Python Labs

Credit Risk Analytics - I

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Credit Default Swap

Credit Default Swap (CDS) is a derivative of a bond or a securitized instruments that involves transfer of a credit risk between two parties. CDS = Credit + Default + Swap, where the 'credit' is linked to bond or loan, 'default' is linked to sensitivity of default event or bankruptcy and 'swap' involves exchange of cashflows. Here, the seller takes the credit risk that the buyer does not wish to shoulder in exchange for a premium over a certain period. The buyer of a CDS contract is seeking insurance against a default of an underlying bond, while the seller is offering a guarantee that the bond will be repaid. The spread of a CDS is the annual amount of the protection buyer must pay the protection seller over the length of the contract, expressed as a percentage of the notional amount.

For example, consider a CDS contract be between two parties say party A and party B. Let Party A be the protection buyer and Party B be the protection seller. The following is the cash flow payment patterns for the above example CDS trade:

Party A

1. No Credit Event: Party A will continue to make premium payment at pre-defined points of time until contract maturity.
2. Credit Event Occurs: Party A will not pay premium payments due in the future anymore.

Party B

1. No Credit Event: Party B will continue to receive premium payment at pre-defined points of time until contract maturity.
2. Credit Event Occurs: Party B will have to make good the loss to Party A owing to the credit event occurring.

Generally, the credit event could be any of the four occurances:

> 1. Complete default on payments
> 2. Partial default on payments
> 3. Credit rating downgrade
> 4. Widening credit spreads

However, as these trades are executed on Over the Counter (OTC), both parties can define credit events as per their requirements in the contract at the time of entering into the trade.

CDS are generally quoated as spreads. All thing being equal, at any given time, if the maturity of two CDS is the same, then the CDS with a company with a higher (CDS) spread is considered more likely to default by the market.

Application of CDS includes Hedging, Speculation and Capital Structure Arbitrage (CSA).

Default Modeling

To model the *arrival of a credit event*, we need to model an unknow random point in time $\tau \in \mathbb{R}_+$. For default risk modeling, we use the *default indicator* function and *survival indicator* function which is one minus the default indicator function. The survival indicator function is thus represented as,

$$I(t) = 1_{\tau > T} = \begin{cases} 1, & \text{if } \tau > nT \\ 0, & \text{if } \tau \leq T \end{cases}$$

where τ is the time of default.

Poisson Processes

For analysis of credit events we use Poisson processes. Poisson processes are usually used to model rare events and discretely countable events such as defaults. It is a discrete time process which gives the probability of default between two time intervals conditional on surviving uptil the initial time point and these inter-arrival times of Poisson process are exponentionally distributed.

We can model the arrival of default as the first jump of a Poisson process with intensity λ and its associated survival probability $P(t, T)$ can be obtained as,

