Introduction to FX Options

History of market

During the mid 1980's a confluence of events gave birth to the FX Options market as we know it today, namely: a demand for the product, the ability to price the product, a market place to trade, and, with the advent of computer power, the ability to manage risk.

To have an option market, first it is necessary to have a liquid market in the underlying rate (usually called just the underlying) upon which the options are based. Before the 1970's, when exchange rates were in general fixed to specific values and adjusted at intervals, there would be no possibility of an option in the market. But as different countries gradually abandoned the increasingly unworkable fixed FX rate regime which had been implemented after the post war Bretton-Woods agreement, risk appeared, and the first to take note and act upon this risk were the corporations of the world. Companies with income and liabilities in other countries are highly sensitive to exchange rate fluctuations and seek ways to minimise them. Corporate treasurers initially used forward FX contracts to lock in rates but then realised they could sell them if the contracts entered very negative territory, assuming a trending market, and replace them if they became close to positive once more. This crudely replicates the protective properties of an option, though it was a cumbersome and imprecise process. The idea of a product where another company took over this adjustment process was attractive. The very early currency overlay companies did exactly this, calling it option replication. As the markets started to swing wildly during the 1980's the demand for this increased. True options in FX began to be bought and sold though the correct price for an option was hotly debated.

Equity option traders began to use the Black-Scholes-Merton model shortly after its publication in 1973 [3], but there was at that time little thought of using it for FX contracts. In 1983 Garman and Kolhagen published the extension to the Black-Scholes-Merton model which enabled FX options to be clearly and simply valued for the first time, as it included dual interest rates. [1]

With the demand for the product, and the ability to price it, came the distribution. The first FX option was dealt on the Philadelphia Stock Exchange in November 1982 [2]. At that time they were a small futures exchange who courageously introduced the new instrument when there was no OTC market at all, and virtually no other instruments available to use as pricing references. These options, consistent with similar equity products at the time, were American-style, exercisable by the option purchaser any time to expiry, which would have made them even more challenging to value. But clearly they showed promise; by the mid 1980's the exchange in Chicago was also actively trading contracts on FX options, and the number of boutique option houses grew.

However, the growth didn't stop there. The contracts offered on the exchanges were well defined in terms of contract size, a list of available strikes and set maturity dates offered throughout the year. But this was too limiting for most FX users who wanted to tailor-make the option to match their exact risk profile, and to have the ability to combine options in different strategies. Hence, the OTC market was born. Investment banks bought boutique option houses for their knowledge and with the increased access to corporate hedging activity, option trade flow increased tremendously.

Fx option ‘trading day’

The flow reported is not constant through the day. The FX market is almost 24-hour. Beginning in Asia, the markets start to trade. Europe enters as the Asian markets slow down, smoothly taking over. When the US markets join in the European afternoon, peak flow conditions prevail. As Europe closes the USA maintains trading for several more hours, with San Francisco finally pulling the curtains closed for a few minutes until Asia awakes. All regions will maintain at least a skeleton overnight coverage so even these few minutes are not so much closed as quiet.

Hourly flow for EURUSD options, for 7th July 2014, from a London trading desk

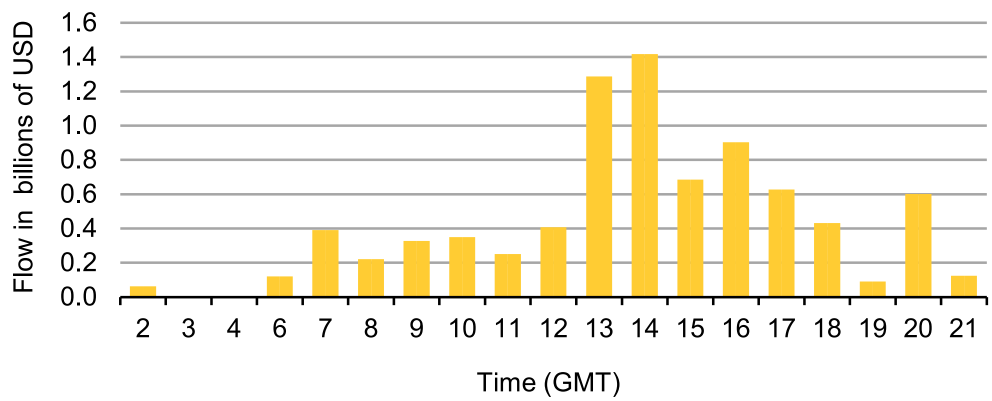

Figure 1

What is an FX option?

Definitions

Let’s look at a definition from a popular website...

‘A foreign-exchange option is a derivative financial instrument that gives the owner the right but not the obligation to exchange money denominated in one currency into another currency at a pre-agreed exchange rate on a specified future date’.

The price or cost of this right is called the premium, in analogy with the insurance market, and it is usually (depending on the tenor and the market at the time) a few per cent of the insured amount (notional amount). The specified future date is called the expiry or expiry date.[1] The payoff profile at expiry of the simplest type of option is shown schematically below in **Figure 2**.

Payoff profile at expiry for a call option

[1] The markets delight in detail; the expiry date will define the payoff of the option but settlement, when cash is transferred, will occur a day or so later, depending on the currency pair.

