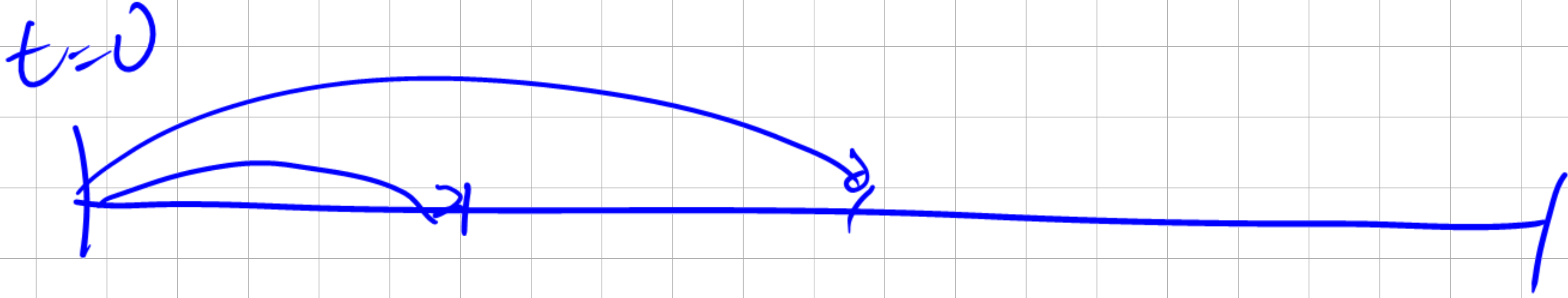


$C \in \mathbb{R}^{n \times n}$ $\qquad c_{ii} = 1$ $\qquad \lambda_i \approx 0$

$$C = S \cdot \Lambda \cdot S^T$$

$$\Lambda = \mathrm{diag}(\vec{\lambda})$$

$$m = \sum_{i=1}^{n} \mathbb{1}_{\{\lambda_i > 0\}}$$

= Rank of C

$$A = S \cdot \sqrt{\Lambda} = {}^{n}\begin{pmatrix} & \end{pmatrix} \cdot \begin{pmatrix} \ddots & \\ & 0\,0\,0\,0 \end{pmatrix}$$

$$A = \begin{pmatrix} & \end{pmatrix}^{n}_{m}$$

$$\underbrace{\hspace{6em}}_{n}$$

$$\begin{array}{cc} & n \\ n & \left(\begin{array}{c|c} \begin{array}{c} \text{non} \\ \text{zero} \end{array} & 0 \end{array}\right) \\ & \underbrace{\hspace{3em}}_{m} \quad \Lambda \end{array}$$

$$A \cdot A^T$$

X : $E^{\mathcal{M}(\ )}\left[X(T)\right] = X(0) \quad \forall T > 0$

$$E^{\mathcal{M}(N)}\left[\frac{Asset_i(t)}{N(t)}\right] = \frac{Asset_i(0)}{N(0)}$$

$\forall\ t > 0$

$v_i(t)$

$X_i := \left(\frac{v_i(t)}{N(t)}\right)$: martingale

$v_i(t) = X_i(t) \cdot N(t)$

"T-forward measure"

Numeraire: $P_T(t) > 0 \quad \forall\ t \le T$

$$\left(\frac{v(0)}{P_T(0)}\right) = E^{\mathbb{M}(P_T)}\left[\frac{c(t)}{P_T(t)}\right]$$

$$v(0) = P_T(0) \cdot E\left[c(t) \cdot \frac{1}{P_T(t)}\right] \qquad 0 \le t \le T$$

$$P_T(T) = 1$$

$$v(0) = P_T(0) \cdot E^{Q(P_T)}\left[\frac{c(T)}{\cancel{P_T(T)}}\right] \to 1$$

$$E^{Q(P_T)}\left[\frac{f(T) \cdot \cancel{P_T(T)}}{\cancel{P_T(T)}}\right] = \frac{v(0)}{P_T(0)}$$

$$v(0) = E^{Q(P_T)}\left[(f-k)_+\right] \cdot \boxed{P_T(0)}$$

$\frac{v(t)}{P_T(t)}$: martingale

"money market account measure"

