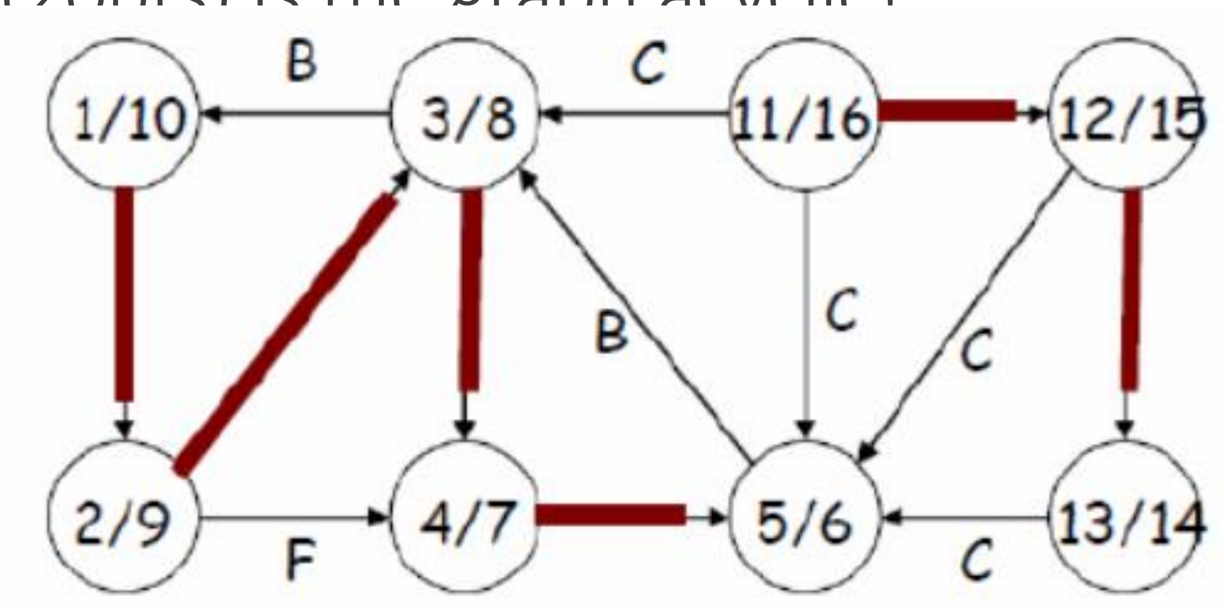
Algorithm 2017 Spring
Quiz Solutions

指 導 教 授 : 謝 孫 源 教 授

助 教 : 許 景 添 陳 琮 皓 林 玉 陞 何 岱 璇

1. (20pts) Is the graph acyclic?

