The Nelson-Siegel-Svensson Model for U.S. Treasury Securities and Its Interpretation

By Lisa Patrick

1 Introduction

Whether you are an investor in equities, bonds, real estate, or other financial securities, the U.S. Treasury yield curve can be an important tool for the evaluation of investment decisions. As an indicator of market sentiment, the yield curve can show expected inflation, movement of interest rates, and the overall direction of the economy.

In this paper, we will examine the meaning of yield and its three main characteristics. Then we will explore interpretations of the different shapes of yield curves. Using the Nelson-Siegel-Svensson model, we can actually model the different yield curves for over 4000 daily observations between 1/2/1996 and 1/3/2012. This paper will show that the Nelson-Siegel-Svensson model provides yield curves that do indeed indicate future economic performance within the United States.

2.1 Yield and its characteristics

The term *yield* simply means the annual return on an investment. In this case, we are exploring the yields on U.S. Treasury bonds. There are three important characteristics of yield: level, slope, and curvature.

The *level* is the average yield across maturities. For U.S. Treasury bonds, we consider short term, medium term, and long term yields. See Figure 1. These yields tend to rise and fall together, making them one of the first indicators of market sentiment. (Veronesi, 2010)

$$Level = \frac{\sum_{i=1}^{n} y_i}{n}\ , where\ n = number\ of\ observed\ yields \tag{1}$$

The *slope*, also known as the *term spread*, of the yield is the difference between long term and short term yields. Typically, the slope of U.S. Treasury bond yields is positive. However, the slope is dependent upon many factors including expected future inflation and expected growth of the economy. In rare cases, these sentiments have resulted in negative term spreads. See Figure 2. One important observation is that a change in term spread does not necessarily correspond to a change in the level of yields. We can see that in the beginning of 1999 and the end of 2000, the level of yields was about 5%. However, in the beginning of 1999, the slope was about 0.50% while it was approximately -0.50% in the end of 2000.

$$Slope = y_{Long\ term} - y_{Short\ term} \tag{2}$$

The *curvature* of a yield is a measure of the relative pricing of short term, medium term, and long term securities. The *butterfly spread* is a popular method used to calculate the curvature. (Equation 3) Curvature can be an important indicator of expected future interest rates.

$$Curvature = -y_{Short\ term} + 2 * y_{Medium\ term} - y_{Long\ term} \qquad (3)$$

Using daily treasury yields from the U.S. Department of the Treasury, the following figures were obtained using equations (1), (2), and (3) showing the level, slope, and curvature for U.S. Treasury bond yields from 1/2/1996 to 1/3/2012. (U.S. Department of the Treasury, 2015)

Shows the data

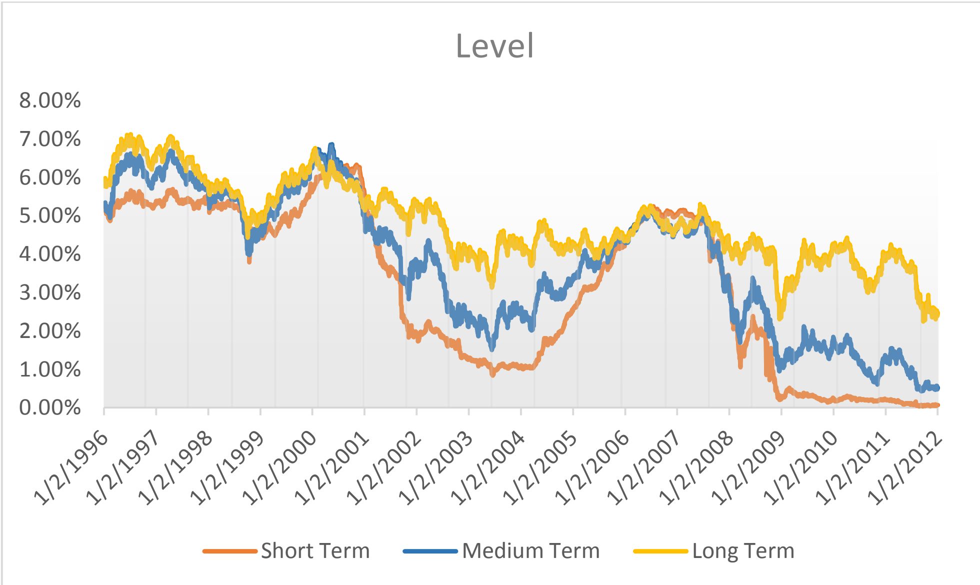
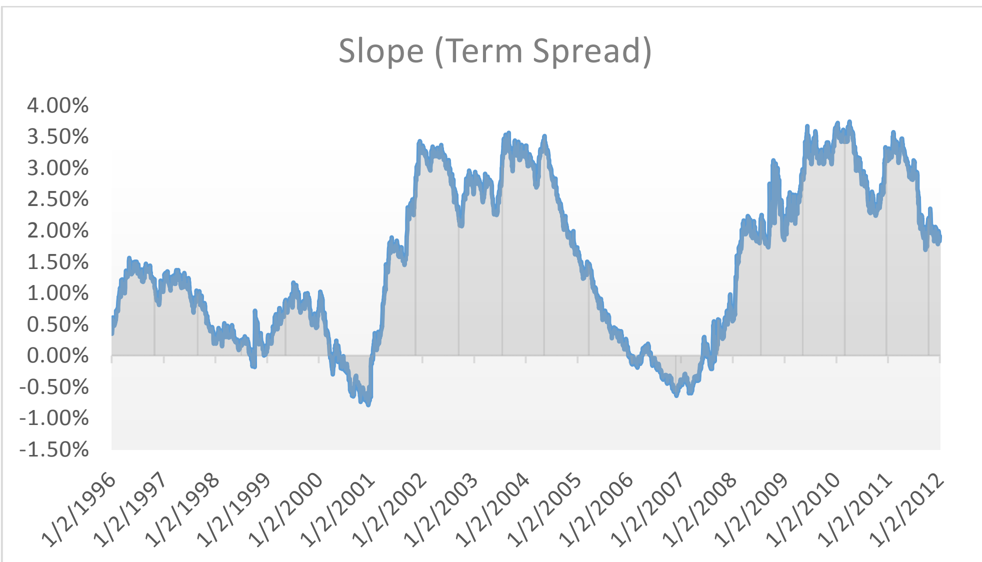


