NCKU CSIE Discrete Mathematics (2015 Spring) Midterm I *(total 110 pts)*

1. (*20 pts*) For each of the following statements, **determine** and **explain** whether it is correct or not.
   (1). Suppose A={1, 2, 3, 4, 5}. Two of the following statements are false: $(a)\{\{3\}\} \subseteq P(A), (b)\emptyset \subseteq A, (c)\{\emptyset\} \subseteq P(A), (d)\emptyset \subseteq P(A), (e)\{2,4\} \in AXA$
   (2). $\{\emptyset, \{a\}, \{\emptyset, a\}\}$ is the power set of some set.
   (3). $2\binom{n}{0} + \binom{n}{1} + 2\binom{n}{2} + \binom{n}{3} + 2\binom{n}{4} + \binom{n}{5} + \cdots + 2\binom{n}{n-2} + \binom{n}{n-1} + 2\binom{n}{n} = 2^{n-1} + 2^n$
   (4). $\neg(p \leftrightarrow q) \Leftrightarrow (p \wedge q) \vee (\neg p \wedge \neg q)$.
   (5). $f: \boldsymbol{R} \to \boldsymbol{R^2}, f(x) = (2x+1, x^2)$ is an one-to-one function.

2. (*15:10,5 pts*) Solve the equation $x_1+x_2+x_3+x_4 < 9$. (a) Find the integer solutions where x_1, $x_2 > 0, x_3 > 2, x_4 > -2$. (b) in (a), if $x_1, x_2, x_3 \in N, x_4 \in Z$.

3. *(15 pts)* Let A = {1, 2, 3, 4, 5, 7, 8, 10, 11, 14, 17, 18}. (a) How many 6-element subsets of A contain four even integers and two odd integers? (b) How many 5-element subsets of A that has the smallest element less than 4? (c) How many binary relations on A? (d) How many functions *f*: A→A? (e) in (d), how many one-to-one functions?

4. *(10 pts)* If a, b are relatively prime and a > b, prove that gcd(a-b, a+b) = 1 or 2. [Hint: if w = gcd(x, y), w|px+qy for all p, q $\in Z$]

5. (*15 pts*) Frances spends $6.20 on candy for prizes in a contest. If a 10-ounce box of this candy costs $.50 and a 3-ounce box costs $.20, how many boxes of each size did she purchase?

6. *(15 pts)* Define the connective "Nor" by $(p \downarrow q) \Leftrightarrow \neg(p \vee q)$, for any statements *p*, *q*. Represent the following using only this connective. (a) $\neg p$ (b) $p \wedge q$, (c) $p \rightarrow q$.

7. (*10:2,2,2,4 pts*) For the complete expansion of $(2x - y + 3z^{-1} + 1)^5$, determine the following value (a) the coefficient of xyz^{-2} (b) the number of the distinct terms (c) the sum of all coefficients, and (d) if we change the constant term '1' to '$1+x^{-1}$',what's the coefficient of xyz^{-1}.

8. *(10 pts)* Use a combinatorial argument to show that $\binom{3n}{3} = 6n\binom{n}{2} + 3\binom{n}{3} + n^3$