Exam: General Physics (II)
Department of Computer Science & Information Engineering
Exam Time : 2017 Jun 19 18:00-20:55

Student ID No.: ______________
Name: ______________

Notice:

(1) Derivation process is required for both Problem 1 and 2.

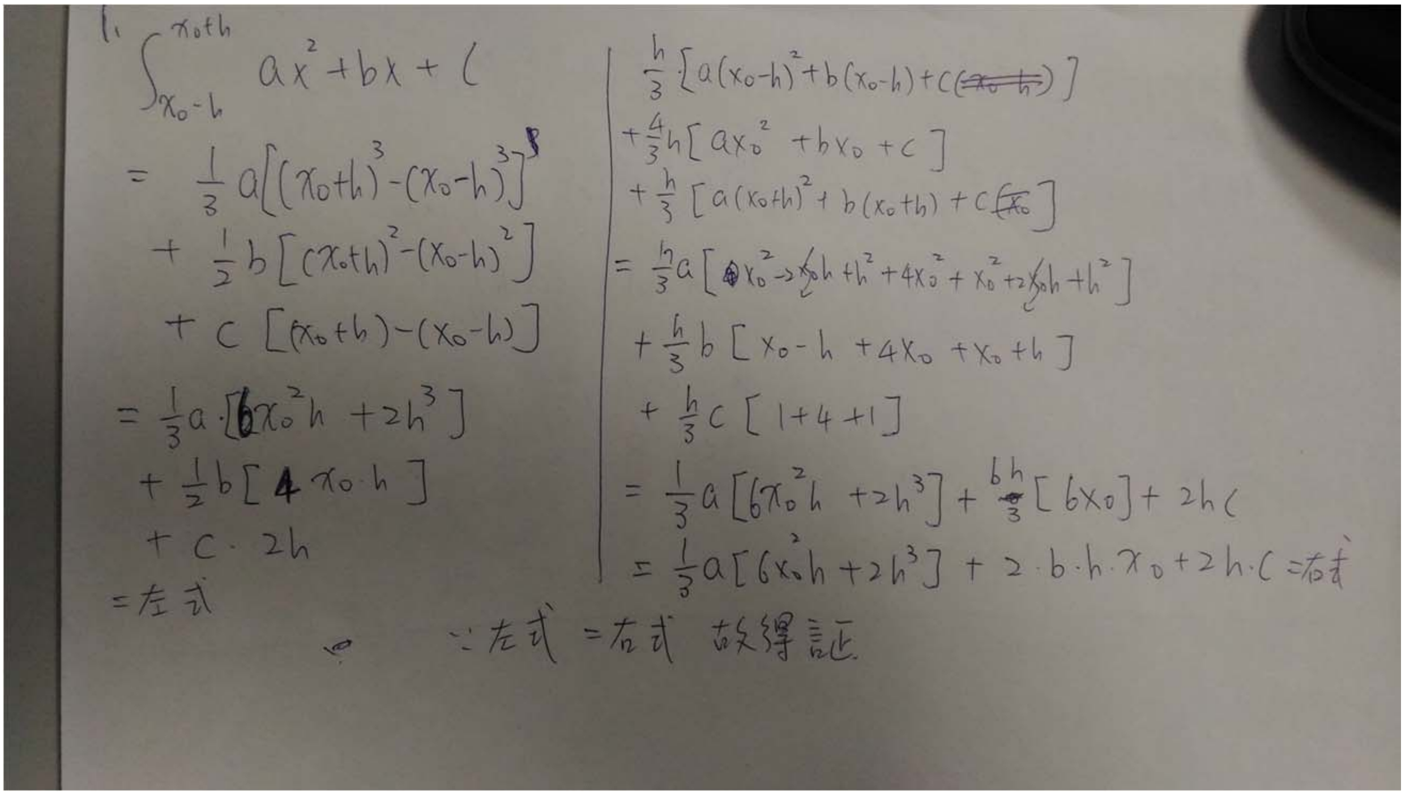
(2) Please follow the naming rule for the m-files.

(3) Print out each answer with designated precision on the command window.

