P6.12 From Figure P6.12, we see that the period of the signals is 20 ms. Therefore, the frequency is 50 Hz. Because the input reaches a positive peak at t = 8 ms, it has a phase angle of

$$\theta_{in} = -(t_d / T) \times 360^\circ = -(8/20) \times 360^\circ = -144^\circ$$

The output reaches its peak at 4 ms, and its phase angle is

$$\theta_{out} = -(t_d / T) \times 360^\circ = -(4/20) \times 360^\circ = -72^\circ$$

The transfer function is

$$H(100) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1\angle -72^\circ}{2\angle -144^\circ} = 0.5\angle 72^\circ$$

P6.19 The input signal is

$$v_{in}(t) = 2 + 3\cos(1000\pi t) + 3\sin(2000\pi t) + \cos(3000\pi t)$$

which has components with frequencies of 0, 500, 1000, and 1500 Hz. We can determine the transfer function at these frequencies by dividing the corresponding output phasor by the input phasor.

The output is

$$v_{out}(t) = 3 + 2\cos(1000\pi t + 30^\circ) + 4\sin(3000\pi t)$$

Thus, we have

$$H(0) = 3/2 = 1.5 \quad H(500) = \frac{2\angle 30^\circ}{3\angle 0^\circ} = 0.6667\angle 30^\circ$$

$$H(1000) = \frac{0}{3\angle -90^\circ} = 0 \quad H(1500) = \frac{4\angle -90^\circ}{1\angle 0^\circ} = 4\angle -90^\circ$$

P6.26* The half-power frequency of the filter is

$$f_B = \frac{1}{2\pi RC} = 500 \text{ Hz}$$

The transfer function is given by Equation 6.9 in the text:

$$H(f) = \frac{1}{1 + j(f/f_B)}$$

The given input signal is

$$v_{in}(t) = 5\cos(500\pi t) + 5\cos(1000\pi t) + 5\cos(2000\pi t)$$

which has components with frequencies of 250, 500, and 1000 Hz.

Evaluating the transfer function for these frequencies yields:

$$H(250) = \frac{1}{1 + j(250/500)} = 0.8944\angle -26.57^\circ$$

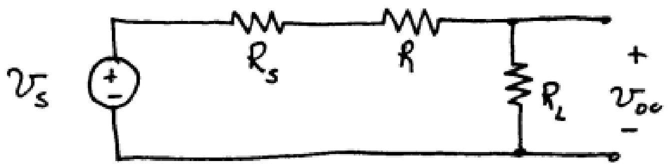
$$H(500) = 0.7071\angle -45^\circ$$

$$H(1000) = 0.4472\angle -63.43^\circ$$

Applying the appropriate value of the transfer function to each component of the input signal yields the output:

$$v_{out}(t) = 4.472\cos(500\pi t - 26.57^\circ) + 3.535\cos(1000\pi t - 45^\circ) + 2.236\cos(2000\pi t - 63.43^\circ)$$

P6.33 (a) First, we find the Thévenin equivalent for the source and resistances.



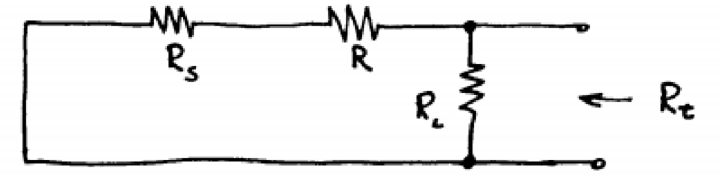
The open-circuit voltage is given by

$$v_t(t) = v_{oc}(t) = v_s(t)\frac{R_L}{R_s + R + R_L}$$

In terms of phasors, this becomes:

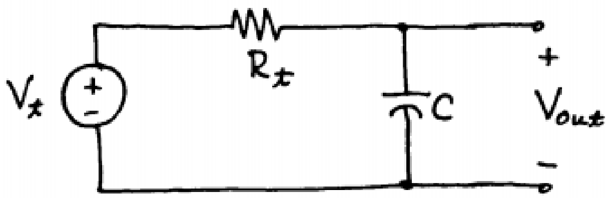
$$\mathbf{V}_t = \mathbf{V}_s \frac{R_L}{R_s + R + R_L} \qquad (1)$$

Zeroing the source, we find the Thévenin resistance:



$$R_t = \frac{1}{1/R_L + 1/(R + R_s)}$$

Thus, the original circuit has the equivalent:



The transfer function for this circuit is:

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_t} = \frac{1}{1 + j(f/f_B)} \qquad (2)$$

where, $f_B = \dfrac{1}{2\pi R_t C}$

Using Equation (1) to substitute for $\mathbf{V}_t$ in Equation (2) and rearranging, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_s} = \frac{R_L}{R_s + R + R_L} \times \frac{1}{1 + j(f/f_B)} \qquad (3)$$

(b) Evaluating for the circuit components given, we have:

$$R_t = 980\ \Omega$$

$$f_B = 812.0\ \text{Hz}$$

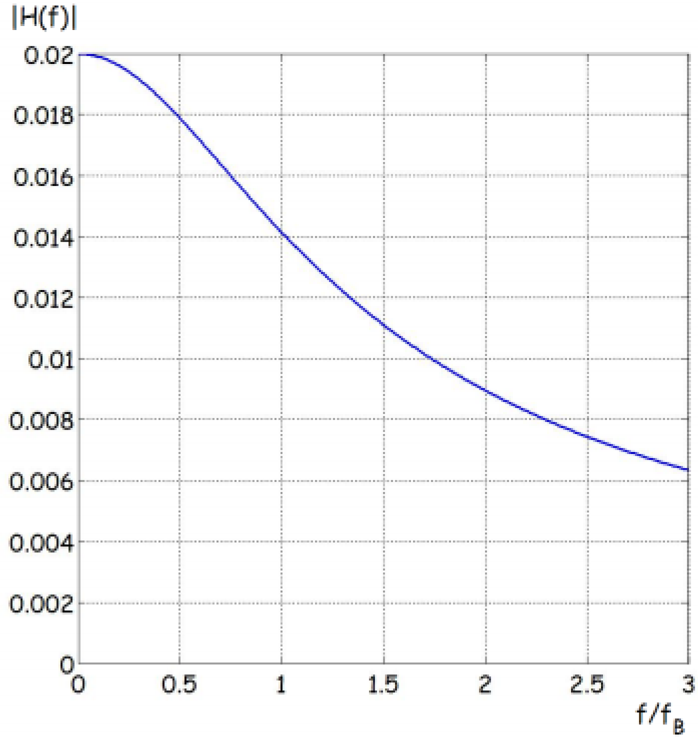
$$H(f) = \frac{0.02}{1 + j(f/f_B)}$$

A MATLAB program to plot the transfer-function magnitude is:

```
foverfb=0:0.01:3;
```

```
Hmag=abs(0.02./(1 + i*foverfb));
plot(foverfb,Hmag)
axis([0 3 0 0.02])
```

The resulting plot is:



P6.48* (a) The overall transfer function is the product of the transfer functions of the filters in cascade:

$$H(f) = H_1(f) \times H_2(f) = \frac{1}{[1 + j(f/f_B)]^2}$$

(b)

$$|H(f)| = \frac{1}{1 + (f/f_B)^2}$$

$$|H(f_{3dB})| = \frac{1}{\sqrt{2}} = \frac{1}{1 + (f_{3dB}/f_B)^2}$$

$$(f_{3dB}/f_B)^2 = \sqrt{2} - 1$$

$$f_{3dB} = f_B\sqrt{\sqrt{2} - 1} = 0.6436 f_B$$

P6.49 For filters in cascade, the transfer functions in decibels are added.

$$|H(f_1)|_{db} = |H_1(f_1)|_{db} + |H_2(f_1)|_{db}$$

$$20log|H(f_1)| = 40$$

$$log|H(f_1)| = 2$$

$$|H(f_1)| = 10^2 = 100$$

P6.56 This is a first-order lowpass *RC* filter. The break frequency is:

$$f_B = \frac{1}{2\pi RC} = 5.895 \text{ MHz}$$

The Bode plots are like Figures 6.15 and 6.16 in the text.

