Basic Intro to CDO

owner/sponsor

Asset pool
Bonds, mortgage, etc
100m

Waterfall

SPV

Loss

100%

ST (AAA) 93m

7%

MT (invest grade) 4m

3%

ET (unrated) 3m

0

	Apt	Dpt	Size	Loss Responsible	yield
ET	0	3%	3m	0 – 3m	10%
MT	3%	7%	4m	3 – 7m	5%
ST	7%	100%	93m	7+m	1%

In Good Times (yearly premium)

ST: $93m \times 1\% = 0.93\ m$

MT: $4m \times 5\% = 0.2\ m$

ET: $3m \times 10\% = 0.3\ m$

In Bad Times

① PLoss = 1m

$(3-1)m \times 10\% = 0.2m$

MT / ST ? Intact, recieve same payment.

(2) PLoss = 4 m

ET is wiped out

MT $(4-1)m \times 5\% = 0.15m$

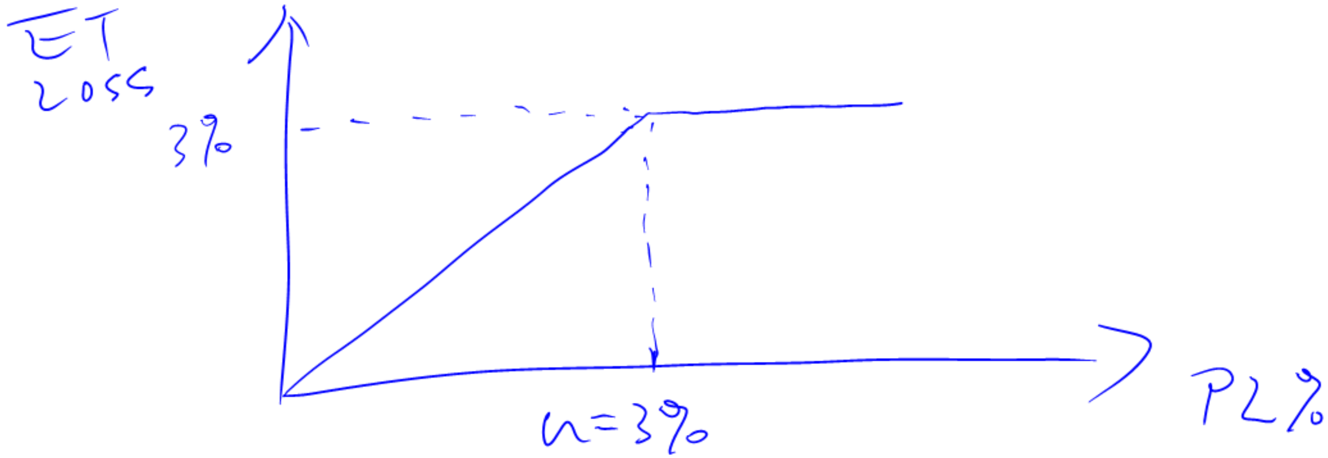
ST: same as before.

CDO Tranch Loss f^n

$$L(t; d, u) = \max[\min(L, u) - d, 0]$$

① $d = 0$ (ET)

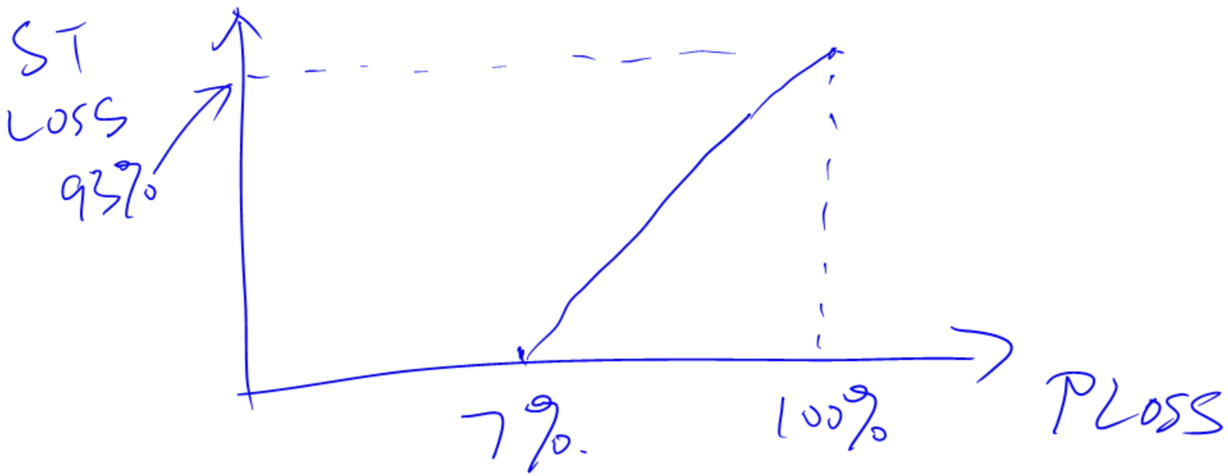
$$L(t; 0, u) = \max[\min(L, u), 0]$$
$$= \min(L, u)$$



