Interest Rate Markets – Multiple Curves

In this lecture. . .

- tenor basis effect
- firm knowledge of FRA, swap maths (under dual-curve)
- update on post-LIBOR world of RFRs
- LOIS spread: evidence and modelling
- Curve bootstrapping (stripping) and interpolation
- Tenor basis effect and its link to credit

By the end of this lecture you will:

- understand why, and in what sense there are multiple curves
- develop awareness about changes to LIBOR rates and migration to new RFR benchmarks
- understand that different tenors produce different curves
- get familiar with classical curve stripping
- consider interpolation and its impact on forward curve

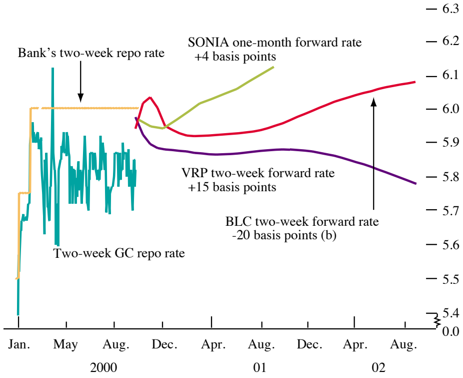
Introduction

Great credit crunch 2007-09 and LIBOR-rigging scandal changed the textbook behaviour of interest rates.

First, the liquidity crisis widened the basis between similar-purpose rates (1M LIBOR vs 1M OIS). Second, it triggered the reform towards transaction-based benchmarks as opposed to quotation-based LIBOR surveys.

The new RFRs do not mitigate **the basis risk** between long-term assets (loans that banks extend) and short-term liabilities (funding).

Historic Curves



A variety of curves was always available. Today's Fed Funds rate includes GC repos (O/N) and **resembles the old BOE GC rate** (two-week term).

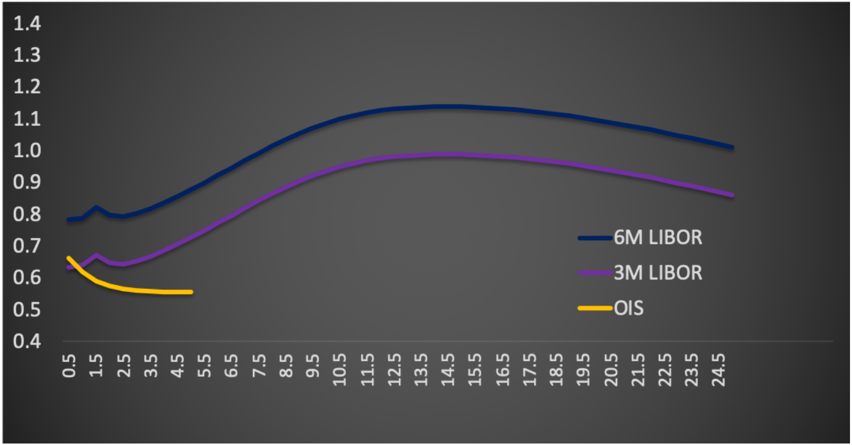
Source: Inferring market interest rate expectations, BOE Quarterly, Nov 2000

LIBOR Curve & Discounting Curve

- Bank Liability Curve (BLC) to 25Y tenor, from LIBOR-linked instruments such as FRA and IRS. The rates equivalent to yields on AA-rated bonds for lending/borrowing banks.

- OIS Curve is available for up to 5Y maturity. Traded OIS swaps, cashflow computed by the geometric averaging of future O/N rates.

  Implied *OIS discounting*: subtract LOIS spread from LIBOR curve (actual or simulated) to get a discounting curve.



Source: Bank of Engand curves as of 30 Oct 2019. BLC assumed as L^{6M}. Proceed by Richard Diamond

Short-term rate	$r(t)$
Term rates	$L^{1M}(0, 1M)$, L^{3M}, L^{6M}
Forward term rates	$L^{1M}(T_{i-1}, T_i)$, L^{3M}, L^{6M}

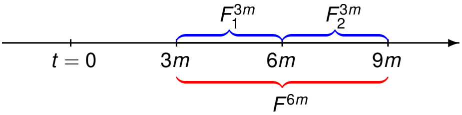
Money Markets reflect instruments for all these rates. In the past, it was possible to narrow down to **one well-defined spot curve** which gave both L^{3M} and L^{6M}.

Distinction: Tenor Basis

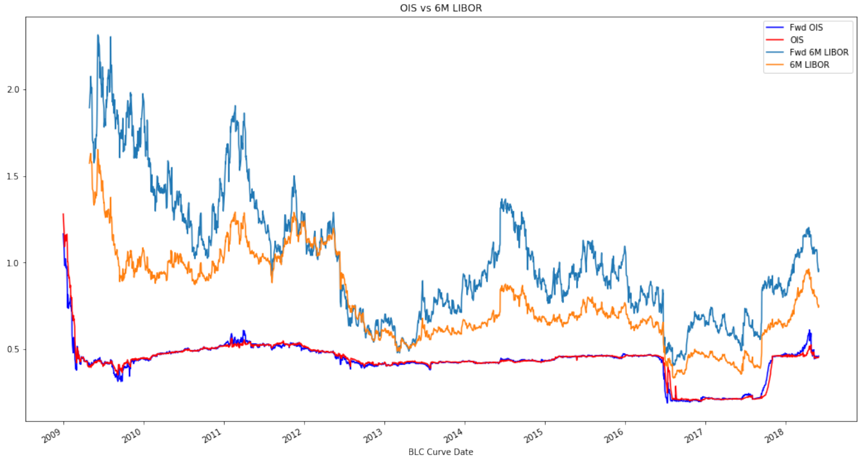
Before **the credit crunch**, interest rates followed textbook behaviour, which allowed to construct one well-defined curve: compounding two consecutive 3M LIBOR rates yielded 6M LIBOR rate

