CQF: Certificate in Quantitative Finance

Valuation Adjustments (‘xVAs’)

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Content

- Historical background and xVA overview
- From discounting to valuation adjustments
- CVA and DVA
- Exposure simulation
- Wrong-way risk
- FVA, MVA and KVA
- Speeding up xVA calculations

The Birth of xVA

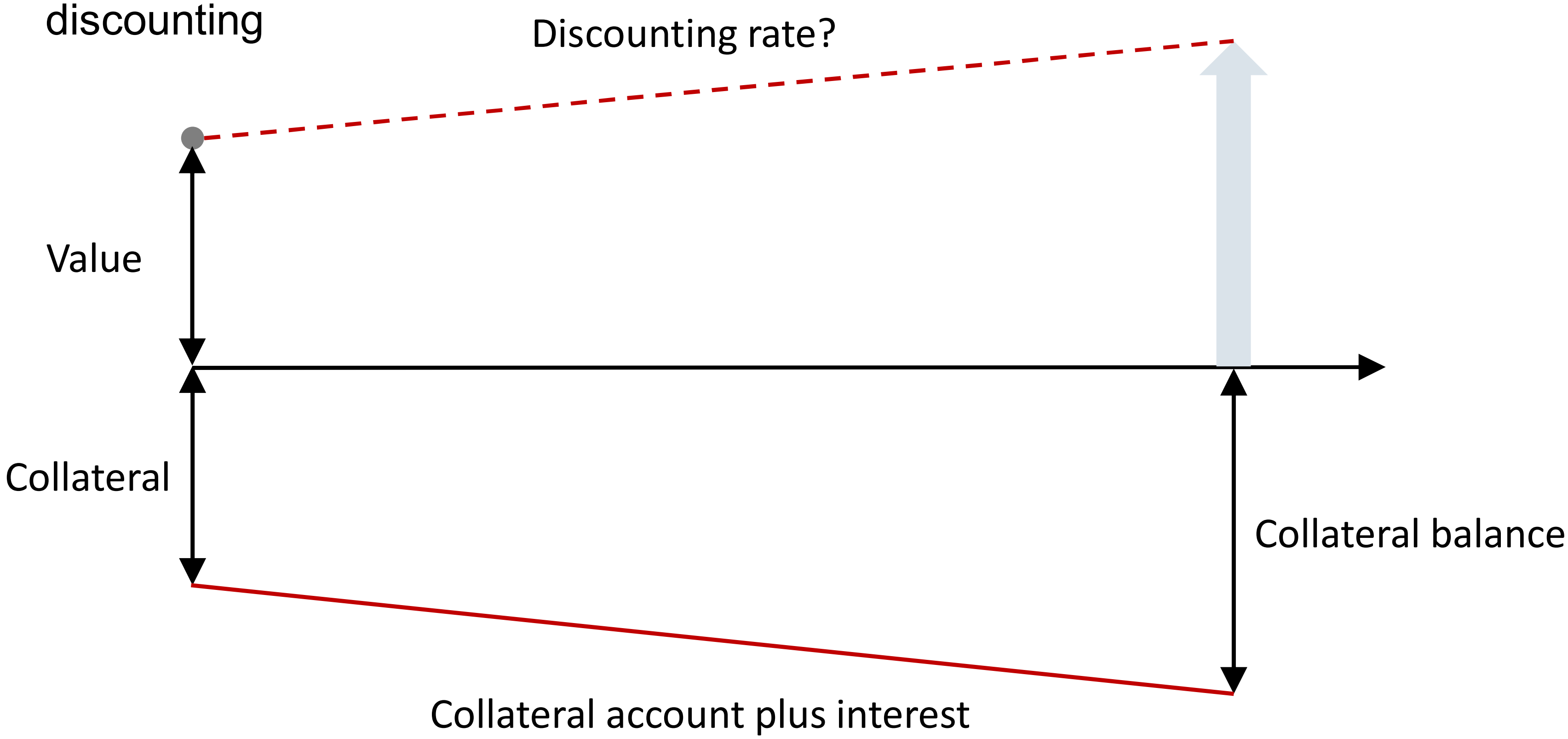
- Derivatives pricing and valuation was historically seen as being cashflow driven
- Now it is seen as being also related to
  - Credit risk
  - Funding
  - Collateral
  - Capital
  - Initial margin

$$V_{actual} = V_{base} + xVA$$

- These aspects are not mutually exclusive and often require portfolio level calculations
  - The has led to the birth of the "xVA desk" or "central resource desk"
  - This desk typically deals with most of the complexity in derivatives pricing

The Starting Point - Collateral Discounting

- OTC derivative collateral agreements specify a (cash) collateral remuneration rate
  - This is normally the OIS (overnight index swap) rate (in most collateral agreements)
  - This leads to ‘collateral discounting’ or ‘OIS discounting’ (Piterbarg 2010)
  - In bilateral (not centrally cleared) derivatives, securities collateral implies repo rate discounting



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Beyond Basic Discounting

- Suppose the discounting rate is different which could occur for a number of reasons
  - Collateral agreement specifies a different rate (e.g. OIS minus 20 bps)
  - Collateralised in a different currency
  - Choice over collateral posted
- Two options
  - Discount at the different (correct) rate!
  - Work out an (xVA) adjustment formula
- We will see that the second – whilst sometimes unnecessary – is the more intuitive and general solution

Collateral Adjustments

"In line with market practice, the methodology for discounting collateralised derivatives takes into account the nature and currency of the collateral that can be posted within the relevant credit support annex (CSA). The CSA aware discounting approach recognises the 'cheapest to deliver' option that reflects the ability of the party posting collateral to change the currency of the collateral."

Source: Barclays Annual Report 2021

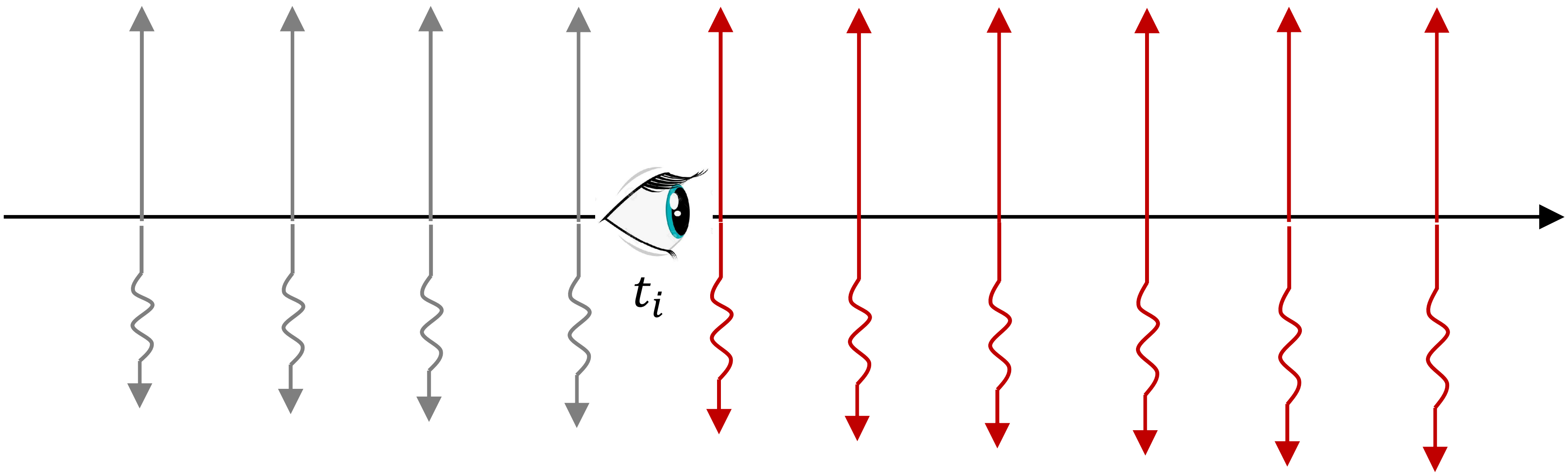
Collateral Valuation Adjustment (CollVA)

Collateral Valuation Adjustment is a derivative valuation adjustment capturing specific features of CSA (Credit Support Annex) with a counterparty that the regular valuation framework does not capture. Non-standard CSA features may include deviations in relation to the currency in which ING posts or receives collateral, deviations in remuneration rate on collateral which may pay lower or higher rate than overnight rate or even no interest at all. Other deviations can be posting securities rather than cash as collateral.

Source: ING Group Annual Report 2021

Expected Future Value (EFV)

- What is the (discounted) expected value of a transaction at a future date?



- Present value of all cashflows after that date
- This is the value of a forward starting transaction

$$EFV(t_i) = \sum_{i+1} CF_{t_i} DF_{t_i}$$

$$CF_{t_i} DF_{t_i} = EFV_{t_{i-1}} - EFV_{t_i}$$

xVA and Discounting (I)

- Suppose we define xVA as the difference between discounting at two different rates (the difference between the rates is a spread *s*)

$$xVA = \sum_i CF_{t_i} DF_{t_i} \underbrace{\exp[-s_{t_i} \times t_i]}_{\text{Additional discounting term}} - \sum_i CF_{t_i} DF_{t_i}$$

$$= \sum_i CF_{t_i} DF_{t_i} \{\exp[-s_{t_i} \times t_i] - 1\}$$

- Also define the EFV as the sum of all the future cashflows after a given date:

$$EFV(t_i) = \sum_{i+1} CF_{t_i} DF_{t_i}$$

$$CF_{t_i} DF_{t_i} = EFV_{t_{i-1}} - EFV_{t_i}$$

xVA and Discounting (II)

- xVA can then be written as:

$$xVA = \sum_i \left[EFV_{t_{i-1}} - EFV_{t_i}\right]\left\{\exp\left[-s_{t_i} \times t_i\right] - 1\right\}$$

- Which gives an xVA-like formula

$$= -\sum_i EFV_{t_{i-1}} \underbrace{\left\{\exp\left[-s_{t_{i-1}} \times t_{i-1}\right] - \exp\left[-s_{t_i} \times t_i\right]\right\}}_{\textit{Forward spread}}$$

- Conclusion
  - The difference is discounting assumptions can be written as an xVA formula (although this may be unnecessarily complex and the reverse is not true)
  - Situations this is relevant
    - ➢ Collateral discounting (discounting in collateral currency)
    - ➢ FVA (discounting at own cost of funding)

Discounting and ColVA - Example

- ColVA formula based on expected collateral balance (ECB = EFV)
- This example is simple due to the symmetry (default is not symmetric)

$$ColVA = -\sum_i ECB_{t-1}\{\exp[-s_{t_{i-1}} \times t_{i-1}] - \exp[-s_{t_i} \times t_i]\}$$

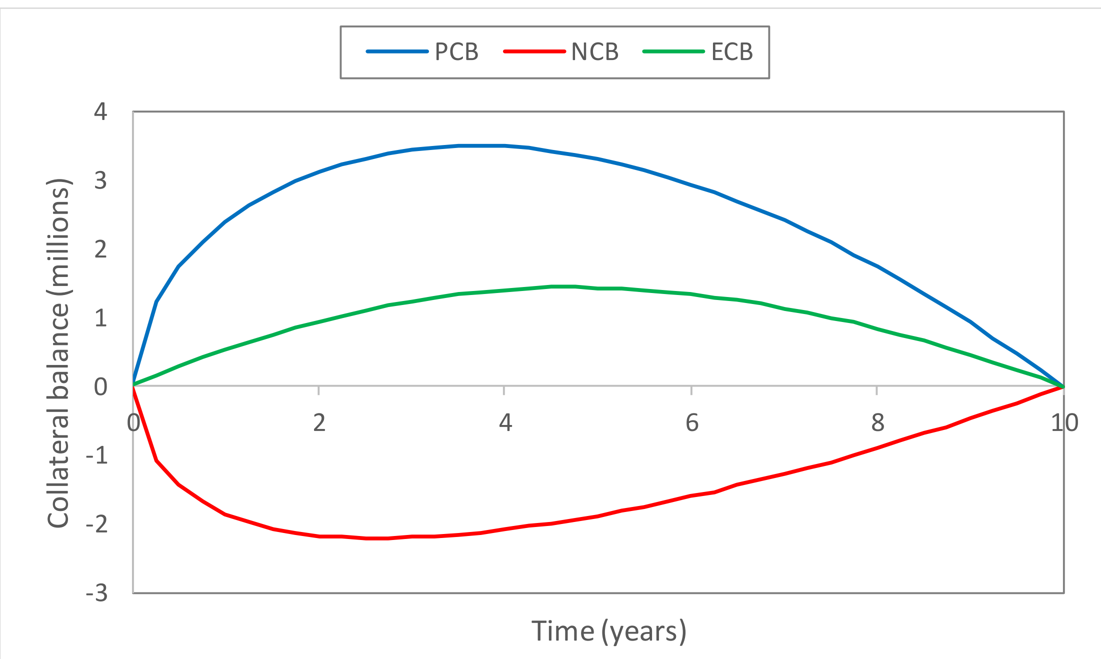
	Pay fixed	Receive fixed
Swap value discounted at base rate	-303,032	303,032
Swap value discounted at alternate rate	-311,074	311,074
Difference (using spread s_t)	-8,043	8,043
ColVA formula	-8,043	8,043

What's the point of a ColVA formula?

- Expand the ColVA formula into cost and benefit terms (*ECB* = *PCB* + *NCB*)
  - Positive collateral balance (PCB) and negative collateral balance (NCB)

Collateral received adjustment $$ColRA = -\sum_i PCB_{t-1}\{\exp[-s_{t_{i-1}} \times t_{i-1}] - \exp[-s_{t_i} \times t_i]\}$$

Collateral posted adjustment $$ColPA = -\sum_i NCB_{t-1}\{\exp[-s_{t_{i-1}} \times t_{i-1}] - \exp[-s_{t_i} \times t_i]\}$$



	Pay fixed
Difference (as before)	-8,043
ColRA	-23,569
ColPA	15,526
ColRA + ColPA	-8,043

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Traditional CVA Formula

- CVA formula (see Appendix) compared to ColVA formula (hazard rate is *h*)

$$ColVA = -\sum_i EFV_{t_{i-1}}\{\exp[-s_{t_{i-1}} \times t_{i-1}] - \exp[-s_{t_i} \times t_i]\}$$

$$CVA \approx -LGD \sum_i EPE_{t_{i-1}}[\exp(-h_{t_{i-1}} \times t_{i-1}) - \exp(-h_{t_i} \times t_{t_i})]$$

$$EFV_{t^*} = E_t[V(t^*, T)] \qquad EPE_{t^*} = E_t[V(t^*, T)_+]$$

- Note EPE is asymmetric (option-like) quantity compared to EFV
  - No discounting formula for CVA
- Since (deterministic) hazard rate is linked to spread and RR: $h_t = s_t/LGD$

$$CVA \approx -LGD \sum_i EPE_{t_{i-1}}\left[\exp\left(-\frac{s_{t_{i-1}} t_{i-1}}{1-RR}\right) - \exp\left(-\frac{s_{t_i} t_i}{1-RR}\right)\right]$$

  - Refer to CDS lecture for discussion on default probability and RR and proxy credit spreads

Example

The Firm estimates derivatives CVA using a scenario analysis to estimate the expected credit exposure across all of the Firm's positions with each counterparty, and then estimates losses as a result of a counterparty credit event. The key inputs to this methodology are (i) the expected positive exposure to each counterparty based on a simulation that assumes the current population of existing derivatives with each counterparty remains unchanged and considers contractual factors designed to mitigate the Firm's credit exposure, such as collateral and legal rights of offset; (ii) the probability of a default event occurring for each counterparty, as derived from observed or estimated CDS spreads; and (iii) estimated recovery rates implied by CDS, adjusted to consider the differences in recovery rates as a derivative creditor relative to those reflected in CDS spreads, which generally reflect senior unsecured creditor risk. As such, the Firm estimates derivatives CVA relative to the relevant benchmark interest rate.

JP Morgan 2015 Annual Report

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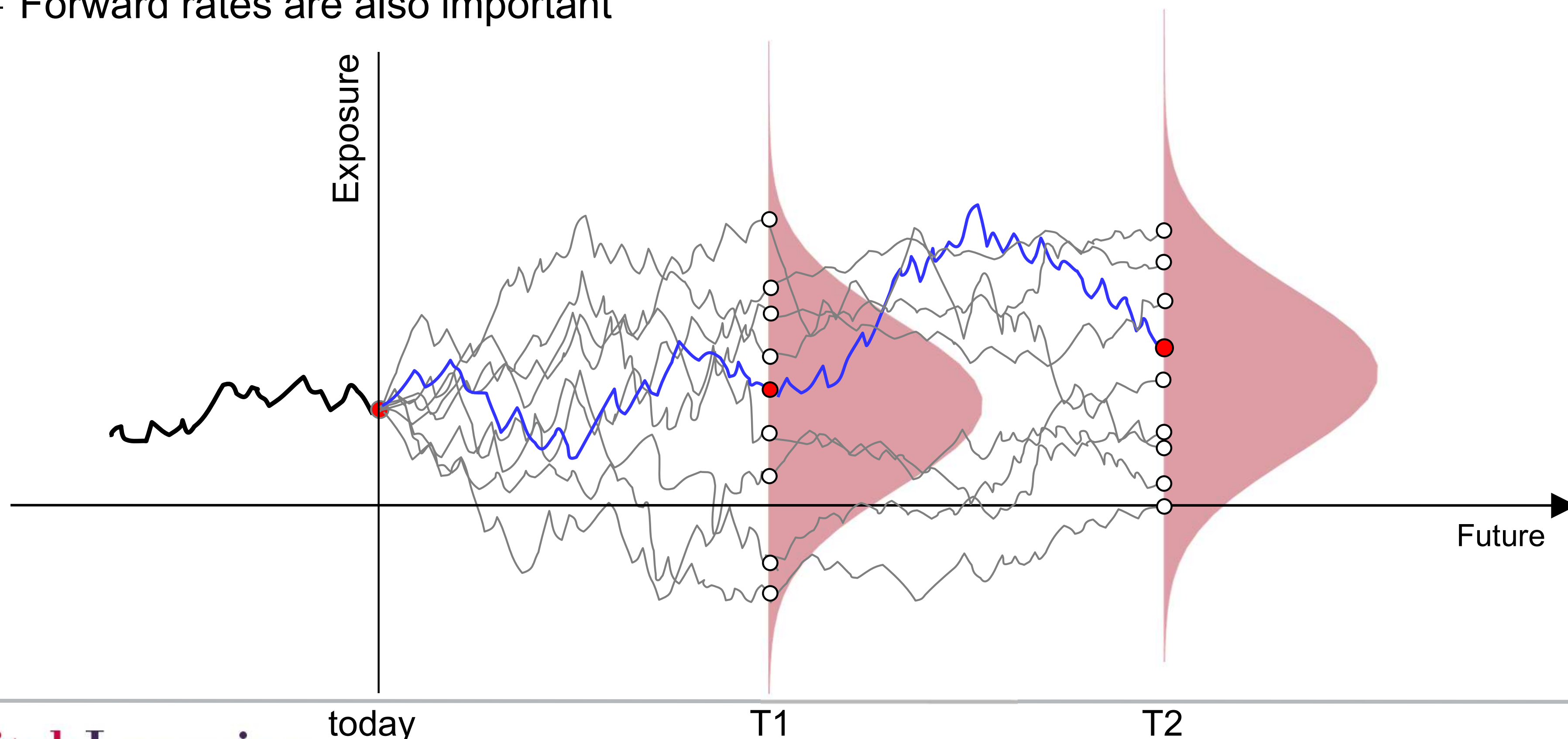
Multi-Step Exposure Simulation