$$P(t, T) = e^{-\lambda(T-t)}$$

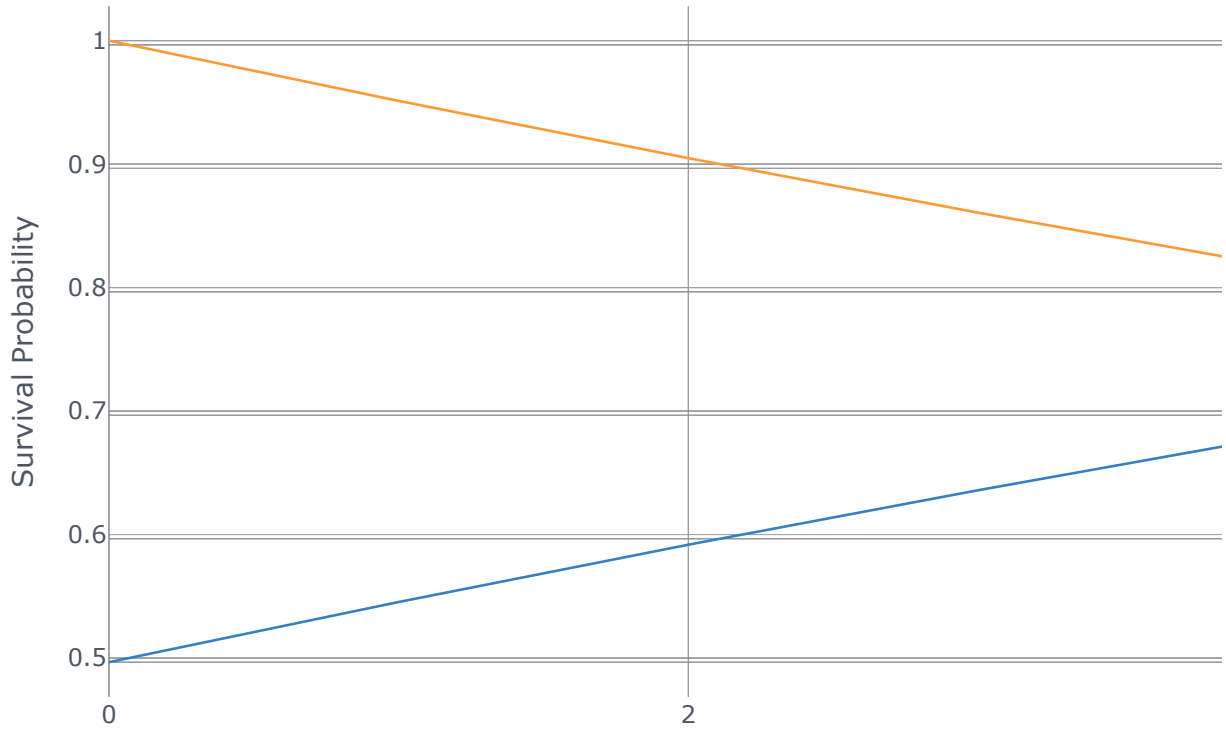
where, λ (also known as) the hazard rate is constant and does not depend on time. Thus, the risk of default is modeled using the survival probability.

In [1]:
```
# Import libraries
import pandas as pd
import numpy as np

# Plotting library
import cufflinks as cf
cf.set_config_file(offline=True)
```

In [2]:
```
# specify lambda
lambda_ = 0.05

# specify tenors
tenor = np.arange(15)

# calculate survival probability
```

```
data = pd.DataFrame({'Survival': np.exp(-lambda_*tenor)})
data['Default'] = 1.-data['Survival']
```

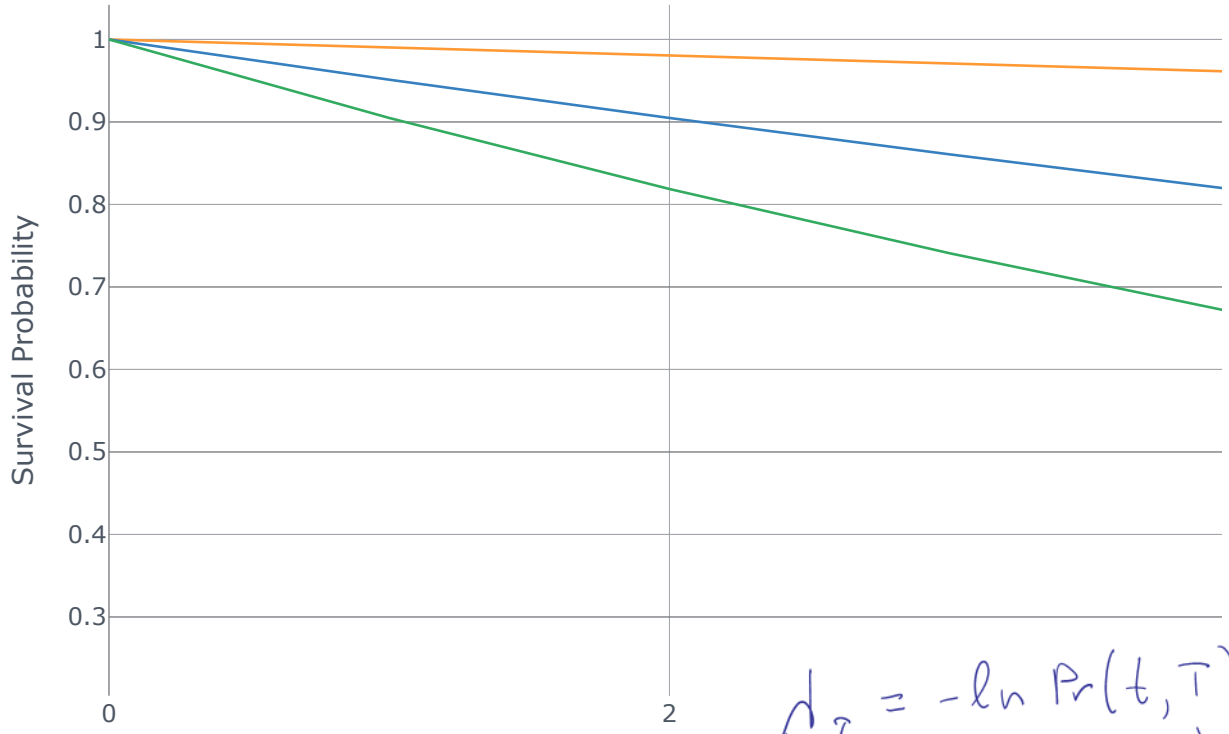
In [3]:
```
# visualize the plot
data.iplot(title='Survival vs Default Probability with Constant Lambda',
           xTitle='CDS Maturity',
           yTitle='Survival Probability',
           secondary_y = 'Default',
           secondary_y_title='Default Probability')
```



