# QUIZ 5

**NAME:**____________________________ **NCKU id:**____________________

**1**. Let $f(x) = \displaystyle\int_x^{\cos x} \sqrt{t^3+1}\, dt$. Find $f'(x) =$ ____________________________

**Answer:** By the fundamental theorem of calculus I,

$$\begin{aligned} f(x) &= \int_0^{\cos x} \sqrt{t^3+1}\, dt - \int_0^{x} \sqrt{t^3+1}\, dt \\ \Rightarrow\ f'(x) &= \sqrt{\cos^3 x + 1}\cdot(-\sin x) - \sqrt{x^3+1} \\ &= -(\sin x\sqrt{\cos^3 x+1} + \sqrt{x^3+1}) \end{aligned}$$

**2**. Evaluate the integration $\int x\sqrt{1-x}\, dx$.

**Answer :** Let $u = \sqrt{1-x}$, $1-u^2 = x \Rightarrow -2udu = dx$

$$\begin{aligned} &\therefore\ \int (1-u^2)\cdot u\cdot(2udu) = \int (2u^2-2u^4)du \\ &= \frac{2u^3}{3} - \frac{2u^5}{5} + C \\ &= \frac{2(x-1)^{3/2}}{3} - \frac{2(x-1)^{5/2}}{5} + C \end{aligned}$$

**3**. Find the area bounded by $y = \cos x + 1$ , $y = \frac{3}{2}$ , $x = 0$ and $x = \pi$.

**Answer:** Solve the intersection point

$$\begin{cases} y = \cos x + 1 \\ y = \frac{3}{2} \end{cases} \Rightarrow\ \cos x = \frac{1}{2}\ \Rightarrow\ x = \frac{\pi}{3}$$

*Date*: December 29th.

Since

$$|(\cos x+1)-\frac{3}{2}|=\begin{cases}(\cos x+1)-\frac{3}{2} & ,\ \text{if } 0\le x<\frac{\pi}{3}\\ \frac{3}{2}-(\cos x+1) & ,\ \text{if } \frac{\pi}{3}\le x\le\pi\end{cases},$$

we have

$$\begin{aligned}
&\int_0^{\pi}|(\cos x+1)-\frac{3}{2}|dx\\
=&\int_0^{\frac{\pi}{3}}\left(\cos x+1-\frac{3}{2}\right)dx+\int_{\frac{\pi}{3}}^{\pi}\left(\frac{3}{2}-(\cos x+1)\right)dx\\
=&(\sin x-\frac{1}{2}x)|_0^{\frac{\pi}{3}}+(\frac{1}{2}x-\sin x)|_{\frac{\pi}{3}}^{\pi}\\
=&\sqrt{3}+\frac{\pi}{6}
\end{aligned}$$