Engineering Mathematics

Midterm Exam, Fall 2009

請詳細列出計算過程，如用到公式,請列出公式的通式。請記得在答案卷上簽名。

1. (8%) Please give the names of the following differential equations in terms of ?階?次?性?微分方程式

   (a) $y'(x)=\left(y(x)\right)^{\frac{1}{3}}$ (b) $\frac{\partial^2 u(x,y)}{\partial x^2}+x\cdot\frac{\partial u(x,y)}{\partial y}=0$

   Ans:

   (a) 一階三次非線性常微分方程式(O.D.E)

   (b) 二階一次線性偏微分方程式(P.D.E)

2. (5%) 下列各問題,何者有唯一解？(Please indicate which problems have unique solutions？)

   (1) $y'=-\sqrt{1-y^2},\ y(0)=1$

   (2) $y'=-\sqrt{1-y^2},\ y(0)=0$

   (3) $y'=e^{xy^2},\ y(0)=1$

   (4) $y'=\sqrt{y},\ y(0)=1$

   (5) $y'=\sqrt{y},\ y(0)=0$

   Ans:

   (1) $f(x,y)=-\sqrt{1-y^2}$ (0,1)

   $\frac{\partial f(x,y)}{\partial y}=\frac{1}{\sqrt{1-y^2}}$ (0,1)

   $\Rightarrow$不具唯一解

   (2) (0,0) $\Rightarrow$具唯一解

   (3) $f(x,y)=e^{xy^2}$ (0,1)

   $\frac{\partial f(x,y)}{\partial y}=2xye^{xy^2}$ (0,1)

   $\Rightarrow$具唯一解

(4) $f(x,y)=\sqrt{y}$ $(0,1)$

$$\frac{\partial f(x,y)}{\partial y}=\frac{1}{2\sqrt{y}} \qquad (0,1)$$

$\Rightarrow$具唯一解

(5) $(0,0)$ $\Rightarrow$不具唯一解

$\Rightarrow$(2) (3) (4) 具唯一解

3. (12%) 求通解(Find general solution)

(a) $\left(e^{y}x+6y+5x\right)dy+\left(e^{y}+5y\right)dx=0$

(b) $\left(xy^{2}-\cos x\sin x\right)dx+\left(x^{2}-1\right)ydy=0$

Ans:

(a)

$$\left(e^{y}x+6y+5x\right)dy+\left(e^{y}+5y\right)dx=0$$

N M

$$\frac{\partial \mathrm{N}}{\partial y}=e^{y}+5 \quad \Leftrightarrow \quad \frac{\partial M}{\partial x}=e^{y}+5$$

(相等代表為正合)

$$\mathrm{N}=\frac{\partial u}{\partial y} \qquad\qquad \mathrm{M}=\frac{\partial u}{\partial x}$$

$$\partial u=\left(e^{y}x+6y+5x\right)\partial y \qquad\qquad \partial u=\left(e^{y}+5y\right)\partial x$$

$$u=\int\left(e^{y}x+6y+5x\right)\partial y+g(x) \quad \Leftrightarrow \quad \mathrm{u}=\int\left(e^{y}+5y\right)\partial x+f(y)$$

$$=\mathrm{e}^{\mathrm{y}}x+3y^{2}+5xy+g(x) \qquad\qquad =\mathrm{e}^{\mathrm{y}}x+5xy+f(y)$$

因此 $f(y)=3y^{2},g(x)=0$

$$\Rightarrow \mathrm{u}(\mathrm{x},\mathrm{y})=\mathrm{e}^{\mathrm{y}}x+5xy+3y^{2}=C$$

(b)

$$\left(xy^2 - \cos x \sin x\right)dx + \left(x^2 - 1\right)y\,dy = 0$$

$$\Rightarrow \frac{\partial M}{\partial y} = 2xy = \frac{\partial N}{\partial x} \qquad (\text{正和})$$

$$\Rightarrow \frac{\partial f}{\partial y} = y(x^2 - 1)$$

$$f(x, y) = \frac{1}{2}y^2(x^2 - 1) + h(x)$$

$$\frac{\partial f}{\partial x} = xy^2 + h'(x) = xy^2 - \cos x \sin x$$

$$\Rightarrow h'(x) = \cos x \sin x$$

$$\Rightarrow h(x) = \frac{1}{2}\cos^2 x = -\frac{1}{2}\sin^2 x = \frac{1}{4}\cos 2x$$

$$\Rightarrow u = \frac{1}{2}y^2(x^2 - 1) + \frac{1}{2}\cos^2 x$$

$$= \frac{1}{2}y^2(x^2 - 1) - \frac{1}{2}\sin^2 x$$

$$= \frac{1}{2}y^2(x^2 - 1) + \frac{1}{4}\cos 2x$$

4. (12%) 求通解(Find general solution)

