

DISCRETE MATHEMATICS – CH3 Homework3

Textbook assignment (30 pts)

3-1

8. For $A = \{1, 2, 3, 4, 5, 6, 7\}$, determine the number of
- e) subsets of A containing three elements
 - f) subsets of A containing 1, 2
 - g) subsets of A containing five elements, including 1, 2
 - h) subsets of A with an even number of elements
 - i) subsets of A with an odd number of elements
 - j) subsets of A containing m elements, including n of 7 elements
- e) $C(7,3)=35$
- f) $2^5 = 32$
- g) $C(5,3)=10$
- h) $C(7,0)+C(7,2)+C(7,4)+C(7,6)=64$
- i) $C(7,1)+C(7,3)+C(7,5)+C(7,7)=64$
- j) $C(7-n,m-n)$

3-2

8. Using Venn diagrams, investigate the truth or falsity of each of the following, for sets $A, B, C \subseteq U$.

a) $A \triangle (B \cup C) = (A \triangle B) \cup (A \triangle C)$

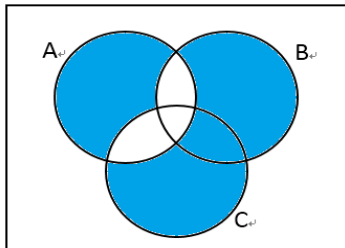
b) $A - (B \cap C) = (A - B) \cup (A - C)$

c) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$

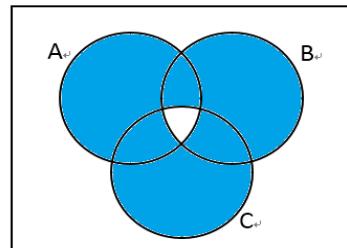
a.

False

$$A \triangle (B \cup C)$$



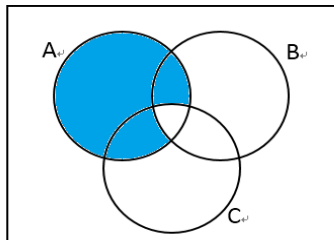
$$(A \triangle B) \cup (A \triangle C)$$



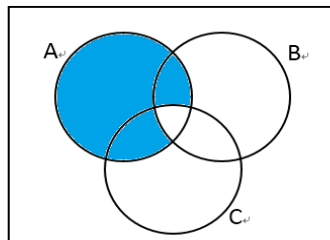
b.

True

$$A - (B \cap C)$$



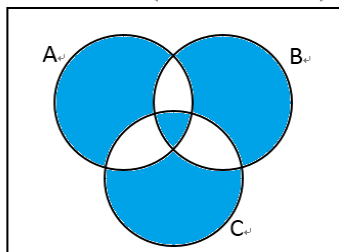
$$(A - B) \cup (A - C)$$



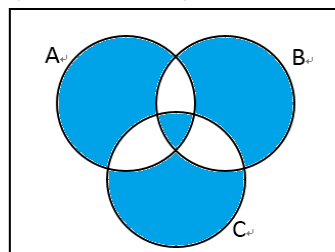
c.

True

$$A \triangle (B \triangle C)$$



$$(A \triangle B) \triangle C$$



3-3

- 9 How many arrangements of the letters in **AATTCCCCGG** have no pair of consecutive identical letters? (brute-force method gets 5pts only)

$$\begin{aligned} \text{Total} &= |A| + |T| + |C| + |G| + |A \cap T| + |A \cap C| + |A \cap G| + |T \cap C| + |T \cap G| + |C \cap G| \\ &\quad - |A \cap T \cap C| - |A \cap T \cap G| - |A \cap C \cap G| + |A \cap T \cap C \cap G| \\ &= (10!/2!2!2!4!) - 3 \cdot (9!/2!4!2!) - \{ (6!/2!2!2!) \cdot [2 \cdot (7!/5!2!) + (7!/5!2!) + (7!/4!3!) \cdot \\ &\quad (3!/2!) + (7!/6!)] \} + 3 \cdot (8!/4!2!) + 3 \cdot (5!/2!2!) - (7!/4!) - \\ &\quad 3 \cdot \{ (4!/2!) \cdot [2 \cdot (5!/2!3!) + (5!/2!3!) + (5!/3!2!) \cdot (3!/2!) + (4!/3!)] \} + 3! \cdot [2 \cdot (4!/2!2!) + (4! \\ &\quad /2!2!) + (4!/2!2!) \cdot (3!/2!) + (4!/3!)] \end{aligned}$$

Advanced assignment (20, 10 pts)

- Read Example 3.9 (page 138, page 128 in old textbook) and write your comments.
 - What does this example say?
 - What are its extension and application?
 - What does you get after this reading?
- Enhance Exercise 3-(10), design new rules and calculate it. More difficult more scores!