Algorithm 2016 fall

Homework 4

1. Modify FASTER-ALL-PAIRS-SHORTEST-PATHS so that it can detect the presence of a negative-weight cycle. (10%)

2. What does the matrix

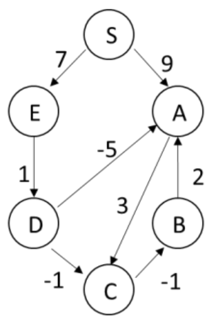
$$L^{(0)} = \begin{pmatrix} 0 & \infty & \infty & \cdots & \infty \\ \infty & 0 & \infty & \cdots & \infty \\ \infty & \infty & 0 & \cdots & \infty \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \infty & \infty & \infty & \cdots & 0 \end{pmatrix}$$

used in the shortest-paths algorithms correspond to in regular matrix multiplication? (10%)

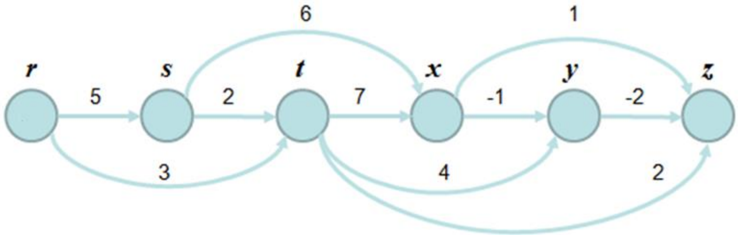
3. Give an O(*VE*)-time algorithm for computing the transitive closure of a directed graph *G* = (*V*, *E*). (10%)

4. Proof or disproof the following statement:
Any subpath of a shortest path is a shortest path. (10%)

5. Illustrate the progress of the Bellman-Ford Algorithm on the following graph. (10%)



6. Run DAG-SHORTEST-PATHS step by step on the directed graph of the figure, using vertex *s* as the source. (10%)

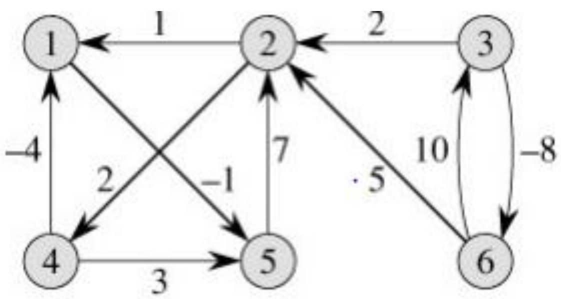


7. Find a feasible solution or determine that no feasible solution exists for the following system of difference constraints (10%)

$x1 - x2 \leq 4$, $x1 - x5 \leq 5$,
$x2 - x4 \leq -6$, $x3 - x2 \leq 1$,
$x4 - x1 \leq 3$, $x4 - x3 \leq 5$,
$x4 - x5 \leq 10$, $x5 - x3 \leq -7$,
$x5 - x4 \leq -8$

8. Can any shortest-path length from the new node, node 0, in the augmented constraint graph, be positive? Explain. (10%)

9. Run SLOW-ALL-PAIRS-SHORTEST-PATHS on the weighted, directed graph of Figure 25.2, showing the matrices that result for each iteration of the loop. Then do the same for FASTER-ALL-PAIRS-SHORTEST-PATHS. (10%)



10. True or False (10%)

(I). The FASTER-ALL-PAIRS-SHORTEST-PATHS procedure requires us to store $\lceil lg(n-1) \rceil$matrices, each with n^2 elements, for a total space requirement of $\Theta(n^2 \lg n)$. (3%)

(II) We can add positive constant to all negative edges, so that all weights become positive, then we apply Dijkstra's algorithm at node *S*. We can always find the correct shortest path. (3%)

(III) Running time of slow-APSP is $O(V^4)$. (2%)

(IV) Given a graph, suppose we have calculated shortest path from a source to all other vertices. If we modify the graph such that weights of all edges is becomes double of the original weight, then the shortest path remains same only the total weight of path changes. (2%)