(a) $\left(y\cos x - \sin 2x\right)dx + dy = 0$

(b) $\left(xy + y^2 + 1\right)dx + \left(xy + x^2 + 1\right)dy = 0$

Ans:

(a)

$$\frac{\partial M}{\partial y} = \cos x, \frac{\partial N}{\partial x} = 0$$

$$\frac{0 - \cos x}{-N} = \frac{-\cos x}{-1} = \cos x$$

$$\cos x\,dx = \frac{dI}{I}$$

$$I = e^{\sin x}$$

$$\Rightarrow (y\cos x\,e^{\sin x} - \sin 2x\,e^{\sin x})dx + e^{\sin x}dy = 0$$

$$\frac{\partial u}{\partial x} = y\cos x e^{\sin x} - \sin 2x e^{\sin x}$$

$$\partial u = (y\cos x e^{\sin x} - \sin 2x e^{\sin x})dx$$

$$\int \partial u = \int y\cos x e^{\sin x}dx - \int \sin 2x e^{\sin x}dx$$

$$u = \underset{(1)}{\underset{\wedge}{\int y\cos x e^{\sin x}dx}} - \underset{(2)}{\underset{\wedge}{\int \sin 2x e^{\sin x}dx}}$$

(1) :

$$\int y\cos x e^{\sin x}dx$$
$$= y\int \cos x e^{\sin x}dx$$
$$= ye^{\sin x}$$

(2) :

$$\int \sin 2x e^{\sin x}dx$$
$$= \int 2\sin x\cos x e^{\sin x}dx$$
$$= 2\int te^{t}dt$$
$$= 2(te^{t} - e^{t})$$

(令 t=sinx)

$$u = ye^{\sin x} - 2\sin x e^{\sin x} + 2e^{\sin x} + f(y)$$

$$\frac{\partial u}{\partial y} = e^{\sin x}$$

$$\partial u = e^{\sin x}dy$$

$$\int \partial u = \int e^{\sin x}dy$$

$$u = \int e^{\sin x}dy + g(x)$$

$$= ye^{\sin x} + g(x)$$

$$f(y) = 0, g(x) = -2\sin x e^{\sin x} + 2e^{\sin x}$$

$$\Rightarrow u = ye^{\sin x} - 2\sin x e^{\sin x} + 2e^{\sin x}$$

(b) $\Rightarrow x(ydx+xdy)+y(ydx+xdy)+dx+dy=0$

$$xd(xy)=yd(xy)+d(x+y)=0$$

$$(x+y)d(xy)+d(x+y)=0$$

$$d(xy)+\frac{1}{x+y}d(x+y)=0$$

$$\int d(xy)+\int\frac{1}{x+y}d(x+y)=\int 0$$

$$\Rightarrow u(x,y)=xy+\ln(x+y)=C$$

5. (8%) 試著由正合概念說明線性微分方程 $y'(x)+p(x)y(x)=r(x)\neq 0$ 的通解如下(Prove through the exact differential equation)

$$y=Ce^{-\int p(x)dx}+e^{-\int p(x)dx}\int e^{\int p(x)dx}r(x)dx$$

Ans:

$$y'(x)+p(x)y(x)=r(x)\neq 0$$

$$\Rightarrow \frac{dy}{dx}+p(x)y(x)-r(x)=0$$

$$\big(p(x)y(x)-r(x)\big)dx+dy=0$$

$$\frac{\partial M}{\partial y}=p(x),\frac{\partial N}{\partial x}=0$$

$$\frac{0-p(x)}{-N}=p(x)$$

$$p(x)dx=\frac{dI}{I}$$

$$I=e^{\int p(x)dx}$$

$$e^{\int p(x)dx}\big(p(x)y(x)-r(x)\big)dx+e^{\int p(x)dx}dy=0$$

$$\frac{\partial u}{\partial x}=e^{\int p(x)dx}\big(p(x)y(x)-r(x)\big)$$

$$\partial u=e^{\int p(x)dx}\big(p(x)y(x)-r(x)\big)\partial x$$

(兩邊同積分)

