
CQF: Certificate in Quantitative Finance

Credit Default Swaps

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Content

- Definition of a CDS
- CDS and credit spreads
- CDS valuation
- Proxy CDS curves
- Stochastic hazard rate models
- Counterparty Risk



Definition of a CDS

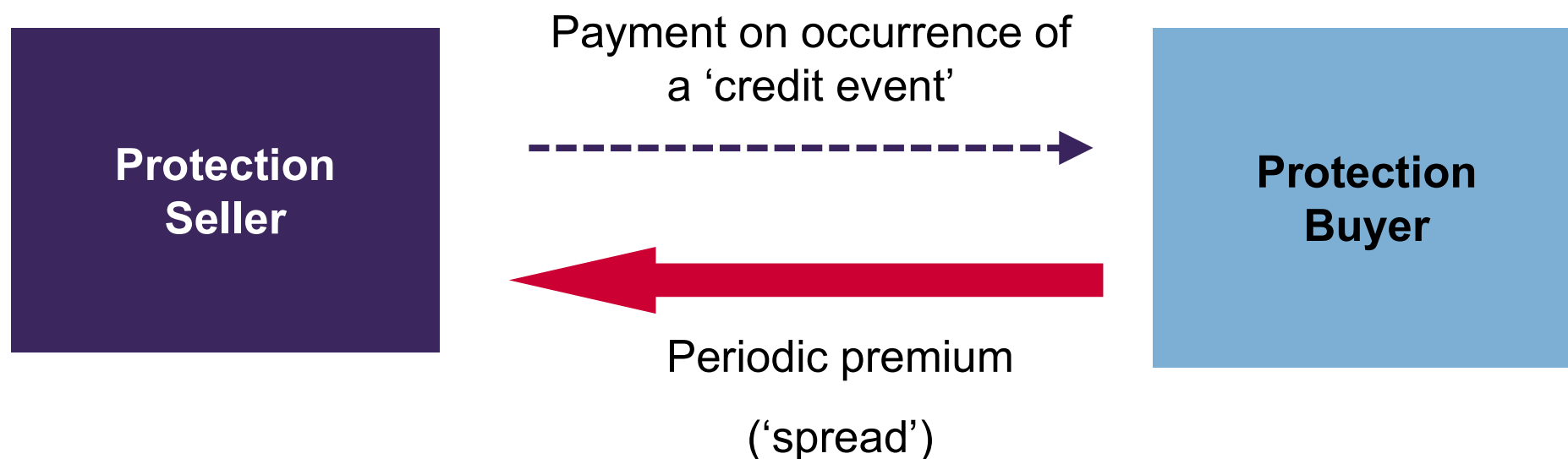
Credit Default Swap: Aim

- A product to buy or sell protection against a 'credit event' such as a default
- Similar to an insurance product but documented as a derivative

Uses:

- Buy protection to hedge credit risk on existing credit exposure:
 - Loans
 - Bonds
 - Other credit exposure (e.g. derivatives)
- Sell or buy protection to create a synthetic long or short position in particular counterparty/group of counterparties (portfolio management)
- Note that naked CDS positions have seen regulatory scrutiny
 - For example, in the EU buying 'naked' CDS protection on sovereign names is banned

CDS Structure



- Spread is usually paid quarterly and is quoted in basis points (bps) per annum
- Usually CDS trade with a fixed spread (100 bps) and upfront payments
 - This makes them more standardised - **Portfolio Compression**
- The market still quotes on an all running spread basis sometimes using "Quoted Spreads"

CDS: Credit Event Definitions

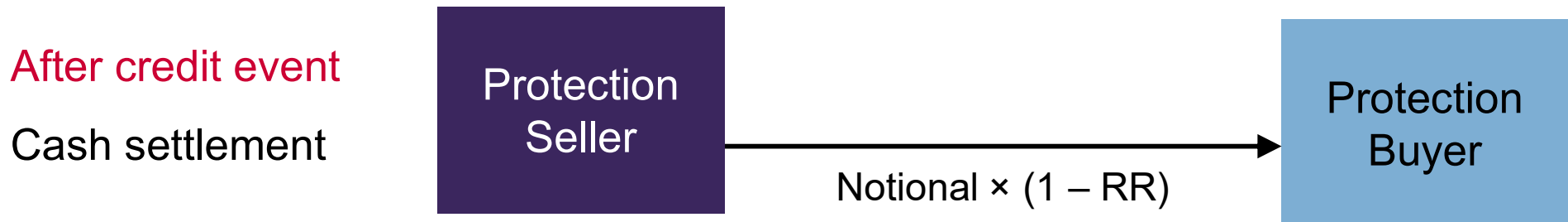
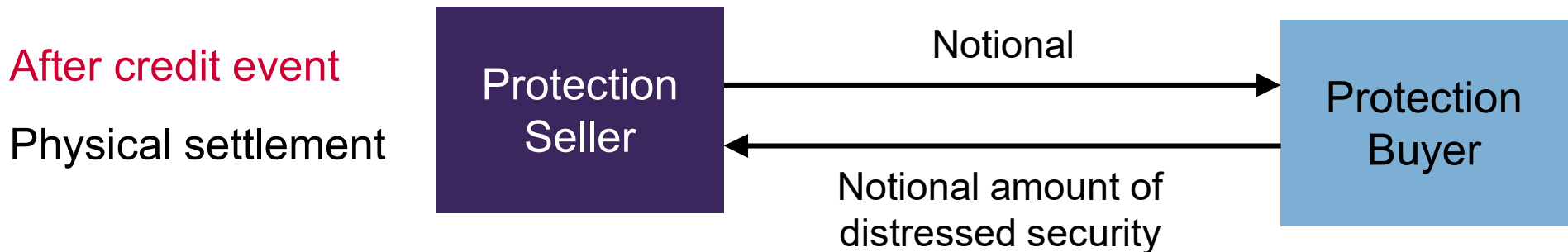
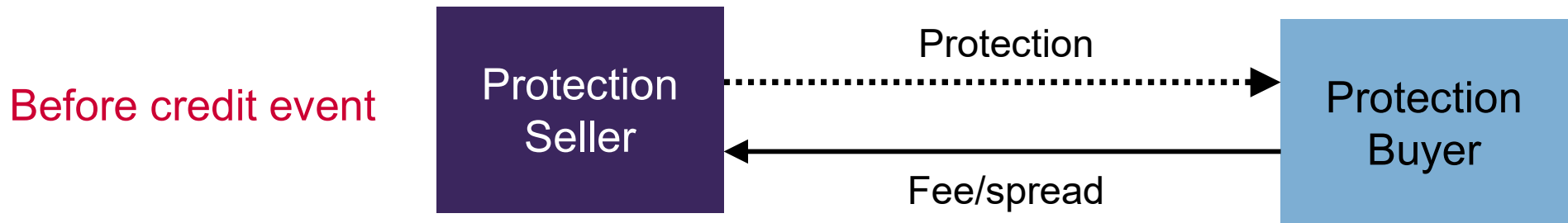
- Bankruptcy
 - Failure to pay
 - Obligation cross default (or acceleration)
 - Repudiation/moratorium
 - Restructuring
-
- Note that CDS payoffs are difficult to document due to the problem defining a default or not (especially for sovereigns) and also determining the default loss
 - This is in contrast to most derivatives where the quantities (fixing, strike, rate) can be defined relatively objectively

Recovery Rates and CDS Payoff

- If you own a bond then in default you still receive some recovery value
 - This is usually defined as the **recovery rate (RR)** and can take any value between 0% and 100%
 - The RR is never known prior to default but is seen in the secondary market after the default (e.g. Lehman Brothers bonds traded at around 10% after default)
- It therefore makes sense for the payoff of a CDS contract to be equal to $(1 - RR)$
- For example, buy a bond and hedge by buying CDS protection:

	Scenario 1	Scenario 2	Scenario 3
Bond	40	80	10
CDS	60	20	90
Total	100	100	100

Physically or Cash Settled CDS Structures



CDS Settlement

- Settlement process has a number of potential problems
 - Cheapest-to-deliver optionality (Conseco Corp, Railtrack restructurings 2001)
 - Delivery squeeze (Parmalat 2003, Delphi 2005), Lehman had \$400 billion of (gross) CDS notional and \$155 billion of debt – **Portfolio Compression important**
- Most large credit events are now settled by an Auction
- Settlement process is aided by reducing the gross notional as much as possible (portfolio compression)

Difficulty in Defining a CDS *Credit Event*

- Switzerland cancelled Credit Suisse Tier 1 bonds as a condition to the UBS deal
- But the CDS reference the Tier 2 bonds which were unaffected
- Tier 1 bonds are not suitable to be referenced in CDS contracts (e.g., perpetual maturities)

Schrödinger's swap: the audacious plan to trigger Credit Suisse's CDS

So crazy it might just work?



