Algorithm 2017 Spring
Homework 4 Solutions

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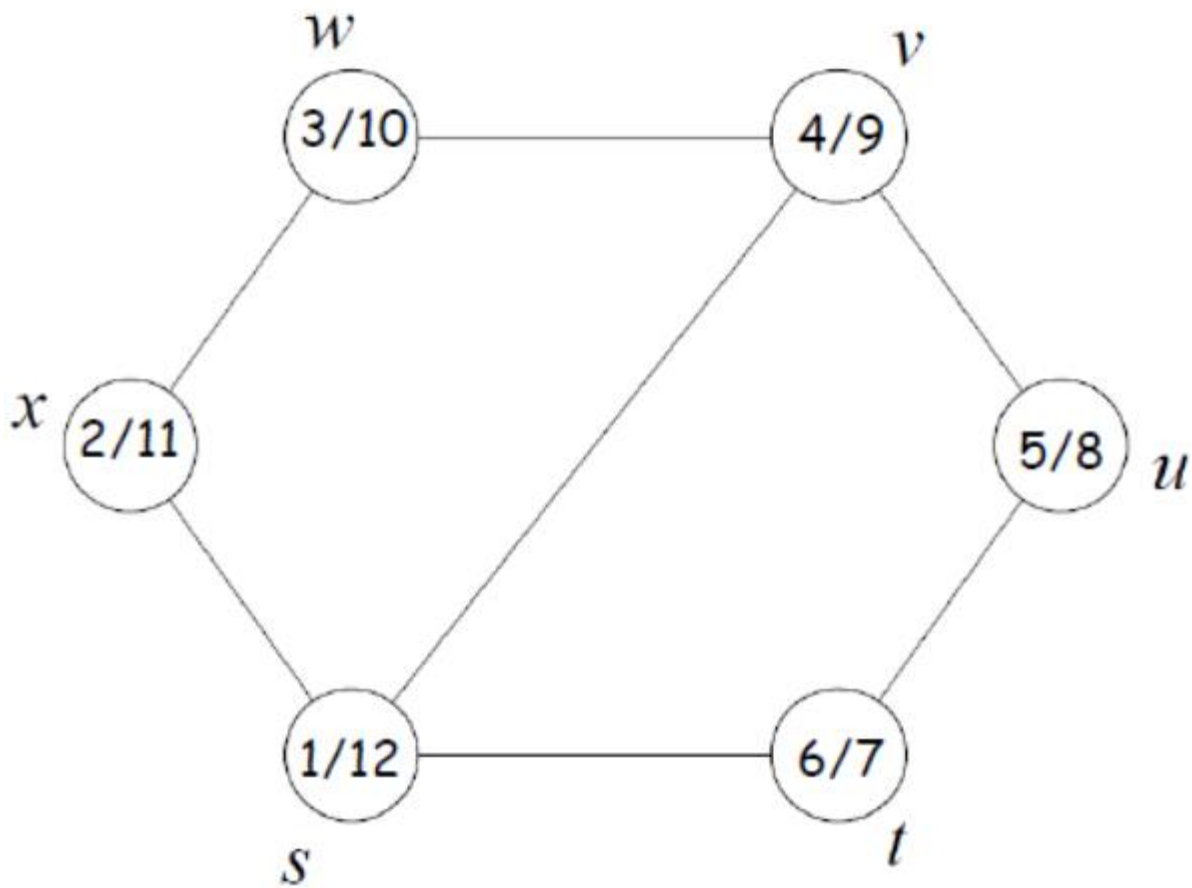
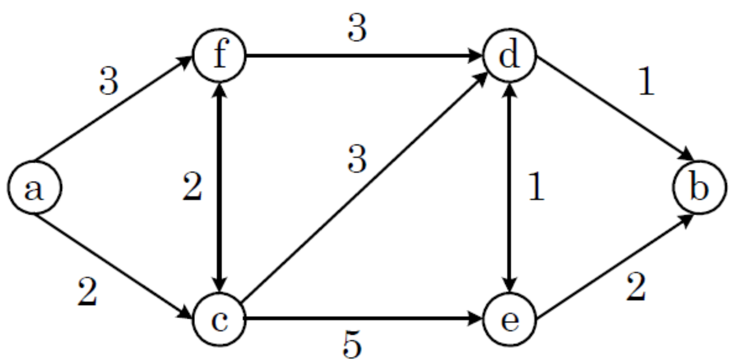
1. Depth-First-Search algorithm
the timestamp (discovery time & finish time)



2.