$$u=\int e^{\int p(x)dx}\big(p(x)y(x)-r(x)\big)dx+f(y)$$

令 $t=\int p(x)dx$

$$dt = p(x)dx \;,\quad dx = \frac{dt}{p(x)}$$

$$u = \int e^t y(x)dt - \int e^{\int p(x)dx} r(x)dx + f(y)$$

$$= y(x)e^t - \int e^{\int p(x)dx} r(x)dx + f(y)$$

$$= y(x)e^{\int p(x)dx} - \int e^{\int p(x)dx} r(x)dx + f(y)$$

$$\frac{\partial u}{\partial y} = e^{\int p(x)dx}$$

$$\partial u = e^{\int p(x)dx}\partial y + g(x)$$

(兩邊同積分)

$$u = \int e^{\int p(x)dx} dy + g(x)$$

$$= ye^{\int p(x)dx} + g(x)$$

$$g(x) = -\int e^{\int p(x)dx} r(x)dx \quad , \quad f(y) = 0$$

$$u = ye^{\int p(x)dx} - \int e^{\int p(x)dx} r(x)dx = C \;\#$$

$$\Rightarrow y = Ce^{-\int p(x)dx} + e^{-\int p(x)dx}\int e^{\int p(x)dx} r(x)dx$$

6. (5%)設一齊性微分方程式的特性方程式的根,分別為

$\lambda_{1\sim16} = (-1\pm5i), (-1\pm5i), (-1\pm5i), -3\pm2i, -2\pm3i, 4, 4, 4, 1, 2, 3$

則其通解為何?(Find general solution)

Ans:

$$\begin{aligned} y = {} & C_1e^x + C_2e^{2x} + C_3e^{3x} + C_4e^{4x} + C_5xe^{4x} \\ & + C_6x^2e^{4x} + e^{-2x}(C_7\cos 3x + C_8\sin 3x) \\ & + e^{-3x}(C_9\cos 2x + C_{10}\sin 2x) \\ & + e^{-x}(C_{11}\cos 5x + C_{12}\sin 5x) \\ & + xe^{-x}(C_{13}\cos 5x + C_{14}\sin 5x) \\ & + x^2e^{-x}(C_{15}\cos 5x + C_{16}\sin 5x) \end{aligned}$$

7. (14%)求通解(Find General Solution)

(a) $y''' + 6y'' + 11y' + 6y = e^x$

(b) $y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$

Ans:

(a)

$$y_h = C_1 e^{-x} + C_2 e^{-2x} + C_3 e^{-3x}$$
$$y_p = e^{-2x}\int e^{3x}(e^{-2x}\int e^{2x}(e^{-x}\int e^x e^x dx)dx)dx$$
$$= \frac{1}{24}e^x$$
$$\Rightarrow y = C_1 e^{-x} + C_2 e^{-2x} + C_3 e^{-3x} + \frac{1}{24}e^x$$

(b)

$$y'' - 2y' - 3y = 0$$
$$\Rightarrow y_c = c_1 e^{-x} + c_2 e^{3x}$$
$$y_p = y_{p1} + y_{p2}$$
$$y_{p1} = Ax + B, y_{p2} = Cxe^{2x} + Ee^{2x}$$
$$y_p = Ax + B + Cxe^{2x} + Ee^{2x}$$
$$y_p'' - 2y_p' - 3y_p = -3Ax - 2A - 3B - 3Cxe^{2x} + (2C - 3E)e^{2x} = 4x - 5 + 6xe^{2x}$$
$$-3A = 4, -2A - 3B = -5, -3C = 6, 2C - 3E = 0$$
$$\Rightarrow y_p = \frac{-4}{3}x + \frac{23}{9} - 2xe^{2x} - \frac{4}{3}e^{2x}$$
$$y = y_c + y_p = c_1 e^{-x} + c_2 e^{3x} - \frac{4}{3}x + \frac{23}{9} - \left(2x + \frac{4}{3}\right)e^{2x}$$

8. (10%) $6y'' + 54y = 2\cos 3x + 3\sin 3x$
分別用微分運算子法(Differential Operator)及變數變換法(Variation of Variable)求出 $y = y_h + y_p$

Ans:

[微分運算子法]

$$6y'' + 54y = 2\cos 3x + 3\sin 3x$$
$$\Rightarrow y'' + 9y = \frac{1}{3}\cos 3x + \frac{1}{2}\sin 3x$$
$$\lambda^2 + 9 = 0 \quad \lambda = \pm 3i$$
$$y_h = C_1 \cos 3x + C_2 \sin 3x$$

$$y_p = y_{p1} + y_{p2}$$

$$\Rightarrow y_{p1} = \frac{1}{3} \times \frac{1}{D^2 + 3^2} \cos 3x$$

$$= \frac{1}{3} \lim_{\Delta \to \infty} \frac{1}{-(3+\Delta)^2 + 3^2} \cos(3+\Delta)x$$

$$= \frac{1}{3} \lim_{\Delta \to \infty} \frac{1}{-2 \times 3 \times \Delta - \Delta^2} \cos(3+\Delta)x$$

$$\left(\begin{array}{l} \cos t \text{ 於 } t = ax \text{ 之 Taylor展開} \\ \cos t = \cos ax - \sin ax(t-ax) - \frac{1}{2!} \cos ax(t-ax)^2 + \frac{1}{3!} \sin ax(t-ax)^3 + \cdots \end{array} \right)$$

$$\Rightarrow y_{p1} = \frac{1}{3} \lim_{\Delta \to \infty} \frac{1}{-\Delta(2 \times 3 + \Delta)} \left[\cos 3x - \Delta x \sin 3x - \frac{1}{2!} (\Delta x)^2 \cos 3x + \frac{1}{3!} (\Delta x)^3 \sin 3x + \cdots \right]$$

(因為 y_h 已含 $\cos 3x$, 故可忽略)

$$= \frac{1}{3} \lim_{\Delta \to \infty} \frac{1}{-\Delta(2 \times 3 + \Delta)} \left[-x \sin 3x - \frac{1}{2!} \Delta x^2 \cos 3x + \frac{1}{3!} \Delta^2 x^3 \sin 3x + \cdots \right]$$

$$= \frac{1}{3} \times \frac{1}{-2 \times 3} x \sin 3x$$

$$= \frac{1}{18} x \sin 3x$$

同理可證

$$\Rightarrow y_{p2} = \frac{1}{2} \times \frac{1}{-2 \times 3} \cos 3x = -\frac{1}{12} x \cos 3x$$

$$\Rightarrow y = y_h + y_p = C_1 \cos 3x + C_2 \sin 3x + \frac{1}{18} x \sin 3x - \frac{1}{12} x \cos 3x$$

[變數變換法]

$$r = \frac{1}{3} \cos 3x + \frac{1}{2} \sin 3x$$

令 $\lambda^2 + 9 = 0, \lambda = \pm 3i$

令 $y_1 = \cos 3x, y_2 = \sin 3x$

$$y_h = C_1 \cos 3x + C_2 \sin 3x$$

$$w(y_1, y_2) = \begin{vmatrix} \cos 3x & \sin 3x \\ -3 \sin 3x & 3 \cos 3x \end{vmatrix} = 3 \cos^2 3x + 3 \sin^2 3x = 3$$

$$y_p = y_1 \int \frac{-r y_2}{w} dx + y_2 \int \frac{r y_1}{w} dx$$

$$= \cos 3x \int \frac{-(\frac{1}{3} \cos 3x + \frac{1}{2} \sin 3x)(\sin 3x)}{3} dx + \sin 3x \int \frac{(\frac{1}{3} \cos 3x + \frac{1}{2} \sin 3x)(\cos 3x)}{3} dx$$

$$= \frac{-1}{9} \cos 3x \int \cos 3x \sin 3x dx - \frac{1}{6} \cos 3x \int \sin^2 3x dx + \frac{1}{9} \sin 3x \int \cos^2 3x dx + \frac{1}{6} \sin 3x \int \sin 3x \cos 3x dx$$

