Discrete Mathematics (2015 Spring) Final

1. **(5, 5 points)** Determine the coefficient of (a) y^2x^{-1} in the complete expansion of $(2x - y + 3x^{-1} + 1)^5$. (b) x^{10} in $(x^3-5x)/(1-2x)^3$.

2. **(10 points)** Define the connective "Nand" by $(p \uparrow q) \Leftrightarrow \neg(p \wedge q)$, for any statements p, q. Represent the following using only this connective. (a) $\neg p$ (b) $p \to q$.

3. **(5 points)** Use a combinatorial argument to show that $\binom{2n}{2} = 2\binom{n}{2} + n^2$

4. **(15 points)** (a) How many nine-digit sequences have each of digits 1, 3, 5 appearing at least once? (b) How many derangements of 1, 2, 3, 4, 5, 6, 7 start with 1, 2, 3 in some order? (c) In how many ways can we arrange the integers 1, 2, 3, 4, 5, 6 in a line so that there are no occurrences of the patterns 12, 23, 34, 45, 56, 61?

5. **(5, 5 points)** (a) In how many ways can 30030 (2*3*5*7*11*13) be factored into 2, 4, and 6 factors, each greater than 1 and the order of the factors is relevant? (b) In (a), if no factor is prime? (exhaustively list all answers is not allowed.)

6. **(4,3,3 points)** Let A = $\{a, b, c, d, e, f\}$, determine the number of relations on A that are (a) antisymmetric and do not contain (a, b), (c, d) and contain (e, f) (b) reflexive and symmetric but not transitive, (c) equivalence relations where $a \in [b], c \in [d]$.

7. **(3,3,4 points)** Find (a) the generating function for the number of solutions of 2w + 3x + 5y + 7z = n, $0 \le w$, $2 \le x, y$, $3 \le z \le 6$, (b) the exponential generating function for the number of ways to arrange n letters, $n \geqq 0$, selected from the word "ISOMORPHISM" and the arrangement must contain at least one S and one I, (c) the generating function for the sequence 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, … .

8. **(2, 5, 3 points)** Let A={a, b, c, d, e}, B={1, 2, 3, 4, 5, 6}. (a) How many one-to-one functions are there from A to B and f (a)$\neq$ 1? (b) How many one-to-one functions from A to B where f (a)$\neq 2, 3$, f (b)$\neq 1, 3$, f (c)$\neq 4$, and f (d)$\neq 2$ (c) How many onto functions from B to A satisfying f(1)=a?

9. **(5, 5 points)** Please determine how many positive integer solutions for $x_1+x_2+x_3+x_4 < n$, where (a) n=8, x_1, $x_2 > 0$, $x_3 > 2$. (b) n=10, $1 \le x_1 \le 4$, $0 \le x_2 \le 5$. (exhaustively list all answers is not allowed.)

10. **(3,3,4 points)** (a) For $n \geqq 1$, A={00, 1}, let a_n count the number of strings in A^* of length n. Find the recurrence relation for a_n. (b) In (a), if A={0, 01, 011, 0111, 1111}. (c) Find a recurrence relation for the number of **4**-ary {0,1,2,3} sequences (e.g., 021, 0113) of length n that have no consecutive 0's.

11. **(6 points)** (a) more small quizzes is good for you? (b) a roll call at the class is good for you? (c) Please list 2 examples/methods/strategies to improve your (or others') learning motivation/performance.

(Stirling number of the second kind: S(4, 2)=7, S(4, 3)=6, S(5, 2)=15, S(5, 3)=25, S(5, 4)=10, S(6, 2)=31, S(6, 3)=90, S(6, 4)=65)