The image above is Craiyon.AI generated in response to "predicting market return".

Predicing Asset Price Direction

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Foreword. This demonstration is a stylised, simplified example of running classifiers (supervised learning) on equity returns. The demo be relevant to CQF Assignment/Examination/one kind of Final Project. For that purpose, is a very starting point. The task is purposefuly limited to prediction of return sign and binomial classification (up/down).

Here, we consider a large-cap equities, however in practice, the high frequency returns of large caps and market indices are the most difficult quantities to predict. Otherwise, the 70-90% traders in CFD accounts would not routinely lose their depostis.

One is more likely to find a sensible scheme in predicting specialised assets and co-movement (spread) rather than returns (prices) of an individual assets. Academic asset pricing theory concerns itself with estimating (prediction) of beta, which is relation between asset returns and the market.

- To organise the prediction (your machine learning workflow) we start with features: generate 7-10 price-related features (columns) and run Classifier.fit() on them. This is no different than running a regression on several variables. For each classifier, you can produce ROC, Confusion Matrix, Transition Probabilities -- the startard evaluation techniques of supervised learning.
- This demo derives features (lagged returns) from the asset price. Price-derived information is limited and there is large discussion about its predictive powers vs. fundamentals, macro economic, and alternative data.

Improving on price information

However, the forecasting can be technically improved by using the measures of Average and Momentum. You will need to make reasonable choices of **time period** (rolling window) for SMA, EMA, Momentum. This link, gives one example on how to do Exponential Moving Average from the first principles. The choice of smoothing factor $\alpha = 2/(N_{obs} + 1)$.

- Setting up prediction classes, you are likely to encounter the high count of uncertain moves/'no move', where the movement took less than 1/10th of a percentage point.

Remember that for a global market index, the daily average return can be $O(0.04\%)$. $[-1, 0, 1]$ trinomial classification might be more appropriate, but there is **no trading action** that follows from the prediction of no move in the asset.

One but not a universal approach, is to lable 'no moves' as positive. This keeps continuity because most of the moves are positive and we are interested in:

```
1) isolating information carried by empirical negative returns;

2) better quality in negative market moves prediction.
```

The second is because of the assymetry of negative feeling reward for losing money (which one wants to avoid), as compared to the reward for gaining.

Note that we can't simply drop 'no move' observations from the data because that will our ability to predict for each next trading day -- therefore, P\&L backtesting will be affected.

NOT Least Squares

- This demo is **NOT** about autoregression analysis in itself. Autoregression is the application of regression on past returns. The regression is essentially an application a linear model and the method of computation (Maximum Likelihood) involves "the Ordinary Least Squares", $\epsilon_t^2 = (y_t - \hat{\boldsymbol{\beta}} \boldsymbol{x_t})^2$, hence the abbreviation OLS.

If you formally attempt to use the autogression (or vector autoregression for multiple tickers at the same time) in order to predict the daily return *quantity* -- you will encounter the forecasting error of 150-200%.

```
Example: you are predicting return of −1.5%, the order of error
above means that realised return will be anything up to +1.5%,
eg 3% absolute difference.
```

- OLS is **NOT** a valid regression model for binary dependent variable $0, 1$ or for y_t, x_t any other than Normal variables. Strictly speaking OLS is applicable to finanical asset **returns** (not prices) and **quarterly changes** in such variables as GDP and other economic macro.

Change in the nature of dependent variable leads to the change in Maximum Likelihood -- which is 'under the hood' method of how regression works, how regression coefficients are derived and computed. We will consider that today.

Genearalized Linear Models (GLM) is both, extension and the substitution of the linear regression (OLS). It implies a non-linear **link function** between dependent variable, such as Binary $0, 1$ and implied probability $[0.01..0.99]$.

Logistic Regression (Classification) and its Maximum Likelihood -- considered today -- is a good illustration and introduction to the GLM.

We will also demonstrate that prediction results (for asset price direction) that are coming from OLS linear regression are *a fudge*.

In [1]:
```
import numpy as np
import pandas as pd
from pylab import plt
plt.style.use('seaborn')
%matplotlib inline
```

Generating simple features: lagged returns

In [25]:
```
data = pd.read_excel('data/EquitiesDataGOOG.xlsx', sheet_name="Sheet1", index_c
data.columns = ['price']  # Adj Close Price
```

In [21]:
```
data.head()
```

Out[21]:

	price
Date	
2010-09-30	261.91
2010-10-01	261.83
2010-10-04	260.20
2010-10-05	268.11
2010-10-06	266.18

In [7]:
```
data.plot(figsize=(10, 6));
```

In [26]:
```
# SEPARATE for Features Correlation
data_advf = data.copy() #save a copy for advanced look at lagged returns over l

data_advf['Returns'] = np.log(data_advf).diff()

data_advf.head()
```

Out[26]:

Date	price	Returns
2010-09-30	261.91	NaN
2010-10-01	261.83	-0.000305
2010-10-04	260.20	-0.006245
2010-10-05	268.11	0.029947
2010-10-06	266.18	-0.007225

In [19]:
```
data['return'] = np.log(data / data.shift(1))
```

In [9]:
```
data.head()
```

Out[9]:

Date	price	return
2010-09-30	261.91	NaN
2010-10-01	261.83	-0.000305
2010-10-04	260.20	-0.006245
2010-10-05	268.11	0.029947
2010-10-06	266.18	-0.007225