The liquidity crisis widened the basis between previously-near rates: $\left(F^{3M} + \text{spread}\right)$

$$(1 + 0.25F_1^{3M}) \times (1 + 0.25F_2^{3M}) = 1 + 0.5F_1^{6M}$$



where the rates are [3M,6M], [6M, 9M] and [3M, 9M] respectively.

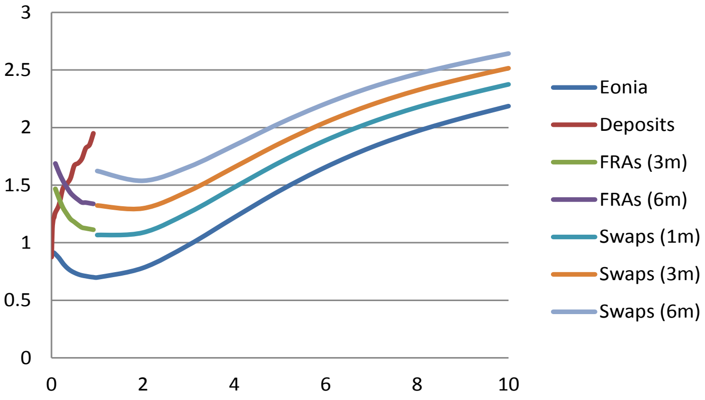


Source: BOE/Bloomberg data processed by Richard Diamond

Comparison of Bank of England (inst fwd) GBP 6M LIBOR vs OIS since 2009. Note **2016 Brexit Referendum** impact

Compounded OIS, or compounded 3M LIBOR **will not produce** 6M LIBOR.

Different tenor basis for a funding rate produces different curve. Eg, in Bloomberg terminal you choose tenor, and curve will be built from **cash rates/spot FRAs/swap rates**.

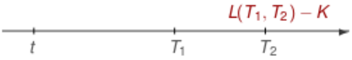


EUR multiple curves.

Source: Rates Modeling After the Credit Crunch, F. Mercurio, 2012

Curve is modelled as **a sequence of forward-spanning rates** $F(0; T_{i-1}, T_i)$ or, in instantaneous terms $f(t, T_i)$.

- HJM and LMM SDEs are for *df*



$$\mathrm{FRA}(0; T_{i-1}, T_i) = \frac{1}{\tau_i}\left(\frac{1}{Z(T_{i-1}, T_i)} - 1\right).$$

rather than overlapping discount factors $Z(0, T_1), Z(0, T_2)$ ....

- for which there will be a problem to evolve SDEs for *dZ*

Distinction: Index Curve/Discounting Curve

a. Index curve – forward-spanning LIBOR rates.

- Rates f_j from IBOR fixings, Futures, FRAs, and Swaps.
- 'LIBOR curve' of different tenor basis L^{3M}, L^{6M} do not match without a basis spread

b. Discounting curve – implied by the Index curve $Z(T_i, T_{i+1}) = 1/(1 + f_i\tau_i)$ past formula.

- Discounting Factors D_i from OIS swaps curve (up to 5Y). After LOIS spread.
- New RFRs will eventually have term rates.

In the past, Money Markets were represented by one index curve.

Swap. Vanilla Interest Rate Swap (under Dual Curve)

PV for the fixed leg with fixed rate k – Payer PAYS. $k\,\tau_i$ paid at T_i

$$\mathrm{PV}_{\text{fixed}} = k\,N \sum_{i=1}^{n} \tau_i\, D_i$$

PV for the floating leg – Payer RECEIVES. Rate plus **basis spread**

$$\mathrm{PV}_{\text{float}} = N \sum_{j=1}^{m} \tau_j \left(f_j + s\right)\, D_j$$

Swap contracts are equivalent to a spanning series of FRAs: each FRA beginning when the previous one matures.

GBP markets refer to OIS swaps and 6M LIBOR swaps.

$$\begin{aligned} PV_{\text{float}} &= 1 - Z(0, T_n) \\ PV_{\text{swap}} &= PV_{\text{fixed}} - PV_{\text{float}} \end{aligned}$$

$$r_s^0 = \frac{1 - Z(0;5)}{\sum_{j=1}^{10} \tau_j \, Z(0;\, T_j)}$$

Today's swap rate obtained from $PV_{\text{fixed}} = PV_{\text{float}}$, for a typical 5-year swap referencing 6M LIBOR rate on *rhs*.

The sum of **all floating legs** totals to $1 - Z(0;5)$.

Denominator $\sum_{j=1}^{n} \tau_j \, D(T_j)$ is called the level or DV01 of the swap.

