Discrete Mathematics (2015 Spring) Midterm II

1. **(24 points)** For each of the following statements, determine and explain (required) whether it is correct or not.
   (a). R is the smallest equivalence relation on {1, 2, 3, 4} that contains {(1,2), (1,4), (3, 3), (4,1)}. |R|=9.
   (b). For Σ={0,1}, the string 00010 belong to two of the following languages $\{0,1\}^*$, $\{0\}^*\{1\}^*\{0\}^*$, $\{00\}\{0\}^*\{10\}$, $\{00\}^*\{10\}^*$.
   (c). Let A={1, 10} the number of strings in {λ, 1101, 1110111, 10110, 110011} that belong to A^* is 3.
   (d). The least upper bound of {1, 2, 5, 10, 15} in the poset(Z^+, |) does not exist.
   (e). The number of different equivalence relations on a set of 4 elements is 16.
   (f). Let *f*: A→B and *g*: B→C, $g \circ f$ is one-to-one if and only if *f* and *g* are one-to-one.

2. **(3,3,4 points)** (a) Let S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. What is the smallest integer k such that any subset of S of size k contains (a1) at least one pair of numbers add up to 9. (a2) two disjoint subsets of size two, $\{x_1, x_2\}$ and $\{y_1, y_2\}$, such that $x_1 + x_2 = y_1 + y_2 = 9$? (b) How many times must we roll a single die in order to get the same score at least n times?

3. **(3,3,4 points)** (a) Determine the following relations are reflexive, symmetric, antisymmetric, or transitive.
   (a1) Let $x, y \in \mathbf{Z}, and\ xRy\ if\ and\ only\ if\ x|y.$ (a2) $a, b \in \mathbf{Z}, and\ aRb\ if\ and\ only\ if\ |a - b| \leq 1.$ (b) Give an example of equivalence relation in your real life and explain the meaning of equivalence class of your example

4. **(10 points)** Let p, q be distinct primes. Please draw the Hasse diagram of all positive divisors of p^3q^2 for the relation "|".

5. **(4, 4 points)** (a) How many two-factor ordered factorizations, where each factor is greater than 1, are there for 312,018 (2*3*7*17*19*23)? (b) In how many ways can 312,018 be factored into two or more factors, each greater than 1 and the order of the factors is relevant?

6. **(2, 3, 5 points)** Let A={a, b, c, d}, B={1, 2, 3, 4, 5, 6}. (a) How many one-to-one functions are there from A to B? (b) How many functions in (a) such that $f(a) \neq 1$? (c) How many onto functions from B to A satisfying $f(1)$=a?

7. **(3,3,4 points)** Let A={a, b, c, d, e} (a) how many closed binary operations *f* on A satisfy $f(a, b) \neq c$? (b) How many closed binary operations *f* on A have e as an identity and $f(a, b)$=c? (c) How many *f* in (b) are commutative?

8. **(2,2,3,3 points)** Let A = {a, b, c, d, e}, determine the number of relations on A that are (a) antisymmetric and do not contain (a, b), (b) reflexive and symmetric but not transitive, (c) equivalence relations, (d) equivalence relations where $a \in [b], c \in [d]$.

9. **(8 points)** Let U={1, 2, 3, 4}, with A be the proper subsets of U, and let R be the *subset relation* on A. For B={{1}, {2}, {1, 2}, {2, 3}}⊆A, determine each of the following. (a) The maximal element of A, (b) The minimal element of A, (c) The greatest element of A, (d) The set of upper bounds that exist for B.

10. **(4, 6 points)** (a) Construct a state diagram for a finite state machine with I=O={0, 1} that recognizes all strings in the language $\{0, 1\}^*\{01\}$ U $\{0, 1\}^*\{10\}$. (b) Design a finite state machine with {0, 1, 2} as its input alphabets and show the remainder of sum divided by 4.

(Stirling number of the second kind: S(4, 2)=7, S(4, 3)=6, S(5, 2)=15, S(5, 3)=25, S(5, 4)=10, S(6, 2)=31, S(6, 3)=90)