Probability & Statistics 2015

Midterm exam solution

1. [8%]

```
d1=[2.0 3.0 0.3 3.3 1.3 0.4];%data in 1st row
d2=[0.2 6.0 5.5 6.5 0.2 2.3];%data in 2nd row
d3=[1.5 4.0 5.9 1.8 4.7 0.7];%data in 3rd row
d4=[4.5 0.3 1.5 0.5 2.5 5.0];%data in 4th row
d5=[1.0 6.0 5.6 6.0 1.2 0.2];%data in 5th row
d=[d1 d2 d3 d4 d5];

disp(['mean of sample: ', num2str(mean(d)),' years']);
disp(['standard deviation of sample: ', num2str(std(d)), ' years']);

[n,xout] = hist(d,[0.45:1:6.45]); %use 7 bins for the histogram
bar(xout,n/sum(n)); %relative frequency is n/sum(n)

set(gca,'xtick',0.45:1:6.45);
title('relative frequency histogram');
xlabel('years')
ylabel('relative frequency')
```

2. [8%]

Consider the events:

A: two nondefective components are selected,

N: a lot does not contain defective components, $P(N) = 0.6$, $P(A \mid N) = 1$,

O: a lot contains one defective component, $P(O) = 0.3$, $P(A \mid O) = \frac{\binom{19}{2}}{\binom{20}{2}} = \frac{9}{10}$,

T: a lot contains two defective components, $P(T) = 0.1$, $P(A \mid T) = \frac{\binom{18}{2}}{\binom{20}{2}} = \frac{153}{190}$.

(a) $P(N \mid A) = \frac{P(A \mid N)P(N)}{P(A \mid N)P(N)+P(A \mid O)P(O)+P(A \mid T)P(T)} = \frac{(1)(0.6)}{(1)(0.6)+(9/10)(0.3)+(153/190)(0.1)}$
$= \frac{0.6}{0.9505} = 0.6312;$

3. [16%]

3.1

We can select x defective sets from 2, and $3-x$ good sets from 5 in $\binom{2}{x}\binom{5}{3-x}$ ways. A random selection of 3 from 7 sets can be made in $\binom{7}{3}$ ways. Therefore,

$$f(x) = \frac{\binom{2}{x}\binom{5}{3-x}}{\binom{7}{3}}, \qquad x = 0, 1, 2.$$

In tabular form

x	0	1	2
$f(x)$	2/7	4/7	1/7

3.2

$$P(0 < X \leq 2) = P(X \leq 2) - P(X \leq 0) = 1 - 2/7 = 5/7.$$

4. [16%]

4.1

$1 = k\int_0^2\int_0^1\int_0^1 xy^2z\;dx\;dy\;dz = 2k\int_0^1\int_0^1 y^2z\;dy\;dz = \frac{2k}{3}\int_0^1 z\;dz = \frac{k}{3}$. So, $k = 3$.

4.2

$P\left(X < \frac{1}{4}, Y > \frac{1}{2}, 1 < Z < 2\right) = 3\int_1^2\int_{1/2}^1\int_0^{1/4} xy^2z\;dx\;dy\;dz = \frac{9}{2}\int_0^{1/4}\int_{1/2}^1 y^2z\;dy\;dz$
$= \frac{21}{16}\int_1^2 z\;dz = \frac{21}{512}$.

5. [12%]

5.1

1. The experiment consists of n repeated trials.
2. Each trial results in an outcome that may be classified as a success or a failure.
3. The probability of success, denoted by p, remains constant from trial to trial.
4. The repeated trial are independent.

5.2

1. The number of outcomes occurring in one time interval or specified region is independent of the number that occurs in any other disjoint time interval or region of space. In this way we say that the Poisson process has no money.
2. The probability that a single outcome will occur during a very short time interval or in a small region Is proportional to the length of time interval or the size of the region and does not depend on the number of outcomes occurring outside this time interval or region.
3. The probability that more than one outcome will occur in such a short time interval or fall in such a small region is negligible.

6. [16%]

6.1

Using Binomial distribution.

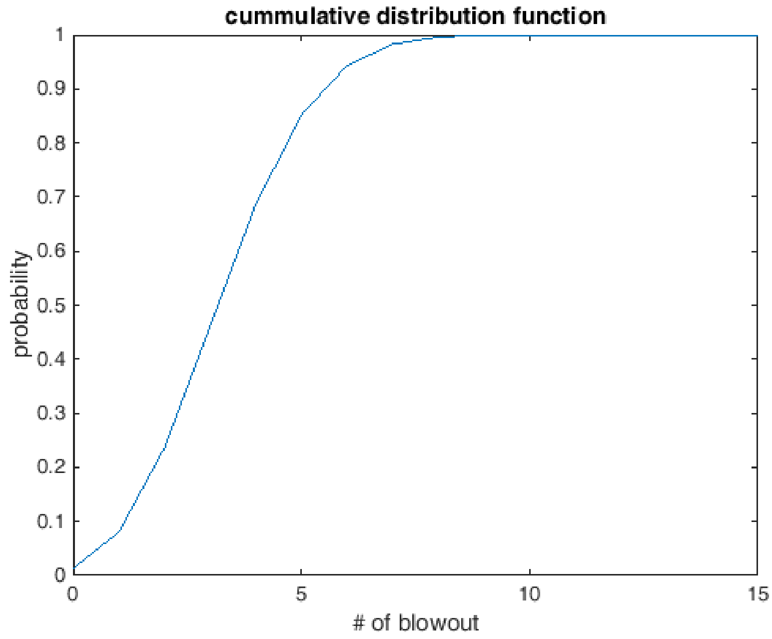
There are n (n=15) repeated tests that are independent, and the probability remains constant (p=0.25) which the test results can classified as a success (without a blowout) or a failure (blowout). From Table A.1 with n =15 and p = 0.25,

we have P(X<=6) = 0.9434(r=6) , P(X<=2) = 0.2361(r=2).

$$P(3 \leq X \leq 6) = P(X \leq 6) - P(X \leq 2) = 0.9434 - 0.2361 = 0.7073.$$

6.2

```
clear all;
n = 15;
p = 0.25;
x = 0:n;
y = binocdf(0:n,n,p);
figure;
plot(x,y);
xlabel('# of blowout');
ylabel('probability');
title('cummulative
distribution function');
```



7. [8%]

Using Poisson distribution.

There are number of outcomes (the average number is estimated by 12) occurring in specified region of space (5-acre wheat field) which is independent of the number occurred in other disjoint region. A single outcome occurs in a small region which is proportional to size of it. And the probability which is more than one outcome occurred in small region is negligible. The average number of field mice is estimated to be 12. It takes μ = 12, and the question is asked the probability fewer than 7 mice which means finding the number of 0~6. Therefore r = 6.

Using the Poisson distribution with $\mu = 12$, we find from Table A.2 that $P(X < 7) = P(X \leq 6) = 0.0458.$

8. [16%]

8.1

Using binomial distribution

Let X represent the number of defects found in 20 samples. The problem can calculated using a binomial distribution *b*(X;20,0.1). From Table A.1,we have *b*(X;20,0.1) = 0.9568(r=4)

$$P(X>= 5) = 1 - P(X<=4) = 1 – 0.9568 = 0.0432$$

8.2

This is a rare probability and thus the original claim that p=0.1 is questionable.