After forming the augmented constraint graph and seeking the shortest path from node 0 to all other nodes, using an algorithm with negative length cycle detection, one finds there is a negative length cycle (2, 3, 5, 4, 2) with length $1 - 7 + 10 - 6 = -2$. Thus the system is infeasible.

3a. (10pts) Describe such a process clearly on the following di-graph with vertex a as the source.

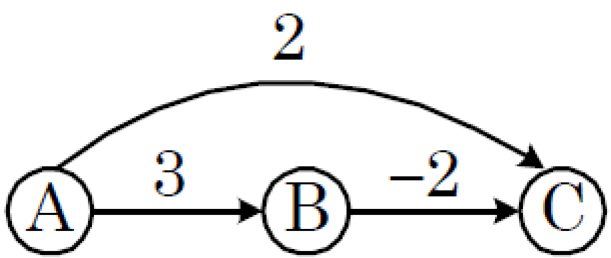


s	u	d[b]	d[c]	d[d]	d[e]	d[f]
{a}	c	∞	$\underline{2}$	∞	∞	3
{a, c}	f	∞	2	5	7	$\underline{3}$
{a, c, f}	d	∞	2	$\underline{5}$	7	3
{a, c, f, d}	b	$\underline{6}$	2	5	6	3
{a, c, f, d, b}	e	6	2	5	$\underline{6}$	3
{a, c, f, d, b, e}		6	2	5	6	3

3b. (10pts) Under what condition Dijkstra’s algorithm will not work? Given an example to explain your answer.

在有negative edge 時Dijkstra’s algorithm 可能失效

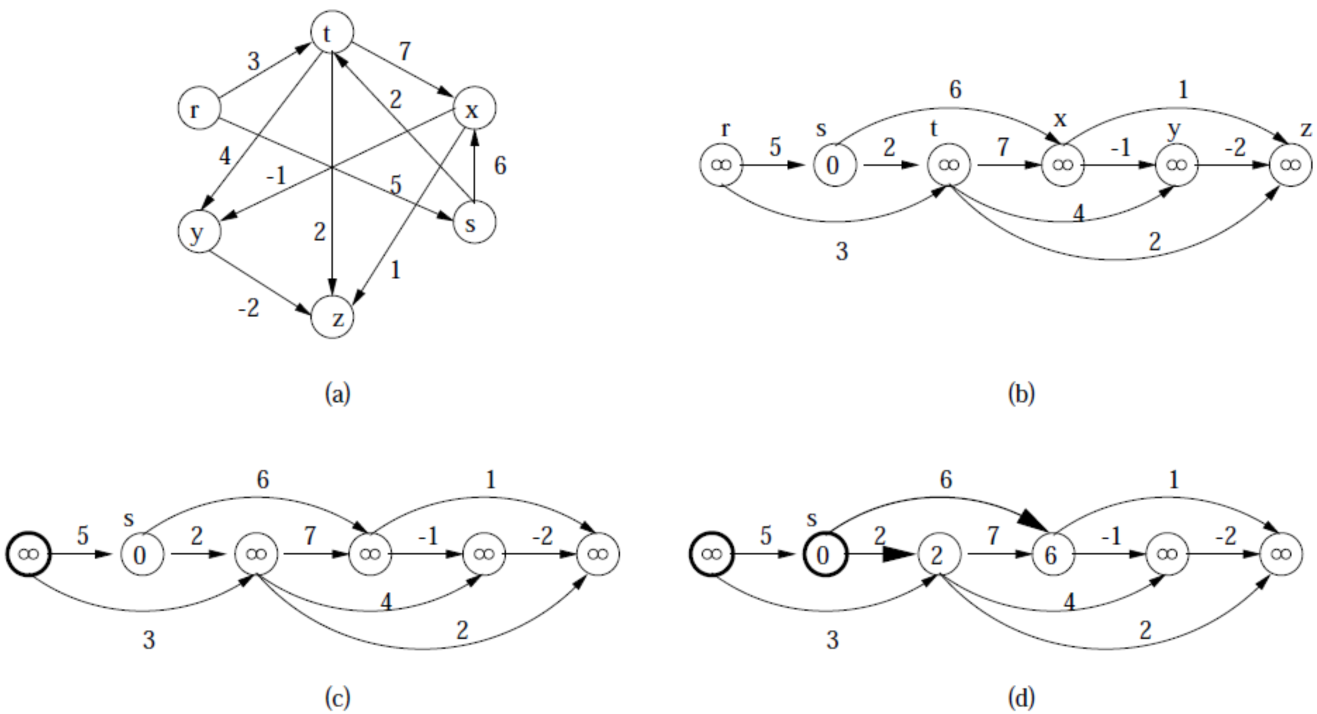
EX:



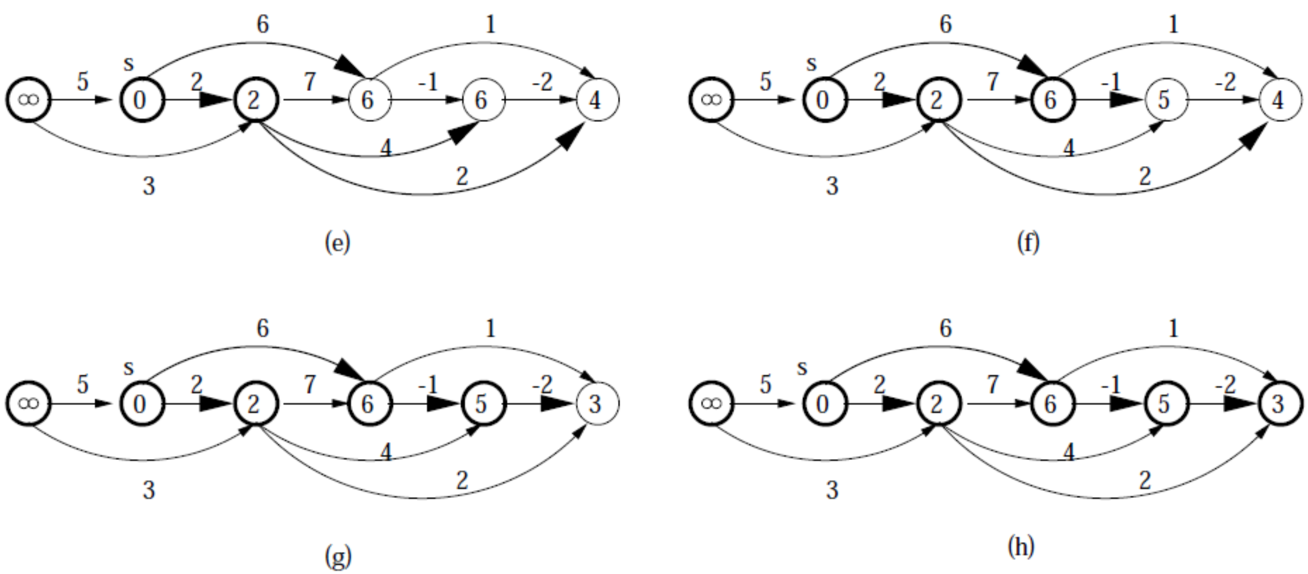
用Dijkstra’s algorithm時得到A到C的最短路徑為(A → C)

但實際答案為(A → B → C)

4.Run DAG-SHORTEST-PATHS step by step on the directed graph of the figure, using vertex s as the source. (10%)



4. Run DAG-SHORTEST-PATHS step by step on the directed graph of the figure, using vertex s as the source. (10%)



5. Give an algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle. Your algorithm should run in $O(V)$ time, independent of $|E|$.

- An undirected graph is acyclic (i.e., a forest) if and only if a DFS yields no back edges.
  - If there is a back edge, there is a cycle.
  - If there is no back edge, then by Theorem 22.10, there are only tree edges.
- Hence, the graph is acyclic.
- Thus, we can run DFS: if we find a back edge, there is a cycle.
- Time: $O(V)$.(We can simply DFS. If find a back edge, there is a cycle. The complexity is $O(V)$ instead of $O(E + V)$. Since if there is a back edge, it must be found before seeing $|V|$ distinct edges. This is because in a acyclic (undirected) forest, $|E| \le |V| - 1$, If it has back edge, $|E| \le |V|$)

5. Give an algorithm that determines whether or not a given undirected graph G = (V ,E) contains a cycle. Your algorithm should run in O(V) time, independent of |E|.

Pseudocode: Uses a global timestamp *time*.

```
DFS(V,E)
for each u ∈ V
    do color[u] ← WHITE
time ← 0
for each u ∈ V
    do if color[u] = WHITE
        then DFS - Visit(u)
```

```
DFS - Visit(u)
color[u] ← GRAY                  ❑ discover u
time ← time +1
d[u] ← time
for each v ∈ Adj[u]              ❑ explore (u, v)
    do if color[v] = WHITE
        then DFS - Visit(v)
color[u] ← BLACK
time ← time +1
f[u] ← time                      ❑ finish u
```