(4) Please also write the DEDUCTIVE PROCESS, the ANSWERS with proper precision and unit, and the METHODS/PARAMETERS for numerical solutions. (e.g. the derivative of f'(0) = xx.xxx with $\Delta t = 0.1$ using forward difference)

1. [15%] Prove that if $f(x) = ax^2 + bx + c$,

$$\int_{x_0-h}^{x_0+h} f(x) = \frac{h}{3} f(x_0 - h) + \frac{4h}{3} f(x_0) + \frac{h}{3} f(x_0 + h)$$



1. $\int_{x_0-h}^{x_0+h} ax^2+bx+c$

$= \frac{1}{3}a[(x_0+h)^3-(x_0-h)^3]$

$+ \frac{1}{2}b[(x_0+h)^2-(x_0-h)^2]$

$+ c\,[(x_0+h)-(x_0-h)]$

$= \frac{1}{3}a\cdot[6x_0^2h+2h^3]$

$+ \frac{1}{2}b[4x_0\cdot h]$

$+ c\cdot 2h$

= 左式

$\frac{h}{3}[a(x_0-h)^2+b(x_0-h)+c(\cancel{x_0-h})]$

$+\frac{4}{3}h[ax_0^2+bx_0+c]$

$+\frac{h}{3}[a(x_0+h)^2+b(x_0+h)+c(\cancel{x_0})]$

$= \frac{h}{3}a[x_0^2-2x_0h+h^2+4x_0^2+x_0^2+2x_0h+h^2]$

$+\frac{h}{3}b[x_0-h+4x_0+x_0+h]$

$+\frac{h}{3}c[1+4+1]$

$= \frac{1}{3}a[6x_0^2h+2h^3]+\frac{bh}{3}[6x_0]+2hc$

$= \frac{1}{3}a[6x_0^2h+2h^3]+2\cdot b\cdot h\cdot x_0+2h\cdot c$ =右式

∵左式=右式 故得証

2. [15%] If $\frac{d^2}{dx^2} f(x) = A\,f(x-h) + Bf(x) + Cf(x+h) + O(h^2)$, find the coefficients A, B and C.

2.

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \frac{h^4}{4!} f^{(4)}(x) + \cdots,$$

$$f(x) = f(x)$$

$$f(x-h) = f(x) - h f'(x) + \frac{h^2}{2!} f''(x) - \frac{h^3}{3!} f^{(3)}(x) + \frac{h^4}{4!} f^{(4)}(x)$$

~~A f(x+~~ T. Approximate $f''(x)$ by $A f(x-h) + B f(x) + C f(x+h)$

$$\Rightarrow \begin{cases} A + B + C = 0 & \cdots f(x) \\ -A + C = 0 & \cdots f'(x) \\ \frac{h^2}{2} \cdot A + \frac{h^2}{2} \cdot C = 1 & \cdots f''(x) \\ -A + C = 0 & \cdots f^{(3)}(x) \end{cases}$$

$$\Rightarrow A = C = \frac{1}{h^2} \quad \text{then } B = -\frac{2}{h^2}$$

$$\Rightarrow A = \frac{1}{h^2},\ B = \frac{-2}{h^2},\ C = \frac{1}{h^2}$$

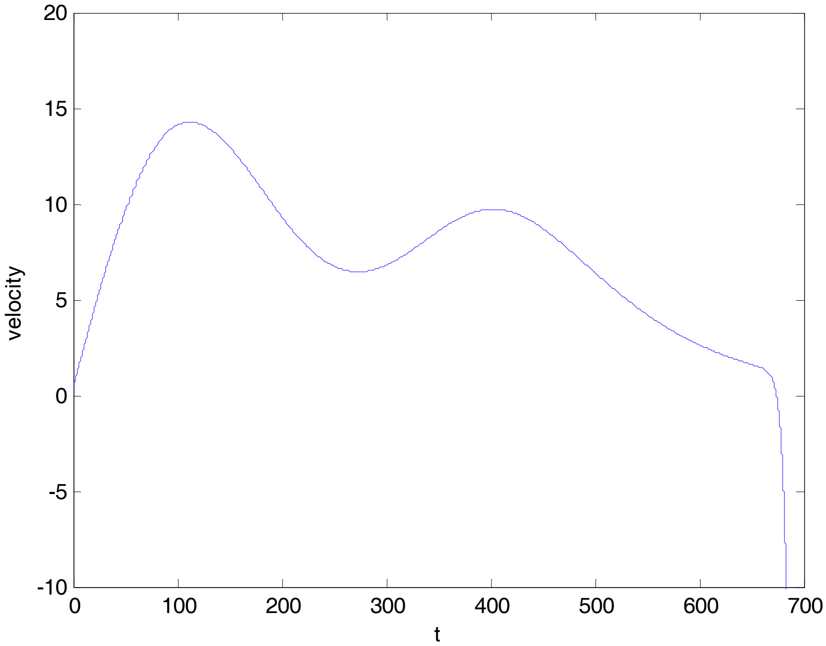
3. [F7xxxxxxx_ prob3.m]

The vertical speed of a ballistic missile is designed as follows:

$$v_y(t) = 20\, e^{-\left(\frac{t}{200}\right)^2} \sin\left(\frac{t}{100}\right) + \frac{20}{e^{\frac{t-400}{100}} + e^{-\frac{t-400}{100}}}$$

$$- 4.9(t-650)\left(1 + \frac{e^{\frac{(t-700)}{10}} - e^{-\frac{(t-700)}{10}}}{e^{\frac{(t-700)}{10}} + e^{-\frac{(t-700)}{10}}}\right) \quad m/s$$

You may notice that after fuel exhaustion, the missile is only affected by gravity. Solve following problems with precision of <u>6 significant digits</u> except for (c).