Difficulty in Defining a CDS *Payout*

Russia CDS to pay out finally following auction

12 Sep 2022 16:19

5 min read

EMEA, Emerging Markets

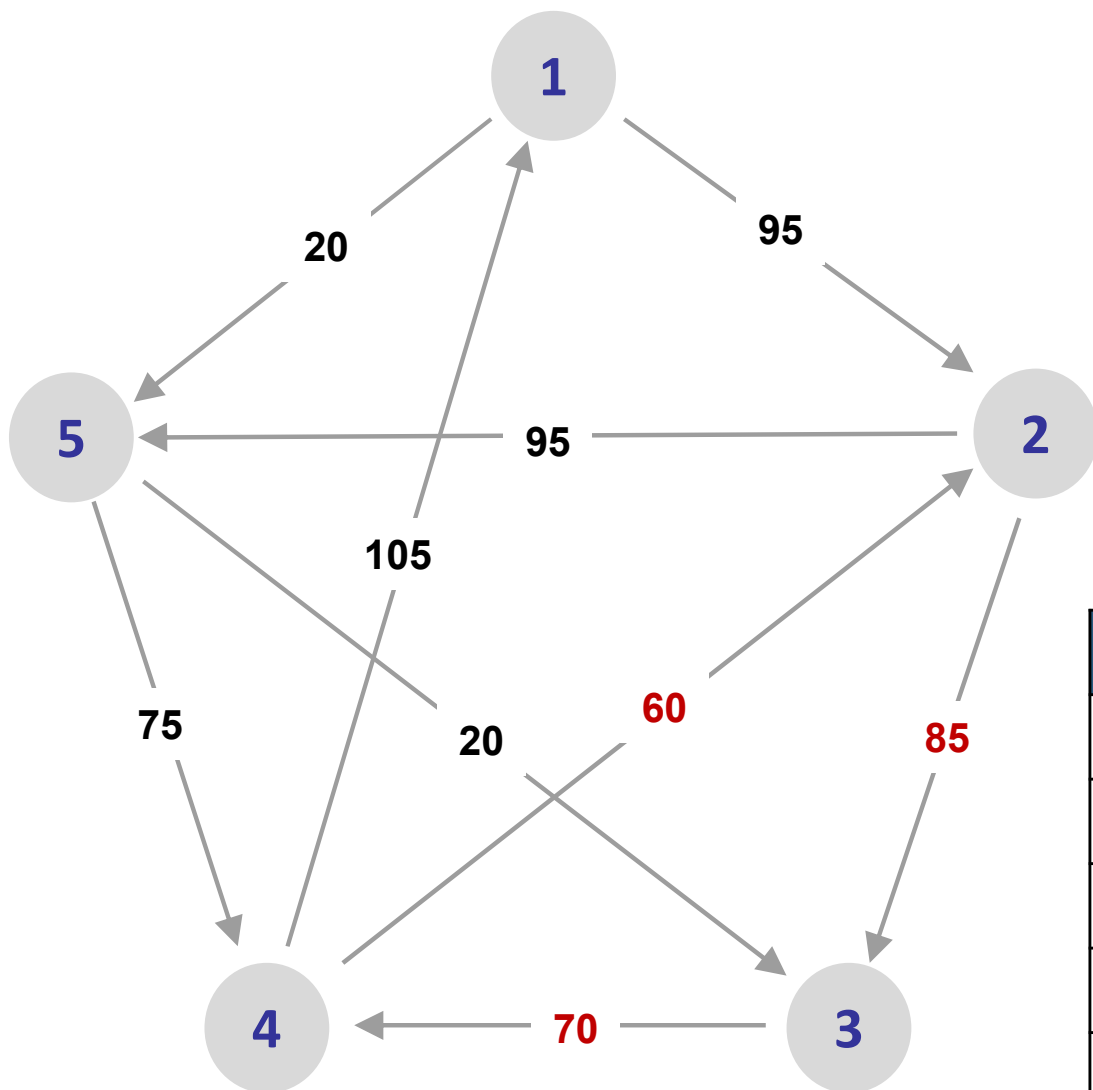
Christopher Whittall

Russia credit default swap protection holders are set to receive what many may consider a smaller-than-expected payout following the conclusion of a closely watched auction process on Monday to establish the value of the contracts.

[After months of painstaking preparations](#), which have been complicated by Western sanctions against Moscow, the auction determined a settlement price of 56 cents for Russia CDS. That means CDS holders will recoup 44 cents for every dollar of default protection they bought on Russian hard currency debt.

While substantial, that is still well below levels implied by Russian bond prices in the immediate aftermath of Moscow's invasion of Ukraine, some of which traded below 20% of face value (suggesting a potential CDS payout of more than 80 cents for every dollar). The final settlement price of 56 is also above the initial market midpoint of 48 cents established in the first round of the auction, where dealers provide indicative bids and offers on Russian bonds.

Portfolio Compression and CDS



As discussed in Collateral and Margins lecture

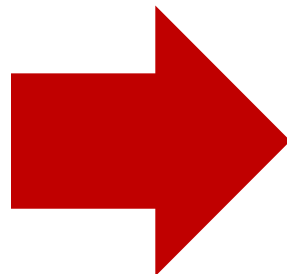
Must be held fixed

	C1	C2	C3	C4	C5	Total
C1	-	95	0	-105	20	10
C2	-95	-	85	-60	95	25
C3	0		-	70	-20	-35
C4	105		-70	-	-75	20
C5	-20		20	75	-	-20

Portfolio Compression Example

Instrument details	
Underlying	Nestle SA
Fixed coupon	100 bps
Maturity	20 September 2026
Dealer Bank's position	100 (net long), 400 (gross longs)

Counterparty	Long/(Short)
A	100
B	(50)
C	150
D	(100)
E	50
F	(150)
G	100




























Counterparty	Long/(Short)
Any of A, C, G or combination including E	100



CDS and Credit Spreads

CDS Spreads

- Reflect the cost of credit risk
- Compared to ratings, may be more forward-looking
- These quotes are in basis points (bps) per annum (5-year maturity)

Country	S&P Rating	5 Years Credit Default Swaps				
		5Y CDS	Var 1m	Var 6m	PD (*)	Date
 Netherlands	AAA	12.88	-2.87 %	+40.77 %	0.21 %	18 Jun
 Denmark	AAA	13.49	+4.82 %	0.00 %	0.22 %	18 Jun
 Austria	AA+	14.01	0.00 %	+83.38 %	0.23 %	18 Jun
 Germany	AAA	14.09	+5.70 %	+85.88 %	0.23 %	18 Jun
 Sweden	AAA	14.66	+5.24 %	0.00 %	0.24 %	18 Jun
 Japan	A+	16.02	-28.67 %	-6.43 %	0.27 %	18 Jun
 United Kingdom	AA	18.24	-4.45 %	+167.45 %	0.30 %	18 Jun
 Finland	AA+	19.82	-3.18 %	0.00 %	0.33 %	18 Jun
 Belgium	AA	22.55	-5.85 %	+120.65 %	0.38 %	18 Jun
 Ireland	AA	23.67	-6.03 %	+59.39 %	0.39 %	18 Jun
 France	AA	25.00	-8.73 %	-9.06 %	0.42 %	18 Jun
 Australia	AAA	26.67	-0.07 %	-24.23 %	0.44 %	18 Jun
 United States	AA+	29.68	-54.47 %	+18.72 %	0.49 %	18 Jun
 Canada	AAA	39.44	+0.28 %	+0.48 %	0.66 %	18 Jun
 Portugal	BBB+	46.10	-10.42 %	+19.40 %	0.77 %	18 Jun
 Spain	A	48.80	-10.11 %	-11.77 %	0.81 %	18 Jun
 China	A+	57.96	-19.48 %	+7.63 %	0.97 %	18 Jun
 Indonesia	BBB	82.90	-11.59 %	-14.50 %	1.38 %	18 Jun
 Italy	BBB	92.57	-17.89 %	-28.57 %	1.54 %	18 Jun
 Greece	BB+	92.63	0.00 %	0.00 %	1.54 %	18 Jun
 Mexico	BBB	105.91	-12.22 %	-16.41 %	1.77 %	18 Jun
 Brazil	BB-	182.84	-17.61 %	-26.39 %	3.05 %	18 Jun
 Turkey	B	471.98	-14.51 %	-8.74 %	7.87 %	18 Jun
 Egypt	B	1561.58	-19.90 %	+187.74 %	26.03 %	18 Jun
 Russia	NR	13775.17	0.00 %	0.00 %	100.00 %	18 Jun

(*) Implied probability of default, calculated on the hypothesis of a 40% recovery rate.

Source: www.worldgovernmentbonds.com

CDS Spread History (US)

Historical Data

Data Source: from 16 Apr 2017 to 18 Jun 2023

The **United States 5 Years Sovereign CDS** reached a maximum value of 69.76 (19 May 2023) and a minimum yield of 8.20 (22 June 2021).

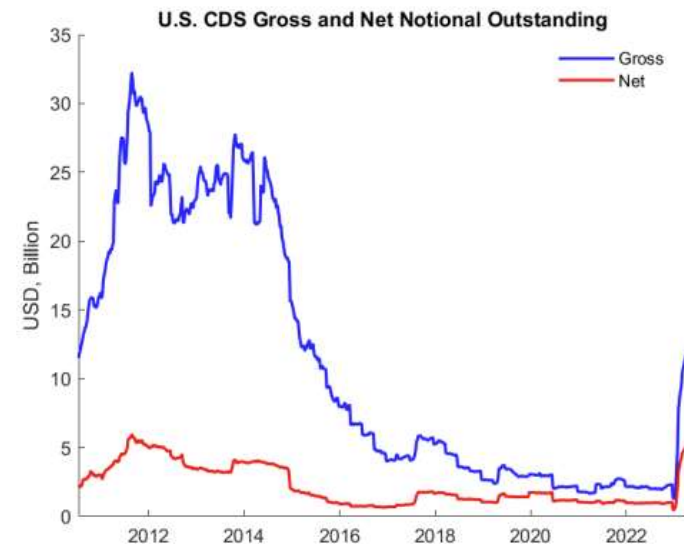
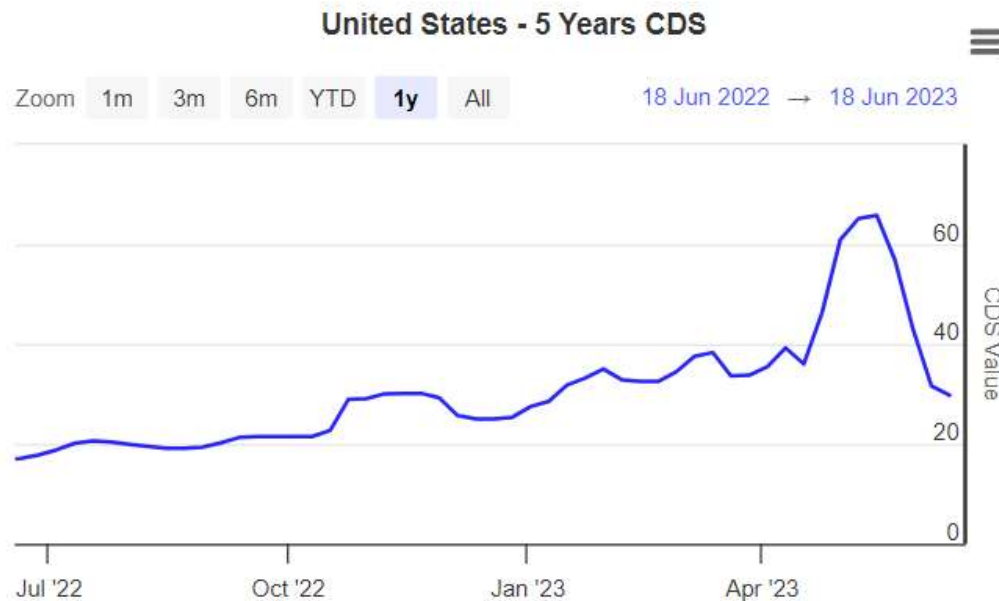


Figure 2: Gross and Net Notional Outstanding in U.S. CDSs. The chart shows the gross and net notional outstanding amounts in U.S. CDS contracts. Source: DTCC Kinetics.