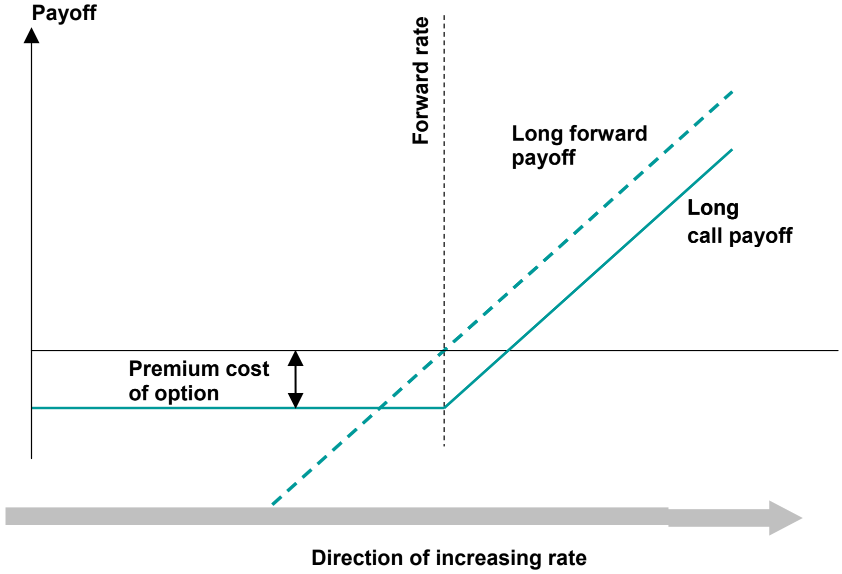

Figure 2

The figure shows the payoff received by the holder of an at-the-money-forward (ATMF) call option on an FX rate. This means that the strike of the option is the forward rate, and the option is the right to buy the base currency, or in other words, an option to buy the FX rate. In other markets such as commodities and equities it is obvious what the call or put is applied to but in FX more clarity is needed. For instance a call option associated with the currency pair USDJPY could be a call on USD (and thereby a put on JPY) or a call on the JPY (and therefore a put on the USD). A put option would be the right to sell the base currency, or FX rate. If the interest rates for the period of the option were identical in both currencies involved in the FX rate, then the forward rate would be identical to today's FX rate. Because they usually are not the same, the rate which one may lock in an exchange without risk for a future date will be somewhat different from today's rate.

The figure shows the premium cost of the option. At all FX rates at expiry which are less than the forward rate, this will be what the option holder loses, meaning that he or she paid a premium to buy the option and will make no money from it. The net result is the loss of the premium. At the forward rate, the payoff begins to rise, at first reducing the overall cost and then taking the owner of the option into profitable territory for higher FX rates at expiry. We have also shown the payoff from a forward contract, which is simply when the owner of the contract locks in the forward rate at the expiry date. This will lose money when the rate at expiry is less than the forward, and make money when the rate is higher. The forward rate is costless to lock in other than bid-offer costs.

The essential thing to grasp about the payoff to an option contract is that it is asymmetric. There is limited loss (the owner of the option can only lose the premium) but in theory unlimited gain. Conversely, the seller of the option stands to make a limited gain but an unlimited loss. Thus the option payoff looks very much like that of an insurance contract; we expect to pay a fixed premium to cover a variety of different loss types, up to and including very large losses indeed.

Who uses them?

One way of looking at the question of who uses FX options would be to think of option suppliers (sellers of risk) and option consumers (buyers of risk). The former might be balance sheet holders who can sell a 'covered option' – essentially, if they hold the underlying currency, they can make money by selling an option which pays out if the currency rises but not if it falls. If it rises, their holdings will increase in value so they can pay the option holder. If it falls, they do not have to pay but they collect the premium. The option consumers have unwanted currency risk they need to reduce, like an investor with an international portfolio of bonds, or a corporation selling goods in another country. Additional to option suppliers, there are market makers like the option desks of larger banks, which both buy and sell options to make a profit from the bid offer cost. Also there are purely profit focussed entities like hedge funds which take views on direction or inefficiencies in the market to make money. Finally the world's central banks can direct massive FX flow, sometimes using options, to execute policy aims like currency strength or weakness. And each of these has properties of the others; a portfolio manager may wish to protect against currency risk but derive some return, and even a central bank may maintain a trading arm to smooth volatility and influence currency levels.

Pricing and trading FX options

Black-Scholes-Merton-Garman-Kohlhagen

If we make some assumptions, then we can model FX spot rates as evolving according to geometric Brownian motion. That is, we assume that spot prices change according to a stochastic process given by:

$$ = \qquad + $$

where S_t is the spot price at time t, μ gives the strength of any drift, σ is the volatility of returns of the underlying asset and W_t is a Wiener process (commonly known as Brownian motion).

By imposing risk neutral pricing so that all risk-free investments earn the same rate of return, this price process, along with a little algebra, leads us to the Black-Scholes-Merton equation:

where V is the price of the derivative, σ is the volatility of returns of the underlying asset, S is the spot price and r and r_b the interest rates on the quote and base currencies respectively. The Black-Scholes-Merton partial differential equation (PDE) describes how the price of an option varies with the price of the underlying, the expected volatility of the underlying and risk free interest rates for the two currencies in the exchange rate.[2]

Solving the Black-Scholes-Merton PDE for known final conditions allows us to write down analytical solutions for the theoretical prices of European put and call options:

[2] Folk are most likely to have come across the Black-Scholes-Merton model as applied to equity derivative pricing where only a single risk free rate applies. For FX option pricing the underlying principles are the same, though the application of risk free pricing during the derivation of the equation is slightly different, leading to minor differences in resulting PDEs. Garman and Kolhagen adapted the Black-Scholes-Merton equation to incorporate the dual interest rates involved in an FX option.

$$c = S_0 e^{-r_b T} N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 e^{-r_b T} N(d_1)$$

with:

$$\frac{\quad + \qquad + \quad}{\sqrt{\ }}$$

$$\frac{\quad + \quad}{\sqrt{\ }}$$

$= d_1 \square \sigma \sqrt{T}$

and:

S_0 = FX rate at inception,

K = strike rate,

r = interest rate for tenor of the option in the quote currency,

r_b = interest rate for the tenor of the option in the base currency,

T = tenor of the option, and

σ = implied volatility of the option.

N(.) denotes the standard normal cumulative distribution function.

We say ‘theoretical’ prices because the solutions do not account for either the term structure of implied volatility or the term structure of interest rates – these theoretical values act only as a starting point for option traders. Nonetheless, for our purposes these expressions for European put and call option prices will be adequate to allow us to identify the main drivers of option premiums.

