Foreign Exchange pt2

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Learning Outcomes: Part 2

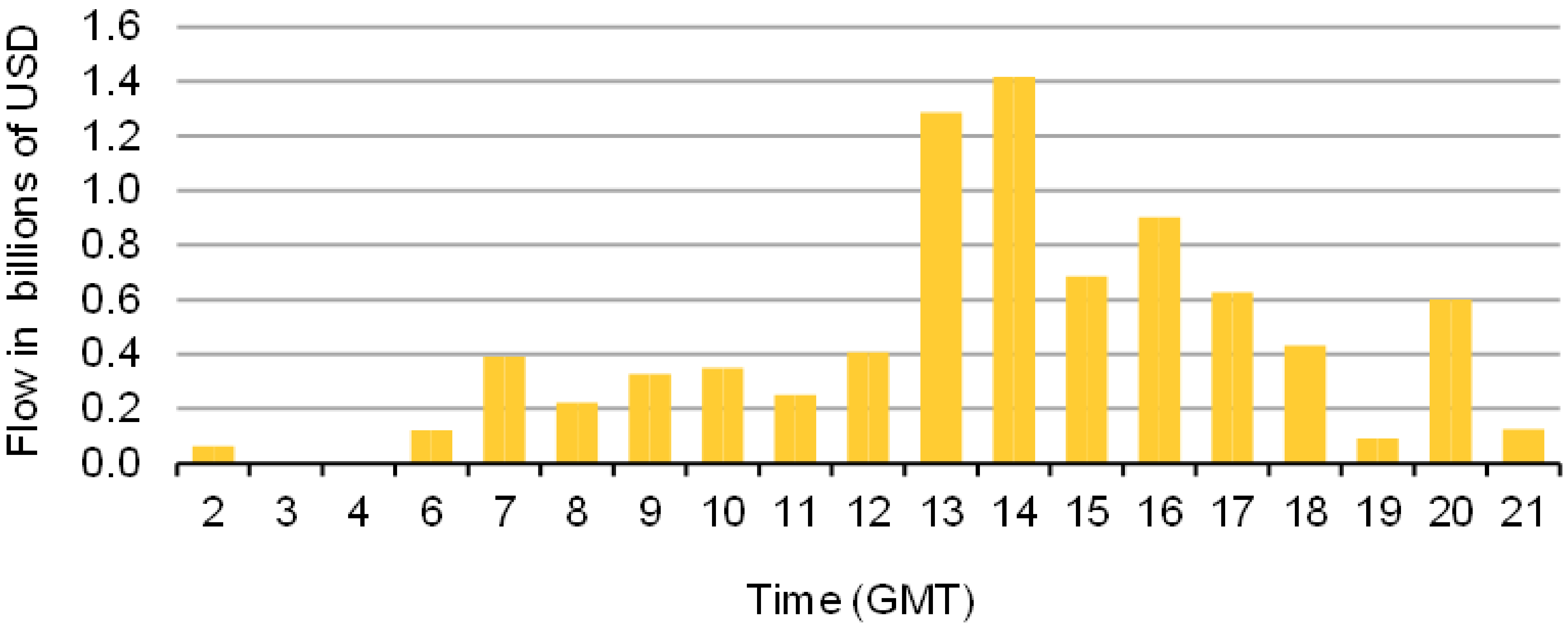
Uses of FX options

Volatility, “volatility surface”, “out of the money”

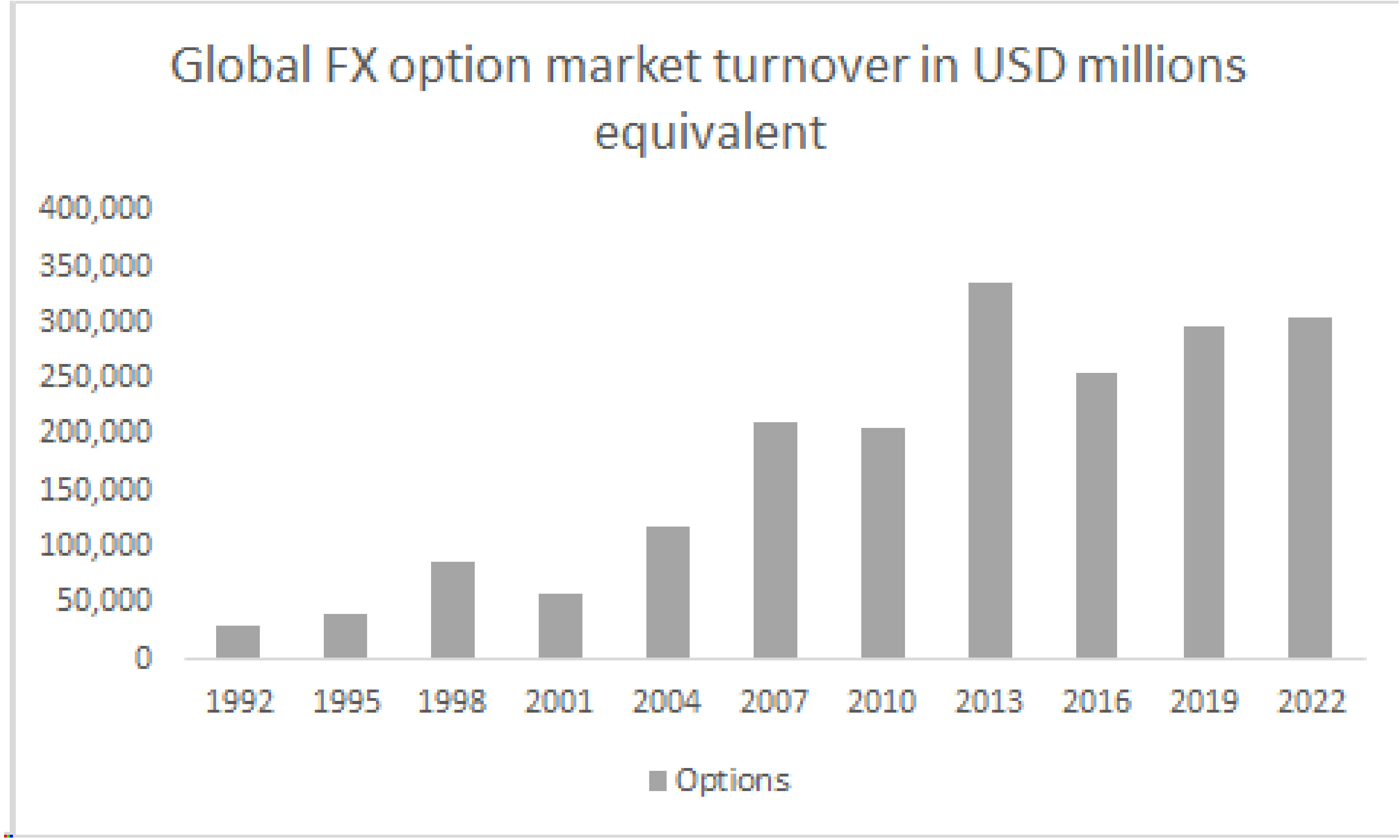
Pricing simple options

Risk management and basic Delta hedging

Hourly flow for EURUSD options, for 7th July 2014, from a London trading desk



Global FX option flow



The Basics

To begin at the beginning…

A Definition

‘A foreign-exchange option is a derivative financial instrument that gives the owner the right but not the obligation to exchange money denominated in one currency into another currency at a pre-agreed exchange rate on a specified future date’.

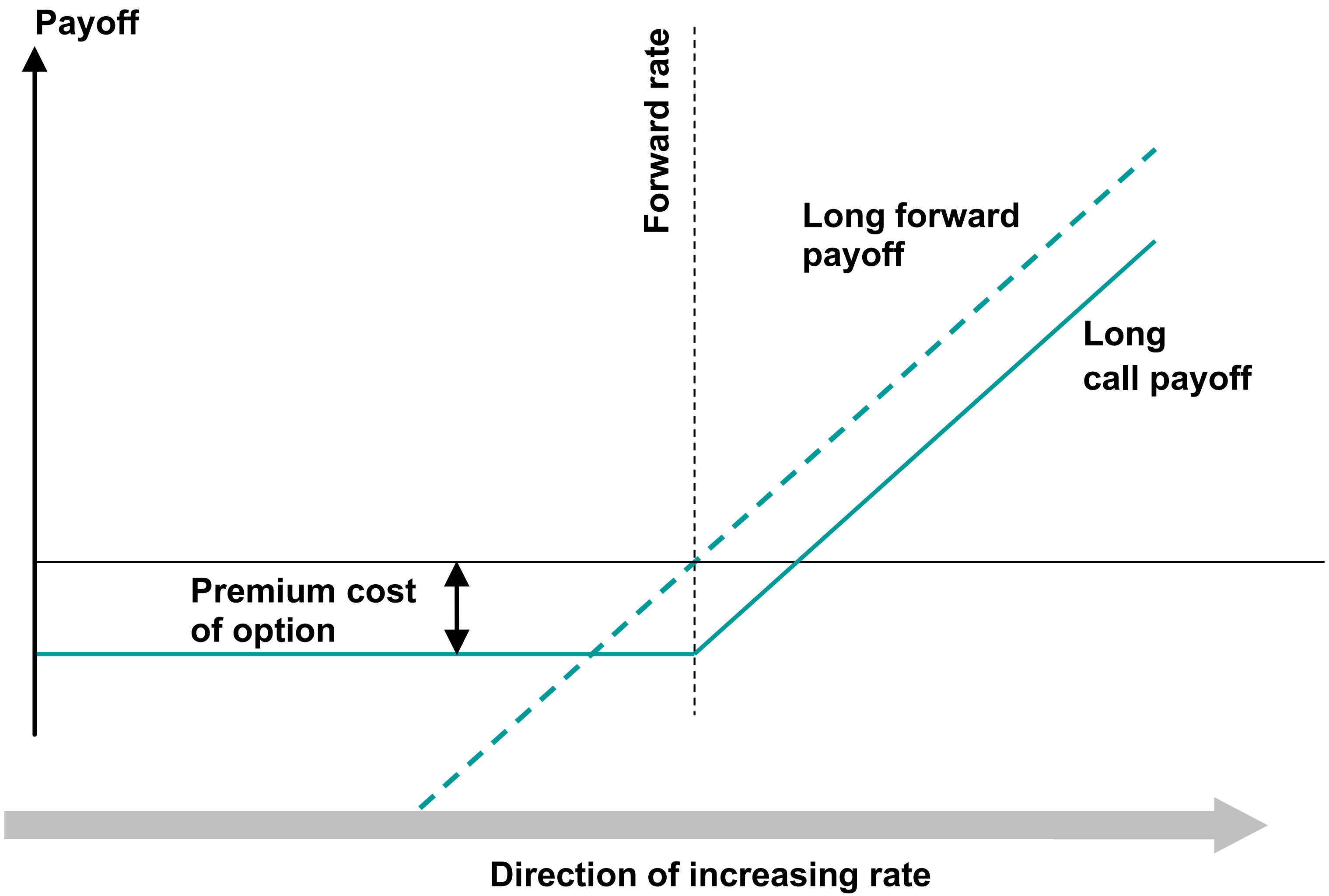
…at a cost (the “premium”)