If the hazard rate λ is known, we can have the entire term structure of probabilities and use these to estimate the likelihood of the various cashflows in the CDS. Premium payments are estimated using survival probability and default payments using probability of default.

In [4]:
```
# specify lambdas
lambda1 = 0.01
lambda2 = 0.05
lambda3 = 0.10

# subsume into dataframe
data_ = pd.DataFrame({'$\lambda = 1\%$': np.exp(-lambda1*tenor)})
data_['$\lambda = 5\%$'] = np.exp(-lambda2*tenor)
data_['$\lambda = 10\%$'] = np.exp(-lambda3*tenor)
data_.iplot(title='Survival Probabilities',
```

```
            xTitle='CDS Maturity',
            yTitle='Survival Probability')
```



$d_2 = -\ln Pr(t,T)$

$d = -\frac{\ln Pr}{\tau}\Big|_{\tau=1} = -\ln Pr$

Inhomogeneous Poisson Processes

For a inhomogeneous Poisson processes, if the intensity $\lambda(t)$ is a non-negative function of time, we can derive the survival probability as

1 - survive after d

$$P(t,T) = e^{-\int_0^T \lambda(s)ds}$$

Here, the hazard rate is non constant and depends on the time horizon T and the term structure of hazard rates is not flat, but given by $\lambda(T)$, so we can reach every term structure we desire so as to match the market exactly.

Survival Probability

The survival probability is a key component in calculation of cash flows and therefore the valuation of a CDS contract. As we know, the occurrence of a credit event is related to the concept of probability of default, which in itself is complement of probability of survial.

Thus, the **Probability of Survival + Probability of default = 1**

The survival probability is not directly observable in the market and has to be implied from traded credit spreads in the market. We then use the CDS spreads and run a bootstrapping algorithm to calculate the survival probability. These survival probabilities are then used to price the CDS contracts.

CDS Pricing: Basic Model

In CDS pricing, we bootstrap the hazard rates given the premium leg, default leg and the fair spread. The simplified pricing model for CDS is called as the 'Standard Model' or popular known as the 'JP Morgan Approach'

Mathematical Setup

1. Discretize the maturity into number of Δt so that the corresponding end of period maturities are then expressed as

$$T_n = n\Delta t$$

The objective is to calculate the cashflow associated with CDS interms of payment that will be made at these time points.

1. Discount factors are written as functions of forward rates r_n

$$D(0, T_n) = e^{-\sum_{k=1}^{n} r_k \Delta t}$$ DF

1. Probability of survival given discretized time and (piecewise constant) hazard rates will then be represented as

$$P(T_n) = e^{-\sum_{k=1}^{n} \lambda_k \Delta t}$$ Pr Surv

Assuming that at time zero, the obligor is solvent, we have $P(T_0) = P(0) = 1$.

1. Premium legs are calculated by multiplying with the corresponding discount factors and probability of survival at each payment dates (time points).

$$PL_N = S_N \sum_{n=1}^{N} D(0, T_n) P(T_n)(\Delta t_n) \times \text{Notional}$$

pay Year 1, Year 2 ← N

rate ↑ S_N | DF ↑ $D(0, T_n)$ | Prob ↑ $P(T_n)$ | time ↑ Δt_n | value ↑ Notional

where, Δt_n is the year fraction corresponding to $T_{n-1} - T_n$ and $P(T_n)$ is the survival probability up to time T_n. This accounts for the expected present value of payments made from the buyer **A** to the seller **B**.