Implied and realised volatility

Volatility can mean many things depending on context, but in this instance we are concerned with a specific type of volatility, namely the standard deviation of log returns. Even once we have settled on this definition, we still need to be more specific as there are broadly speaking two main classes of volatility that may be of interest to those active in FX markets.

Realised (or historical) volatility as the name suggests is the volatility that has actually been measured in the market on the underlying. It is a known, calculated, backward-looking value. As the FX market is dominated by spot transactions the realised volatility is usually based on the underlying spot rate. Depending on when the data point is collected (in the middle of the day or at the market close for example) or which data point is taken (the mid-point of the price, the bid or the offer) daily realised volatility can vary slightly. Either way, the measure gives an indication of the day-to-day variability in an exchange rate over a recent period of time.

Implied volatility on the other hand is a forward looking number. It is the expected volatility of (usually) daily changes in an exchange rate over a quoted period. Because the level is forward looking, nobody knows for certain at the point of dealing if it is accurate or fair value. It is the market's best estimation based on past performance and expected events. Hence, as a baseline, implied volatility will normally be relatively close to realised volatility. If a currency pair moved at 1% a day over the last month and has done so over the last year it would be reasonable to assume it would move at 1% a day over the next month. But, there is always the financial cliché: past performance is no guide to future returns. As shown in **Figure 3**, while the series are similar it is obvious that forward looking implied volatility rarely provides an accurate forecast of spot rate volatility for the subsequent month.[3] Future volatility is impacted by events such as elections and data releases (higher volatility) or holidays (lower volatility).

Annualized daily 1M EURCHF implied volatility and subsequent 1M realised volatility

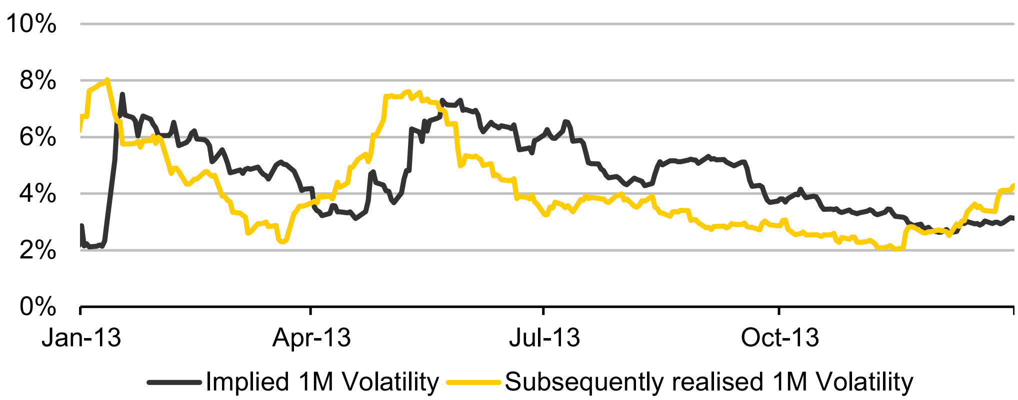

Figure 3

In general when option traders discuss volatility they are talking about implied volatility measured on an annualised basis. This is what is of most interest since, as we have already seen, the value of an option is closely related to the expected volatility of the exchange rate underlying over the lifetime of the contract. In short, higher implied volatility leads to higher option prices. We can immediately see that human nature comes into play at this point. For traders to accurately price options of differing tenors, they must be able to rationally evaluate risks over wildly different periods of time. We all know however that in financial markets, it is often immediate risks that tend to dominate the news, with longer term problems conveniently downgraded.

The volatility surface

In order to derive the Black-Scholes-Merton option pricing model one must make a number of simplifying assumptions relating to both the way in which financial markets function and the way in which market participants behave. Despite the need for these assumptions, the model has proved extremely successful, albeit that we have to account for a number of discrepancies between what the model predicts and what we observe in markets. One could say that in reality, the Black-Scholes-Merton

[3] While the realised volatility of changes in an exchange rate over a one month period will appear to lag 1M implied volatility, this is a consequence of the backward looking calculation of historical volatility. In reality, it is implied volatility that lags market developments as traders react to changes in the dynamics of the spot FX rate.

equations represent a market quoting convention for the more complex models of volatility that now tend to be used.[4]

One key feature of the Black-Scholes-Merton model is the way in which a single volatility is assigned to the underlying. By assuming a constant-volatility log-normal process in the derivation of Black-Scholes-Merton, the model is consistent with a volatility surface like the one shown in **Figure 4**. That is, plotting the implied volatility as a function of expiry and strike price produces a flat surface.

Figure 5 shows a more realistic volatility surface, one that we might actually construct by plotting implied volatilities observed in the market. Implied volatility is clearly not constant, neither with the level of the underlying, nor the passage of time. This is of course not surprising. The world is too complex to be modelled in the simplistic way that the Black-Scholes-Merton model suggests, though as we will discover in the following section, we are able to rationalise deviations from constant log-normal volatility based on some of the properties of markets.

The implied volatility surface

Under the Black-Scholes-Merton model

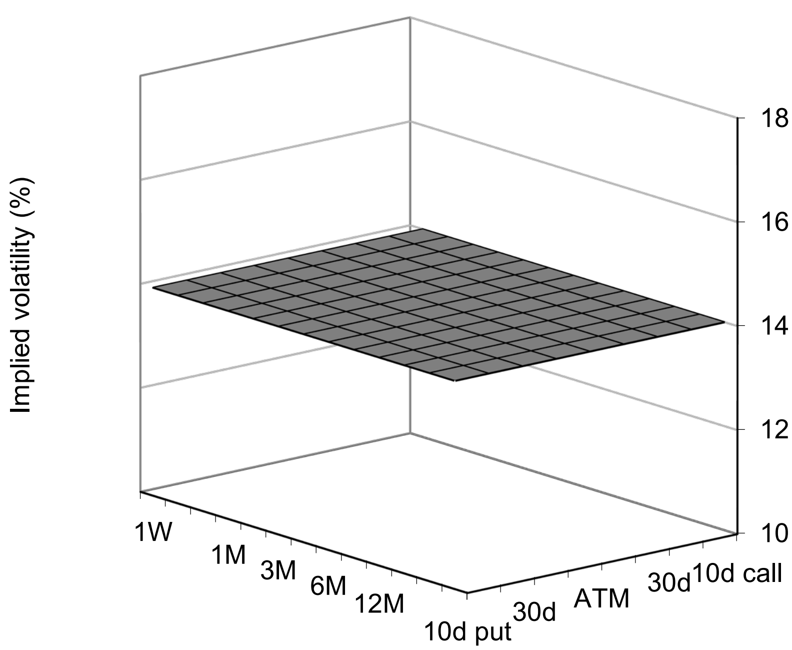

Figure 4

[4] For exotic options the market calculates a theoretical value which is a benchmark, but no standard model exists for pricing. Most traders use a stochastic local volatility model but everyone's calibration is slightly different.