Implied Default Probabilities (I)

- Approx. annual default probability:

$$PD(1) = 1 - \exp\left(-\frac{s}{1 - RR}\right) \approx \frac{s}{1 - RR}$$

Denmark $PD(1) = 1 - \exp\left(-\frac{0.1349\%}{1 - 40\%}\right) = 0.22\%$

Greece $PD(1) = 1 - \exp\left(-\frac{0.9263\%}{1 - 40\%}\right) = 1.53\%$

Egypt $PD(1) = 1 - \exp\left(-\frac{15.62\%}{1 - 40\%}\right) = 22.92\%$

Russia $PD(1) = 1 - \exp\left(-\frac{137.75\%}{1 - 40\%}\right) = 89.93\%$

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(*) Implied probability of default, calculated on the hypothesis of a 40% recovery rate.

- What does the risk-neutral ('implied') default probability mean?

Implied Default Probabilities (II)

This chart shows how blatantly negative Credit Suisse is perceived by the markets. CDS markets are pricing in a probability of default of 38%.



11:02 AM • Mar 15, 2023 • 1.4M Views

Credit Suisse $PD(5) = 1 - \exp\left(-\frac{5.5381\%}{1 - 40\%} 5\right) = 37.0\%$

Deutsche Bank $PD(5) = 1 - \exp\left(-\frac{0.9948\%}{1 - 40\%} 5\right) = 7.96\%$

$$PD(T) = 1 - \exp\left(-\frac{s}{1 - RR} T\right) \approx \frac{s \times T}{1 - RR}$$

$s = \text{spread}$

$RR = \text{Recovery rate}$

$T = \text{CDS maturity}$

The Black Scholes Formula

$$C = S_0 N(d_1) - K e^{-rT} N(d_2)$$

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left[\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T \right] = d_2 + \sigma\sqrt{T}$$

- We also have:

$$\Pr(S_T > K) = N(d_2)$$

- This is the risk-neutral probability of exercise for a call option
- How often do option traders look at this?

Risk-Neutral (Implied) Default Probabilities

- Risk-neutral PDs should not be expected to coincide with real world default probabilities
- Also, if we change the RR then the risk-neutral PD changes
- There is (effectively) no traded RR and the 40% figure typically used is just a market convention
- So, like $N(d_2)$ the risk-neutral PD is an intermediate calculation in a valuation formula and has limited meaning
- Also notice that a CDS is like an out-of-the-money (OTM) option
 - Would you ever buy an OTM option if you thought the expected payoff was zero?

Distressed Credits

- For names near or at default, periodic premiums don't make sense

 Russia	NR	13775.17	+27.50 %	+10949.31 %	100.00 %	27 Jun
(*) Implied probability of default, calculated on the hypothesis of a 40% recovery rate.						

- The above calculation more accurately:

$$h \approx \frac{S_{CDS}}{LGD} = \frac{13775.17}{10000 \times 60\%} = 230\%$$

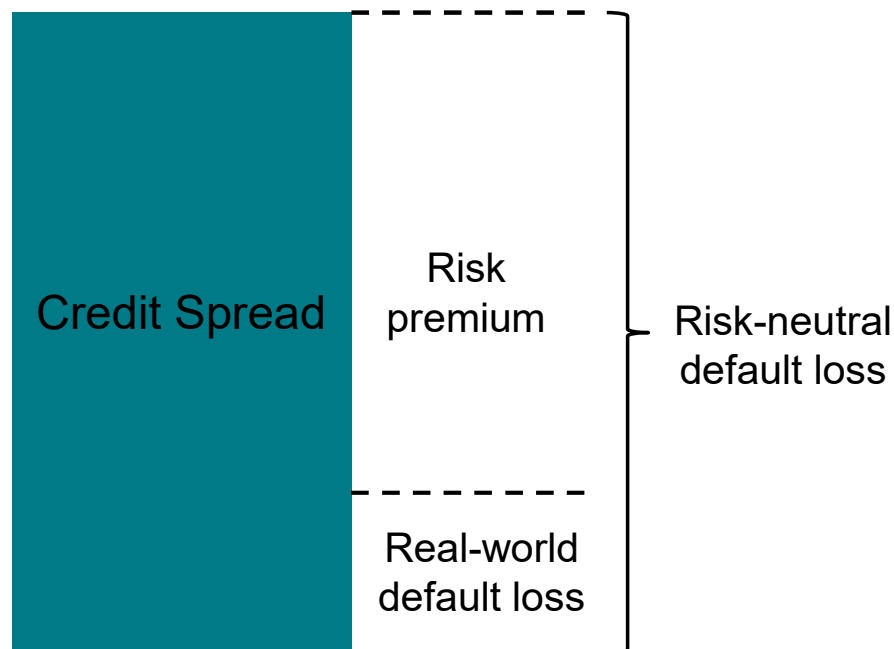
- Probability of default in next year is $1 - \exp(-h) = 89.9\%$
- (Note that a hazard rate of more than 100% is ok)
- Five-year default probability
$$1 - \exp(-h \times 5) = 99.999\%$$
- Spread of 13,775 bp pa is relatively meaningless and we need to know the RR!
- Distressed credits trade with up-front premiums due to annuity risk (this would be close to the expected $(1 - R)$ for a name very close to default)

Risk-Neutral and Real World PDs

Giesecke et al. report an average credit spread of 153 bps from 1866 to 2008

The average default loss was bps

- ‘Credit risk premium’ is substantial



	Real world loss (bps)	Risk neutral loss (bps)	Ratio
Aaa	4		
Aa	6		
A	13		
Baa	47		
Ba	240		
B	749		
Caa	1690		

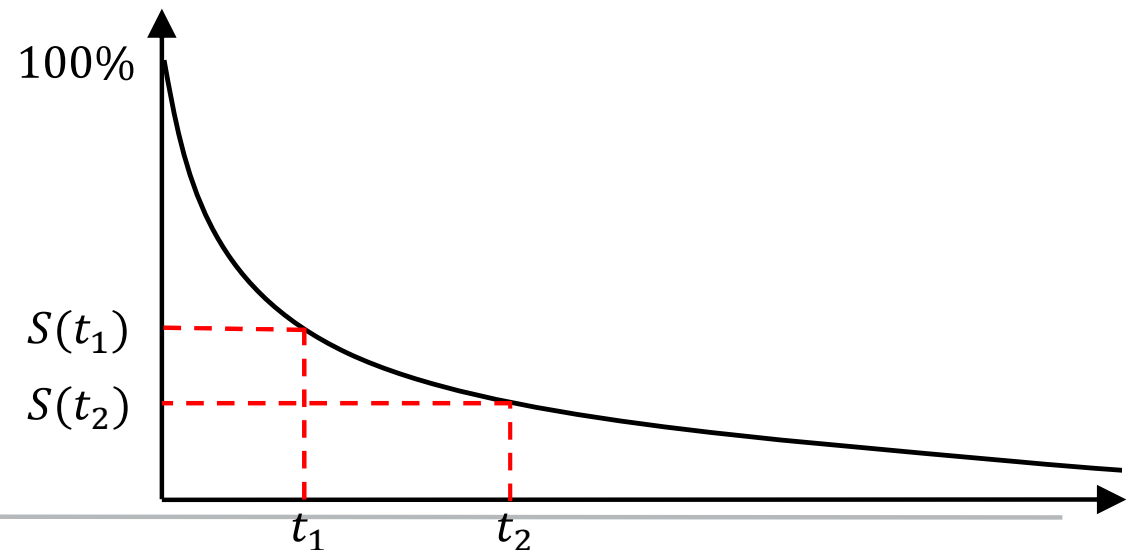
Source: Hull, J., M. Predescu and A. White, 2004



CDS Valuation

CDS Valuation

- A CDS can be split into two legs
 - Premium payments that occur contingent on no prior default (aka credit event)
 - A default payment that occurs contingent on a credit event
- Generally, it is assumed that interest rates, defaults and RRs are independent
- Note that some cashflows are contingent on survival
 - Denote the survival probability for t as $S(t)$
 - The default probability between dates t_1 and t_2 is then $S(t_1) - S(t_2)$
- Discount factors $B(t, u) = \exp(-rt)$ with r being a continuously compounded interest rate



Mathematical Model for Default

- Default can be modelled as a Poisson process
 - The process is driven by an intensity or hazard rate h
 - Probability of default in an infinitively small period conditional on survival is hdt
 - Probability of no default is given by $S(t) = \exp[-ht]$
- The value of a unit cashflow can be written as:
 - $\exp[-ht] \times \exp[-rt]$
 - Since this can be written as $\exp[-(r + h)t]$ we can interpret $r + h$ as a risky rate
- A CDS contract is two legs
 - Default leg (if there is a default)
 - Premium leg (contingent on there not being a default)

CDS Default Leg

- Default can happen from now until the maturity date (T)
- Default payment is $(1 - RR)$

$$PV_{default} = (1 - RR) \int_0^T B(t) dS(t) = -LGD \int_0^T h \exp(-(r + h)u) du$$
$$= -(1 - RR) \cdot h \cdot \frac{1 - \exp(-(r + h)T)}{(r + h)}$$

- This is negative so for selling CDS protection

CDS Premium Leg

- Premium is paid conditional on no default
- It is usually only quarterly (discrete) but there is an accrued premium at default which means it can be considered continuous
- Unit premium leg

$$PV_{premium}^1 = \int_0^T \underbrace{\exp(-ru)}_{\text{Discount factor}} \underbrace{\exp(-hu)}_{\text{Survival probability}} du = \int_0^T \exp(-(r+h)u) du = \frac{1 - \exp(-(r+h)T)}{(r+h)}$$

- Fair CDS spread

$$s_{CDS} = \frac{PV_{default}}{PV_{premium}^1} \approx (1 - RR) \cdot h$$
$$h \approx \frac{s_{CDS}}{1 - RR}$$

Calibration to CDS Curve (I)

- CDS quotes can be seen at several tenors (5-year is most liquid)
- In order to fit a term structure of CDS quotes then we need a (deterministic) functional form for the hazard rate


$$S(u) = \exp \left[- \int_0^u h(x) dx \right]$$

- Assume a grid of observable quotes given by the grid $[t_0 = 0, t_1, t_2, \dots \dots \dots t_n)$
- For simple forms, can calibrate a set of hazard rates sequentially (bootstrap)

Calibration to CDS Curve (II)

- Can still calculate integrals analytically – for example piecewise constant h and r
- Default leg for period i

$$(1 - RR) \cdot h_i \cdot \frac{1 - \exp(-(r_i + h_i)\Delta t_i)}{(r_i + h_i)} \prod_{j=1}^{i-1} \exp(-(r_j + h_j)\Delta t_j)$$