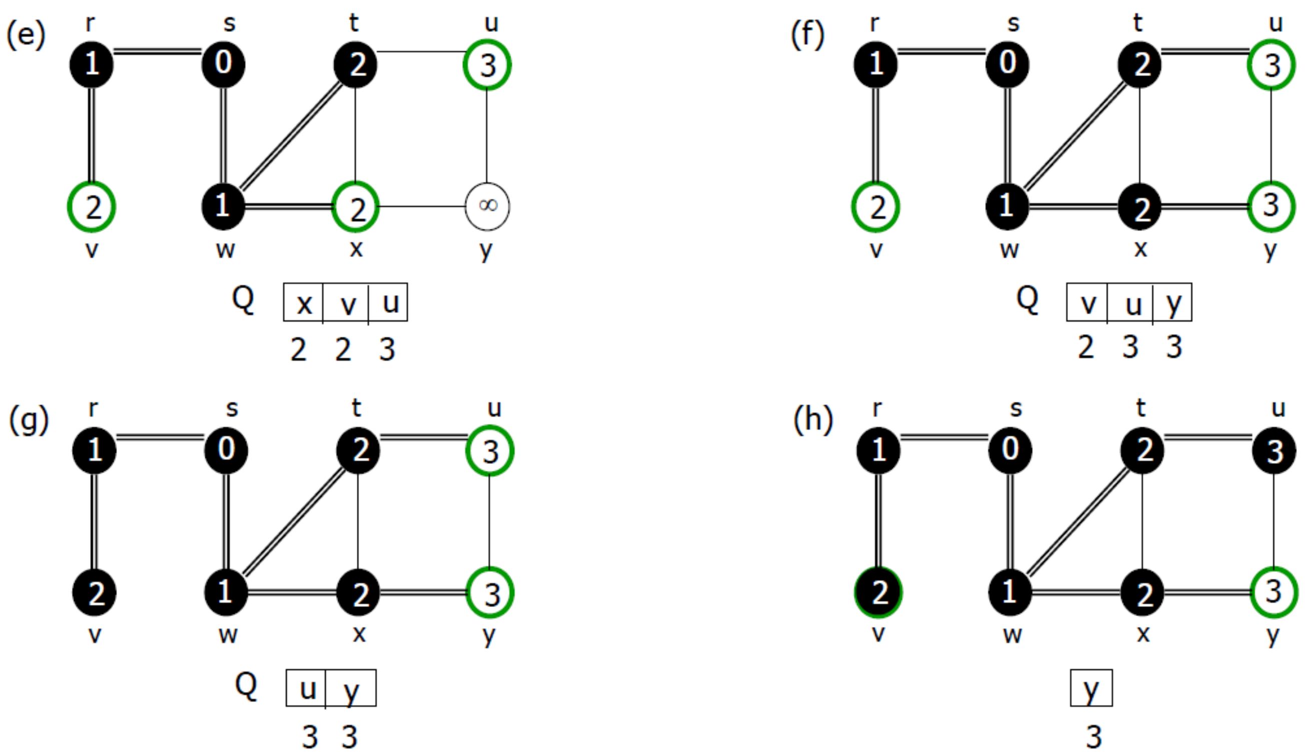


No, a directed graph is not acyclic because it has "back" edges.

2.

BFS(V, E, s)
for each $u \in V - \{s\}$
    do $d[u] \leftarrow \infty$
$d[s] \leftarrow 0$
$Q \leftarrow \phi$
ENQUEUE(Q, s)
While $Q \neq \phi$
   do $u \leftarrow$ DEQUEUE(Q)
      for each $v \in Adj[u]$
            do if $d[v] = \infty$
                  then $d[v] \leftarrow d[u] + 1$
                        ENQUEUE(Q, v)

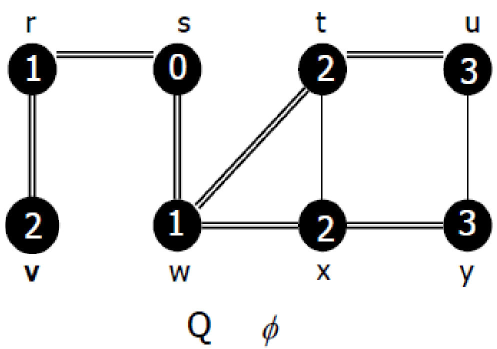
2.



(e)
r
s
t
u
1
0
2
3
2
1
2
∞
v
w
x
y
Q
x
v
u
2
2
3
(f)
r
s
t
u
1
0
2
3
2
1
2
3
v
w
x
y
Q
v
u
y
2
3
3
(g)
r
s
t
u
1
0
2
3
2
1
2
3
v
w
x
y
Q
u
y
3
3
(h)
r
s
t
u
1
0
2
3
2
1
2
3
v
w
x
y
y
3

2.

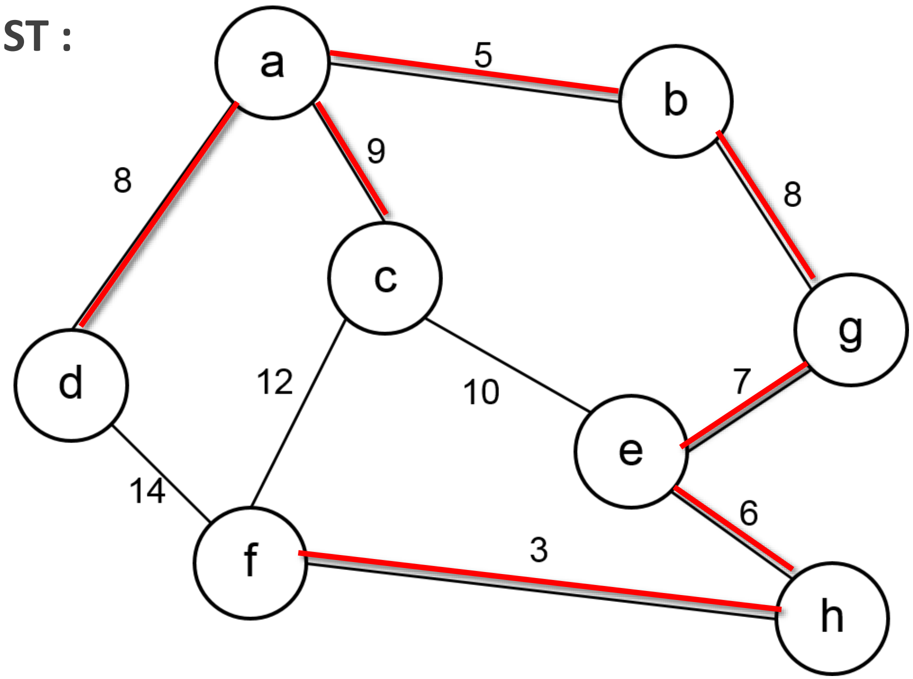
(i)