In [10]:
```
lags = 5

cols = []
for lag in range(1, lags+1):
    col = 'ret_%d' % lag
    data[col] = data['return'].shift(lag)
    cols.append(col)

# Column indexation is from 0, so we effectively have range(1, 6) for returns
# CHECK NOT TO USE Return IN ACTUAL PREDICTION
```

In [11]:
```
data.head(10)
```

Out[11]:

Date	price	return	ret_1	ret_2	ret_3	ret_4	ret_5
2010-09-30	261.91	NaN	NaN	NaN	NaN	NaN	NaN
2010-10-01	261.83	-0.000305	NaN	NaN	NaN	NaN	NaN
2010-10-04	260.20	-0.006245	-0.000305	NaN	NaN	NaN	NaN
2010-10-05	268.11	0.029947	-0.006245	-0.000305	NaN	NaN	NaN
2010-10-06	266.18	-0.007225	0.029947	-0.006245	-0.000305	NaN	NaN
2010-10-07	264.02	-0.008148	-0.007225	0.029947	-0.006245	-0.000305	NaN
2010-10-08	267.17	0.011860	-0.008148	-0.007225	0.029947	-0.006245	-0.000305
2010-10-11	268.41	0.004631	0.011860	-0.008148	-0.007225	0.029947	-0.006245
2010-10-12	269.68	0.004720	0.004631	0.011860	-0.008148	-0.007225	0.029947
2010-10-13	270.64	0.003553	0.004720	0.004631	0.011860	-0.008148	-0.007225

In [12]:
```
data.dropna(inplace=True)

data = data.drop(columns="price")

data['return_sign'] = np.sign(data['return'].values)

data_adv = data.copy() #save a copy for advanced methods (classifiers), as comp
```

In [13]:
```
data.head()
```

Out[13]:

Date	return	ret_1	ret_2	ret_3	ret_4	ret_5	return_sign
2010-10-08	0.011860	-0.008148	-0.007225	0.029947	-0.006245	-0.000305	1.0
2010-10-11	0.004631	0.011860	-0.008148	-0.007225	0.029947	-0.006245	1.0
2010-10-12	0.004720	0.004631	0.011860	-0.008148	-0.007225	0.029947	1.0
2010-10-13	0.003553	0.004720	0.004631	0.011860	-0.008148	-0.007225	1.0
2010-10-14	-0.004370	0.003553	0.004720	0.004631	0.011860	-0.008148	-1.0

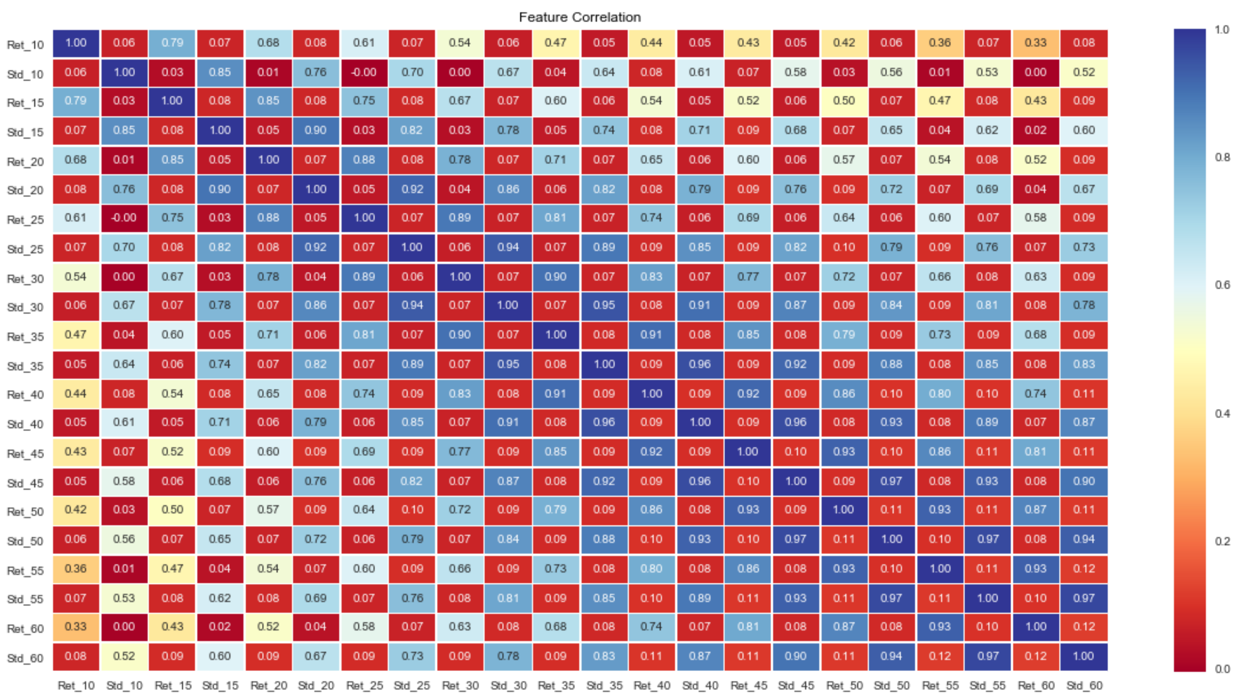
ASIDE: Correlation in Lagged Returns

In [27]:

```
# Create features (predictors) list
features_list = []
for r in range(10, 65, 5):
    data_advf['Ret_'+str(r)] = data_advf.Returns.rolling(r).sum()
    data_advf['Std_'+str(r)] = data_advf.Returns.rolling(r).std()
    features_list.append('Ret_'+str(r))
    features_list.append('Std_'+str(r))

# Drop NaN values
data_advf.dropna(inplace=True)
```

