2016 Algorithm HW2 Solutions

指導教授 ： 謝孫源 教授

助教： 盧緯 張耕華 楊順翔 許景添

Question 1(10pts)

- Let 's say that two elements at indices $i_1<i_2$ are equal to each other. In the sorted array, they take place at indices j_1+1=j2. Since the COUNTING-SORT processes the input array in reverse order, A[i_2] is put in B[j_2] first and then A[i_1] is put in A[j_2]. Since the two elements preserve their order, the algorithm is stable.

Question 1(10pts)

- 解釋 COUNTING-SORT 不寫 proof 扣五分
- 語意不清楚扣五分

Question 2(10pts)

COW		SEA		TAB		BAR
DOG		TEA		BAR		BIG
SEA		MOB		EAR		BOX
RUG		TAB		TAR		COW
ROW		DOG		SEA		DIG
MOB		RUG		TEA		DOG
BOX		DIG		DIG		EAR
TAB	$\Rightarrow$	BIG	$\Rightarrow$	BIG	$\Rightarrow$	FOX
BAR		BAR		MOB		MOB
EAR		EAR		DOG		NOW
TAR		TAR		COW		ROW
DIG		COW		ROW		RUG
BIG		ROW		NOW		SEA
TEA		NOW		BOX		TAB
NOW		BOX		FOX		TAR
FOX		FOX		RUG		TEA

Question 3(10pts)

	j	0	1	2	3	4	5	6
i		y_j	B	D	C	A	B	A
0	x_i	0	0	0	0	0	0	0
1	A	0	↑ 0	↑ 0	↑ 0	↖ 1	← 1	↖ 1
2	B	0	↖ 1	← 1	← 1	↑ 1	↖ 2	← 2
3	C	0	↑ 1	↑ 1	↖ 2	← 2	↑ 2	↑ 2
4	B	0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3	← 3
5	D	0	↑ 1	↖ 2	↑ 2	↑ 2	↑ 3	↑ 3
6	A	0	↑ 1	↑ 2	↑ 2	↖ 3	↑ 3	↖ 4
7	B	0	↖ 1	↑ 2	↑ 2	↑ 3	↖ 4	↑ 4

Question 3(10pts)

沒箭頭，沒字母扣三分

重大錯誤扣五分

Question 4

w[i][j]

	1	**2**	**3**	**4**	**5**	**6**
1	0	0.25	0.4	0.6	0.95	1
2		0	0.15	0.35	0.7	0.75
3			0	0.2	0.55	0.6
4				0	0.35	0.4
5					0	0.05
6						0

e[i][j]

	1	**2**	**3**	**4**	**5**	**6**
1	0	0.25	0.55	1.05	1.85	2
2		0	0.15	0.5	1.2	1.3
3			0	0.2	0.75	0.85
4				0	0.35	0.45
5					0	0.05
6						0

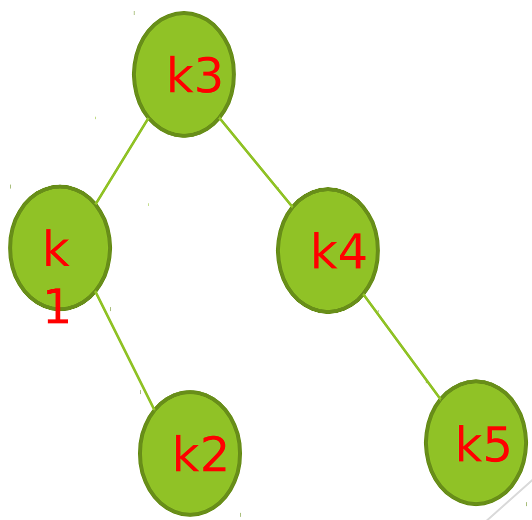
Question 4

Root[i][j]

	1	**2**	**3**	**4**	**5**
1	1	1	2	3	3
2		2	3	3 or 4	4
3			3	4	4
4				4	4
5					5

(a) Cost =
2

(b) Structure of an optimal binary search tree :



* 注意是” binary search tree”：小的在左，大的在右

Question 5

M[i][j]

	1	**2**	**3**	**4**	**5**	**6**
1	0	15750	7875	9375	11875	15125
2		0	2625	4375	7125	10500
3			0	750	2500	5375
4				0	1000	3500
5					0	5000
6						0

S[i][j]

	2	**3**	**4**	**5**	**6**
1	1	1	3	3	3
2		2	3	3	3
3			3	3	3
4				4	5
5					5

So, ANS= $((A_1(A_2A_3))((A_4A_5)A_6))$

minimum number of scalar multiplications = 15125

Question 6(10pts)

解答：

m

i \ j	1	2	3	4
1	0	30	66	102
2		0	60	80
3			0	48
4				0

Cost = 102

s

i \ j	2	3	4
1	1	2	2
2		2	2
3			3

$(A_1A_2)(A_3A_4)$

$$m[1,2] = m[1,1] + m[2,2] + p_0p_1p_2 = 30 \quad ,k=1$$

$$m[2,3] = m[2,2] + m[3,3] + p_1p_2p_3 = 60 \quad ,k=2$$

$$m[3,4] = m[3,3] + m[4,4] + p_2p_3p_4 = 48 \quad ,k=3$$

$$m[1,3] = \min\begin{cases} m[1,1] + m[2,3] + p_0p_1p_3 = 150 & ,k=1 \\ m[1,2] + m[3,3] + p_0p_2p_3 = \underline{66 \quad ,k=2} \end{cases}$$

$$m[2,4] = \min\begin{cases} m[2,2] + m[3,4] + p_1p_2p_4 = \underline{88 \quad ,k=2} \\ m[2,2] + m[4,4] + p_1p_3p_4 = 180 & ,k=3 \end{cases}$$

$$m[1,4] = \min\begin{cases} m[1,1] + m[2,4] + p_0p_1p_4 = 148 & ,k=1 \\ m[1,2] + m[3,4] + p_0p_2p_4 = \underline{102 \quad ,k=2} \\ m[1,3] + m[4,4] + p_0p_3p_4 = 138 & ,k=2 \end{cases}$$

Question 7(10pts)

- overlapping subproblem ༀ
- optimal substructure

Question 8(10pts)

Solution:

No

子問題有無限多個，且當背包承重為小數時無法查表

只有答案給 5 分

Question 9(10pts)

- LCS 長度 = 6
  <1,0,0,1,1,0> or <1,0,1,1,0,1> or <1,0,1,0,1,1>
- 若兩序列擺相反則答案為 <0,1,0,1,0,1>
- 配分
  - 沒箭頭扣三分

Question 10(10pts)
fractional-knapsack problem(5pts)

item	value	weight	Value/weight
1	8	6	4/3
2	6	5	6/5
3	3	3	3/3

- Maximum capacity of knapsack is 8
- (4/3)*6 + (6/5)*(8-6) = 8 + 12/5 = 10.2

Question 10(10pts) cont.
0/1-knapsack problem(5pts)

Solution:

- $profit[i][j]$:

  is maximum value that can be attained with weight le ss than or equal to j using item s up to I

- $weight[i]$:

  Weight of item i

- $v[i]$:

  Value of item i

$$profit[i][j] = \begin{cases} profit[i-1][j], & if\ \ weight[i] > j \\ \max\begin{pmatrix} profit[i-1][j], \\ profit[i-1][j-weight[i]] + v[i] \end{pmatrix}, & otherwise \end{cases}$$

Question 10(10pts) cont.
0/1-knapsack problem

Solution:

j i	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	8	8	8
2	0	0	0	0	6	8	8	8
3	0	0	3	3	6	8	8	9