QUIZ 4

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1. Find the limit of $\lim_{x\to 0^+} \sin(x)^{\tan(x)}$.

Answer: Since $\sin(x)^{\tan(x)} \xrightarrow{x\to 0^+} 0^0$ is in indeterminate form, we will try L'Hopital's rule. Since e^x is continuous, we have

$$\lim_{x\to 0^+} e^{\ln(\sin(x))^{\tan(x)}} = \lim_{x\to 0^+} e^{\tan(x)\ln(\sin(x))} = e^{\lim_{x\to 0^+} \frac{\ln(\sin(x))}{\cot x}}.$$

To compute $\lim_{x\to 0^+} \dfrac{\ln(\sin(x))}{\cot x}$, we check that

(1) Both $\ln(\sin(x))$ and $\cot(x)$ are differentiable on $(0, \epsilon)$ for some $\epsilon > 0$;
(2) $(\cot(x))' - \csc^2 x \not 0$ on $(0, \epsilon)$ if ϵ is very small;
(3) the limit

$$\lim_{x\to 0^+} \frac{(\ln(\sin(x)))'}{(\cot x)'} = \lim_{x\to 0^+} \frac{\cos(x)/\sin(x)}{-\csc^2 x} = \lim_{x\to 0^+} -\cos(x)\sin(x) = 0.$$

Hence we can apply L'Hospital Rule and

$$\lim_{x\to 0^+} \frac{\ln(\sin(x))}{\cot x} = \lim_{x\to 0^+} \frac{\frac{\cos(x)}{\sin(x)}}{-\csc^2 x} = 0,$$

which then implies that

$$\lim_{x\to 0^+} e^{\ln(\sin(x))^{\tan(x)}} = e^{\lim_{x\to 0^+} \frac{\ln(\sin(x))}{\cot x}} = e^0 = 1.$$

Date: December 1st.

2. Sketch the graph of $f(x) = xe^x$ and give answers to the following items:

(1) Domain of f: $(-\infty, \infty)$

(2) The $x-$ and $y-$intercepts: 0, 0

(3) Horizontal and vertical asymptotes: $y = 0$ is the only horizontal asymptote.

(4) Domain of increasing and decreasing: Increasing on $(-1, \infty)$ and decreasing on $(-\infty, 1)$

(5) Local maximum and minimum: Only one local minimum at $x = -1$

(6) Domain of concavity: CD on $(-\infty, -2)$, CU on $(-2, \infty)$

(7) Inflection point: $(-2, -2/e^2)$

(8) Absolute maximum and minimum: Only absolute minimum at $x = -1$, $f(-1) = -1/e$.

Answer : See Section 4.5 of the textbook.

$f'(x) = (x + 1)e^x$, $f''(x) = (x + 2)e^2$