In [28]:

```
# Derive features correlation
import seaborn as sns
corrmat = data_advf.drop(['price', 'Returns'],axis=1).corr()

# Visualize feature correlation
fig, ax = plt.subplots(figsize=(20,10))
sns.heatmap(corrmat, annot=True, annot_kws={"size": 10}, fmt="0.2f", linewidths
ax.set_title('Feature Correlation', fontsize=12, color='black');
```



In [ ]:

Maximum Likelihood for Regression (Ordinary Least Squares)

Below work is illustration-only. Do not run OLS in your assignments and projects.

When the assumption of Normality of residuals holds: ϵ_t is *iid* $N(0, \sigma^2)$, **the linear regression** $y_t = \hat{\boldsymbol{\beta}} \boldsymbol{x_t} + \epsilon_t$ has MLE properties.

That means estimated coefficients $\hat{\boldsymbol{\beta}}$ are

- consistent (i.e., close to unknown true estimates β with low tolerance) and
- asymptotically efficient (i.e., their variance is known and minimised).

Estimates $\hat{\boldsymbol{\beta}}$ in fact, maximise the following **joint Normal** likelihood:

$$\mathbf{L} = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^T \exp -\frac{1}{2}\left[\frac{\epsilon_1^2}{\sigma^2} + \frac{\epsilon_2^2}{\sigma^2} + \cdots + \frac{\epsilon_T^2}{\sigma^2}\right]$$

Substituting $\epsilon_t = y_t - \hat{\boldsymbol{\beta}}\boldsymbol{x_t}$ and taking log gives for an individual observation -- I call this quantity a contribution of likelihood from an observation (data row values of features) at time *t*

$$\log L_t = -\frac{1}{2}\log 2\pi\sigma^2 - \frac{1}{2}\frac{(y_t - \hat{\boldsymbol{\beta}}\boldsymbol{x_t})^2}{\sigma^2}$$

Total log-likelihood for a regression model is the sum of contributions from each observation $\log \mathbf{L} = \sum_{t=1}^{T} L_t$.

Numerical MLE varies $\hat{\boldsymbol{\beta}}$ to maximise $\log \mathbf{L}$. This can be done by any non-specific optimisation routine, such as Excel Solver. It is clear to spot that log-likelihood is maximised by **minimising** the residual sum of squares

$$RSS = \sum_{t=1}^{T} \epsilon_t^2 = \sum_{t=1}^{T} (y_t - \hat{\boldsymbol{\beta}}\boldsymbol{x_t})^2$$

.

Residual sum of squares (RSS) is also known as the sum of squared residuals (SSR) or the sum of squared estimate of errors (SSE).

CAUTION OLS is an **invalid** model for binary dependent variable {0, 1}. Think about a change in MLE function for such variable.

In [13]:
```
# Regression from NUMPY library
reg_coef = np.linalg.lstsq(data[cols].values, data['return_sign'])[0]

# PREFER delegates to use STATSMOTELS
#import statsmodels.api as sm
```

```
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:2: FutureWarning:
`rcond` parameter will change to the default of machine precision times ``max
(M, N)`` where M and N are the input matrix dimensions.
To use the future default and silence this warning we advise to pass `rcond=No
ne`, to keep using the old, explicitly pass `rcond=-1`.
```

In [14]:
```
reg_coef
```

Out[14]:
```
array([ 1.13255353,  0.34409898, -2.87238464, -0.40975749, -1.38671844])
```

In [15]:
```
data['ols_pred'] = np.sign(np.dot(data[cols].values, reg_coef)) #dot product
```

In [16]:
```
data.head(15)
```

Out[16]:

Date	return	ret_1	ret_2	ret_3	ret_4	ret_5	return_sign	ols_pre
2010-10-08	0.011860	-0.008148	-0.007225	0.029947	-0.006245	-0.000305	1.0	-1.
2010-10-11	0.004631	0.011860	-0.008148	-0.007225	0.029947	-0.006245	1.0	1.
2010-10-12	0.004720	0.004631	0.011860	-0.008148	-0.007225	0.029947	1.0	-1.
2010-10-13	0.003553	0.004720	0.004631	0.011860	-0.008148	-0.007225	1.0	-1.
2010-10-14	-0.004370	0.003553	0.004720	0.004631	0.011860	-0.008148	-1.0	-1.
2010-10-15	0.106028	-0.004370	0.003553	0.004720	0.004631	0.011860	1.0	-1.
2010-10-18	0.026677	0.106028	-0.004370	0.003553	0.004720	0.004631	1.0	1.
2010-10-19	-0.016119	0.026677	0.106028	-0.004370	0.003553	0.004720	-1.0	1.
2010-10-20	0.000231	-0.016119	0.026677	0.106028	-0.004370	0.003553	1.0	-1.
2010-10-21	0.006582	0.000231	-0.016119	0.026677	0.106028	-0.004370	1.0	-1.
2010-10-22	0.000885	0.006582	0.000231	-0.016119	0.026677	0.106028	1.0	-1.
2010-10-25	0.006468	0.000885	0.006582	0.000231	-0.016119	0.026677	1.0	-1.
2010-10-26	0.003381	0.006468	0.000885	0.006582	0.000231	-0.016119	1.0	1.
2010-10-27	-0.003446	0.003381	0.006468	0.000885	0.006582	0.000231	-1.0	1.
2010-10-28	0.003446	-0.003446	0.003381	0.006468	0.000885	0.006582	1.0	-1.