Figure 1: The level of yields from 1/2/1996 to 1/3/2012[1]

Slope & Curvature are calculated by those two formula



[1] Short term securities have maturities of one year or less, medium term securities have maturities greater than one year but less than ten, and long term securities have maturities ten years or greater.

Figure 2: The slope of yields from 1/2/1996 to 1/3/2012[2]

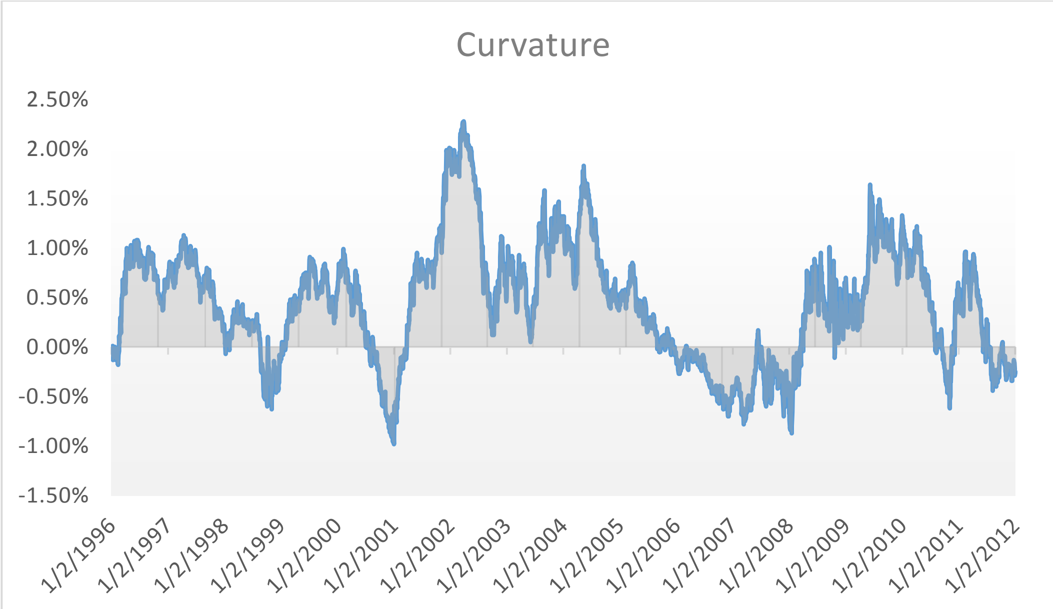


Figure 3: The curvature of yields from 1/2/1996 to 1/3/2012[3]

There are different shape of the yield curve: Increasing/Decreasing/Humped

2.2 What is a yield curve, and why is it important?

While understanding the level, slope, and curvature of yields is useful, yield curves are a popular tool for understanding overall economic performance. There are two main components to a yield curve: time to maturity and yield. In a standard yield curve, yield is plotted against time to maturity for financial securities of the same level of risk.[4] Because U.S. Treasury bonds are considered to be default-free or riskless, their yields are not affected by certain variables like the yields corporate bonds or movements in the stock market. For this reason, the most commonly reported yield curve is that of U.S. Treasury bonds.