(a) [15%] How long from launching does this missile reach the apogee (highest altitude)?

At t = 673.986 or 673.987s, the missile reaches the apogee

(b) [15%] How high is the apogee?

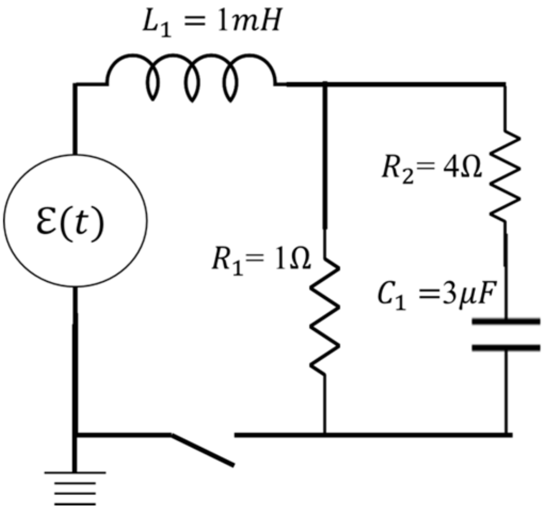
The altitude of apogee is 5123.19 or 5123.20 m

(c) [10%] Find the acceleration at t = 70 s with 9 digit of precision.

Using Central difference with $\Delta t = 2^{-11}$, v'(70) = 0.102782653 m/s^2

4. [F7xxxxxxx_ prob4.m]

Given the following circuit and assume there is no current or charge inside the loop before the switch is on. Solve following problems with precision of 4 significant digits.



(a) [5%] Assume that the current flowing through L_1 is I_L, and C_1 is I_c. Write down the loop equation(s) in terms of $\mathcal{E}(t), I_c(t), I_L(t), L_1, C_1, R_1,$ and R_2.

$$\begin{cases} \mathcal{E} - V_L - (I_L - I_C)R_1 = 0 \\ -(I_C - I_L)R_1 - I_C R_2 - V_C = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \mathcal{E} - L\dfrac{dI_L}{dt} - (I_L - I_C)R_1 = 0 \\ \dfrac{d}{dt}(I_C - I_L)R_1 + \dfrac{d}{dt}(I_C R_2) + \dfrac{d}{dt}V_C = 0 \end{cases}$$

$$\Rightarrow I_L' = \frac{1}{L}\left(\mathcal{E} - (I_L - I_C)R_1\right) \#$$

All acceptable

$$R_1 C_1 [I_C' - I_L'] + C_1 R_2 I_C' + I_C = 0 \#$$

$$\Rightarrow I_C' [R_1 C_1 + R_2 C_1] - R_1 C_1 I_L' + I_C = 0 \#$$

$$I_C' = \frac{1}{R_1 C_1 + R_2 C_1} \cdot \left(-I_C + R_1 C_1 \cdot I_L'\right) \#$$

$$= \frac{1}{R_1 C_1 + R_2 C_1} \left[\frac{R_1 C_1}{L}\left(\mathcal{E} - (I_L - I_C)R_1\right) - I_C\right] \#$$

(b) [2%] Find the voltage across R_1 at t = 2 s if $\mathcal{E}(t) = 3V$.

It's at steady state, so V_{R1} = 3.000V or 2.999V(if you got the answer from simulation)

(c) [8%] Find the voltage across R_2 at t = 2 ms if $\mathcal{E}(t) = 3V$.

$V_{R2}(2ms) = 0.4931$ *or* $0.4932\ V$ with dt = 1e-9s and Euler's method

(d) [10%] Find the voltage across L_1 at t = 20ms if $\mathcal{E}(t) = \cos(2\pi \times 500\ t)V$.