- Usually used to calculate quantities needed for valuation adjustments
  - EFV, EPE, ENE, ECB, PCB, NCB, ……
  - Ageing is important (e.g., a 10-year swap becomes a 9-year swap in 1-year and options may get exercised)
  - Forward rates are also important

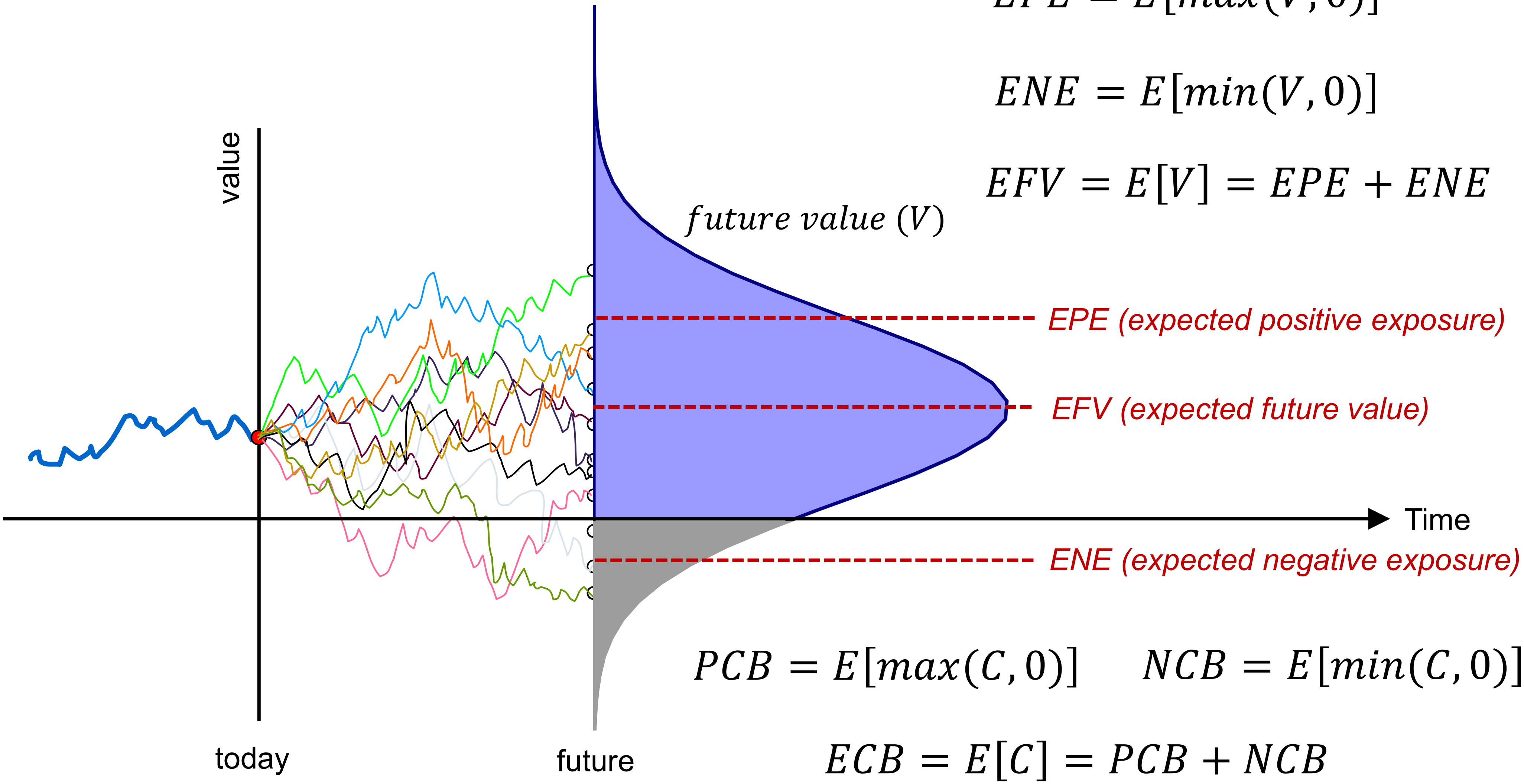


Metrics for xVA

$$EPE = E[max(V, 0)]$$

$$ENE = E[min(V, 0)]$$

$$EFV = E[V] = EPE + ENE$$



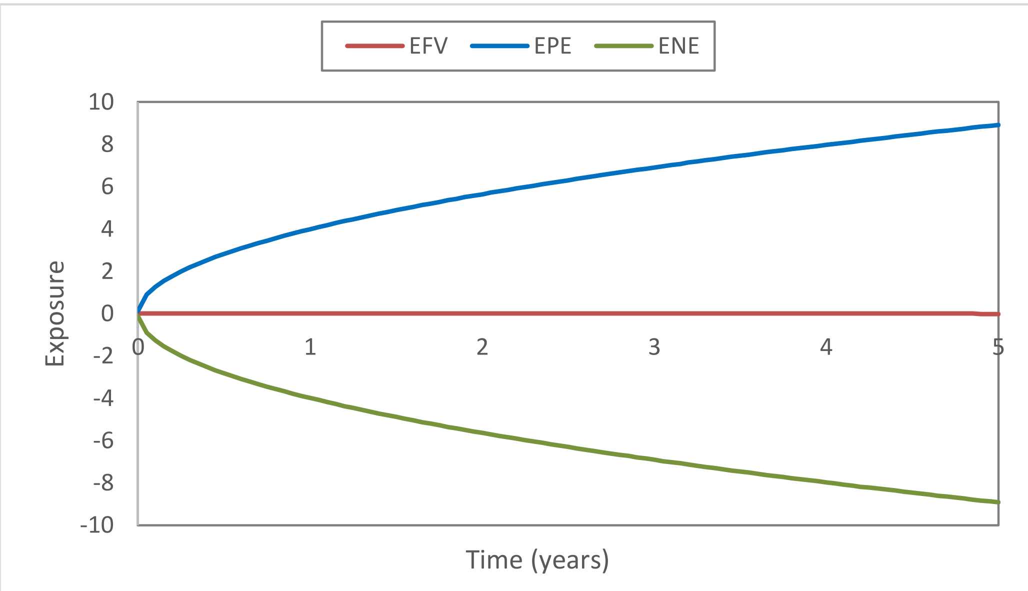
$$PCB = E[max(C, 0)] \qquad NCB = E[min(C, 0)]$$

$$ECB = E[C] = PCB + NCB$$

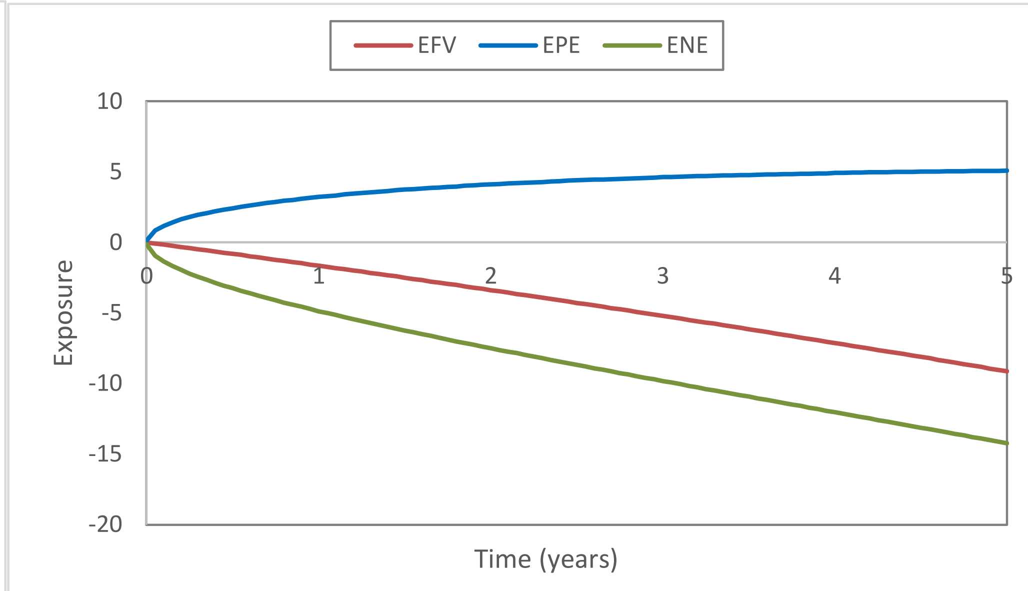
- Strongly collateralised implies $C \approx V$ but may need to model the margin period of risk (MPoR)

Example – FX Forward

Risk-neutral drift



Zero drift



Multi-Dimensional Exposure Simulation – Modelling

- Interest rates
  - One factor Gaussian is quite common
  - Closed-form calculations of discount factors, caps, floors and swaptions which is helpful
  - Constant basis
  - Time-dependent volatility (swaptions)
- FX
  - Lognormal
  - Choose base currency - only need to simulate FX pairs with this base currency involved
  - Constant (cross-currency) basis (risky FX forwards)
  - Time-dependent volatility (FX options)
  - Must calibrate FX options with stochastic interest rate term structure (hybrid model)
- Other
  - Inflation, equity, commodity often less exposure but simple models also used
  - Credit (wrong-way risk, see later)

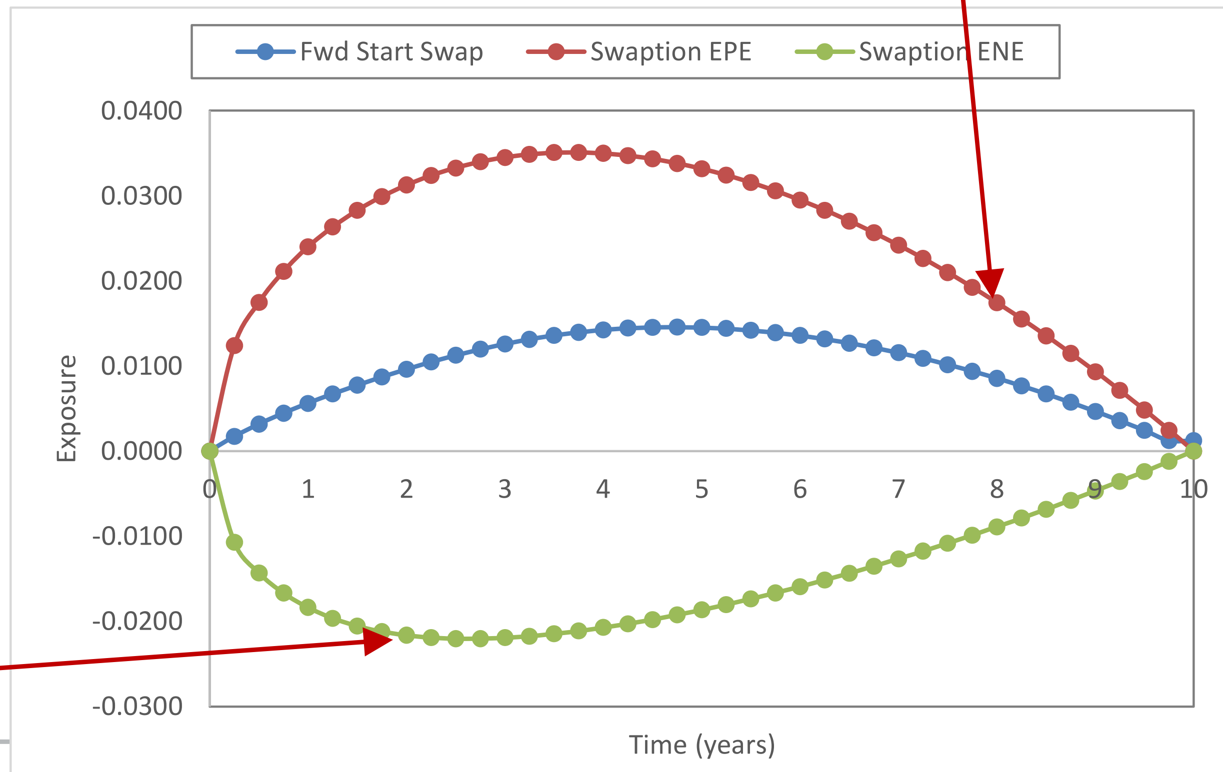
Thoughts on Volatility Calibration

- Sorenson and Bollier (1994)
  - Exposure (EPE) for a swap can be constructed from a series of European swaptions
- Intuition
  - Counterparty has the option to “cancel” the trade when they default

$$Exposure = max(value, 0)$$

Value of 8-year maturity European payer swaption on a 2-year swap

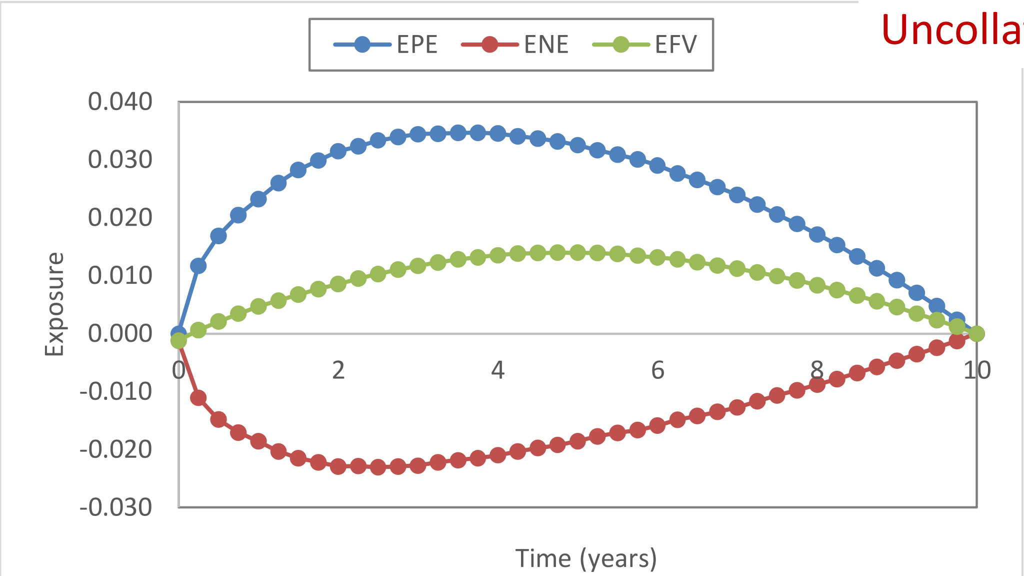
- Would imply a calibration to the ‘co-terminal swaptions’
  - 2x8, 4x6, 5x5, 6x4, 8x2 etc…..
  - However there are several problems with this?

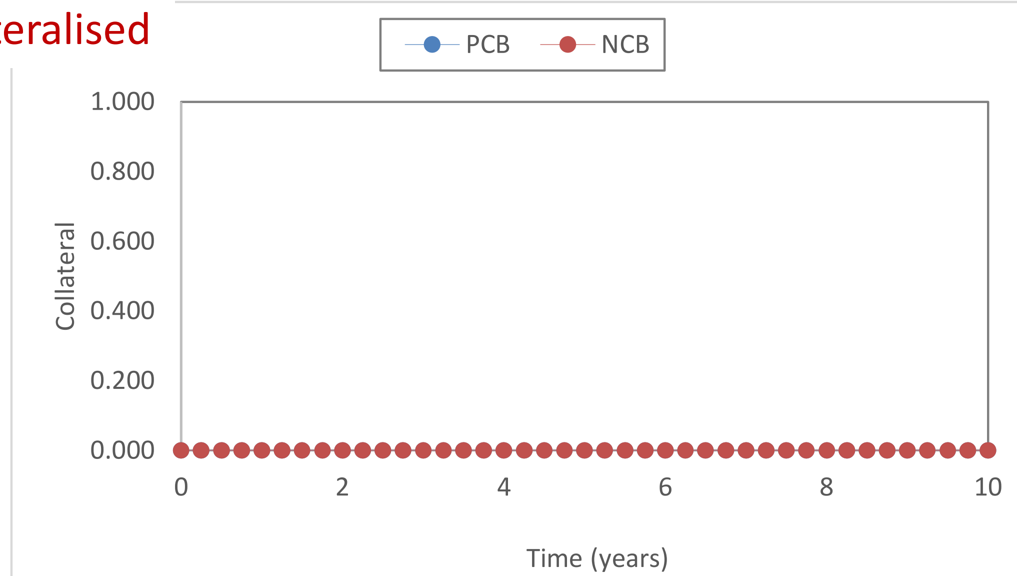


Value of 2-year maturity European receiver swaption on an 8-year swap

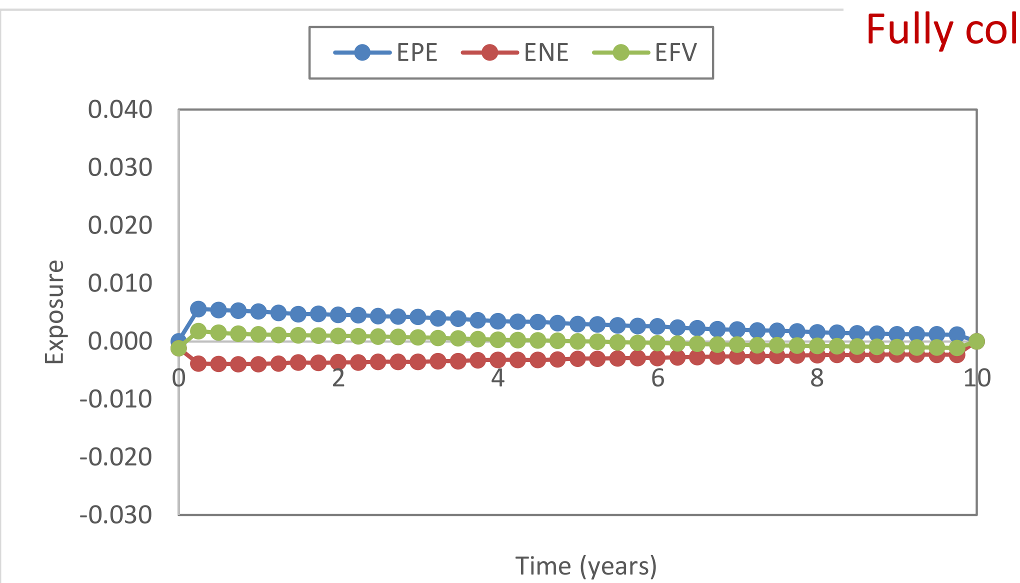
Exposure and Collateral (I)