1. Default legs are conditional on occurrence of the default event. The exepcted present value of this payment depends on the recovery rate R in the event of default. The loss payment on default is then equal to $(1 - R)$ for every $\$1$ of notional principal. Thus, the expected present value of loss payments is given as

$$DL_N = (1-R)\sum_{n=1}^{N} D(0,T_n)(P(T_{n-1}) - P(T_n))$$

The probability is given by the probability of surviving until period $n-1$ and then defaulting in periond n.

1. The fair quots of the spread S_N is such that the expected present value of the payments made by buyer and seller are equal, i.e. $PL_N = DL_N$

Thus, we obtain

$$S_N = \frac{(1-R)\sum_{n=1}^{N} D(0,T_n)(P(T_{n-1}) - P(T_n))}{\sum_{n=1}^{N} D(0,T_n)P(T_n)(\Delta t_n)}$$

where, $P(T_{n-1}) - P(T_n)$ is the marginal probability of survival.

CDS Boostrapping

We assume that we have a CDS market spreads for increasing maturities $S_1, S_2, \ldots, S_N$, we can then determine their associated survival probabilities $P(T_1), P(T_2), \ldots, P(T_N)$

First Step (N=1),

$$P(T_1) = \frac{L}{L + \Delta t_1 S_1}$$

where, $L = 1 - R$

Second Step (N=2),

$$P(T_2) = \frac{D(0,T_1)[L(1) - (L + \Delta t_1 S_2)P(T_1)]}{D(0,T_2)(L + \Delta t_2 S_2)} + \frac{P(T_1)L}{L + \Delta t_2 S_2}$$

In general, we can write down the expression for $P(T_N)$ as

$$P(T_N) = \frac{\sum_{n=1}^{N-1} D(0,T_n)[LP(T_{n-1}) - (L + \Delta t_n S_N)P(T_n)]}{D(0,T_N)(L + \Delta t_n S_N)} + \frac{P(T_{N-1})L}{L + \Delta t_N S_N}$$

Thus, we begin with $P(T_1)$ and through a process of bootstrapping, we arrive at all $P(T_n)$ where, $n = 1, 2, \ldots, N$.

Implementation

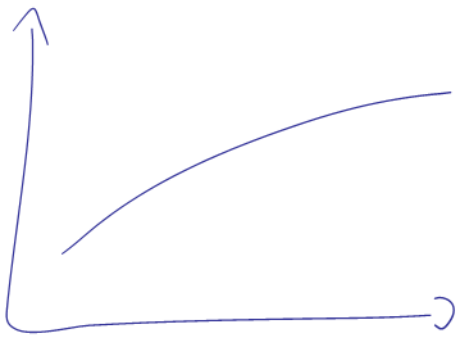
We'll now execute a bootstrapping algorithm to determine the survival probabilities.

In [5]:
```
# Read the cds spreads data
df = pd.read_csv('cds_spreads_1.txt', sep='\t')
df
```

Out[5]:

	Maturity	Df	Spread

	Maturity	Df	Spread
0	0	0.00	0.0
1	1	0.97	50.0
2	2	0.94	77.0
3	3	0.92	94.0
4	4	0.89	109.5
5	5	0.86	125.0



In [6]:
```
# convert bps to decimal
df['Spread'] = df['Spread']/10000

# specify delta_t
df['Dt'] = df['Maturity'].diff().fillna(0)

# specify recovery rate
RR = 0.40

# loss rate
L = 1.0 - RR

df
```

Out[6]:

	Maturity	Df	Spread	Dt
0	0	0.00	0.00000	0.0
1	1	0.97	0.00500	1.0
2	2	0.94	0.00770	1.0
3	3	0.92	0.00940	1.0
4	4	0.89	0.01095	1.0
5	5	0.86	0.01250	1.0