Default leg for this period Risky discount factor

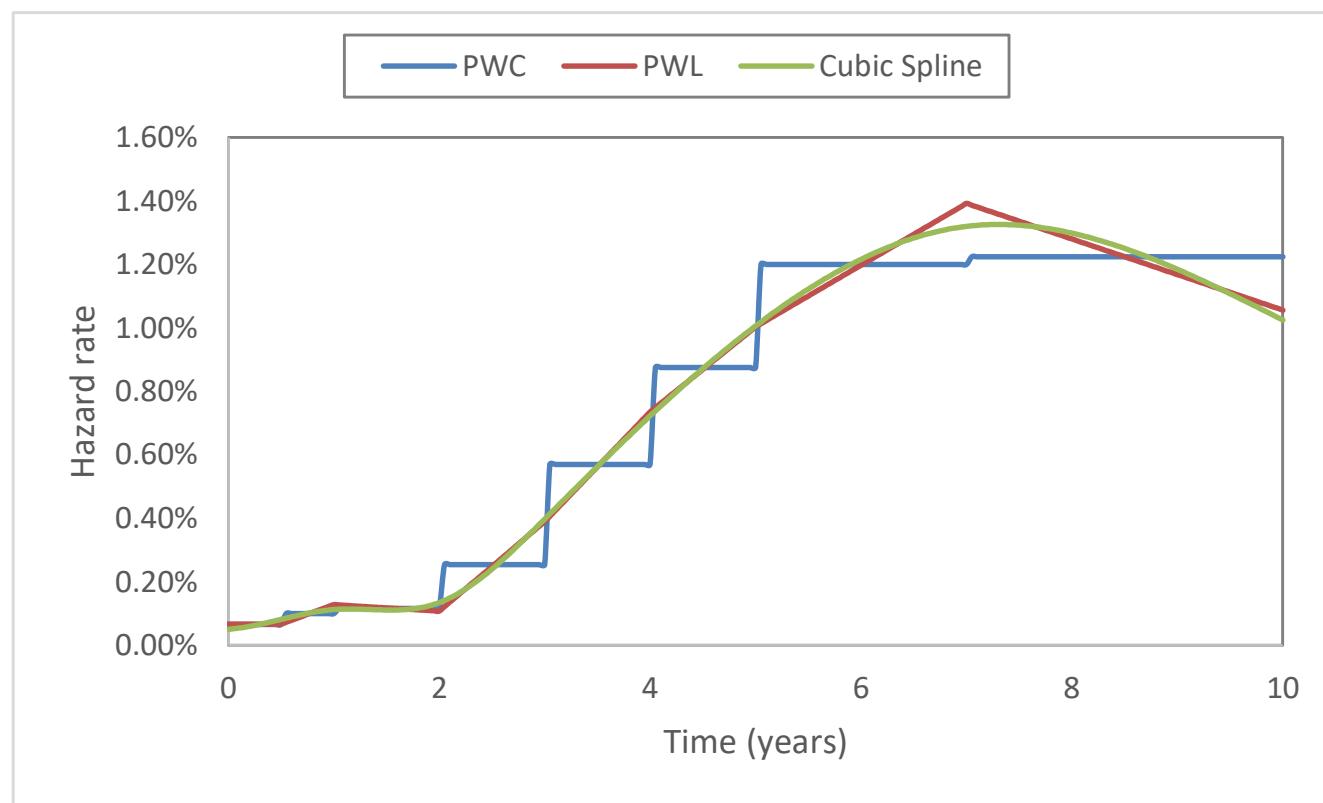
- Premium leg for period i (>1)

$$\frac{1 - \exp(-(r_{i-1} + h_{i-1})\Delta t_{i-1})}{(r_i + h_i)} \prod_{j=1}^{i-1} \exp(-(r_j + h_j)\Delta t_j)$$

- Can also do numerically (spreadsheet does both)

Calibration to CDS Curve (Excel Example)

- Comparison for different hazard rate functions
 - Piecewise constant
 - Piecewise linear
 - Cubic spline (global optimisation)



	CDS
6M	0.12%
1Y	0.18%
2Y	0.32%
3Y	0.50%
4Y	0.75%
5Y	0.92%
7Y	1.20%
10Y	1.35%



Proxy CDS Curves

CDS Market Over Time

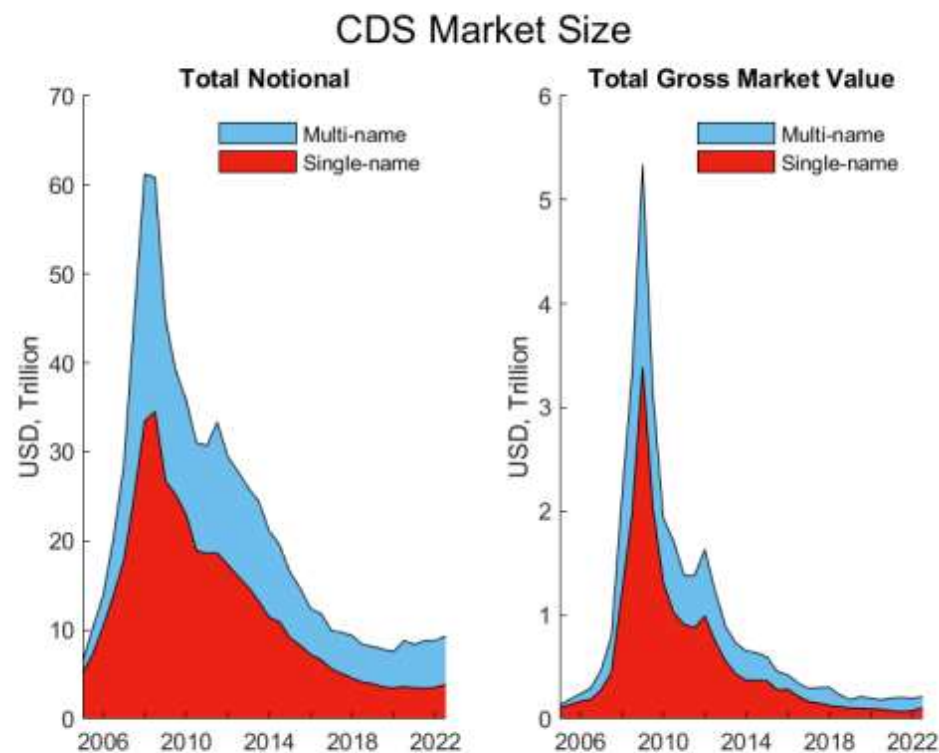


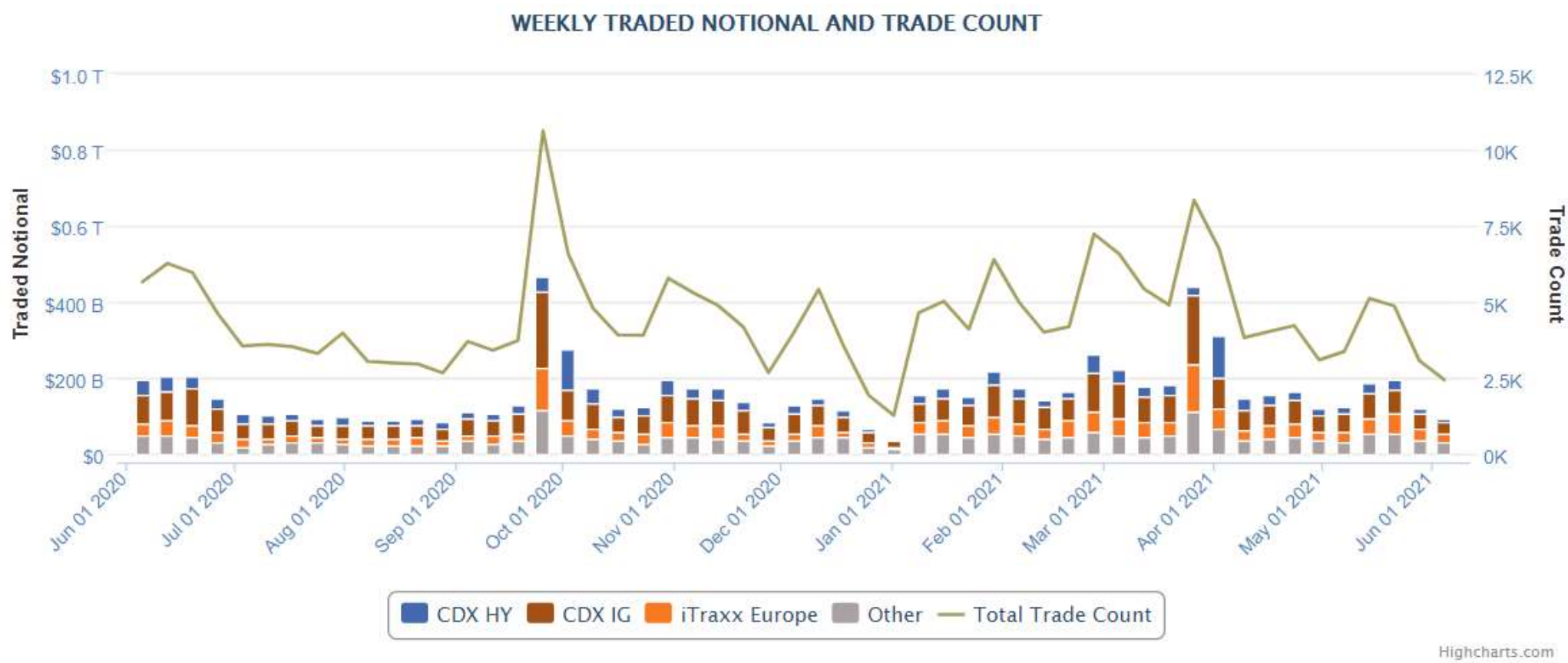
Figure 1: Total Notional and Market Value Outstanding Across the Entire CDS Market. The chart shows the total gross notional amount outstanding (left panel) and the associated market value (right panel) across the entire CDS market. Source: BIS.