Count number of predicted moves UP and DOWN

In [17]:
```
data['ols_pred'].value_counts()

#c.value_counts()[1] / (c.value_counts().sum()) #We were UP this percentage of
```

Out[17]:
```
-1.0    938
 1.0    819
Name: ols_pred, dtype: int64
```

False Negatives -- Type II Error

The probelm transpires: the model is likely to be bad at predicting negative returns, there are a lot of **false negatives** with -1.0 label.

In terms of generated asset path: in case of bad prediction of negative returns (moves down) we can observe the path drifting downwards and downwards. But that is for later.

Vectorised Backtesting = Rebalancing

'ols_pred'	'return'	Result P\&L
NEGATIVE 'ols_pred'	NEGATIVE 'return'	POSITIVE P\&L
NEGATIVE 'ols_pred'	POSITIVE 'return' (move up)	NEGATIVE P\&L, Loss
POSITIVE 'ols_pred'	NEGATIVE 'return'	NEGATIVE P\&L, Loss
POSITIVE 'ols_pred'	POSITIVE 'return' (move up)	POSITIVE P\&L

Exercise care with Dr Hilpisch code, particularly on 'vectorised backtesting' where correctly predicted negative sign translates into the Positive P\&L -- that assumes daily rebalancing (betting) rather than replication of the actual asset path.

In [56]:
```
data['ols_pred_backtest'] = data['return'] * data['ols_pred']
```

So above multiplication represents a sequence of daily bets, based on the sign (up/down move) predicted from past returns.

The actual return that realises today t, is \% P\&L that one makes (loses) on the bet of $100, for example if return today is POSITIVE 0.018 and predcited sign was POSITIVE, then P\&L is 1.18%.

$$Return_{pred} = Z^* \times \sigma$$

Assume we want to have more model-like prediction. Then, we will use standard deviation, which gives some measure of randomness.

- We use std dev from the dataset but that can be estimated from any prevoius holdout period/window. Such backtesting requires past data but not regular daily update of return -- the latter is historical backtesting.
- $Z^* = \pm 0.7$ of the standard deviation translates into betting \%70 of one sigma -- the Actual P\&L which would still depend on actual return. But we can plot cumulative P\&L from 'ols_pred_move' to see if matches with the asset path. It also is possible to do P\&L Attribution test on such std dev model:

'ols_pred_move' is in effect, our **Theoretical P\&L**

'return' is the **Actual P\&L**

- Why z-score of ± 0.7? This is because empirical asset returns are not well-Normal and between ± 2 standard deviation, but their density/histogram is high-peak (high mode). If you standardise returns $z_t = (r_t - \mu)/\sigma$ the histogram of z_t will have bars within ± 0.7.
- Instead of ± 0.7 and to provide negative outcomes, we can use simulated values of Random Normal ϕ which can be positive or negative.

In [58]:
```
stdev = data['return'].std()  #from a whole dataset but you can introduce Trair

data['ols_pred_move'] = 0.7 * stdev * (data['ols_pred'])
data['ols_pred_move_FUDGED'] = 0.7 * stdev * ( - data['ols_pred']) #PREDICTION

data['ols_pred_ABSOLUTE'] = abs(data['return']) * data['ols_pred']
```

In [53]:
```
data.head(20)
```

Out[53]:

Date	return	ret_1	ret_2	ret_3	ret_4	ret_5	return_sign	ols_pre
2010-10-08	0.011860	-0.008148	-0.007225	0.029947	-0.006245	-0.000305	1.0	-1
2010-10-11	0.004631	0.011860	-0.008148	-0.007225	0.029947	-0.006245	1.0	1
2010-10-12	0.004720	0.004631	0.011860	-0.008148	-0.007225	0.029947	1.0	-1
2010-10-13	0.003553	0.004720	0.004631	0.011860	-0.008148	-0.007225	1.0	-1
2010-10-14	-0.004370	0.003553	0.004720	0.004631	0.011860	-0.008148	-1.0	-1
2010-10-15	0.106028	-0.004370	0.003553	0.004720	0.004631	0.011860	1.0	-1
2010-10-18	0.026677	0.106028	-0.004370	0.003553	0.004720	0.004631	1.0	1
2010-10-19	-0.016119	0.026677	0.106028	-0.004370	0.003553	0.004720	-1.0	1
2010-10-20	0.000231	-0.016119	0.026677	0.106028	-0.004370	0.003553	1.0	-1
2010-10-21	0.006582	0.000231	-0.016119	0.026677	0.106028	-0.004370	1.0	-1
2010-10-22	0.000885	0.006582	0.000231	-0.016119	0.026677	0.106028	1.0	-1
2010-10-25	0.006468	0.000885	0.006582	0.000231	-0.016119	0.026677	1.0	-1
2010-10-26	0.003381	0.006468	0.000885	0.006582	0.000231	-0.016119	1.0	1
2010-10-27	-0.003446	0.003381	0.006468	0.000885	0.006582	0.000231	-1.0	1
2010-10-28	0.003446	-0.003446	0.003381	0.006468	0.000885	0.006582	1.0	-1
2010-10-29	-0.007950	0.003446	-0.003446	0.003381	0.006468	0.000885	-1.0	-1
2010-11-01	0.002124	-0.007950	0.003446	-0.003446	0.003381	0.006468	1.0	-1
2010-11-02	0.000979	0.002124	-0.007950	0.003446	-0.003446	0.003381	1.0	-1
2010-11-03	0.007408	0.000979	0.002124	-0.007950	0.003446	-0.003446	1.0	1
2010-11-04	0.006582	0.007408	0.000979	0.002124	-0.007950	0.003446	1.0	1