Numeraire : $\beta(t) = e^{\int_{s=0}^{t} r(s)\, ds}$

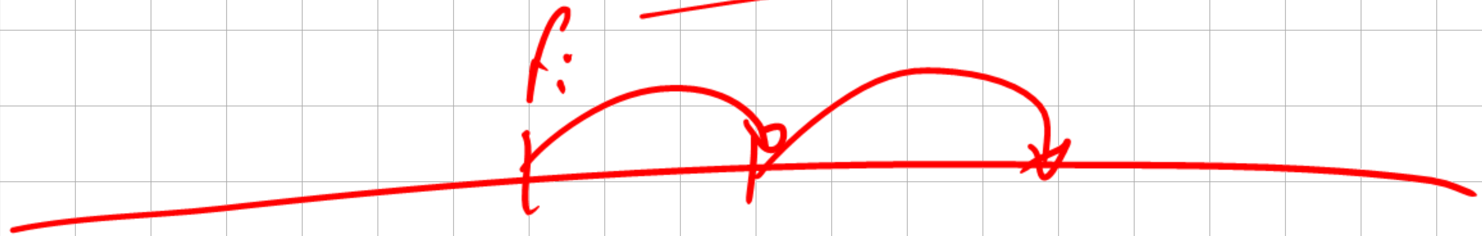
$$\frac{v(0)}{\cancel{\beta(0)}} = E^{\mu(\beta)}\left[\frac{c(t)}{\beta(t)}\right]$$

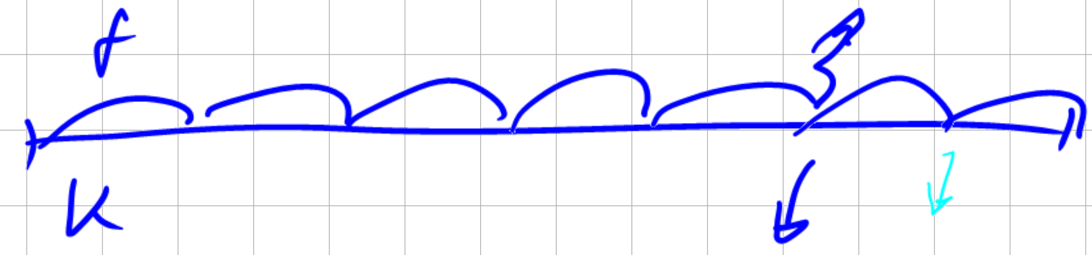
$\beta(0) \to 1$

$$= E^{\mu(\beta)}\left[c(t) \cdot e^{-\int_{s=0}^{t} r(s)\, ds}\right]$$

discount along the short rate path

$$\beta_{3M}(T_i) = \prod_{j=1}^{i} \left(1 + f_i \cdot \tau_j \right)$$





t'

T

$$\sum \left(f_i(t) - K\right)\cdot \tau_i \cdot P_{T_{i+1}}(t)$$

$$= \left(SR(t) - K\right)\cdot \underbrace{\sum \tau_i\, P_{T_{i+1}}(t)}_{\text{Annuity}}$$

$$\frac{\left(SR(t_0) - K\right)\cdot \cancel{\text{Annuity}(t_0)}}{\cancel{\text{Annuity}}} = E^{M(A)}\left[\frac{\left(SR(t)-K\right)\cdot \cancel{A(T)}}{\cancel{A(t)}}\right]$$

$$SR(0) - \cancel{K} = E^{M(A)}\left[SR(T)\right] - \cancel{K}$$

$$SR(0) = E^{M(A)}\left[SR(T)\right]$$

$$x_i = \left(\frac{v_i(t)}{N(t)} \right) = \left(\frac{v_0(t+dt)}{N(t+dt)} \right)$$

