DISCRETE MATHEMATICS – CH2 Homework2

2.2

4. For primitive statements p, q, r, and s, simplify the compound statement

$$[[[(p \land q) \land r] \lor [(p \land q) \land \neg r]] \lor \neg q] \rightarrow s.$$

Based on distribution law:

5) $p \lor (q \land r) \Leftrightarrow (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \Leftrightarrow (p \land q) \lor (p \land r)$	*Distributive* Laws

[[(p^q)^r]V[(p^q)^-r]] ←→ (p^q)^(rV-r)

Based on Inverse law:

8) $p \lor \neg p \Leftrightarrow T_0$ $p \land \neg p \Leftrightarrow F_0$	*Inverse* Laws

(p^q)^(rV-r) ←→ (p^q)^T_0

Based on Identity law:

7) $p \lor F_0 \Leftrightarrow p$ $p \land T_0 \Leftrightarrow p$	*Identity* Laws

(p^q)^T_0 ←→ p^q

Now our statement is: [(p^q)V-q]->s

Based on Commutative law:

3) $p \lor q \Leftrightarrow q \lor p$ $p \land q \Leftrightarrow q \land p$	*Commutative* Laws

[(p^q)V-q] ←→[-qV(p^q)]

Based on Distributive law:

5) $p \lor (q \land r) \Leftrightarrow (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \Leftrightarrow (p \land q) \lor (p \land r)$	*Distributive* Laws

[-qV(p^q)] ←→(-qVp)^(-qVq)

Based on Inverse law:

8) $p \lor \neg p \Leftrightarrow T_0$ $p \land \neg p \Leftrightarrow F_0$	*Inverse* Laws

(-qVp)^(-qVq) ←→(-qVp)^T_0

Based on Identitiy law:

7) $p \vee F_0 \Leftrightarrow p$ $p \wedge T_0 \Leftrightarrow p$	*Identity* Laws

(-qVp)^T_0←→(-qVp)

Final Statement is

(-qVp)-> s or (pV-q)->s

p	q	s	-q	pV-q	(pV-q)->s	q->p	(q->p)->s
0	0	0	1	1	0	1	0
0	0	1	1	1	1	1	1
0	1	0	0	0	1	0	1
0	1	1	0	0	1	0	1
1	0	0	1	1	0	1	0
1	0	1	1	1	1	1	1
1	1	0	0	1	0	1	0
1	1	1	0	1	1	1	1

Therefore (pV-q)->s ← → (q->p)->s

16. The connective "Nor" or "Not … or…" is defined for any statements *p*, *q* by *(p ↓ q)* ⇔ ¬ *(p* ∨ *q)*. Represent the statements in parts (a) through (e) of Exercise 15, using only this connective.

a. –p ←→ -(pVp)

p	-p	Pvp	-(pVp)
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

b. pVq←→--(pVq)←→ -(p↓q) ←→(p↓q) ↓(p↓q)

p	q	pvq	-(pvq)	(p↓q)	(p↓q) ↓(p↓q)
0	0	0	1	1	0
0	1	1	0	0	1
1	0	1	0	0	1
1	1	1	0	0	1

c. p^q ←→-(-pV-q)←→-p↓-q

p	q	pvq	-(pvq)	(p↓q)	p^q	-p	-q	-pV-q	-(-pV-q)

0	0	0	1	1	0	1	1	1	0
0	1	1	0	0	0	1	0	1	0
1	0	1	0	0	0	0	1	1	0
1	1	1	0	0	1	0	0	0	1

d. p→q ←→ -pvq ←→ (-p↓q) ↓ (-p↓q)

p	q	pvq	-(pvq)	(p↓q)	p→q	-p	-pvq	-p↓q	(-p↓q) ↓ (-p↓q)
0	0	0	1	1	1	1	1	0	1
0	1	1	0	0	1	1	1	0	1
1	0	1	0	0	0	0	0	1	0
1	1	1	0	0	1	0	1	0	1

e. p←→q -[(p↓q) ↓(-p↓-q)]

or

(e) $p \leftrightarrow q \Longleftrightarrow (r \downarrow r) \downarrow (s \downarrow s)$ where r stands for $[(p \downarrow p) \downarrow q] \downarrow [(p \downarrow p) \downarrow q]$ and s for $[(q \downarrow q) \downarrow p] \downarrow [(q \downarrow q) \downarrow p]$

p	q	pvq	-(pvq)	(p↓q)	p←→q	-p↓-q	(p↓q) ↓(-p↓-q)	-[(p↓q) ↓(-p↓-q)]
0	0	0	1	1	1	0	0	1
0	1	1	0	0	0	0	1	0
1	0	1	0	0	0	0	1	0
1	1	1	0	0	1	1	0	1