In [59]:

```
data[['return', 'ols_pred_ABSOLUTE', 'ols_pred_move']].cumsum().apply(np.exp).p
```

- 'ols_pred_ABSOLUTE' and 'ols_pred_move' from std dev do not reproduce the asset price path at all. This plot reveals the poor quality of prediction. **FINDING:** correct negatives plus false negatives make for a bad prediction. OLS seems to produce a lot of negative predicitons which do not realise.

In [60]:
```
data[['return', 'ols_pred_backtest', 'ols_pred_move_FUDGED']].cumsum().apply(np
```

INTERIM QUESTIONS

- Why 'pred_return' does not match the asset path ('return'). ANSWER: because our prediction is not a prediction but Daily Rebalanced P\&L, where correctly predicted Negative move results in Positive P\&L increment.

- We have done **worse** than buy and hold: simply investing $100 and holding the position.

FUDGED inverted sign applied to OLS prediction worked better at reproducing the asset price path. Whoa!

ANSWER: that's likely because of cancelled false negatives, ie, OLS produces a lot of negative predicitons which do not realise.

Remember OLS is an **invalid** regression model for binary dependent variable {0, 1}. **DO NOT** run OLS in your ML assignments and projects.

Logistic Classifier (in detail) // Support Vectors (comparison)

In [35]:
```
from sklearn import linear_model

from sklearn.svm import SVC  #you can import ANY OTHER CLASSIFIER and proceed i
```

In [36]:
```
lm = linear_model.LogisticRegression(C = 1e5)

svcm = SVC(C = 1e5, probability=True)
```

The points below can be considered/have been asked for analysis in an ML Assignment (Exam 3), *in the past*. The one and only hyperparameter of interest is C, which relates to penalisation or soft/hard margins.

- **Logistic Classifier**

C= 1e5 implies nearly no L2 Penalty because parater set in inverse. Try stronger penalisation C = 0.01 to 0.5

L1 vs L2 type penalty can be investigated but the answer is: impact of L1 penalisation is very strong on zero-ing out the coefficients.

- **SVM Classifier**

C= 1e5 **NOT** inverse for Support Vector Machines. This is Hard Margins.

Prediction with Soft Margins is supposed to work better on time series. Also, SVM requires much selectivity in features choosen because it becomes noticeably slow and because the separability (of classes) is problematic across multiple dimensions (features) at the same time.

Maximum Likelihood in Binomial Classification (Logistic Classifier/Regression)

Let's find an analytical solution to the maximum likelihood estimation problem for a mix of independent identically distributed Bernoulli draws in the regression (prediction) setting.

Each {0,1} outcome has a set of its own explanatory variables $\boldsymbol{X_i}$.

$$y_i|\boldsymbol{X_i} \ \sim \ \text{Bernoulli}(p_i)$$

$$\mathbb{E}\left[y_i|\boldsymbol{X_i}\right] \ = \ p_i$$

$$\Pr\left(y_i = 1, 0\right) \ = \ \begin{cases} p_i \\ 1 - p_i \end{cases}$$

$$f(y_i; p_i) \ = \ p_i^{y_i}(1-p_i)^{1-y_i}$$

Each outcome is determined by the probability of default p_i, which is unobserved (latent) in the regression model.

Logistic ML Part 1

The Bernoulli density above translates to the log-likelihood (contribution from one observation i).

$$\log L_i = \log f(y_i; p_i) = y_i \log p_i + (1-y_i)\log(1-p_i)$$

The joint log-likelihood for multiple events observed together and treated as independent, is given by the Product Rule of probabilties:

$$\begin{aligned} \log f(y_1, y_2, \ldots, y_N) &= \log \prod_{i=1}^{N_{obs}} f(y_i; p_i) \\ &= \sum_{i=1}^{N_{obs}} \log f(y_i; p_i) \end{aligned}$$

$$\log L = \sum_{i=1}^{N_{obs}} \left[y_i \log p_i + (1-y_i)\log(1-p_i)\right] \tag{1}$$

y_i is known from dataset.

p_i requires inverse of the link $p_i = g^{-1}(\boldsymbol{X_i}\boldsymbol{\beta}')$

Logistic ML Part 2

We can express Bernoulli density for a random variable $y = \{1, 0\}$ in a more canonical form -- as a member of the Exponential family of distributions.

$$f(y;p) = p^y(1-p)^{1-y} = \exp\left[y\log\left(\frac{p}{1-p}\right) + \log(1-p)\right]$$

Choice of a link function is \underline{the same for any categorical $\boldsymbol{Y}$}

$$g(p) = \log\left(\frac{p}{1-p}\right)$$

This is a logit function (different from logistic function!), which can be read as the ``log of odds''. The inverse of logit function is logistic function, which we are interested in.