The “expiry” or “expiry date”

In more detail

- Look at an at-the-money-forward (ATMF) call option on an FX rate
- Strike = forward rate
- Option is on buying the FX rate (i.e. base currency)
- Call = buy, put = sell but with currency pairs (e.g. USDJPY) which is which?
- Need to specify e.g. call on USD (which is a put on JPY)
- Forward rate depends on difference in interest rates (and today's FX rate)

Payoff profile at expiry for a call option



Uses of FX Options

“Very good, but what is it for?”

Sellers

Balance sheet holder selling “covered option” as hold underlying currency

Currency rises:
Pay out on option (ouch!)
But assets rise (yay!)

Currency falls:
Collect premium (yay!)
But assets fall (ouch!)

Sellers

Market Makers e.g. large banks

Rate rises:
Pay out on call options
Get premium on put options
Collect on bid-offer spread

Rate falls:
Pay out on put options
Get premium on call options
Collect on bid-offer spread

Sellers

Hedge funds: view takers on direction or inefficiencies

Central Banks: to direct large FX flow

Buyers

Investors with a currency risk e.g. international bonds

Corporations with currency risk e.g. car-maker buying parts in one country, making cars in another and selling in a third

Pricing Simple Options

Black-Scholes-Merton-Garman-Kohlhagen

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}W_t$$

$S_t =$ spot price at time t

$\mu =$ strength of drift

$\sigma =$ volatility of returns of underlying asset

$W_t =$ a Wiener process

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma S^2 \frac{\partial^2 V}{\partial S^2} + (r - r_b)S\frac{\partial V}{\partial S} - rV = 0$$

V = price of derivative

σ = volatility of returns of underlying asset

S = spot price

r = interest rate on quote currency

r_b = interest rate on base currency

Solution for price of call (c) and put (p) options

$$c = S_0 e^{-r_b T} \mathrm{N}(d_1) - K e^{-rT} \mathrm{N}(d_2)$$

$$p = K e^{-rT} \mathrm{N}(-d_2) - S_0 e^{-r_b T} \mathrm{N}(d_1)$$

$$d_1 = \frac{\ln\left({}^{S_0}/_{K}\right) + (r - r_b + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left({}^{S_0}/_{K}\right) + (r - r_b - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

$$c = S_0 e^{-r_b T} \mathrm{N}(d_1) - K e^{-rT} \mathrm{N}(d_2)$$

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$$d_1 = \frac{\ln\left({}^{S_0}/_{K}\right) + (r - r_b + \sigma^2\,/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left({}^{S_0}/_{K}\right) + (r - r_b - \sigma^2\,/2)T}{\sigma\sqrt{T}}$$

$$= d_1 - \sigma\sqrt{T}$$

S_0 = FX rate at inception

K = strike rate

r = interest rate for tenor of the option in the quote currency

r_b = interest rate for the tenor of the option in the base currency

T = tenor of the option

σ = implied volatility of the option

$\mathrm{N}(d)$ denotes the standard normal cumulative distribution function

Theoretically…

- Starting point
- Need to consider
  - Term structure of volatility
  - Term structure of interest rates
- European not American

- But sufficiently good to see main drivers

Volatility

The Devil is in the detail

Define:

σ is the standard deviation of log returns

Choose:

past or future

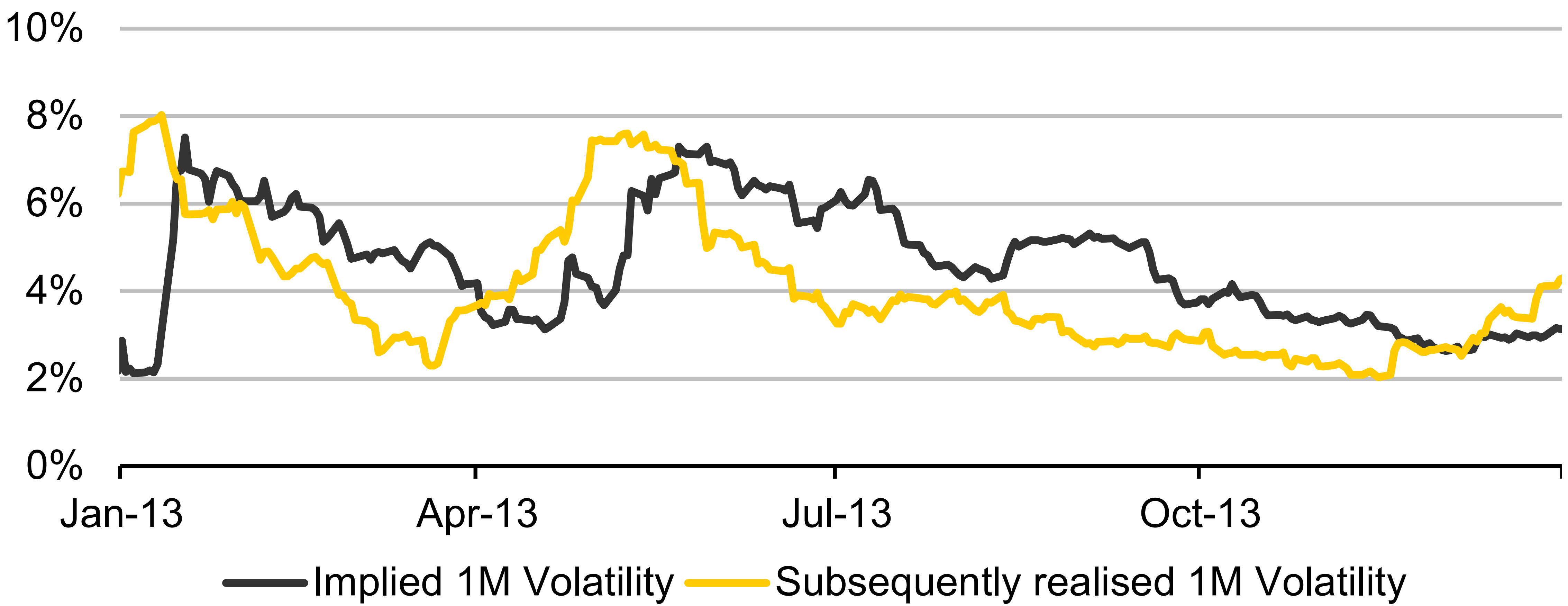
“Realised”

or “historic”

“Implied”

or “historic”

Annualized daily 1M EURCHF implied volatility and subsequent 1M realised volatility



Which lags, which leads?

Choose:

past or future

~~"Realised"~~

~~or "historic"~~

"Implied"

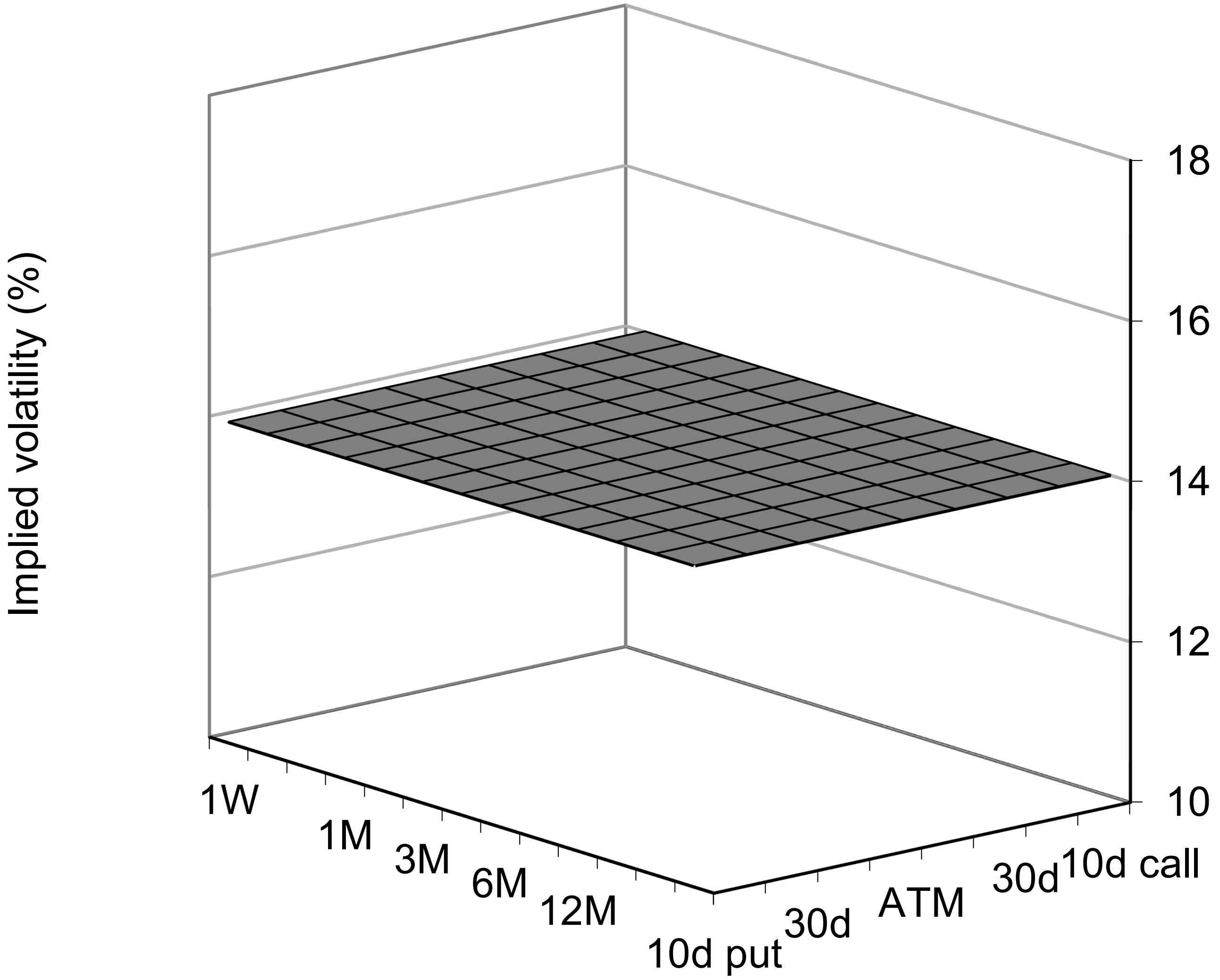
or "historic"

- Effect of higher volatility?
- Over what time period?
- Evaluate risks?