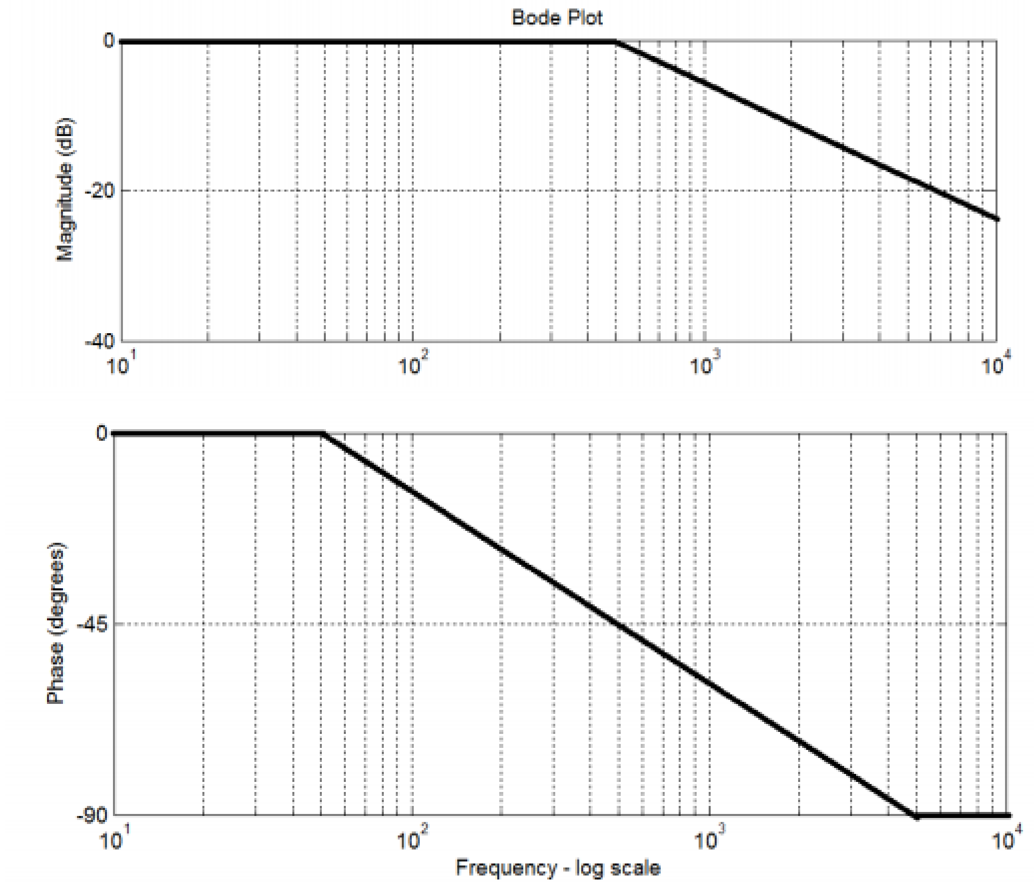
P6.59 Applying the voltage-division principle, we find the transfer function:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{R}{R + j2\pi fL} = \frac{1}{1 + j(f/f_B)}$$

where $f_B = R/2\pi L$. Thus, this is a first-order lowpass filter.

The break frequency is $f_B = R/2\pi L = 500 \text{ Hz}$.