Observed volatility surface for USDSEK as at 28 June 2013

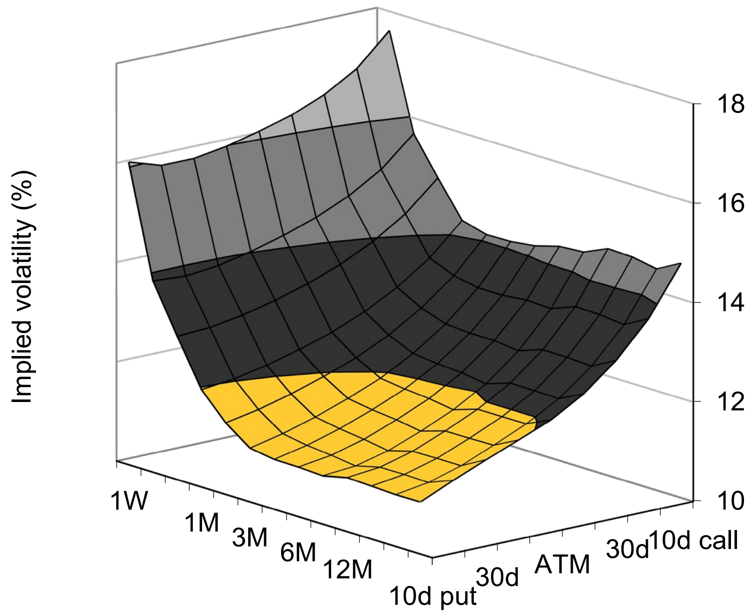

Figure 5

Causes of volatility smiles and surface variations

Implied volatilities tend to be greater when related to moves that are strongly risk-averse. While rising risk aversion invariably means a sell-off in equity markets, the direction in which a spot FX rate develops as risk assets decline in value is not always obvious.

For a currency pair that includes a 'hard' currency such as the US dollar and a 'soft' currency (e.g. an emerging market currency) safe-haven flows are most likely to mean the selling of the latter and increased demand for the former. Thus, in cases where a currency pair contains a clear safe-haven currency we may expect the shape of the volatility smile to be skewed so that it resembles and equity index implied volatility 'smirk', with volatility higher for strike prices that involve a strong depreciation of the more risky currency. Depending on how we quote the exchange rate, the smirk may slope in the opposite direction; we should expect the exchange rate to rise in times of crisis if the US dollar is specified as the base currency.

For cross rates where two 'hard' currencies appear, the volatility structure may be more symmetrical: closer to a smile than a smirk. **Figure 6** shows two volatility smiles that support this argument. For USDMXN the skew structure has a definite smirk shape, with OTM calls on the rate implying a higher volatility than OTM puts. For EURGBP however, a case where it is less clear whether a rise of fall in the exchange rate should be considered a risk-off event, the skew structure is almost symmetric.

Volatility smiles for USDMXN and EURGBP as at 28 June 2013

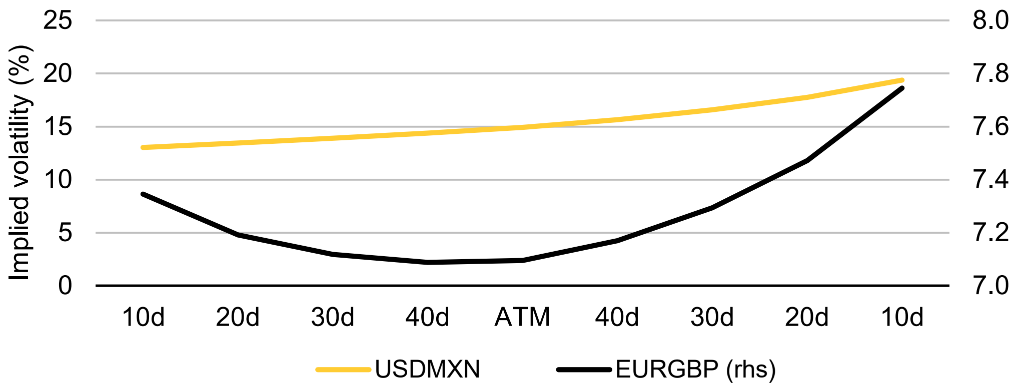

Figure 6

Finally, to complete our ride on the emotional roller coaster of volatility structures, in addition to the possibility of smiles and smirks, we must also consider the possibility of a volatility frown. A volatility frown may arise in the case of a binary event that may cause a sharp step either higher or lower in an asset price. For example, a company subject to the outcome of litigation may see its stock price step higher or lower by 10% depending on the announcement of the result.

In currency markets, a volatility frown is relatively rare. The phenomenon is normally restricted to very short dated options (under one week maturity) around the time of significant events. Such an example would be EURUSD options over the release of non-farm payroll (NFP) data in the US. This is typically a market moving event so volatility is expected, but once the event risk has passed volatilities tend to decline. Traders look to buy ATM options and sell the wings of the volatility smile to take advantage of this characteristic.

Measuring the volatility smile

How do we identify the various shapes of implied volatility skews in order to gauge what markets are pricing in regarding tail risks?

To introduce a degree of confusion, as is often the case in finance the exact terminology used to this end is dependent on the market in question.