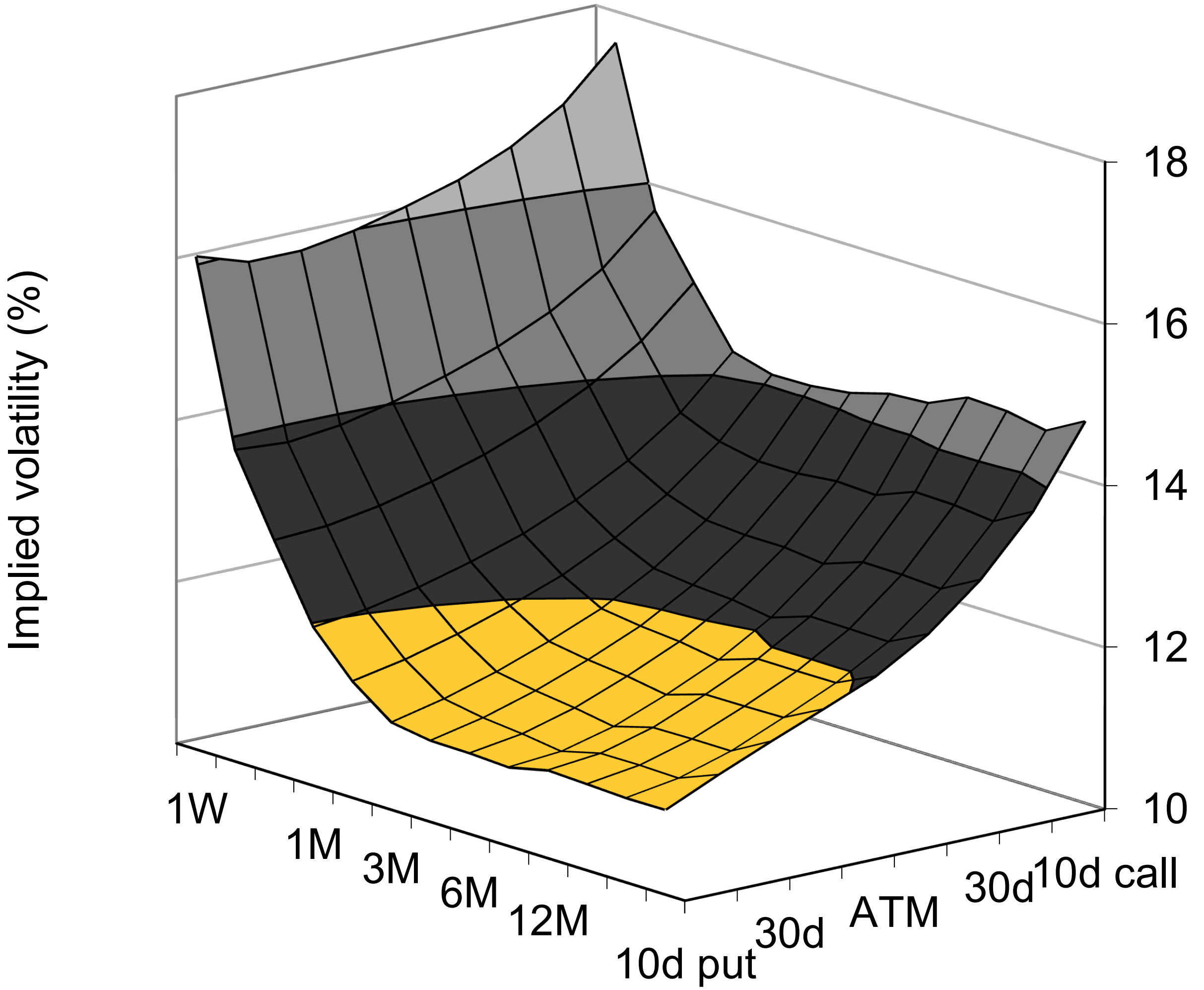
Assumptions in more detail

- Assumptions allow a solution to be found
- But market observations differ from model predictions
- Single volatility not adequate

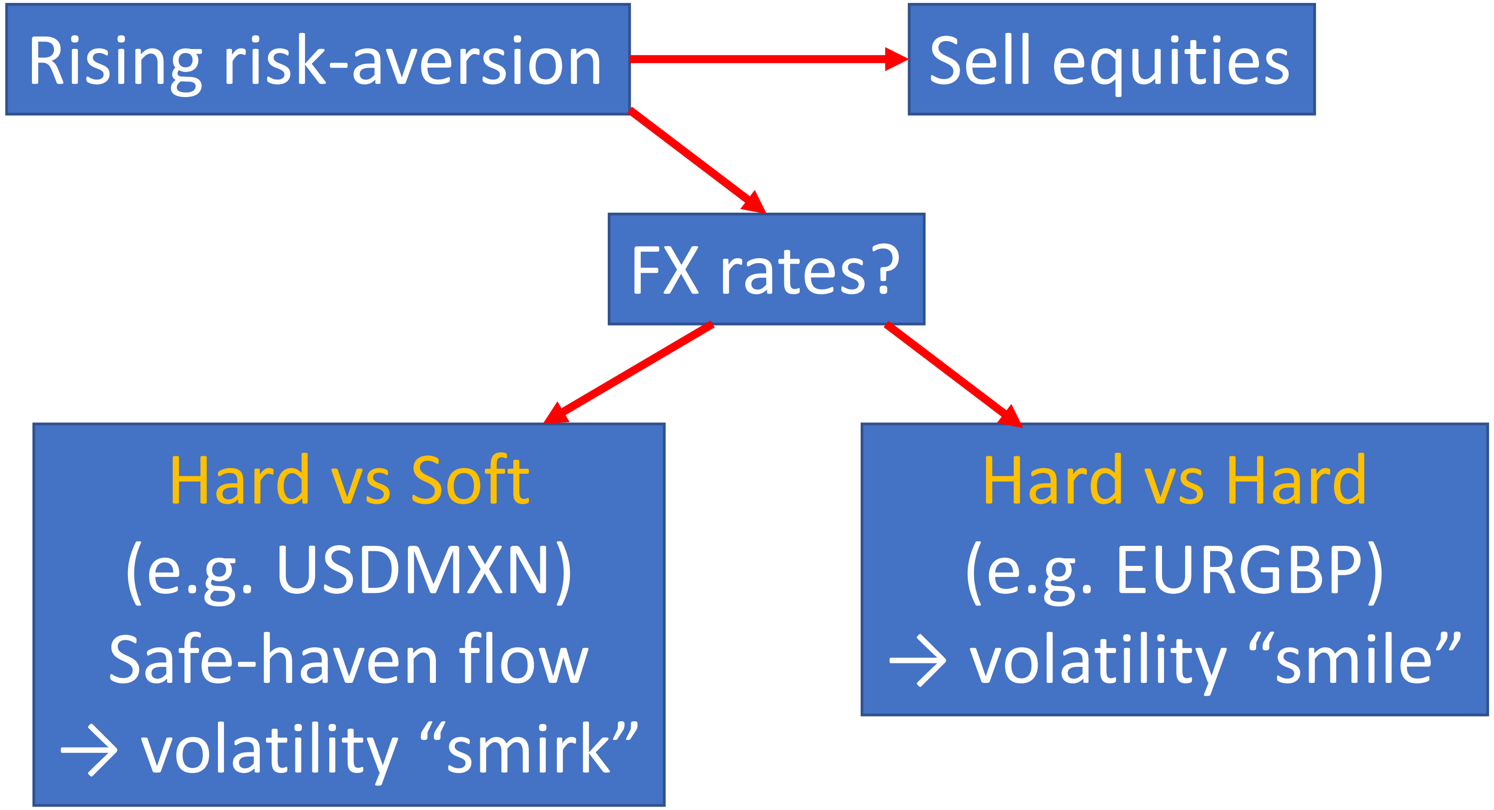
Implied Volatility Surface under Black-Scholes-Merton



