2016 Algorithm Quiz Solutions

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Question 1(10pts)

- PPT CH22 P14
- BFS(V, E, s)
- for each $u \in V$ - {s}
- do $d[u] \leftarrow \infty$
- $d[s] \leftarrow 0$
- $Q \leftarrow \phi$
- ENQUEUE(Q, s)
- While $Q \neq \phi$
- do $u \leftarrow$ DEQUEUE(Q)
- for each $v \in Adj[u]$
- do if $d[v] = \infty$
- then $d[v] \leftarrow d[u] + 1$
- ENQUEUE(Q, v)

- Complexity O(V + E).

Question 2(10pts)

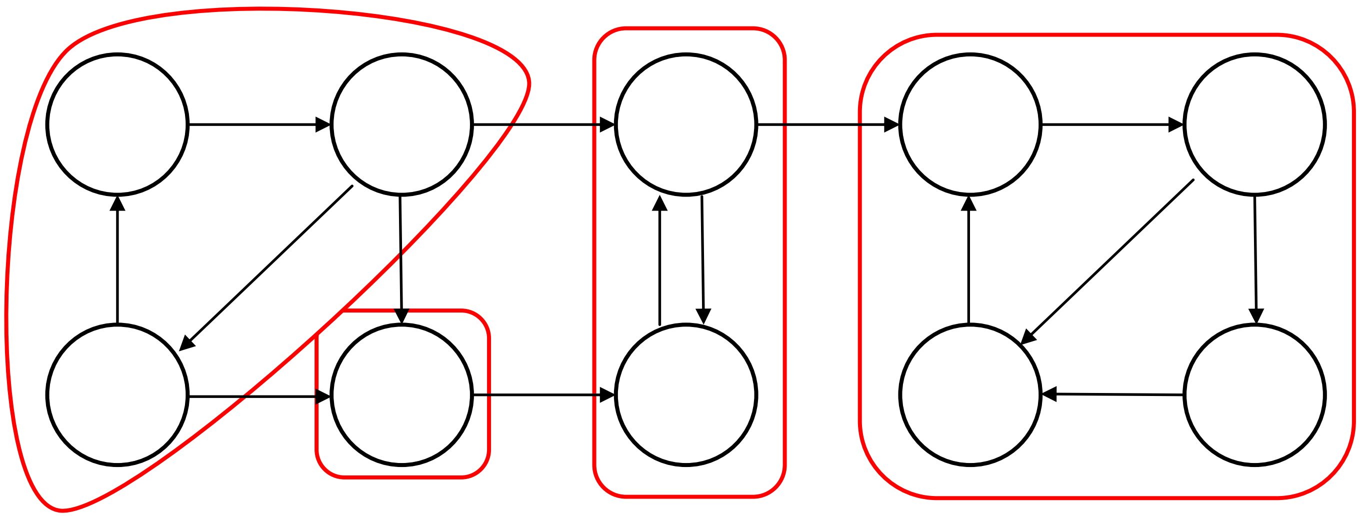


Figure 1

Question 3(20pts)

```
n ← length[p] − 1
for i ← 1 to n do
    m[i, i] ← 0
end for
for ℓ ← 2 to n do
    for i ← 1 to n − ℓ + 1 do
        j ← i + ℓ − 1
        m[i, j] ← ∞
        for k ← i to j − 1 do
            q ← m[i, k] + m[k + 1, j] + p_{i−1} · p_k · p_j
            if q < m[i, j] then
                m[i, j] ← q
                s[i, j] ← k
            end if
        end for
    end for
end for
```

$$m[i,j] = \begin{cases} 0 & \text{if } i = j \\ \min_{i \le k < j}\{m[i,k] + m[k+1,j] & \text{if } i < j \\ +p_{i-1} \cdot p_k \cdot p_j\} & \end{cases}$$

We have three nested loops:

1. ℓ, length, $O(n)$ iterations
2. i, start, $O(n)$ iterations
3. k, split point, $O(n)$ iterations

Body of loops: constant complexity.