τ_j year fractions or 'coverages' do not need to be equal; can be based on calendar times $T_1, T_2, T_3, \ldots$

Forward-starting swap rate (you might need this for swaptions)

$$r_s^t = \frac{Z(t, T_0) - Z(t, T_n)}{\sum_{j=1}^{n} \tau_j Z(t, T_j)}$$

Swap valuation principle becomes

$$\mathrm{PV}_{\text{float}} \quad = \quad Z(0, T_{\text{start}}) - Z(0, T_{\text{mature}})$$

$$r_s(T_{\text{start}}, T_{\text{mature}}) = \frac{1 - Z(T_{\text{start}}, T_{\text{mature}})}{\sum_{j=1}^{n} \tau_j Z(T_{\text{start}}, T_j)}.$$

IRS Pricing under multiple curves

Swap rate	Formulas
OLD	$\frac{\sum_{k=1}^{b} \tau_k P(0,T_k) F_k(0)}{\sum_{j=1}^{d} \tau_j^S P(0,T_j^S)} = \frac{1 - P(0,T_d^S)}{\sum_{j=1}^{d} \tau_j^S P(0,T_j^S)}$
NEW	$\frac{\sum_{k=1}^{b} \tau_k P_D(0,T_k) L_k(0)}{\sum_{j=1}^{d} \tau_j^S P_D(0,T_j^S)}$

Now formulae on Forward LIBOR rates $L_k(t) = \mathrm{FRA}(t; T_{k-1}, T_k)$.
Can't return to a single spot curve, which gives discounting factors for both, L^{3M} and L^{6M}.

In practice, swap valuation give us classic bootstrap for $L_k(0)$.

Source: IR Modeling After the Credit Crunch, F. Mercurio, 2012

GBP Money Markets - Swaps segment

ICE Swap Rate Historical Rates

GBP Rates 1100 30-Oct-2019	
1 Year	No Publication
2 Years	0.798
3 Years	0.799
4 Years	0.806
5 Years	0.818
6 Years	0.83
7 Years	0.846
8 Years	0.864
9 Years	0.884
10 Years	0.904
12 Years	0.938
15 Years	0.973
20 Years	1.003
25 Years	1.008
30 Years	1.011

Source: ICE Swap Rate

`https://www.theice.com/marketdata/reports/180`

USD Money Markets (1997)

	Period	Rate	Maturity Date
LIBOR	o/n	5.59375	9/10/97
	1m	5.625	10/11/97
	3m	5.71875	8/1/98
Futures	Oct-97	94.27	15/10/97
	Nov-97	94.26	19/11/97
	Dec-97	94.24	17/12/97
	Mar-98	94.23	18/3/98
	Jun-98	94.18	17/6/98
	Sep-98	94.12	16/9/98
	Dec-98	94	16/12/98
Swaps	2	6.01253	
	3	6.10823	
	4	6.16	
	5	6.22	
	7	6.32	
	10	6.42	
	15	6.56	
	20	6.56	
	30	6.56	

Source: Interest Rate Models - Lectures, University of Munich, D. Filipovic

FRA Discrete forward rate $\mathrm{FRA}(t, T_1, T_2)$ which is **a traded quantity** – a strike of Forward Rate Agreement written today at par

$$[f_1 - \mathrm{FRA}(t, T_1, T_2)] = 0$$

at time t agree to

- pay \$1 at T_1 to receive $e^{\int_{T_1}^{T_2} f(t,T)dT}$ at T_2
- pay $e^{-\int_{T_1}^{T_2} f(t,T)dT}$ at T_1 to receive \$1 at T_2.

at its reset time T_1 that FRA rate will coincide with LIBOR fixing $L(T_1, T_2)$.

Futures Futures rates are slightly *larger* than than forward rates – that difference is called the convexity adjustment.

$$\text{fwd} = \text{futures rate} - \frac{1}{2}\sigma^2\tau^2$$

$$100 \times (1 - F(t_0, T, T + \delta)) \quad \text{where } \delta = 91/360$$

Three Month Sterling (Short Sterling) Future is interest rate futures contract that settles on LIBOR 3M. Currently traded on ICE. Options available.

One can even trade CFD on it!

For USD, there are one-months and three-months SOFR futures.

LIBOR-rigging scandal

> "duuuude... what's up with ur guys 34.5 3m fix... tell him to get it up!"
> (A Barclays trader)

In June 2012, Barclays PLC admitted it had manipulated LIBOR to gain profits and/or limit losses from derivative trades. In addition, between 2005 and 2009 derivatives traders made a total of 257 documented requests to fix Libor and Euribor rates, particularly to dampen media comments about the firm's viability during the crisis.

In settling the firm agreed to pay $450 million in fines.

BBC Timeline: Libor-fixing scandal
`https://www.bbc.co.uk/news/business-18671255`

FT ongoing coverage `https://www.ft.com/libor-scandal`

London-based USD LIBOR is offshore for the US. It follows the City tradition to reflect **unsecured borrowing**.