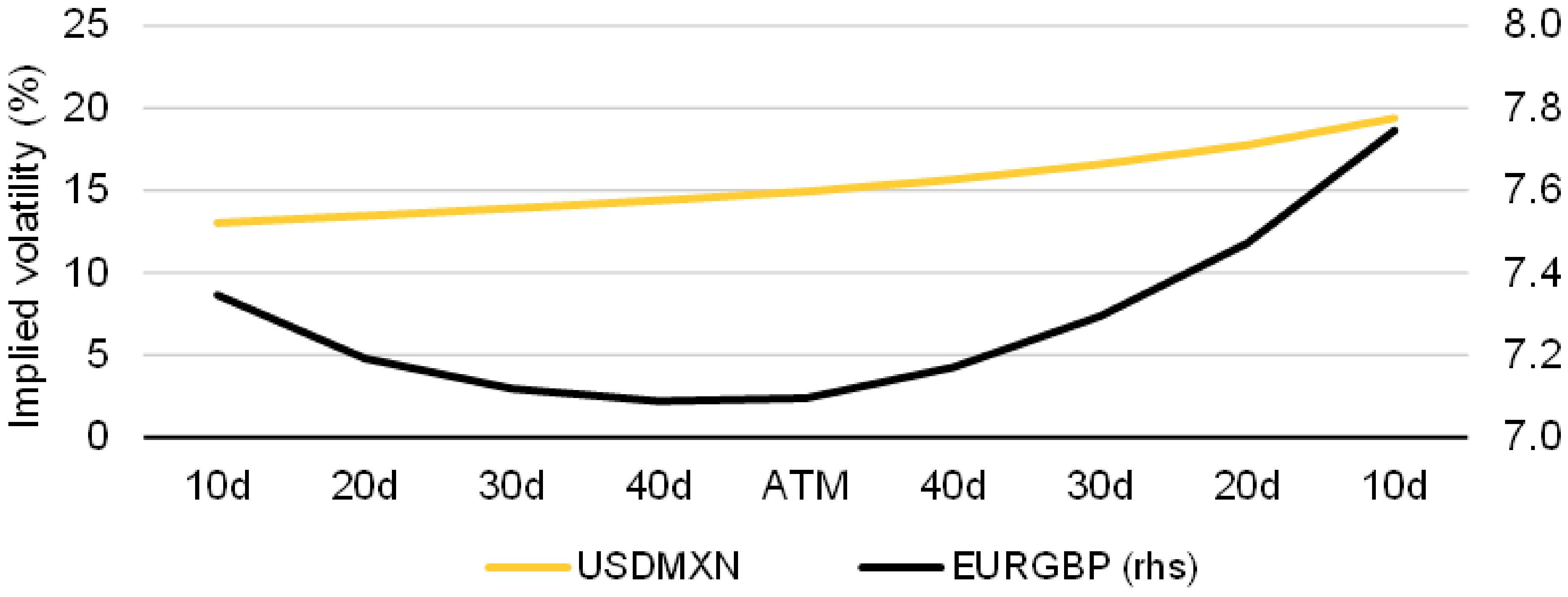
Observed volatility surface for USDSEK as at 28 June 2013



How do Spot FX rates develop?



Volatility smiles for USDMXN and EURGBP as at 28 June 2013



Do “frowns” ever happen?

- Usually an event causing a step in asset price e.g. result of litigation causes share price to jump (up or down) 10%
- Very rare in FX, typically associated with
  - Very short dated (<1wk maturity) options
  - At time of significant effect
  - E.g. EURUSD at release of non-farm payroll (NFP) data in US
- Market will shift so volatility will temporarily increase
- Trade: buy ATM, sell wings of volatility smile

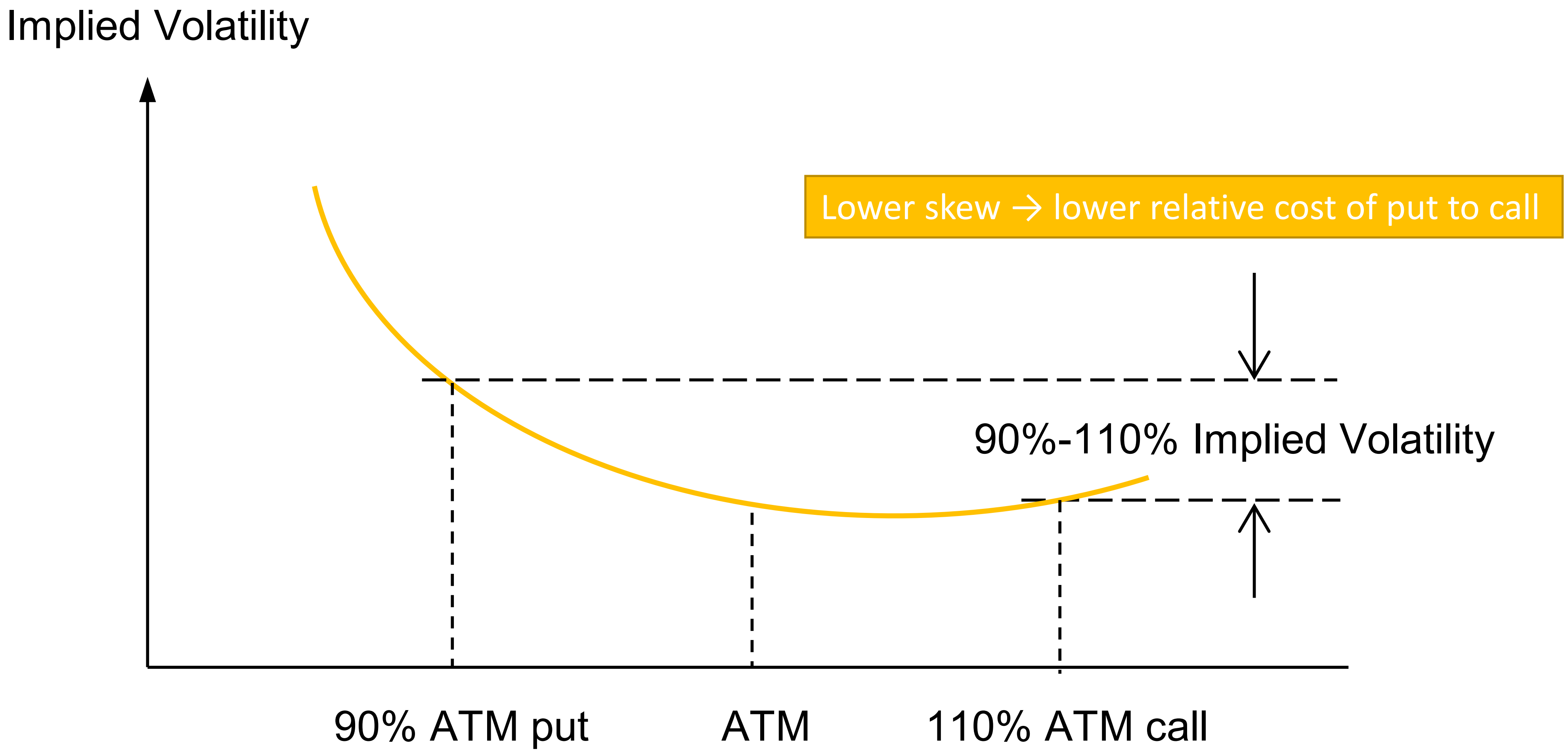
Measuring the volatility smile: skew

First: How to quote strike price?

Equities: % of spot rate

∴ 90%-110% measures skew

Schematic illustration of the equity index 90%-110% implied volatility skew (equity price on x-axis)



Measuring the volatility smile

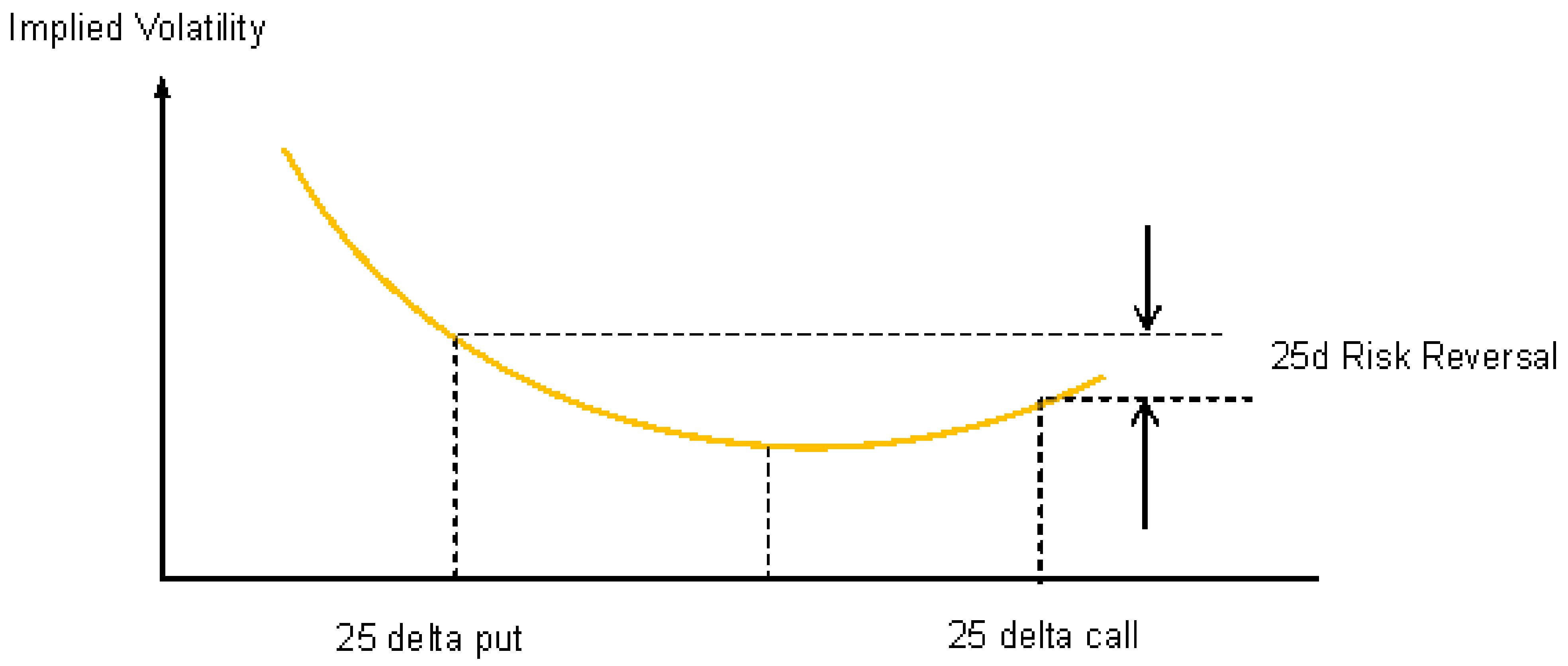
First: How to quote strike price?

FX: Use delta*

∴ 25-delta OTM put vs 25-delta OTM call measures skew

* Will come back to this shortly!

Measures of skewness for the volatility smile: Schematic representation of a 25-delta risk reversal (FX rate on x-axis)

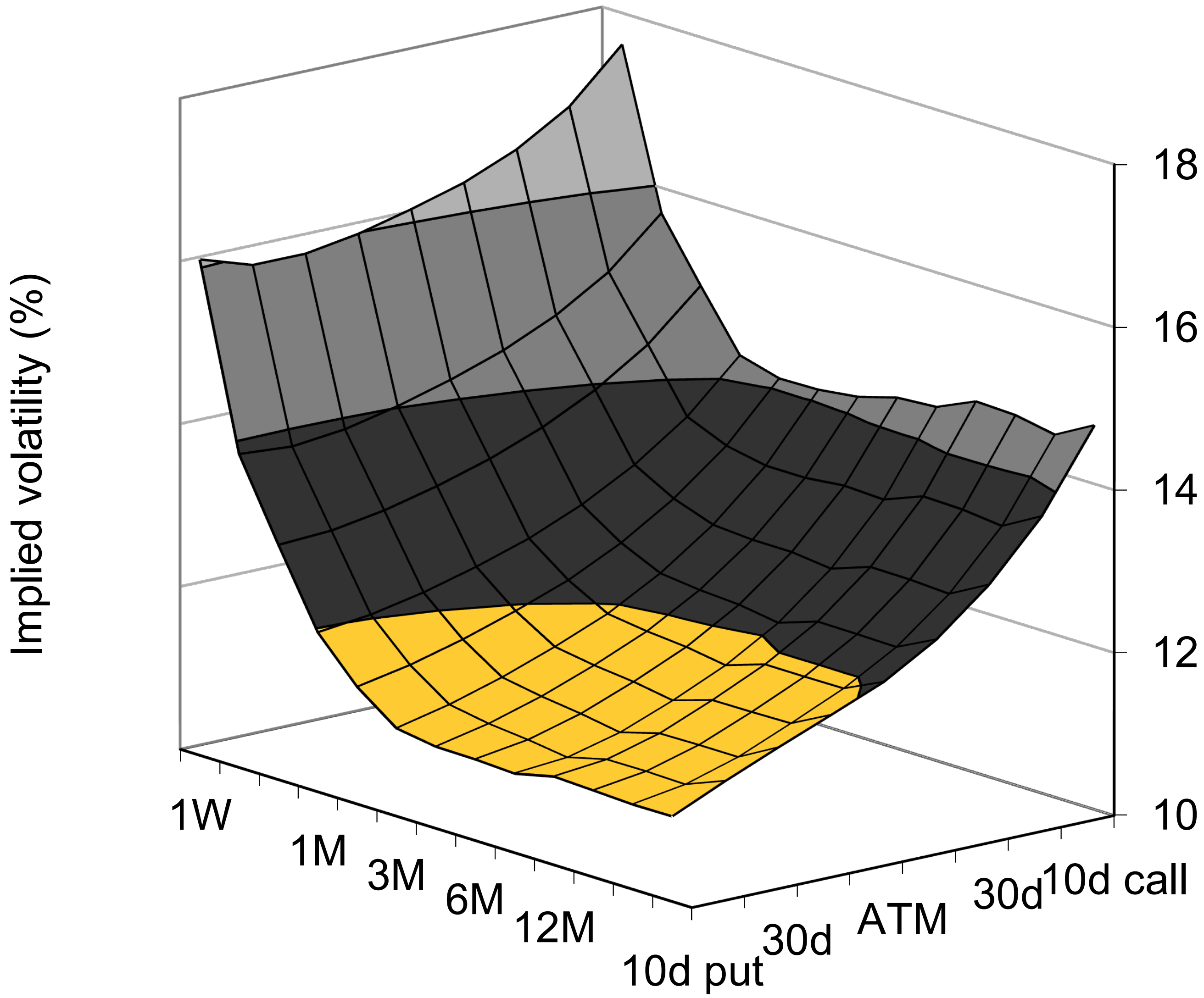


Delta (one of the ‘greeks’)

Formally: change in price of a derivative, such as an options contract, given a unit change in its underlying, expressed as a percentage

Also: It is the price sensitivity of the FX option with regard to a unit change in the FX rate

Intuitively: a proxy to the probability of an option to end in the money at expiration. An option with Delta of 10 has a 10 percent probability of being in-the-money at expiry.



Implied volatility (%)
18
16
14
12
10
1W
1M
3M
6M
12M
10d put
30d
ATM
30d
10d call

Measuring the volatility smile: curvature

Liquid market e.g. EURUSD

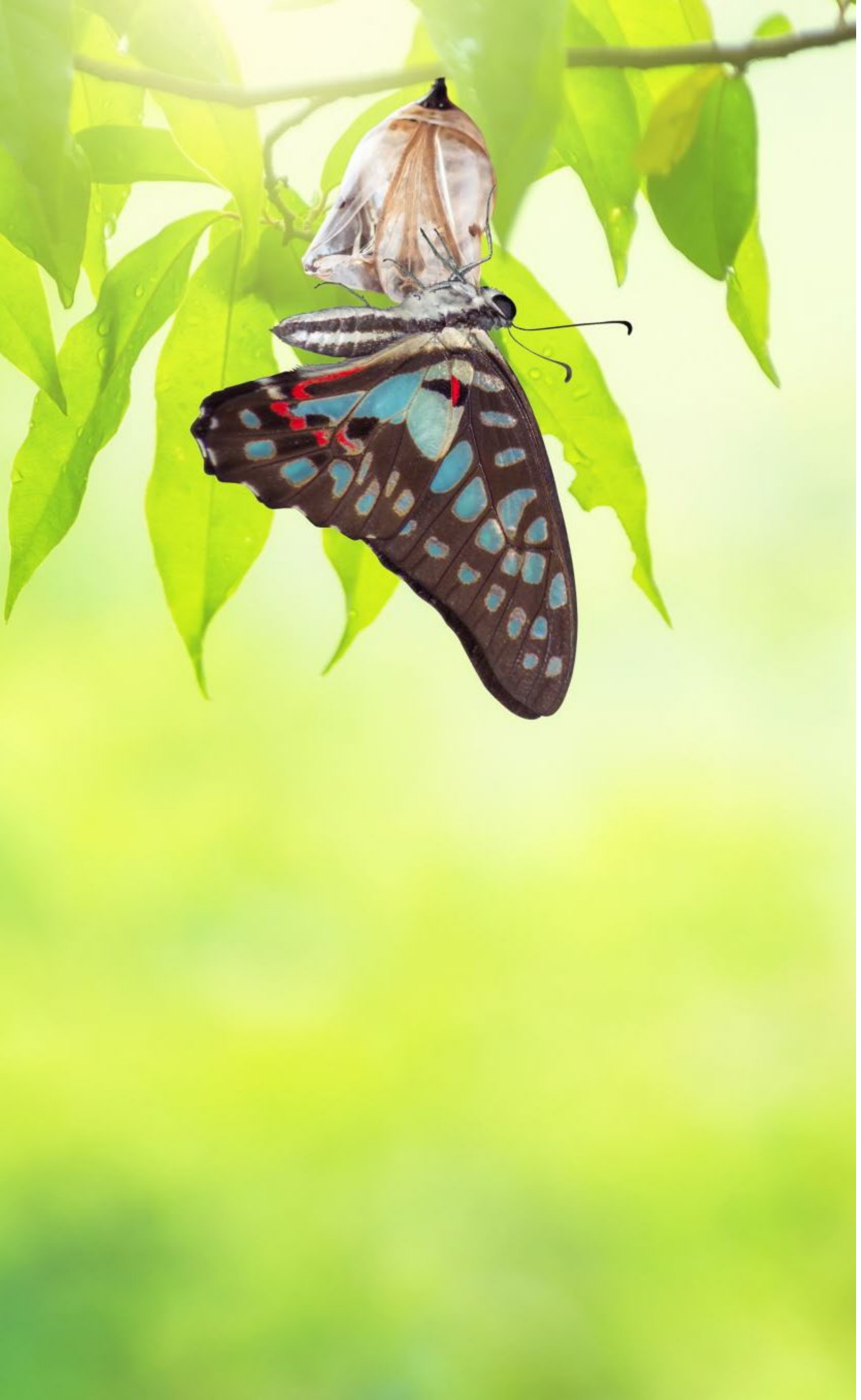
Volatility does not change (much) with underlying

Shallow smile

Emerging market

Large movements in FX and volatility

Deeper smile



Butterfly spread (curvature measure)

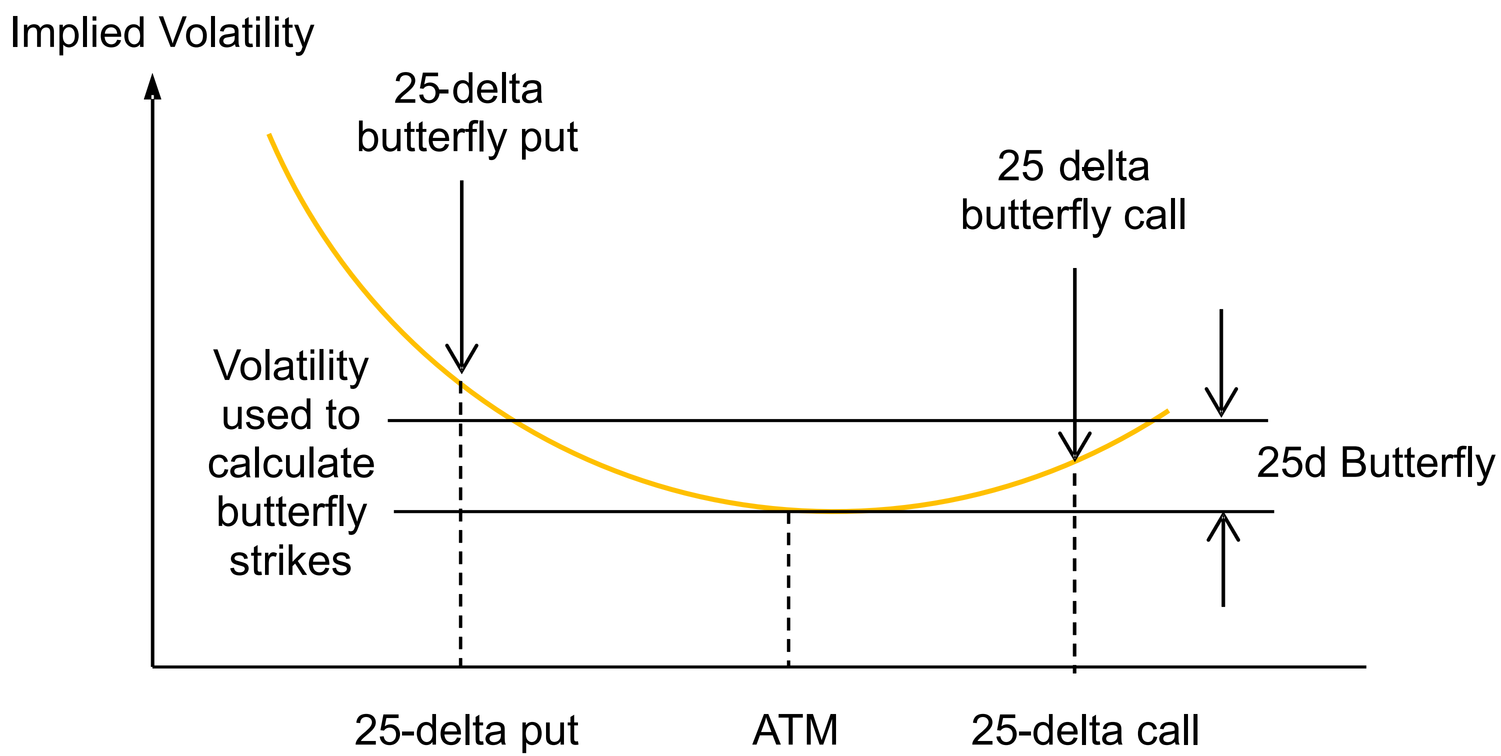
- (Long butterfly spread)
- Buy ATM straddle
- Sell put and call options of equal delta

- Traded so can see the probability of larger moves priced in to FX market

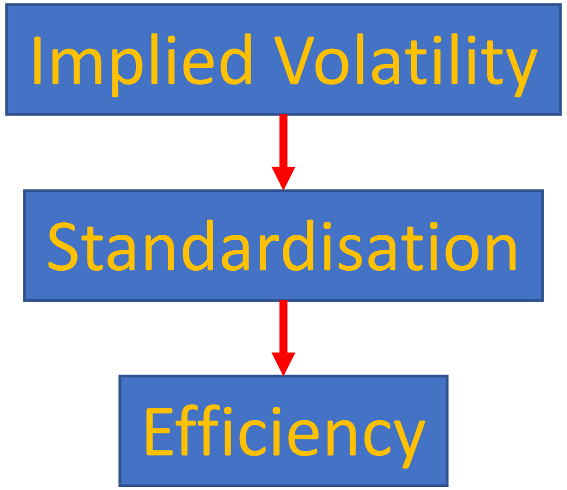
Butterfly Volatility

$$\sigma_{butterfly/25-delta} = \left[\left(\sigma_{call/25delta} + \sigma_{put/25delta}\right)/2\right] - \sigma_{ATM}$$

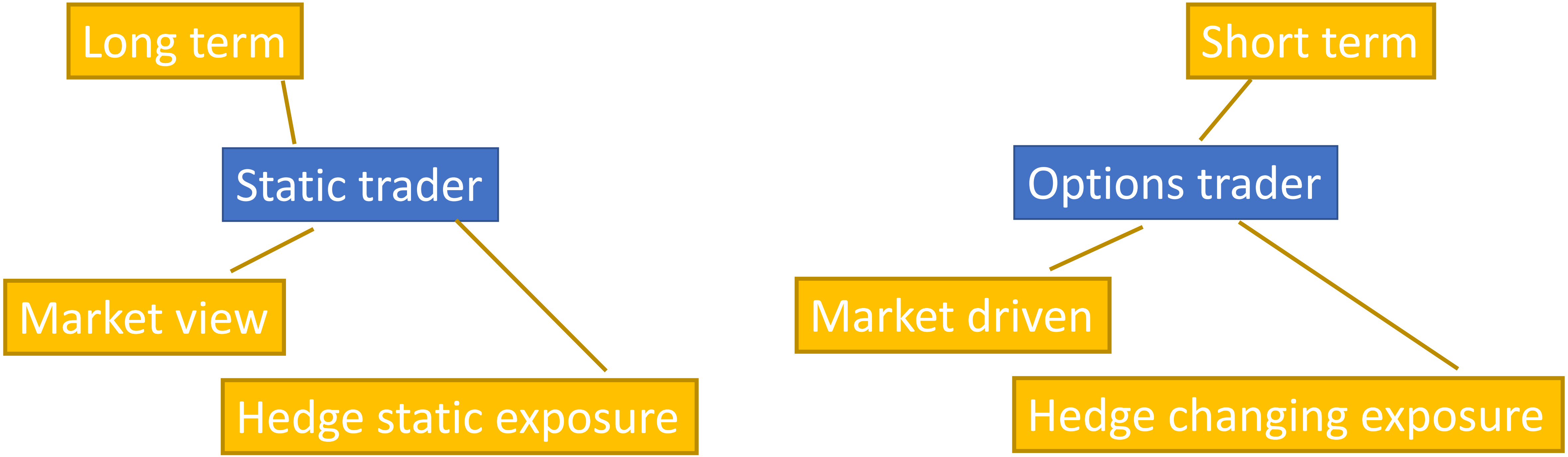
A butterfly spread can be used to gauge the steepness of the volatility smile



Trading Practice



Trading Practice



Static vs Dynamic

Volatility

Waiting for expiry…how much am I paying for protection? Is it right?

During the journey, how much can I make from movements?

Static Trader

Hedging predefined currency exposure

E.g. corporate treasurer

Buying insurance so set strike at worst level they'll tolerate and option kicks in beyond that

Needs relatively little terminology

Static Trader

- ATMF: At The Money Forward
    - Strike set by forward rate at time of purchase
    - Call pays off if FX rate above strike at expiry
    - Put pays off if FX rate below strike at expiry
    - Breakeven when option value overtakes premium
- Out Of The Money (OTM) and In The Money (ITM)
    - Strike is unfavourable (OTM) or favourable (ITM) compared to forward
    - ATMF options become OTM or ITM as spot rates and forward rates move
    - Delta gives a handle on how far ITM or OTM option is
    - Delta = 50% ATMF

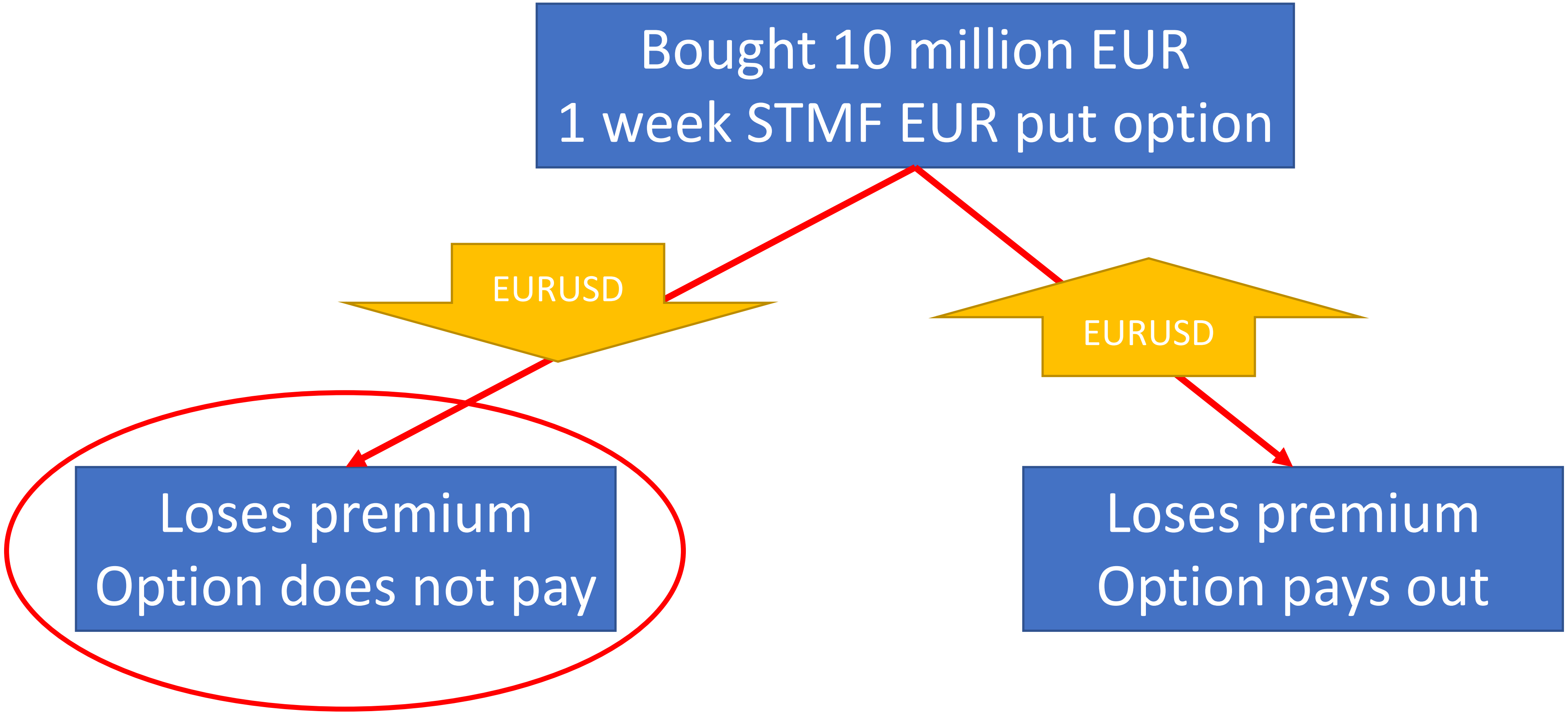
Dynamic Trader

- Trading options all the time
- E.g. FX dealer for a bank
- Dealing with short term day-to-day or intraday on lots of positions
- Interested in volatility as changes in underlying impact them

A Crude Example

- Trader in EURUSD
- Only variable (for this example) is spot rate
- Buys a one week ATMF EUR call (USD put)
- 10 million EUR notional
- Expects volatility but no idea of direction of market moves

Crude Example cont...



