AKSHAT MEHTA

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Assignment 1

Q1) $P(A) = 0.3$ $\quad P(B) = 0.4$ $\quad P(A \cap B) = 0.2$

a) $P(\text{exactly one occurs}) = P(A) - P(A \cap B) + P(B) - P(A \cap B)$
$= \underline{0.3}$

b) $P(\text{at least one occurs}) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \underline{0.5}$

c) $P(\text{None occur}) = 1 - P(A \cup B) = \underline{0.5}$

Q2) Initially $P(\text{selecting car}) = 1/3$

$P(\text{selecting goat}) = 2/3$

~~If I select car,~~

If I select goat, I should definitely switch. ~~thus~~
~~I should switch 2/3 times.~~
Thus, $P(\text{switching is beneficial}) = P(\text{selecting goat}) = 2/3$

Using conditional probability,

~~Lets select D_1 initially and assume that Monty opens D_2.~~

$D_i \rightarrow$ car behind D_i

We select D_1 initially

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$B \rightarrow$ event Monty opens D_2.

$P(D_1) = 1/3$ $\quad P(D_2|B) = 0$ $\quad$ ~~$P(D_3|B)=$~~ $\quad P(Di) = 1/3$

$P(B|D_1) = 1/2$

$P(B|D_2) = 0$

$P(B|D_3) = 1/2$

$$P(D_3|B) = \frac{P(B|D_3) \times P(D_3)}{P(B)}$$

$$= \frac{P(B|D_3) \times P(D_3)}{P(B|D_1)P(D_1) + P(B|D_2)P(D_2) + P(B|D_3)P(D_3)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2} \times \frac{1}{3} + 0 \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{3}} = \frac{2}{3}$$

$P(D_3|B) = 2/3$

Hence, he should switch.

Q3) 6 balls $\qquad$ 3 balls are red

$6R \rightarrow$ 6 Red present $\qquad 3O \rightarrow$ 3 Red taken out

$$P(6R|3O) = \frac{P(3O|6R) \times P(6R)}{P(3O)}$$

$$= \frac{\frac{4}{4} \times \frac{5}{5} \times \frac{6}{6}}{\frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} + \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} + \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} + \frac{6}{6} \times \frac{5}{5} \times \frac{4}{4}}$$

$$= \frac{20}{35} = \boxed{\frac{4}{7}}$$

Q4) a) $P(X < 0.5) = P(x=0.2) + P(x=0.4)$

$= \underline{0.3}$

b) $P(0.25 < X < 0.75) = P(x=0.4) + P(x=0.5)$

$= \underline{0.4}$

c) $P(X=0.2 | X<0.6) = \dfrac{P(X=0.2) \times \overset{1}{P(X<0.6 | x=0.2)}}{P(X<0.6)}$

$= \dfrac{0.1}{0.1+0.2+0.2} = \dfrac{1}{5} = \underline{\underline{0.2}}$

~~Q5) $\int_{-\infty}^{\infty} F(x)\,dx = 1$~~

~~$\int_{\infty}^{0} F(x)\,dx + \int_{0}^{1} F(x)\,dx + \int_{1}^{2} F(x)\,dx + \int_{2}^{3} F(x)\,dx + \int_{3}^{\infty} F(x)\,dx = 1$~~

Q5) Probability distribution functions are rightward continuous

$\Rightarrow F(3) = F(3^+)$

$\Rightarrow \dfrac{4c^2-9c+6}{4} = 1 \Rightarrow c = 1/4$ or 2

$c \neq 2$ as then $\dfrac{7-6c}{6} < 0 \rightarrow$ not possible

$\underline{F(x) > 0}$

$\Rightarrow \boxed{c = 1/4}$

$(h \to 0)$

$P(1 < X < 2) = F(2-h) - F(1+h) = \frac{11}{12} - \frac{11}{12} = 0$

$P(2 \leq X < 3) = F(3-h) - F(2) = 1 - 1 = 0$

$P(0 < X \leq 1) = F(1) - F(h) = \frac{11}{12} - \frac{2}{3} = \frac{1}{4}$

$P(1 \leq X \leq 2) = F(2) - F(1) = 1 - \frac{11}{12} = \frac{1}{12}$

$P(X \geq 3) = F(\infty) - F(3) = 1 - 1 = 0$

$\dfrac{7-6c}{6} = \dfrac{11}{12}$

$\dfrac{4c^2-9c+6}{4} = 1$

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6) a) $E[X] = \int_0^1 p(x) \cdot x \, dx = \int_0^1 1 \cdot x \, dx = 1/2$

b) $Var[X] = E[X^2] - (E[X])^2$

$E[X^2] = \int_0^1 p(x) \cdot x^2 \cdot dx = \int_0^1 x^2 dx = 1/3$

$Var[X] = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

c) $E[X^2 + Y^2] = E[X^2] + E[Y^2] = 1$ (Linearity of Expectation)

$E[Y^2] = 1 - E[X^2] = 2/3$

$Var[Y] = E[Y^2] - (E[Y])^2 = 5/9$

$= \frac{2}{3} - (E[Y])^2 = \frac{5}{9}$

$E[Y] = \frac{1}{3}$

d) ~~E[X] +~~ $E[X+Y] = E[X] + E[Y] = \frac{1}{2} + \frac{1}{3}$

$= \frac{5}{6}$