$$p = \frac{1}{1+e^{-g}}$$

Under the hood: the choice of Link Function

To adapt the linear regression to non-linear output variable $y_i = 0, 1$ or Binomial $y_i = 0, 1, 2, 3, 4, \ldots$ or in general case to probability we introduce a non-linear link funciton $y = g(p)$

$$\text{Dependent Varialbe } = \text{ Link Function}\,(\,\text{Probability}\,)$$

$$\begin{aligned}
\mathbf{Y} &= \boldsymbol{X}\boldsymbol{\beta}' + \boldsymbol{\epsilon} && (2)\\
PD \equiv \mathbb{E}[\mathbf{Y}|\mathbf{X}] &= \boldsymbol{X}\boldsymbol{\beta}' && (3)\\
g(p) &= \boldsymbol{X}\boldsymbol{\beta}' \qquad \text{and} && (4)\\
p &= g(\boldsymbol{X}\boldsymbol{\beta}')^{-1} && (5)
\end{aligned}$$

$$\text{Probability } = \textbf{ Inverse Link Function}\,(\,\text{Dependent Variable}\,)$$

- This covers default/no default $y_i = \{1, 0\}$ and ordinal ratings $y_i = 1, 2, 3, 4, 5$. In fact, response variable $\boldsymbol{Y}$ can have any distribution from Exponential family (quasi MLE).
- Linear part $\boldsymbol{\beta X}$ is linked to a non-linear, latent variable (probability).

$$p = g(\boldsymbol{X}\boldsymbol{\beta}')^{-1}$$

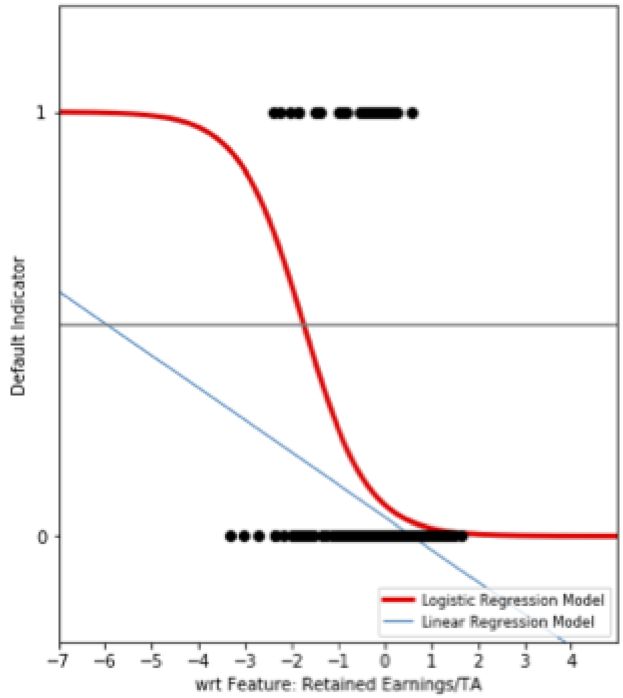
A link function is the clever bit that allows to convert a categorical event indicator $y_i = \{1, 0\}$ to the probability p_i

$$p_i = g^{-1}(\boldsymbol{X_i}\boldsymbol{\beta}')$$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \Rightarrow \begin{pmatrix} g(\boldsymbol{X_1}\boldsymbol{\beta}')^{-1} \\ \vdots \\ g(\boldsymbol{X_n}\boldsymbol{\beta}')^{-1} \end{pmatrix} \Rightarrow \begin{pmatrix} \text{Prob}_1 \\ \vdots \\ \text{Prob}_n \end{pmatrix}$$

Let's make a step forward and say that the inverse of our link will be **the logistic sigmoid** function,

$$p = \frac{1}{1+e^{-\boldsymbol{X}\beta'}} = \frac{e^{\boldsymbol{X}\beta'}}{1+e^{\boldsymbol{X}\beta'}}$$



In [37]:
```
# use data_adv copy of the dataset

lm.fit(data_adv[cols], data_adv['return_sign'])
```

Out[37]:
```
LogisticRegression(C=100000.0, class_weight=None, dual=False,
                   fit_intercept=True, intercept_scaling=1, l1_ratio=None,
                   max_iter=100, multi_class='auto', n_jobs=None, penalty='l
2',
                   random_state=None, solver='lbfgs', tol=0.0001, verbose=0,
                   warm_start=False)
```

In [41]:
```
svcm.fit(data_adv[cols], data_adv['return_sign'])
```

Out[41]:
```
SVC(C=100000.0, break_ties=False, cache_size=200, class_weight=None, coef0=0.
0,
    decision_function_shape='ovr', degree=3, gamma='scale', kernel='rbf',
    max_iter=-1, probability=True, random_state=None, shrinking=True, tol=0.00
1,
    verbose=False)
```

