Discrete Mathematics (2011 Spring) Final

1. **(10 points)** Negate and simplify the statement $p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)$.

2. **(5 points)** What is the number of the equivalence relations on A = {*a*, *b*, *c*, *d*, *e*, *f*, *g*} that have exactly one equivalence class of size 3?

3. **(10 points)** Let A = {1, 2, 3, …,10}, and B = {1, 2, 3, …,7}. (a) How many functions f : A→B satisfy $|f(A)| = 4$? (b) How many have $|f(A)| \leq 4$?

4. **(20 points)** Determine the following coefficient of (a) $x^3y^3z^{-3}$ in the complete expansion of $(x - 2y)^6(3z^{-1} + 4)^4$, (b) x^4y^3 in the complete expansion of $(x - 2y)^3(3x + 4y)^4$, (c) x^{50} in $(x^7 + x^8 + x^9 + \ldots)^6$, (d) x^{15} in $(x^3 - 5x)/(1 - x)^3$.

5. **(5 points)** If A = {*w*, *x*, *y*, *z*}, determine the number of relations on A that are reflexive and symmetric but not transitive. [Note: S(3,2)=3, S(4,2)=7]

6. **(10 points)** Determine how many integer solutions for $x_1 + x_2 + x_3 = 10$, $0 \leq x_1 \leq 5$, $0 \leq x_2 \leq 6$, $3 \leq x_3 \leq 7$.

7. **(3+3+4 points)** Let A = {*a*, *b*, *c*}, and B = {*u*, *v*, *w*, *x*, *y*, *z*}. (a) If f : A→B is a randomly generated function, what is the probability that f is one-to-one? (b) How many closed binary operations on A that have c as the identity? (c) How many closed binary operations on A that are commutative and have an identity?

8. **(2+3+5 points)** (a) What is the sequence the generation function $a + (d - a)x$ generate? (b) What is the exponential generating function for the sequence 0!, 1!, 2!, 3!, … (c) Find a generation function to generate sequence a, $a + d$, $a + 2d$, $a + 3d$, ….

9. **(8+7 points)** (a) Find a recurrence relation for the number of **4**-ary sequences (e.g., 0213, 0113) of length n that have no consecutive 0's. (b) Solve the recurrence relation in (a).

10. **(10 points)** Using the method of *generating functions* to solve the recurrence relation $a_{n+2} - 2a_{n+1} + a_n = 2^n$, where $n \geq 0$, $a_0 = 1$, $a_1 = 2$.

11. **(5 points)** Please list 2 examples/methods/strategies to improve your (or others') learning motivation/performance.