2.3

10. Establish the validity of the following arguments. ((a)(f) for students with odd ID, (b)(g) for even ID)

a) $[(p \wedge \neg q) \wedge r] \rightarrow [(p \wedge r) \vee q]$

b) $[p \wedge (p \rightarrow q) \wedge (\neg q \vee r)] \rightarrow r$

f)
$$\begin{array}{l} p \wedge q \\ p \rightarrow (r \wedge q) \\ r \rightarrow (s \vee t) \\ \neg s \\ \hline \therefore t \end{array}$$

g)
$$\begin{array}{l} p \rightarrow (q \rightarrow r) \\ p \vee s \\ t \rightarrow q \\ \neg s \\ \hline \therefore \neg r \rightarrow \neg t \end{array}$$

(a)

(1)	$p \wedge \neg q$	Premise
(2)	p	Step (1) and the Rule of Conjunctive Simplification
(3)	r	Premise
(4)	$p \wedge r$	Steps (2), (3) and the Rule of Conjunction
(5)	$\therefore (p \wedge r) \vee q$	Step (4) and the Rule of Disjunctive Amplification

(b)

(1)	$p,\ p \rightarrow q$	Premises
(2)	q	Step (1) and the Rule of Detachment
(3)	$\neg q \vee r$	Premise
(4)	$q \rightarrow r$	Step (3) and $\neg q \vee r \Longleftrightarrow (q \rightarrow r)$
(5)	$\therefore r$	Steps (2), (4) and the Rule of Detachment

(f)

(1)	$p \wedge q$	Premise
(2)	p	Step (1) and the Rule of Conjunctive Simplification
(3)	$p \rightarrow (r \wedge q)$	Premise
(4)	$r \wedge q$	Steps (2), (3) and the Rule of Detachment
(5)	r	Step (4) and the Rule of Conjunctive Simplification
(6)	$r \rightarrow (s \vee t)$	Premise
(7)	$s \vee t$	Steps (5), (6) and the Rule of Detachment
(8)	$\neg s$	Premise
(9)	$\therefore t$	Steps (7), (8) and the Rule of Disjunctive Syllogism

(g)

(1)	$\neg s,\ p \vee s$	Premises
(2)	p	Step (1) and the Rule of Disjunctive Syllogism
(3)	$p \rightarrow (q \rightarrow r)$	Premise
(4)	$q \rightarrow r$	Steps (2), (3) and the Rule of Detachment
(5)	$t \rightarrow q$	Premise
(6)	$t \rightarrow r$	Steps (4), (5) and the Law of the Syllogism
(7)	$\therefore \neg r \rightarrow \neg t$	Step (6) and $(t \rightarrow r) \Longleftrightarrow (\neg r \rightarrow \neg t)$

2.4

20. Rewrite each of the following statements in the if-then form. Then write the converse, inverse, and contrapositive of your implication. For each result in parts (a) and (c) give the truth value for the implication and the truth values for its converse, inverse, and contrapositive. [In part (a) "divisibility" requires a remainder of 0.] (a)
a) [The universe comprises all positive integers.] Divisibility by 21 is a sufficient condition for divisibility by 7.

Implication: 若正整數可以被 21 整除，則可被 7 整除 (TRUE)
Converse: 若正整數可被 7 整除，則可被 21 整除 (FALSE)
Inverse: 若正整數不能被 21 整除，則不能被 7 整除 (FALSE)
Contrapositive: 若正整數不能被 7 整除，則不能被 21 整除 (TRUE)

Supplementary

4. Express the negation of the statement $p \leftrightarrow q$ in terms of the connectives $\wedge$ and $\vee$.

$p \leftrightarrow q \iff (p \rightarrow q) \wedge (q \rightarrow p) \iff (\neg p \vee q) \wedge (\neg q \vee p)$, so $\neg(p \leftrightarrow q) \iff \neg(\neg p \vee q) \vee \neg(\neg q \vee p) \iff (p \wedge \neg q) \vee (q \wedge \neg p)$

Advanced assignment (僅供參考)

- Write an argument (include statement description) and prove it is valid. (More complicated argument gets a higher score)
  - A 2pts-example:
    - p: It rains. q: absent from discrete mathematics
    - Argument $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$
      - » If it rains, I will be absent from discrete mathematics.
      - » I join the discrete mathematics today.
      - » Conclusion: Today is not rainy.