Total complexity: $O(n^3)$

- 配分(10%)
    - Algorithm(or Pseudocode) 8分
    - Time complexity 2分

Question 3(20pts)

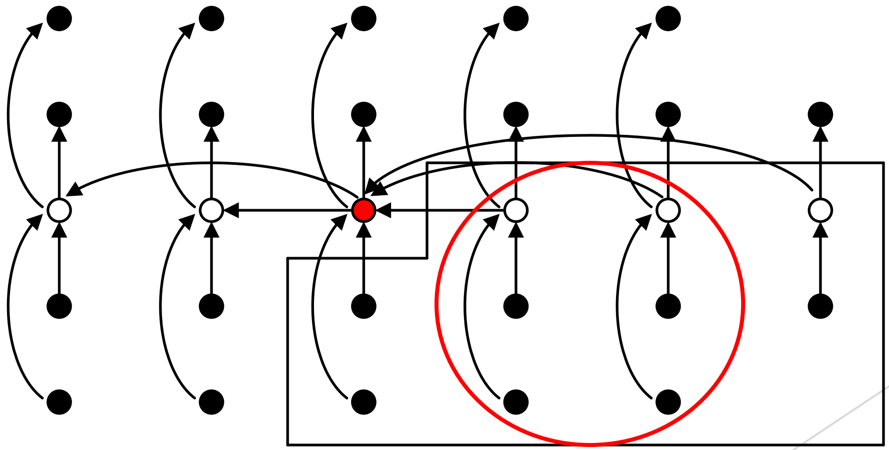
	1	2	3	4	5	6
1	0	15750	7875	9375	11875	15125
2		0	2625	4375	7125	10500
3			0	750	2500	5375
4				0	1000	3500
5					0	5000
6						0

	2	3	4	5	6
1	1	1	3	3	3
2		2	3	3	3
3			3	3	3
4				4	5
5					5

- So, ANS= $((A_1(A_2A_3))((A_4A_5)A_6))$
- minimum number of scalar multiplications = 15125

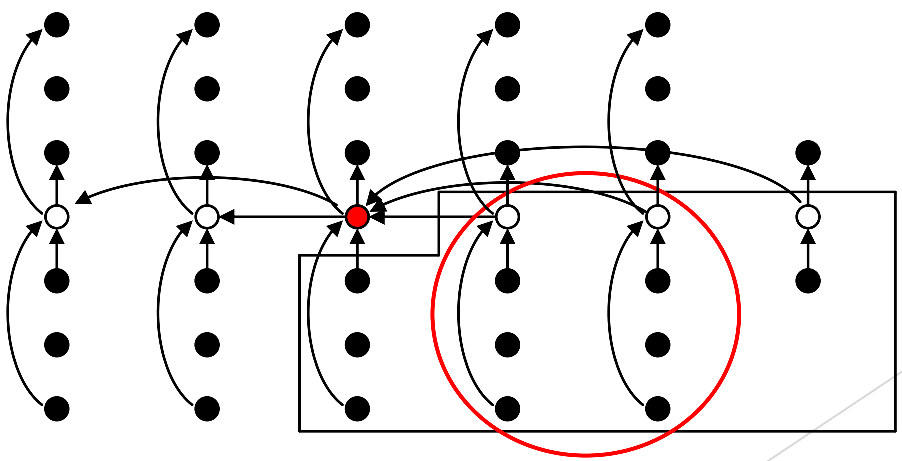
Question 4(10pts)

- Ans: group of 7 work, but groups of 3 does not work
- 以課堂上所教之group of 5 為例，決定median of medians x後，至少有$\frac{3n}{10}-6$個 elements大於x，所以至多有$\frac{7n}{10}+6$個elements小於x，worst case為在$\frac{7n}{10}+6$中找i-th smallest element
  Total cost:$T(n) \le T\left(\left\lceil\frac{n}{5}\right\rceil\right)+T\left(\frac{7n}{10}+6\right)+O(n)$=>$T(n)=O(n)$
  示意圖:



Question 4(10pts)cont.

