HW4 參考解答

3.14 (a)
$$P(X < 0.2) = F(0.2) = 1 - e^{-1.6} = 0.7981;$$

(b)
$$f(x) = F'(x) = 8e^{-8x}$$
. Therefore, $P(X < 0.2) = 8 \int_0^{0.2} e^{-8x} dx = -e^{-8x} \Big|_0^{0.2} = 0.7981$.

3.49 (a)
$$\begin{array}{c|ccccc} x & 1 & 2 & 3 \\ \hline g(x) & 0.10 & 0.35 & 0.55 \end{array}$$

(b)
$$\begin{array}{c|cccc} y & 1 & 2 & 3 \\ \hline h(y) & 0.20 & 0.50 & 0.30 \\ \end{array}$$

(c)
$$P(Y = 3 \mid X = 2) = \frac{0.1}{0.05 + 0.10 + 0.20} = 0.2857.$$

3.63 (a)
$$g(x) = \int_0^\infty y e^{-y(1+x)} dy = -\frac{1}{1+x} y e^{-y(1+x)} \Big|_0^\infty + \frac{1}{1+x} \int_0^\infty e^{-y(1+x)} dy$$

 $= -\frac{1}{(1+x)^2} e^{-y(1+x)} \Big|_0^\infty$
 $= \frac{1}{(1+x)^2}$, for $x > 0$.
 $h(y) = y e^{-y} \int_0^\infty e^{-yx} dx = -e^{-y} e^{-yx} \Big|_0^\infty = e^{-y}$, for $y > 0$.

(b)
$$P(X \ge 2, Y \ge 2) = \int_2^\infty \int_2^\infty y e^{-y(1+x)} dx dy = -\int_2^\infty e^{-y} e^{-yx} \Big|_2^\infty dy = \int_2^\infty e^{-3y} dy = -\frac{1}{3} e^{-3y} \Big|_2^\infty = \frac{1}{3e^6}.$$