Algorithm 2015
fall Homework 1

1. Below the figure as a model, illustrate the operation of merge sort on the array A = 〈5, 22, 76, 92, 32, 1, 63, 21〉.
2. Draw the recursion tree for $T(n) = 2T(\sqrt{n}) + log_2 n$ and provide a tight asymptotic bound on its solution.
3. Given tight asymptotic bounds for $T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{3n}{4}\right) + n$.
4. Solve $T(n) = 9T\left(\frac{n}{3}\right) + n$ using Θ
5. (1).Give two functions that in $O(n^2)$ but not in $o(n^2)$.
   (2). Give two functions that in $\Omega(n^2)$ but not in $\omega(n^2)$.
6. Prove $\log(n!) = n \log n$
7. Use the master method to give tight asymptotic bounds for the following recurrences

   (1).$T(n) = 4T\left(\frac{n}{2}\right) + n$.

   (2).$T(n) = 4T\left(\frac{n}{2}\right) + n^2$

   (3).$T(n) = 4T\left(\frac{n}{2}\right) + n^3$
8. Let f(n) + g(n) be asymptotically nonnegative functions. Using the basic definition of Θ-notation, prove that max(f(n),g(n))=Θ(f(n)+g(n)).
9. Prove $a^{log_b c} = c^{log_b a}$
10. Show that if $T(n) = \sqrt{n}T(\sqrt{n}) + n$,T(m)=k and $\sqrt[2^i]{n} \leq m$,

    then $T(n) = k \cdot n^{\frac{2^i - 1}{2^i}} + i \cdot n$.