

UNIVERSITY PARTNER



4MM013 - Computational Mathematics

Mathematics Assignment-1

Full Marks: 10

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1. State the definition of a function and a composite function. **[2Marks]**

Let f and g be functions defined as follows:

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = \frac{x-3}{x+1}, f(2) = ? \text{ and}$$

$$g: \mathbb{R} \rightarrow \mathbb{R}, g(x) = \frac{1}{x}, x \neq 0$$

Calculate $(f \circ g)(x)$ and $(g \circ f)(x)$.

1. State the definition of a function and a composite function.

⇒ A function is a rule that operates on an input and produce single output from that input.

⇒ A composite function is a function obtained by combining two or more functions, where the output of one function is used as the input of another function. It can be represented as $f(g(x))$ or $g(f(x))$, where f and g are two functions.

Let f and g be functions defined as follows:

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x - 3, f(2) = ? \text{ and}$$

$$g: \mathbb{R} \rightarrow \mathbb{R}, g(x) = \frac{1}{x}, x \neq 0$$

Calculate $(f \circ g)(x)$ and $(g \circ f)(x)$.

$$\text{when } f(x) = x - 3/x + 1$$

$$f(2) = \frac{2-3}{2+1} = \frac{-1}{3}$$

$$(f \circ g)(x) = f(g(x))$$

$$= f(1/x)$$

$$= \frac{1/x - 3}{1/x + 1}$$

$$= \frac{1 - 3x}{1 + x}$$

$$= \frac{1 - 3x}{1 + x} \times \frac{x}{x}$$

$$\therefore (f \circ g)(x) = \frac{1 - 3x}{1 + x}$$

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And,
 $(g \circ f)(x) = g(f(x))$
 $= g\left(\frac{x-3}{x+1}\right)$

$$= \frac{1}{\left(\frac{x-3}{x+1}\right)}$$

$$= \frac{x+1}{x-3}$$

$$\therefore (f \circ g)(x) = \frac{1-3x}{1+x} \text{ and } (g \circ f)(x) = \frac{x+1}{x-3} //$$

2. Solve the following using the inverse matrix method:

[2 Marks]

$$6x - y = 0$$

$$2x - 4y = 1$$

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2. Solve the following using the inverse matrix method:

$$6x - y = 0$$

$$2x - 4y = 1$$

Drawing row and column pictures,

$$\begin{bmatrix} 6 & -1 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Let $AX = B$

$$\text{So, } A = \begin{bmatrix} 6 & -1 \\ 2 & -4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

To calculate inverse of A

$$|A| = \begin{vmatrix} 6 & -1 \\ 2 & -4 \end{vmatrix} = 6(-4) - (-1) \cdot 2 = -24 + 2 = -22$$

Since, $|A| \neq 0$, it is invertible

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj}[A]$$

$$= \frac{-1}{22} \begin{bmatrix} -4 & 1 \\ -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4/22 & -1/22 \\ 2/22 & -6/22 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 2/11 & -1/22 \\ 1/11 & -6/22 \end{bmatrix}$$

Now,

$$X = A^{-1}B$$

$$= \begin{bmatrix} 2/11 & -1/22 \\ 1/11 & -6/22 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} (2/11) \cdot 0 + (-1/22) \cdot 1 \\ (1/11) \cdot 0 + (-6/22) \cdot 1 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -1/22 \\ -6/22 \end{bmatrix}$$

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$$\text{i.e. } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/22 \\ -6/22 \end{bmatrix}$$

So, the values of x and y are $(-1/22)$ and $(-6/22)$ respectively.

3. Calculate the inverse of the following functions:

[2 Marks]

a. $f(x) = \frac{6+x}{7}$

b. $f(x) = \frac{3}{2-x}$

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3. Calculate the inverse of the following functions:-

a) $f(x) = \frac{6+x}{7}$

Solution:-

To find: $f^{-1}(x)$

Let $f(x)$ be y , then

$$y = \frac{6+x}{7}$$

Interchanging the values of x and y

$$x = \frac{6+y}{7}$$

$$\text{or, } 7x = 6+y$$

$$\text{or, } 7x - 6 = y$$

$$\text{or, } y = 7x - 6$$

$$\therefore f^{-1}(x) = 7x - 6$$

So, the inverse of $f(x) = \frac{6+x}{7}$ is $f^{-1}(x) = 7x - 6$.

b) $f(x) = \frac{3}{2-x}$

Solution:-

To find: $f^{-1}(x)$

Let $f(x)$ be y , then

$$y = \frac{3}{2-x}$$

Interchanging the values of x and y .

$$x = \frac{3}{2-y}$$

$$\text{or, } 2x - xy = 3$$

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$$\text{or } 2x - xy = 3$$

$$\text{or } 2x - 3 = xy$$

$$\text{or } \frac{2x - 3}{x} = y$$

$$\text{or } y = 2 - \frac{3}{x}$$

$$\therefore f^{-1}(x) = 2 - \frac{3}{x}$$

So, the inverse of $f(x) = \frac{3}{2-x}$ is $f^{-1}(x) = 2 - \frac{3}{x}$

4. Sketch the

4. Sketch the graph of the following functions:

[2 Marks]

$f(x) = 2(x - 1)^2 + 3$, $-2 < x < 2$ In the interval of $x=0.5$

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4. Sketch the graph of the following gradient functions:

$$f(x) = 2(x-1)^2 + 3, -2 < x < 2 \text{ in the interval of } x = 0.5$$

Solution:-

$$\text{For the function: } f(x) = 2(x-1)^2 + 3$$

$$\text{Values of } x = [-1.5, -1, -0.5, 0, 0.5, 1, 1.5]$$

To calculate $f(x)$

$$f(-1.5) = 2(-1.5-1)^2 + 3 = 2(6.25) + 3 = 12.5 + 3 = 15.5$$

$$f(-1) = 2(-1-1)^2 + 3 = 2(4) + 3 = 8 + 3 = 11$$

$$f(-0.5) = 2(-0.5-1)^2 + 3 = 2(2.25) + 3 = 4.5 + 3 = 7.5$$

$$f(0) = 2(0-1)^2 + 3 = 2(1) + 3 = 5$$

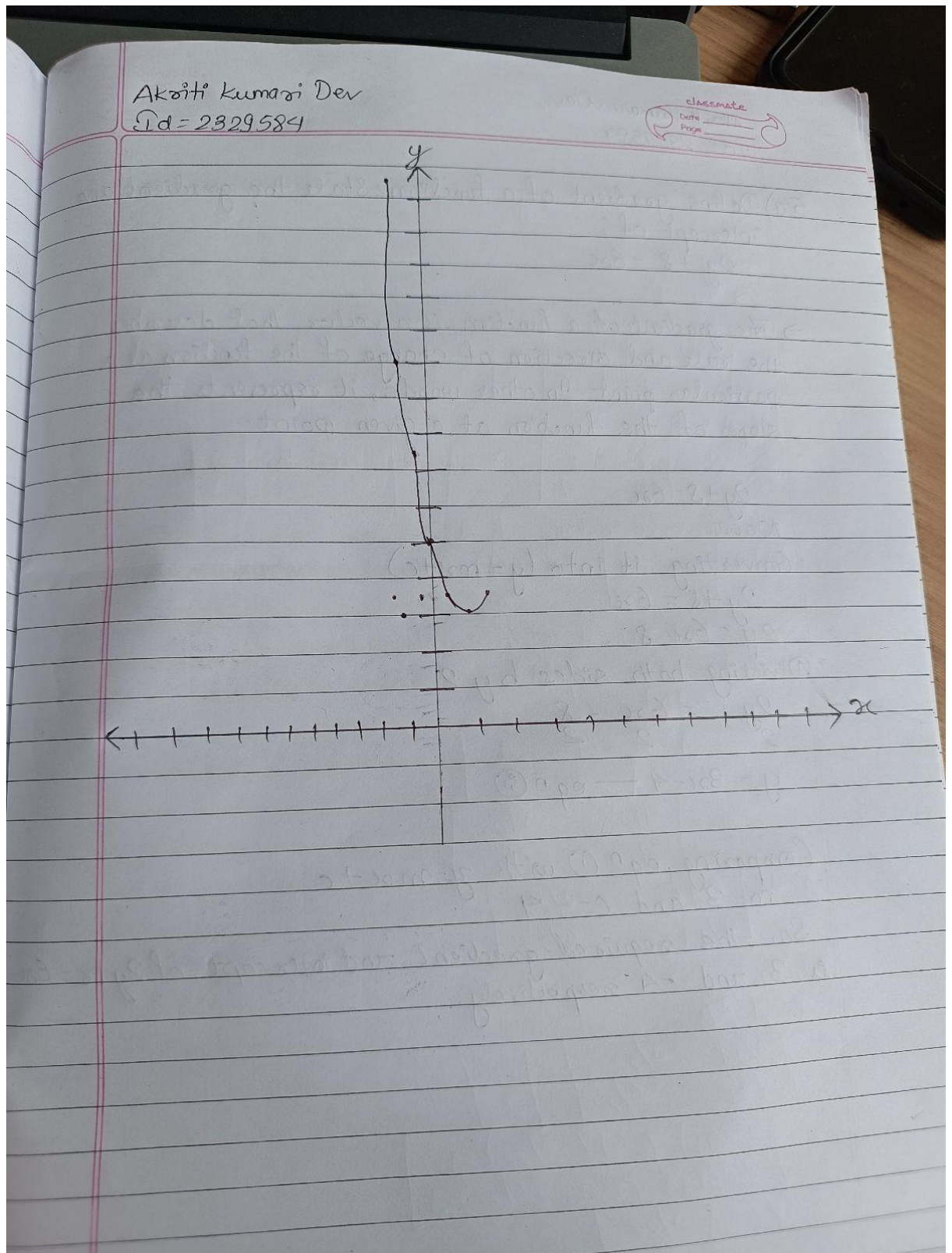
$$f(0.5) = 2(0.5-1)^2 + 3 = 2(0.25) + 3 = 3.5$$

