# DISCRETE MATHEMATICS – CH1 Homework1

## 1-1 & 1-2

22. How many positive integers $n$ can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want $n$ to exceed 5,440,000?

$$n > 5{,}000{,}000 \Rightarrow \frac{6!}{2!} + \frac{6!}{2!2!} + \frac{6!}{2!2!} = 720$$

$$5{,}000{,}000 < n < 5{,}400{,}000 \Rightarrow \frac{5!}{2!} = 60$$

$$5{,}400{,}000 < n < 5{,}440{,}000 \Rightarrow 4! = 24$$

Result = 720 - 60 - 24 = 636

**30.** A sequence of letters of the form abcba, where the expression is unchanged upon reversing order, is an example of a palindrome (of five letters).
(a) If a letter may appear more than twice, how many palindromes of five letters are there? of six letters? (b) Repeat part (a) under the condition that no letter appears more than twice.

(a) for 5 letter: $26 \times 26 \times 26 \times 1 \times 1 = 26^3 = 17576$
for 6 letter: $26 \times 26 \times 26 \times 1 \times 1 \times 1 = 26^3 = 17576$

(b) for 5 letter: $26 \times 25 \times 24 = 15{,}600$
for 6 letter: $26 \times 25 \times 24 = 15{,}600$

## 1-3

**18.** For the strings of length 10 in Example 1.24, how many have (a) four 0's, three 1's, and three 2's; (b) at least eight 1's; (c) weight 4?

(a) 10!/(4!*3!*3!)
(b) $\binom{10}{8} * 2^2 + \binom{10}{9} * 2 + 1$
(c) $\binom{10}{4} + \binom{10}{2} * \binom{8}{1} + \binom{10}{2}$

**22.** a) In the complete expansion of $(a + b + c + d) \cdot (e + f + g + h)(u + v + w + x + y + z)$ one obtains the sum of terms such as $agw$, $cfx$, and $dgv$. How many such terms appear in this complete expansion?
b) Which of the following terms do not appear in the complete expansion from part (a)?
**i)** $afx$ **ii)** $bvx$ **iii)** $chz$ **iv)** $cgw$ **v)** $egu$ vi) $dfz$

(a) 4*4*6 = 96
(b) **ii)** $bvx$ , **v)** $egu$

**1-4**

**7.** Determine the number of integer solutions of

$x_1 + x_2 + x_3 + x_4 = 32$,

where a) $x_i \geq 0, \quad 1 \leq i \leq 4$ b) $x_i > 0, \quad 1 \leq i \leq 4$ c) $x_1, x_2 \geq 5, \quad x_3, x_4 \geq 7$

d) $x_i \geq 8, \quad 1 \leq i \leq 4$ e) $x_i \geq -2, \quad 1 \leq i \leq 4$ f) $x_1, x_2, x_3 > 0, \quad 0 < x_4 \leq 25$

(a) ( n=4, r=32) $C_{4-1}^{32+4-1} = C_{32}^{32+4-1} = \binom{35}{32}$

(b) $C_{28}^{28+4-1} = \binom{31}{28}$

(c) $C_{8}^{8+4-1} = \binom{11}{8}$

(d) 1

(e) $y_i = x_i + 2$, $y_1 + y_2 + y_3 + y_4 = 40$, $C_{40}^{40+4-1} = \binom{43}{40}$

(f) $C_{28}^{28+4-1} - C_{3}^{3+4-1} = \binom{31}{28} - \binom{6}{3}$

**18.** a) How many nonnegative integer solutions are there to the pair of equations

$x_1 + x_2 + x_3 + \cdots + x_7 = 37, \quad x_1 + x_2 + x_3 = 6$?

b) How many solutions in part (a) have $x_1, x_2, x_3 > 0$?

(a) $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 37$

$6 + x_4 + x_5 + x_6 + x_7 = 37$

The first condition： $x_1 + x_2 + x_3 = 6$

The second condition： $x_4 + x_5 + x_6 + x_7 = 31$

$C_{31}^{31+4-1} * C_{6}^{6+3-1} = \binom{34}{31} * \binom{8}{6}$

(b) from (a)

The first condition：$x_1 + x_2 + x_3 = 6$, and $x_1, x_2, x_3 > 0$ ( $x_1, x_2, x_3$ at least 1)

The second condition：$x_4 + x_5 + x_6 + x_7 = 31$

$C_{3}^{3+3-1} * C_{31}^{31+4-1} = \binom{5}{3} * \binom{34}{31}$

**25.** Consider the $2^{19}$ compositions of 20. (a) How many have each summand even? (b) How many have each summand a multiple of 4?