(請參考附註)

$$= \frac{-1}{9}\cos 3x(\frac{1}{6}\sin^2 3x) - \frac{1}{6}\cos 3x(\frac{1}{2}x - \frac{\sin 6x}{12}) + \frac{1}{9}\sin 3x(\frac{1}{2}x + \frac{\sin 6x}{12}) + \frac{1}{6}\sin 3x(\frac{-1}{6}\cos^2 3x)$$

$$= \frac{-1}{54}\cos 3x\sin^2 3x - \frac{1}{12}x\cos 3x + \frac{1}{72}\cos 3x\sin 6x + \frac{1}{18}x\sin 3x + \frac{1}{108}\sin 3x\sin 6x - \frac{1}{36}\sin 3x\cos^2 3x$$

$$= \underset{(1)}{\frac{-1}{54}\cos 3x\sin^2 3x} - \frac{1}{12}x\cos 3x + \underset{(2)}{\frac{1}{36}\cos^2 3x\sin 3x} + \frac{1}{18}x\sin 3x + \underset{(1)}{\frac{1}{54}\sin^2 3x\cos 3x} - \underset{(2)}{\frac{1}{36}\sin 3x\cos^2 3x}$$

$$= -\frac{1}{12}x\cos 3x + \frac{1}{18}x\sin 3x$$

$$\Rightarrow y = y_h + y_p = C_1\cos 3x + C_2\sin 3x + \frac{1}{18}x\sin 3x - \frac{1}{12}x\cos 3x$$

附註：

$$\int \cos 3x\sin 3x dx \quad 令 y = \sin 3x, dx = \frac{dy}{3\cos 3x}$$

$$= \int (\cos 3x) y \frac{dy}{3\cos 3x}$$

$$= \frac{1}{3}\int y dy = \frac{-1}{6}\cos^2 3x$$

$$若令 y = \cos 3x, 則 \int \cos 3x\sin 3x dx = \frac{1}{6}\sin^2 3x$$

$$\int \sin^2 3x dx = \int \frac{1-\cos 6x}{2}dx = \frac{1}{2}x - \frac{\sin 6x}{12} \qquad \int \cos^2 3x dx = \int \frac{1+\cos 6x}{2}dx = \frac{1}{2}x + \frac{\sin 6x}{12}$$

$$\sin 6x = 3\sin(3x)\cos(3x)$$

9. (10%) 求通解(Find general solution) $\begin{cases} (a) y'' + 3y' + 2y = \cos x + x \\ (b) y'' + 8y' + 16y = 3e^{-4x} \end{cases}$

Ans:

(a)

$$\lambda^2 + 3\lambda + 2 = 0$$

$$\lambda = -1, -2$$

$$y''_{p1} + 3y'_{p1} + 2y_{p1} = r_1(x) = \cos x$$

$$y''_{p2} + 3y'_{p2} + 2y_{p2} = r_2(x) = x$$

$$(y_{p1} + y_{p2})'' + 3(y_{p1} + y_{p2})' + 2(y_{p1} + y_{p2}) = r_1(x) + r_2(x)$$

$$y_p = y_{p1} + y_{p2}$$

$$y''_{p1} + 3y'_{p1} + 2y_{p1} = \cos x$$

$$(D^2 + 3D + 2) y_{p1}(x) = \cos x$$

$$y_{p1}(x) = \frac{\cos x}{D^2 + 3D + 2} \qquad (a = 1)$$

$$L(D^2)\cos ax = L(-a^2)\cos ax$$

$$y_{p1}(x) = \frac{\cos x}{-1+3D+2} = \frac{\cos x}{3D+1}$$

$$= \frac{1-3D}{(1-3D)(1+3D)}\cos x$$

$$= \frac{1-3D}{1-9D^2}\cos x$$

$$= \frac{1-3D}{1-9(-1)}\cos x = \frac{1}{10}(1-3D)\cos x$$

$$= \frac{1}{10}\cos x + \frac{3}{10}\sin x$$

$$y''_{p2} + 3y'_{p2} + 2y_{p2} = x$$

$$(D^2+3D+2)y_{p2}(x) = x$$

$$y_{p2}(x) = \frac{x}{D^2+3D+2}$$

$$= \frac{x}{2\left(1+\dfrac{D^2+3D}{2}\right)}$$

$$= \frac{1}{2}\left(1 - \frac{D^2+3D}{2} + \left(\frac{D^2+3D}{2}\right)^2 - \cdots\right)x$$

$$= \frac{1}{2}\left(x - \frac{3}{2}\right) = \frac{1}{2}x - \frac{3}{4}$$

$$y_p = y_{p1} + y_{p2} = \frac{1}{10}\cos x + \frac{3}{10}\sin x + \frac{1}{2}x - \frac{3}{4}$$

$$\Rightarrow y = y_h + y_p = C_1e^{-x} + C_2e^{-2x} + \frac{1}{10}\cos x + \frac{3}{10}\sin x + \frac{1}{2}x - \frac{3}{4}$$