Index and Single Name Liquidity



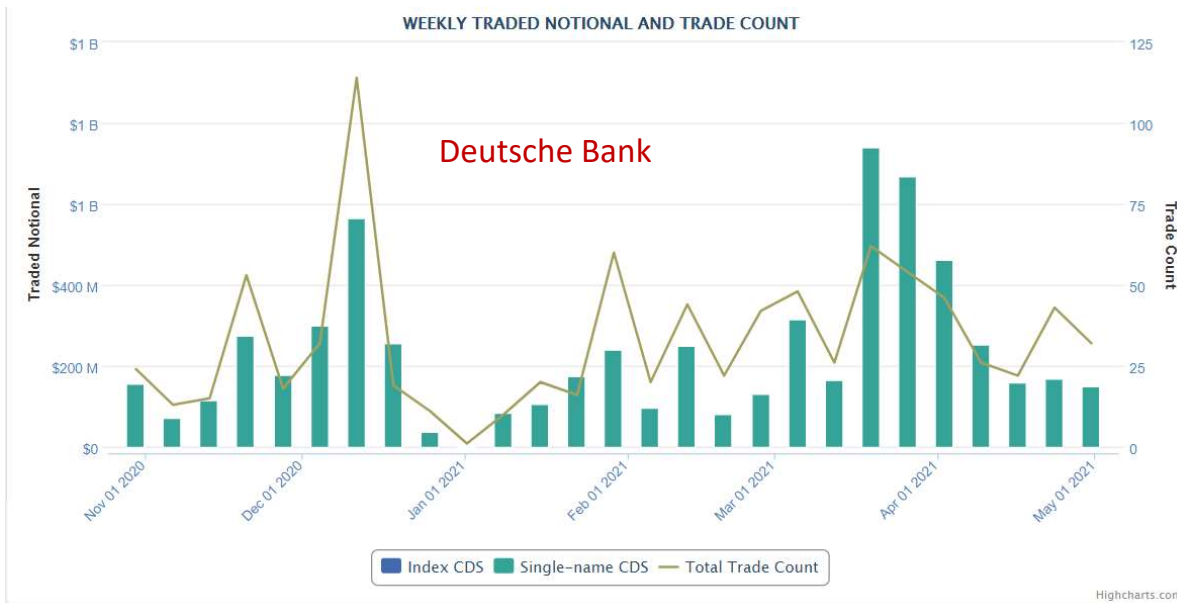
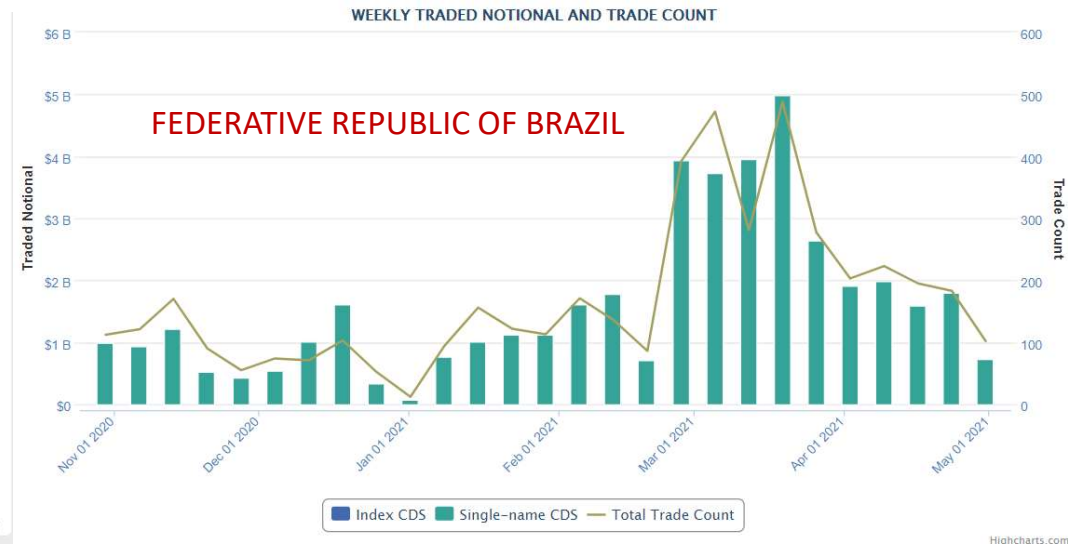
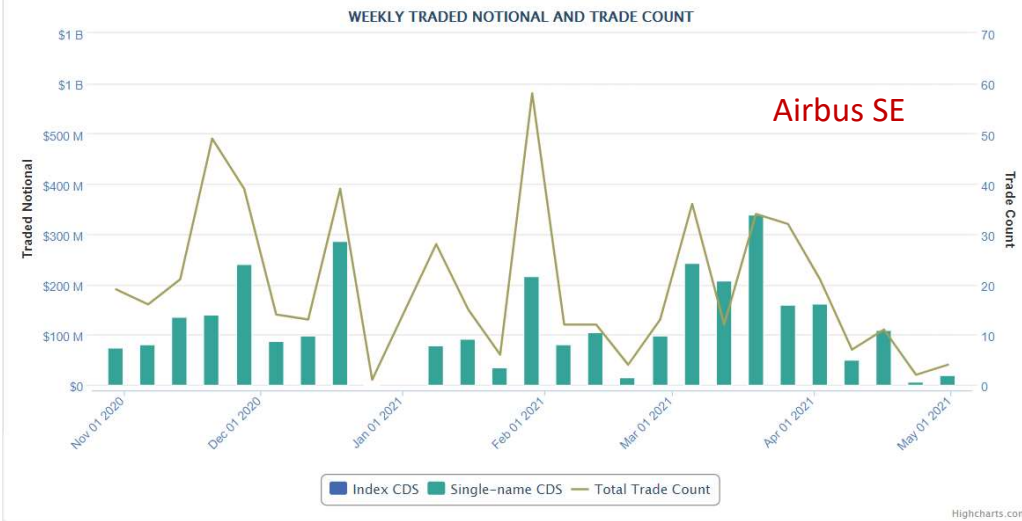
Source: ISDA SwapInfo

Index Liquidity



Source: ISDA SwapInfo

The CDS Market Liquidity



Source: ISDA SwapInfo

Example Rationale – CVA Capital Requirements

Not all counterparties have traded credit spreads. However, the FRTB-CVA framework must capitalise CVA risk arising from dealing with all counterparties, including ones that are not actively traded in credit markets (“illiquid counterparties”). Therefore, in order to use the FRTB-CVA framework, a bank is required to have a methodology for approximating the credit spreads of illiquid counterparties (see Section B.1(f) of the draft Accord text).

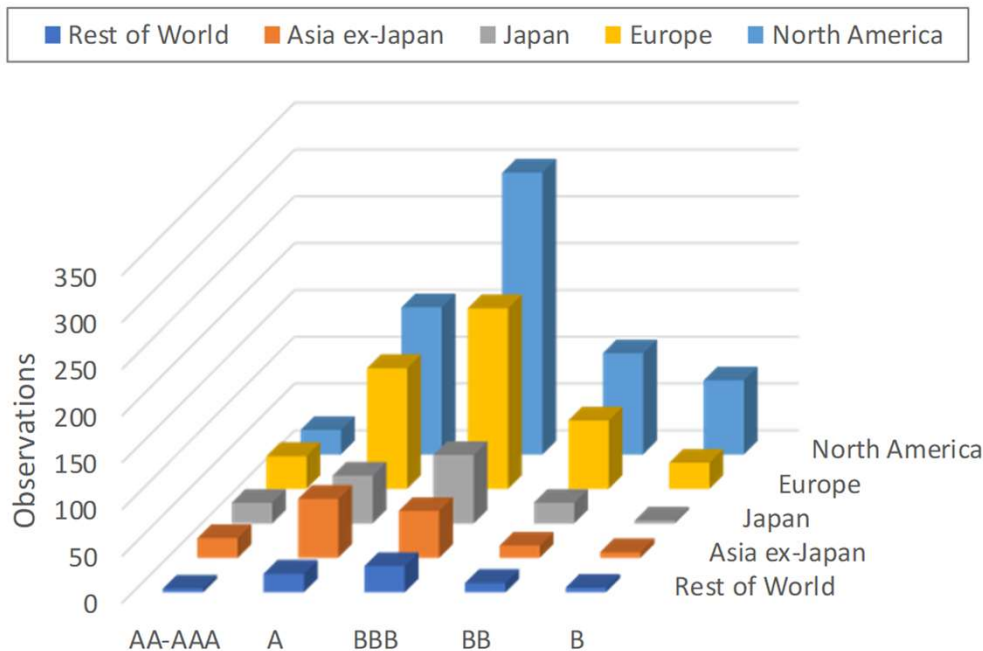
Banks normally develop the capability of calculating CVA sensitivities in order to manage their CVA risk. Typically, CVA risk management is performed by a dedicated function, such as the CVA desk. CVA sensitivities calculated by a bank without any internal function to use them would not be deemed reliable. Thus, the existence of a dedicated CVA risk management function will be a requirement.

Estimation of CDS Proxy Curves

- Two potential approaches
 - Bucketing approach (cannot use very granular buckets due to lack of data)
 - Regression approach

$$\text{Log Spread}_i = X_{\text{global}} + X_{\text{sector}(i)} + X_{\text{region}(i)} + X_{\text{rating}(i)}$$