$n(\,)$

$$x_i(t) = E\left[x_i(t+dt) / \mathcal{F}_t \right]$$

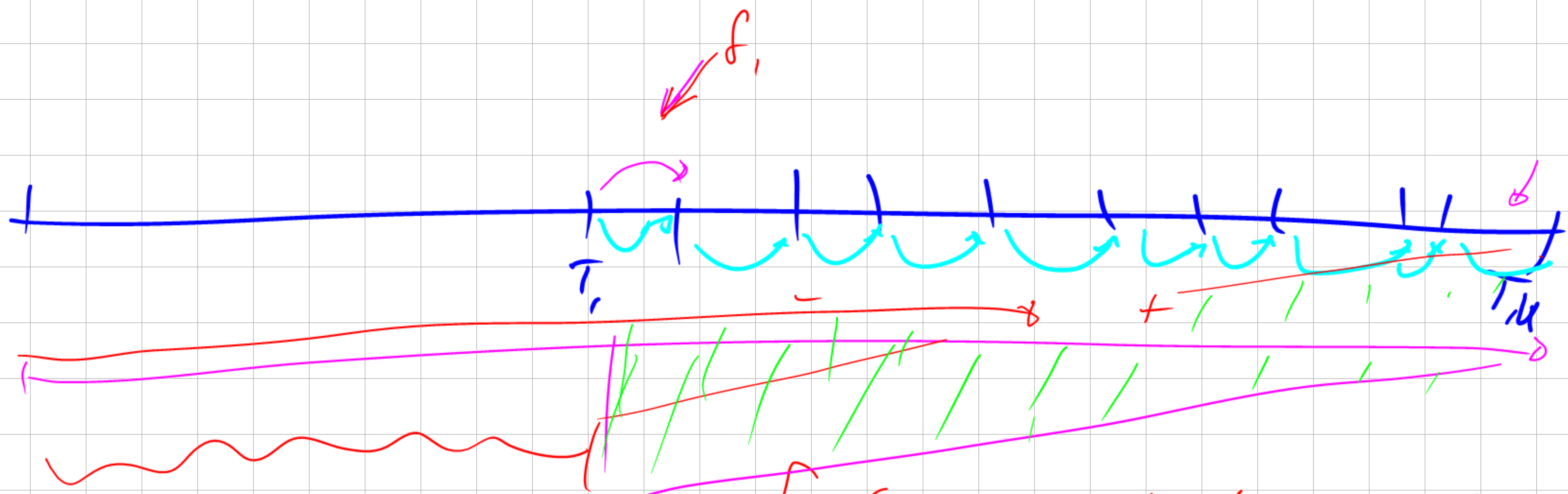
$$0 = E\left[\underbrace{x_i(t+dt) - x_i(t)}_{d\,x_i(t)} / \mathcal{F}_t \right]$$

