3-3-21

Find the general solution of the given higher-order differential equation.

$$y''' + 3y'' + 3y' + y = 0$$ +

Sol:

21. From $m^3 + 3m^2 + 3m + 1 = 0$ we obtain $m = -1$, $m = -1$, and $m = -1$ so that

$$y = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x}.$$

3-4-3

Solve the given differential equation by undetermined coefficients.

$$y'' - 10y' + 25y = 30x + 3$$

Sol:

From $m^2 - 10m + 25 = 0$ we find $m_1 = m_2 = 5$. Then $y_c = c_1 e^{5x} + c_2 x e^{5x}$ and we assume $y_p = Ax + B.$ Substituting into the differential equation we obtain

$25A = 30$ and

$-10A + 25B = 3$. Then $A = \frac{6}{5}, B = \frac{3}{5}, yp = \frac{6}{5}x + \frac{3}{5}$, and

$$y = c_1 e^{5x} + c_2 x e^{5x} + \frac{6}{5}x + \frac{3}{5}$$

在這裡鍵入方程式。

3-5-15

Solve each differential equation by variation of parameters.

$$y'' + 2y' + y = e^{-t}\ln t$$

Sol:

15. The auxiliary equation is $m^2 + 2m + 1 = (m+1)^2 = 0$, so $y_c = c_1 e^{-t} + c_2 t e^{-t}$ and

$$W = \begin{vmatrix} e^{-t} & te^{-t} \\ -e^{-t} & -te^{-t} + e^{-t} \end{vmatrix} = e^{-2t}.$$

Identifying $f(t) = e^{-t}\ln t$ we obtain

$$u_1' = -\frac{te^{-t}e^{-t}\ln t}{e^{-2t}} = -t\ln t$$

$$u_2' = \frac{e^{-t}e^{-t}\ln t}{e^{-2t}} = \ln t.$$

Then

$$u_1 = -\frac{1}{2}t^2\ln t + \frac{1}{4}t^2$$

$$u_2 = t\ln t - t$$

and

$$\begin{aligned} y &= c_1 e^{-t} + c_2 te^{-t} - \frac{1}{2}t^2 e^{-t}\ln t + \frac{1}{4}t^2 e^{-t} + t^2 e^{-t}\ln t - t^2 e^{-t} \\ &= c_1 e^{-t} + c_2 te^{-t} + \frac{1}{2}t^2 e^{-t}\ln t - \frac{3}{4}t^2 e^{-t}. \end{aligned}$$

3-6-32

Solve the given boundary-value problem.

$$x^2y'' - 3xy' + 5y = 0, \;\; y(1) = 0, y(e) = 1$$

Sol:

32. Proceeding in the same manner as exercise 31, we assume the solution $y = x^m$ and force it into the Cauchy-Euler equation which leads to

$$x^m[m^2 - 4m + 5] = 0$$

The roots to the auxiliary equation are complex conjugates $m = 2 \pm i$ and so the general solution is $y(x) = c_1 x^2\cos(\ln x) + c_2 x^2 \sin(\ln x)$. Next we apply the boundary conditions. The first gives us $y(1) = c_1 + 0 = 0$ so $c_1 = 0$. The second condition gives us $y(e) = c_2 e^2 \sin(\ln e) = 1$. Solving this for the constant leads to $c_2 = (e^2 \sin 1)^{-1}$. The final solution therefore is $y(x) = (e^2 \sin 1)^{-1} x^2 \sin(\ln x)$.