Crude example: delta hedging

Bought 10 million EUR
1 week STMF EUR put option

Sell 5 million EUR at spot rate

If rate falls, buy back and
lock in profit

If rate returns to original
level, repeat

Put-Call Parity

- Buy an ATMF call option and sell full notional amount of forward contract
  - Gives same profile as a put option
- Buy an ATMF put option and buy full notional amount of forward contract
  - Gives same profile as a call option

- Also true for selling options (reverse buy and sell)
- Hence vol and strike matter, not if option is put or call

Put-Call Parity

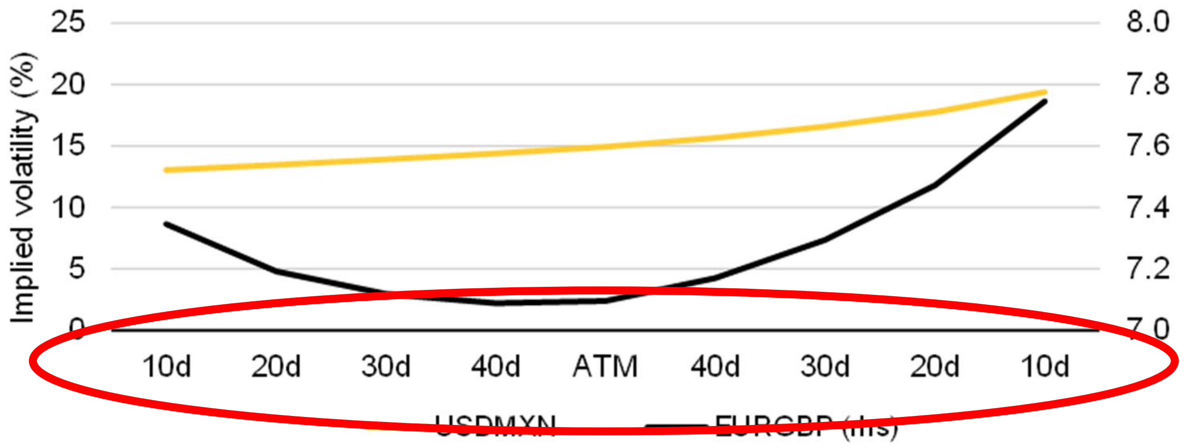
- Strictly Delta has a sign
- Based on trader *selling* option (i.e. -1 amount of option) and *buying* Δ amount of underlying to hedge
- Selling a call (e.g. on USD) is hedged by buying USD i.e. going long on rate with dollar base. $\therefore \Delta$ +ve
- Selling a put (e.g. on USD) is hedged by selling USD i.e. going short on rate with dollar base. $\therefore \Delta$ –ve

Put-Call Parity

$$\Delta_{\text{call}} = N(d_1)$$

$$\Delta_{\text{put}} = N(d_1) - 1$$

$$\therefore \Delta_{\text{call}} - \Delta_{\text{put}} = 1$$



Risk Management

Basic Delta hedging

Timeo Danaos Et Dona Ferentes…

Δ

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Γ

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Θ

Delta hedging

- Delta (Δ) measures the rate of change of option value with respect to changes in the underlying asset's price
- Delta is the *first derivative* of the value of the option with respect to the underlying instrument's price

- Consider: option with 5 days to maturity, 10% implied volatility

Long Side
(trading desk bought the option)

Customer collects premium; desk has no view on spot movement.

Desk thinks value is good at that level of volatility.

Exact Delta calculated and underlying traded in open market.

Spot moves up and down with desk adjusting position rapidly to make money.

Short Side
(trading desk sold the option)

Customer pays premium; desk has no view on spot movement.

Desk thinks value is good at that level of volatility.

Exact Delta calculated and underlying traded in open market.

Desk wants to protect premium so does not trade (incurs costs)

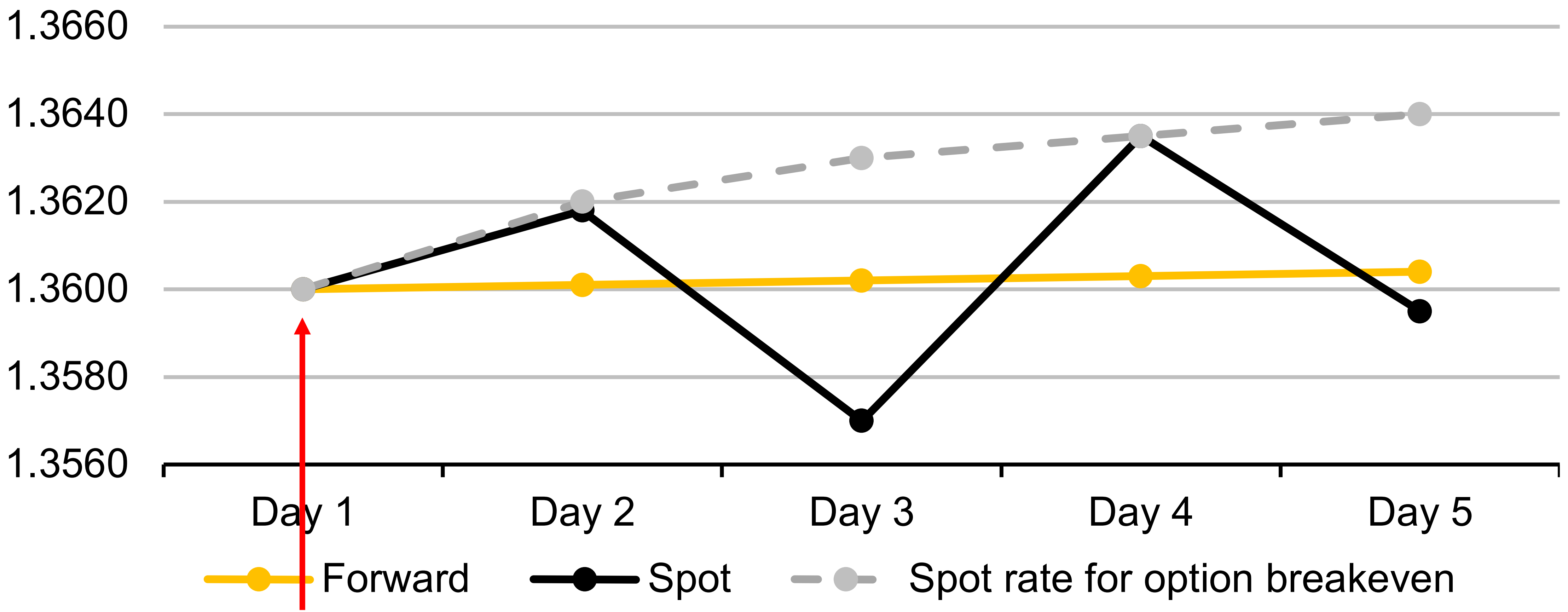
How and when to adjust position?

- Gamma (Γ) measures the rate of change of Delta with respect to changes in the underlying asset's price
- Gamma is the *second derivative* of the value of the option with respect to the underlying instrument's price

- Gamma tells you how much Delta hedging to be doing

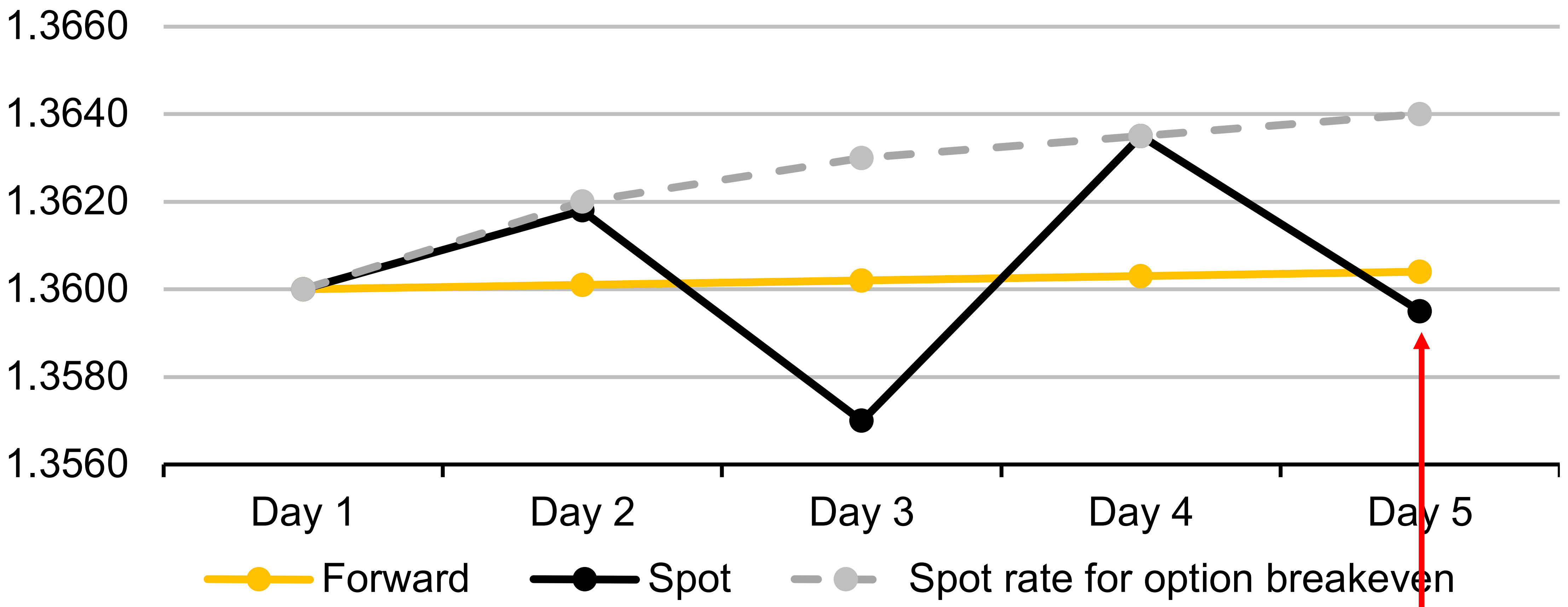
Regime	Likely gamma	Likely trading strategy for LONG option position
High volatility	Low gamma	Wait for the larger swings to develop to trade the delta as the high volatility makes the option expensive and therefore the premium paid large.
Low volatility	High gamma	You haven’t paid much for the option. If you see a chance, take it.
Long maturity	Low gamma	Relax. You have plenty of time to see how the underlying develops.
Short maturity	High gamma	Trade as much as you can, you are running out of time.