ICE LIBOR Historical Rates

Tenor	Publication Time*	USD ICE LIBOR 30-Oct-2019
Overnight	11:55:05 AM	1.80438
1 Week	11:55:05 AM	1.68000
1 Month	11:55:05 AM	1.78138
2 Month	11:55:05 AM	1.85900
3 Month	11:55:05 AM	1.90913
6 Month	11:55:05 AM	1.91950
1 Year	11:55:05 AM	1.97988

Source: `https://www.theice.com/marketdata/reports/170`

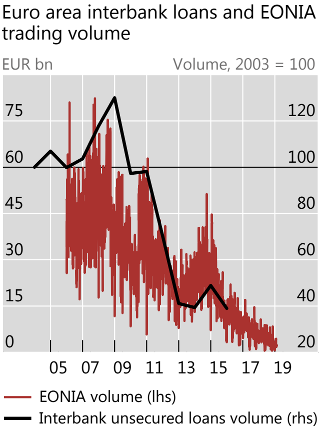
Methodology and Transition: `https://www.theice.com/iba/libor`

NEW US Money Markets

FRBNY publishes **SOFR** a broad measure of the overnight cost of borrowing cash collateralized by Treasures.
`https://apps.newyorkfed.org/markets/autorates/SOFR`

US money market volume shifted to short-term and refers to **Fed funds and reverse repos** rate.

- Banks tilted their funding mix towards less risky sources of wholesale funding (GC repos).
- For example, reverse repo of US Treasuries can be used to cover a short position in the stock.
- SOFR is like an old Bank of England two-week GC (Gilts) repo rate!



Core bank funding before and after the crunch (US Money Markets)
Source: *Beyond LIBOR*, BIS Quarterly March 2019, Schrimpf & Sushko

New Risk-Free Rates I

- The new risk-free rates (RFRs) aim to provide robust and accurate representation of interest rates in core money markets. Derived from actual transactions in **active** and liquid markets.

  LIBOR was constructed from a small panel of large banks reporting non-binding quotes. Made an ample scope to manipulate LIBOR submissions.

- Asset-Liability Management may **still** require benchmarks (and numeraires) that provide closer match to marginal funding costs (6M, 12M, ...).
  - RFRs or term rates linked to them via fixed spread are unlikely to deliver.
  - Future **segmentation** of Money Markes remains unclear.

New Risk-Free Rates II

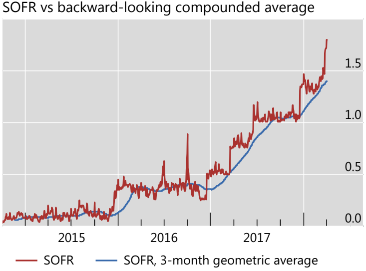
The main problem for **RFRs** is their use as benchmark for term **lending and funding**. The basis risk between long-term assets (loans that banks extend) and short-term liabilities (funding) has to be mitigated.

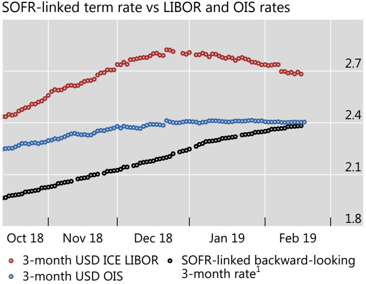
Adding fixed spread for each term is not a solution: those spreads will change daily will be as guesswork as old LIBOR submissions.

Being **reference rates** for financial contracts that extend beyond traded maturities is also problematic: requires longer-term SOFR swaps, futures (now to one-month and three-months only) and further derivatives referenced to RFRs (FRA-like?).

New Risk-Free Rates III (RFR-linked Term Rates)

Computation of cashflows likely to be based on **backward-looking** compounded rates.





Towards SOFR Term Rates [NOT Forward Term Rates]
Source: *Beyond LIBOR*, BIS Quarterly March 2019, Schrimpf & Sushko

New Risk-Free Rates IV (Capital Requirements)

Interbank lending is sparse, and has been such even before the credit crunch (longer LIBOR). However, benchmark curve (index curve, LIBOR curve) is necessary to price many traded derivatives.

Banks have a strong incentive to deposit extra cash at CB instead of lending to others: reserves contribute to the high-quality liquid asset requirement (HQLA). **Banks don't deposit with one another.**

Short-term unsecured wholesale funding is unattractive under the Liquidity Coverage Ratio.

Banks have been **turning to non-banks** to source unsecured term funding. Attempt to **lengthen their funding maturities** is incentivised by the Net Stable Funding Ratio. Consider all that push for issuance of subordinate debt, AT1 capital/CoCos.

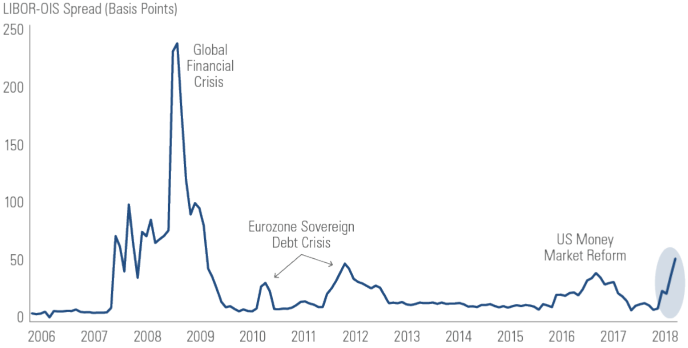
OIS Spread over LIBOR (LOIS)

Has an established meaning of credit risk, natural credit spread.