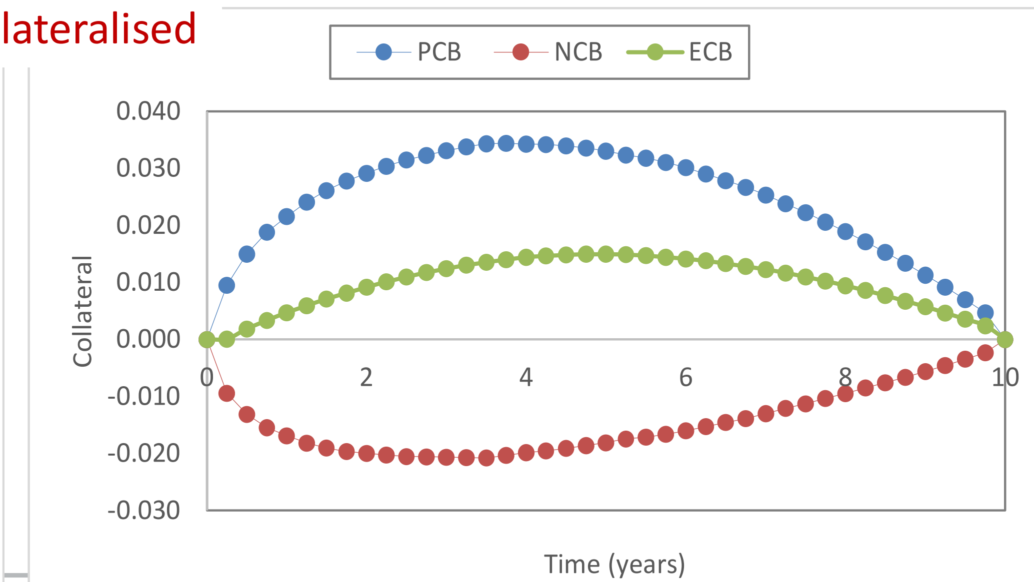
Uncollateralised





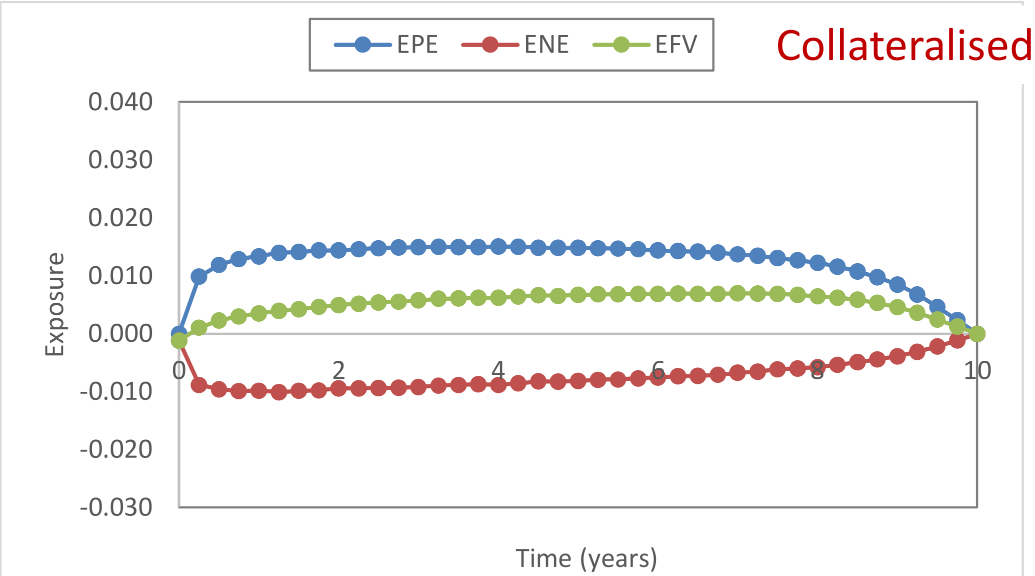
Fully collateralised

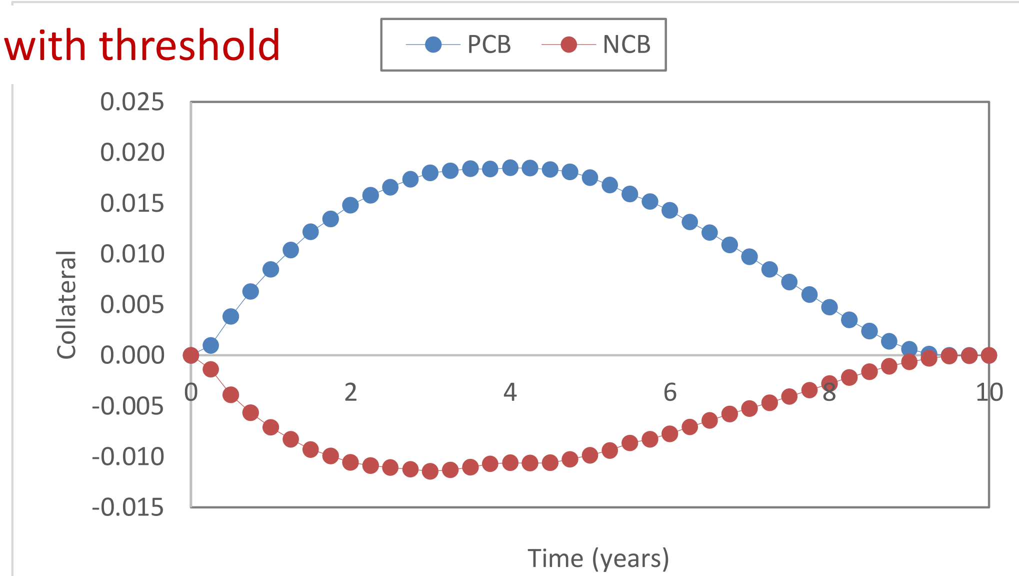




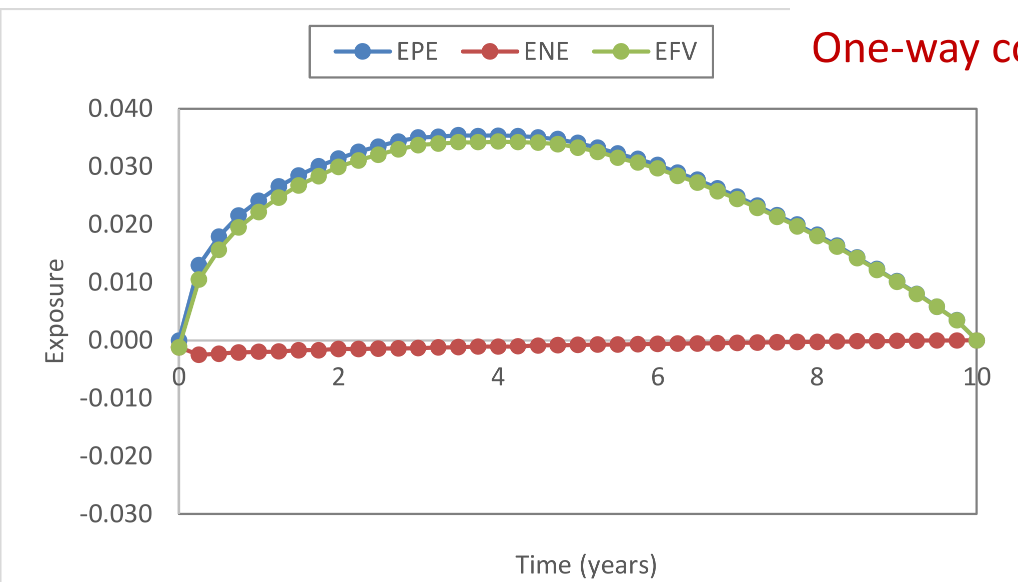
Exposure and Collateral (II)

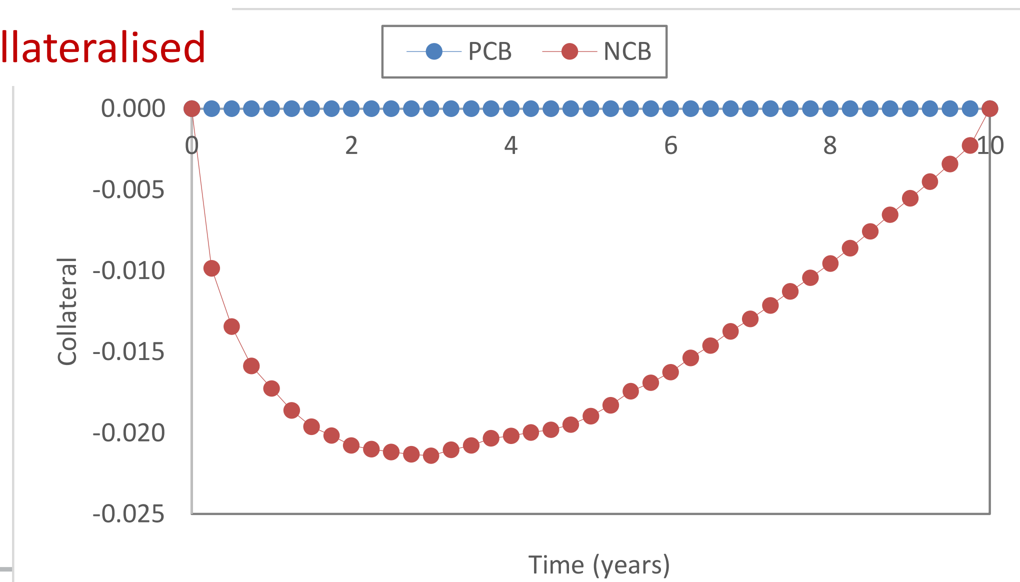
Collateralised with threshold





One-way collateralised





Exposure Allocation

- Two obvious ways to allocate exposure (and xVA)
- Incremental
  - Useful for pricing
  - Each trade is added sequentially

$$EPE_{inc,i} = EPE_{p+i} - EPE_p$$

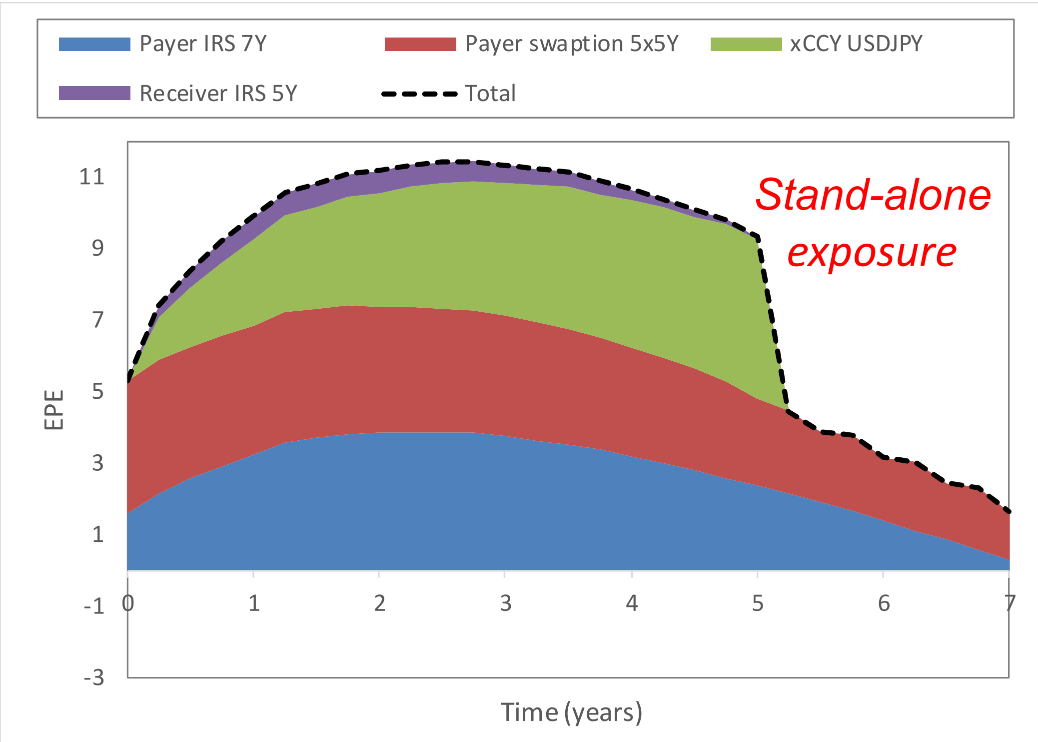
- Marginal
  - Useful for accounting / risk analysis
  - Euler allocation

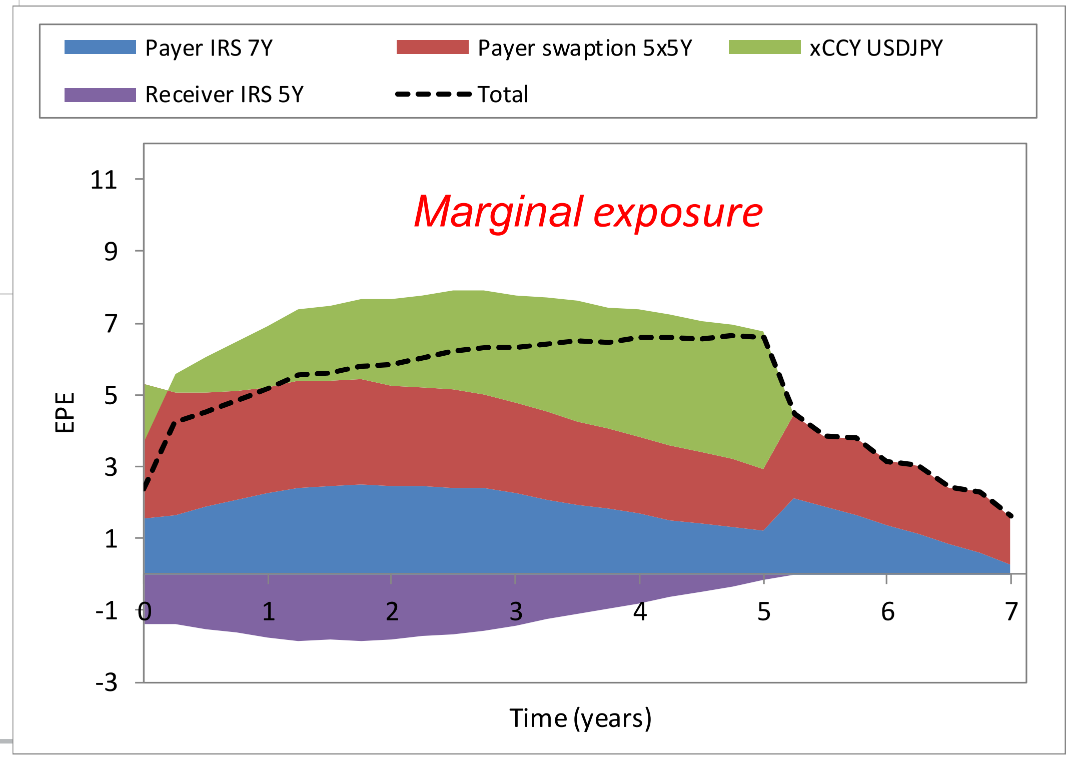
$$\underbrace{EPE_i'(s)}_{\text{Marginal exposure for trade } i} = E[V_i | V_{NS} > 0]$$

Future value of trade *i*

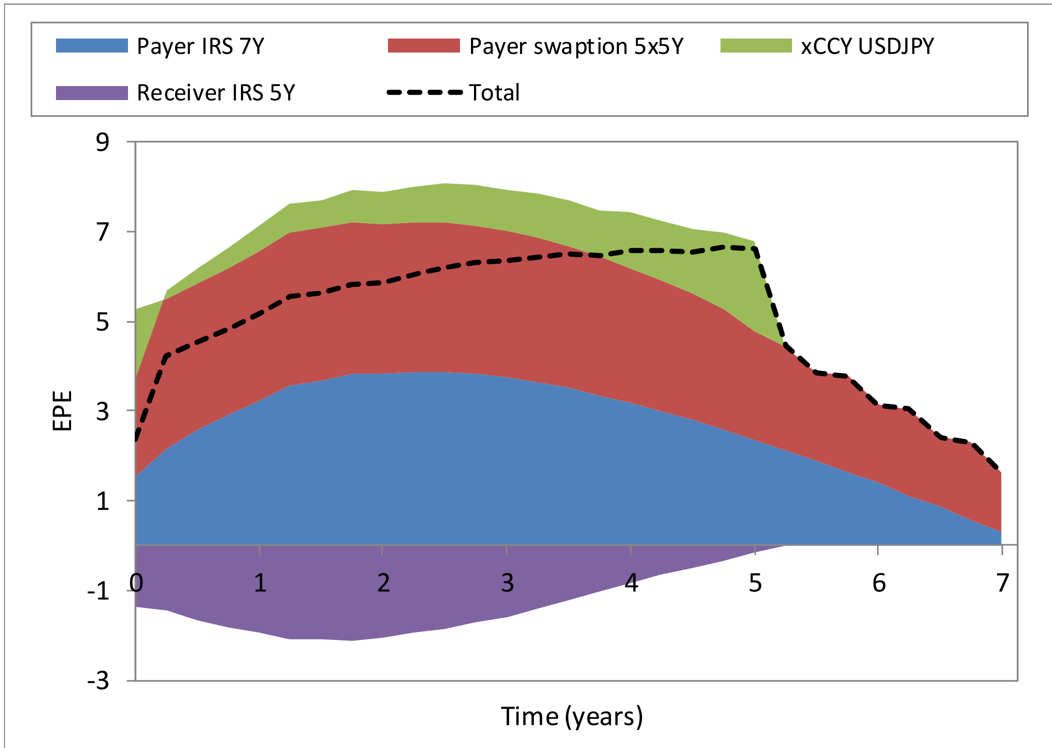
Future value of netting set

Stand-alone vs. Marginal Exposure



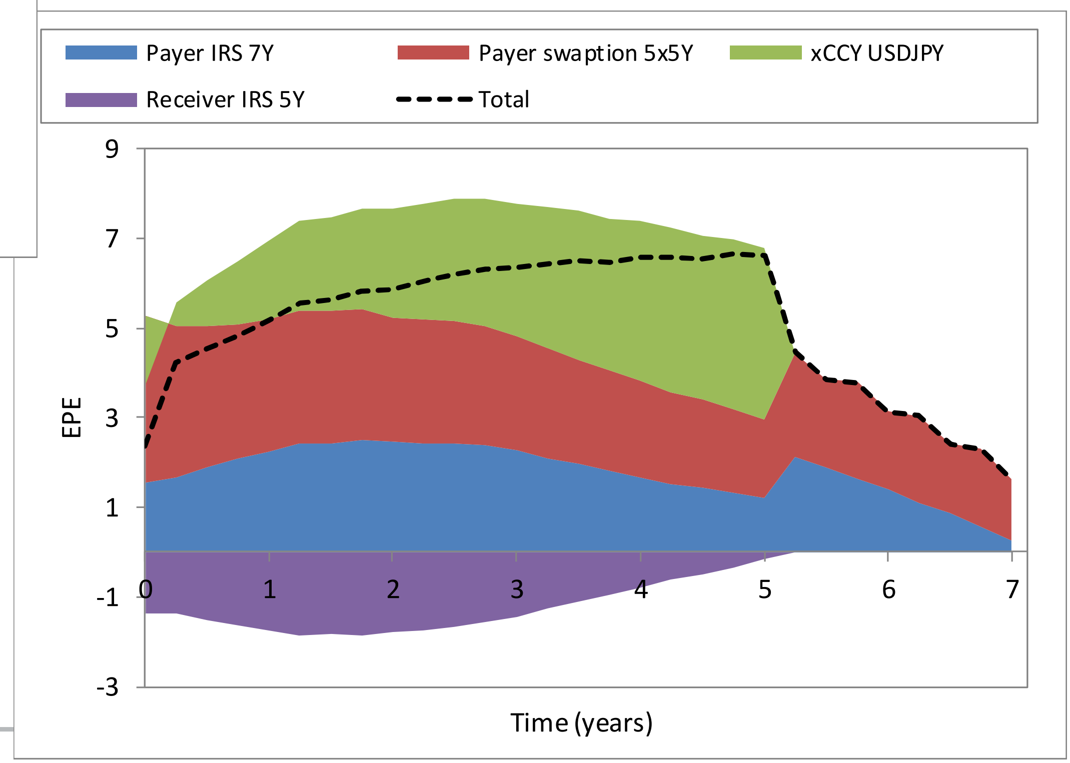


Marginal vs. Incremental Exposure



Incremental exposure (1-2-3-4)