The basic shapes of yield curves are increasing, decreasing, hump, and flat. An increasing yield curve means that long term yields are higher than short term yields, a common occurrence in U.S. Treasury yield curves. This means investors tend to expect higher future inflation, leading to higher nominal interest rates, and they need to be compensated for the length of their investment.

A decreasing yield curve means short term rates are higher than long term rates. This corresponds to a negative slope in yields. This type of yield curve is fairly rare and typically followed by a recession within the next two years. (Estrella & Mishkin, 1996) Looking at Figure 2, we can see that there was a negative spread beginning in early 2000 and a negative spread beginning in the middle of 2006. In both of these cases, negative slopes were followed by recession from March 2001 to November 2001 and December 2007 to June 2009. (The National Bureau of Economic Research, 2015)

The humped yield curve means that medium term yields tend to be higher than short and long term yields. This can be a sign of economic slowing, but does not necessarily mean a recession is coming. The other

[2] The U.S. Treasury discontinued 30 year bonds from Feb 18, 2002 to Feb 9, 2006. Due to this, the 10 year yield is used as the long term yield. Also, due to a lack of consistency in 3 month data availability, the 6 month yield is used as the short term yield.
[3] Again, due to lack of consistency in the data, the previously mentioned short and long term yields are used. The 5 year yield serves as the medium term.
[4] Yield is on the vertical axis and maturity is on the horizontal axis

case we need to consider is a flat yield curve. This kind of curve shows rates that are about the same for all maturities. A flat yield curve expresses the current market sentiment that interest rates are not expected to change.

3 Estimations of the yield curve

We are going to be using the Nelson-Siegel-Svensson Model to create the term structure of interest rates for U.S. Treasury securities. However, to ensure accuracy, we need a measure of how well the model fits the data. Our objective is to minimize the errors in yields created by the model using the sum of least squares method.

$$Minimize\ residual = \sum_{i=1}^{n}(y_{observed_i} - y_{model_i})^2 \qquad (4)$$

It is also important to consider the range of the yield curve. If the observed data only goes up to ten year maturities, the model may not be a great indicator of 15 or 20 year yields.

4 The Nelson-Siegel-Svensson Model

The Nelson and Siegel group of models are based on the original model by Nelson and Siegel, which originally had four parameters. The Svensson extension introduced two new "slope" parameters to allow for a better variety of shapes for both instantaneous forward rate curves and yield curves. (Yallup, 2011) Equation (5) shows how the instantaneous forward rate can be expressed as a function of the six parameters. . The Nelson-Siegel-Svensson model provides a well behaved, smooth forward rate curve. The original Nelson-Siegel model is the same as the Svensson with $\beta_4 = 0$. Equation (6) shows how the model can be expressed as a closed form expression of yields.

$$f(t) = \beta_1 + \beta_2 \exp\left(\frac{-t}{\lambda_1}\right) + \beta_3\left(\frac{t}{\lambda_1}\right)\exp\left(\frac{-t}{\lambda_1}\right) + \beta_4\left(\frac{t}{\lambda_2}\right)\exp\left(\frac{-t}{\lambda_2}\right) \qquad (5)$$

$$r(t) = \beta_1 + \beta_2\left(\frac{1-\exp\left(\frac{-t}{\lambda_1}\right)}{\frac{t}{\lambda_1}}\right) + \beta_3\left(\frac{1-\exp\left(\frac{-t}{\lambda_1}\right)}{\frac{t}{\lambda_1}} - \exp\left(\frac{-t}{\lambda_1}\right)\right) + \beta_4\left(\frac{1-\exp\left(\frac{-t}{\lambda_2}\right)}{\frac{t}{\lambda_2}} - \exp\left(\frac{-t}{\lambda_2}\right)\right) \qquad (6)$$

A benefit of the Nelson-Siegel-Svensson model is the interpretations of β_1 and $\beta_1 + \beta_2$. β_1 can be interpreted as the long term interest rate, while $\beta_1 + \beta_2$ is the instantaneous short term interest rate. (Yallup, 2011)

5.1 Data

The data used is from the U.S. Department of the Treasury. The rates are called "Constant Maturity Treasury" rates. Using the closing market bid yields on actively traded Treasury securities, the Federal Reserve Bank of New York computes yields at fixed maturities, i.e., 3 and 6 months and 1, 2, 3, 5, 7, 10, 20, and 30 years. (U.S. Department of the Treasury, 2015) Through the estimation of the yield curve with the Nelson-Siegel-Svensson model, we are able to find the expected yield on securities with maturities outside of the fixed set computed by the Federal Reserve Bank of New York. This is also beneficial when considering other investments that do not have the same fixed maturities.

