小考參考解答

1

(a) A stem-and-leaf plot is shown next.

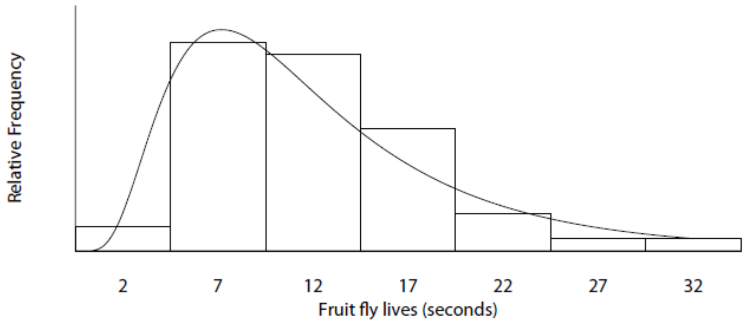
Stem	Leaf	Frequency
0*	34	2
0	56667777777889999	17
1*	0000001223333344	16
1	5566788899	10
2*	034	3
2	7	1
3*	2	1

(b) The relative frequency distribution table is shown next.

Relative Frequency Distribution of Fruit Fly Lives

Class Interval	Class Midpoint	Frequency, f	Relative Frequency
$0-4$	2	2	0.04
$5-9$	7	17	0.34
$10-14$	12	16	0.32
$15-19$	17	10	0.20
$20-24$	22	3	0.06
$25-29$	27	1	0.02
$30-34$	32	1	0.02

(c) A histogram plot is shown next.



2.

(a) $P(A \cap B \cap C) = P(C \mid A \cap B)P(B \mid A)P(A) = (0.20)(0.75)(0.3) = 0.045.$

(b) $P(B' \cap C) = P(A \cap B' \cap C) + P(A' \cap B' \cap C) = P(C \mid A \cap B')P(B' \mid A)P(A) + P(C \mid A' \cap B')P(B' \mid A')P(A') = (0.80)(1-0.75)(0.3) + (0.90)(1-0.20)(1-0.3) = 0.564.$

3.

(a) $P(Z > 20) = \frac{1}{10}\int_{20}^{\infty} e^{-z/10}\, dz = -\, e^{-z/10}\Big|_{20}^{\infty} = e^{-20/10} = 0.1353.$

4.

(a) $g(x) = \int_1^2 \left(\frac{3x-y}{9}\right)\, dy = \frac{3xy-y^2/2}{9}\Big|_1^2 = \frac{x}{3} - \frac{1}{6}$, for $1 < x < 3$, and

$h(y) = \int_1^3 \left(\frac{3x-y}{9}\right)\, dx = \frac{4}{3} - \frac{2}{9}y$, for $1 < y < 2$.

(b) No, since $g(x)h(y) \neq f(x,y)$.

5.

(a)

The variance of a random variable X is

$$\sigma^2 = E(X^2) - \mu^2.$$

: For the discrete case, we can write

$$\begin{aligned}\sigma^2 &= \sum_x (x-\mu)^2 f(x) = \sum_x (x^2 - 2\mu x + \mu^2) f(x) \\ &= \sum_x x^2 f(x) - 2\mu \sum_x x f(x) + \mu^2 \sum_x f(x).\end{aligned}$$

Since $\mu = \sum_x x f(x)$ by definition, and $\sum_x f(x) = 1$ for any discrete probability distribution, it follows that

$$\sigma^2 = \sum_x x^2 f(x) - \mu^2 = E(X^2) - \mu^2.$$

For the continuous case the proof is step by step the same, with summations replaced by integrations.

(b)

The covariance of two random variables X and Y with means μ_X and μ_Y, respectively, is given by

$$\sigma_{XY} = E(XY) - \mu_X \mu_Y.$$

: For the discrete case, we can write

$$\begin{aligned}\sigma_{XY} &= \sum_x \sum_y (x-\mu_X)(y-\mu_Y) f(x,y) \\ &= \sum_x \sum_y xy f(x,y) - \mu_X \sum_x \sum_y y f(x,y) \\ &\quad - \mu_Y \sum_x \sum_y x f(x,y) + \mu_X \mu_Y \sum_x \sum_y f(x,y).\end{aligned}$$

Since

$$\mu_X = \sum_x x f(x,y), \quad \mu_Y = \sum_y y f(x,y), \quad \text{and} \quad \sum_x \sum_y f(x,y) = 1$$

for any joint discrete distribution, it follows that

$$\sigma_{XY} = E(XY) - \mu_X \mu_Y - \mu_Y \mu_X + \mu_X \mu_Y = E(XY) - \mu_X \mu_Y.$$

For the continuous case, the proof is identical with summations replaced by integrals.