Out-of-the-money equity index options have historically tended to have their strike prices quoted in percent of the spot rate. The measure of asymmetry of the smile commonly used in this market is the 90%-110% equity implied volatility skew. As one might imagine, this quantity represents the difference in implied volatility for options with strike prices of 90% and 110% of the spot rate.

Figure 7 illustrates the construction of the measure. The lower the value (for a given prevailing level of volatility), the lower the relative cost of put options to call options. One may interpret a particularly low level of equity implied volatility skew as an indicator of complacency among investors.

Measures of skewness for the volatility smile: Schematic illustration of the equity index 90%-110% implied volatility skew (equity price on x-axis)

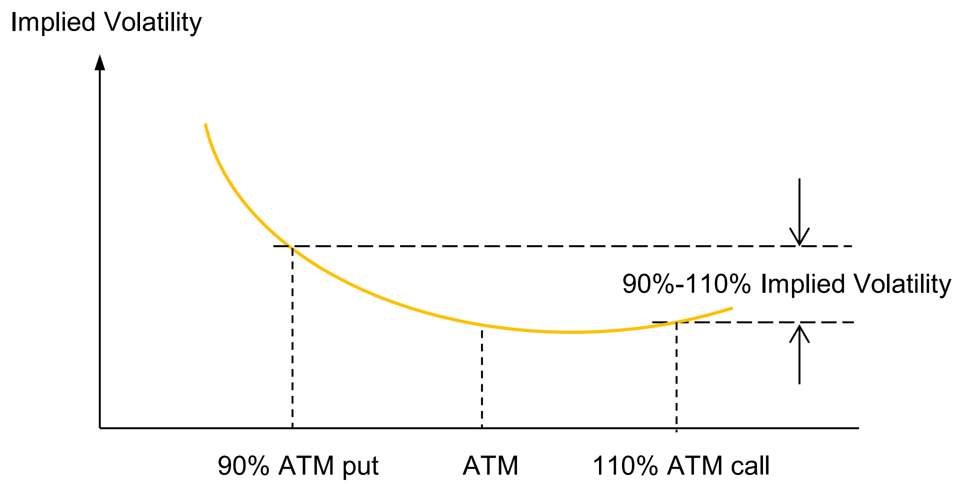

Figure 7

Measures of skewness for the volatility smile: Schematic representation of a 25-delta risk reversal (FX rate on x-axis)

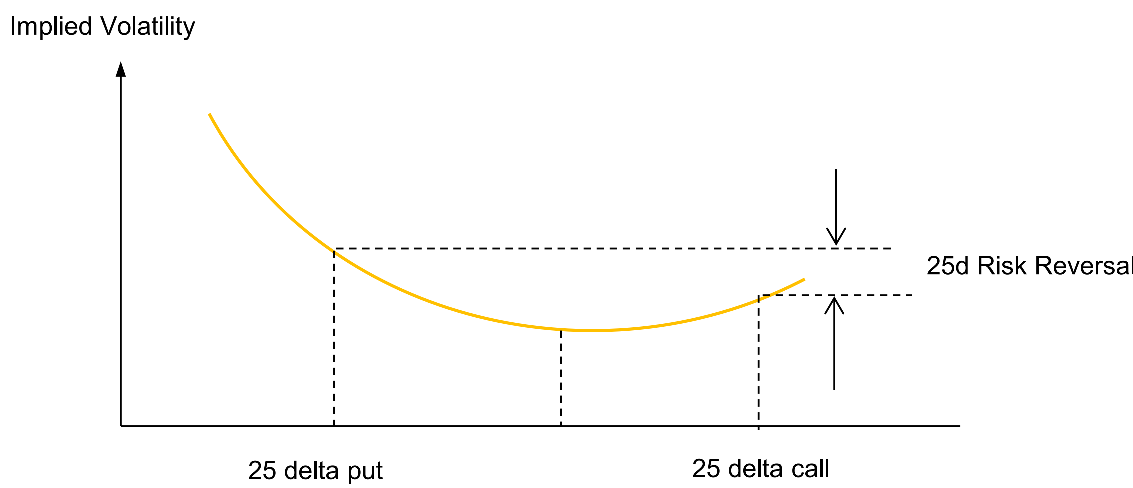

Figure 8

In foreign exchange markets, OTM options tend to be quoted in terms of delta rather than as a percentage of spot. Considering the difference in implied volatility for 25-delta OTM put and call options gives a similar indication of the amount of skew in the FX option volatility smile. For foreign exchange options a particular terminology is adopted for this spread, with market participants often referring to the '25-delta risk reversal'. By analogy to the equity implied volatility skew for equity index options, FX risk reversals contain information about the directional outlook of market participants.[5]

[5] In addition to being a convenient label for the difference in implied volatility of put and call options of the same delta, the term risk reversal also refers to an option strategy that involves buying an OTM call option and selling an OTM put option.

Delta is one of the option ‘greeks’ and has various definitions. Formally it is change in price of a derivative, such as an options contract, given a unit change in its underlying, expressed as a percentage. It is the price sensitivity of the FX option with regard to a unit change in the FX rate. It is also a proxy to the probability of an option to end in the money at expiration. An option with Delta of 10 has a 10 percent probability of being in-the-money at expiry.

Beyond the simple measure of symmetry provided by risk reversals, we can gauge the curvature of the implied volatility smile by considering the butterfly spread. If markets are liquid and we can expect the same volatility for all values of the underlying, we should anticipate a flat Black-Scholes-Merton volatility surface. However, in reality unlikely events can occur and markets can react wildly to shocks. In such cases we should expect much higher volatility away from the current spot price and the smile curve will be shaped more like a letter ‘U’ than a horizontal line.

To provide an example, the market in EURUSD is a well-established. There is great liquidity and even if the spot rate moves strongly we can expect an orderly market. These facts are reflected in a shallow smile – imagine the cross section of a saucer for a teacup. In contrast, some emerging markets have much wilder gyrations in exchange rates and as a result a much more convex volatility smile – imagine the cross section of the teacup rather than its saucer. This is a relatively easy concept to imagine and is captured by an FX option butterfly. A long butterfly strategy consists of buying an ATM straddle and selling put and call options of equal delta.