Marginal exposure



Bilateral CVA Formula

- Considering a bank's own default

Do we include survival probabilities (grey terms)?

$$BCVA = CVA + DVA =$$

$$-LGD_C \sum_j \underbrace{EPE(t_j)}_{\text{Expected positive exposure}} S_I(t_j) \underbrace{\Delta PD_C(t_{j-1}, t_j)}_{\text{Probability counterparty defaults}} \quad \text{CVA}$$

$$-LGD_I \sum_j \underbrace{ENE(t_j)}_{\text{Expected negative exposure}} S_C(t_j) \underbrace{\Delta PD_I(t_{j-1}, t_j)}_{\text{Probability we default}} \quad \text{DVA}$$


CUTTING EDGE. COUNTERPARTY RISK

Closing out DVA

The choice of a close-out convention applicable on the default of a derivatives counterparty can have a significant effect on the credit and debit valuation adjustments, as can the order of defaults. Jon Gregory and Ilya German examine this phenomenon in detail

The Debate Around DVA

Quant Congress USA: Ban DVA, counterparty risk quant says

Author: Laurie Carver

Source: Risk magazine | 16 Jul 2010

Categories: Credit Risk

Being two-faced over counterparty credit risk

A recent trend in quantifying counterparty credit risk for over-the-counter derivatives has involved taking into account the bilateral nature of the risk so that an institution would consider their counterparty risk to be reduced in line with their own default probability. This can cause a derivatives portfolio with counterparty risk to be more valuable than the equivalent risk-free positions. In this article, ***Jon Gregory*** *discusses the bilateral pricing of counterparty risk and presents an approach that accounts for default of both parties. He derives pricing formulas and then argues that the full implications of pricing bilateral counterparty risk must be carefully considered before it is naively applied for risk quantification and pricing purposes*

Counterparty credit risk is the risk that a counterparty in a financial contract will default prior to the expiry of the contract and fail to make future payments. Counterparty risk is taken by each party in an over-the-counter derivatives contract and is present in all asset classes, including interest rates, foreign exchange, equity derivatives, commodities and credit derivatives. Given the recent decline in credit quality and heterogeneous concentration of credit exposure, the high-profile defaults of Enron, Parmalat, Bear Stearns and Lehman Brothers, and writedowns associated with insurance purchased from monoline insurance companies, the topic of counterparty risk management remains ever-important.

A typical financial institution, while making use of risk mitigants such as collateralisation and netting, will still take a significant amount of counterparty risk, which needs to be priced and risk-managed appropriately. Over the past decade, some financial institutions have built up their capabilities for handling counterparty risk and active hedging has also become common, largely in the form of buying credit default swap (CDS) protection to mitigate large exposures (or future exposures). Some financial institutions have a dedicated unit that charges a premium to each business line and in return takes on the counterparty risk of each new trade, taking advantage of portfolio-level risk mitigants such as netting and collateralisation. Such units might operate partly on an actuarial basis, utilising the diversification benefits of the exposures, and partly on a risk-neutral basis, hedging key risks such as default and forex volatility.

A typical counterparty risk business line will have significant reserves held against some proportion of expected and unexpected losses, taking into account hedges. The recent significant increases in credit spreads, especially in the financial markets, will have increased such reserves and/or future hedging costs associated with counterparty risk. It is perhaps not surprising that many institutions, notably banks, are increasingly considering the two-sided or bilateral nature when quantifying counterparty risk. A clear advantage of doing this is that it will dampen the impact of credit spread increases by offsetting mark-to-market losses arising, for example, from increases in required reserves. However, it requires an institution to attach economic value to its own default, just as it may expect to make an economic loss when one of its counterparties defaults. While it is true a corporation does 'gain' from its own default, it might seem strange to take this into account from a pricing perspective. In this article, we will make a quantitative analysis of the pricing of counterparty risk and use this to draw conclusions about the validity of bilateral pricing.

Unilateral counterparty risk

The reader is referred to Pykhtin & Zhu (2006) for an excellent overview of measuring counterparty risk. We denote by $V(s, T)$ the value at time s of a derivatives position with a final maturity date of T. The value of the position is known with certainty at the current time $t(< s \leq T)$. We note that the analysis is general in the sense that $V(s, T)$ could indicate the value of a single derivatives position or a portfolio of netted positions[1], and could also incorporate effects such as collateralisation. In the event of default, an institution must consider the following two situations:

- $V(s, T) > 0$. In this case, since the netted trades are in the institution's favour (positive present value), it will close out the position but retrieve only a recovery value, $V(s, T)\delta_c$, with δ_c a percentage recovery fraction.
- $V(s, T) \leq 0$. In this case, since the netted trades are valued against the institution, it is still obliged to settle the outstanding amount (it does not gain from the counterparty defaulting).

[1] We note that since exposures within netted portfolios are linear then this case is suitably general

Risk February 2009

Banks' profits boosted by DVA rule

The profits of British banks could be inflated by as much as £4bn due to a bizarre accounting rule that allows them to book a gain on the fall in the value of their debt.

Using debt value adjustment to inflate profits

Financial results in large banks have been inflated in the third quarter due to an accounting rule called "debt value adjustment" (DVA). DVA states that banks are allowed to mark their debt to market. In other words, if their debt decreases in price on the market, this is interpreted as a decrease in liabilities and is reported as profit. In the third quarter, this rule created £10 billion in profits in the biggest U.K. banks and $12 billion in profits in the biggest U.S. banks.

Overlap Between DVA and FVA

- In January 2014, JP Morgan reported FVA for the first time
  - $1.5 billion pre-tax loss (delta around -$25 million per bp assuming a funding spread of 60 bps)

"The adjustment this quarter is largely related to uncollateralized derivatives receivables, as

- Collateralized derivatives already reflect the cost or benefit of collateral posted in valuations

- Existing DVA for liabilities already reflects credit spreads, which are a significant component of funding spreads that drive FVA"

- DVA sensitivity?
  - Q4 loss of $536 million on DVA (JPM CDS spread had tightened from 93 bps to 70 bps)
  - Delta around +$23.3 million per bp?

"P&L volatility of combined FVA/DVA going forward is expected to be lower than in the past."

- What JP Morgan calls FVA partially offsets their DVA results

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Wrong-Way Risk Modelling Approaches

- The previous modelling was unconditional with respect to the counterparty
- Conditioning needs to be included for wrong-way risk
- In other words, the exposure is conditioned on the date in question being the counterparty default time (τ_C)

$$EPE(t|\tau_C = t)$$

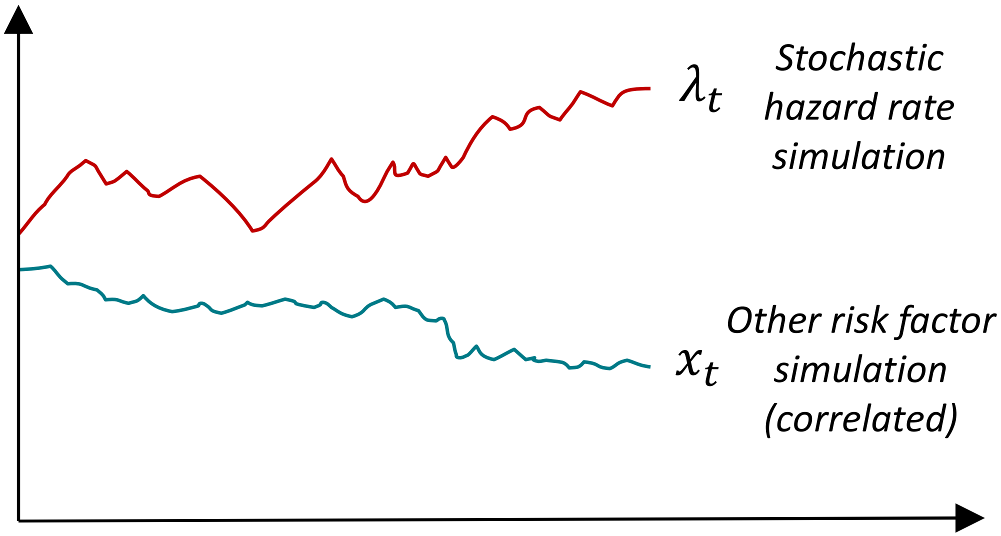
- This requires a model to generate τ_C alongside the exposure model
- Broadly speaking two types of model here:
  - Stochastic intensity (reduced-form)
  - Structural (firm value)
- We will concentrate on the former in line with market practice

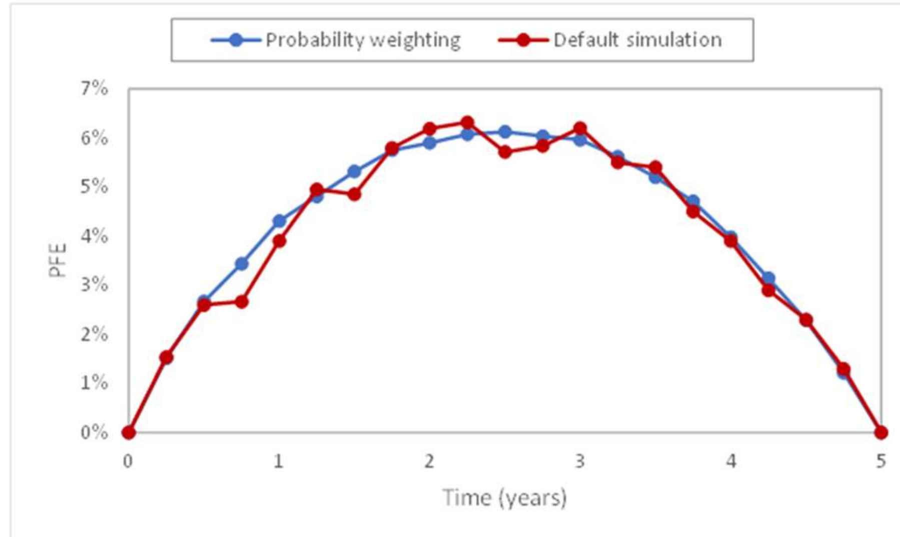
FRTB-CVA Text

measured ES via a conservative multiplier. The proposed default level of the multiplier is [1.5]. The value of the multiplier can be increased from its default value by a bank's supervisory authority if a bank fails to capture the dependence between counterparty credit quality and exposure in its CVA calculations, or if it determines that a bank's CVA model risk is higher than its peer's.

Stochastic Intensity Simulation

- It is not necessary to simulate defaults





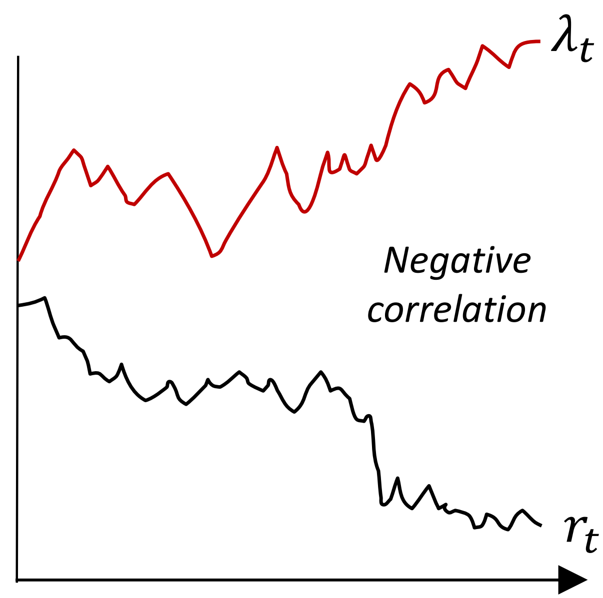
- The probability of default between t_{i-1} and t_i on a given path is:

$$\exp\left(-\int_{t_0}^{t_{i-1}} \lambda_s ds\right)\left[1-\exp\left(-\int_{t_{i-1}}^{t_i} \lambda_s ds\right)\right]$$

- The conditional exposure distribution is the unconditional distribution weighted by the above factors for each path
  - The weights will tend to be larger for large values of λ

Interest Rate Swap Example – General WWR model

- Interest rate model (r)
  - One-factor Hull-White
- Hazard rate model (λ)
  - Cox-Ingersoll-Ross
- Processes are correlated

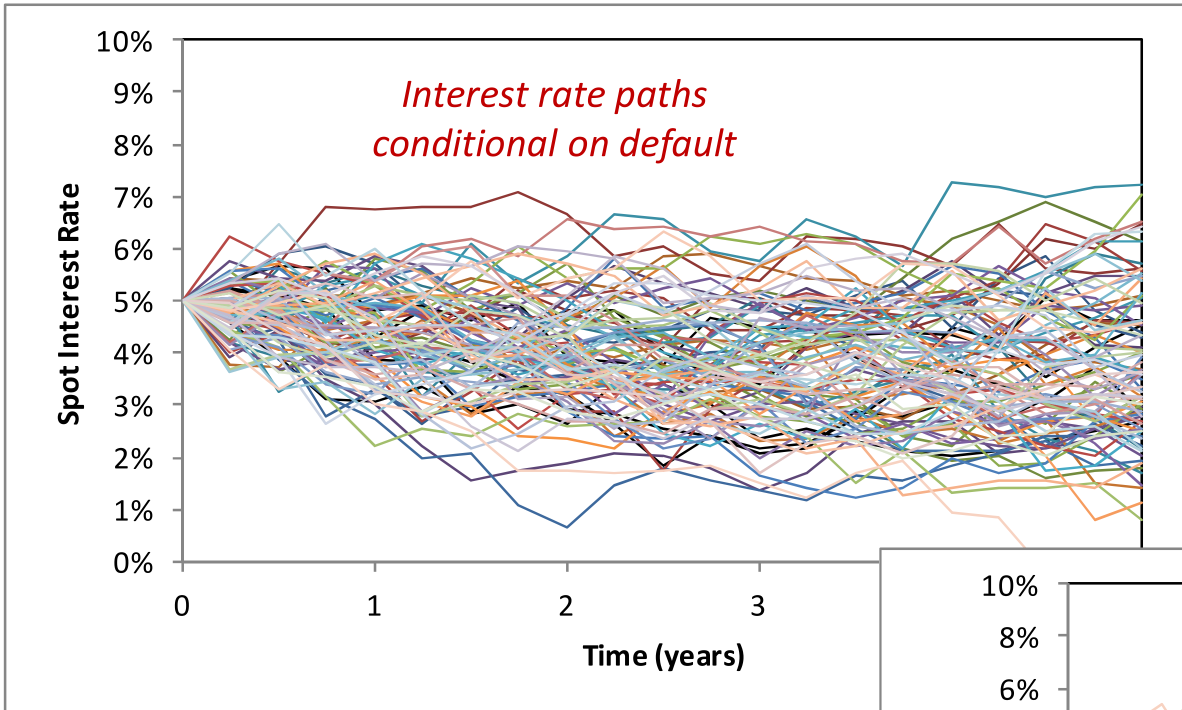


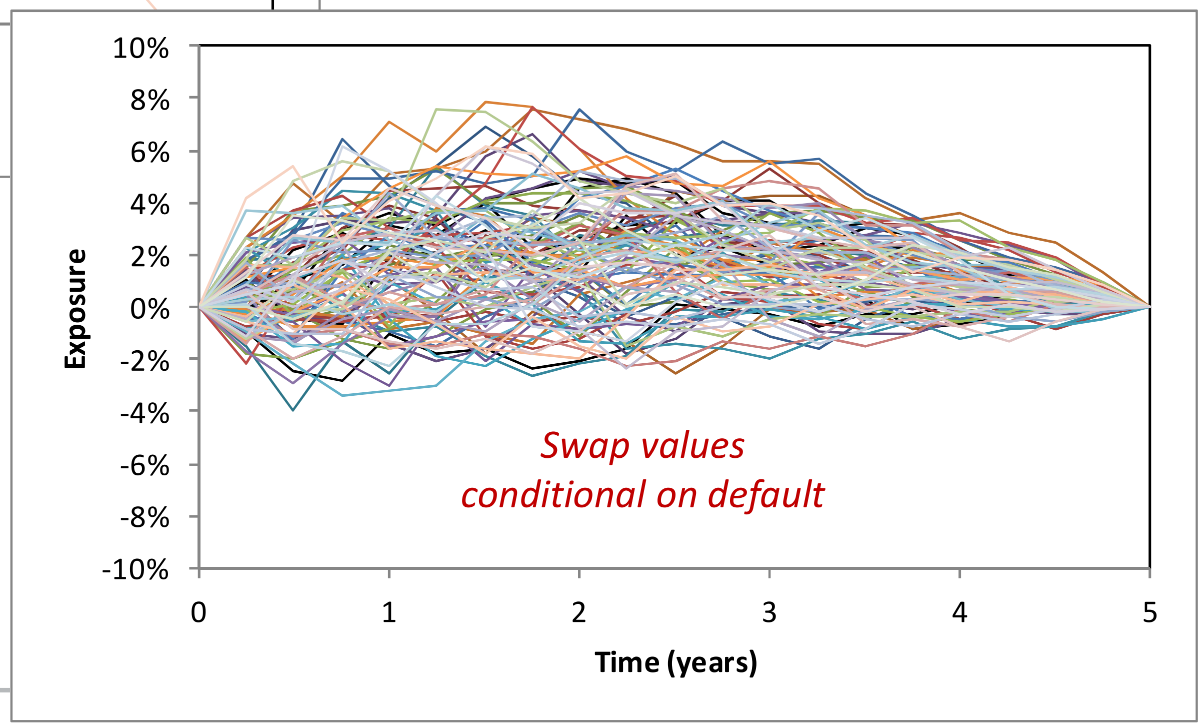
Large hazard rate means more chance of default