An example of Delta hedging



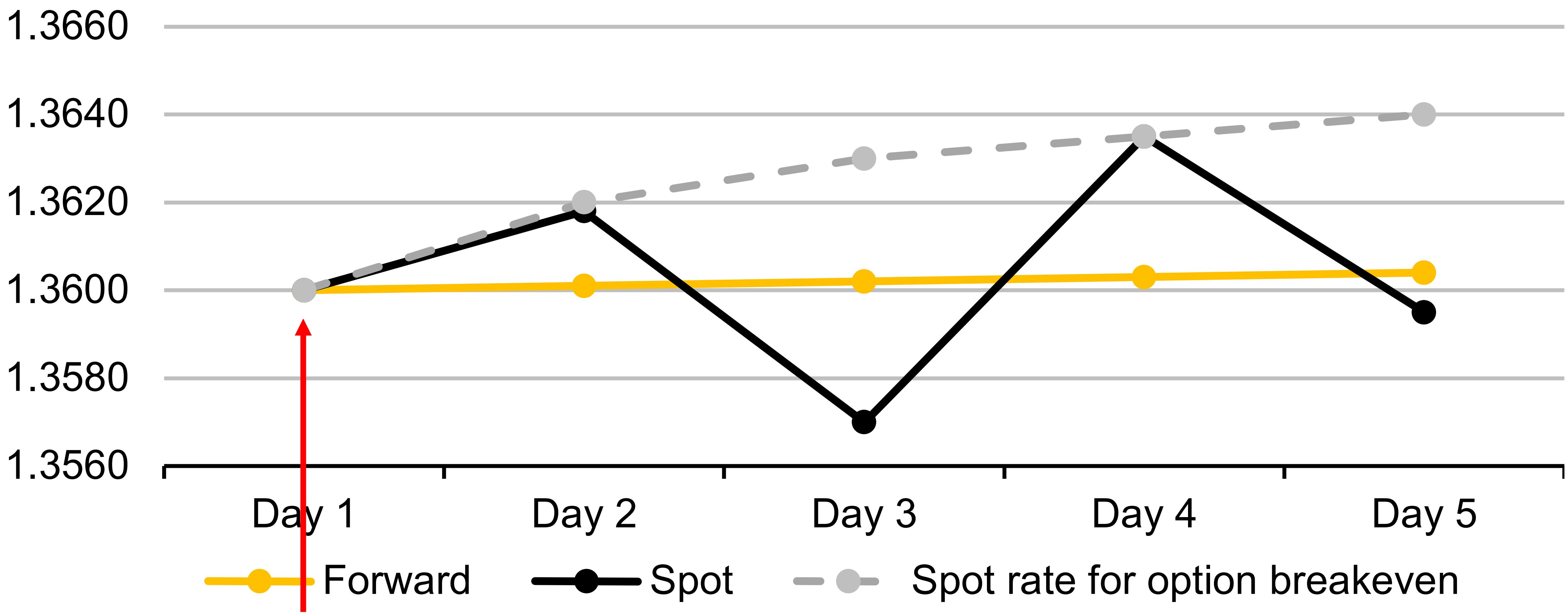
Spot rate = 1.3600. Forward rate slightly higher. Customer sells call option for 1.3640

An example of Delta hedging



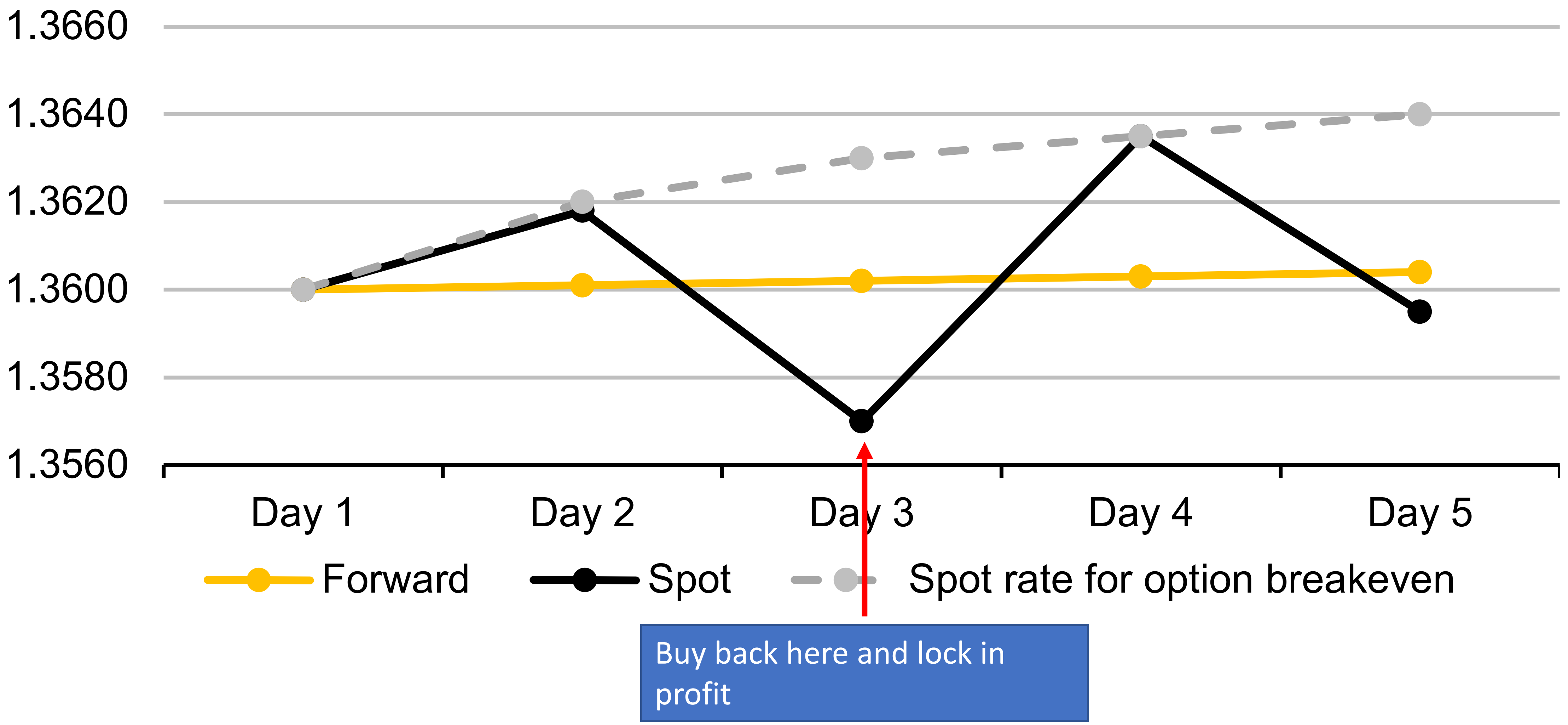
Spot rate < breakeven so seller keeps premium and pays nothing

An example of Delta hedging



Trader who bought option sells (some) underlying to Delta hedge

An example of Delta hedging



Time matters…

- Theta (Θ) measures the sensitivity of option value to the passage of time
- Theta can be estimated by valuing an option and repeating with one less trading day (everything else the same)
- Gives the value of the next trading day or “time decay”

Beat the Theta

High Gamma means money to be made from Gamma adjustments

Low Theta means option costs little in time decay

Low Theta means option costs little in time decay

Ideal is high Gamma:Theta

Back to volatility

- Vega (ν) measures the rate of change of Delta with respect to changes in the implied volatility
- Vega is the *first derivative* of the value of the option with respect to the underlying instrument's volatility

- Vega is important for long-dated options (Gamma:Theta for short-dated)

And finally…

- Rho (ρ) measures the sensitivity to interest rates
- Rho is the *first derivative* of the value of the option with respect to the base or quote currencies (i.e. two exposures)

	Calls	Puts
Value (p or c)	$c = S_0 e^{-r_b T} N(d_1) - K e^{-rT} N(d_2)$	$p = K e^{-rT} N(-d_2) - S_0 e^{-r_b T} N(-d_1)$
Delta Δ	$\Delta = e^{-r_b T} N(d_1)$	$\Delta = e^{-r_b T} [N(d_1) - 1]$
Gamma Γ	$\Gamma = e^{-r_b T} \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$	
Theta Θ	$\Theta = \frac{-S_0 N'(d_1) \sigma e^{-r_b T}}{2\sqrt{T}} + r_b S_0 N(d_1) e^{-r_b T} - rK e^{-rT} N(d_2)$	$\Theta = \frac{-S_0 N'(d_1) \sigma e^{-r_b T}}{2\sqrt{T}} - r_b S_0 N(-d_1) e^{-r_b T} + rK e^{-rT} N(-d_2)$
Vega ν	$S_0 \sqrt{T}\, N'(d_1) \sigma e^{-r_b T}$	
Rho ρ	$-S_0 e^{-rT} N(d_1)$	$S_0 e^{-rT} N(-d_1)$

$$d_1 = \frac{ln(S_0/K) + (r - r_b + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{ln(S_0/K) + (r - r_b - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

S_0 = FX rate at inception,

K = strike rate,

r = interest rate for tenor of the option in the quote currency,

r_b = interest rate for the tenor of the option in the base currency,

T = tenor of the option, and

σ = implied volatility of the option.

N(.) denotes the standard normal ***cumulative*** distribution function.

$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ is the normal distribution, mean 0, sd 1

What didn’t we cover?

- Using the Greeks as trading and hedging tools
- Designing trading strategies
  - Combinations of puts and calls (straddles, strangles, risk reversals etc)
  - Rule-based to make money off a statistical understanding of FX rate and option evolution
- Back-testing strategies using historical data
  - Many pitfalls of historical data series
- More exotic FX options including American, Bermudan etc