Trading a butterfly spread in the market is relatively easy even if understanding exactly how the spread is constructed is a little more complicated. The market calculates the butterfly volatility as:

$$= \qquad + \qquad .$$

Using this derived volatility the strikes for the butterfly are then calculated. Hence the 25-delta put strike for a butterfly is not the same as the 25-delta put strike for a risk reversal (**Figure 9**).

In simple terms, the butterfly spread allows us to observe the probability of larger moves in the exchange rate that are priced into the options market.

A butterfly spread can be used to gauge the steepness of the volatility smile

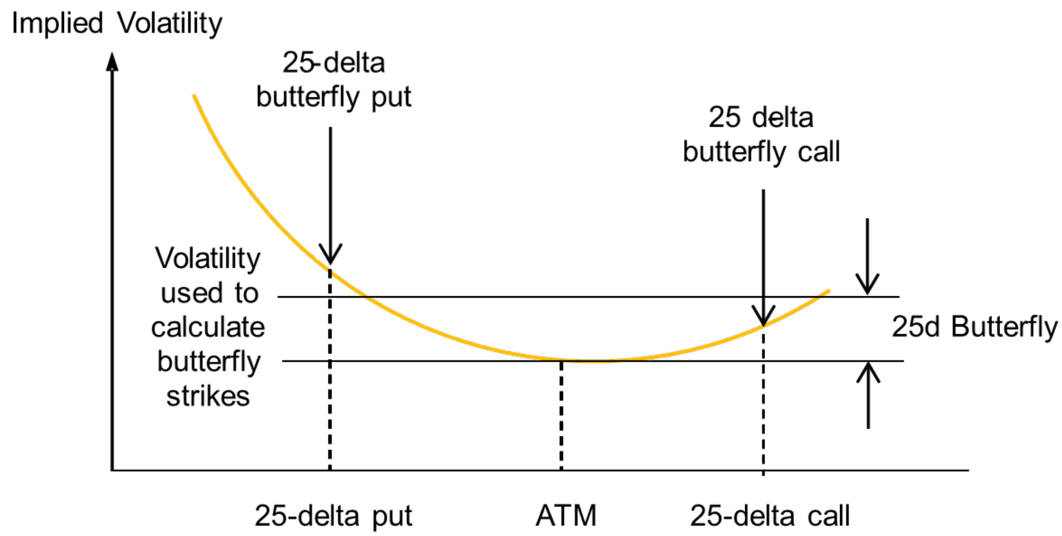

Figure 9

Trading practice

Early option trading was dominated by prices given in premium terms. This was perfect for an occasional hedger wanting to buy or sell an option to cover a risk and know the exact cost. Indeed for a hedger implementing static option positions there is little need to understand much more. However, market makers wanting to cover their 'options' exposure wanted to find the contract offering the best value. As the underlying was constantly moving it was a cumbersome task to feed the price in to a formula in order to calculate an implied volatility for each contract. Each time spot moved you needed to start the calculation again. However, if the market makers started with the implied volatility then they had a standardised way of comparing contracts with different strikes and different maturities. Traders understood that as volatility was directly related to premium this was a more efficient way to deal. After agreeing the volatility all the underlying parameters would be set at the prevailing market conditions to determine the premium. Hence, for dynamic traders the market evolved to implied volatility quoting and trading to identify the relative cost of different options. This still holds true for the over-the-counter options market where the majority of the volume is transacted in the form of vanilla options.

This simple example of how the option market developed gives us a first indication that there are two fundamental approaches to trading options. On the one hand the static trader has a long term view and as described in more detail below will tend to think in terms of matching an option with an underlying view or exposure over a relatively long horizon. An option dealer on the other hand is far more likely to act in the much shorter term to continually hedge risk exposure.

One of the beautiful things about options is that both the buyer and seller can make money from the same trade. Conversely they can both lose! This is all down to risk management of the option, the skill of the trader and sometimes luck. We will explore in more detail how the buyer and seller can both make money as we consider different types of trading style.

Two views of volatility

Depending on the market participant and the reason for trading an option, volatility can be viewed differently. A corporate hedger wishing to protect a risk may enquire about the price of an option. From this the breakeven of the contract at maturity can be determined – how far will the spot rate have to move before the payoff from the option covers the premium? The hedger will have an appreciation of whether in his mind the price is too high or too low. In essence he is judging how far from the current expectation the underlying will finish. As such, the corporate treasurer may rely on intuition, economic research or forecasts to determine whether the implied volatility is too high or too low and if the contract offers good value.

For a dynamic trader volatility means something rather different. It is how volatile the underlying is during the journey to maturity of the contract and how much value can be extracted from this movement. An underlying could start and finish at the same level, but if it has been extremely volatile in the intervening period the dynamic trader can make money.

Hence there is a different concept of ‘good value’ which is one of the reasons there is such an active market in FX options. We consider the two strongly diverging outlooks in greater detail when we consider static and dynamic trading approaches below.

Static trading

A static trader or hedger will usually not be interested in a lot of option market jargon. A typical static trader might be a corporate treasurer. They could have a predefined currency exposure in a ‘risk currency’ which they want to hedge back into a base currency. They set a strike rate based on the worst level that they can tolerate and an option acts like an insurance policy. A premium is paid upfront and if the market moves against the hedger, the option is exercised.

For such applications, a relatively small amount of terminology is required, specifically relating to the setting of the option strike rate. The alternatives are listed below.

ATMF: At the money forward

ATMF options are the simplest options to understand and to value of all the FX option contracts. The strike of the option is determined by the forward rate at the time the option is purchased. A person who buys an ATMF call option on an FX rate will receive a payoff if the FX rate is above the strike rate on the expiry date; if instead they have bought a put option then they will receive a payoff if the FX rate is below the strike rate. The forward rate at inception is the critical point at which the option holder begins to make some return. The breakeven is the point where the option value overtakes the premium.