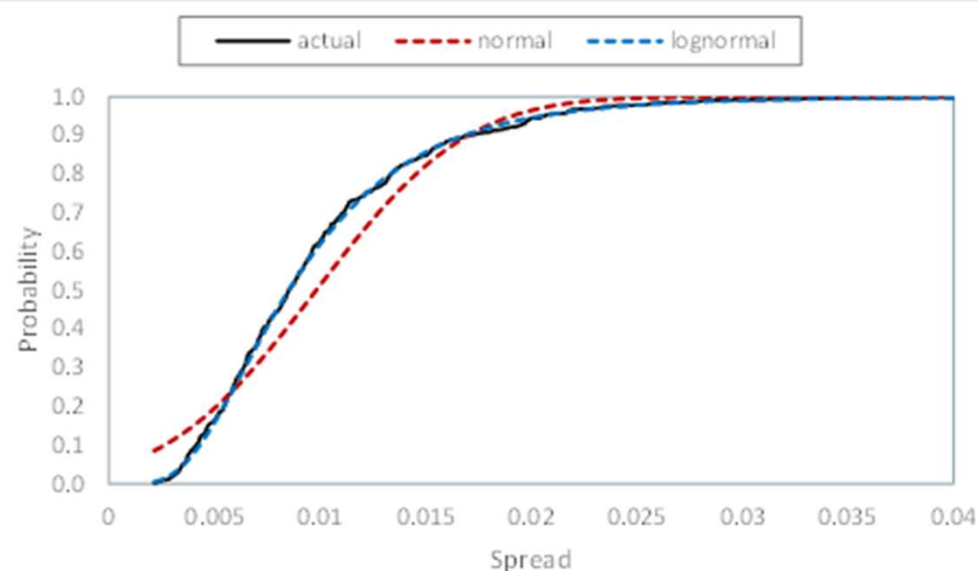
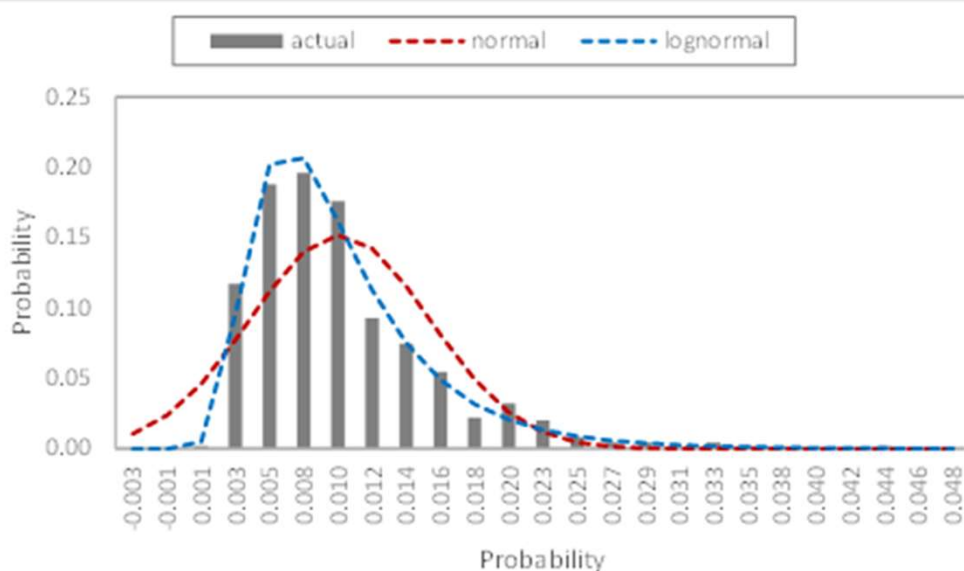
Financial CDS population



Region	Industry	Rating
Asia	Basic materials	Aaa
Eastern Europe	Consumer goods	Aa
Europe	Consumer services	A
Japan	Energy	Baa
Middle East	Financials	Ba
Latin America	Government	B
North America	Healthcare	Caa
Oceania	Industrials	
	Technology	
	Telecommunications	
	Utilities	

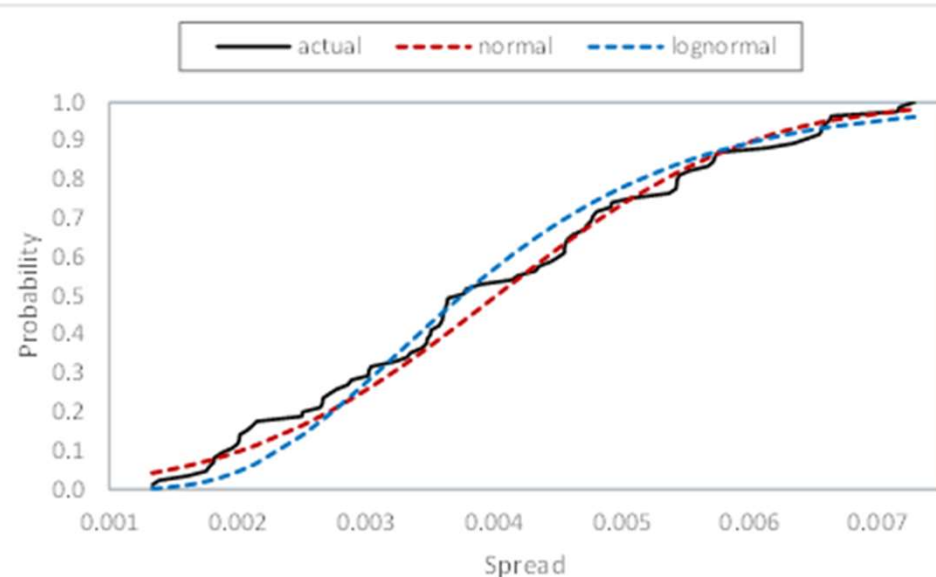
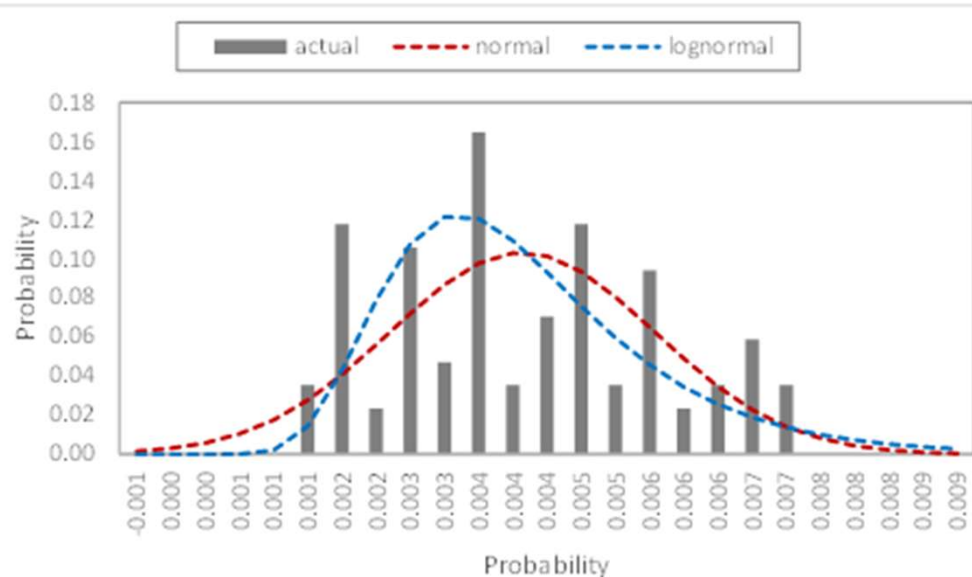
CDS Spread Distributions (I)

- Often well approximated by a lognormal distribution
 - For example, BBB in 2018



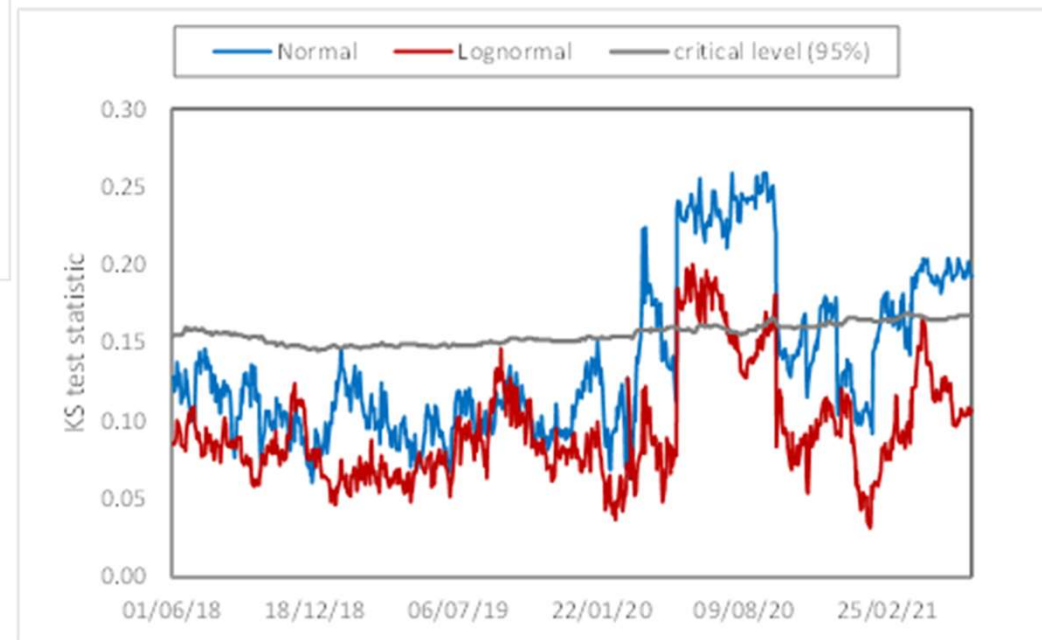
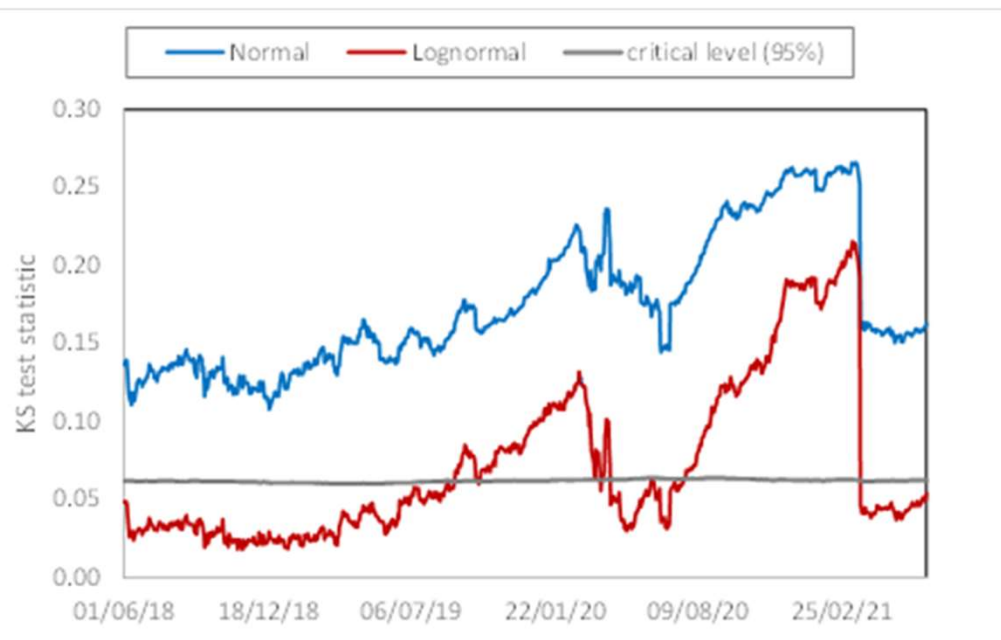
CDS Spread Distributions (II)

- Sometimes (but not often) normal distribution is closer
 - For example AA in 2018



