Discrete Mathematics (2009 Spring) Midterm II

1. (30 points) For each of the following statements, **determine** and **explain** whether it is correct or not.
   (1). [6.1-12]The string 00010 is in the language {00}{0}*{10} also in {000}*{1}*{0}
   (2). [7.3-23]Let (A, **R**) be a poset. If (A, **R**) is a lattice, then it is a total order.
   (3). [5.2-20]If A={1, 2, 3, 4} and there are 1680 injective functions *f*: A→B, then |B|=8.
   (4). [5.5-20]The maximal number of rolling a single die to get the same score at least 4 times is 25.
   (5). [5.6-18]Function *f* denotes a closed binary operations on $\mathbf{P}(\mathbf{Z}^+)$. For A, B$\subseteq\mathbf{Z}^+$, *f*(A, B)=A∩B then *f* is one-to-one.
   (6). [7.5] Two states are not 2-equivalent if and only if they are not 3-equivalent.

   Ans:

   (1) True, $00010=\{00\}\{0\}\{10\}\in\{00\}\{0\}^*\{10\}$
   $00010=\{000\}\{1\}\{0\}\in\{000\}^*\{1\}^*\{0\}$

   (2) False,
   $let\ u=\{1,2\}, A=p(u),$ and R be the inclusion relation. Then (A,R) is a latice where for all S, T $\in$ A,
   lub{S,T}=S∪T and glb{S,T}=S∩T, However,{1}and {2} are not related, sp {A,R} is not a total order.

   (3) True, $\dfrac{8!}{(8-4)!}=1680$

   (4) False, 6(n-1)+1$\Rightarrow$ n=4 $\Rightarrow$6(4-1)+1=19$\neq$25

   (5) False, f(ϕ,ϕ)=ϕ=f(ϕ,{1}) and (ϕ,ϕ)$\neq$(ϕ,{1})
   so f is not one-to -one

   (6) False, let p: not 2-equivalent and q: not 3-equivalent
   p$\rightarrow$q is true, but q$\rightarrow$p is false $\therefore$ p$\leftrightarrow$q is incorrect

2. [5.4-5,6 7.4-12] (20 points, 5+5+5+5) Let A={a, b, c, d}. Determine the following values (a) the number of functions *f*: A x A→A, (b) the number of closed binary operations *f* on A satisfy *f*(a, b)=b and have an identity, (c) the number of closed binary operations in (b) are commutative, (d) the number of relations on A that are reflexive and symmetric but not transitive.

   Ans:

   (a) 4^{16}

   (b) a is identity : 4^9
   b is identity : 0
   c,d is identity : 4^8
   $\therefore 4^9+4^8+4^8$

   (c) a is identity : 4^6
   b is identity : 0
   c,d is identity : 4^5
   $\therefore 4^6+4^5+4^5$

(d) Reflexive and symmetric : 2^6

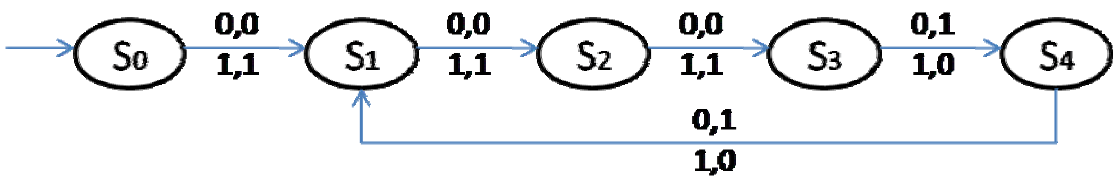
transitive: $\sum_{i=1}^{4} S(4,i)$

$$\therefore 2^6 - \sum_{i=1}^{4} S(4,i) = 49$$

3. [7 ex7.58] (10 points) Suppose R is an equivalence class relation on {1, 2, 3, 4, 5, 6} and the equivalence class induced by R are {1, 5, 6}, {2, 4}, {3}. What is the value of |R|?
Ans:

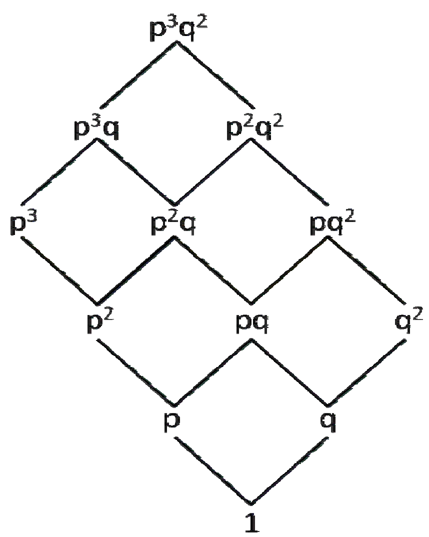
$$3^2 + 2^2 + 1^2 = 9 + 4 + 1 = 14$$

4. [6supp 9] (15 points) Let **I** = **O** = {0, 1}. Construct a state diagram for a finite state machine that reverses (from 0 to 1 or from 1 to 0) the symbols appearing in the *4kth* and (*4k+1*)*th* positions of an input string $x \in \mathbf{I}^+$, where k≥1. For example, if s_0 is the starting state, then $w(s_0,$ 00010111)=00001110.
Ans:



5. [7.3-27] (15 points, 5+10) Let *p*, *q*, *r* be three distinct primes. We denote relation $x\mathbf{R}y$ if *x* divides *y*. Under this relation **R**, (1) please draw the Hasse diagram of all positive divisors of p^3q^2; (2) and answer how many edges are there in the Hasse diagram of all positive divisors of $p^mq^nr^k$, *m*, *n*, $k \in$ Z+.
Ans:

(1)



(2)

Consider the vertex $p^a q^b r^c$, $0 \le a < m$, $0 \le b < n$, $0 \le c < k$. There are mnk such vertices; each determines three edges — going to the vertices $p^{a+1}q^b r^c, p^a q^{b+1} r^c, p^a q^b r^{c+1}$. This accounts for $3mnk$ edges.

Now consider the vertex $p^m q^b r^c$, $0 \le b < n$, $0 \le c < k$. There are nk of these vertices; each detemines two edges — going to the vertices $p^m q^{b+1} r^c$, $p^m q^b r^{c+1}$. This accounts for $2nk$ edges. And similar arguments for the vertices $p^a q^n r^c (0 \le a < m, 0 \le c < k)$ and $p^a q^b r^k (0 \le a < m, 0 \le b < n)$ account for $2mk$ and $2mn$ edges, respectively.

Finally, each of the k vertices $p^m q^n r^c$, $0 \le c < k$, determines one edge (going to $p^m q^n r^{c+1}$) and so these vertices account for k new edges. Likewise, each of the n vertices $p^m q^b r^k$, $0 \le b < n$, determines one edge (going to $p^m q^{b+1} r^k$), and so these vertices account for n new edges. Lastly, each of the m vertices $p^a q^n r^k$, $0 \le a < m$, determines one edge (going to $p^{a+1} q^n r^k$) and these vertices account for m new edges.

The preceding results give the total number of edges as $(m+n+k)+2(mn+mk+nk)+3mnk$.

6. [Table 5.1] (5 points) Considering the Stirling number of the second kind S(m, n), we have S(7, 4)=350, S(6, 4)=65, S(6,5)=15. What is S(8, 5) ?

Ans:

$$\because S(n,m) = S(n-1,m-1) + mS(n-1,m)$$
$$\begin{aligned}\therefore S(8,5) &= S(7,4) + 5S(7,5)\\ &= 350 + 5[S(6,4) + 5S(6,5)]\\ &= 350 + 325 + 375\\ &= 1050\end{aligned}$$

7. [7.5-1(c)] (15 points, 10+5) (1) Apply the minimization process to the finite state machine in the following state table. (2) What is the minimal distinguishing string for s_1 and s_5?

	ν		ω	
	0	1	0	1
s_1	s_6	s_3	0	0
s_2	s_3	s_1	0	0
s_3	s_2	s_4	0	0
s_4	s_7	s_4	0	0
s_5	s_6	s_7	0	0
s_6	s_5	s_2	1	0
s_7	s_4	s_1	0	0

Ans:

(1)

$$P_1 = \{S_1\ S_2\ S_3\ S_4\ S_5\ S_7\}\{S_6\}$$
$$P_2 = \{S_1\ S_5\}\{S_2\ S_3\ S_4\ S_7\}\{S_6\}$$
$$P_3 = \{S_1\ S_5\}\{S_2\ S_7\}\{S_3\ S_4\}\{S_6\}$$
$$P_4 = \{S_1\}\{S_5\}\{S_2\ S_7\}\{S_3\ S_4\}\{S_6\}$$

S_2 and S_7 are equivalent; S_3 and S_4 are equivalent

(2)

Consequently, 1100 is a distinguishing sequence since $w(S_1, 1100) = 0000 \ne 0001 = w(S_5, 1100)$