# Discrete Mathematics (2012 Spring) Midterm I

1. (20%) For each of the following statements, determine whether it is correct or not.
(1) $\phi \in \phi$ (2) $\phi \subset \phi$ (3) $\phi \subseteq \{\phi\}$ (4) $\phi \subset \{\phi\}$ (5) $\phi \in P(\phi)$ (6) $A\Delta(B\Delta C) = (A\Delta B)\Delta C$ (7) $A\Delta(B \cup C) = (A\Delta B) \cup (A\Delta C)$ (8) $\overline{\overline{(A \cup B) \cap C} \cup \overline{B}} = B \cap C$ (9) $R^+ \subseteq Q$
(10) $P(A \cap B) = P(A) \cap P(B)$.

2. (15%) Define the connective "Nand" by $(p \uparrow q) \Leftrightarrow \neg(p \wedge q)$, for any statements $p$, $q$. Represent the following using only this connective. (a) $\neg p$, (b) $p \wedge q$, (c) $p \rightarrow q$.

3. (10%) For all $n \in Z^+$, show that if $n \geq 64$, then $n$ can be written as a sum of 5's and/or 17's. (hints: Use Mathematical induction and try 5 initial cases)

4. (10%) Prove that gcd($n$, $n$+2) = 1 or 2.

5. (10%) Determine the value of $c \in Z^+, 13 \leq c \leq 20$ such that equation $84x + 990y = c$ has solutions. Determine the solutions for this $c$ value.

6. (10%) (a) Determine the coefficient of $x^3y^2z^{-1}$ for the complete expansion of $(x - 4y)^5(3z^{-1} + 3)^4$ (b) Determine the number of nonnegative integer solutions of $x_1 + x_2 + x_3 + \ldots + x_6 = 27$ and $x_1 + x_2 + x_3 = 6$ where $x_1, x_2, x_3 > 0$, $x_4 > 3$.

7. (10%) (a) How many positive integers $n$ can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want $n$ to exceed 4,500,000? (b) How many arrangements of the letters in "MISCELLANEOUS" have no pair of consecutive identical letters?

8. (15%) Establish the validity of the following argument. (Don't need to write reasons.)

$$\begin{array}{l} \forall x[p(x) \vee q(x)] \\ \underline{\forall x[(\neg p(x) \wedge q(x)) \rightarrow r(x)]} \\ \quad \therefore \forall x[\neg r(x) \rightarrow p(x)] \end{array}$$