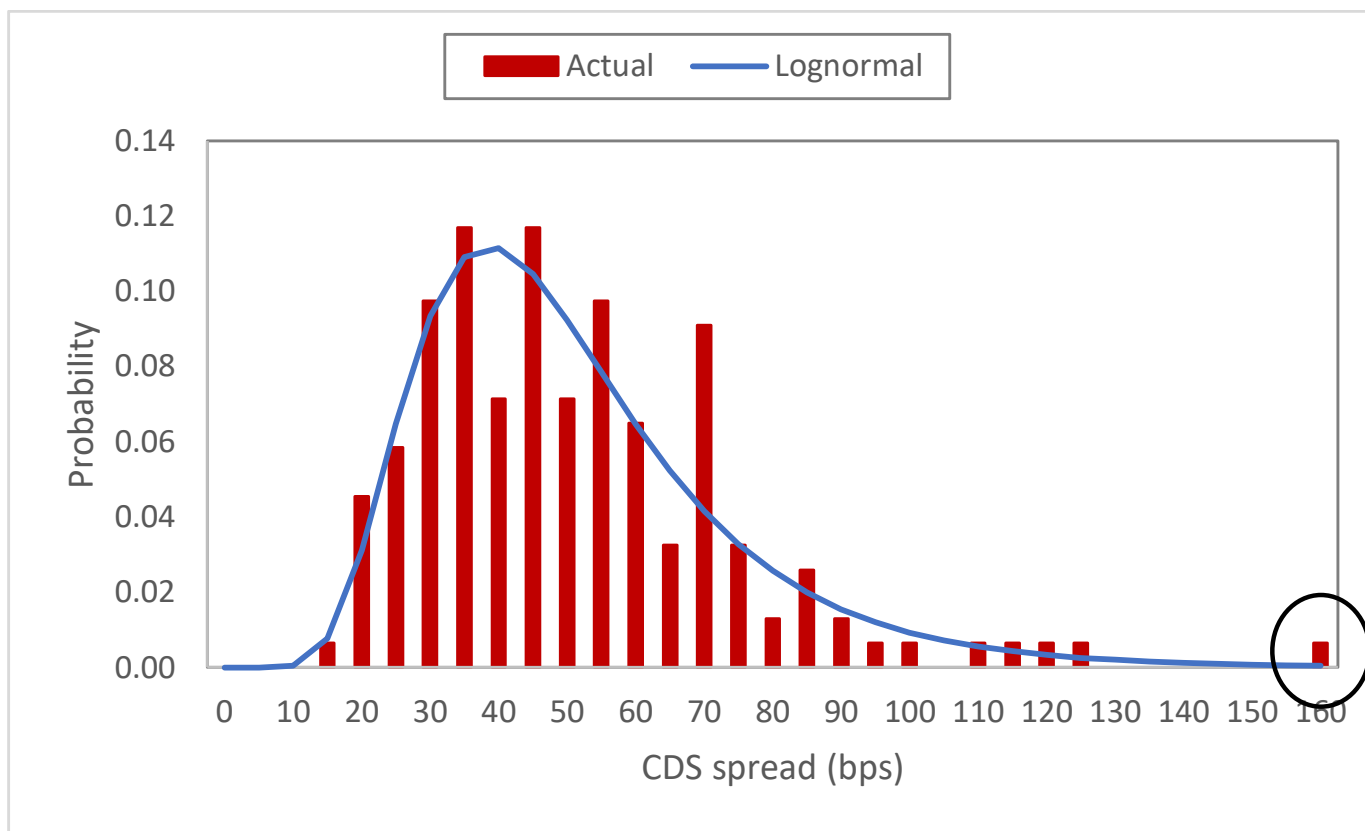
Kolmogorov–Smirnov Tests

- BBB (top right) and AA (bottom left)
 - Failures can be often linked to one or more “outliers”



Outlier Removal

- Deviation from lognormal suggests removing outliers (single-A example)



- Note a general problem here – cannot observe rating class credit spreads precisely

CDS Proxy Curves – Example

Rating	Coeff
Aaa	0.0000
Aa	0.4672
A	0.7611
Baa	1.2448
Ba	1.9351
B	2.5882
Caa	3.9599

Region	Coeff
Asia	-0.1820
Eastern Europe	-0.0444
Europe	-0.4152
Japan	-0.6069
Middle East	0.1509
Latin America	0.0000
North America	-0.2574
Oceania	-0.2106

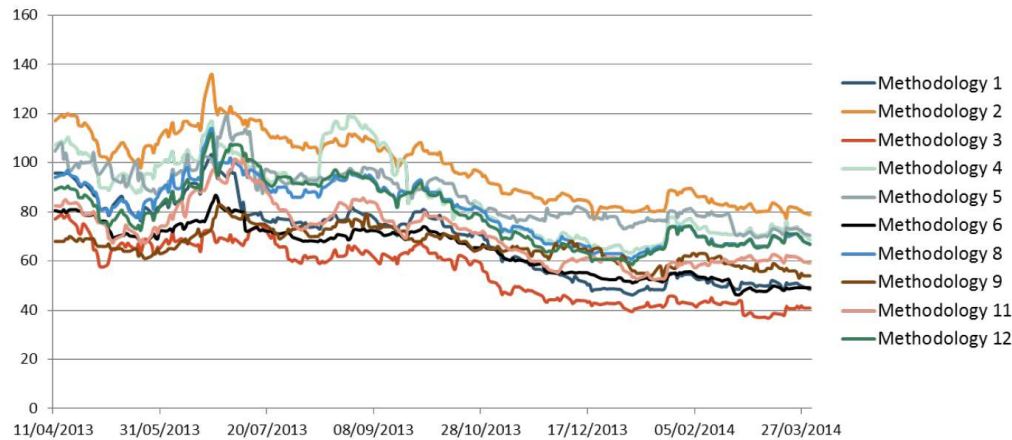
Industry	Coeff
Basic materials	-0.0025
Consumer goods	-0.0168
Consumer services	-0.0093
Energy	0.2062
Financials	0.1814
Government	-0.1635
Healthcare	-0.2806
Industrials	-0.0650
Muni	0.3385
Technology	0.0000
Telecommunications	-0.1149
Utilities	-0.1191

Rating	Region	Industry	Intercept	Rating	Region	Industry	Spread
Aaa	N America	Financial	-5.9364	0.0000	-0.2574	0.1814	0.245%
Aaa	Europe	Financial	-5.9364	0.0000	-0.4152	0.1814	0.209%
A	N America	Healthcare	-5.9364	0.7611	-0.2574	-0.2806	0.330%
A	N America	Muni	-5.9364	0.7611	-0.2574	0.3385	0.613%
A	Middle East	Financial	-5.9364	0.7611	0.1509	0.1814	0.788%
BB	Europe	Energy	-5.9364	1.9351	-0.4152	0.2062	1.484%
CCC	Europe	Healthcare	-5.9364	3.9599	-0.0444	-0.2806	6.910%
CCC	E Europe	Energy	-5.9364	3.9599	-0.0444	0.2062	16.290%