5.2 Findings and Figures

Through the use of Visual Basic for Applications and the Solver feature in Microsoft Excel, we can iterate through the data and find the seven parameters for each daily observation that minimize the residual error. In our period of interest – 1/2/1996 to 1/3/2012 – we have 4006 daily observations for which parameters were computed. Table 1 provides the parameters from the beginning of each year, along with the residual error, R^2. The values for $\beta_1 + \beta_2$ are also given. Recall that this can be interpreted as the instantaneous short term interest rate.

Date	β_1	β_2	β_3	β_4	λ_1	λ_2	$\beta_1 + \beta_2$	Residuals (R^2)
1/2/1996	0.060951	-0.00909	0.015292	-0.03298	1.031532	1.535411	5.19%	8.86952E-07
1/2/1997	0.068865	-0.01846	-0.01106	-0.00898	0.503528	2.6177	5.04%	1.73378E-07
1/2/1998	0.059688	-0.00721	-0.00313	-0.00804	0.474959	3.619166	5.25%	2.22952E-07
1/4/1999	0.053833	-0.00898	0.014807	-0.02977	1.747907	2.783943	4.49%	1.71582E-06
1/3/2000	0.189902	-0.13429	0.473635	-0.71774	8.779111	11.93454	5.56%	9.69966E-06
1/2/2001	0.061292	0.001726	-0.03476	-0.02938	1.027548	7.499287	6.30%	1.74563E-06
1/2/2002	0.055385	-0.0388	-0.03948	0.009609	0.768588	7.538776	1.66%	1.11642E-06
1/2/2003	-0.02697	0.038787	0.003374	0.215619	1.314499	7.530469	1.18%	8.69950E-08
1/2/2004	0.002931	0.005398	-0.00562	0.140871	0.82157	7.440909	0.83%	1.59009E-07
1/3/2005	0.0214	-0.00551	0.017547	0.071705	0.280875	8.206162	1.59%	6.31597E-07
1/3/2006	0.042483	-0.06403	0.074376	0.002848	0.088283	7.971521	-2.15%	3.25250E-07
1/2/2007	0.042255	0.00635	0.01712	0.017363	0.397061	13.25251	4.86%	2.69600E-07
1/2/2008	0.012653	0.01449	0.037687	0.104563	0.34166	12.57114	2.71%	6.12400E-07
1/2/2009	0.031319	-0.03099	-0.00881	-0.02298	1.032614	1.532737	0.03%	2.16722E-06
1/4/2010	0.051204	-0.0506	-0.0286	-0.02748	1.137607	1.454381	0.06%	1.20534E-06
1/3/2011	0.049757	-0.04747	0.005905	-0.08358	1.041972	1.453937	0.23%	1.33130E-06
1/3/2012	0.035162	-0.03603	0.046129	-0.10588	1.085415	1.602535	-0.09%	2.43244E-07

Table 1: Sample parameter values from the beginning of each year

The sum total of residual errors for all 4006 observations is 0.471%, and the mean squared error (MSE) is 0.000117%. Figures 4-A through 4-Q show sample yield curves and forward rate curves from the beginning of each year. The Nelson-Siegel-Svensson model produced smooth yield curves and instantaneous forward rate curves.

