Discrete Mathematics (2014 Spring) Final
Total 110 points (max 100 points)

1. **(12 points)** For each of the following statements, determine and explain (required) whether it is correct or not.
   (1). Only one of $(Z, =)$, $(Z, \geq)$, $(Z, \nmid)$ are posets.
   (2). $3\binom{n}{0} + \binom{n}{1} + 3\binom{n}{2} + \binom{n}{3} + 3\binom{n}{4} + \binom{n}{5} + \cdots + 3\binom{n}{n-2} + \binom{n}{n-1} + 3\binom{n}{n} = 2^{n+1}$
   (3). If $12 \mid 3a+5b$ then $12 \mid 12a+4b$.

2. **(15 points)** Let A={a, b, c, d, e, f} (a) how many closed binary operations f on A satisfy $f(a, b) \neq \{c, d\}$? (b) How many closed binary operations f on A have an identity and $f(a, b)=a$? (c) B={a, b, c, d}, how many onto function from A to B satisfying $f(a)=a$?

3. **(10 points)** Find the integer solutions of the equation (a) $x_1+x_2+x_3+x_4 < 9$ where $x_1, x_2 > 0$, $x_3 > 1$, $x_4 > 3$, (b) $x_1+x_2+x_3+x_4 = 9$ where $x_1 < 3$, $x_3 > 1$, $x_4 < 2$. (exhaustively list all answers is not allowed.)

4. **(10 points)** Use the rook polynomials to find the number of permutation of the letters x, x, y, y, z, z so that no x appears in the first and second positions, no y appears in the third position and no z appears in the fifth and sixth positions.

5. **(15 points)** (a) How many derangements of 1, 2, 3, 4, 5, 6, 7 start with 1, 2, 3 in some order? (b) 5, 6, 7 in some order? (c) How many derangements only for odd numbers?

6. **(10 points: 7, 3)** A ship carries 30 flags, 10 each of the colors red, white, and blue. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. We want to know the number of the signals have at least three white flags or no white flags at all. Please answer it by (a) exponential generating function, (b) one other method.

7. **(15 points: 3,4,4,4)** Determine the following coefficients (a) x^2yz^{-2} in $(2x - y + 3z^{-1} + 2)^7$, (b) x^2yz^{-2} in $(2x - y + 3z^{-1} + 2 + x^2)^7$, (c) x^{48} in $(x^5+x^6+x^7+x^8+\ldots)^6$, (d) x^{12} in $(1 + x)^3/(1 - x)^3$.

8. **(15 points:10, 5)** (a) Let a_n be the number of possible cases that toss a coin n times and no two consecutive heads occurs. Please write the recurrence relation for a_n, and determine the value of a_8. (b) Solve the recurrence relation $a_n - 2a_{n-1} = 3^n$, for $n \geq 1$ and $a_0 = 1$.

9. **(8 points)** Please answer (a) more small quizzes is good for you? (b) a roll call at the class is good/helpful for you? (c) 10 chapters are too much for you? (d) 2 examples/methods/strategies to improve your (or others') learning motivation/performance.

(Some values of Stirling number of the second kind: S(4, 2)=7, S(4, 3)=6, S(5, 2)=15, S(5, 3)=25, S(5, 4)=10, S(6, 2)=31, S(6, 3)=90)