DISCRETE MATHEMATICS – CH4 Homework4

Textbook assignment (30 pts)

4-1

23. *a) Let n $\in \mathbf{Z}^+$, where n $\neq$ 1,3. Prove that n can be expressed as a sum of 2's and/or 5's.*

b) For all n $\in \mathbf{Z}^+$, show that if n $\geq$ 24, then n can be written as a sum of 5's and/or 7's.

(a) The result is true for $n = 2, 4, 5, 6$. Assume the result is true for all $n = 2, 4, 5, \ldots, k-1, k$, where $k \geq 6$. If $n = k+1$, then $n = 2 + (k-1)$, and since the result is true for $k-1$, it follows by induction that it is true for $k+1$. Consequently, by the Alternative Form of the Principle of Mathematical Induction, every $n \in \mathbf{Z}^+, n \neq 1, 3$, can be written as a sum of 2's and 5's.

(b) $24 = 5 + 5 + 7 + 7$ $\qquad$ $25 = 5 + 5 + 5 + 5 + 5$ $\qquad$ $26 = 5 + 7 + 7 + 7$

Hence the result is true for all $24 \leq n \leq 28$. Assume the result true for $24 \leq n \leq 28 \leq k$, and consider $n = k+1$. Since $k+1 \geq 29$, we may write $k+1 = [(k+1)-5]+5 = (k-4)+5$, where $k-4$ can be expressed as a sum of 5's and 7's. Hence $k+1$ can be expressed as such a sum and the result follows for all $n \geq 24$ by the Alternative Form of the Principle of Mathematical Induction.

4-2

18. *Consider the permutations of 1, 2, 3, 4. The permutation 1432, for instance, is said to have one ascent—namely, 14 (since 1 < 4). This same permutation also has two descents—namely, 43 (since 4 > 3) and 32 (since 3 > 2). The permutation 1423, on the other hand, has two ascents, at 14 and 23—and the one descent 42.*

(b) How many permutations of 1, 2, 3, 4 have k ascents, for k = 0, 1, 2, 3?

(c) If a permutation of 1,2,3,4,5,6,7 has four ascents, how many descents does it have?

(e) Consider the permutation p = 12436587. This permutation of 1,2,3,…,8 has four ascents. In how many of the nine locations (at the start, end or between two numbers) in p can we place 9 so that the result is a permutation of 1,2,3,…,8,9 with (i) four ascents; (ii) five ascents?

b) k=0 共 1 個 4321

k=1 共 11 個 1432, 2143, 2431, 3142, 3214, 3241, 3421, 4132, 4213, 4231, 4312

k=2 共 11 個 1243, 1324, 1342, 1423, 2134, 2314, 2341, 2413, 3124, 3412, 4123

k=3 共 1 個 1234

c) 2 descents

e) (i) 5 locations

1) In front of 1

2) Between 1, 2

3) Between 2, 4

4) Between 3, 6

5) Between 5, 8

[The five locations are determined by the four ascents and the one location at the start (in front of 1) of p]

(ii) 4 locations

1) Between 4, 3

2) Between 6, 5

3) Between 8, 7

4) Following 7

[The four locations are determined by the three descents and the one location at the end (following 7) of p]

4-4

14. *An executive buys $2490 worth of presents for the children of her employees. For each girls she gets an art kit costing $33; each boy receives a set of tools costing $29. How many presents of each type did she buy?*

(also solve $29→$22)

a)

Let x = numbers of art kit, y = numbers of set of tools => 33x+29y=2490

gcd(33,29)=1

33=1*29+4

29=7*4+1

1=27-7*4=29-7(33-29)=8*29-7*33

1=33(-7)+29*8

⇨ 33(-7*2490)+29*(8*2490)

= 33(-17430+29k)+29(19920-33k) for all $k \in \mathbf{Z}$

x=-17430+29k, y=19920-33k

$x \geq 0 \Rightarrow 29k \geq 17430 \Rightarrow k \geq 602$

$y \geq 0 \Rightarrow 19920 \geq 33k \Rightarrow 603 \geq k$

k = 602: x = 28, y = 54;

k = 603: x = 57, y = 21

b)33x+22y=2490

gcd(33,22)=11

2490 = 11*226+4 (不能被整除) => 用2486算

33x + 22y = 2486 => 3x + 2y = 226

gcd(3,2)=1

1 = 3*1 + 2*(-1)

⇨ 226 = 3*226+2*(-226) = 3*(226-2k)+2*(-226+3k) for all k ∈ **Z**

x=226-2k, y=-226+3k

$x \geq 0 \Rightarrow k \leq 75.3$

$y \geq 0 \Rightarrow k \geq 113$

$113 \geq k \geq 75.3$

x = 226-2k, y = -226+3k

共38組最近似解，餘4元

Advanced assignment (20 pts)

- *For n>=1, show that if n>=64, then n can be written as a sum of 5's and/or 17's. (92,95 nthu.cs)*

  n = 64, 64=5*6+17*2

  n = 65, 65=5*13

  n = 66, 66=5*3+17*3

  n = 67, 67=5*10+17

  n = 68, 68=17*4

  Suppose 64 <= n <= k-1 can be written as a sum of 5's and/or 17's

  Let k >= 69, n=k

  ∵ k = (k-5)+5 and k-5 can be written as a sum of 5's and/or 17's

  ∴ k also can be written as a sum of 5's and/or 17's

  Thus, n>=64 can be written as a sum of 5's and/or 17's