3M LIBOR − 3M OIS

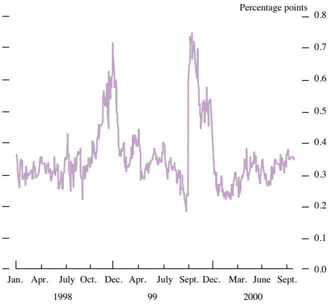
- LIBOR indicates money-market rates for **actual loans** between banks for up to 18M.
- OIS targets the Federal Funds Rate, an average over the period. Reflects uncollaterised borrowing overnight.

The spread for LIBOR fixings vs. OIS prices reveals the short-term **credit risk in the financial system**.



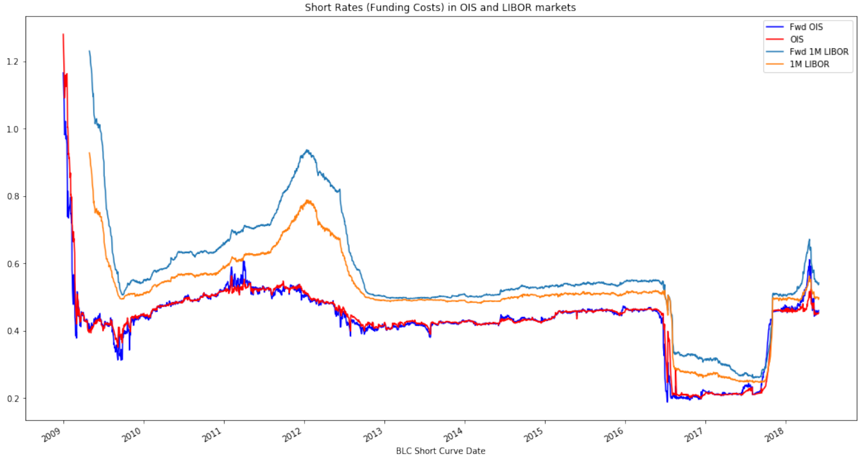
Source: Goldman Sachs Asset Management. FAQ: What's Happening in Short-Term Rates

Three-month Libor minus three-month GC repo



An old way to look at LIBOR minus RFR spread – *35 bps* on average, decomposed into 15 bps **liquidity** and 20 bps **credit risk**.

Source: Inferring market interest rate expectations, BOE Quarterly, Nov 2000



Compounding OIS will never give you 1M LIBOR unless you add a spread $(OIS + LOIS)^n$

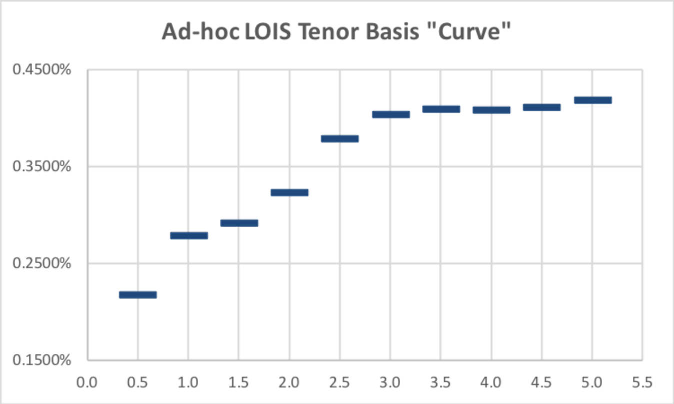
Source: BOE/Bloomberg data processed by Richard Diamond

LOIS computation – not really 'a spread'.

		0.5	1.0	1.5	2.0
LIBOR-OIS (spot)	61.10	**0.2285%**	0.3381%	0.4552%	0.5833%
LIBOR - Fwd OIS	35.78	**0.2179%**	0.2798%	0.2935%	0.3259%
Fwd Inst -Fwd OIS	35.36	**0.2168%**	0.2783%	0.2912%	0.3227%
	Average, bps. Evaluate the range as well				
LIBOR-Fwd OIS	spread	35.78	bps		

First, we end up with a variety of spreads for the important short end (funding), depending on convention and shortcuts.

Second, Stabilises for the long end, or $>$ 5Y sensitivity to tenor lost?



Source: *Yield Cuve v4.xls* by Richard Diamond

Implied OIS Discounting

OIS Discounting was a popular topic as Discounting Curve diverged from Index Curve(s).

$$L_i - \text{LOIS} \qquad \forall \; tenors$$

Fwd LIBOR		0.6617%	0.9422%	1.2346%	1.5090%
However, no new OIS curve available (or the new OIS data is stale)!					
Implied Fwd OIS		0.3039%	0.5843%	0.8768%	1.1512%

The recipe was 'just' to subtract LOIS, from each simulated curve of Forward LIBORs, which can be up to 25Y maturity – while OIS swaps are to 5Y maturity maximum.

Bootstrap Framework

The following process is referred to as **yield curve stripping**

- Setting up par bootstrap equations
- Solving for the unknown curves by stepping through time.