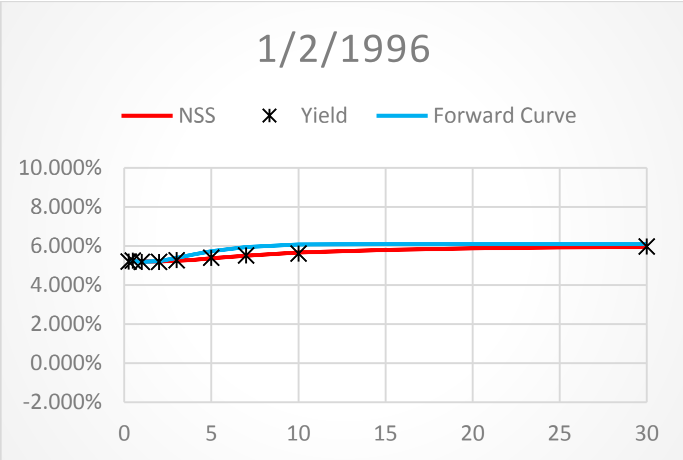


Figure 4 - A: Residual = 8.8695E-07

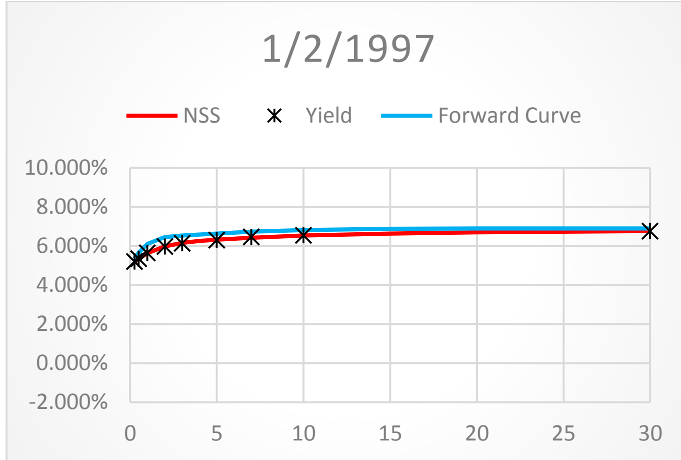


Figure 4 - B: Residual = 1.7338E-07

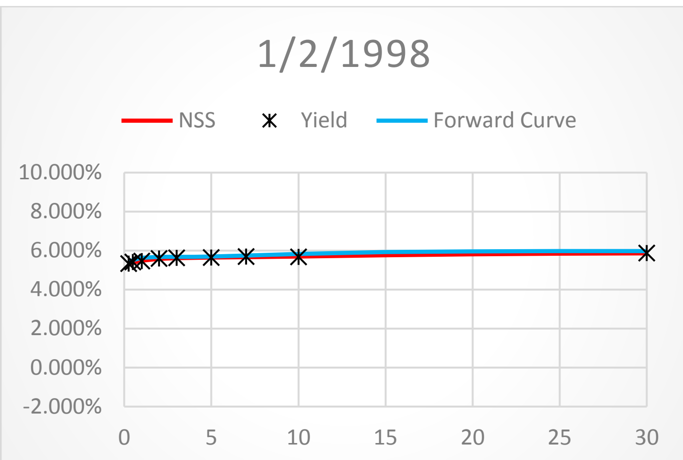


Figure 4 - C: Residual = 2.2295E-07

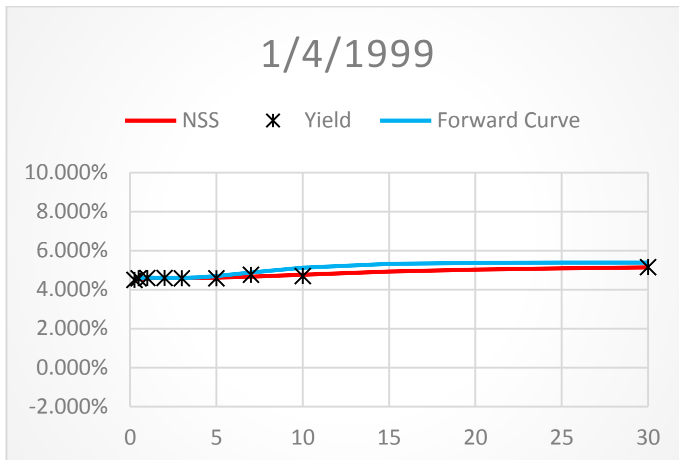


Figure 4 - D: Residual = 1.7158E-06

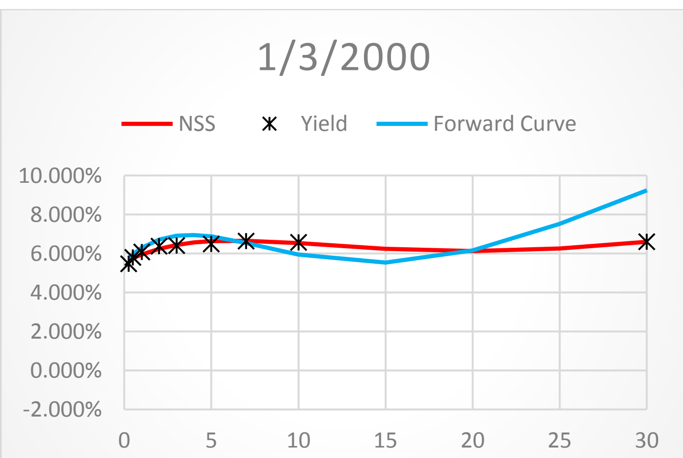


Figure 4 - E: Residual = 9.6997E-06

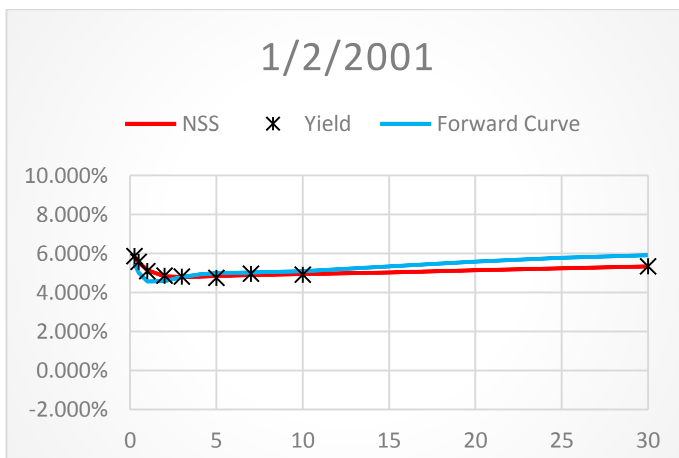


Figure 4 - F: Residual = 1.7456E-06

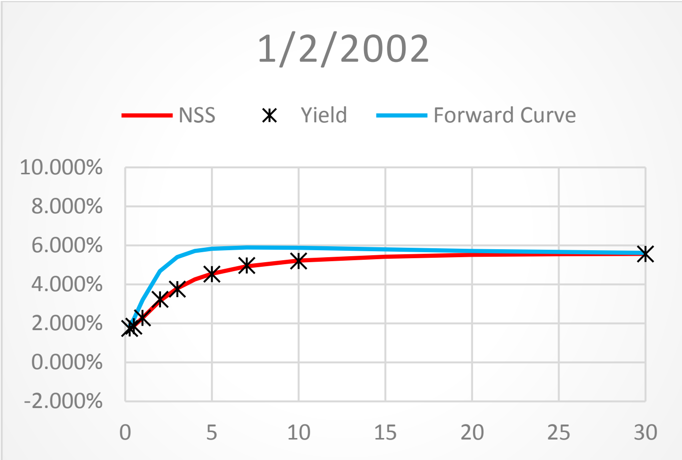


Figure 4 - G: Residual = 1.1164E-06

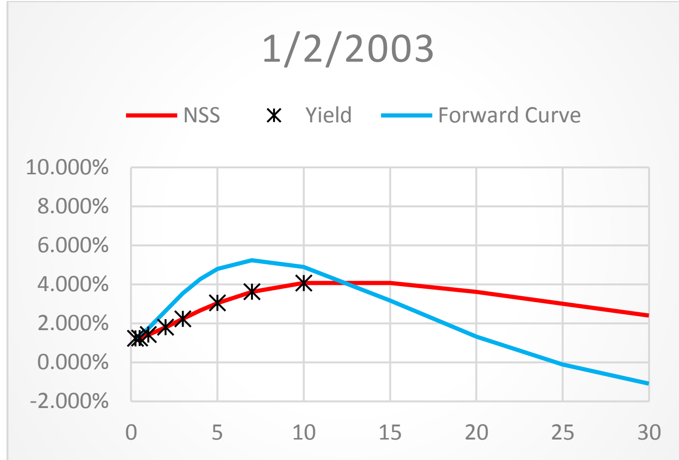


Figure 4 - H: Residual = 8.6995E-08

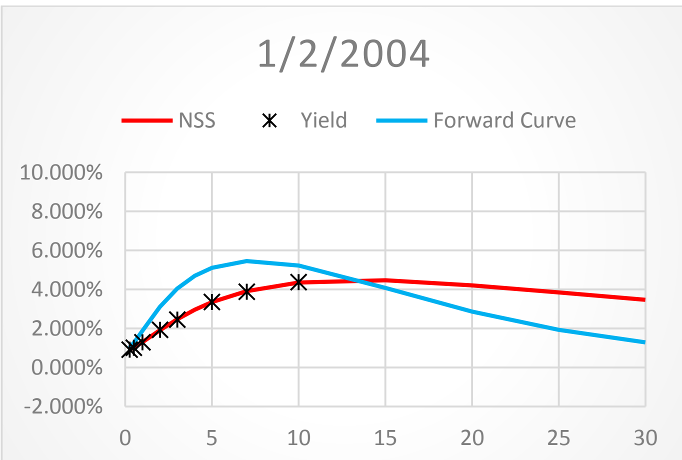


Figure 4 - I: Residual = 1.5901E-07

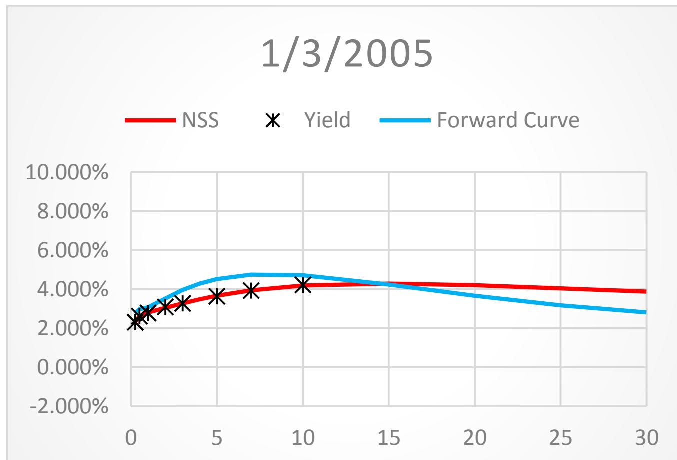


Figure 4 - J: Residual = 6.3160E-07

