PS1: You need write down detail calculation process. Answer only can't get any score.
PS2: You can write answer on the reverse side of this test paper, if the front space is not enough.

NCKU CSIE Discrete Mathematics (2016 Spring) Homework Quiz 1

1. **(15 pts)** Find the coefficient of $w^2x^2y^2z^2$ in the expansion of (a) $(w+x+y+z+1)^{10}$, (b) $(2w-x+3y+z-2)^{12}$, (c) $(2w-x+3y+z^{0.5}-2)^{12}$

(a) $\binom{10}{2,2,2,2,2} = \frac{10!}{(2!)^5} = \frac{10!}{32} = 113400$ ※

(b) $\binom{12}{2,2,2,2,4}\cdot 2^2\cdot(-1)^2\cdot 3^2\cdot 1^2\cdot(-2)^4$

$= \frac{12!}{(2!)^4 4!}\cdot 2^2\cdot 3^2\cdot 2^4 = 718502400$ ※

(c) $\binom{12}{2,2,2,4,2}\cdot 2^2\cdot(-1)^2\cdot 3^2\cdot 1^4\cdot(-2)^2 = \frac{12!}{(2!)^4 4!}\cdot 2^2\cdot 3^2$

$= 449625600$ ※

2. **(15:10, 5 pts)** (a) How many nonnegative integer solutions are there to the pair of equations $x_1+x_2+x_3+\ldots+x_7 = 37$, $x_1+x_2+x_3 = 6$. (b) How many solutions in (a) have $x_1, x_2 > 0$.

(a) $x_1+x_2+x_3=6$ $\binom{3+6-1}{6}=\binom{8}{6}$

$x_4+x_5+x_6+x_7 = 37-6 = 31$ $\binom{4+31-1}{31}=\binom{34}{31}$

$\therefore Ans = \binom{8}{6}\binom{34}{31} = 28\cdot 5984 = 167552$

(b) $x_1'+x_2'+x_3 = 4$ $\binom{3+4-1}{4}=\binom{6}{4}$

$\therefore Ans = \binom{6}{4}\binom{34}{31} = 15\cdot 5984 = 89760$

3. **(10 pts)** (a) How many compositions of 20 that have each summand a multiple of 4? (b) Let n, m, k be positive integers with $n=mk$. How many compositions of n have each summand a multiple of k?

(a) $20 = 4(1+2+1+1)$

$1+2+1+1$ is a composition of 5

$\therefore$ the solution = the number of composition of 5

$= 2^{5-1} = 2^4 = 16$ ※

(b) Each such composition can be factored as k times a composition of m

$\therefore 2^{m-1}$ ※

4. **(10 pts)** In how many ways can 16 be written as a sum of 2's and 3's if the order of the summands is (a) not relevant? (b) relevant?

(a) $16 = 3\times 0+2\times 8$
$= 3\times 2+2\times 5$
$= 3\times 4+2\times 2$ $\therefore 3$ ※

(b) $16 = 3\times 0+2\times 8$ $\binom{8}{0}=1$
$= 3\times 2+2\times 5$ $\binom{7}{2}=21$
$= 3\times 4+2\times 2$ $\binom{6}{4}=15$ $\therefore 37$ ※

5. **(15 pts)** Negate each of the following and simplify the resulting statement. (a) $p\wedge(q\vee r)\wedge(\neg p\vee\neg q\vee r)$ (b) $p\to(\neg q\wedge r)$ (c) $\exists x[(p(x)\vee q(x))\to r(x)]$

(a) $\neg[p\wedge(q\vee r)\wedge(\neg p\vee\neg q\vee r)]$

$\Leftrightarrow \neg p\vee(\neg q\wedge\neg r)\vee(p\wedge q\wedge\neg r)$

$\Leftrightarrow (\neg q\wedge\neg r)\vee[\neg p\vee(p\wedge q\wedge\neg r)]$

$\Leftrightarrow (\neg q\wedge\neg r)\vee[T_0\wedge(\neg p\vee(q\wedge\neg r))]$

$\Leftrightarrow (\neg q\wedge\neg r)\vee[\neg p\vee(q\wedge\neg r)] \Leftrightarrow \neg p\vee[(\neg q\vee q)\wedge\neg r]$

$\Leftrightarrow \neg p\vee\neg r$

(b) $p\wedge(q\vee\neg r)$

(c) $\forall x[(p(x)\vee q(x))\wedge\neg r(x)]$

6. **(10 pts)** Express the negation of the statement p ↔ q in terms of the connectives ∧ and ∨.

$p\leftrightarrow q \Leftrightarrow (p\to q)\wedge(q\to p)$

$\Leftrightarrow (\neg p\vee q)\wedge(\neg q\vee p)$

so $\neg(p\leftrightarrow q)\Leftrightarrow\neg(\neg p\vee q)\vee\neg(\neg q\vee p)$

$\Leftrightarrow (p\wedge\neg q)\vee(q\wedge\neg p)$

7. **(5 pts)** How many distinct four-digit integers can one make from the digits 3, 3, 3, 7, 7, and 8?

no 7 : $\frac{4!}{3!} = 4$

one 7 and two 3 : $\frac{4!}{2!} = 12$

one 7 and three 3 : $\frac{4!}{3!} = 4$

two 7 and one 3 : $\frac{4!}{2!} = 12$

two 7 and two 3 : $\frac{4!}{2!2!} = 6$

$4+12+4+12+6 = 38$ ※

8. **(20 pts)** Define the connective "Nor" by $(p\downarrow q)\Leftrightarrow\neg(p\vee q)$, for any statements p, q. Represent the following using only this connective. (a) $\neg p$ (b) $p\vee q$, (c) $p\wedge q$ (d) $p\to q$.

(a) $\neg p\Leftrightarrow(p\downarrow p)$

(b) $p\vee q\Leftrightarrow\neg\neg(p\vee q)\Leftrightarrow\neg(p\downarrow q)$

$\Leftrightarrow (p\downarrow q)\downarrow(p\downarrow q)$

(c) $p\wedge q\Leftrightarrow\neg\neg p\wedge\neg\neg q\Leftrightarrow(\neg p\downarrow\neg q)$

$\Leftrightarrow (p\downarrow p)\downarrow(q\downarrow q)$

(d) $p\to q\Leftrightarrow\neg p\vee q\Leftrightarrow(\neg p\downarrow q)\downarrow(\neg p\downarrow q)\Leftrightarrow[(p\downarrow p)\downarrow q]\downarrow[(p\downarrow p)\downarrow q]$