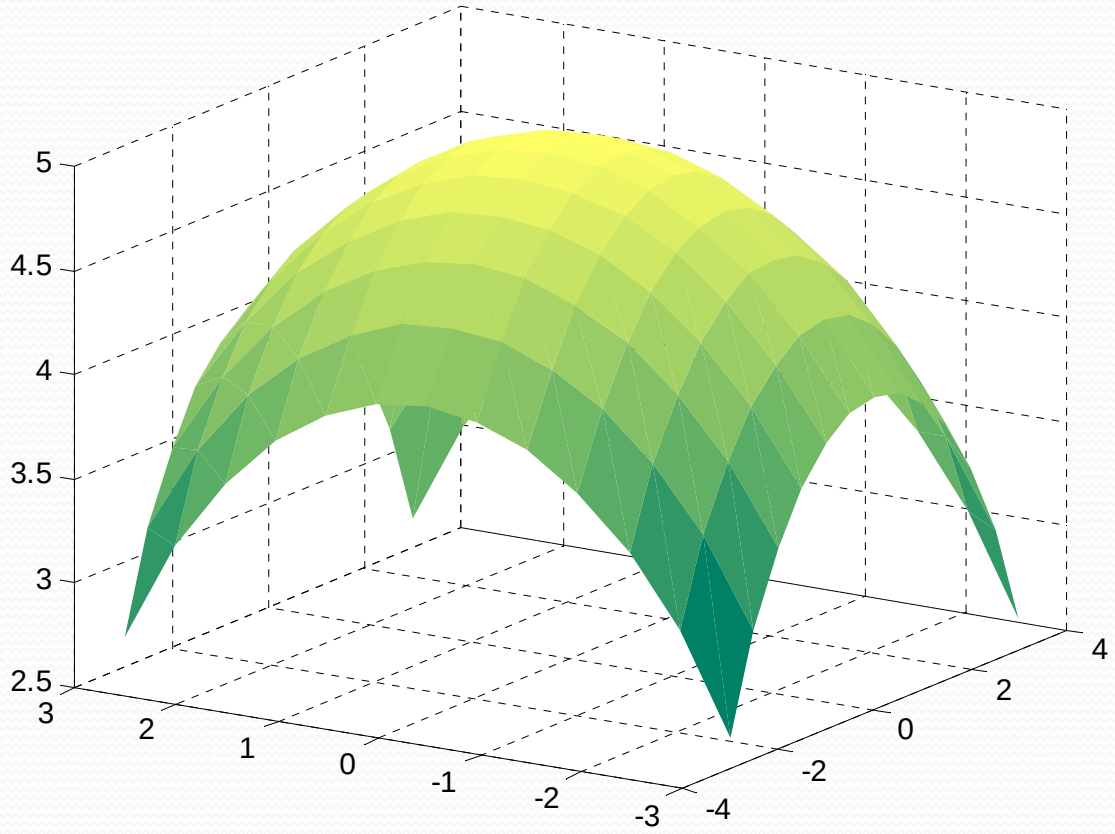
Multiple Integral

Double Integral

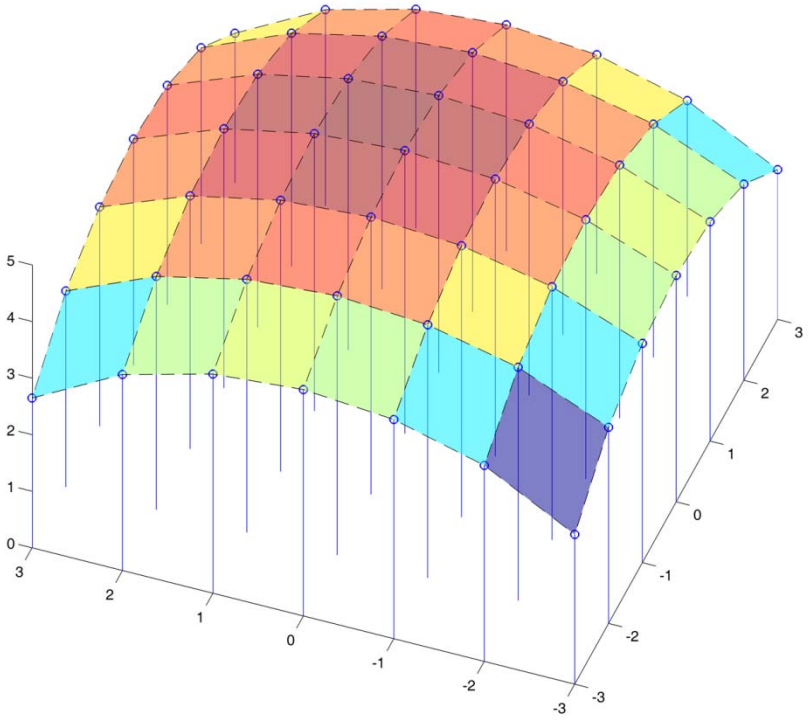
Double Integral

- $\int_{y_0}^{y_1} \int_{x_0}^{x_1} f(x, y)\, dx\, dy$



Double Integral

- $\int_{-3}^{3}\int_{-3}^{3}\sqrt{25-x^2-y^2}\,dx\,dy$



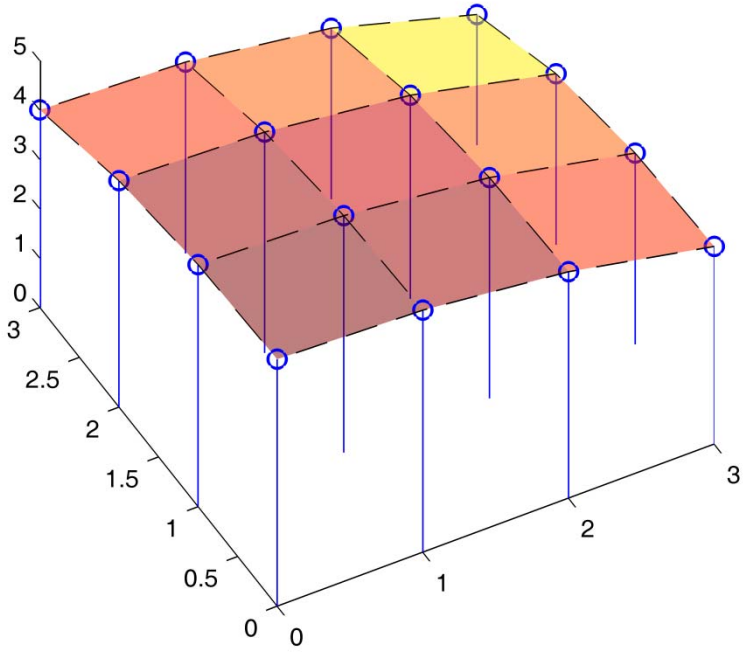
Box Counting

- $\int_{y_0}^{y_1}\int_{x_0}^{x_1} f(x,y)\,dx\,dy$

$$\sim \int_{y_0}^{y_1} [f(x_0,y) + f(x_0+\Delta x,y) + f(x_0+2\Delta x,y) + \cdots]\Delta x\,dy$$

$$= \int_{y_0}^{y_1} \sum_{i=0}^{N-1} f(x_0 + i*\Delta x, y)\,\Delta x\,dy$$

$$\sim \sum_{j=0}^{M-1}\sum_{i=0}^{N-1} f(x_0 + i*\Delta x, y_0 + j*\Delta y)\,\Delta x\Delta y$$

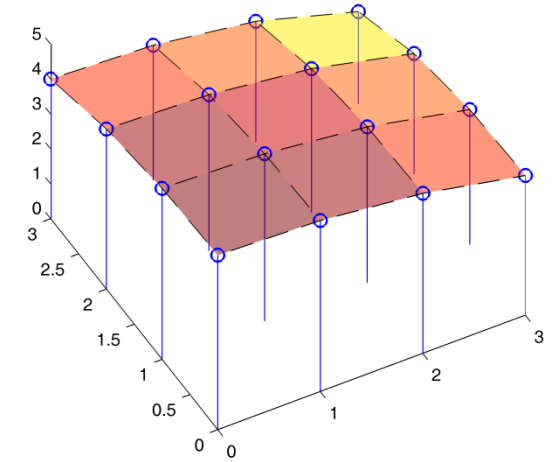


$$\Delta x = \frac{x_1 - x_0}{N}$$

$$\Delta y = \frac{y_1 - y_0}{M}$$

Trapezoid

$$\Delta x = \frac{x_1 - x_0}{N}$$