In [7]:
```
# initialize the variables
term = term1 = term2 = divider = 0
```

In [8]:
```
for i in range(0, len(df.index)):
    if i == 0: df.loc[i,'Survival'] = 1
    if i == 1: df.loc[i,'Survival'] = L / (L+df.loc[i,'Dt']*df.loc[i,'Spread'])
    if i > 1:
        terms = 0
        for j in range(1, i):
            term = df.loc[j,'Df']*(L*df.loc[j-1,'Survival'] - \
                                   (L + df.loc[j,'Dt']*df.loc[i,'Spread']*
                                   df.loc[j,'Survival'])

            terms = terms + term

        divider = df.loc[i,'Df']*(L+df.loc[i,'Dt']*df.loc[i,'Spread'])
        term1 = terms/divider

        term2 = (L*df.loc[i-1,'Survival']) / (L + (df.loc[i,'Dt'] * df.loc[i,'Sp
```

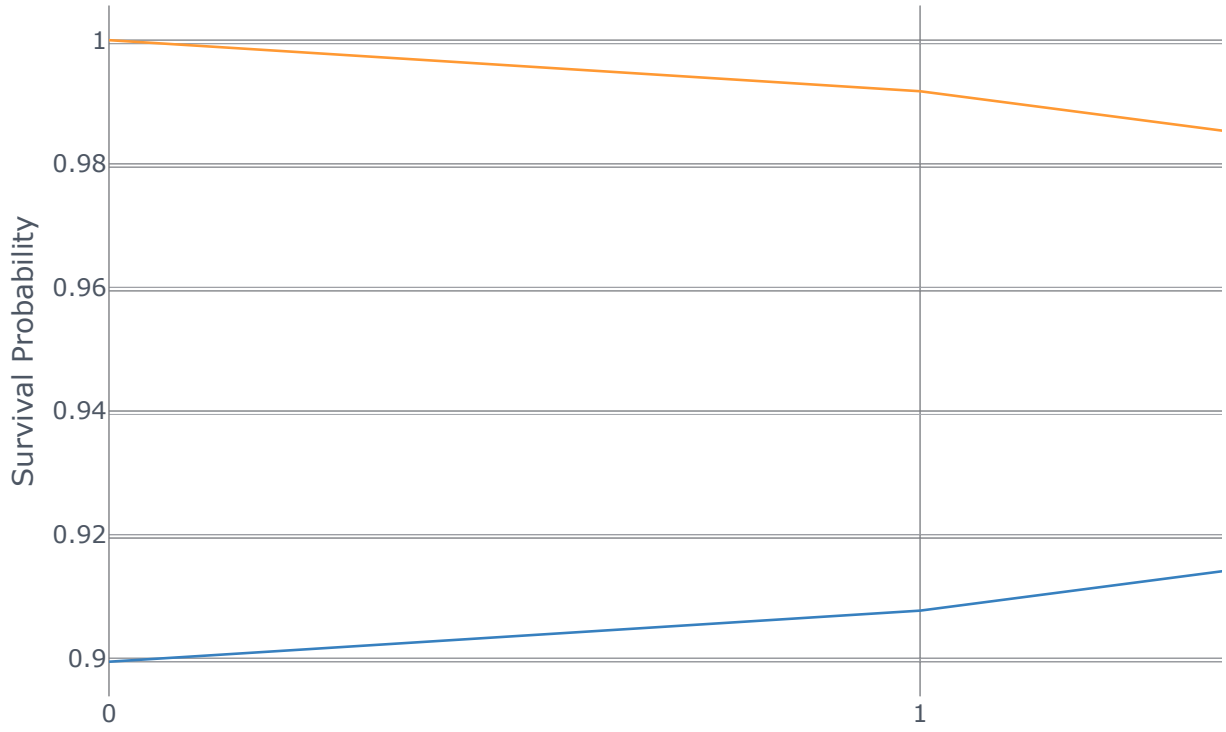
```
        df.loc[i,'Survival'] = term1 + term2
```

In [9]:
```
# derive probability of default
df['Default'] = 1 - df['Survival']

# output the results
df
```

Out[9]:

	Maturity	Df	Spread	Dt	Survival	Default
0	0	0.00	0.00000	0.0	1.000000	0.000000
1	1	0.97	0.00500	1.0	0.991736	0.008264
2	2	0.94	0.00770	1.0	0.974623	0.025377
3	3	0.92	0.00940	1.0	0.953894	0.046106
4	4	0.89	0.01095	1.0	0.928942	0.071058
5	5	0.86	0.01250	1.0	0.899443	0.100557