OTM: Out of the money

Where the strike is unfavourable compared to the forward.

ITM: In the money

Where the strike is favourable compared to the forward.

Given the nature of FX markets, with spot and forward rates constantly moving, options starting out ATMF will soon be either OTM or ITM. What is more, the status of an option's strike rate can oscillate during the life of the contract.

As discussed, the delta of an option is sometimes used to describe how likely it is to be exercised. A 50 delta option will be an ATMF option where the underlying has an equal chance to move up or down and so broadly speaking has a 50% chance of exercise. An option with a delta of 85% or more may be described as deeply ITM and a delta of 15% or lower may be associated with deeply OTM options. We note that this is just a rule of thumb and that one must be careful of using deltas in this fashion – for exotic options delta can easily be ±200%, but this does not imply that the probability of exercise can reach 200%!

Dynamic trading

If we choose to describe a corporate treasurer as a static trader, then an FX option dealer working for a bank might be labelled a dynamic trader. Compared to the static trader who is most interested in the option premium versus the expected size and probability of the payoff, the dynamic trader is far more interested in volatility. In the notation of Chapter 1, our dynamic trader would fall into the class of 'investors', though as a special case, usually focussing on the shorter term.

A dynamic trader will typically have many option positions and may trade day-to-day if not intraday in order to maximise the returns on a portfolio. We can see immediately that unlike for the static trader, the size of daily changes in the underlying are very important for the dynamic trader. To help us understand how an active option dealer goes about making money we can consider a crude example.

Let us imagine a market where the only parameter to vary is the underlying spot rate. Let's also assume that a trader that buys a one week ATMF EUR call USD put option for 10 million EUR notional. He or she thinks there will be a lot of volatility, but he is not sure of the direction of moves in the spot rate.

To protect against the case of EURUSD moving lower and the premium being paid for no return, the trader sells EURUSD in the spot market. Given that the spot rate could equally well move higher or lower, the trader sells 5 million EUR – half of the option notional. During the day the spot rate does indeed move lower and the trader buys back the 5 million EURUSD position at a profit and is left with just the option. If the spot rate retraces back to the original level, this process can be repeated. This is the essence of delta hedging. On each trading day to maturity of the option the trader buys and sells currencies in the spot market, hopefully making a profit. A dynamic trader will judge the cost of an option with reference to its implied volatility and whether they can cover the premium of the option through this delta hedging process. This is clearly a different measure of value than that utilised by the static trader.

It is worth mentioning at this point why it is that most dynamic traders will not be too concerned about the specific characteristics of an option and will instead concentrate on the risk-reward and the payoff. This is specifically due to put-call parity.

In simple terms, the profile of an ATMF call option that has been bought can be turned in to the profile of an ATMF put option that has been bought if the full notional amount of the forward contract is sold. Similarly, the profile of an ATMF put option that has been bought can be turned in to the profile of an

ATMF call option that has been bought if the full notional amount of the forward contract is bought. This also holds true if the options had been sold as opposed to bought.[6]

Hence, a dynamic trader will care more about the volatility and the strike rather than whether the option is a put or a call.

Greeks and Delta hedging

The Greeks

Delta measures the rate of change of option value with respect to changes in the underlying asset's price. Delta is the first derivative of the value of the option with respect to the underlying instrument's price, but what does this mean in practice? To better understand, let us imagine an option with five days to maturity and an implied volatility of 10%.

Trading from the long side (the trading desk has bought the option)

A customer sells an option to a market-making volatility desk and collects the premium. The desk has no view on the direction of spot, but just believes that buying the option at that level of volatility provides good value. Hence, the exact delta of the option will be calculated and the underlying traded in the open market.

As spot moves up and down the desk trades the delta back and forth. It is easy to imagine a market where spot is volatile but mean-reverting and finishes each day where it started. In this environment the purchaser of the option will be adjusting furiously throughout the day and will make more money on the spot deals than they have spent on the premium. At the same instance, the seller of the option waits and collects the premium. Both buyer and seller of the option can make money from the same deal.

Trading from the short side (the desk has sold the option)

A customer buys an option from a market making trading desk to hedge an exposure. The desk has no view on the direction of spot only that selling the option at that level of volatility is an attractive trade. Hence, the exact delta of the option will be calculated and the underlying traded in the open market. The desk will now try to adjust the delta as little as possible, knowing that each time they incur trading costs they will eat into the premium they have received.

Whether you are long or short an option the burning questions are how do you know exactly when to time adjustments and how often to do so? As no-one can foresee market moves this is all down to interpretation of what will happen. But gamma can give a guide to the level of delta hedging required.

Gamma measures the rate of change in delta with respect to changes in the underlying's price. Gamma is the second derivative of the option value function with respect to the underlying price. There is not much to understand about gamma, it's just a number that helps traders evaluate an option and how much delta hedging they should be doing. A simplified guide to interpreting gamma is shown in **Error! Reference source not found.**.

Different regimes, implications for gamma and likely trading strategies

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[6] Put-call parity also holds more generally as long as the option strikes and forward rate are equal. A long call, short put combination gives a synthetic long FX position.

Regime	Likely gamma	Likely trading strategy for LONG option position
High volatility	Low gamma	Wait for the larger swings to develop to trade the delta as the high volatility makes the option expensive and therefore the premium paid large.
Low volatility	High gamma	You haven't paid much for the option. If you see a chance, take it.
Long maturity	Low gamma	Relax. You have plenty of time to see how the underlying develops.
Short maturity	High gamma	Trade as much as you can, you are running out of time.