Lower interest rate benefits receiving fixed

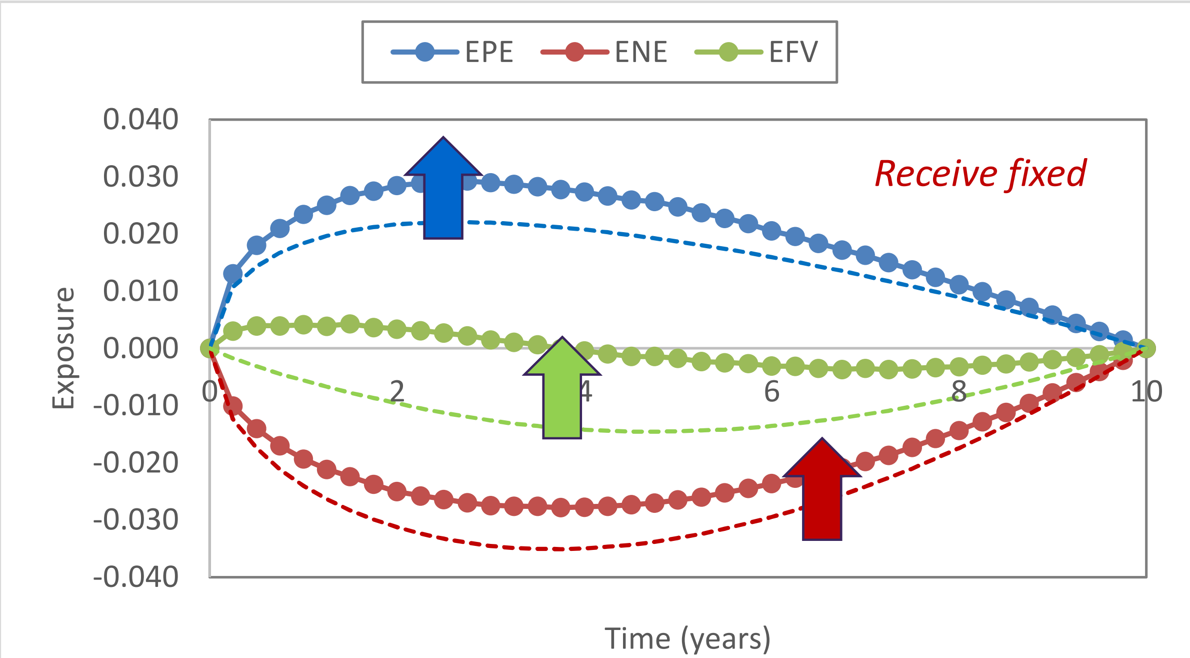
- With negative correlation, interest rates more likely to be low in the event of the counterparty defaulting
  - Receive fixed swap has wrong-way risk
  - Pay fixed swap has right way risk

Interest Rate Swap Example –

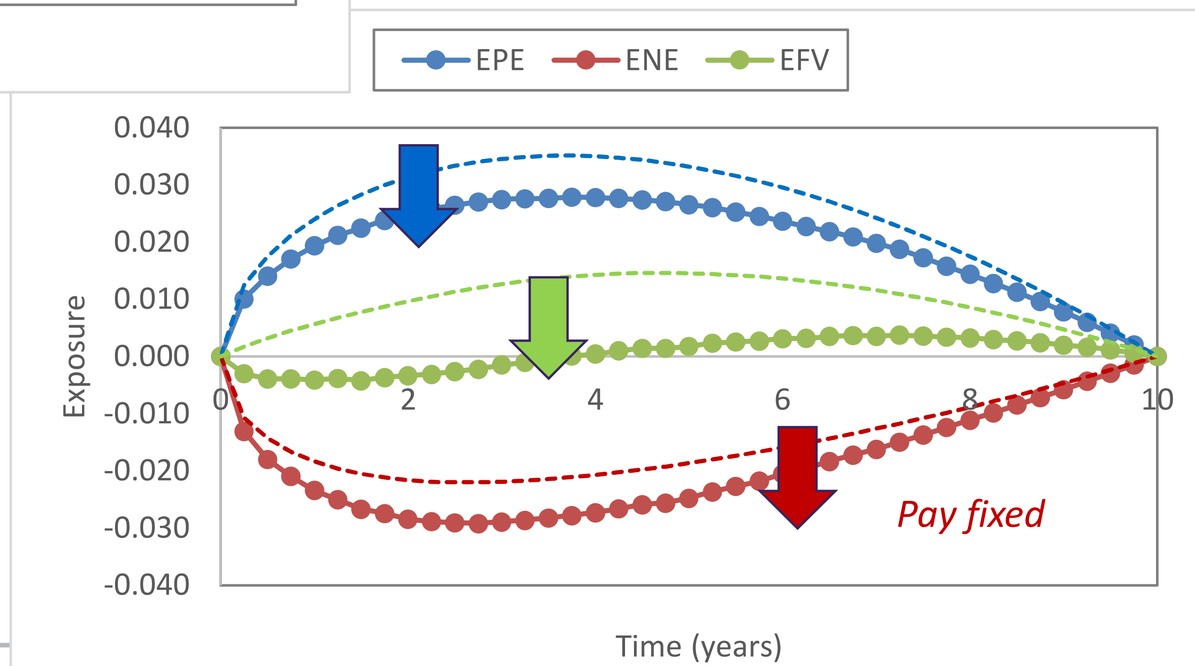




Interest Rate Swap Example – EPE



Dotted line represents case with no WWR (zero correlation)



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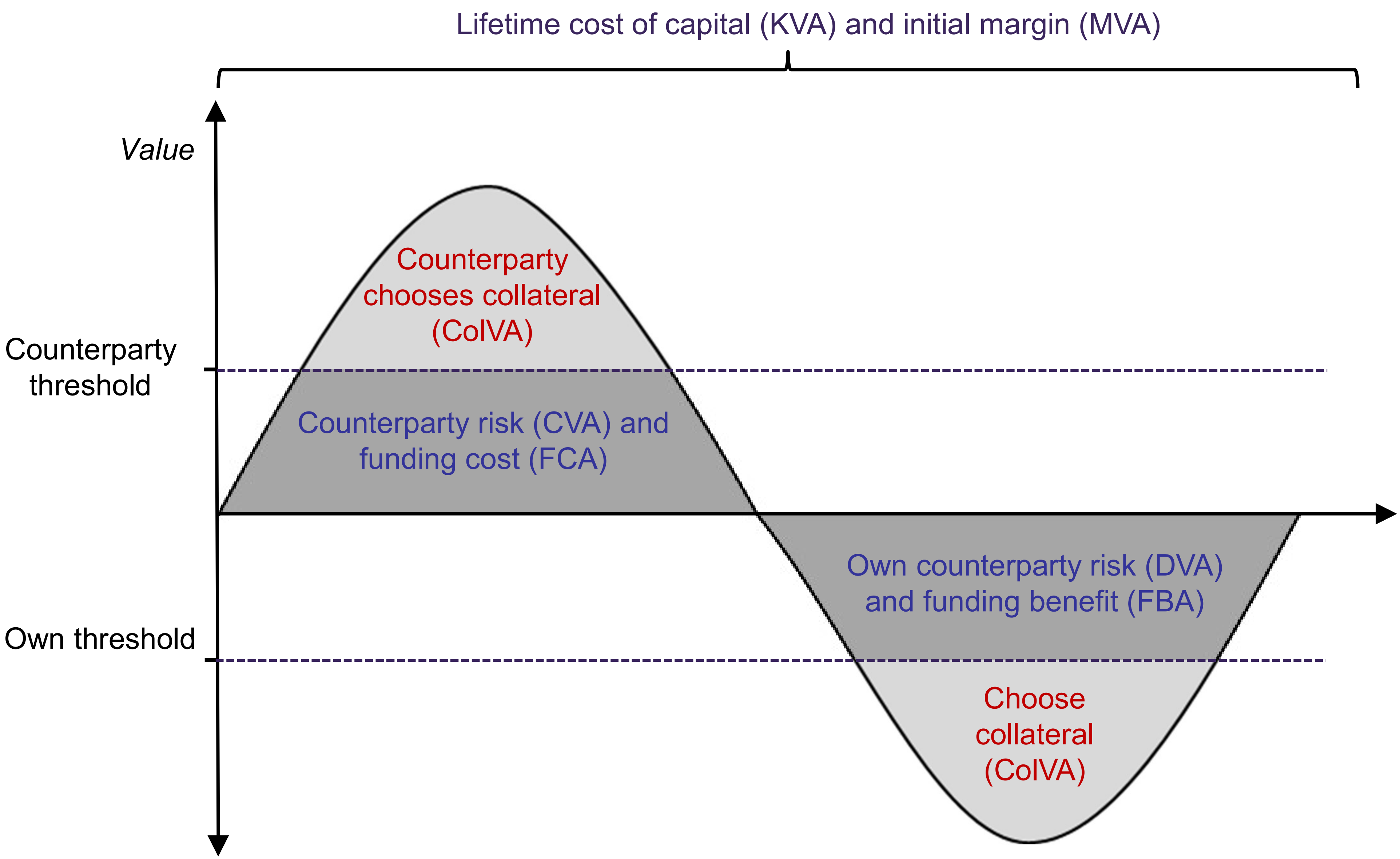
- From discounting to valuation adjustments
- CVA and DVA
- Exposure simulation
- Wrong-way risk
- FVA, MVA and KVA
- Speeding up xVA calculations

Economic Costs of Holding a Derivative Transaction



Beyond CVA - xVA

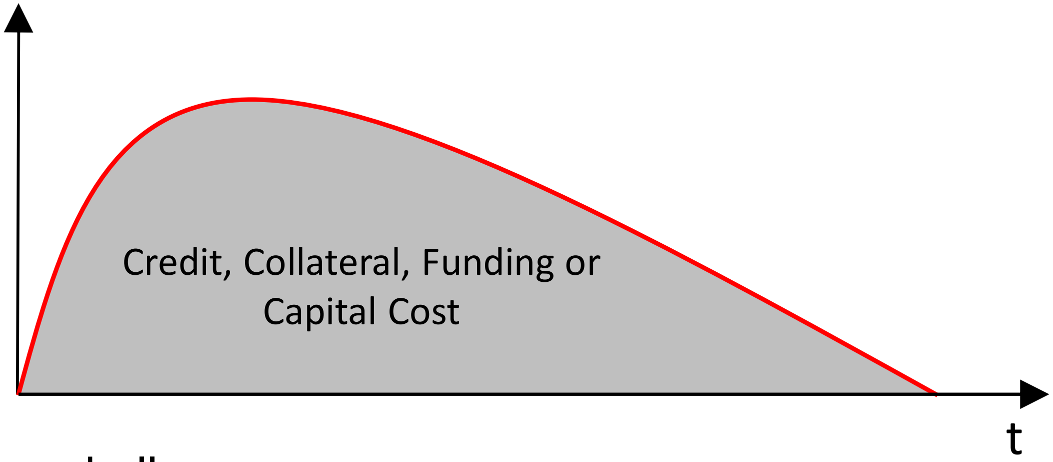
- CVA and DVA
  - Counterparty risk
- FVA (funding value adjustment)
  - Cost of funding derivatives (generally undercollateralisation)
  - Funding benefit is DVA
- ColVA
  - Adjustment for collateral effects (e.g. cheapest-to-deliver, remuneration, floors)
- MVA (margin value adjustment)
  - Cost of posting initial margin (generally overcollateralisation)
- KVA (capital value adjustment)
  - Cost of holding capital against transaction

The xVA Hierarchy

Category	Drivers	xVA	
Capital	Leverage Ratio CVA Capital Charge CCR Capital Charge Market risk	KVA	Profit to generate return on capital
Initial margin	Clearing mandate Bilateral margin rules	MVA	Real costs?
Collateral	Floors Cheapest to deliver	ColVA	Real costs?
Funding	NSFR / LCR Cross currency basis LIBOR-OIS basis Treasury funding	FVA	Real costs?
Credit	IFRS 13 Accounting Credit line utilisation Credit provisioning	CVA / DVA	Real costs?

The xVA Calculation – General Comments

$$xVA = \int_0^{\infty} C(t) e^{-\int_0^t \beta(u)du} E[X(t)]dt$$



- xVA computation involves
  - Determination of curves, $C(t)$ – qualitative challenge
  - Calculation of underlying profile, $X(t)$ – quantitative challenge
- In some special cases, we are only really pricing forward contracts
  - xVA can be implemented by the correct choice of discount factor
- Discounting choices are not easy
  - Relates to questions such as close-out assumptions (e.g. do we close-out at an uncollateralised or collateralised price?)

Content

- Historical background and xVA overview
- From discounting to valuation adjustments
- CVA and DVA
- Exposure simulation
- Wrong-way risk
- FVA, MVA and KVA
- Speeding up xVA calculations

Challenges of Exposure Modelling

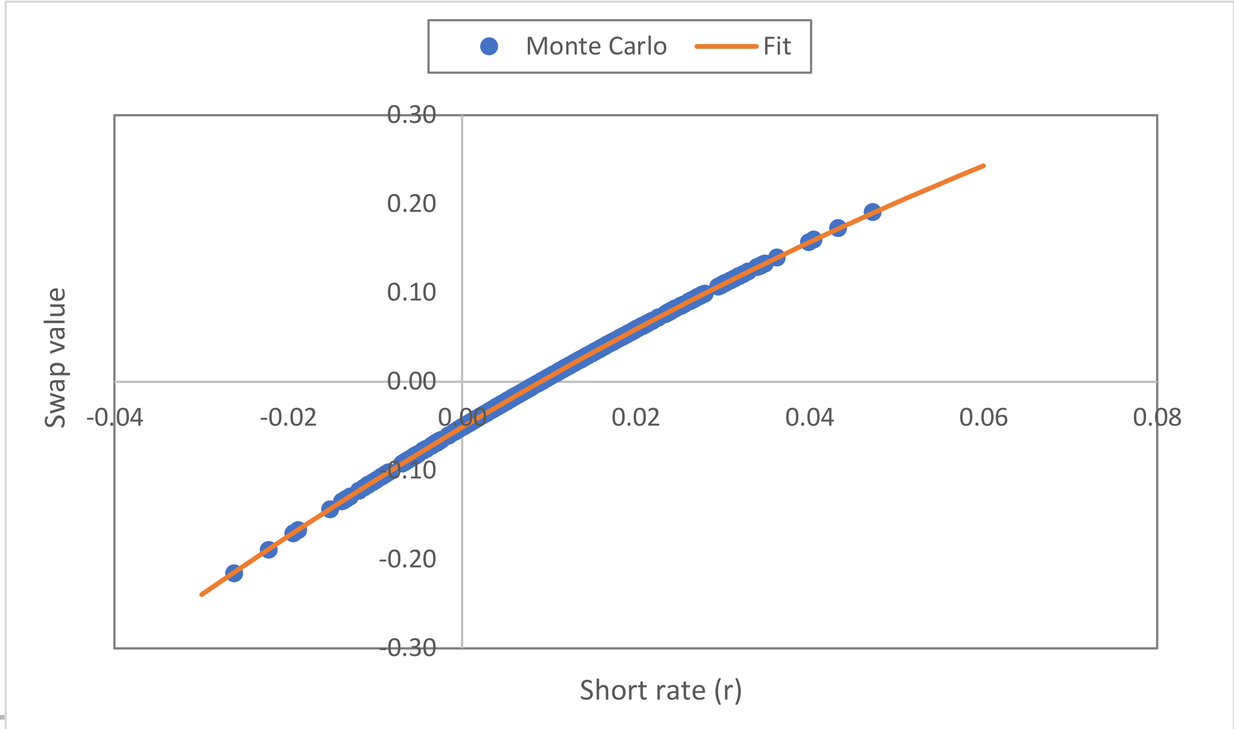
- Computation time
  - Need for full revaluation creates a massive bottleneck

$$\underbrace{10{,}000}_{\text{Simulations}} \times \underbrace{100}_{\text{Time-steps}} \times \underbrace{1{,}000{,}000}_{\text{Trades}} \times \underbrace{100}_{\text{Sensitivities}} = \underbrace{100{,}000{,}000{,}000{,}000}_{\text{Total number of revaluations}}$$

- Solutions
- Speed up revaluation
  - Regression
  - Neural networks
  - <u>Note that xVA calculations are often models with low dimensionality (e.g. one-factor with deterministic basis and volatility)</u>
- Algorithmic differentiation
  - Get sensitivities with finite effort

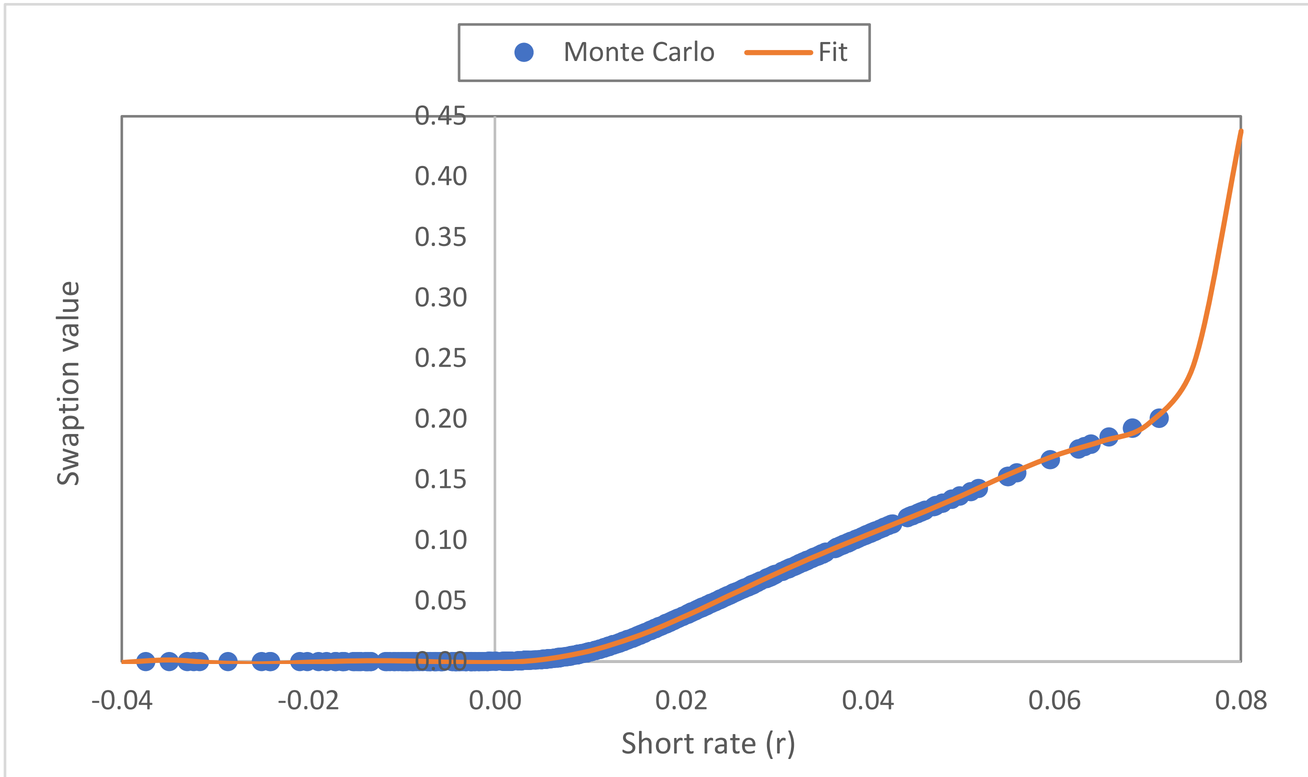
Revaluation Speed-Up Simple Example

- Consider an interest rate swap in a one-factor Hull-White model
  - At a given time, there is a monotonic relationship between short rate and swap value
  - Regression with two basis functions has an R-squared of 0.999997
  - Value of the swap is then given by $V_t^{swap} = \beta_t^0 + \beta_t^1 r + \beta_t^2 r^2$
  - Single-currency portfolio of interest rate products is similar