Bootstrap Framework

PV for the fixed leg with fixed rate k – Payer PAYS. $k\,\tau_i$ paid at T_i

$$\mathrm{PV} = k\,N \sum_{i=1}^{n} \tau_i\, D_i$$

PV for the floating leg – Payer RECEIVES. Rate plus **basis spread**

$$\mathrm{PV} = N \sum_{j=1}^{m} \tau_j \left(I_j + s\right) D_j$$

where s is spread, and I_j is Index curve rate, such as 6M LIBOR.

Bootstrap Framework

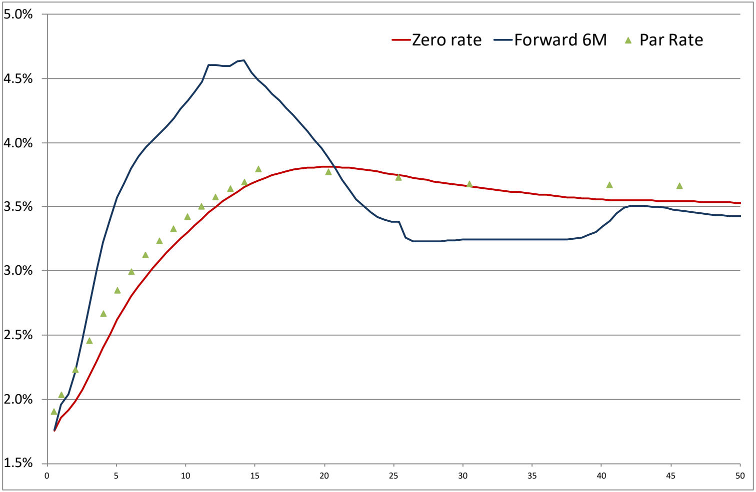
Equating PV of swap legs we obtain **par bootstrap equation**,

$$k \sum_{i=1}^{n} \tau_i \, D_i = \sum_{j=1}^{m} \tau_j \left(l_j + s \right) \, D_j$$

$$\tau_i \neq \tau_j$$

In fact, small $\tau_i \neq \tau_{i+1}$, eg ON, 1M, 2M, 3M, 6M, 12M, 2Y, 3Y curve is converted into 6M basis $\tau_j \approx 0.5$.

Let's explore **LIBOR Bootstrap v1.xlsm**. There it is implemented via re-interpolation of spot so $l_j = r(0, T_j)$, $j = [1..100]$.



Market data points (triangles). Bootstrapped spot curve in red – 6M tenor increment (based on exact day count, so assume EONIA underlying).

Instrument	Par Bootstrap Equation	Unknown Curves to Solve
Standard Swap	$k\sum_i^n \tau_i D_i = \sum_i^m \tau_i (I_i + s) D_i$	Discount curve (D) and index curve (I). As an example, D can be the OIS curve, and I a LIBOR curve.
Tenor Basis Swap	$\sum_i^n \tau_i I_i^x D_i = \sum_i^m \tau_i (I_i^y + s) D_i$	Discount curve (D) and two index curves. As an example, I_i^x can be the 6-month LIBOR and I_i^y the 3-month LIBOR curve.

From: *Manufacturing and Managing Customer-Driven Derivatives*, Dong Qu, 2016

OIS	$k\sum_i^n \tau_i D_i = \sum_j^m \tau_j R_j D_j$ $R_j = \dfrac{\prod_d (1+\tau_d r_d)-1}{\sum_d \tau_d}$	Discount curve (D) is the only unknown, as the daily overnight rate (r_d) is assumed linked to D: $r_d = \dfrac{1}{\tau_d}\left(\dfrac{D_1}{D_2}-1\right)$ R_j is the compound rate of r_d.

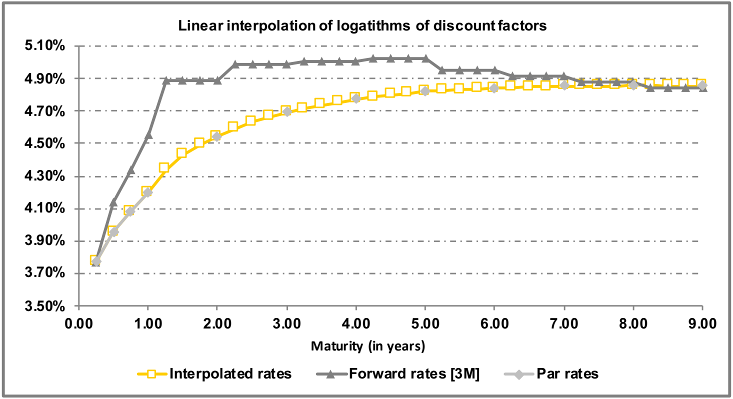
From: *Manufacturing and Managing Customer-Driven Derivatives*, Dong Qu, 2016

Interpolation

Curve stripping and interpolation go together. Instead of boostrapping $\ln Z(t, T) \Rightarrow f(t, T)$, direct interpolation methods are applied to infer Forward LIBOR (simple rates) form par rates $r_s \Rightarrow L(t)_i$

- Linear over zero coupon rates (spot)
- Linear over discount factors
- Linear over **log of discount factors** (raw interpolation)
- Natural cubic spline (also B-splines)
- Monotone-convex spline

Explore **Interpolation and FRA.xlsm** file for the effect of interpolation method on **a.** resulting forward curve and **b.** PV01 of interest rate swap.



