

**Discrete Mathematics (2011 Spring) Midterm I**

1. (12%) For sets  $A, B \subseteq U$ , in each of the following statements, **determine** and **explain** whether it is correct or not.
  - (1).  $P(A \cup B) = P(A) \cup P(B)$
  - (2).  $P(A \cap B) = P(A) \cap P(B)$
  - (3).  $(A - B) \cup (A \cap B) = U$
  - (4).  $\overline{A} \cup \overline{B} \cup (A \cap B \cap \overline{C}) = \overline{A} \cup \overline{B} \cup \overline{C}$
2. (10%) Negate and simplify the statement  $p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)$ .
3. (15%) Define the connective “Nand” by  $(p \uparrow q) \Leftrightarrow \neg(p \wedge q)$ , for any statements  $p, q$ . Represent the following using only this connective. (a)  $p \vee q$ , (b)  $p \wedge q$ , (c)  $p \rightarrow q$ .
4. (10%) In all compositions of 18, what are the probabilities of (a) each summand is a multiple of 3, and (b) the first summand is 3?
5. (15%) Determine the value of  $c \in \mathbb{Z}^+, 10 \leq c \leq 16$  such that equation  $84x + 990y = c$  has solutions. Determine the solutions for this  $c$  value.
6. (10%) Determine the following value (a) the coefficient of  $x^4 y^2 z^{-2}$  for the complete expansion of  $(x - 2y)^6 (3z^{-1} + 4)^4$ , (b) the coefficient of  $x^3 y^4$  for the complete expansion of  $(x - 2y)^3 (3x + 4y)^4$ . [Note: Exhaustive expansion is not allowed.]
7. (8%) How many permutations of the 26 letters of the alphabet contain (a) either the pattern “OUT” or the pattern “DIG”? (b) neither the pattern “MAN” nor the pattern “ANT”?
8. (10%) Please give TWO different solutions to determine the number of integer solutions for  $x_1 + x_2 + x_3 < 10, x_1, x_2, x_3 > 0$ .
9. (10%) If  $n \in \mathbb{N}$ , prove that  $5F_{n+2} = L_{n+4} - L_n$ , where  $F_{n+2} = F_{n+1} + F_n, F_0 = F_1 = 1$  and  $L_{n+2} = L_{n+1} + L_n, L_0 = 2, L_1 = 1$ .