Engineering Mathematics Homework 10 Solution

1. $Find: \mathscr{L}\{\int_0^t e^{\tau}\sin(t-\tau)d\tau\}$

Sol:

$$\mathscr{L}\{\int_0^t e^{\tau}\sin(t-\tau)d\tau\}$$
$$=\mathscr{L}\{e^t \otimes \sin(t)\}$$
$$=\mathscr{L}\{e^t\}\times\mathscr{L}\{\sin(t)\}$$
$$=\frac{1}{s-1}\times\frac{1}{s^2+1}$$

2. $If\ \mathscr{L}\{f(t)\}=\frac{1}{(s^2+k^2)^2}$, $please\ find\ f(t)$!

Sol:

$$Let\ F(s)=G(s)=\frac{1}{s^2+k^2}$$
$$f(t)=g(t)=\frac{1}{k}\mathscr{L}^{-1}\{\frac{1}{s^2+k^2}\}=\frac{1}{k}\sin kt$$
$$\mathscr{L}^{-1}\{\frac{1}{(s^2+k^2)^2}\}=\frac{1}{k^2}\int_0^t \sin k\tau \sin k(t-\tau)d\tau$$
$$=\frac{1}{2k^2}\int_0^t[\cos k(2\tau-1)-\cos kt]d\tau$$
$$=\frac{1}{2k^2}[\frac{1}{2k}\sin k(2\tau-t)-\tau\cos kt]_0^t$$
$$=\frac{\sin kt-kt\cos kt}{2k^3}$$
$$f(t)=\mathscr{L}^{-1}\{\frac{1}{(s^2+k^2)^2}\}=\frac{\sin kt-kt\cos kt}{2k^3}$$