

Engineering Mathematics

Quiz 1, Fall 2014/09/29

請詳細列出計算過程，如用到公式，請列出公式的通式。請記得在答案卷上簽名。A, B 卷

A(1), B(3) Please give the name of the following differential equation in terms of
__ 階 __ 次 __ 微分方程式

$$(1) \frac{\partial^3 u(x, y)}{\partial x^3} + x \frac{\partial u(x, y)}{\partial y} = 0$$

$$(2) y' = \sqrt{y} + 5y$$

Sol:

(1) 三階一次偏微分方程式

(2) 一階二次常微分方程式

A(2), B(4) Please indicate which differential equations have unique solution.

(Hint: $\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$)

$$(1) y' = e^{xy^2}, y(0) = 1$$

$$(2) y' = -\sqrt{1-y^2}, y(0) = 1$$

$$(3) y' = \ln\left(\frac{x+y}{x-y}\right), y(2) = 1$$

$$(4) y' = \sin^{-1}\left(\frac{x+y}{x-y}\right), y(0) = 1$$

Sol:

(1) \Rightarrow unique solution

(2) \Rightarrow not unique solution

(3) \Rightarrow unique solution

(4) \Rightarrow not unique solution

So, the differential equation (1) (3) have unique solution

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A(3),B(1) Find the value k so that the given differential equation is exact.

$$(6xy^3 + \cos y)dx + (2kx^2y^2 - x \sin y)dy = 0$$

Sol:

$$k = \frac{9}{2}$$

A(4),B(2) Solve the given differential equation.

$$(y^3 + 3)dx + (3xy^2 + 5)dy = 0$$

Sol:

$$\Rightarrow u(x, y) = xy^3 + 3x + 5y = C$$