Engineering Mathematics Homework 3-Solution

1. $Solve:(e^{2y} - y\cos xy)dx + (2xe^{2y} - x\cos xy + 2y)dy = 0$

Ans:

$$\frac{\partial M}{\partial y} = 2e^{2y} + xy\sin xy - \cos xy = \frac{\partial N}{\partial x}$$

$$The\ equation\ is\ exact!$$

$$\partial u = (e^{2y} - y\cos xy)\partial x \qquad \partial u = (2xe^{2y} - x\cos xy + 2y)\partial y$$

$$u = xe^{2y} - \sin xy + f(y) \qquad u = xe^{2y} - \sin xy + y^2 + g(x)$$

$$f(y) = y^2 \qquad g(x) = 0$$

$$u = xe^{2y} - \sin xy + y^2 = c$$

2. $Solve: \frac{dy}{dx} = \frac{1}{3y^2 - 3xy^2}$

Ans:

$$dx - (3y^2 - 3xy^2)dy = 0$$

$$\frac{\partial M}{\partial y} = 0 \qquad\qquad \frac{\partial N}{\partial x} = 3y^2$$

$$Not\ exact!$$

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} dy = \frac{3y^2}{1} dy = \frac{dI}{I}$$

$$I = e^{\int 3y^2 dy} = e^{y^3}$$

$$\Rightarrow e^{y^3} dx - (3y^2 e^{y^3} - 3xy^2 e^{y^3})dy = 0$$

$$\partial u = (e^{y^3})\partial x \qquad\qquad \partial u = -(3y^2 e^{y^3} - 3xy^2 e^{y^3})\partial y$$

$$u = xe^{y^3} + f(y) \qquad\qquad u = -e^{y^3} + xe^{y^3} + g(x)$$

$$f(y) = -e^{y^3} \qquad\qquad g(x) = 0$$

$$u = -e^{y^3} + xe^{y^3} = c$$

3. $Solve: xydx + (2x^2 + 3y^2 - 20)dy = 0$

Ans:

$$\frac{\partial M}{\partial y} = x \qquad \frac{\partial N}{\partial x} = 4x$$

$$Not\ exact!$$

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} dy = \frac{3x}{xy} dy = \frac{dI}{I}$$

$$I = e^{\int \frac{3}{y} dy} = e^{\ln y^3} = y^3$$

$$\Rightarrow xy^4 dx + (2x^2 y^3 + 3y^5 - 20y^3)dy = 0$$

$$\partial u = (xy^4)\partial x \qquad \partial u = (2x^2 y^3 + 3y^5 - 20y^3)\partial y$$

$$u = \frac{1}{2}x^2 y^4 + f(y) \qquad u = \frac{1}{2}x^2 y^4 + \frac{1}{2}y^6 - 5y^4 + g(x)$$

$$f(y) = \frac{1}{2}y^6 - 5y^4 \qquad g(x) = 0$$

$$u = \frac{1}{2}x^2 y^4 + \frac{1}{2}y^6 - 5y^4 = c$$