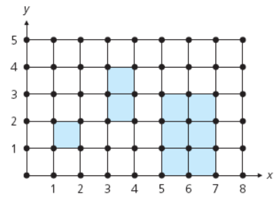
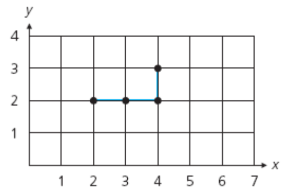
Discrete Mathematics (2008 Spring) Midterm I

1. (16 points, 3 for (6)(7), 2 for others) For each of the following statements, determine and explain whether it is correct or not.
   (1). $Q^* \cap Z = Z$
   (2). $R^+ \cap C = R^+$
   (3). $\phi \subset \phi$
   (4). $\phi \in \phi$
   (5). $\phi \subset \{\phi\}$
   (6). $p \to [q \to (p \wedge q)]$ is a tautology.
   (7). $(p \vee q) \to [q \to (p \wedge q)]$ is a tautology.

2. (10 points) Consider the 8 x 5 grid shown in the right figure. How many different **rectangles** (with integer-coordinate corners, including squares) does this grid contain? How many **squares** does it contain? [For example, there is a rectangle (square) with corners (1, 1), (2, 1), (2, 2), (1, 2), a second rectangle with corners (3, 2), (4, 2), (4, 4), (3, 4)]



3. (12 points) (a) In how many ways can a particle move in the xy-plane from the origin to the point (7, 4) if the moves that are allowed are of the form: (R): (x, y)→(x + 1, y); (U): (x, y)→(x, y + 1)? (b) How many of the paths in part (a) do not use the path from (2, 2) to (3, 2) to (4, 2) to (4, 3) shown in the right figure? (c) Answer parts (a) and (b) if a third type of move (D): (x, y)→(x + 1, y + 1) is also allowed.



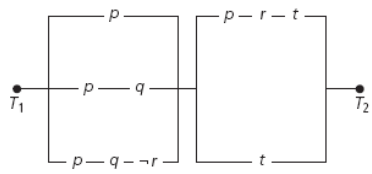
4. (12 points) (a) Find the coefficient of x^2yz^2 in the expansion of $[(x/2) + y - 3z]^5$. (b) How many distinct terms are there in this complete expansion? (c) What is the sum of all coefficients in the complete expansion?

5. (10 points) Using the laws of set theory to simply the expression $(A \cap B) \cup [B \cap ((C \cap D) \cup (C \cap \overline{D}))]$, *where* A, B, C, D $\subseteq U$.

6. (10 points) Express the **negation** of the statement $p \leftrightarrow q$ in term of the connectives $\wedge$ and $\vee$.

7. (10 points) For $n \in N$, prove that $L_0 + L_1 + L_2 + \cdots + L_n = \sum_{i=0}^{n} L_i = L_{n+2} - 1$, where $L_0 = 2, L_1 = 1, L_n = L_{n-1} + L_{n-2}$.

8. (10 points) Simplify the network shown in the following figure.



9. (10 points) Show that $\sqrt{p}$ is irrational for every prime p.