Interpolating over $log\, Z(0, T)$ gives nearly as good curve as advanced methods (spline-based). **Forward curve** (in deep grey) is not zig-zag and 'smooth enough'.

Take **monotone convex splines**, for example.

- Hagan and West [2006] 'ameliorated' the method by making the interpolation less local, ie, using inputs which are two nodes away.

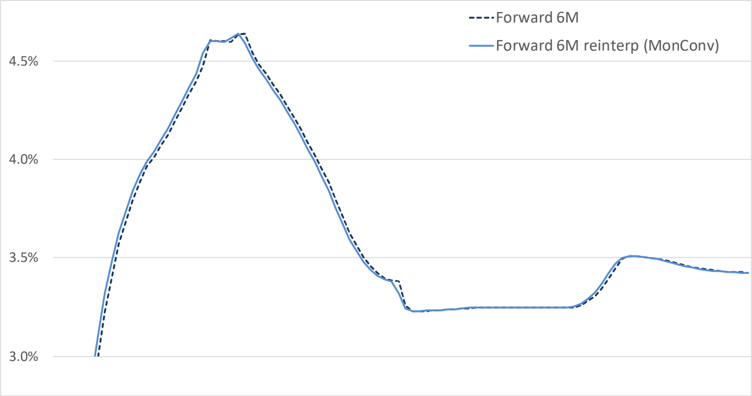
  LIBOR Bootstrap v1.xls has VBA code and use example.

- BIS Paper 25, spot curves (ZCB rates, zero-rates) fitted with

  1) cubic B-spines with knot points, utilised by FRBNY previously

  2) exponential polynomial family (Nelson-Siegel, Svensson) utilised by BOE,

  3) curve fitting with Artificial Neural Nets – see William McGhee (Natwest Markets, 2018) for recipe.



Bootstrapped 6M Fwd re-interpolated using monotone-convex splines. Impact is slight because the rate comes from the already-stripped curve – in regular increment.

Please take away the following important ideas:

- traditional maths of FRA, Forward LIBOR and Interest Rates Swap maths is important and in force,
- new set of RFR benchmarks are emerging but forward curves their produce do not match LIBOR, and hence might not satisfy asset-liability management needs
- LOIS spread is risk indicator of the financial system
- different tenor of financing (overnight, 3M, 6M) means ‘different curve’
- stripping of an index curve might be implemented together with interpolation and smoothing as a single technique.

Additional notes on **Consistent Discounting and Multiple Curve Pricing**

This includes a) link between LOIS and CDS (credit spread) and b) pricing under stochastic models for forward rate.

The consistent multiple curve pricing is desired.

Such pricing takes into account information not only **a.** OIS-LIBOR basis (LOIS) but also **b.** tenor basis spreads:

- implied by the differences in funding from 3M and 6M LIBOR.
- tenor basis swaps are tradeable instruments.

Certain explanation always necessary about how you tackled tenor basis in discounting/IRS pricing/CVA calculation.

Constant Basis to Stochastic Basis

1. There is a potential to co-simulate OIS curves (same RNs) but the structure of factors for [0, 5Y] OIS curves is simpler than longer LIBOR curves.

2. The spread is not tenor-constant: has a ‘humped-shaped curve of its own. The area of modelling known as *stochastic basis*, while attempts to extend LMM and HJM have been very complex.

OIS Curve	**Implied OIS Curve**	**Stochastic Basis**
static	re-simulated curves tenors to $25Y$ **Practitioner**	re-simulated curves and multiple spreads **Academic Quant**

Castagna et al., 2015 on links between basis risks and credit risk defined **spot credit spread** – to model LOIS.

$$\text{LOIS} \propto \frac{1}{\tau} \frac{(\text{LGD} + r(0,\tau)\tau)\ \text{PD}(0,\tau)}{1 - \text{PD}(0,\tau)}$$

where $PD(0,\tau) = 1 - \text{PrSurv} = 1 - P_{CDS}(0,T)$ can be taken from short-term CDS quotes.

Forward credit spread $s(t, t+\tau)$ calculation relies on $PD(t, t+\tau)$ easily bootstrapped from CDS.

The following integration $r(\tau)\tau = \int_0^T \bar{f}(\tau)d\tau$ allows us to compute the input.

HJM Model with OIS and Stochastic Spread

Stochastic spread. OIS-LIBOR spread is customarily between discrete-time variables. Fwd-Fwd spread gives better estimation.

We use rate differences, analysed with the PCA to calibrate the HJM

$$\Delta f = f_{t+1} - f_t \qquad \text{separate for each tenor (column)}$$

$$f_{Fwd,t} - f_{FwdOIS,t} \qquad \text{is also spread} \qquad \Delta f_s$$

then proceed with PCA on covariance of $\Delta f_s^i, \Delta f_s^j$ where i, j are tenors.

HJM SDE allow evolving df_s^j a stochastic spread for each tenor j, and will be sensitive to volatility input: high volatility during a credit crunch.