② $u = 100\%$ (ST)

$$L(t; d, 100\%) = \max[\min(L, u) - d, 0]$$

$$= \max[L - d, 0]$$



③ MT

(i) $l < d < u$

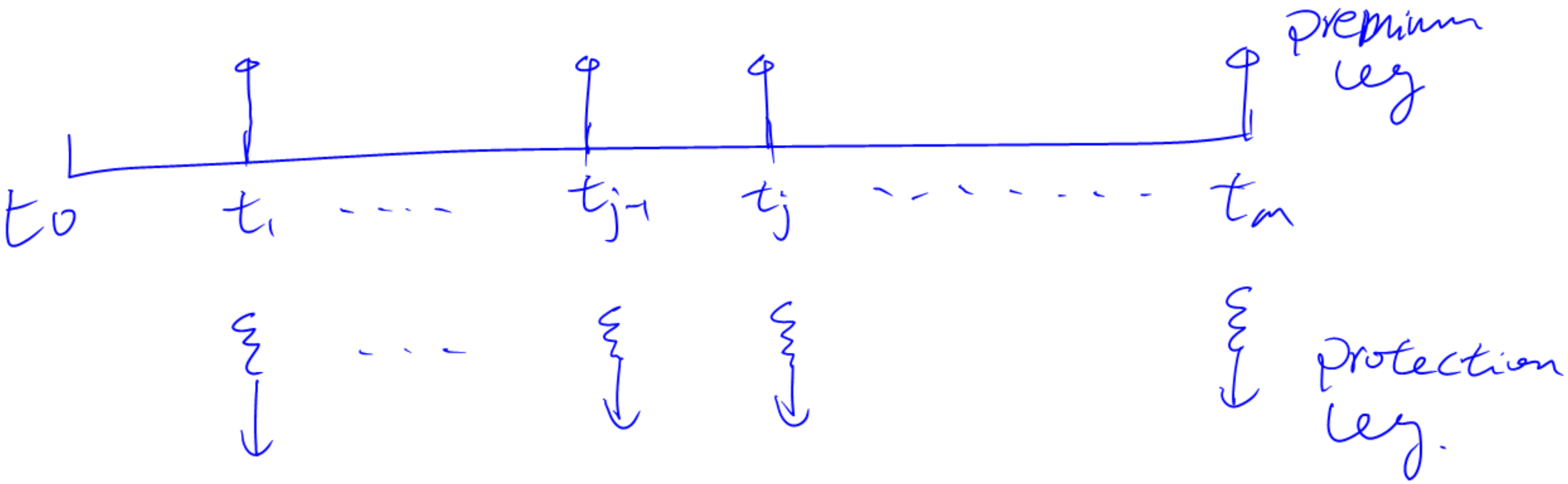
$L(t; d, u) = \max[l - d, 0] = 0.$

(ii) $d < l < u$

$L(t; d, u) = \max[l - d, 0] = l - d$

(iii) $d < u < l$

$L(t; d, u) = \max[u - d, 0] = u - d.$



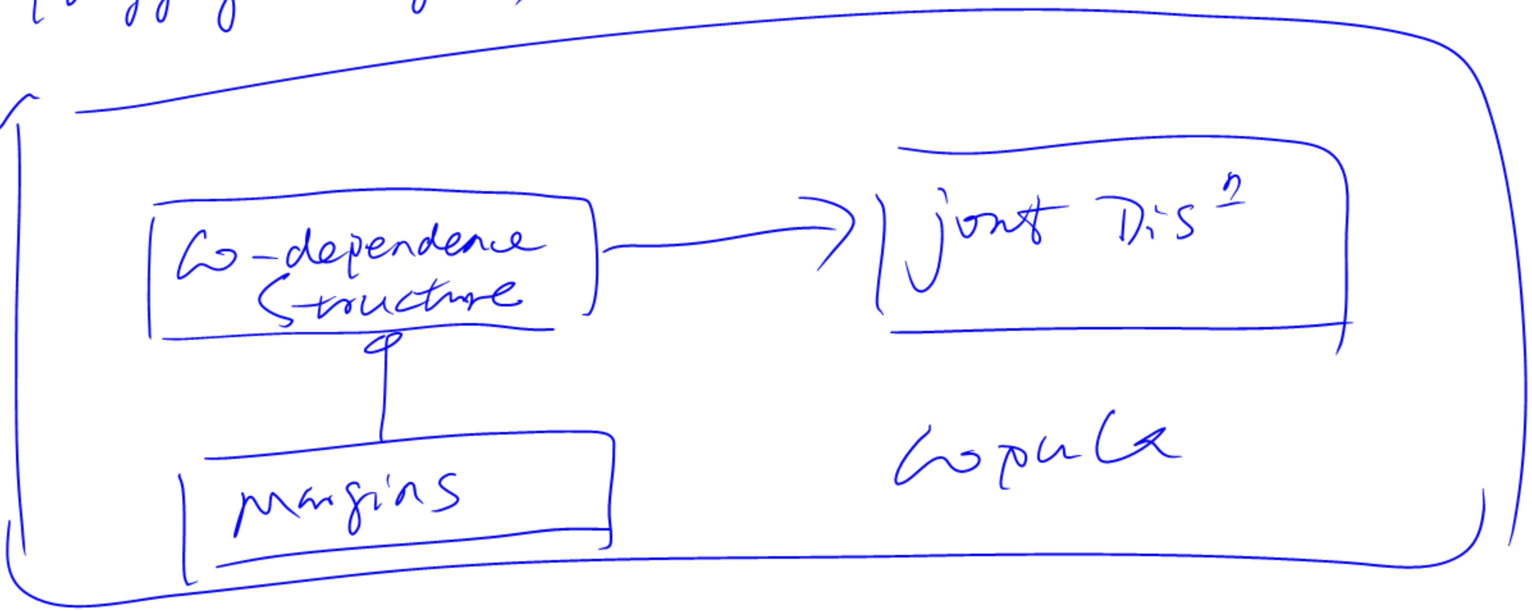
$$E(\text{P.V. premium leg.}) = E(\text{P.V. protection leg.})$$

Concept of Copula

(1) Copula is a generation f^n for joint Dis^n

(2) Copula takes margins as input, tranform it into joint Dis^n f^n

(3) Before plugging margins, it represents a co-dependence structor



Notation

margin : $F(X)$ $F(Y)$.

Copula : $C(u_1, u_2)$

Joint Dis : $F(X, Y)$

$$C(u_1, u_n)$$

$$\downarrow$$

$$C(F(x), F(y))$$

$$\downarrow$$

$$F(x, y)$$

(4) Before plugging margins, Copula is a joint Dis^n f^n for a bunch of uniform R.V.s.

$$\begin{array}{ccc} C(& u_1, & u_2) \\ & \Uparrow & \Uparrow \\ & F(X) & F(Y) \end{array}$$

u_1 & u_2 are uniform r.v.s.

Does it mean $F(x)$ / $F(y)$ is uniform?

Show $F(X) \sim \text{uniform } [0, 1]$

$$Y = F(X) = Pr(X \leq x)$$

① $Y \in [0, 1]$ ✓

