is interesting to note that-

Number of leaf nodes in any binary tree depends only on the number of nodes with 2 children.

Property-04:

**Maximum number of nodes at any level 'L' in a binary tree
= 2^L**

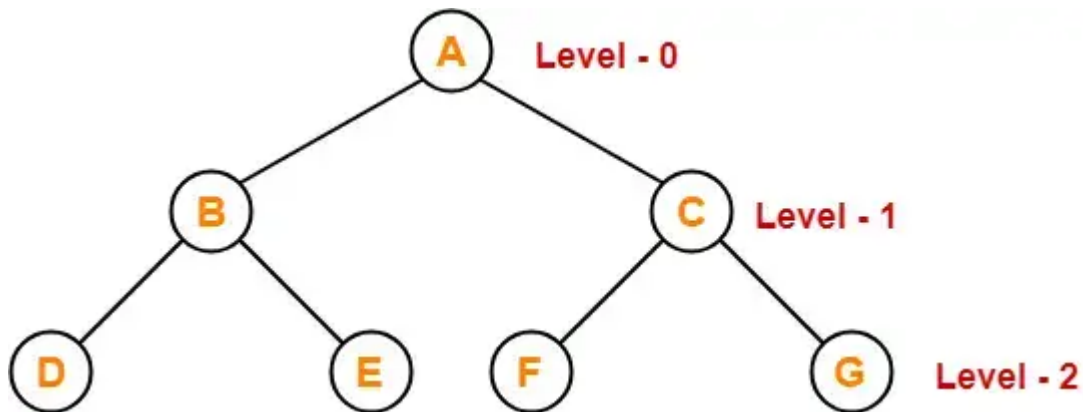
Example-

Maximum number of nodes at level-2 in a binary tree

$$= 2^2$$

$$= 4$$

Thus, in a binary tree, maximum number of nodes that can be present at level-2 = 4.



To gain better understanding about Binary Tree Properties,

[Watch this Video Lecture](#)

PRACTICE PROBLEMS BASED ON BINARY TREE PROPERTIES-

Problem-01:

A binary tree T has n leaf nodes. The number of nodes of degree-2 in T is _____?

1. $\log_2 n$
2. $n-1$
3. n
4. 2^n

Solution-

Using property-3, we have-

Number of degree-2 nodes

$$= \text{Number of leaf nodes} - 1$$

$$= n - 1$$

Thus, Option (B) is correct.

Problem-02:

In a binary tree, for every node the difference between the number of nodes in the left and right subtrees is at most 2. If the height of the tree is $h > 0$, then the minimum number of nodes in the tree is _____?

1. 2^{h-1}
2. $2^{h-1} + 1$
3. $2^h - 1$
4. 2^h

Solution-

Let us assume any random value of h . Let $h = 3$.

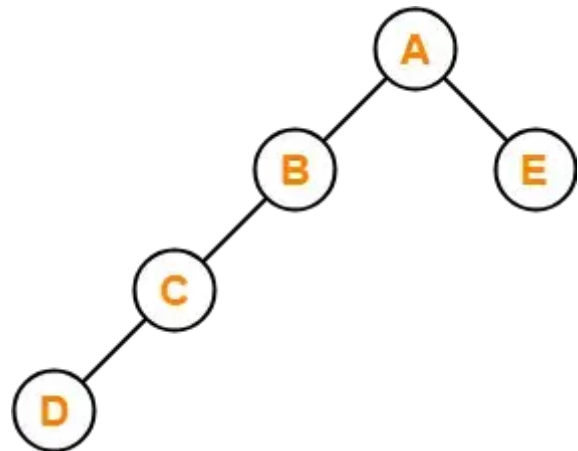
Then the given options reduce to-

1. 4
2. 5
3. 7
4. 8

Now, consider the following binary tree with height $h = 3$ -

- This binary tree satisfies the question constraints.
- It is constructed using minimum number of nodes.

Thus, Option (B) is correct.



Problem-03:

In a binary tree, the number of internal nodes of degree-1 is 5 and the number of internal nodes of degree-2 is 10. The number of leaf nodes in the binary tree is _____?

1. 10
2. 11
3. 12
4. 15

Solution-

Using property-3, we have-

Number of leaf nodes in a binary tree

= Number of degree-2 nodes + 1

= 10 + 1

= 11

Thus, Option (B) is correct.

Problem-04:

The height of a binary tree is the maximum number of edges in any root to leaf path. The maximum number of nodes in a binary tree of height h is _____?

1. 2^h
2. $2^{h-1} - 1$
3. $2^{h+1} - 1$
4. 2^{h+1}

Solution-

Using property-2, Option (C) is correct.

Problem-05:

A binary tree T has 20 leaves. The number of nodes in T having 2 children is _____?

Solution-

Using property-3, correct answer is 19.

To watch video solutions and practice more problems,

[Watch this Video Lecture](#)