HW2 reference answers

3.29

(a) $f(x) \geq 0$ and $\int_1^\infty 3x^{-4}\ dx = -3\ \frac{x^{-3}}{3}\Big|_1^\infty = 1$. So, this is a density function.

(b) For $x \geq 1$, $F(x) = \int_1^x 3t^{-4}\ dt = 1 - x^{-3}$. So,

$$F(x) = \begin{cases} 0, & x < 1, \\ 1 - x^{-3}, & x \geq 1. \end{cases}$$

(c) $P(X > 4) = 1 - F(4) = 4^{-3} = 0.0156$.

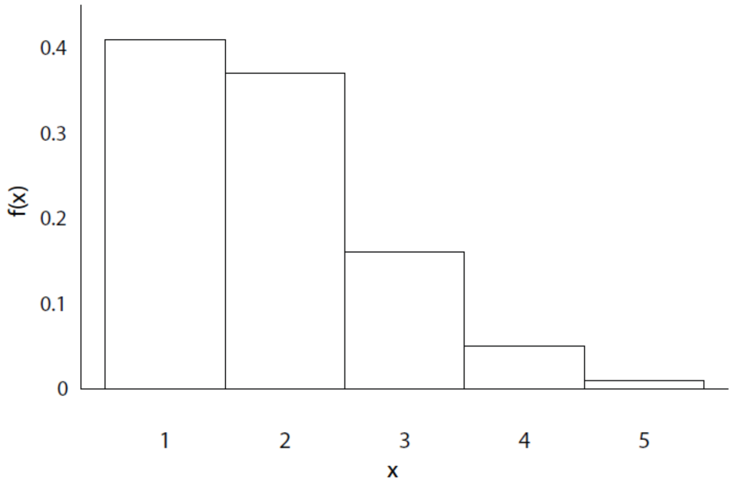
3.64

(a) $P\left(X \leq \frac{1}{2}, Y \leq \frac{1}{2}\right) = \frac{3}{2}\int_0^{1/2}\int_0^{1/2}(x^2 + y^2)\ dydx = \frac{3}{2}\int_0^{1/2}\left(x^2y + \frac{y^3}{3}\right)\Big|_0^{1/2}\ dx$
$= \frac{3}{4}\int_0^{1/2}\left(x^2 + \frac{1}{12}\right)\ dx = \frac{1}{16}$.

(b) $P\left(X \geq \frac{3}{4}\right) = \frac{3}{2}\int_{3/4}^1\left(x^2 + \frac{1}{3}\right)\ dx = \frac{53}{128}$.

4.32

(a) A histogram is shown next.



(b) $\mu = (0)(0.41) + (1)(0.37) + (2)(0.16) + (3)(0.05) + (4)(0.01) = 0.88$.

(c) $E(X^2) = (0)^2(0.41) + (1)^2(0.37) + (2)^2(0.16) + (3)^2(0.05) + (4)^2(0.01) = 1.62$.

4.78

$\mu = E(X) = 6\int_0^1 x^2(1-x)\ dx = 0.5$, $E(X^2) = 6\int_0^1 x^3(1-x)\ dx = 0.3$, which imply $\sigma^2 = 0.3 - (0.5)^2 = 0.05$ and $\sigma = 0.2236$. Hence,

$$\begin{aligned} P(\mu - 2\sigma < X < \mu + 2\sigma) &= P(0.5 - 0.4472 < X < 0.5 + 0.4472) \\ &= P(0.0528 < X < 0.9472) = 6\int_{0.0528}^{0.9472} x(1-x)\ dx = 0.9839, \end{aligned}$$

compared to a probability of at least 0.75 given by Chebyshev's theorem.