② $f_Y = 1$

Y & X has one to one montonic relationship.

$$\frac{dy}{dx} = f_X$$

$$f_Y \, dy = f_X \, dx$$

$$f_Y = f_X \frac{dx}{dy} = f_X \left(\frac{dy}{dx}\right)^{-1}$$

$$= f_X \frac{1}{f_X} = 1$$

Sklar Theorem

$$C(u_1, \ldots, u_n) = \Pr(U_1 \le u_1, \ldots U_n \le u_n)$$

$$u_i = F_i(x_i)$$

$$= \Pr\left(F_1(X_1) \le u_1, \ldots, F_n(X_n) \le u_n\right)$$

$$= \Pr\left(X_1 \le F_1^{-1}(u_1), \ldots, X_n \le F_n^{-1}(u_n)\right)$$

$$= \Pr(X_1 \le x_1, \ldots X_n \le x_n)$$

$$= F_n(x_1, \ldots x_n)$$

Example of Gaussian Copula

Example of Gaussian Copula -

① $u_i = F_i(x_i) = \Phi(x_i)$

$$C(u_1, \cdots u_n) = \Phi_n(\Phi^{-1}(\Phi(x_1)), \cdots \Phi^{-1}(\Phi(x_n)))$$

$$= \Phi_n(x_1, \cdots x_n)$$

② $u_i = F_i(z_i) = 1 - e^{-\lambda_i z_i} = \Pr(Z_i \le z_i) = p_i$

$$C(u_1, u_2) = \Phi_2(\Phi^{-1}(p_1), \Phi^{-1}(p_2))$$

Distance to Default $= \Phi^{-1}(P_i)$

$$= d_1$$

$$\Phi_2(d_1, d_2) = \Pr(z_1 < z_i,\ z_2 \le z_2).$$

$$= \int_{-\infty}^{d_1} \int_{-\infty}^{d_2} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{ - \frac{x^2 - 2\rho x y + y^2}{2(1-\rho)} \right\} dx\, dy.$$