In [29]:
```
def logistic_sigmoid(xb):
    return (1 / (1 + np.exp(-xb)))
```

In [119…
```
#Procedure RELIES X_Features, Y_Response variables to be existing
def logistic_plot(X_min, X_max, FeatureName, FeatureBetaIdx):

    plt.clf() #clears the figure drawing space, nothing to do with classifier!
    fig, ax = plt.subplots(figsize=(18,10))  #fig = plt.figure(figsize=(18,10))

    # 1. Plot two clusters of observations at Y={-1,1} on a scatter
    ax.scatter(data_adv[FeatureName], data_adv['logit_pred'], c=(data_adv['logi

    # 2. Plot CALIBRATED sigmoid function -- with the correctly picked coeffie
    X_Sim = np.linspace(X_min, X_max, 100) #fill in values for the range of Axe
```

```
    Y_Loss = logistic_sigmoid(X_Sim * lm.coef_[0,FeatureBetaIdx] + lm.intercept
    # Y_Loss = logistic_sigmoid(X_Sim * logit.coef_[0,FeatureBetaIdx] + logit.i
    ax.plot(X_Sim, Y_Loss, color='red', linewidth=3) # plot sigmoid in Red

    plt.ylabel('Beta coeff value for sigmoid: -6.138', fontsize=22) # also ax.s
    plt.xlabel('wrt Feature: ' + FeatureName, fontsize=22)
    plt.xticks(np.linspace(X_min, X_max, num=10), fontsize=14)
    plt.yticks([-1, 1], fontsize=14)
    plt.ylim(-1.1, 1.1)
    plt.xlim(X_min, X_max) #Axe X range
    plt.legend(('Logistic Regression',),
           loc="lower right", fontsize=14)
    #plt.show()
    return ax
```

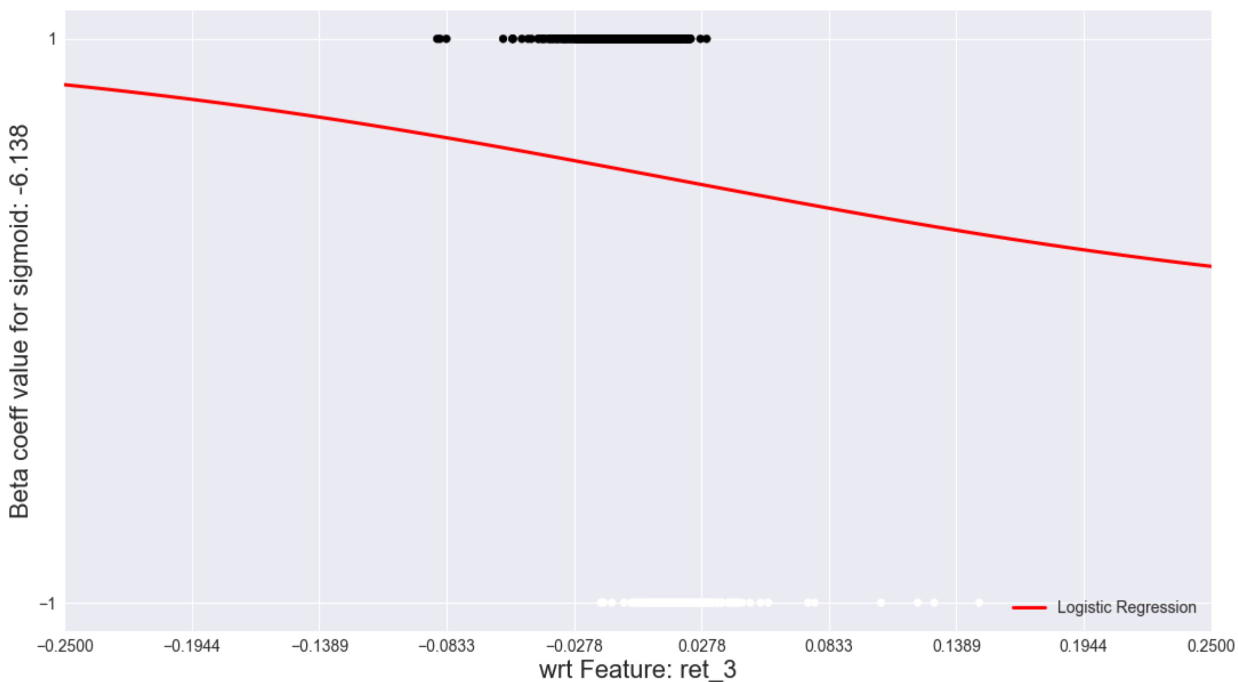
In [123…

```
# Error message below will remain due to difference in data types passed into t
# TypeError: 'float' object cannot be interpreted as an integer

logistic_plot(-0.25, 0.25, 'ret_3', 2)

# 'ret_1' has lm.coef_[0,0]

# 'ret_3' has lm.coef_[0,2], the coefficient for our GOOG is -6.13818103, the l
```

Out[123]:

```
<matplotlib.axes._subplots.AxesSubplot at 0x7fe0dc157950>
```

```
<Figure size 432x288 with 0 Axes>
```



Above plot implements the inverse of our link will be **the logistic sigmoid** function,

$$p = \frac{1}{1 + e^{-X\beta'}}$$

In [122…

```
data_adv['logit_pred'] = lm.predict(data_adv[cols])
data_adv['logit_pred_backtest'] = data_adv['return'] * data_adv['logit_pred']
```

```
data_adv['svm_pred'] = svcm.predict(data_adv[cols])
data_adv['svm_pred_backtest'] = data_adv['return'] * data_adv['svm_pred']
```

In [62]:
```
data_adv.head(20)
```

Out[62]:

