Math's Assignment :

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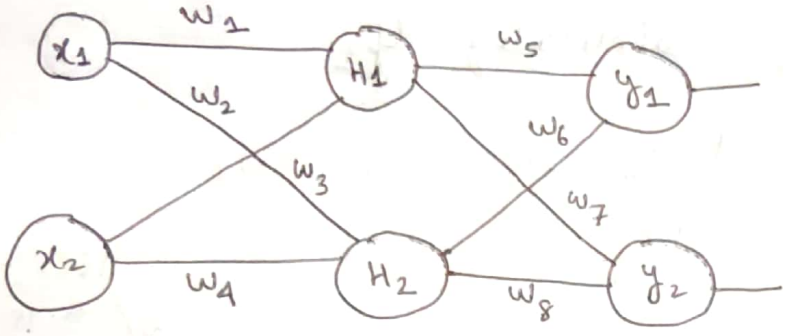
⇒ Back propagation example for 2 iterations.



$$H_1 = x_1 \times w_1 + x_2 \times w_2$$

Activation function is sigmoid for both hidden & output layer $\left\{\frac{1}{1+e^{-x}}\right\}$

$$\text{at } H_1 = \frac{1}{1+e^{-H_1}}$$

Example :-

$x_1 = 0.05$

$x_2 = 0.10$

Target Values

T_1	T_2
0.01	0.99

Initial weights :-

$w_1 = 0.15$
$w_2 = 0.20$
$w_3 = 0.25$
$w_4 = 0.30$
$w_5 = 0.40$
$w_6 = 0.45$
$w_7 = 0.50$
$w_8 = 0.55$

Forward Pass

$$H_1 = x_1 \times w_1 + x_2 \times w_2$$
$$= 0.05 \times 0.15 + 0.10 \times 0.20$$
$$= 0.0275$$

$$H_1 = \frac{1}{1+e^{-0.0275}} = 0.507614213197$$

Now for calculating y_1

$$y_1 = \text{out}\, H_1 \times w_5 + \text{out}\, H_2 \times w_6$$
$$= 0.4 \times 0.507614213197 + 0.596884378 \times 0.45$$

{similarly calculated as H_1}

$$= 1.10590567$$

$$\text{out}\, y_1 = \frac{1}{1+e^{-y_1}} = 0.75136507$$

In the same way ⇒

$$\text{out}\, y_2 = 0.772928465$$

Calculating Total Error

$$E_{Total} = \sum \frac{1}{2}(\text{target} - \text{output})^2$$
$$= \frac{1}{2}(T_1 - \text{out}\, y_1)^2 + \frac{1}{2}(T_2 - \text{out}\, y_2)^2$$
$$= \frac{1}{2}(0.01 - 0.75136507)^2 + \frac{1}{2}(0.99 - 0.772928465)^2$$
$$= 0.274811083 + 0.023560026$$

$$E_{Total} = 0.298371109$$

$$E_1 = \frac{1}{2}(T_1 - out\,y_1)^2$$

$$E_2 = \frac{1}{2}(T_2 - out\,y_2)^2$$

Backward Pass

To update weight.

consider w_5

Error at $w_5 = \frac{\partial E.total}{\partial w_5}$

$$\frac{\partial E\,total}{\partial w_5} = \frac{\partial E_{total}}{\partial out\,y_1} * \frac{\partial out\,y_1}{\partial y_1} * \frac{\partial y_1}{\partial w_5}$$

$$E_{Total} = \frac{1}{2}(T_1 - out\,y_1)^2 + \frac{1}{2}(T_2 - out\,y_2)^2$$

$$\frac{\partial E\,total}{\partial out\,y_1} = 2 * \frac{1}{2}(T_1 - out\,y_1)^2 * -1 + 0$$

$$= -(T_1 - out\,y_1)$$

$$= -(0.01 - 0.75136507)$$

$$\frac{\partial E_{total}}{\partial out\,y_1} = 0.74136507$$

$$out\,y_1 = \frac{1}{1+e^{-y_1}}$$

$$\frac{\partial out\,y_1}{\partial y_1} = out\,y_1(1 - out\,y_1)$$

$$= 0.75136507\,(1 - 0.75136507)$$

$$\frac{\partial out\,y_1}{\partial y_1} = 0.186815602$$

$$\frac{\partial y_1}{\partial w_5} = 1 * out_{H_1} * w_5^{(1-1)} + 0 + 0$$

$$= out_{H_1}$$

$$\frac{\partial y_1}{\partial w_5} = 0.593269992$$

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{y_1}} * \frac{\partial out_{y_1}}{\partial y_1} * \frac{\partial y_1}{\partial w_5}$$

$$= 0.74136507 * 0.186815602 * 0.593269992$$

$$\frac{\partial E_{total}}{\partial w_5} = 0.082167041 \longrightarrow \text{change in } w_5$$

Updating w_5

$$w_5(new) = w_5(old) - \alpha * \frac{\partial E_{total}}{\partial w_5} \quad [\alpha \rightarrow \text{learning Rate}]$$

$$= 0.4 - 0.5 * 0.082167041$$

$$w_5(new) = 0.35891648$$

In the same way

$$w_6 = 0.408666186$$
$$w_7 = 0.511301270$$
$$w_8 = 0.561370121$$

} new updated this.

Now at Hidden layer updating w_1, w_2, w_3, w_4

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{H_1}} \times \frac{\partial out_{H_1}}{\partial H_1} \times \frac{\partial H_1}{\partial w_1}$$

$$\frac{\partial E_{total}}{\partial out_{H_1}} = \frac{\partial E_1}{\partial out_{H_1}} + \frac{\partial E_2}{\partial out_{H_2}}$$

$$\frac{\partial E_1}{\partial out_{H_1}} = \frac{\partial E_1}{\partial y_1} \times \frac{\partial y_1}{\partial out_{H_1}}$$

$$\frac{\partial E_1}{\partial y_1} = \frac{\partial E_1}{\partial out_{y_1}} \times \frac{\partial out_{y_1}}{\partial y_1}$$

$$= 0.74\,136507 \times 0.18\,6815602$$

$$= 0.13849856\,2$$

$$\frac{\partial y_1}{\partial out_{H_1}} = w_5 = 0.40$$

$$\frac{\partial E_1}{\partial out_{H_1}} = 0.138498562 \times 0.40$$

$$= 0.0553\,99425$$

$$\frac{\partial E_2}{\partial out_{H_1}} = -0.019049119$$

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{H_1}} \times \frac{\partial out_{H_1}}{\partial H_1} \times \frac{\partial H_1}{\partial w_1}$$

$= 0.03635 \times 0.241300 \times 0.05$

$$\frac{\partial E_{total}}{\partial w_1} = 0.000438568$$

updating w_1

$$w_1 = w_1 - \alpha \times \frac{\partial E_{total}}{\partial w_1}$$

$$= 0.15 - 0.5 \times 0.000438$$

$$w_1 = 0.149780716$$

In the same way

$w_2 = 0.19956143$

$w_3 = 0.24975114$

$w_4 = 0.29950229$