CDS Proxy Curves – Comparison

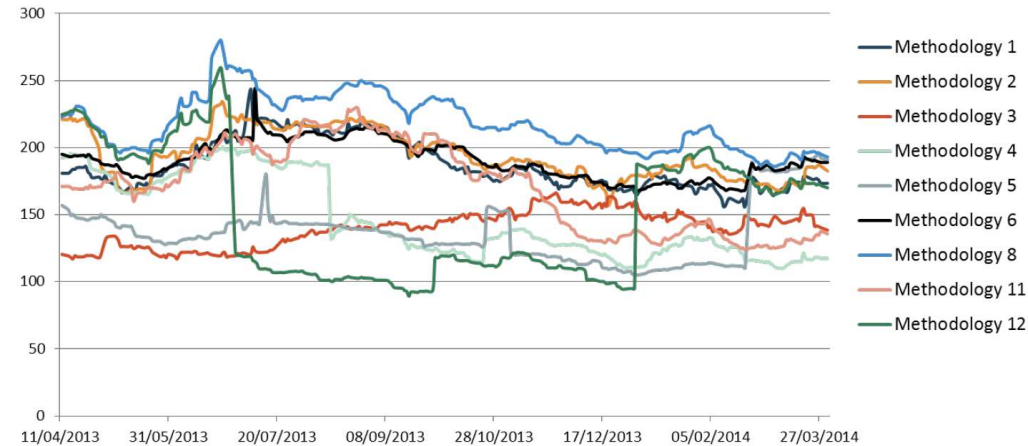
UK AA-rated insurance company

One-year history of 5y proxy spread
April 2013 – March 2014



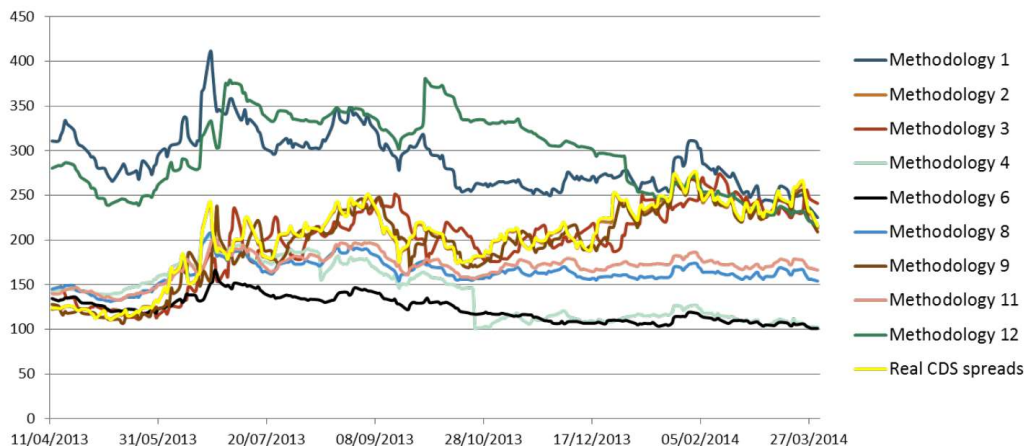
Japanese BB-rated airline

One-year history of 5y proxy spread
April 2013 – March 2014



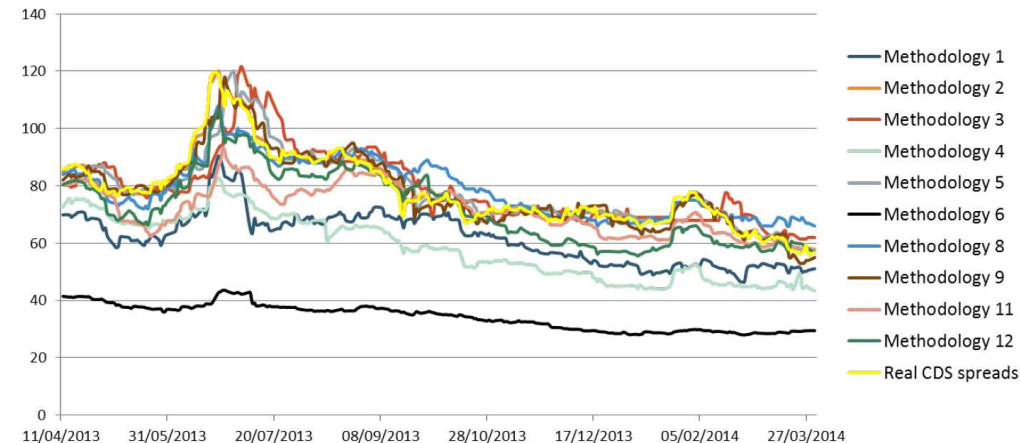
Government of Turkey

One-year history of 5y proxy spread
April 2013 – March 2014



Berkshire Hathaway

One-year history of 5y proxy spread
April 2013 – March 2014



Source: EBA



Stochastic Hazard Rate Models

Stochastic Credit Spread Models

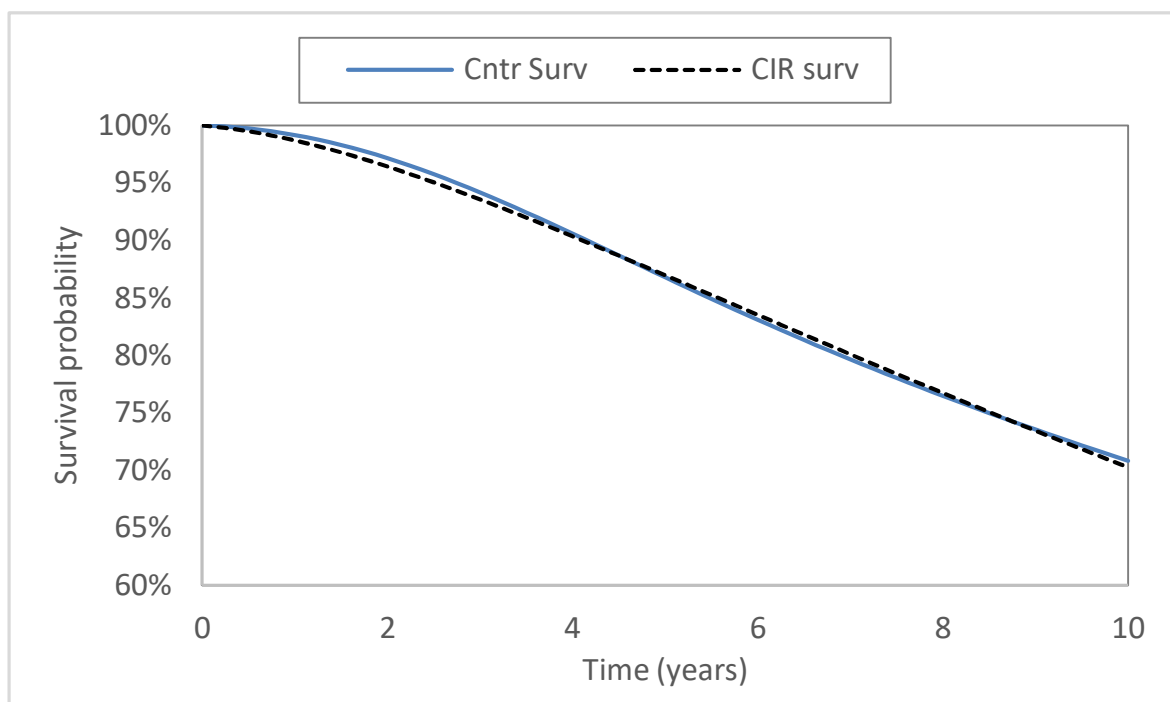
- Hull-White (HW) (extended Vasicek) - normal
 - Fits entire survival curve
 - Allows negative intensities (which will be quite considerable given the likely volatilities required and especially for low spread names)
 - Tractable
- Black-Karasinski (BK) – lognormal
 - Fits survival curve
 - Positive intensities and volatility is not restricted
 - Not tractable (e.g., no closed-form formula for survival probabilities)
- Cox-Ingersoll-Ross (CIR)
 - Positive intensities if ‘Feller Condition’ is met
 - Feller condition in practical terms restricts volatility
 - CIR++ approach for matching whole survival curve
 - Tractable

Stochastic CDS Modelling

- Cox Ingersoll Ross (CIR) model for stochastic default intensity (λ_t)

$$d\lambda_t = k(\theta - \lambda_t)dt + \sigma\sqrt{\lambda_t}dW_t$$

- Four parameters (λ_0, k, θ and σ) calibrated to survival curve and CDS options if available (constrained optimization, Feller condition)
- CIR++ can fit perfectly if required



$$S(t, T) = A(t, T)\exp(-B(t, T) \cdot h_t)$$

$$A(t, T) = \left[\frac{2he^{\frac{(h+k)(T-t)}{2}}}{2h + (h+k)(e^{k(T-t)} - 1)} \right]^{\frac{2k\theta}{\sigma^2}}$$

$$B(t, T) = \left[\frac{2(e^{k(T-t)} - 1)}{2h + (h+k)(e^{k(T-t)} - 1)} \right]$$

$$h = \sqrt{a^2 + 2\sigma^2}$$

Default Contagion

- Default contagion can be introduced by noticing (Lando 1998)

$$\tau = \inf \left\{ t: \exp \left(- \int_0^t \lambda_s ds \right) \geq U \right\}$$

- τ is the default time and $\int_0^t \lambda_s ds$ is the integrated intensity process up to time t and $\exp \left(- \int_0^t \lambda_s ds \right)$ is the integrated survival probability up to time t

- Simulate default intensity processes
 - Link stochastic intensity to other risk factors (e.g., interest rates) – more in xVA lectureOR
 - Link default times through a copula linking the U variables



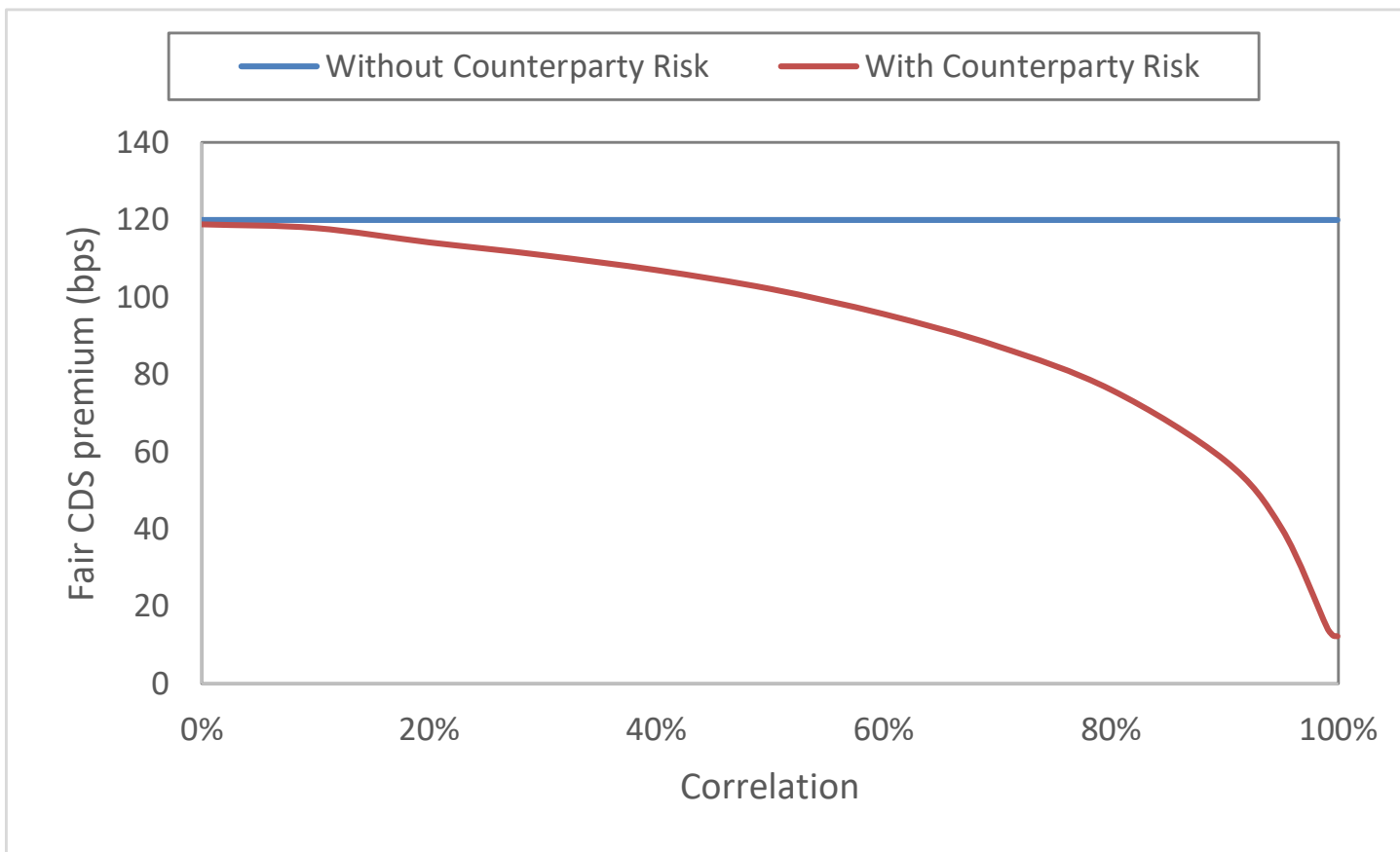
Counterparty Risk

Counterparty Risk

- Would you be happy buying CDS protection from a bank on another bank?
- Buying protection from a US Bank (AA+) on another US Bank (A-)?
- Buying protection from a European Bank (A-) on a US Bank (A-)?
- Massive wrong-way risk (WWR) problem

CDS Counterparty Risk

- Key aspect is correlation



Gregory J., 2011, "Counterparty risk in credit derivative contracts", The Oxford Handbook of Credit Derivatives, A. Lipton and A. Rennie (Eds), Oxford University Press.

Summary

- Single-name and index CDS are important derivatives
 - The definition of the payoff is difficult
- CDS valuation is straightforward assuming independence between default, recovery and interest rates
 - Portfolio problems (e.g., CDOs and credit portfolio risk) are more complex
- Most credits do not have liquid observable CDS quotes
 - Proxy methodologies are important
- Stochastic hazard rate models are difficult due to required positivity
 - But important for CVA computation (see xVA lecture)
- Counterparty risk in CDS contracts is important
 - But also difficult to model given the nature of default dependency