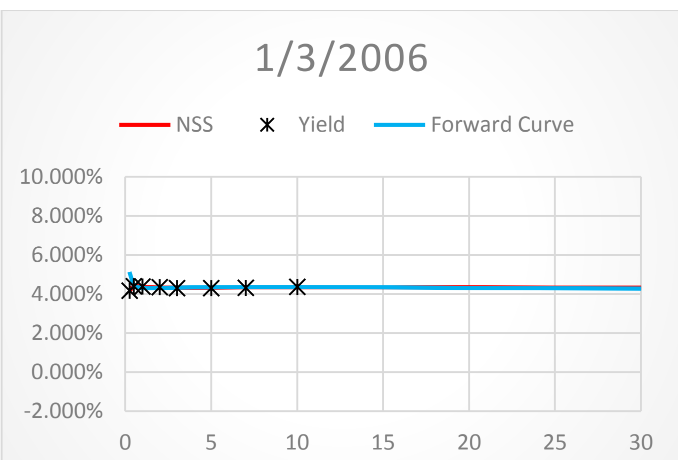


Figure 4 - K: Residual = 3.2525E-07

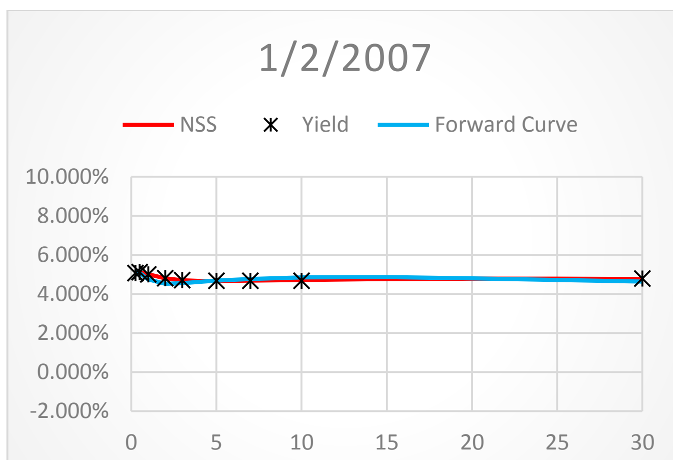


Figure 4 - L: Residual = 2.6960E-07

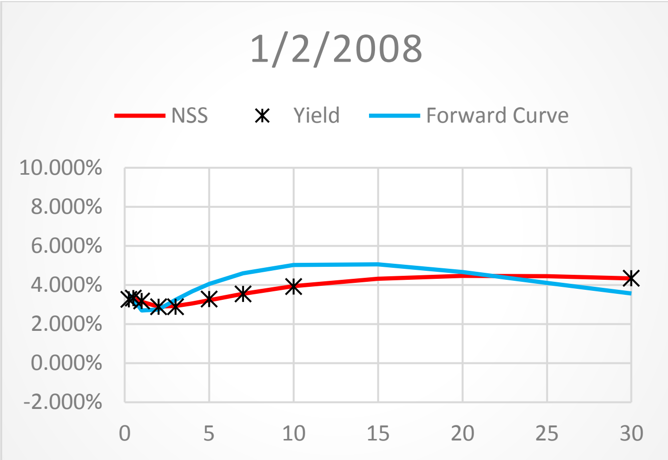


Figure 4 - M: Residual = 6.1240E-07

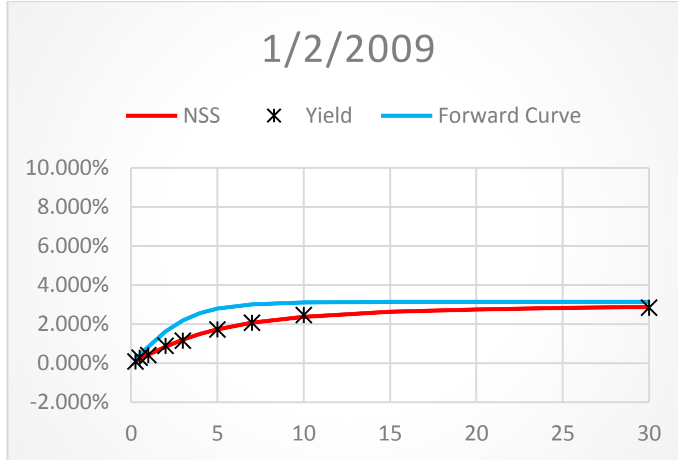


Figure 4 - N: Residual = 2.1672E-06

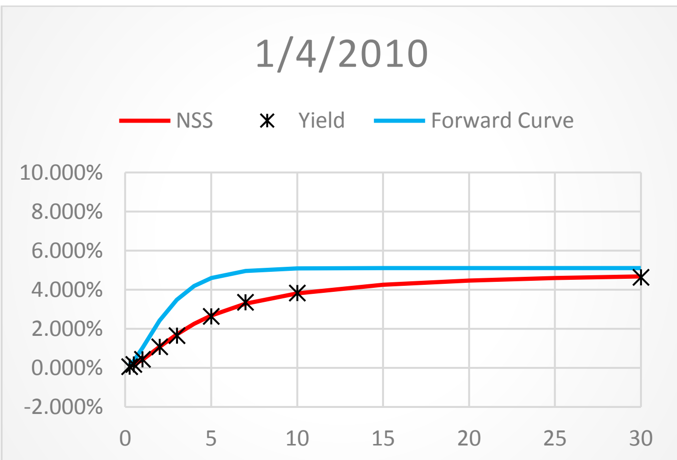


Figure 4 - O: Residual = 1.2053E-06

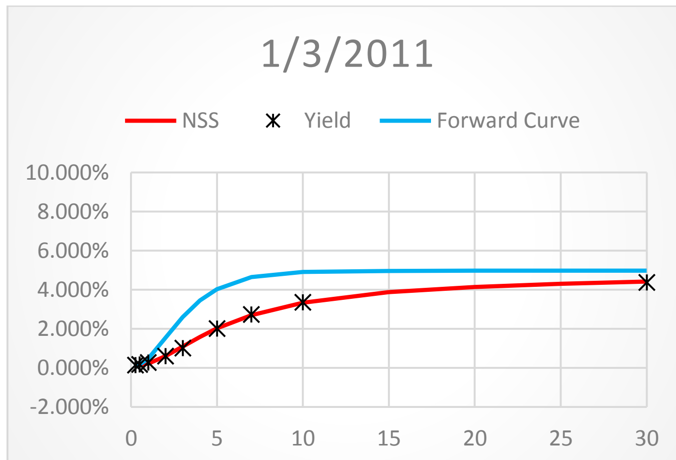


Figure 4 - P: Residual = 1.3313E-06

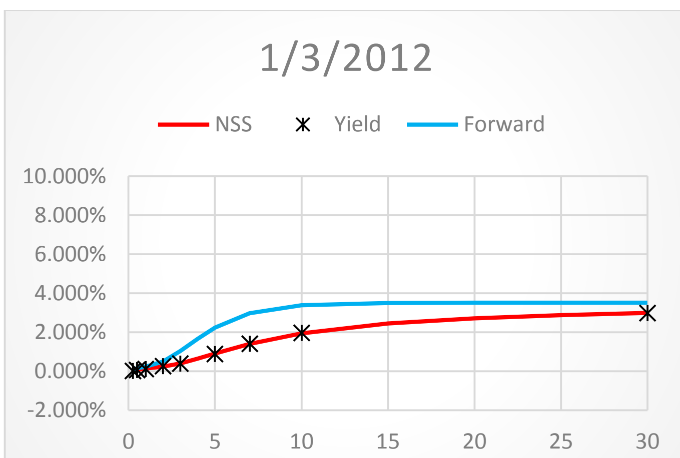


Figure 4 - Q: Residual = 2.4324E-07

5.3 Interpretation of shapes of yield curves produced by the NSS model

As mentioned previously, during 1/2/1996 to 1/3/2012 time period, the U.S. had experienced two recessions – March 2001 to November 2001 and December 2007 to June 2009. Figure 4-E and 4-F show two humped