Algorithm 2017 Spring

Homework 3

範圍：Chapter 15~ Chapter 16

1. **Consider the knapsack problem consists of 3 items, and the capacity of the knapsack is equal to 8. The profits and weights of the three items are (p1, p2, p3) = (8, 6, 3) and (w1, w2, w3) = (6, 5, 3), respectively.**
   - **(a) (5pts) Assume that you are allowed to put in a fraction of an item. Use the greedy method to solve for the maximum profit and show the items to be included in the knapsack.**
   - **(b) (5pts) Now suppose that you must take each item as a whole (i.e 0/1 knapsack problem). Show how you can use dynamic programming to solve the problem.**

2. **The Knapsack problem:**
   - **(a) (4pts) If the size of each object is arbitrary real number, does the dynamic programming method still work?**
   - **(b) (6pts) Explain your answer.**

3. **Given a chain ⟨A1 , A2 , A3 , A4 ⟩ of 4 matrices and their matrix dimensions: A1 : 3 × 5, A2 : 5 × 2, A3 : 2 × 6, A4 : 6 × 4. Please compute the minimum number of scalar multiplications to multiply them.**

4. **Please show the tables produced by LCS-LENGTH on the sequences X = {A, B, C, B, D, A, B} and Y = {B, D, C, A, B, A}**

5. **Determine the longest common subsequence of:**
   - **(a) (5pts) <1, 0, 0, 1, 0, 1, 0, 1> and <0, 1, 0, 1, 1, 0, 1, 1, 0>**
   - **(b) (5pts) s1, s2 and s3**
     - **s1: a, b, c, d, b, c, e, e, a**
     - **s2: c, a, b, d, e, f, g, a**
     - **s3: d, c, e, a**

6. **Find the maximal revenue obtainable with the prices below.**
   (a) **(5pts) The maximal revenue r4**
   (b) **(5pts) The maximal revenue r5**

Length *i*	1	2	3	4	5	6	7	8
Price *pi*	1	5	8	9	10	17	17	20

7. **(10pts) Give a dynamic-programming solution to the 0-1 knapsack problem that runs in $O(nW)$ time, where n is the number of items and W is the maximum weight of items that the thief can put in his knapsack.**

8. **(10pts) Show, by means of a counterexample, that the following "greedy" strategy does not always determine an optimal way to cut rods. Define the density of a rod of length i to be p_i, that is, its value per inch. The greedy strategy for a rod of length n cuts off a first piece of length i, where $1 \leq i \leq n$, having maximum density. It then continues by applying the greedy strategy to the remaining piece of length $n - i$.**

9. **Consider the following statements, answer true or false and explain why.**
   (a) Dynamic programming always provides polynomial time algorithms.
   (b) Dynamic programming uses tables to design algorithm.
   (c) Optimal substructure is an important element of Dynamic programming algorithm.
   (d) The single source shortest path problem has the property of optimal substructure.
   (e) In the Knapsack problem, if the size of each object is arbitrary real number, the Dynamic programming method still work.

10. **What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci number.**
    a:1 b:1 c:2 d:3 e:5 f:8 f:13 h:21