Additional notes on **Forward Rates** and their ‘bootstrapping’

Bootstrapping fwds simple way

No arbitrage relationship between a sequence of *simple* forward rates and ZCB yield at time n (**spot rate**) is given by a geometric average

$$(1 + rs_n)^n = (1 + f_0) \times (1 + f_1) \times ... \times (1 + f_n).$$

where for the first period $f_0 = rs_0$.

You might know this relationship well, and it leads to *a recursive* bootstrapping *from* a set of ZCB prices...

Overlapping discount factors $Z(0,T_1), Z(0,T_2), Z(0,T_3)$ ... encapsulate information about forward rates, also called *hedgeable rates*.

$$
\begin{aligned}
Z(0,T_1) \times Z(0;T_1,T_2) &= Z(0,T_2) \\
Z(0;T_1,T_2) &= \exp\left[-f(0;t_1,t_2)(t_2-t_1)\right] \\
f_2 &= -\frac{\ln Z_2 - \ln Z_1}{t_2 - t_1}
\end{aligned}
$$

Computing forward rates

A scheme for each discrete tenor j follows from the instantaneous forward rate maths

$$f_0 = -\frac{\ln Z_1 - \ln 1}{T_1 - 0} \qquad f_1 = -\frac{\ln(Z_2/Z_1)}{T_2 - T_1}$$

Tenor, T	**0.08**	**0.17**	**0.25**	**0.33**
Spot	0.5052%	0.5295%	0.5500%	0.5682%
Z(0, T)	0.9996	0.9991	0.9986	0.9981
Forward	0.5063%	0.5552%	0.5927%	0.6244%

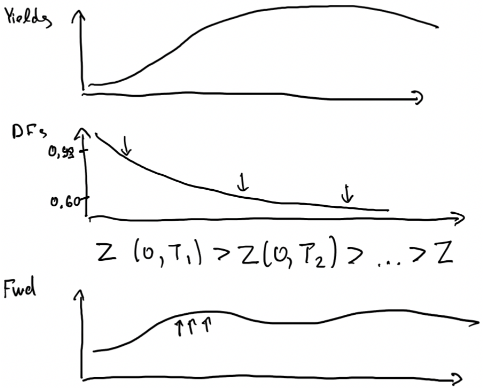
Bootstrapping Fwds

1. Let's explore results of boostrapping fwd rates from spot rates, ZCB factors, and compare to BOE inst fwd curve for the same day

2. It is evident that BOE applies additional VRP smoothing on top of this classical fwds bootstrapping.

3. It is recommended to use **raw interpolation**: linear over log discount factors $\ln Z(T_{i-1}, T_i)$.

Please examine *Yield Curve v4.xlsm* spreadsheet and its VBA.



Source: Drawing by Richard Diamond

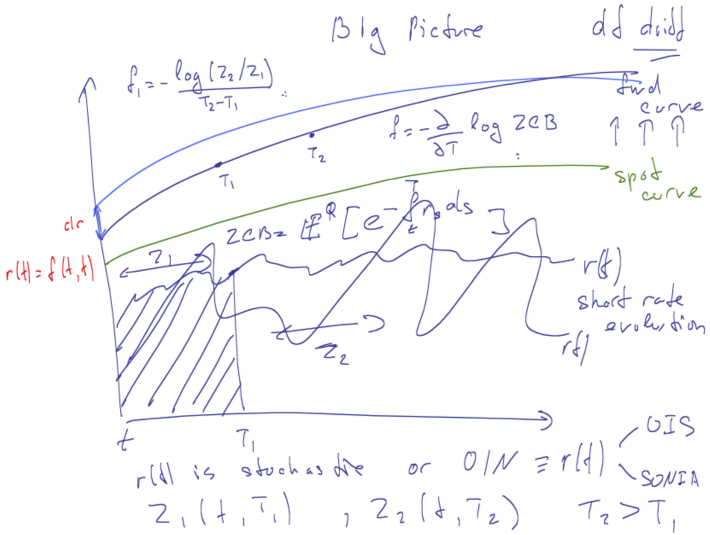
Let's draw a summary of relationship among

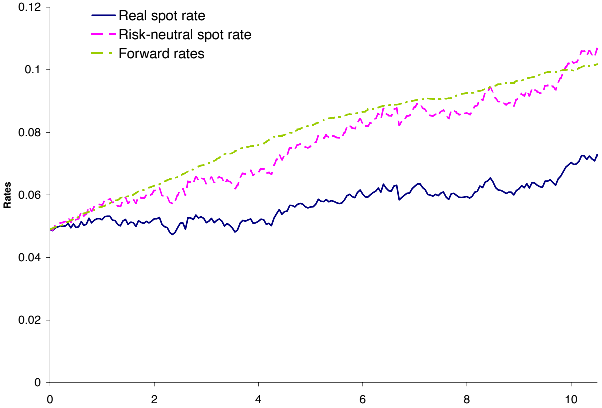
a. the short rate process,

b. spot curve, and

c. forward curve.

From Spot Rate to Forward Curve





Fwd curve is the expectation of risk-adjusted spot rates. Evident in short maturities, eg Fwd 1M LIBOR is risk-adjusted OIS (Slide 33).

Source: CQF IR Calibration Lecture by Paul Wilmott, Slide 72