Engineering Mathematics

Final Exam, 2015 / 01 / 12

1. (10%) $y''+2ty'-4y=6, y(0)=0, y'(0)=0, \quad find\ y(t)=?$

Sol:

$$Y(s)=6s^{-3} \Rightarrow y(t)=3t^2$$

2. (10%) Use the Laplace transform to solve the given system of differential equation.

$$\frac{d^2x}{dt^2}+\frac{dx}{dt}+\frac{dy}{dt}=0 \qquad x(0)=1,\ x'(0)=0$$

$$\frac{d^2y}{dt^2}+\frac{dy}{dt}-4\frac{dx}{dt}=0 \qquad y(0)=-1,\ y'(0)=5$$

Sol:

$$x(t)=e^{-t}\cos 2t+\frac{1}{2}e^{-t}\sin 2t \qquad y(t)=-e^{-t}\cos 2t+2e^{-t}\sin 2t$$

3. (10%) Find the indicial equation and its roots.

$$x^2y''+xe^xy'+(x^2-1)y=0$$

Sol:

The coefficient of x^{n+r} : $r(r-1)+r-1=0 \to r^2-r+r-1=0 \to r^2-1=0 \Rightarrow$ the indicial equation roots: $r=1,-1$

4. (10%) Find the series solution at x=0 for the following equation

$$2x(1-x)y''+(1+x)y'-y=0$$

Sol:

$$y(x)=C_1x^{\frac{1}{2}}a_0+C_2(1+x)a_0$$

5. (10%) Use the Laplace transform to solve the given initial-value

$$y'-2y=\int_0^t e^{2(t-\tau)}\cos 3\tau\, d\tau, \quad y(0)=0$$

Sol:

$$y(t)=\frac{5}{169}e^{2t}+\frac{2}{13}te^{2t}-\frac{5}{169}\cos(3t)-\frac{12}{169}\sin(3t)$$

6. (10%) $F(s)=\dfrac{2s}{(s^2+4)^2}$, find f(t).

Sol:

$$f(t)=\frac{1}{2}t\sin 2t$$

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7. (10%) Find the given inverse transform : $\mathscr{L}^{-1}\{\frac{s}{(s+2)(s^2+4)}\}$

Sol:

$$\mathscr{L}^{-1}\{\frac{s}{(s+2)(s^2+4)}\} = \frac{-1}{4}e^{-2t} + \frac{1}{4}\cos 2t + \frac{1}{4}\sin 2t$$

8. (10%) $y'' + 4y' + 4y = 3H(t-2), y(0) = y'(0) = 0, \ find\ y(t)?$

Sol:

$$y(t) = (\frac{3}{4} - \frac{3}{4}e^{-2(t-2)} - \frac{3}{2}(t-2)e^{-2(t-2)})H(t-2)$$

9. (10%) $\int_0^\infty \frac{\sin t}{t}\,dt = ?$

Sol:

$$\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$$

10. (10%) Please write down the Legendre differential equation and the Bessel differential equation (5%) and give their respective solutions (5%).

Sol:

Legendre differential equation:

$\left(1-x^2\right)y'' - 2xy' + \lambda y = 0$ in which?$-1 \le x \le 1$, and?λ is a real constant

the respective solutions:

$$y(\mathrm{x}) = a_0 y_e\left(x\right) + a_1 y_0\left(x\right),\ a_{n+2} = \frac{n\left(n+1\right)-\lambda}{\left(n+2\right)\left(n+1\right)}a_n$$

Bassel's differential equation

$$x^2 y'' + xy' + \left(x^2 - \upsilon^2\right)y = 0$$

the respective solutions:

$$y = c_1\mathcal{J}_\upsilon + c_2\mathcal{J}_{-\upsilon}$$

$$y_1\left(x\right) = a_0\sum_{n=0}^{\infty}\frac{\left(-1\right)^n}{2^{2n+\upsilon}\cdot n!\Gamma\left(n+\upsilon+1\right)}x^{2n+\upsilon} = \mathcal{J}_\upsilon\left(x\right)$$

$$y_2\left(x\right) = \mathcal{J}_{-\upsilon}\left(x\right) = \sum_{n=0}^{\infty}\frac{\left(-1\right)^n}{2^{(2n-\upsilon)}\cdot n!\Gamma\left(n-\upsilon+1\right)}x^{2n-\upsilon}$$