Probability and Statistics – Quiz 1(Solution)

1.

(a) A stem-and-leaf plot is shown next.
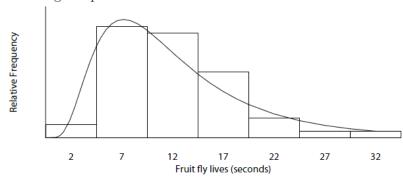
Stem	Leaf	Frequency
0*	34	2
0	56667777777889999	17
1*	0000001223333344	16
1	5566788899	10
2*	034	3
2	7	1
3*	2	1

(b) The relative frequency distribution table is shown next.

Relative Frequency Distribution of Fruit Fly Lives

Class Interval	Class Midpoint	Frequency, f	Relative Frequency
$0-4$	2	2	0.04
$5-9$	7	17	0.34
$10-14$	12	16	0.32
$15-19$	17	10	0.20
$20-24$	22	3	0.06
$25-29$	27	1	0.02
$30-34$	32	1	0.02

(c) A histogram plot is shown next.

2.

(a) $P(A \cap B \cap C) = P(C \mid A \cap B)P(B \mid A)P(A) = (0.20)(0.75)(0.3) = 0.045.$

(b) $P(B' \cap C) = P(A \cap B' \cap C) + P(A' \cap B' \cap C) = P(C \mid A \cap B')P(B' \mid A)P(A) + P(C \mid A' \cap B')P(B' \mid A')P(A') = (0.80)(1-0.75)(0.3) + (0.90)(1-0.20)(1-0.3) = 0.564.$

3.

(a) $g(x) = \int_0^1 (x+y)\, dy = x + \frac{1}{2}$, for $0 < x < 1$, and $h(y) = y + \frac{1}{2}$ for $0 < y < 1$.

(b) $P(X > 0.5, Y > 0.5) = \int_{0.5}^1 \int_{0.5}^1 (x+y)\, dx\, dy = \int_{0.5}^1 \left(\frac{x^2}{2} + xy\right)\Big|_{0.5}^1 \, dy$
$= \int_{0.5}^1 \left[\left(\frac{1}{2} + y\right) - \left(\frac{1}{8} + \frac{y}{2}\right)\right]\, dy = \frac{3}{8}$.

4.

(a) $E(X+Y) = E(X) + E(Y) = 3.5 + 3.5 = 7.0$.

(b) $E(XY) = E(X)E(Y) = (3.5)(3.5) = 12.25$.