3. For the following graph

A. **(10pts) What is a minimum-cost spanning tree? Please write down the a minimum-cost spanning tree.**

- **(5pts) Cost : 46**
- **(5pts) The MST :**



3. For the following graph

B. **(10pts) Write an algorithm to describe how you get the result of (A).**

- ***Method 1 : KRUSKAL(V, E, w)***

$A \leftarrow \emptyset$
for each vertex $v \in V$[G]
    do MAKE-SET(v)
sort E into nondecreasing order by weight w
for each (u, v) taken from the sorted list
    do if FIND-SET(u) $\neq$ FIND-SET(v)
        then $A \leftarrow A \cup \{(u, v)\}$
            UNION(u,v)
return A

3. For the following graph

Method 2 : PRIM(V, E, w, r)

```
Q ← ∅
for each u ∈ V[G]
      do key[u] ← ∞
         π[u] ← NIL
         INSERT(Q, u)
DECREASE-KEY(Q, r, 0)
while Q ≠ ∅
      do u ← EXTRACT-MIN(Q)
         for each v ∈ Adj[u]
              do if v ∈ Q and w(u, v) < key[v]
                    then π[v] ← u
                         DECREASE-KEY(Q, v, w(u, v) )
```

4.

Q: Show, by means of a counterexample, that the following "greedy" strategy does not always determine an optimal way to cut rods. Define the density of a rod of length i to be p_i, that is, its value per inch. The greedy strategy for a rod of length n cuts off a first piece of length i, where $1 \leq i \leq n$, having maximum density. It then continues by applying the greedy strategy to the remaining piece of length $n - i$

Sol.

length i	1	2	3	4
price p_i	1	20	33	36
p_i/i	1	10	11	9

- Here is a counterexample for the "greedy" strategy:
- Let the given rod length be 4.
- According to a greedy strategy, we first cut out a rod of length 3 for a price of 33, which leaves us with a rod of length 1 of price 1.
- The total price for the rod is 34. The optimal way is to cut it into two rods of length 2 each fetching us 40 dollars.

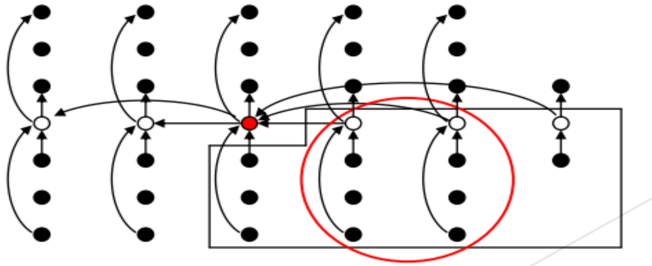
5. (20pts) In the algorithm SELECT, the input elements are divided into groups of 5.

(a) (10pts) What is the purpose of this algorithm?

- The SELECT algorithm determines the *i*th smallest of an input array of n>1 distinct elements.

(b) (10pts) Will the algorithm work in linear time if they are divided into groups of 7?

- Ans : YES
- 決定median of medians x後，至少有 $\frac{2n}{7} - 8$ 個 elements大於x， 所以至多有 $\frac{5n}{7} + 8$ 個elements小於x，worst case為在$\frac{5n}{7} + 8$中找 *i*-th smallest element Total cost:$T(n) \leq T(\left\lceil\frac{n}{7}\right\rceil) + T(\frac{5n}{7} + 8) + O(n)$=>$T\ n = O(n)$
- 示意圖:



5. (20pts) In the algorithm SELECT, the input elements are divided into groups of 5.

(b) (10pts) Will the algorithm work in linear time if they are divided into groups of 7?

- $T(n) \leq T(\left\lceil \frac{n}{7} \right\rceil) + T(\frac{5n}{7} + 8) + O(n)$ 利用substitution method得到的結果為

$T(n) \leq \frac{6cn}{7} + 9c + an \leq cn = cn + (-\frac{cn}{7} + 9c + an)$

- $-\frac{cn}{7} + 9c + an \leq 0$

$\frac{cn}{7} - 9c \geq an$

$cn - 63c \geq 7an$

$c(n - 63) \geq 7an$

取$c \geq 7a(\frac{n}{n-63})$ 得到 $T(n) = O(n)$