$V_{L1}(20ms) = 0.9102$ *or* $0.9103\ V$ with dt = 1e-9s and Euler's method

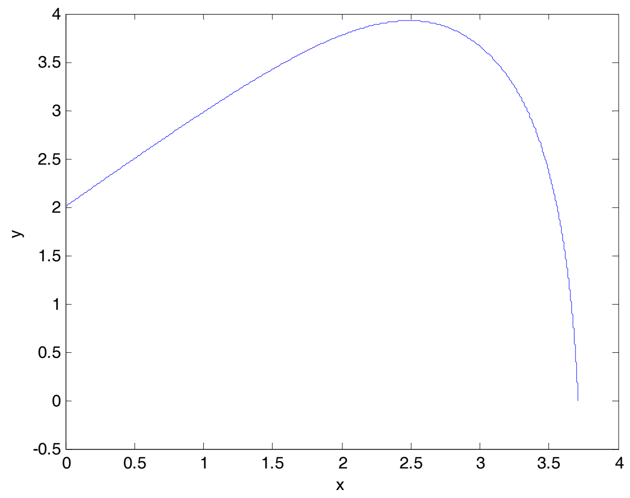
5. [F7xxxxxxx_ prob5.m]

A 0.3-kg ball was thrown from 2.0m level at an angle of 45^0 to the horizontal. The initial speed of the ball was 30m/s. Assume that the drag force from the air resistance can be modelled by $\vec{F}_{drag} = -0.2|\vec{v}|^2\ \hat{v}\ N$ and the acceleration of gravity was $9.8\ m/s^2$. Solve following problems with precision of <u>4 significant digits</u>.

(a) [15%] What was the horizontal range of the projectile when it hit the ground?

The total flying time is around 1.7572 s and the horizontal range is 3.712 m

(b) [5%] Plot the trajectory of the ball (x-y plot). (Let the plot shown on the window. No need to save the figure)



(c) [10%] What was the maximum vertical height?

At Vy = 0, it reaches the highest point 3.926 m

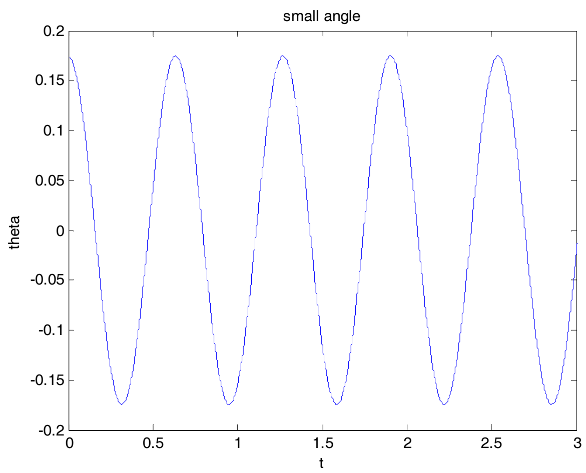
All the solutions above are solved by Euler's method using dt = 10^-7 s;

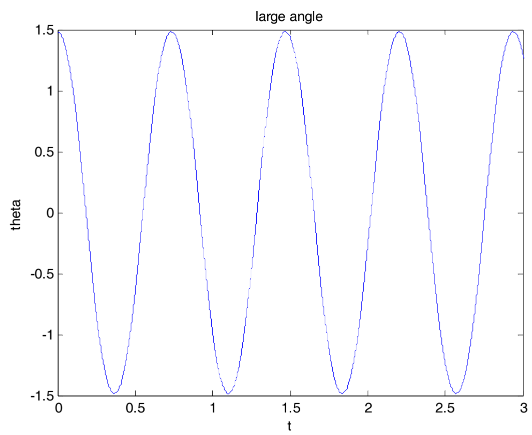
6. [F7xxxxxxx_ prob6.m]

The equation of motion of a pendulum can be described as

$l\frac{d^2}{dt^2}\theta(t) = -g\ \sin\theta(t)$, where l is the length of the pendulum and g is the gravitational acceleration.

If a pendulum has a string length of 0.1m, and g = $9.8\ m/s^2$ Solve following problems with precision of 4 significant digits.





(a) [10%] Find the period of the pendulum with the initial condition:

$$\theta(0) = 5^0\ and\ \frac{d}{dt}\theta(0) = 0\ rad/s$$

Measure the two time points when $\frac{d}{dt}\theta = 0$; The period is twice longer than the measured time difference. The period is 0.6346 or 0.6347 s

(b) [15%] Find the period of the pendulum with the initial condition:

$$\theta(0) = 85^0 \; and \; \frac{d}{dt}\theta(0) = 0 \; rad/s$$

Measure the two time points when $\frac{d}{dt}\theta = 0$; The period is twice longer than the measured time difference. The period is 0.7349 s

All the solutions above are solved by Euler's method using dt = 10^-7 s;