(b)

$$\lambda = -4, -4$$

$$y_h = C_1e^{-4x} + C_2xe^{-4x}$$

$$(D+4)(D+4)y_p = 3e^{-4x}$$

$$I_1 = e^{4x}, I_2 = e^{4x}$$

$$y_p = I_2^{-1}\int I_2I_1^{-1}\int I_1 r\,dxdx$$

$$y_p = e^{-4x}\int e^{4x}e^{-4x}\int e^{4x}3e^{-4x}dxdx$$

$$= e^{-4x}\int 3x\,dx$$

$$= \frac{3}{2}x^2e^{-4x}$$

$$\Rightarrow y = y_h + y_p = C_1e^{-4x} + C_2xe^{-4x} + \frac{3}{2}x^2e^{-4x}$$

10. (16%) $x^2 y'' - xy' - 3y = 4x$

用 4 個方法求出 $y = y_h + y_p$

(未定係數法(Undermined Coefficient)，降階法(reduction of Order))，微分運算子法(Differential Operator)及變數變換法(Variation of Variable))

Ans:

$$y = y_h + y_p$$

$$y_h : x^2 y_h'' - xy_h' - 3y_h = 0$$

$$令 x = e^t, \wp = \frac{d}{dt}$$

$$\wp(\wp - 1)y - \wp y - 3y = 0$$

$$(\wp^2 - 2\wp - 3)y = 0$$

$$\lambda^2 - 2\lambda - 3 = 0$$

$$\lambda = 3, -1$$

$$y_h = C_1 e^{3t} + C_2 e^{-t}$$

$$= C_1 x^3 + C_2 x^{-1}$$

$$y_p : x^2 y_p'' - xy_p' - 3y_p = 4x$$

[未定係數法]

$$y_p'' - 2y_p' - 3y_p = 4e^t$$

$$y_p = Ae^t$$

$$y_p' = Ae^t$$

$$y_p'' = Ae^t$$

$$Ae^t - 2Ae^t - 3Ae^t = 4e^t$$

$$\Rightarrow -4Ae^t = 4e^t$$

$$\Rightarrow A = -1$$

$$y_p = -e^t = -x$$

[降階法]

$$(\wp^2 - 2\wp - 3)y_p = 4e^t$$

$$(\wp - 3)(\wp + 1)y_p = 4e^t, z(t) = (\wp + 1)y_p$$

$$z'(t) - 3z(t) = 4e^t$$

$$z_0(t) = I_1^{-1}\int I_1 r dt$$

$$I_1 = e^{-3t}, r = 4e^t$$

$$(\wp + 1)y_p = z_p = I_1^{-1}\int I_1 r dt$$

$$y_p' + y_p = I_1^{-1}\int I_1 r dt$$

$$y_p = CI_2^{-1} + I_2^{-1}\int I_2 r' dt$$

$$I_2 = e^t$$

$$r' = I_1^{-1}\int I_1 r dt$$

$$y_p = I_2^{-1}\int I_2 I_1^{-1}\int I_1 r dt dt$$

$$= e^{-t}\int e^t e^{3t}\int e^{-3t} 4e^t dt dt$$

$$= e^{-t}\int -2e^{2t} dt$$

$$= e^{-t}(-1)e^{2t} = -e^t = -x$$

[微分運算子法]

$$(\wp^2 - 2\wp - 3) = 4e^t$$

$$y_p = \frac{1}{\wp^2 - 2\wp - 3} * 4e^t$$

$$= \frac{1}{1-2-3}4e^t = -e^t = -x$$

[變數變換法]

$$y_1 = e^{3t}, y_2 = e^{-t}$$

$$w(y_1, y_2) = \begin{vmatrix} e^{3t} & e^{-t} \\ 3e^{3t} & -e^{-t} \end{vmatrix} = -4e^{2t}$$

$$y_p = e^{3t}\int \frac{-4e^t * e^{-t}}{-4e^{2t}} dt + e^{-t}\int \frac{e^{3t} * 4e^t}{-4e^{2t}} dt$$

$$= e^{3t}\int e^{-2t} dt + e^{-t}\int -e^{2t} dt$$

$$= e^{3t}(\frac{-1}{2}e^{-2t}) + e^{-t}(-\frac{1}{2}e^{2t}) = \frac{-1}{2}e^t - \frac{1}{2}e^t = -e^t = -x$$

$$\Rightarrow y = y_h + y_p = C_1 e^{3t} + C_2 e^{-t} - x$$