yield curves before the March 2001 recession. Both curves have higher short and medium term rates than long term rates. Before December 2007, we reference Figure 4-L. We see an inverted yield curve preceding a recession. After both recessions, the subsequent yield curves become more increasing in shape, signaling economic recovery.

6 Conclusion

In this paper, we have examined the important characteristics of yields – slope, level, and curvature, discussed the varying shapes of yield curves along with their different interpretations, and shown that daily treasury data can be modelled with high levels of accuracy by the Nelson-Siegel-Svensson model.

During the 1/2/1996 to 1/3/2012 time period, the Nelson-Siegel-Svensson model correlates to both recessions. We can conclude that the seven parameter model allows for a good fit of U.S. Treasury yields and can be an indicator of future economic performance.

References

Estrella, A., & Mishkin, F. S. (1996). The Yield Curve as a Predictor of U.S. Recession. *Current Issues in Economics and Finance*, 1-4.

The National Bureau of Economic Research. (2015, April 1). Retrieved from US Business Cycle Expansions and Contractions: http://www.nber.org/cycles.html

U.S. Department of the Treasury. (2015, March 12). *Daily Treasury Yield Curve Rates*. Retrieved from http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yieldAll

Veronesi, P. (2010). *Fixed Income Securities: Valuation, Risk, and Risk Management.* Hoboken, NJ: Wiley.

Yallup, P. J. (2011). Models of the Yield Curve and the Curvature of the Implied Forward Rate Function. *Journal of Banking & Finance*, 121-35.