In [10]:
```
# plot survival probability
df[['Survival', 'Default']].iplot(title='CDS Bootsrapping',
                                  xTitle='CDS Maturity', yTitle='Survival Probab
                                  secondary_y = 'Default', secondary_y_title='De
```



User Defined Function

In [11]:
```
def survival_probability(maturity, discountfactor, spread, recovery, is_plot=Fal

    # subsume list of inputs into a dataframe
    df = pd.DataFrame({'Maturity': maturity, 'Df': discountfactor, 'Spread': spr

    # convert bps to decimal
    df['Spread'] = df['Spread']/10000

    # specify delta_t
    df['Dt'] = df['Maturity'].diff().fillna(0)

    # loss rate
    L = 1.0 - recovery

    # initialize the variables
    term = term1 = term2 = divider = 0

    for i in range(0, len(df.index)):
        if i == 0: df.loc[i,'Survival'] = 1
        if i == 1: df.loc[i,'Survival'] = L / (L+df.loc[i,'Dt']*df.loc[i,'Spread
        if i > 1:
            terms = 0
            for j in range(1, i):
                term = df.loc[j,'Df']*(L*df.loc[j-1,'Survival'] - \
                                       (L + df.loc[j,'Dt']*df.loc[i,'Spre
                                       df.loc[j,'Survival'])
                terms = terms + term

            divider = df.loc[i,'Df']*(L+df.loc[i,'Dt']*df.loc[i,'Spread'])
            term1 = terms/divider

            term2 = (L*df.loc[i-1,'Survival']) / (L + (df.loc[i,'Dt'] * df.loc[i

            df.loc[i,'Survival'] = term1 + term2

    # derive probability of default
    df['Default'] = 1. - df['Survival']

    if is_plot:
        # plot survival probability
        df[['Survival', 'Default']].iplot(title='Survival vs Default Probability
                                          xTitle='CDS Maturity',
                                          yTitle='Survival Probability',
                                          secondary_y = 'Default',
                                          secondary_y_title='Default Probability

    return df
```

In [12]:
```
# inputs
maturity = np.arange(6)
discountfactor = [0, 0.97, 0.94, 0.92, 0.89, 0.86]
cds_spread = [0, 50, 77, 94, 109.5, 125]
recovery = 0.40
```

In [13]:
```
# call the user defined function
sp = survival_probability(maturity,discountfactor,cds_spread,recovery,is_plot=Fa
```

```
sp
```

Out[13]:

	Maturity	Df	Spread	Dt	Survival	Default
0	0	0.00	0.00000	0.0	1.000000	0.000000
1	1	0.97	0.00500	1.0	0.991736	0.008264
2	2	0.94	0.00770	1.0	0.974623	0.025377
3	3	0.92	0.00940	1.0	0.953894	0.046106
4	4	0.89	0.01095	1.0	0.928942	0.071058
5	5	0.86	0.01250	1.0	0.899443	0.100557

In [14]:
```
# inputs
maturity = np.arange(6)
discountfactor = [0, 0.9803, 0.9514, 0.9159, 0.8756, 0.8328]
cds_spread = [0, 29, 39, 46, 52, 57]
recovery = 0.50
```

In [15]:
```
# call the user defined function
survival_probability(maturity,discountfactor,cds_spread,recovery,is_plot=False)
```

Out[15]:

	Maturity	Df	Spread	Dt	Survival	Default
0	0	0.0000	0.0000	0.0	1.000000	0.000000
1	1	0.9803	0.0029	1.0	0.994233	0.005767
2	2	0.9514	0.0039	1.0	0.984505	0.015495
3	3	0.9159	0.0046	1.0	0.972636	0.027364
4	4	0.8756	0.0052	1.0	0.958824	0.041176
5	5	0.8328	0.0057	1.0	0.943693	0.056307

Example : Bank CDS

In [16]:
```
# HSBC
maturity = np.arange(6)
discountfactor = [0, 0.9972, 0.9916, 0.9775, 0.9619, 0.9426]
cds_spread = [0, 11.2, 27.7, 36.9, 57.1, 67.8]
recovery = 0.40
```

In [17]:
```
# call function
hsbc = survival_probability(maturity,discountfactor,cds_spread,recovery,is_plot=
hsbc
```

Out[17]:

	Maturity	Df	Spread	Dt	Survival	Default
0	0	0.0000	0.00000	0.0	1.000000	0.000000
1	1	0.9972	0.00112	1.0	0.998137	0.001863
2	2	0.9916	0.00277	1.0	0.990802	0.009198
3	3	0.9775	0.00369	1.0	0.981663	0.018337

	Maturity	Df	Spread	Dt	Survival	Default
4	4	0.9619	0.00571	1.0	0.962224	0.037776
5	5	0.9426	0.00678	1.0	0.944246	0.055754

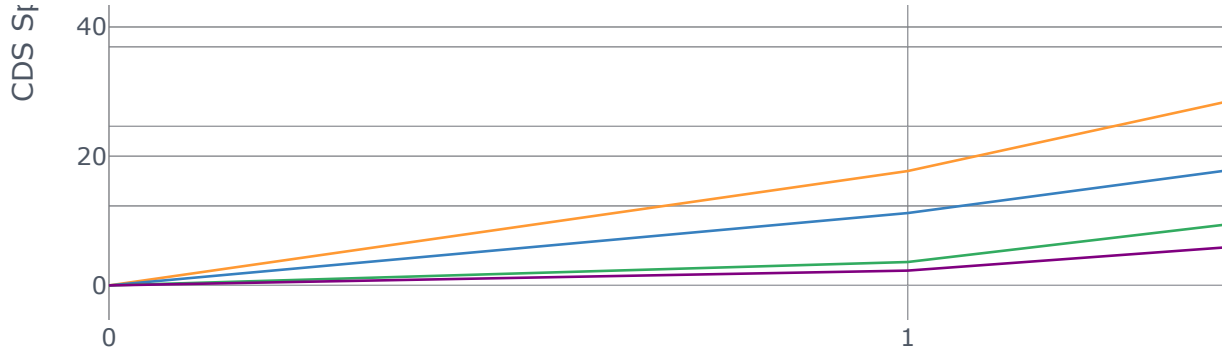
In [18]:
```
# Barclays
maturity = np.arange(6)
discountfactor = [0, 0.9972, 0.9916, 0.9775, 0.9619, 0.9426]
cds_spread = [0, 17.7, 44.6, 54.8, 83.5, 96.2]
recovery = 0.40
```

In [19]:
```
# call function
barclays = survival_probability(maturity,discountfactor,cds_spread,recovery,is_p
barclays
```

Out[19]:

	Maturity	Df	Spread	Dt	Survival	Default
0	0	0.0000	0.00000	0.0	1.000000	0.000000
1	1	0.9972	0.00177	1.0	0.997059	0.002941
2	2	0.9916	0.00446	1.0	0.985240	0.014760
3	3	0.9775	0.00548	1.0	0.972925	0.027075
4	4	0.9619	0.00835	1.0	0.945239	0.054761
5	5	0.9426	0.00962	1.0	0.921855	0.078145

In [20]:
```
# create dataframe of banks cds
bank = pd.DataFrame({'Barclays CDS': barclays['Spread']*10000,
                     'HSBC CDS': hsbc['Spread']*10000,
                     'Barclays PD': barclays['Default']*100,
                     'HSBC PD': hsbc['Default']*100})

# plot bank cds & probability of default
bank.iplot(title='Spread vs Default Probability',
           xTitle='CDS Maturity',
           yTitle='CDS Spread (bps)',
           secondary_y = ['Barclays PD', 'HSBC PD'],
           secondary_y_title='Default Probability (%)')
```