- group of 7 也是相同的概念
- 決定median of medians x後，至少有 $\frac{2n}{7}-8$ 個 elements大於x，所以至多有$\frac{5n}{7}+8$個elements小於x，worst case為在$\frac{5n}{7}$+8中找 i-th smallest element
  Total cost:$T(n) \leq T\left(\left\lceil\frac{n}{7}\right\rceil\right)+T\left(\frac{5n}{7}+8\right)+O(n)$=>$T(n)=O(n)$
  示意圖:

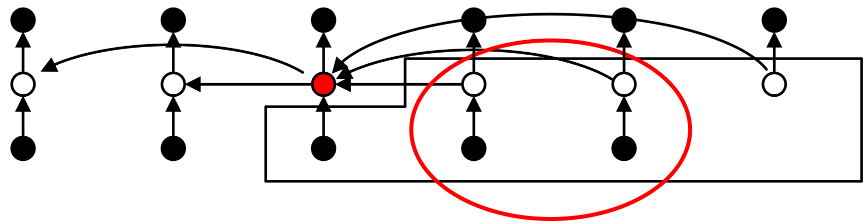


Question 4(10pts)cont.

- $T(n) \le T\left(\left\lceil \frac{n}{7} \right\rceil\right) + T\left(\frac{5n}{7} + 8\right) + O(n)$
  利用substitution method得到的結果為
- $T(n) \le \frac{6cn}{7} + 9c + an \le cn$
  $$= cn + (-\frac{cn}{7} + 9c + an)$$
- $-\frac{cn}{7} + 9c + an \le 0$
  $$\frac{cn}{7} - 9c \ge an$$
  $$cn - 63c \ge 7an$$
  $$c(n - 63) \ge 7an$$
  取$c \ge 7a(\frac{n}{n-63})$ 得到 $T(n) = O(n)$

Question 4(10pts)cont.

- group of 3 則不能達到線性時間
- 決定median of medians x後，至少有$\frac{n}{3}-4$個 elements大於x，所以至多有$\frac{2n}{3}$+4個elements小於x，worst case為在$\frac{2n}{3}+4$中找
  i-th smallest element
  Total cost:$T(n) \le T\left(\left\lceil\frac{n}{3}\right\rceil\right)+T\left(\frac{2n}{3}+4\right)+O(n)$
  示意圖:



Question 4(10pts)cont.

- $T(n) \leq T\left(\left\lceil \frac{n}{3} \right\rceil\right) + T\left(\frac{2n}{3} + 4\right) + O(n)$