$$\Delta y = \frac{y_1 - y_0}{M}$$



- $\int_{y_0}^{y_1} \int_{x_0}^{x_1} f(x,y)\, dx\, dy$

$$\sim \left[0.5 \int_{x_0}^{x_1} f(x, y_0)\, dx + \int_{x_0}^{x_1} f(x, y_0 + \Delta y)\, dx + \int_{x_0}^{x_1} f(x, y_0 + 2\Delta y)\, dx \right.$$
$$\left. + \cdots + 0.5 \int_{x_0}^{x_1} f(x, y_0 + M\Delta y)\, dx \right] \Delta \mathrm{y}$$

$$\int_{x_0}^{x_1} f(x, y_0)\, dx \sim [0.5 f(x_0, y_0) + f(x_0 + \Delta x, y_0) + \cdots$$

$$\int_{y_0}^{y_1}\int_{x_0}^{x_1} f(x,y)\,dx\,dy$$

$$\sim\left[0.5\int_{x_0}^{x_1} f(x,y_0)\,dx + \int_{x_0}^{x_1} f(x,y_0+\Delta y)\,dx\right.$$

$$\left.+\int_{x_0}^{x_1} f(x,y_0+2\Delta y)\,dx + \cdots + 0.5\int_{x_0}^{x_1} f(x,y_0+M\Delta y)\,dx\right]\Delta y$$

$$\int_{x_0} f(x,y_0)\,dx \sim [0.5f(x_0,y_0) + f(x_0+\Delta x, y_0)$$

$x_i = x_0 + i * \Delta x$

$y_j = y_0 + j * \Delta y$