(a) 20 = 2 + 4 + 12 + 2 = 2( 1 + 2 + 6 + 1) = 2*10
The number of composition of 10 – namely , $2^{10-1} = 2^9$

(b) multiple of 4 = {4 , 8 , 12 , 16 , 20}
The number of composition of 5 – namely , $2^{5-1} = 2^4$

**Supplementary**

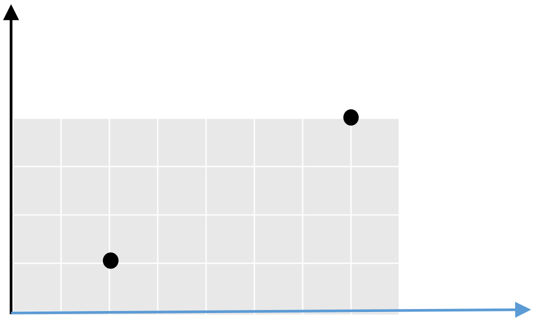
**26.** a) In how many ways can 17 be written as a sum of 2's and 3's if the order of the summands is (i) not relevant? (ii) relevant?
b) Answer part (a) for 18 in place of 17. (學號偶數(a), 奇數(b))

(a) (i) 1(one 3) + 1(three 3's) + 1(five 3's) = 3
(ii) $C_1^8$(one 3) + $C_3^7$(three 3's) + $C_5^6$(five 3's)

(b) (i)1(no 3) + 1(two 3's) + 1(four 3's) + 1(six 3's) = 4
(ii)$C_0^9$(no 3) + $C_2^8$(two 3's) + $C_4^7$(four 3's) + $C_6^6$(six 3's)

**Bonus assignment (僅供參考)**

- In example 14, if we have one new step R-, that means a backward walking $x = x-1$, think about how to calculate the number of paths from (2, 1) to (7, 4). Note that R- can’t follow by a step R.



As long as walker can take one step toward left, he can return from any (8,y) position to (7, y+1) position after one U step. The possible number of step is now 10. As long as it says R-(left) can’t be followed by R, there must be U after R-.

a. So R-U combination is fixed. The number of possible position of R-U is 9. For the remaining 8 positions, 2 U and 6 R should be arranged. Therefore,

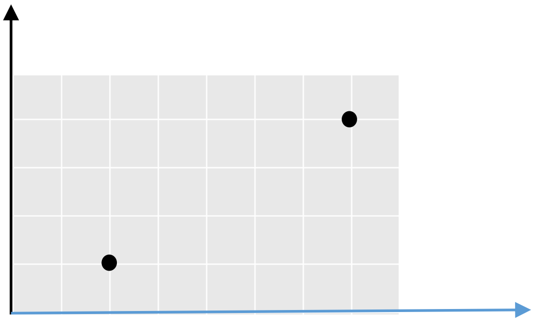
$$9 * \frac{8!}{2! * 6!} = 9 * \frac{8 * 7 * 6!}{2 * 1 * 6!} = 9 * 4 * 7 = 252$$

b. In the case that R- is the last element, the element in the second position from tail is fixed U. So the number of combination is

$$\frac{8!}{2! * 6!} = \frac{8 * 7 * 6!}{2 * 1 * 6!} = 4 * 7 = 28$$

Total possible route is 252+28=280

- Also, if we have U-?



In the case there are one U- and one R-, the number of possible step would be 12. U- should be followed by either R or R-. R- should be followed by either U or U-.

So 4 of 12 position are fixed.

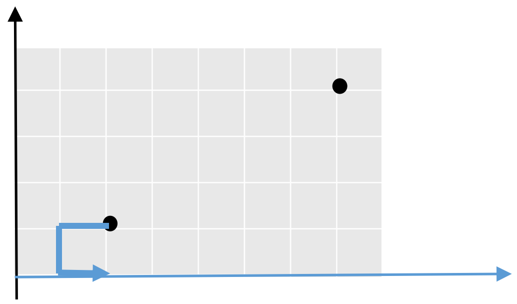
a. U-R and

i. R-U

$$\frac{8!}{3!\,5!} = \frac{8*7*6*5!}{5!*3*2*1} = 56$$

ii. R-U-

$$\frac{9!}{5!*4!} = \frac{9*8*7*6*5!}{5!*4*3*2*1} = \frac{132}{1} = 132$$



b. U-R-

i. R-U

$$\frac{9!}{6!*3!} = \frac{9*8*7*6!}{6!*3*2*1} = 84$$

Totally, 56+132+84=272