Roots of Quadratic Equations

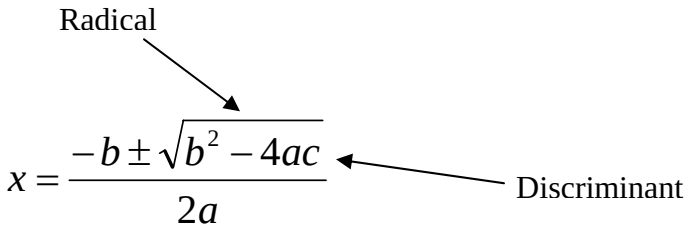
I. Finding Roots of Quadratic Equations

a. The *Standard Form* of a quadratic equation is: $ax^2 + bx + c = 0$.

b. We can use the *Quadratic Formula* to solve equations in standard form:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

c. *Discriminant* – The radical portion of this formula $\sqrt{b^2 - 4ac}$, determines the nature of the roots. This quantity under the radical sign $b^2 - 4ac$, is called the discriminant.



d. Three things may occur regarding the discriminant:

i. If $b^2 - 4ac > 0$

We can take the square root of this positive amount and there will be two different real answers (or roots) to the equation.

ii. If $b^2 - 4ac < 0$

We cannot take the square root of a negative number, so there will be no real roots.

iii. If $b^2 - 4ac = 0$

The amount under the radical is zero and since the square root of zero is zero, we will get only 1 distinct real root.

II. Examples

a. $x^2 - 6x + 9 = 0$

$a = 1 \quad b = -6 \quad c = 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{+6 \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 36}}{2}$$

$$x = \frac{6 \pm \sqrt{0}}{2}$$ ← The Discriminant Equals Zero.

$$x = \frac{6 \pm 0}{2}$$

$x = \frac{6}{2} = 3$ (There is only 1 real root.)

b. $x^2 + 3x + 1 = 0$

$a = 1 \quad b = 3 \quad c = 1$

$$x = \frac{-3 \pm \sqrt{9 - 4(1)(1)}}{2(1)}$$ ← The Discriminant is positive.

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

$x = \frac{-3 + \sqrt{5}}{2}$ and $x = \frac{-3 - \sqrt{5}}{2}$

Since the discriminant is positive (it equals +5) there are two real roots.

c. $x^2 + x + 3 = 0$
$a = 1 \quad b = 1 \quad c = 3$

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(3)}}{2(1)}$$

The Discriminant is negative.

$$x = \frac{-1 \pm \sqrt{-11}}{2}$$

The discriminant is -11. Since we cannot take the square root of a negative number we have no real roots.

III. Practice Problems

By examining the discriminant $= b^2 - 4ac$, determine how many real roots, if any, the following quadratic equations have.

1. $x^2 - 4x + 4 = 0$

2. $x^2 + 4 = 0$

3. $x^2 - 2x + 4 = 0$

4. $x^2 - 4x = 0$

5. $5r^2 - 3r + 2 = 0$

6. $7x^2 - 10x - 5 = 0$

7. $x^2 - 4 = 0$

8. $25t^2 - 10t = -1$

9. $6y^2 - 5y = 21$

10. $2y^2 - 19y = 3$

Answers: Roots of Quadratic Equations

1. 1 real root
2. no real roots
3. no real roots
4. 2 real roots
5. no real roots
6. 2 real roots
7. 2 real roots
8. 1 real root
9. 2 real roots
10. 2 real roots