$$f(1) = 2(1-1)^2 + 3 = 0 + 3 = 3$$

$$f(1.5) = 2(1.5-1)^2 + 3 = 2(0.25) + 3 = 3.5$$

Now,

x	-1.5	-1	-0.5	0	0.5	1	1.5
y	15.5	11	7.5	5	3.5	3	3.5



5.

a. Define gradient of a function. State the gradient and intercept of :

$$2y + 8 = 6x$$

[2 Marks]

b. Solve the following equations:

$$x^2 + \frac{17}{6}x + 2 = 0$$

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5a.) Define gradient of a function. State the gradient and intercept of:

$$2y + 8 = 6x$$

⇒ The gradient of a function is a vector that describes the rate and direction of change of the function at a particular point. In other words, it represents the slope of the function at a given point.

$$2y + 8 = 6x$$

Now,

Converting it into $y = mx + c$

$$2y + 8 = 6x$$

$$2y = 6x - 8$$

Dividing both sides by 2

$$\frac{2y}{2} = \frac{6x}{2} - \frac{8}{2}$$

$$y = 3x - 4 \text{ --- eqn (1)}$$

Comparing eqn (1) with $y = mx + c$
 $m = 3$ and $c = -4$

So, the required gradient and intercept of $2y + 8 = 6x$ is 3 and -4 respectively.

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and

5b.) Solve the following equations:-

$$x^2 + \frac{17}{6}x + 2 = 0$$

$$\text{or, } 6x^2 + 17x + 12 = 0 \text{ --- eqn (1)}$$

Comparing eqn (1) with $ax^2 + bx + c = 0$,
we get

$$a = 6, b = 17, c = 12$$

using quadratic equation,

$$\text{or, } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{or, } x = \frac{-17 \pm \sqrt{(17)^2 - 4 \times 6 \times 12}}{2 \times 6}$$

$$\text{or, } x = \frac{-17 \pm \sqrt{289 - 288}}{12}$$

$$\therefore x = \frac{-17 \pm 1}{12}$$

Either,

$$x = \frac{-17 + 1}{12}$$

$$\therefore x = \frac{-4}{3}$$

Or,

$$x = \frac{-17 - 1}{12}$$

$$\therefore x = \frac{-3}{2}$$

So, the value of x is either $\left(\frac{-4}{3}\right)$ or $\left(\frac{-3}{2}\right)$ //

The End