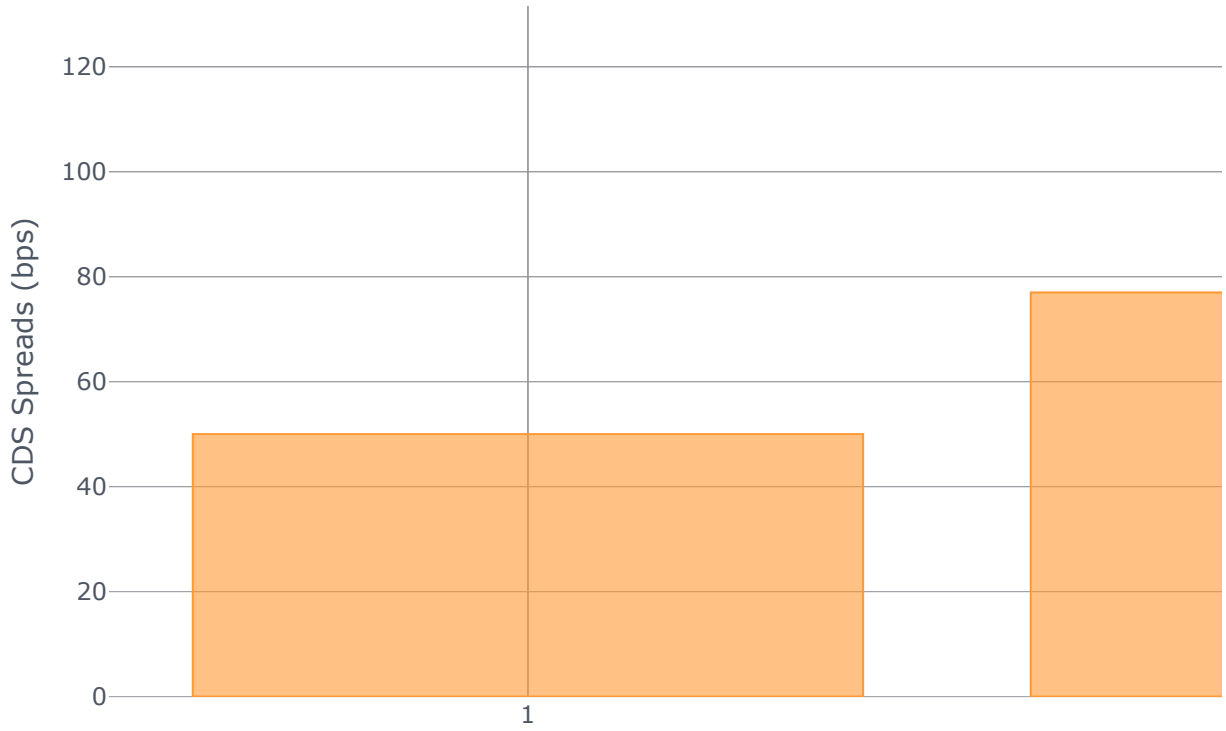


CDS Pricing

In [21]:
```
def get_cds_spread(maturity, discountfactor, probability, recovery, is_plot=Fals

    # subsume list of inputs into a dataframe
    df = pd.DataFrame({'Maturity': maturity, 'Df': discountfactor, 'Survival': p

    # specify delta_t
    df['Dt'] = df['Maturity'].diff().fillna(0)

    # loss rate
    L = 1.0 - recovery

    # initialize the variables
    nterm = dterm = 0

    for i in range(0, len(df.index)):
        if i == 0: df.loc[i,'Spread'] = 0
        else:
            nterms = 0; dterms = 0
            for j in range(1, i+1):
                nterm = L * df.loc[j,'Df']*(df.loc[j-1,'Survival'] - df.loc[j,'S
                nterms = nterms + nterm

                dterm = df.loc[j,'Df'] * df.loc[j,'Survival'] * df.loc[j,'Dt']
                dterms = dterms + dterm

            df.loc[i,'Spread'] = nterms / dterms * 10000

    if is_plot:
        # plot spreads
        df['Spread'][1:].iplot(kind='bar',
                               title='CDS Spreads',
                               xTitle='CDS Maturity',
                               yTitle='CDS Spreads (bps)')

    return df
```

In [22]:
```
# inputs
maturity = np.arange(6)
discountfactor = [0, 0.97, 0.94, 0.92, 0.89, 0.86]
cds_spread = [0, 50, 77, 94, 109.5, 125]
recovery = 0.40
```

In [23]:
```
get_cds_spread(maturity, discountfactor, sp.Survival, recovery, is_plot=True)
```



Out[23]:

	Maturity	Df	Survival	Dt	Spread
0	0	0.00	1.000000	0.0	0.0
1	1	0.97	0.991736	1.0	50.0
2	2	0.94	0.974623	1.0	77.0
3	3	0.92	0.953894	1.0	94.0
4	4	0.89	0.928942	1.0	109.5
5	5	0.86	0.899443	1.0	125.0

References

- CDS Market Quotes https://www.cnbc.com/sovereign-credit-default-swaps/
- Cufflinks documentation https://github.com/santosjorge/cufflinks and https://plotly.com/python/cufflinks/

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Certificate in Quantitative Finance, June 2020