$$\mu_N^{(6)} = \frac{f_N \tau_N}{1 + f_N \tau_N} \cdot \sigma_N^2$$

$$f_N(t)$$

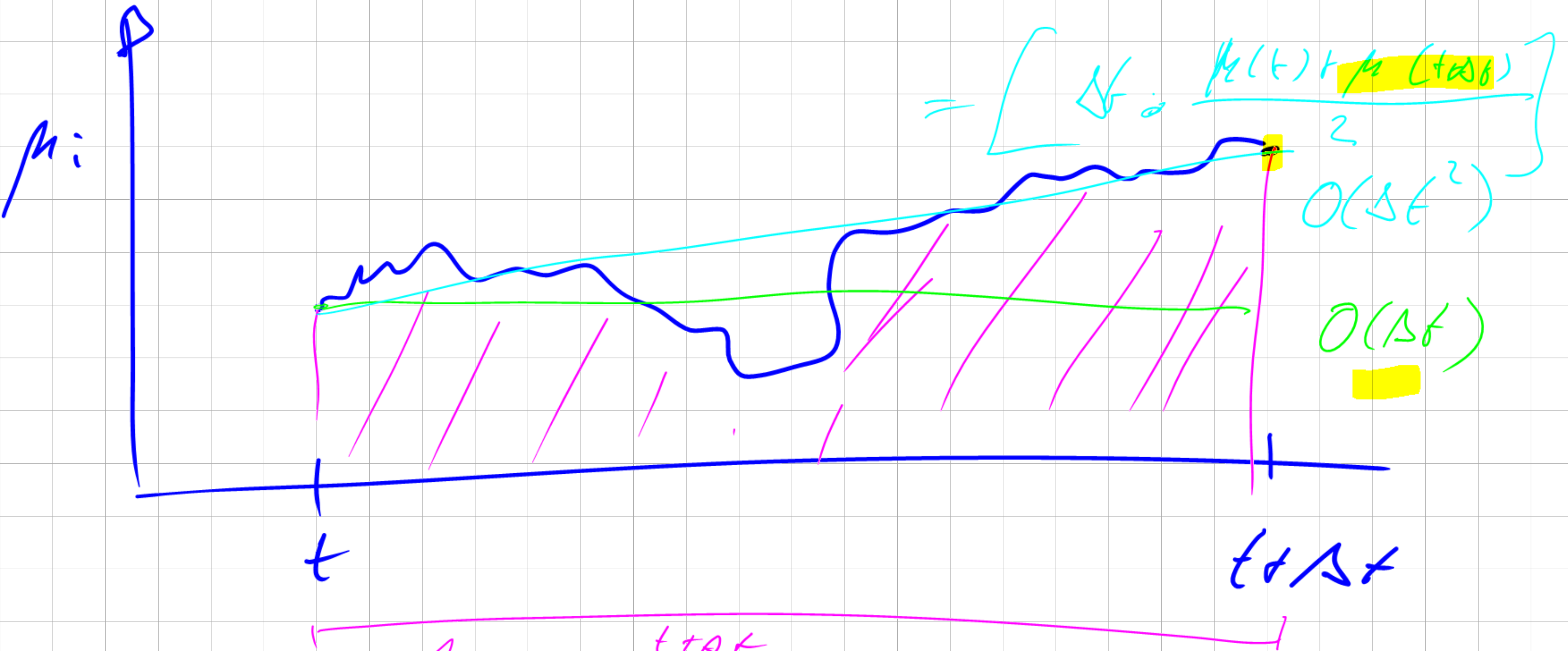
$$df_N = \mu_N \cdot f_N \, dt + \sigma_N f_N \, dW_N$$

$$= \frac{\tau_N}{1 + f_N \tau_N} \cdot f_N^2 \cdot \sigma_N^2 \, dt$$



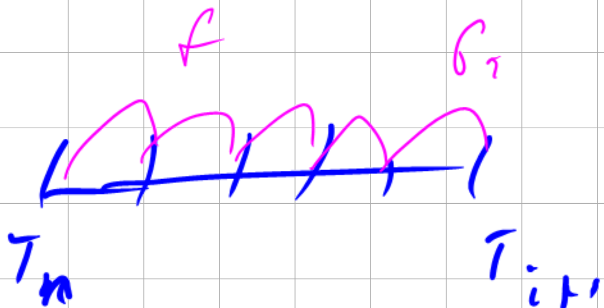
$$E\left[\frac{\left(SR(T_1)-V_1\right)\cdot A(T_1)}{P_N}\cdot \mathbb{1}_{\{\delta_1>0\}}\right]$$