The plots are

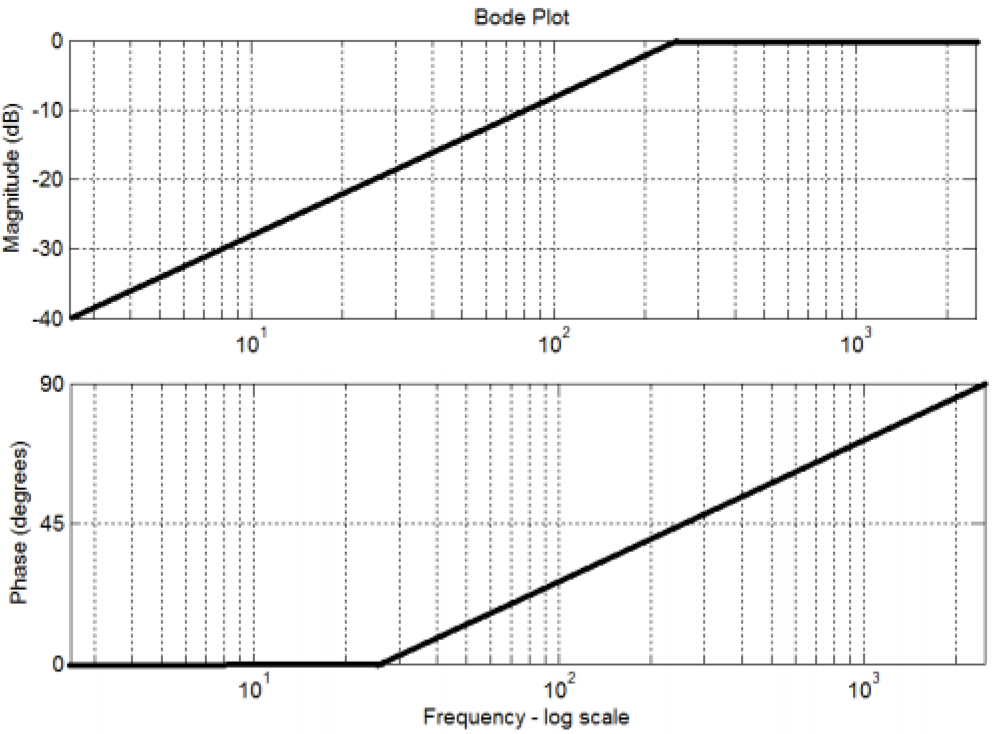


P6.67 This is a first-order high-pass filter analyzed in Section 6.5 in the text. The transfer function is

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{j(f/f_B)}{1 + j(f/f_B)}$$

where $f_B = \dfrac{1}{2\pi RC} = 250\text{ Hz}$.

The Bode plots are



P6.68 Applying the voltage-division principle, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{j2\pi fL}{R + j2\pi fL} = \frac{j2\pi fL/R}{1 + j2\pi fL/R}$$

$$= \frac{j(f/f_B)}{1 + j(f/f_B)}$$

in which $f_B = R/2\pi L = 2\text{ MHz}$. The Bode plots are the same as Figure 6.21 in the text.

P6.71*

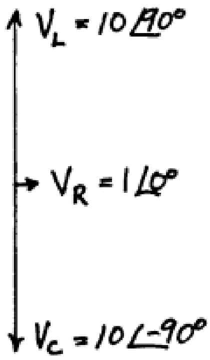
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 1.125\text{ MHz}$$

$$Q_s = \frac{2\pi f_0 L}{R} = 10$$

$$B = \frac{f_0}{Q_s} = 112.5\text{ kHz}$$

$$f_H \cong f_0 + \frac{B}{2} = 1.181\text{ MHz}$$

$$f_L \cong f_0 - \frac{B}{2} = 1.069\text{ MHz}$$



At the resonant frequency: $\mathbf{V}_R = 1\angle 0^\circ$ $\quad \mathbf{V}_L = 10\angle 90^\circ$ $\quad \mathbf{V}_C = 10\angle -90^\circ$

P6.79*

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 1.592 \text{ MHz}$$

$$Q_p = \frac{R}{2\pi f_0 L} = 10.00 \qquad B = \frac{f_0}{Q_p} = 159.2 \text{ kHz}$$