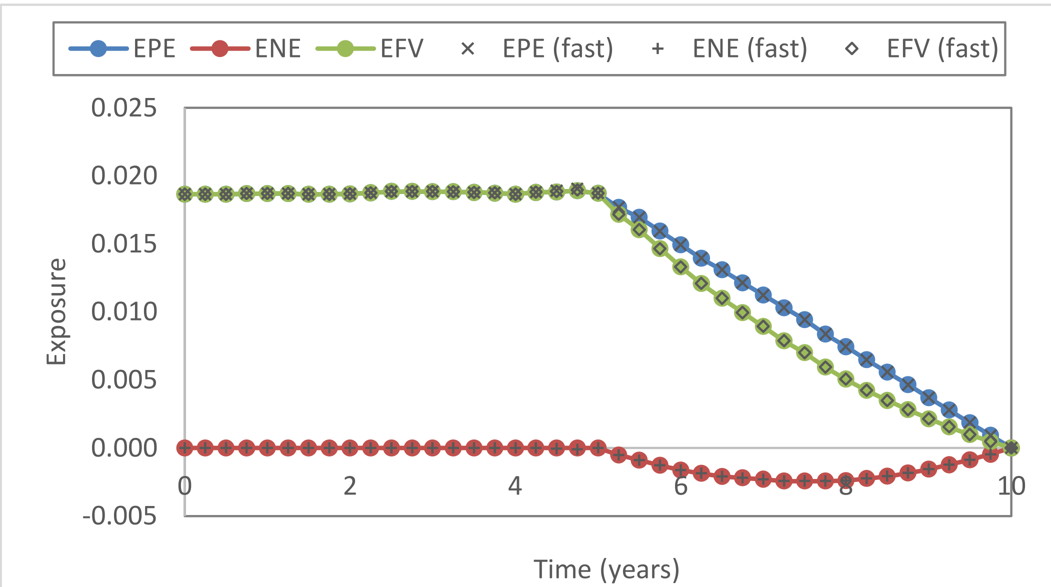
Revaluation Speed-Up Example (cont)

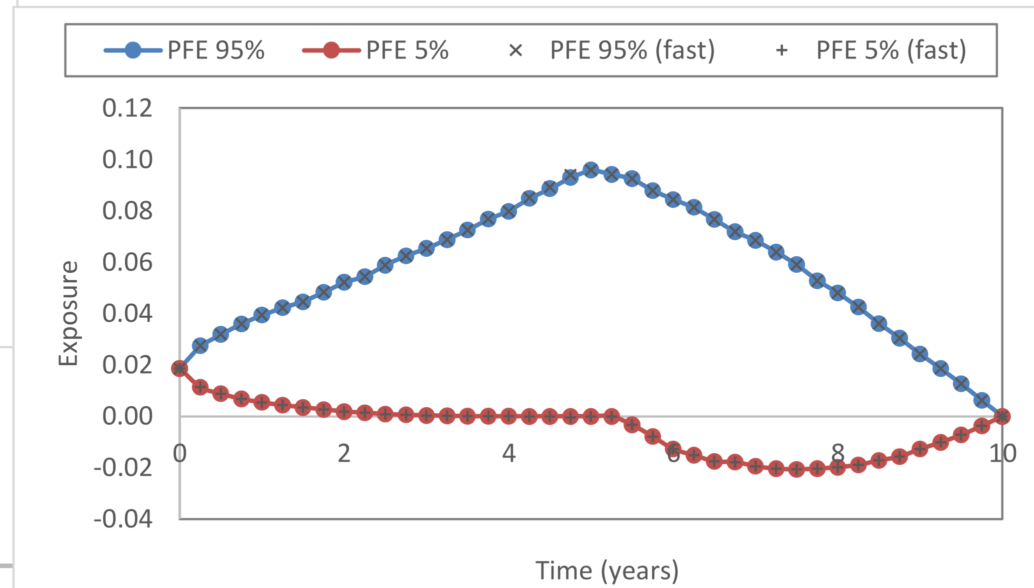
- Interest rate swaption (or portfolio) is more complex
  - Basic polynomial regression needs more basis functions to match shape
  - R squared of 0.9997 for 10 basis functions
  - Need to be careful over extrapolation
  - There are better methods!



Swaption

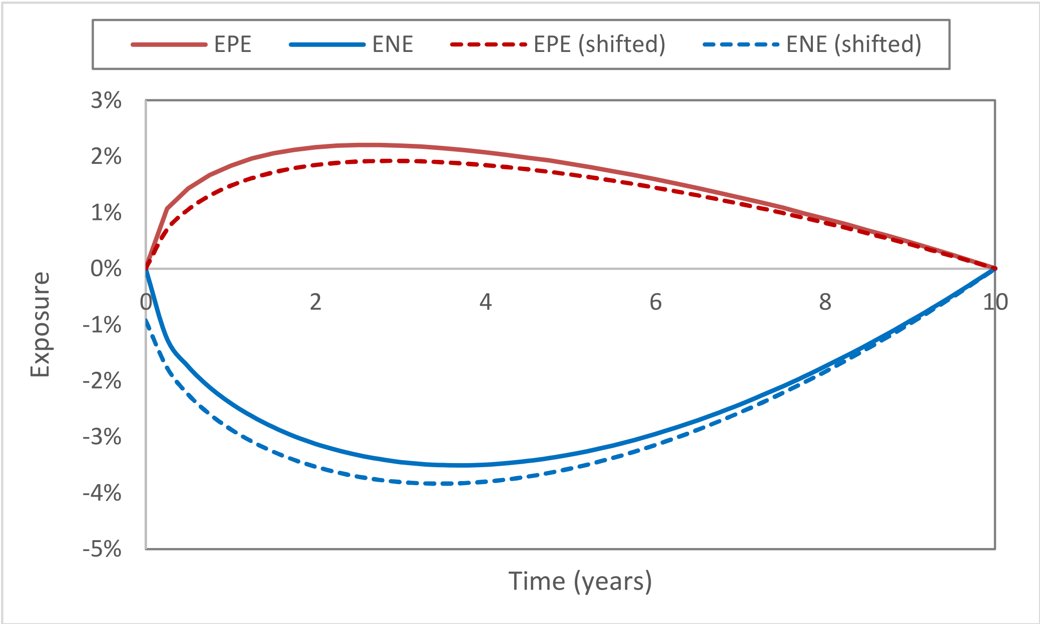
- Speed up is more than an order of magnitude
  - European swaption valuation in Hull White model is quite time consuming (solving non-linear equations and using Black-Scholes like formula for set of bond options)
  - Error in CVA calculation – 0.07%





CVA Hedging

- Interest rate swap (receive fixed) interest rate risk
  - Upwards move in rates (+10 bps)
  - Value of swap goes down (negative sensitivity to rates)
  - CVA goes down also (positive sensitivity to rates)



	6M	1Y	2Y	3Y	5Y	10Y	Total
Base value	(50)	(172)	(385)	(927)	(3,546)	(77,306)	(46,655)
CVA	27	(88)	94	(618)	(1,040)	5,613	3,988
CVA (credit spreads wider)	32	(107)	104	(754)	(1,265)	6,840	4,850

Algorithmic Differentiation

- Calculate derivatives during code using chain rule
- Simple example: start of xVA calculation for a swap in Hull-White model
  - Interest rate delta defined by IR_{tenor}

$$\frac{dFwd_t}{dIR_{tenor}} = \frac{dr_t}{dIR_{tenor}} \times \frac{dDF_t}{dr_t} \times \frac{dFwd_t}{dDF_t}$$

	Calculation	Derivative
Short rate	r_t	$\frac{dr}{dIR_{tenor}}$
Discount factor	$DF_t = \exp(-r_t \times t)$	$\frac{dDF_t}{dr_t} = -t \times \exp(-r_t \times t)$
Forward rate	$\frac{-\ln(DF_{t_i}) - \ln(DF_{t_{i-1}})}{t_i - t_{i-1}}$	$\frac{dFwd_t}{dDF_t} = \frac{1}{(t_i - t_{i-1})}\left\{\frac{dDF_{t-1}/dr_{t-1}}{DF_{t_{i-1}}} - \frac{dDF_t/dr_t}{DF_{t_i}}\right\}$

Algorithmic Differentiation (example)

- Sensitivities for 10-year interest rate swap (IR deltas)
- AD code runs about 4 times slower but this is a fixed cost
- This is a forward mode example – reverse mode adjoint automatic differentiation (AAD) is more efficient for large numbers of sensitivities

	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y
Finite difference	0.0017	-0.0046	0.0109	-0.0014	0.0722	0.0968	0.2110	-0.7653
AD	0.0017	-0.0046	0.0109	-0.0013	0.0723	0.0971	0.2124	-0.7616
Finite difference (1/100 bp bump)	0.0017	-0.0046	0.0109	-0.0013	0.0723	0.0971	0.2124	-0.7617

Summary

- Virtually all derivatives have significant xVA considerations
  - Bilateral OTC uncollateralised – CVA, FVA, KVA
  - Bilateral OTC collateralised – CVA, KVA, ColVA
  - Central cleared – MVA, KVA
  - Exchange-traded – MVA, MVA
- xVA to some extent occupies the ground previously taken by exotic derivatives
- xVA represents a portfolio problem and terms are not mutually exclusive
- Represents a key area for quantitative finance going forward, for example:
  - Proxy credit spreads
  - Numerical methods for exposure simulation
  - Optimisation of xVA (e.g. initial margin compression)
  - Advanced xVA models (volatility skew, stochastic basis)
- xVA is often the core component in derivatives valuation

Technology innovation of the year: Scotiabank

Risk Awards 2021: new risk engine can run nearly a billion XVA calculations per second

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  - Capriotti, L., and M.B. Giles. 2010, "Fast correlation Greeks by adjoint algorithmic differentiation", Risk, 23(4):77-83.
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Additional Slides

Deriving the CVA Formula (I)

- Default is asymmetric (no discounting formula)
- Time of default is denoted by τ and discounted base value (no default risk) of

derivatives portfolio at time t with final maturity T: $V_{ND}(\tau, T)$

- Default payoff

$$\underbrace{\delta}_{\text{Recovery rate (\%)}} V_{ND}(\tau, T)_+ + V_{ND}(\tau, T)_-$$

Note that a payoff of zero ('extinguisher') is a discounting problem again

- Value with counterparty risk

$$V_{CCR}(t, T) = E_t \begin{bmatrix} \overbrace{I(\tau > T)}^{\text{No default before } T} V_{ND}(t, T) + \\ I(\tau \le T) V_{ND}(t, \tau) + \\ \underbrace{I(\tau \le T)(\delta V_{ND}(\tau, T)_+ + V_{ND}(\tau, T)_-)}_{\text{Default payoff}} \end{bmatrix}$$

Full value if no default

Cashflows before default

Deriving the CVA Formula (II)

- Extract the non-default (base) value

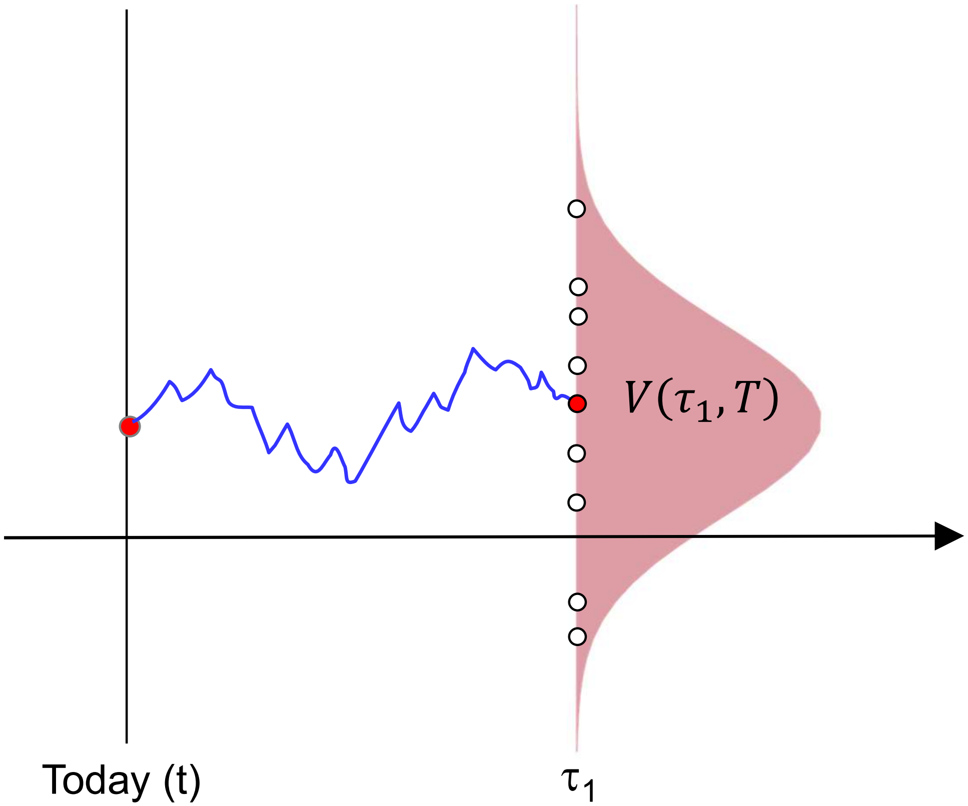
$$V_{CCR}(t,T) = E_t\left[\begin{array}{c} I(\tau > T)V_{ND}(t,T) + \\ I(\tau \leq T)V_{ND}(t,\tau) + \\ I(\tau \leq T)(\delta V_{ND}(\tau,T)_+ + V_{ND}(\tau,T)_-) \end{array}\right]$$

$$V_{CCR}(t,T) = E_t\left[\begin{array}{c} I(\tau > T)V_{ND}(t,T) + \\ I(\tau \leq T)V_{ND}(t,\tau) + \\ I(\tau \leq T)(\delta V_{ND}(\tau,T)_+ + V_{ND}(\tau,T) - V_{ND}(\tau,T)_+) \end{array}\right]$$

$$V_{CCR}(t,T) = V_{ND}(t,T) - \underbrace{E_t[I(\tau \leq T)V_{ND}(\tau,T)_+(1-\delta)]}_{\text{CVA}}$$

Direct CVA Formula

$$CVA = -E_t[\underbrace{I(\tau \leq T)}_{\text{Default time } (\tau) \text{ is prior to final maturity } (T)} \times \underbrace{V(\tau, T)_+}_{\text{Exposure at default (discounted)}} \times \underbrace{(1-\delta)}_{\text{Loss given default \% (LGD)}}]$$



Possible scheme

1) Simulate time of default
2) Calculate discounted exposure at default $V(\tau, T)_+$
3) Multiply by LGD $= (1-\delta)$
4) Repeat and average

Traditional CVA Formula

$$CVA = -E_t[\underbrace{I(\tau \le T)}_{\text{Default}}\underbrace{(1-\delta)}_{\text{Loss given default (LGD)}}\underbrace{V(t,T)_+}_{\text{Positive exposure}}]$$

- If we assume independence (no wrong-way risk)

$$CVA = -LGD \int_t^T \underbrace{EPE(u)}_{\text{Discounted expected positive exposure}} \underbrace{dPD_C(u)}_{\text{Default probability}} \qquad EPE(t) = E_t[V(t,T)_+]$$

Credit spread

$$\approx -E[LGD] \sum_i^T EPE(t_i) \left[\exp\left(-\frac{s_{i-1} t_{i-1}}{E[LGD]} \right) - \exp\left(-\frac{s_i t_i}{E[LGD]} \right) \right]$$

- Refer to CDS lecture for discussion on default probability and LGD and determining proxy credit spreads

Wrong-Way Risk Modelling Approaches

- Intensity (hazard rate) models
  - Process (e.g. CIR) for default intensity which can be correlated to other market variables
  - Can be formulated as a conditional (on default) EPE calculation
  - Most obvious and straightforward but the effect can be weak
- Structural models
  - Merton approach for default which can be correlated to market variables
  - Stronger effect but correlation is less accessible
  - Can be explicit or implicit
- Jump approaches
  - Tractable and strong dependency
  - Requires market prices to calibrate (e.g. CDS)

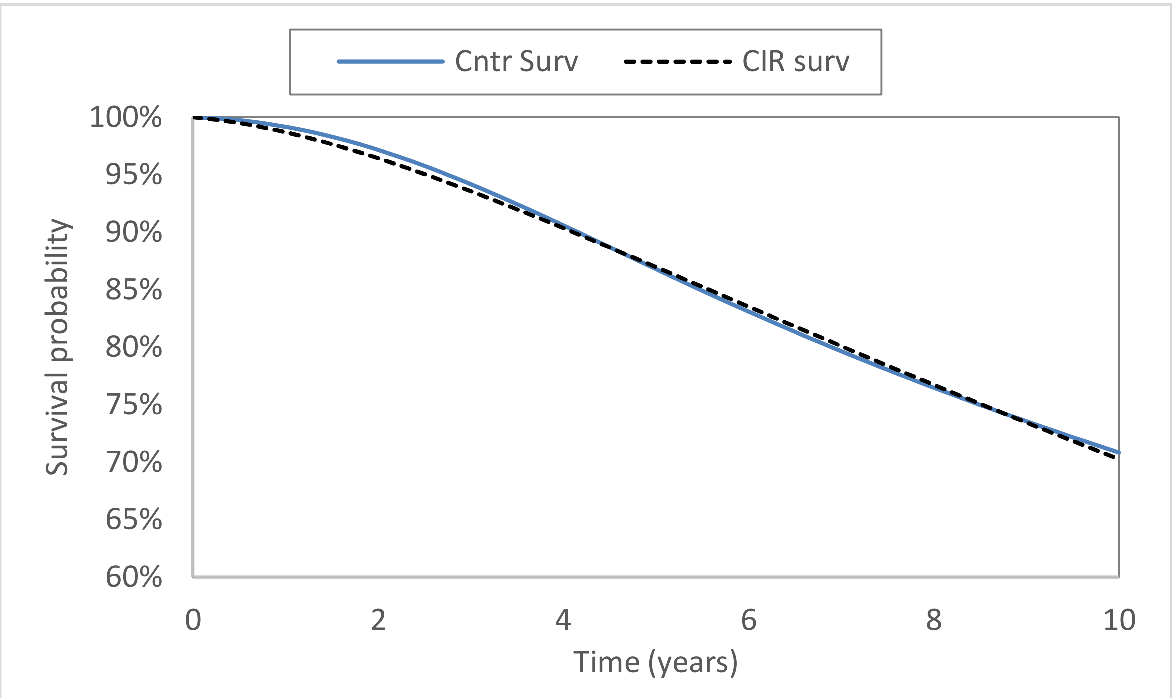
Stochastic CDS Modelling

- Cox Ingersoll Ross (CIR) model for stochastic default intensity (λ_t)

$$d\lambda_t = k(\theta - \lambda_t)dt + \sigma\sqrt{\lambda_t}dW_t$$

  – Four parameters ($\lambda_0, k, \theta \; and \; \sigma$) calibrated to survival curve and CDS options if available (constrained optimization, Feller condition)

  – CIR++ can fit perfectly if required



$$S(t,T) = A(t,T)\exp(-B(t,T).h_t)$$

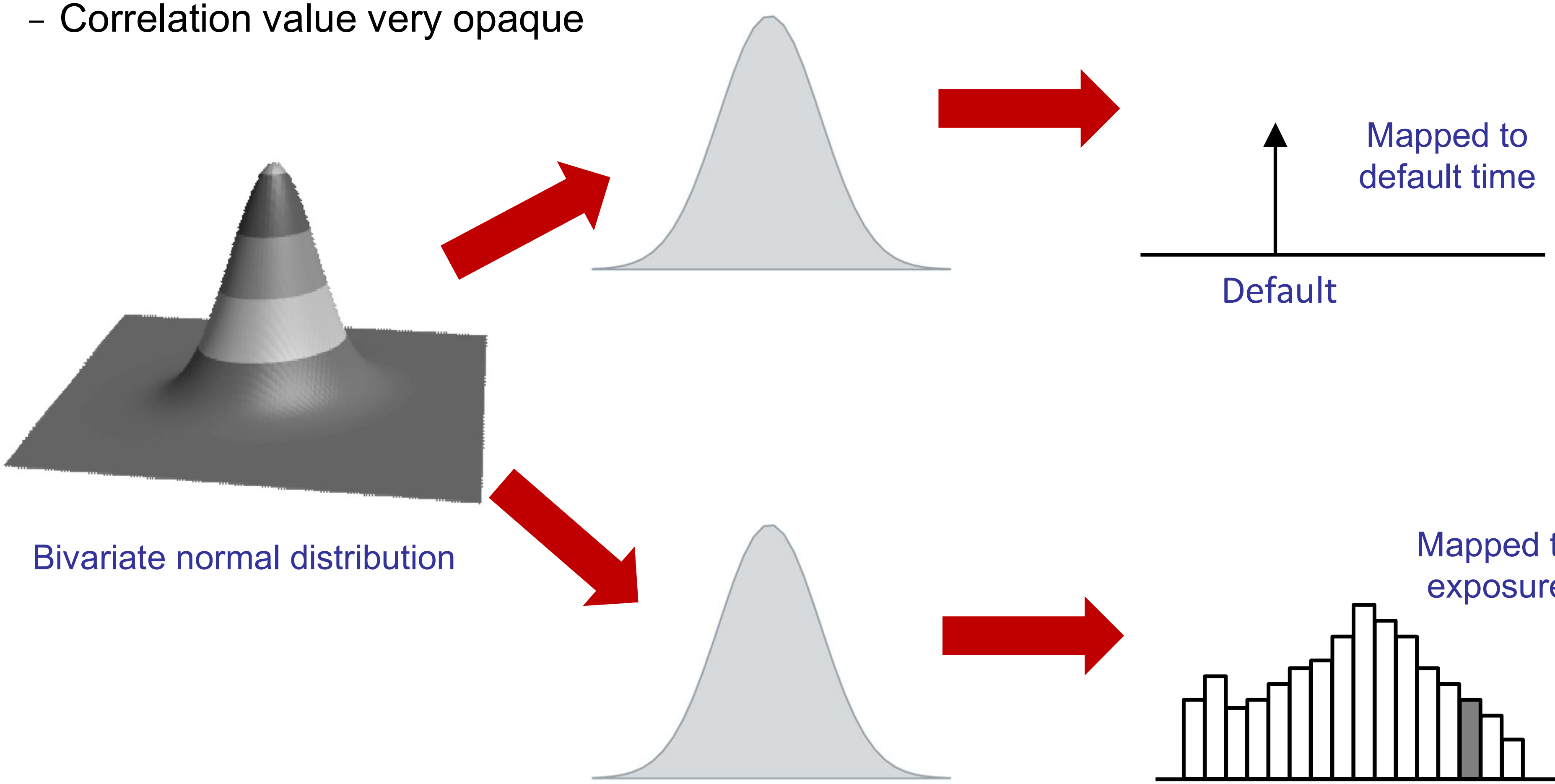
$$A(t,T) = \left[\frac{2he^{\frac{(h+k)(T-t)}{2}}}{2h + (h+k)(e^{k(T-t)} - 1)}\right]^{\frac{2k\theta}{\sigma^2}}$$

$$B(t,T) = \left[\frac{2\left(e^{k(T-t)} - 1\right)}{2h + (h+k)(e^{k(T-t)} - 1)}\right]$$

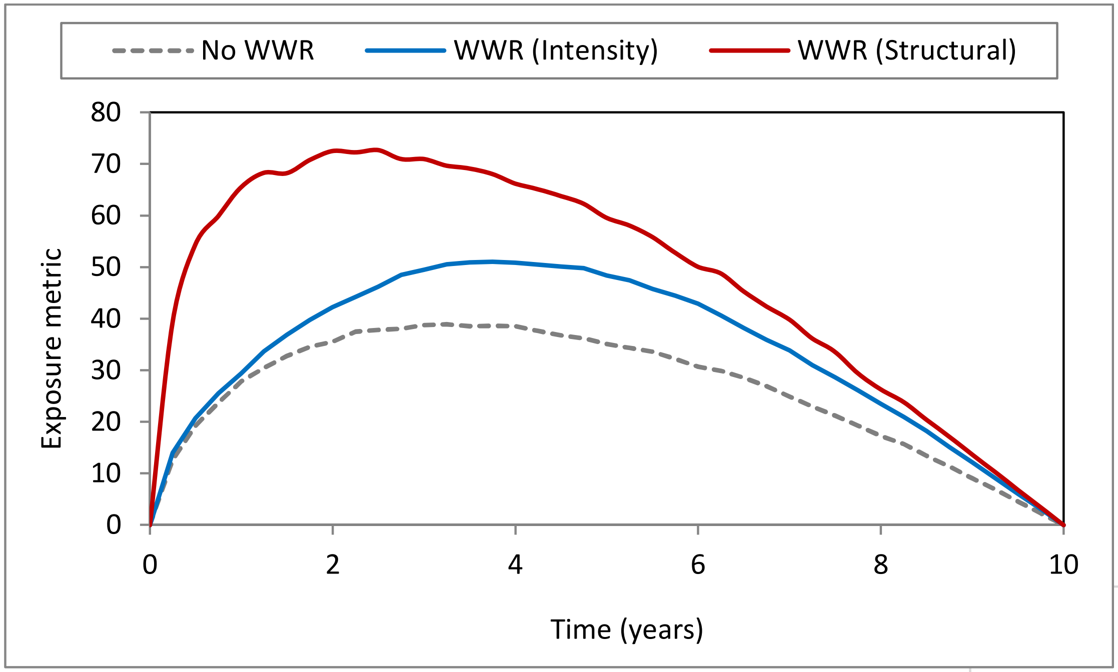
$$h = \sqrt{a^2 + 2\sigma^2}$$

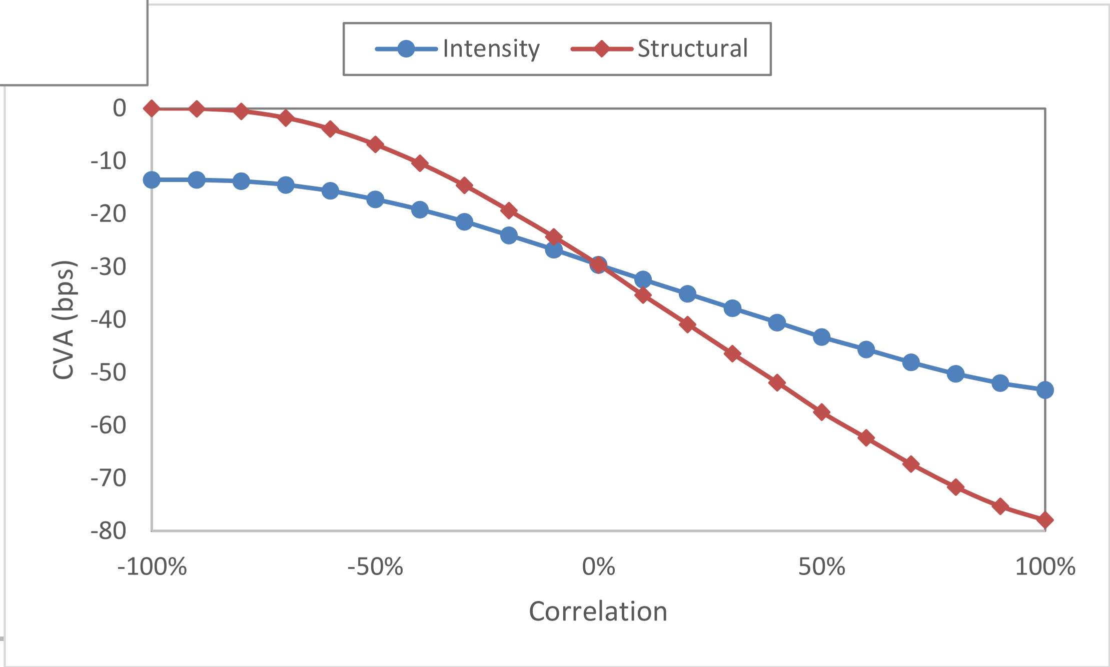
WWR – Implicit Structural Approach

- Gaussian (or other) copula drives dependency between exposure and default time directly
  - Correlation value very opaque



Intensity vs. Structural Model





Wrong-Way Risk

CVA wrong-way risk: calibration using a quanto CDS basis

Tsz-Kin Chung and Jon Gregory discuss the calibration of a wrong-way risk (WWR) model using information from the quanto credit default swap market. Empirical evidence shows that implied foreign exchange jump sizes are significant for a wide range of corporates. For systemic counterparties, the credit valuation adjustment WWR add-on could be 40% higher than in the standard case, and choosing a proper jump-at-default WWR model is critical for capturing the impact

The wrong-way risk (WWR) modelling of credit valuation adjustments (CVAs) is known to be a challenging, if not intractable, problem. Aziz *et al* (2014) summarise the two main difficulties with modelling WWR as being (1) the lack of relevant historical data and (2) the potential misspecification of the dependency between credit spreads and exposures. In particular, it is not straightforward to link a binary default event to continuous risk factor movement and an exposure distribution. Traditional modelling approaches and the use of correlation are very prone to the misspecification problem due to the difficulty of estimating co-dependency parameters using relevant historical data. These correlation-based approaches also fail to address the specific WWR driven by causal linkages between exposure and counterparty default.

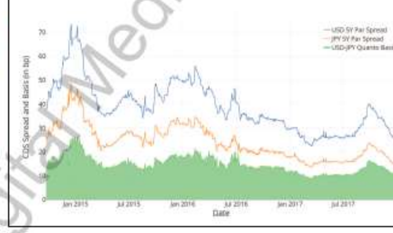
Against this background, recent studies have attempted to tackle the WWR problem from a different perspective: via an event risk modelling approach. Pykhtin & Sokol (2013) argue that WWR behaves differently for exposure to systemic important counterparties (Sics); they use a risk factor jump-at-default to capture the market impact of a Sic default on the economy. Turlakov (2013) discusses the challenges in correlation calibration for WWR and proposes a simple add-on approach to account for the increase in tail risk upon a counterparty default. Mercurio & Li (2015) introduce the jump-to-default (JtD) approach to CVA WWR modelling by using an additive risk factor jump or proportional jump at the time of default.[1]

In this article, we revisit WWR modelling by calibrating to information from the credit default swap (CDS) market. In particular, we study the market prices of a quanto CDS contract, which is designed to provide credit protection against the default of a reference entity and is denominated in a non-domestic currency (Brigo *et al* 2015). By analysing the contingent payout of a quanto CDS contract, we demonstrate how one can extract the market-implied information of the interaction between a foreign exchange jump risk and a credit default event. This important piece of market-implied information helps us to explain how WWR is being priced in the market, and it leads to an appropriate calibration of the WWR model for a forex-sensitive CVA portfolio.

The modelling of WWR will likely increase in importance for banks, especially since the proposals for a revised CVA capital charge have recommended an increased multiplier if a bank does not account for the dependence between exposure and counterparty credit quality in its CVA calculations.[2] Therefore, the modelling approaches proposed in this article not only let banks analyse the impact of CVA WWR in their portfolios but also help them to meet regulatory requirements on CVA capital calculation.

The rest of this article is structured as follows. The next section discusses a bivariate jump-diffusion model to explain the quanto basis. After that, we report our empirical analysis on calibrating the WWR model parameters to a quanto CDS basis under various assumptions. The difference between implied correlation and historical correlation regarding the forex risk factor and the credit spread is discussed. For illustration, we also compare the implied forex jump sizes from quanto CDS bases and over-the-counter forex options. We then provide numerical examples to illustrate the impact of CVA WWR using various modelling assumptions, calibration procedures and hypothetical portfolios.

1 Japanese sovereign quanto basis for the USD/JPY pair



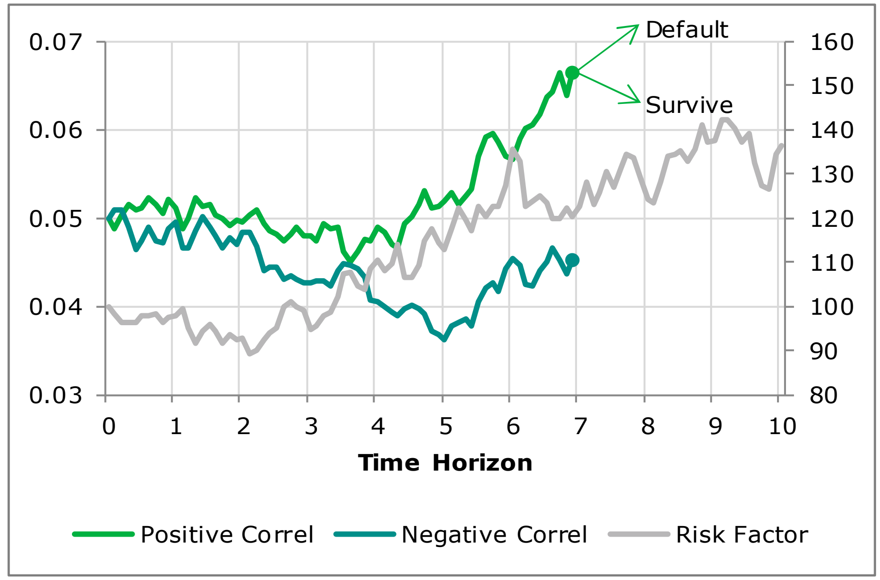
A tale of two spreads

Figure 1 shows the Japanese sovereign CDS premium (five-year par spread), which represents the cost of buying credit protection against a Japanese sovereign default. The USD and JPY spreads correspond to the CDS contracts in which the currencies are denominated in USD and JPY, respectively. Note that these contracts should always trigger simultaneously, so the only major difference between them is the settlement currency in which the contingent default payment is made. There is a persistent basis between the two CDS spreads despite the fact that they reference the same entity. Indeed, the persistent quanto basis reflects a strong market-implied devaluation jump of JPY against USD upon a Japanese sovereign default, reflecting the fact that it is more expensive to buy protection in USD than in JPY. This shows the CDS market has been consistently pricing in WWR via a forex-credit interaction as a devaluation jump upon a credit default.

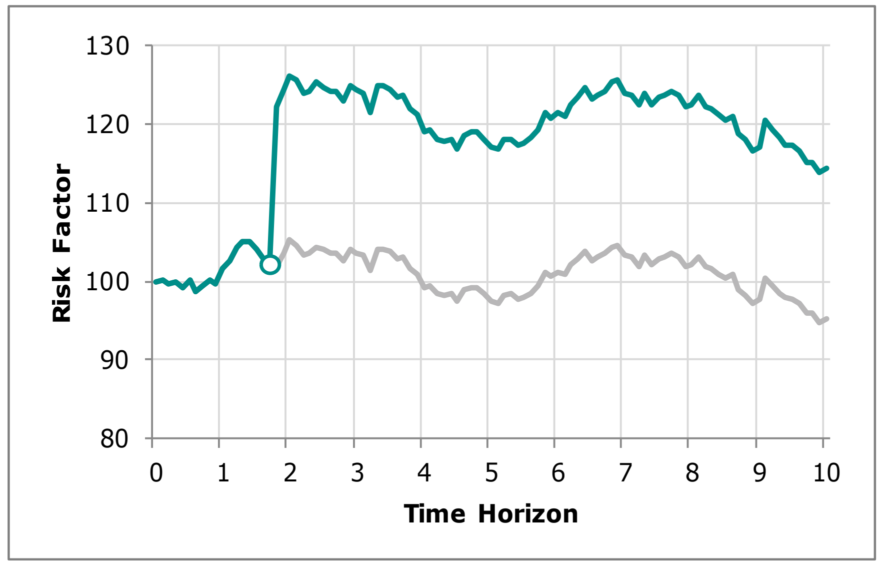
[1] *The JtD modelling approach has been widely used in financial engineering problems: see Chung & Kwok (2016) and the references therein.*
[2] *See www.bis.org/bcbs/publ/d424.htm.*

Wrong-Way Risk Case Study (I)

- Model 1
  - Soft WWR model correlating credit spread (~hazard rate) with FX process
  - Correlation estimated historically



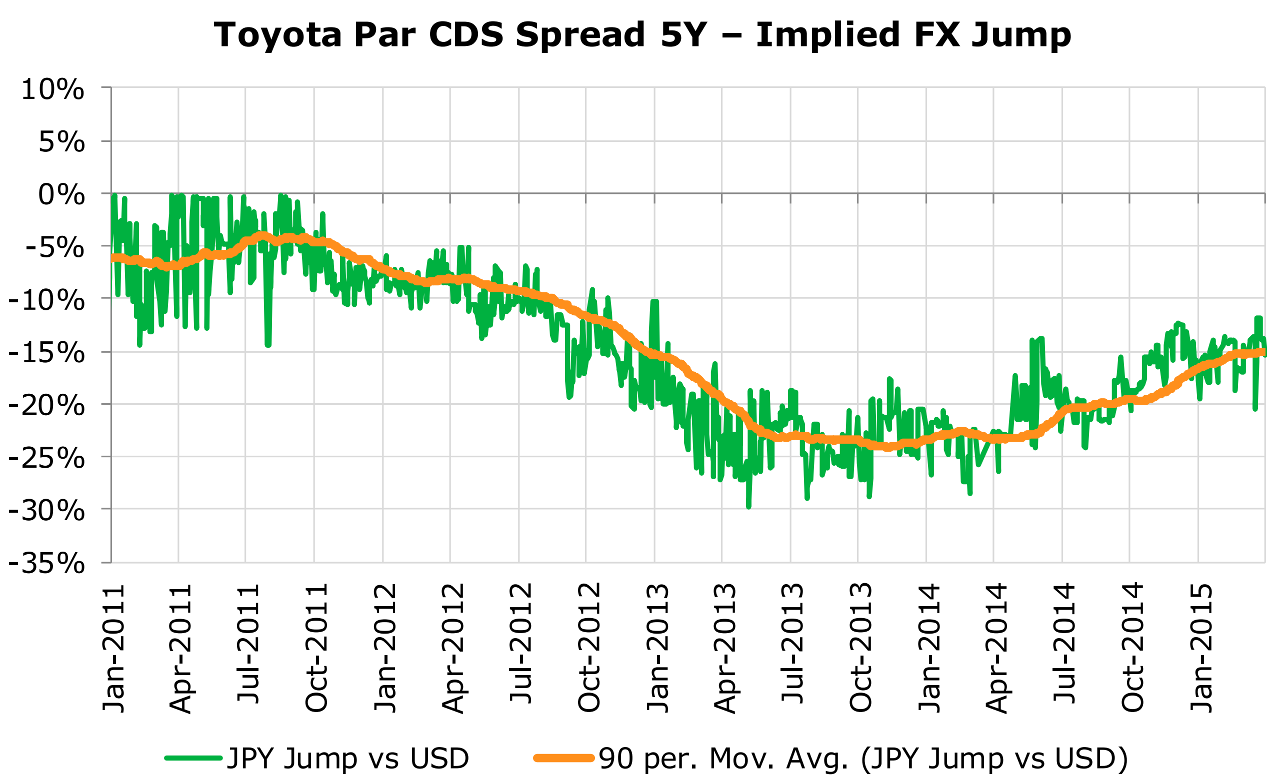
- Model 2
  - 'Hard' causal WWR model where FX rate jumps when the counterparty defaults
  - Correlation calibrated from CDS market



Source: Chung and Gregory [2019]

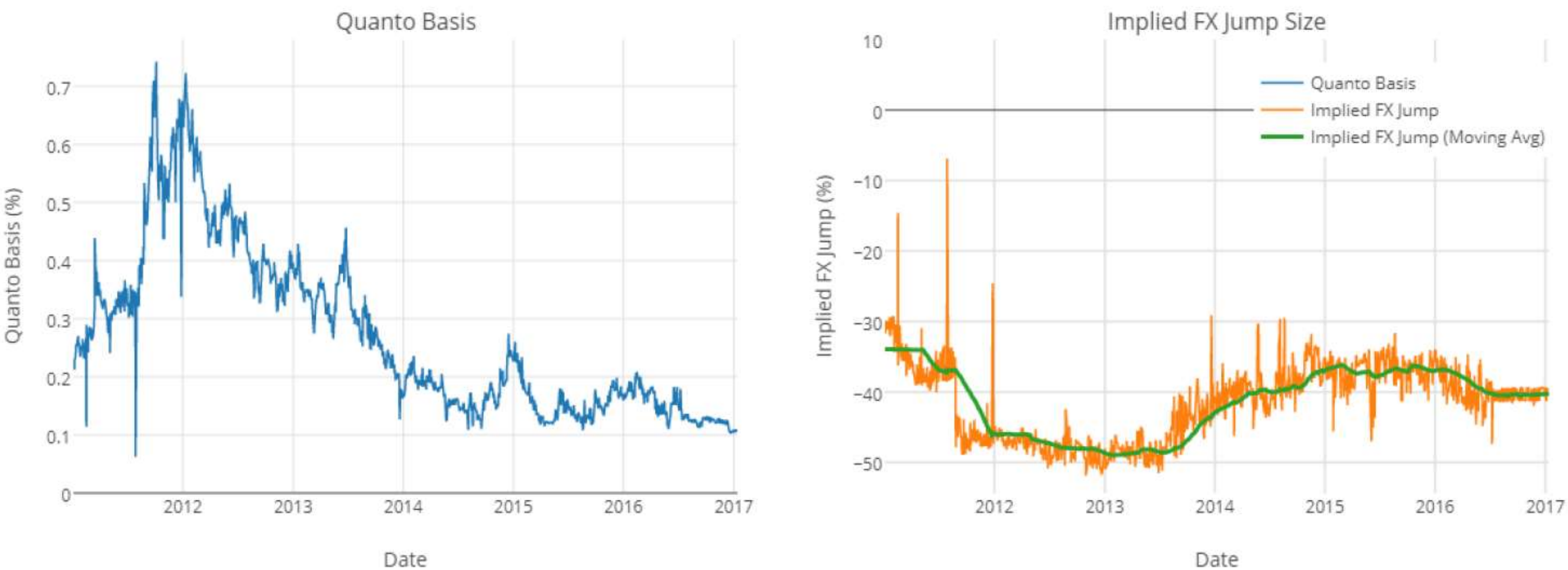
Wrong-Way Risk Case Study (II)

- Model 2 calibration
  - Implied jump can be calibrated from CDS in local current and USD
  - Similar jump size can be calibrated from the FX market



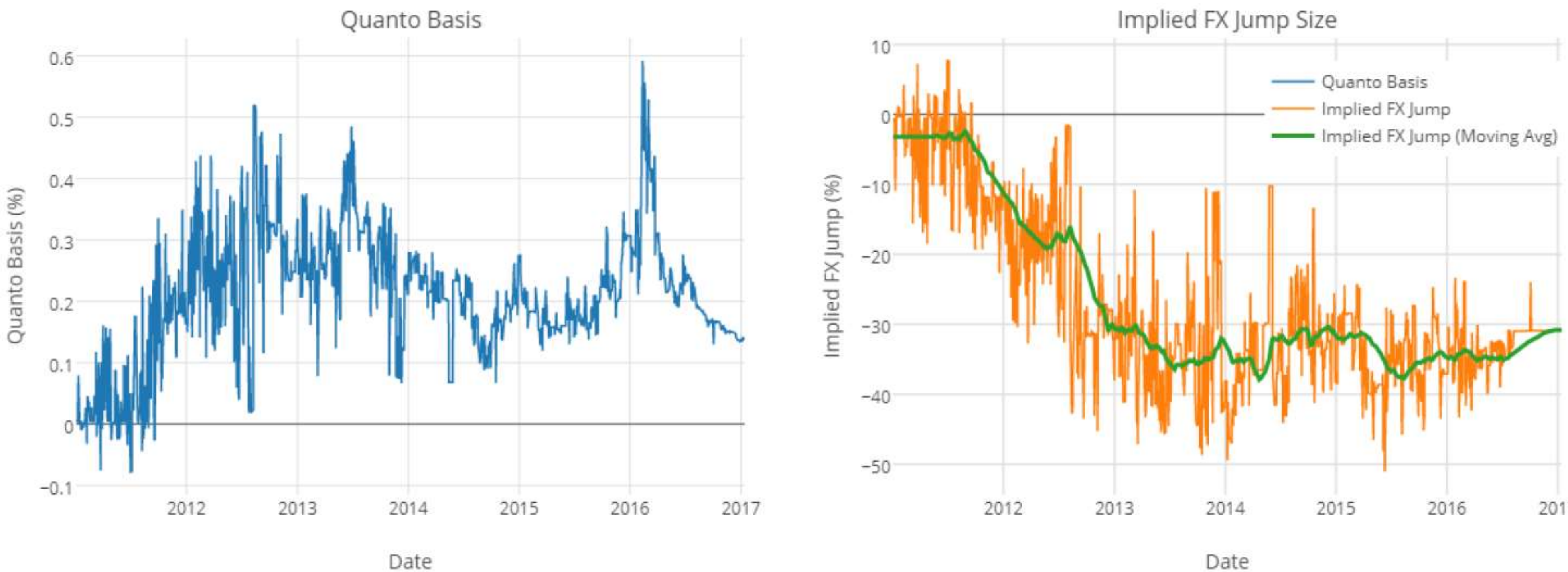
$$Implied\ Jump \approx \frac{CDS\ JPY}{CDS\ USD} - 1$$

Jump Calibration (I)



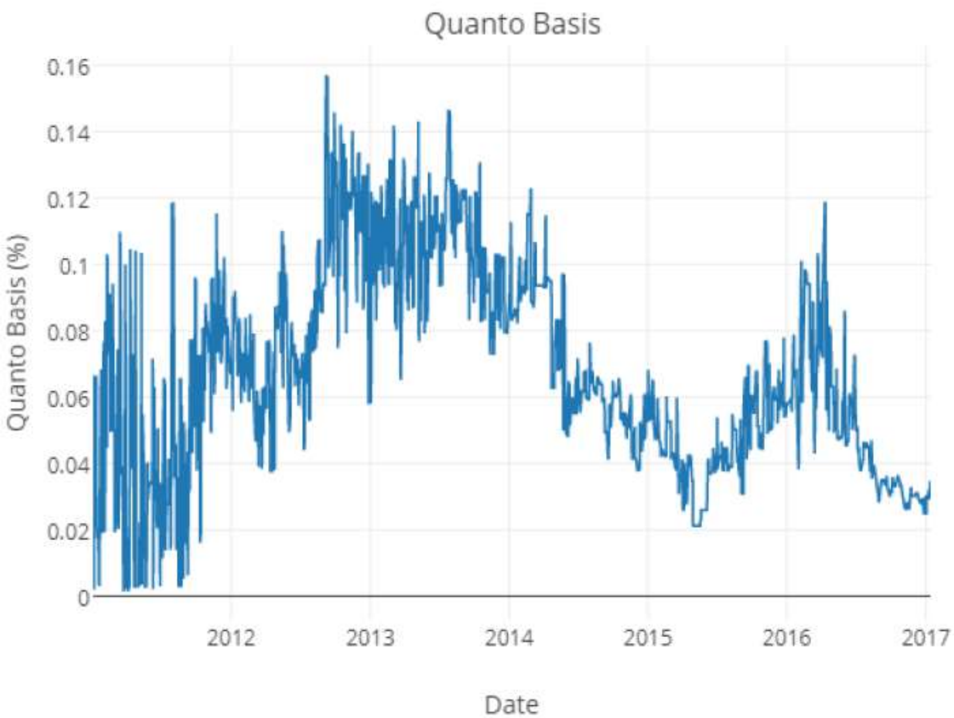
(a) Japan sovereign.

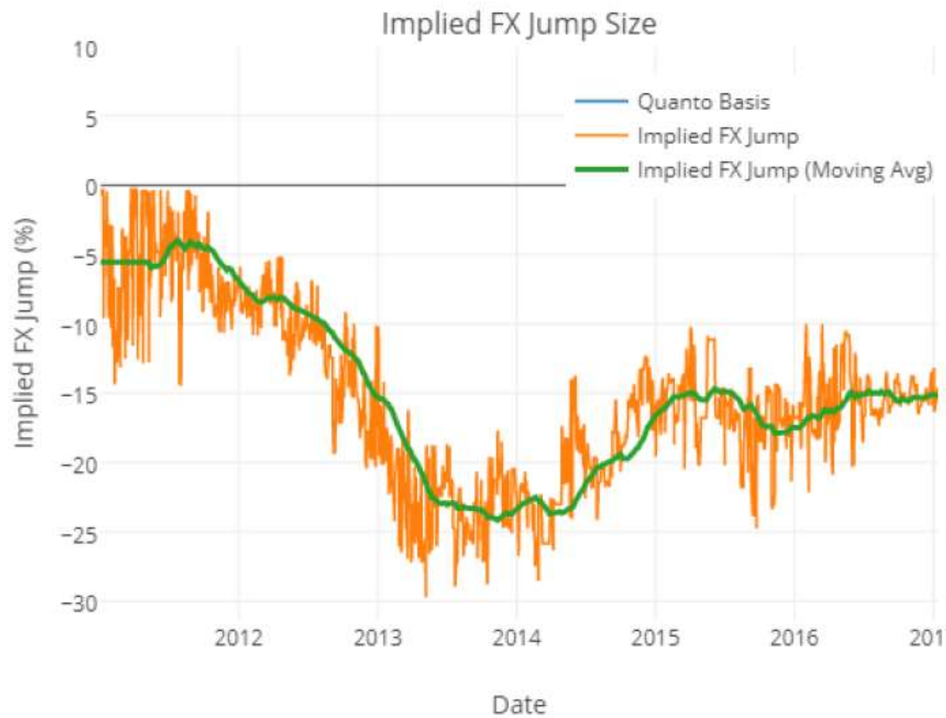
Jump Calibration (II)



(b) Bank of Tokyo Mitsubishi UFJ Ltd.

Jump Calibration (III)

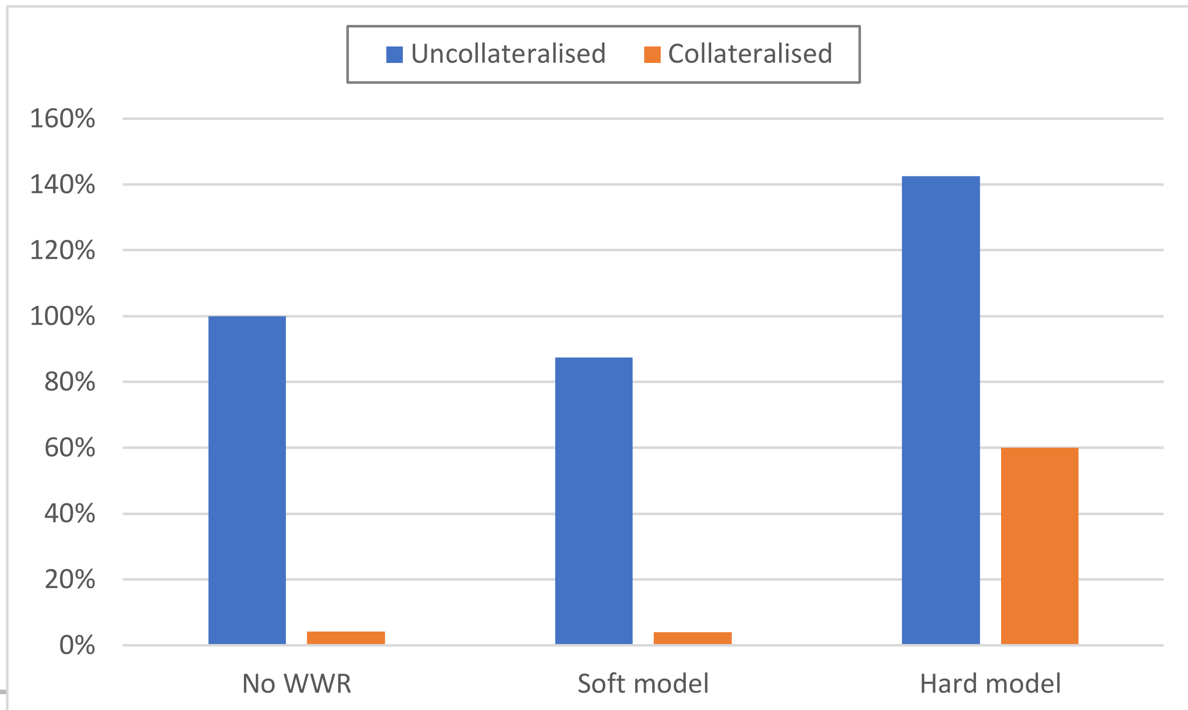




(c) Toyota Motor Corp.

Results

- Comparison of models for a directional portfolio
  - Soft WWR model gives lower CVA since correlation implies a weakening of JPY will be beneficial for the corporate
  - Hard WWR model gives much higher CVA since default of corporate implies devaluation of JPY
  - Soft WWR model cannot reproduce market prices



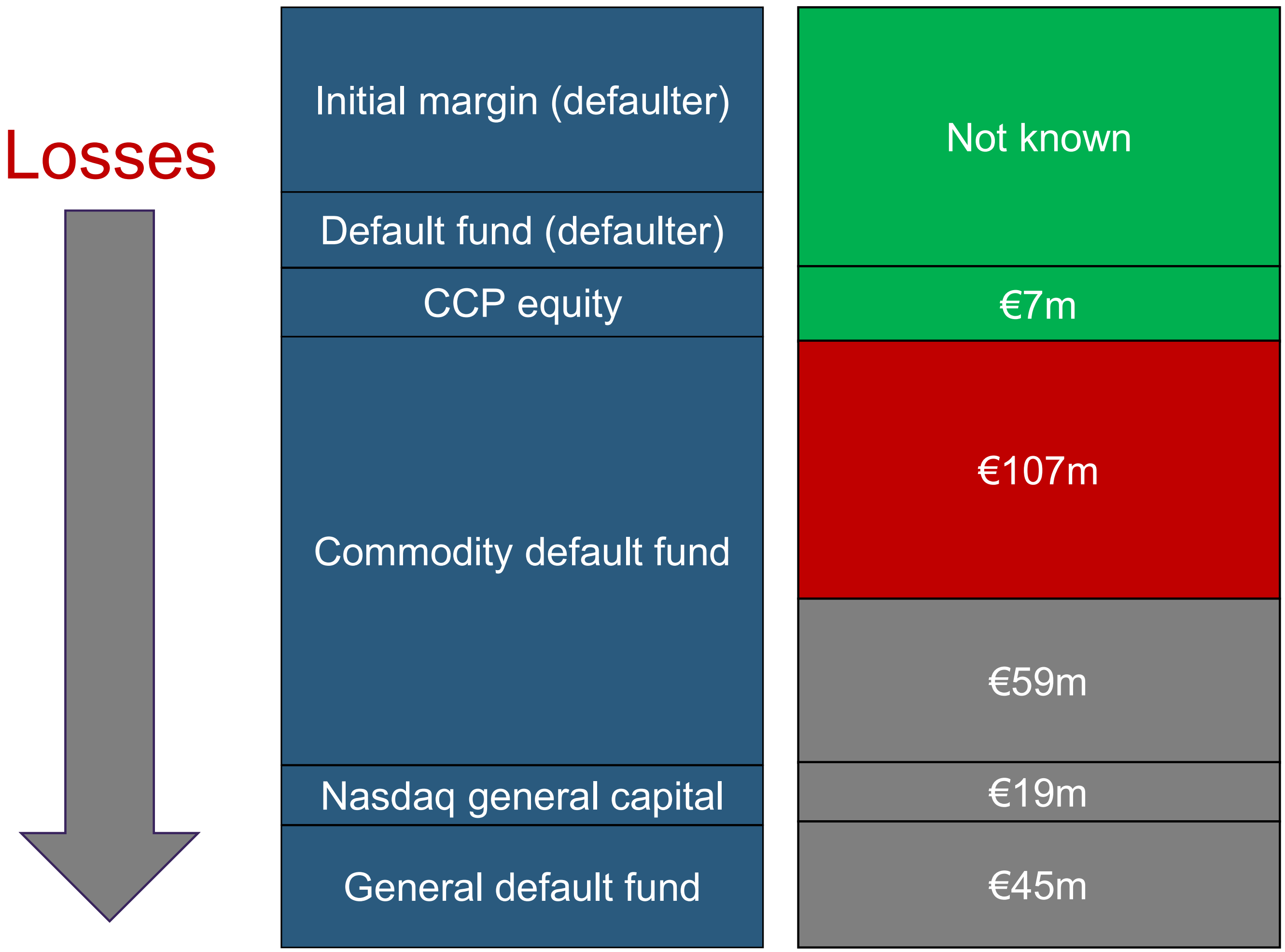
CCP Waterfall and Exposure at Default

- In the event one or more clearing members defaults …….

Waterfall	Outcome	Surviving member
Initial margin (defaulter)	No Loss	*Surviving member losses nothing*
Default fund (defaulter)		
CCP equity		
Default Fund (non-defaulting members)	Loss	*Surviving member exposed to losses*
Other loss allocation methods		
CCP Capital		
Liquidity Support or CCP Fails		

Will the Nasdaq default spur CVA for CCPs?

Default Fund – Nasdaq and Einar Aas (2018)



CVA to CCPs

- Default probability
  - Of one or more than one clearing member
  - Bank CDS? Financial CDS index
- Exposure
  - Loss as a result of the above
  - The overall loss (L) less the 'defaulter pays' (DP) resources
  - The proportion of the total CCP loss passed onto us (α)

$$CVA = -\sum_{j} \underbrace{\alpha . E\left[L_{t_i} - DP_{t_i}\right]_+}_{\text{Loss as a result of the default}} \underbrace{\Delta PD(t_{i-1}, t_i)}_{\text{Probability of at least one default}}$$

- Wrong-way risk!
- References
  - Arnsdorf [2013]
  - Andersen and Dickinson [2018]