  當n足夠大時$T(\frac{2n}{3} + 4)$可視為$T\left(\frac{2n}{3}\right)$

  利用substitution method得到的結果為

  $T(n) \leq T(n/3) + T(2n/3) + cn$

  $\leq d\,(n/3)\lg(n/3) + d(2n/3)\lg(2n/3) + cn$

  $= (d\,(n/3)\lg n - d\,(n/3)\lg 3) + (d\,(2n/3)\lg n - d\,(2n/3)\lg(3/2)) + cn$

  $= d\,n\lg n - d\,((n/3)\lg 3 + (2n/3)\lg(3/2)) + cn$

  $= d\,n\lg n - d\,((n/3)\lg 3 + (2n/3)\lg 3 - (2n/3)\lg 2) + cn$

  $= d\,n\lg n - d\,n\,(\lg 3 - 2/3) + cn$

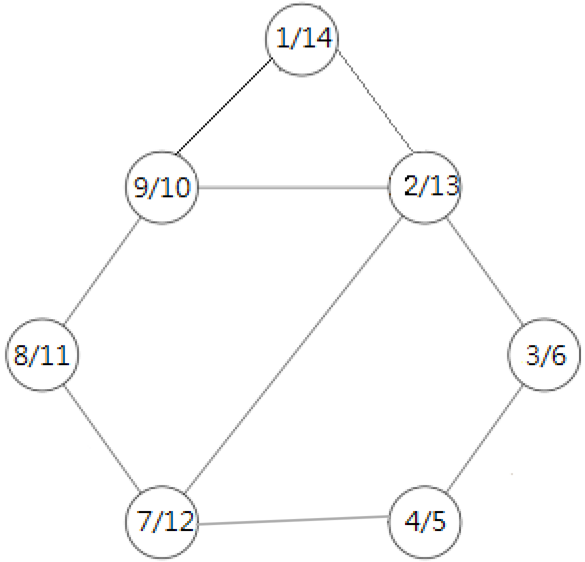
  $\leq d\,n\lg n \qquad$ if $-d\,n\,(\lg 3 - 2/3) + cn \leq 0,$

- $T(n) = O(n\log n)$

Question 4(10pts)cont.

- 配分方式(10%)
  - Group 7:yes 5%(yes or no 一分/解釋 四分)
  - Group 3:no 5%(yes or no 一分/解釋 四分)

Question 5(10pts)



Question 6(10pts)

item	value	weight	Value/weight
1	8	6	4/3
2	6	5	6/5
3	3	3	3/3

- Maximum capacity of knapsack is 8
- $(4/3)*6 + (6/5)*(8-6) = 8 + 12/5 = 10.4$

Question 7(10pts)

False

不必記錄所有子問題也能計算出所需的解答

以fibonacci number為例, 我們可以只用O(1) 空間去計算

令n為想求出的fibonacci number sequence的index(index從0開始)

```
prev = curr = 1;
i=2;
while(i<n){
    next = prev + curr;
    prev = curr;
    curr = next;
}
```

Question 8(10pts)

- (I) false

  fractional knapsack才能以greedy strategy解決

- (II)false

  Theorem[CH22, p31]

  In DFS of an undirected graph, we get only tree and

  back edges. No forward or cross edges.

- (III)true

  [CH15, p16]

  Two basic approaches: top-down with memoization, and bottom-up.

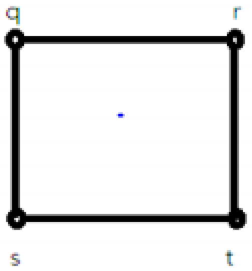
- (IV)true

  從圖可得知 q到t的最長path是q-r-t

  但是q-r並不是q到r的最長path (應該是q-s-t-r)

  而r-t也不是r到t的最長path (應該是r-q-s-t)

  所以不具有optimal substructure.



- (V)true

  Lemma[CH22, p35]

  A directed graph G is acyclic if and only if a DFS

  of G yields no back edges.

Question 9(10pts)

根據題意為0-1背包問題

利用公式建立表格, 選取 item 1 + item 2 + item 3 + item 7 ,最佳總價值是38

$$profit[i][j] = \begin{cases} profit[i-1][j], & if \;\; weight[i] > j \\ \max\begin{pmatrix} profit[i-1][j], \\ profit[i-1][j-weight[i]] + v[i] \end{pmatrix}, & otherwise \end{cases}$$

size / Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
2	0	8	8	8	14	14	14	14	14	14	14	14	14	14	14	14
3	0	8	8	8	15	15	23	23	23	29	29	29	29	29	29	29
4	0	8	8	8	15	15	23	23	23	29	29	29	29	32	32	32
5	0	8	8	8	15	15	23	23	23	29	29	29	31	32	32	32
6	0	8	8	13	15	15	23	23	28	29	29	34	34	34	36	37
7	0	8	8	13	15	15	23	23	28	29	29	34	34	34	37	38