Table 1

Delta hedging an option is also sometimes called 'trading the gamma' and trades can be entered manually or automatically. In an automated approach, as the delta is a calculated quantity it is possible to leave buy and sell orders in the market at pre-defined levels based on the implied volatility. This is more prevalent now with electronic platforms and loop orders.[7]

Most trading desks, however, prefer to have some manual input based on experience and/or ego. Because long and short positions in a vanilla option have equal and opposite gamma, sometimes the decision of whether or not to trade boils down to who blinks first: the traders who are long options or those who are short. If the market is quiet and range-bound, the short option holders are in control and the longs need to actively trade to cover the premium they have paid. Conversely, if the market is breaking out of a trading range, moves can be accelerated by short gamma players all rushing to cover their positions at the same time.

The potential rush to cover positions by those who are short gamma is the reason the market has a fascination with, and a desire to find out, where the strikes with big notional value are, ie, are there large trades done which are waiting for the strike rate to be reached? If an underlying starts to trade near a strike close to expiry then even people who are not connected with the option will start to trade around that level, buying the underlying below and selling above. This can help concertina the movements of the underlying so much that at exercise time the underlying is right at the strike level. This phenomenon is known to traders as 'pinning the strike'.

If we look at a simple example of delta hedging we can see explicitly how it is that both the option buyer and seller can make money (Figure 10). At inception of the trade the spot rate is at 1.3600 and the forward rate is slightly higher than spot. The option seller sells a call option for 1.3640 – the breakeven

[7] A loop order may be created when a cyclical movement in the market is expected. Two orders may be placed with each triggered in turn as specific conditions occur. The process is repeated until the order is cancelled by the trader.

on the premium is mapped out for each day in the figure. As spot finishes below the breakeven the seller collects the premium and does not have to make any payments to the buyer. Thus, the seller ends in profit, but what about the other side of the trade? While the trader who bought the option will have initially been out of pocket due to paying the option premium, by actively 'trading the gamma' this premium could have been covered and a small profit generated. Both sides of the trade can win!

A simple example of delta hedging. With the spot rate finishing below the option breakeven rate the seller makes a profit. Spot rate volatility means that the option buyer could have realised a profit too

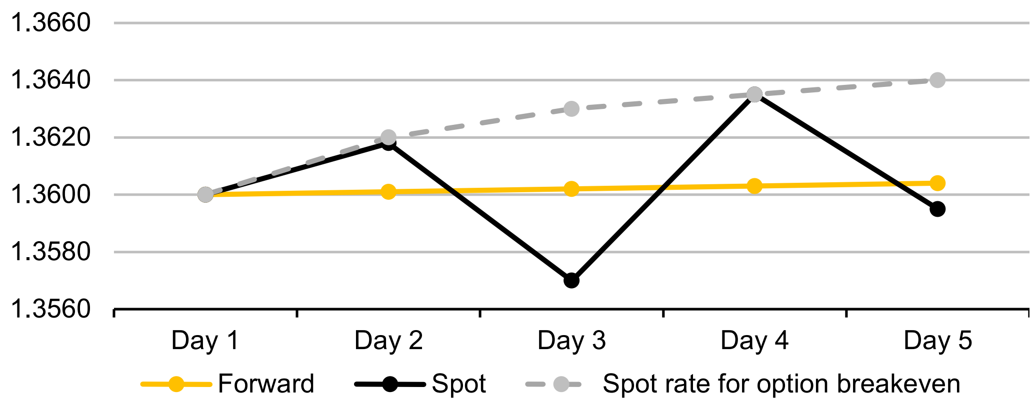

Figure 10

Theta measures the sensitivity of option value to the passage of time.

An initial estimate of theta can be calculated by valuing the option and then revaluing the option with all the parameters the same but with one trading day less. This gives the value attached to the next trading day or the 'time decay' of an option. The target for most active gamma traders is to 'beat the theta' every day and thereby lock in a profit as time passes.

When valuing different options, traders will often compare the gamma-to-theta ratio to determine the most attractive option to buy. In other words, they ask which option has the best potential to allow money to be made from gamma adjustments, but will cost the least each day in time decay. Options with shorter maturities will have higher gamma *and* theta numbers. Hence, these Greeks are most relevant for vanilla options that have a maturity of less than one month.

Vega is another important Greek for option traders. It gives the sensitivity of an option to changes in implied volatility. More formally, vega is the derivative of option value with respect to the volatility of the underlying asset. Whereas for shorter dated options traders are most concerned about the gamma-to-theta ratio, for longer-dated options vega has a much greater impact.

Rho measures sensitivity to interest rates; it is the derivative of the option value with respect to the interest rate. For currency trading there are two exposures, relating to the rates for the base and quote currencies.

	Calls	Puts
Value (p or c)	$c = S_0 e^{-r_b T} N(d_1) - K e^{-rT}\, N(d_2)$	$p = K e^{-rT} N(-d_2) - S_0 e^{-r_b T}\, N(-d_1)$
Delta Δ	$\Delta = e^{-r_b T} N(d_1)$	$\Delta = e^{-r_b T}[N(d_1) - 1]$
Gamma Γ	$\Gamma = e^{-r_b T} \dfrac{N'(d_1)}{S_0 \sigma \sqrt{T}}$	
Theta Θ	$\Theta = \dfrac{-S_0 N'(d_1) \sigma e^{-r_b T}}{2\sqrt{T}} + r_b S_0 N(d_1) e^{-r_b T} - rK e^{-rT} N(d_2)$	$\Theta = \dfrac{-S_0 N'(d_1) \sigma e^{-r_b T}}{2\sqrt{T}} - r_b S_0 N(-d_1) e^{-r_b T} + rK e^{-rT} N(-d_2)$
Vega υ	$S_0 \sqrt{T}\, N'(d_1) \sigma e^{-r_b T}$	
Rho ρ	$-S_0 e^{-rT} N(d_1)$	$S_0 e^{-rT} N(-d_1)$

Where

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$$\frac{\quad + \quad}{\sqrt{\ }}$$

$= d_1 \square \sigma\sqrt{T}$

and:

S_0 = FX rate at inception,

K = strike rate,

r = interest rate for tenor of the option in the quote currency,

r_b = interest rate for the tenor of the option in the base currency,

T = tenor of the option, and

σ = implied volatility of the option.

N(.) denotes the standard normal cumulative distribution function.

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

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