Date	return	ret_1	ret_2	ret_3	ret_4	ret_5	return_sign	logit_p
2010-10-08	0.011860	-0.008148	-0.007225	0.029947	-0.006245	-0.000305	1.0	
2010-10-11	0.004631	0.011860	-0.008148	-0.007225	0.029947	-0.006245	1.0	
2010-10-12	0.004720	0.004631	0.011860	-0.008148	-0.007225	0.029947	1.0	
2010-10-13	0.003553	0.004720	0.004631	0.011860	-0.008148	-0.007225	1.0	
2010-10-14	-0.004370	0.003553	0.004720	0.004631	0.011860	-0.008148	-1.0	
2010-10-15	0.106028	-0.004370	0.003553	0.004720	0.004631	0.011860	1.0	
2010-10-18	0.026677	0.106028	-0.004370	0.003553	0.004720	0.004631	1.0	
2010-10-19	-0.016119	0.026677	0.106028	-0.004370	0.003553	0.004720	-1.0	
2010-10-20	0.000231	-0.016119	0.026677	0.106028	-0.004370	0.003553	1.0	
2010-10-21	0.006582	0.000231	-0.016119	0.026677	0.106028	-0.004370	1.0	
2010-10-22	0.000885	0.006582	0.000231	-0.016119	0.026677	0.106028	1.0	
2010-10-25	0.006468	0.000885	0.006582	0.000231	-0.016119	0.026677	1.0	
2010-10-26	0.003381	0.006468	0.000885	0.006582	0.000231	-0.016119	1.0	
2010-10-27	-0.003446	0.003381	0.006468	0.000885	0.006582	0.000231	-1.0	
2010-10-28	0.003446	-0.003446	0.003381	0.006468	0.000885	0.006582	1.0	
2010-10-29	-0.007950	0.003446	-0.003446	0.003381	0.006468	0.000885	-1.0	
2010-11-01	0.002124	-0.007950	0.003446	-0.003446	0.003381	0.006468	1.0	
2010-11-02	0.000979	0.002124	-0.007950	0.003446	-0.003446	0.003381	1.0	
2010-11-03	0.007408	0.000979	0.002124	-0.007950	0.003446	-0.003446	1.0	
2010-11-04	0.006582	0.007408	0.000979	0.002124	-0.007950	0.003446	1.0	

In [65]:

```
data_adv[['return', 'logit_pred_backtest', 'svm_pred_backtest']].cumsum(
          ).apply(np.exp).plot(figsize=(10, 6));
```

In [ ]:
```
# Instead of 0.7 we should use simulated Normal Random Variable
#or we end up with just gives exponentially rising plots, no down moves predict

#data_adv['logit_pred_move'] = 0.7 * stdev * (data_adv['logit_pred'])
#data_adv['svm_pred_move'] = 0.7 * stdev * (data_adv['svm_pred'])
```

Further Steps

- Do not rush to a quick conclusion as to which is better Logistic Classifer/SVM/Decision Tree Regressor. That would be dependent on data history, frequency, historical regime (eg high volatility) and the model itself. SVM estimation with more than 2-3 features becomes very slow.

- Consider the accuracy and pattern of prediction *within each class*. This is necessary and can be done in the form of

  1) common tools to check the output of classifiers: **confusion matrix** and **area under the ROC curve**;

  2) investigating the Recall for negative moves, ie, False Negatives problem.

- Rethink of advantages and disadvantages of moving onto multinomial classification $\{-1, 0, 1\}$. For example what would you do if most of observations (daily return) will fall into category of 'no move'.

END OF DEMONSTRATION

In [ ]:

More Features from Price Information

Below is an initial set of features, which econometricans typically utilise, and a good starting-level textbook is *Forecasting Methods and Applications* by Hyndman, Makridakis, and Wheelwright. They dilligently consider lags, each relevant test such as F-statistic, ARIMA and how to implement seasonality. However that is typical econometrics aimed at quarterly, cyclically-dependent indicators such as GDP.

It is possible to get professional and utilise a Python wrapper for something like TA-Lib. However, we would like to be able to compupte technical analysis indicators from the first princiles, whenever possible.

There are also a variety of estimators for volatility, which unlilike the EWMA and GARCH -- focus on capturing the realised variance and the ghi -- some you are very familiar with, such as EWMA and GARCH, however

In [ ]:
```
def createFeatures(df):
    '''
    Below code features to an existing data frame
    '''
#    df = df.rename(columns={df.columns[0]: 'price'})
#    df['return'] = df.pct_change()
#    df['log_return'] = np.log(df['price']/df['price'].shift(periods=1))
#    df['sign'] = df['log_return'].apply(lambda x: 0 if x<0 else 1)
    df['return_1d']= df['return'].shift(periods=1)
    df['return_2d']= df['return'].shift(periods=2)
    df['return_5d']= df['return'].shift(periods=5)
    df['momentum_1d']=df['price']-df['price'].shift(periods=1)
    df['momentum_2d']=df['price']-df['price'].shift(periods=2)
    df['momentum_5d']=df['price']-df['price'].shift(periods=5)
    df['MA_5d']=df['price'].rolling(5).mean()
    df['MA_10d']=df['price'].rolling(10).mean()
    df['MA_20d']=df['price'].rolling(20).mean()
    df['MA_50d']=df['price'].rolling(50).mean()
    df['EMA_5d']=df['price'].ewm(5, adjust=False).mean()
    df['EMA_10d']=df['price'].ewm(10, adjust=False).mean()
    df['EMA_20d']=df['price'].ewm(20, adjust=False).mean()
    df['EMA_50d']=df['price'].ewm(50, adjust=False).mean()
    return df
```

In [ ]:

In [ ]: