Discrete Mathematics (2009 Spring) Midterm I

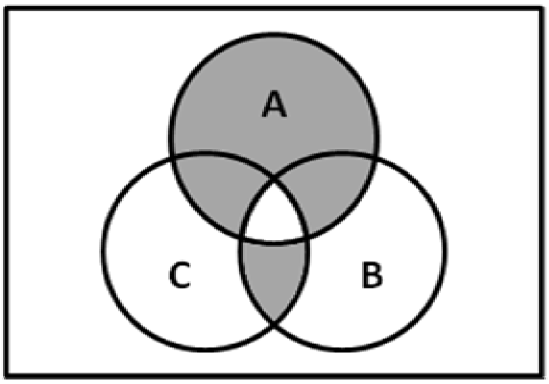
1. (ch3.1 4, ch2.2 6(b), ch3.2 4, ch1.4 18) (25 points) For each of the following statements, **determine** and **explain** whether it is correct or not.
   (1). $\phi \subset \phi$
   (2). $\phi \subseteq \{\phi\}$
   (3). $\neg[(p \wedge q) \rightarrow r] \Leftrightarrow (p \wedge q) \vee \neg r$
   (4). $A = \{2n \mid n \in Z\}, B = \{6n \mid n \in Z\}, \textit{then } \overline{B} \subseteq \overline{A}.$
   (5). The number of integer solutions for $x_1+x_2+x_3=6$ and $x_1, x_2, x_3 > 0$ is 10.
   Ans:
   (1) False, $\phi \not\subset \phi$, ϕ不爲ϕ的子集合
   (2) True, ϕ包含於$\{\phi\}$
   (3) False, $\neg((p \wedge q) \rightarrow r) = \neg(\neg(p \wedge q) \vee r) = (p \wedge q) \wedge \neg r$
   (4) False, $\because B \subseteq A \ \therefore \overline{A} \subseteq \overline{B}$, not $\overline{B} \subseteq \overline{A}$
   (5) True, let $y_1=x_1-1$, $y_2=x_2-1$, $y_3=x_3-1$
   $\Rightarrow x_1+x_2+x_3=6$
   $\Rightarrow y_1+y_2+y_3=3$
   $\therefore H_3^3 = C_3^5 = 10$

2. (ch1.3 25, ch1 supp. 16) (15 points) For the complete expansion of $(2x - 2y + 3z^{-1}+1)^4$, determine the following value (a) the coefficient of yz^{-2} (b) the number of the distinct terms (c) the sum of all coefficients.
   Ans:
   (a) $\frac{4!}{2!1!1!}(2x)^0(-2y)^1(3z^{-1})^2(1)^1 = -216yz^{-2}$
   (b) $H_4^4 = C_4^7 = 35$
   (c) $\mathrm{x}=\mathrm{y}=\mathrm{z}=1$ 代入 $(2-2+3+1)^4 = 256$

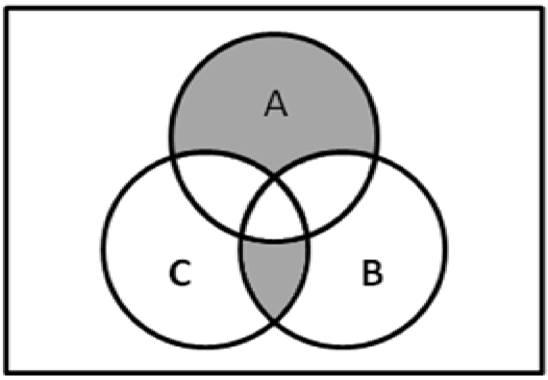
3. (ch1.4 25) (15 points) What is the probability of each summand even in all compositions of 20?
   Ans:
   $$P = \frac{\#\textit{compositions of } 10}{\#\textit{compositions of } 20} = \frac{2^9}{2^{19}} = \frac{1}{2^{10}}.$$
   Consider that with + or not:
   $1+1+1+\cdots+1+1=20$
   ⇨ 2^{19} (all of possible)
   $2+2+2+\cdots+2+2=20$
   ⇨ 2^9 (even of possible)
   $P=2^9/2^{19}=1/2^{10}=1/1024$

4. (ch3.2 8) (15 points) Using Venn diagrams to prove the truth or falsity of $A\Delta(B \cap C) = (A\Delta B) \cap (A\Delta C)$, for sets $A, B, C \subseteq U$.

Ans:



A Δ (B ∩ C)



(AΔB) ∩ (AΔC)

Ans:

False, $A\Delta(B \cap C) \neq (A\Delta B) \cap (A\Delta C)$

5. (ch2.2 4) (20 points) For primitive statements *p*, *q*, *r*, and *s*, simplify the compound statement $[[[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r]] \vee \neg q] \rightarrow s$.

Ans:

$$[[[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r]] \vee \neg q] \rightarrow s$$
$$\Leftrightarrow [[(p \wedge q) \wedge (r \vee \neg r)] \vee \neg q] \rightarrow s$$
$$\Leftrightarrow [(p \wedge q) \vee \neg q] \rightarrow s$$
$$\Leftrightarrow [(p \vee \neg q) \wedge (q \vee \neg q)] \rightarrow s$$
$$\Leftrightarrow [(p \vee \neg q) \wedge T_0] \rightarrow s$$
$$\Leftrightarrow (p \vee \neg q) \rightarrow s \text{ or } (q \rightarrow p) \rightarrow s \text{ or } (\neg p \wedge q) \vee s$$

6. (ch4.4 10) (20 points) If *a*, *b* are relatively prime and $a > b$, prove that $gcd(a\text{-}b, a\text{+}b)$=1 or 2.

Ans:

$\gcd(a, b) = 1$

let $\gcd(a - b, a + b) = \alpha$

$\alpha | a\text{-}b$ and $\alpha | a\text{+}b \Rightarrow \alpha | (a\text{-}b)x + (a\text{+}b)y$

if $x = y = 1$, $\alpha | 2a$

$x = -1$, $y = 1$, $\alpha | 2b$

$\gcd(2a, 2b) = 2\gcd(a, b) = 2$

$\therefore \alpha | 2$ and $\alpha = 1$ or 2