half way between last observation time and last payment time

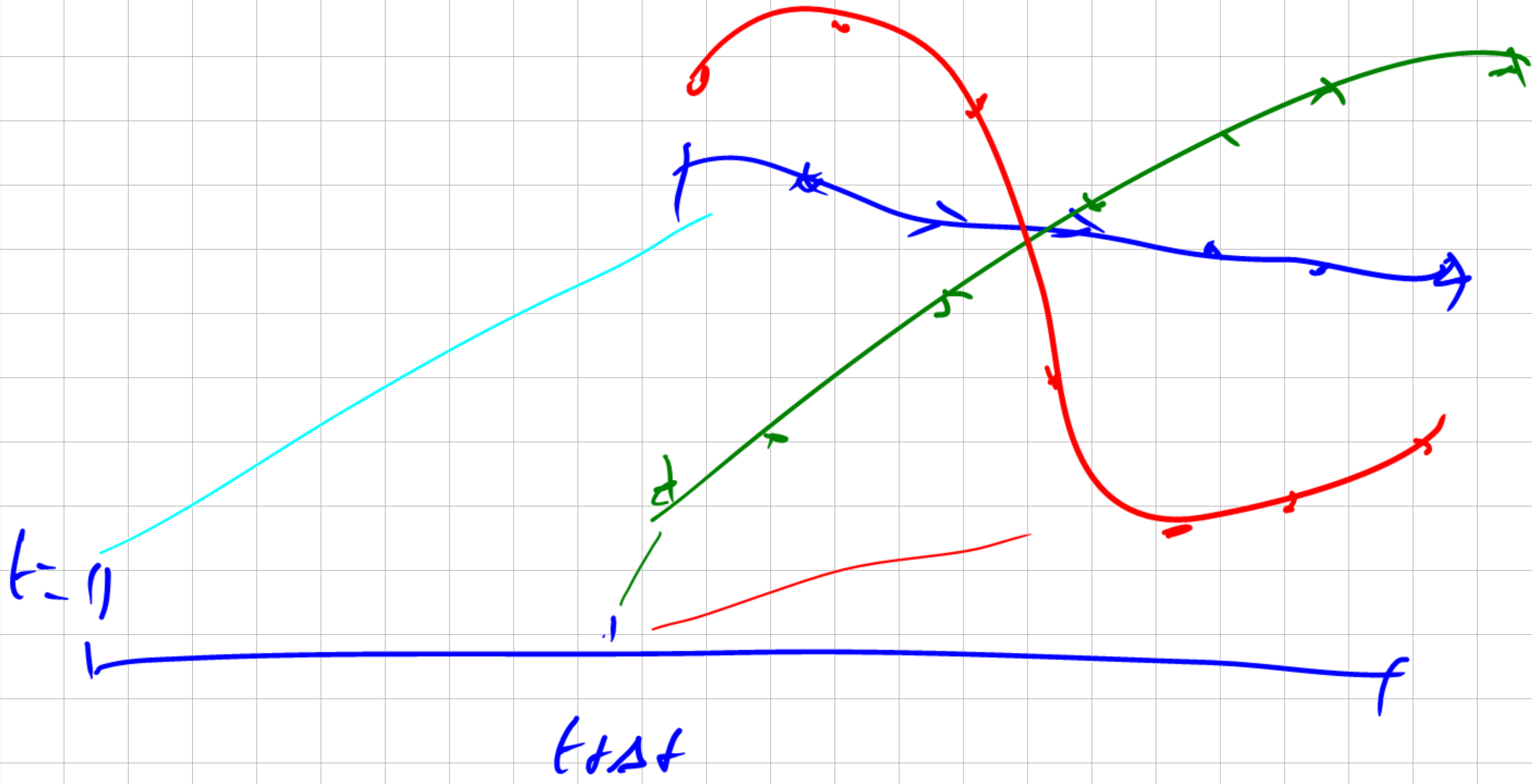


$$\hat{\mu}\Delta t = \int_t^{t+\Delta t} \mu_i(t')\,dt'$$

$$E\left[e^{\mu_i \cdot \Delta t}\right] = e^{\mu_i(t) \cdot \Delta t}$$



$$\sum_{k=n}^{i} \frac{f_k \tau_k}{1+f_k \tau_k} c_{ik} = \mu_i$$



$$\Delta t \cdot \left(\frac{\vec{\mu}(\vec{f}(t), \quad) + \vec{\mu}(\vec{f}(t+\Delta t), \ldots)}{2} \right)$$