HW9 solution

6.25

$n = 100$.

(a) $p = 0.01$ with $\mu = (100)(0.01) = 1$ and $\sigma = \sqrt{(100)(0.01)(0.99)} = 0.995$.
So, $z = (0.5 - 1)/0.995 = -0.503$. $P(X \le 0) \approx P(Z \le -0.503) = 0.3085$.

(b) $p = 0.05$ with $\mu = (100)(0.05) = 5$ and $\sigma = \sqrt{(100)(0.05)(0.95)} = 2.1794$.
So, $z = (0.5 - 5)/2.1794 = -2.06$. $P(X \le 0) \approx P(X \le -2.06) = 0.0197$.

6.53

$\alpha = 5$; $\beta = 10$;

(a) $\alpha\beta = 50$.

(b) $\sigma^2 = \alpha\beta^2 = 500$; so $\sigma = \sqrt{500} = 22.36$.

(c) $P(X > 30) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_{30}^{\infty} x^{\alpha-1} e^{-x/\beta}\, dx$. Using the incomplete gamma with $y = x/\beta$, then

$$1 - P(X \le 30) = 1 - P(Y \le 3) = 1 - \int_0^3 \frac{y^4 e^{-y}}{\Gamma(5)}\, dy = 1 - 0.185 = 0.815.$$

6.58

$\beta = 1/5$ and $\alpha = 10$.

(a) $P(X > 10) = 1 - P(X \le 10) = 1 - 0.9863 = 0.0137$.

(b) $P(X > 2)$ before 10 cars arrive.

$$P(X \le 2) = \int_0^2 \frac{1}{\beta^\alpha} \frac{x^{\alpha-1} e^{-x/\beta}}{\Gamma(\alpha)}\, dx.$$

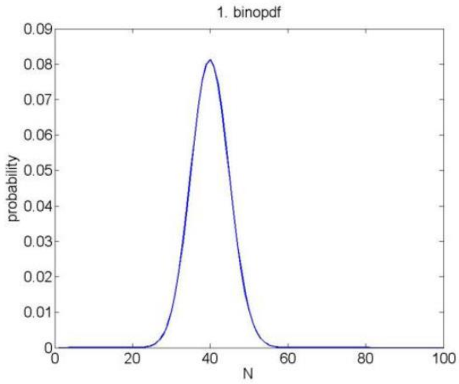
Given $y = x/\beta$, then

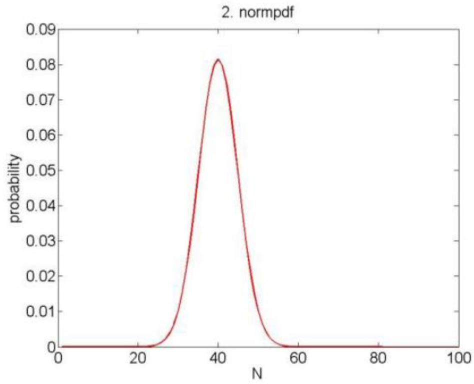
$$P(X \le 2) = P(Y \le 10) = \int_0^{10} \frac{y^{\alpha-1} e^{-y}}{\Gamma(\alpha)}\, dy = \int_0^{10} \frac{y^{10-1} e^{-y}}{\Gamma(10)}\, dy = 0.542,$$

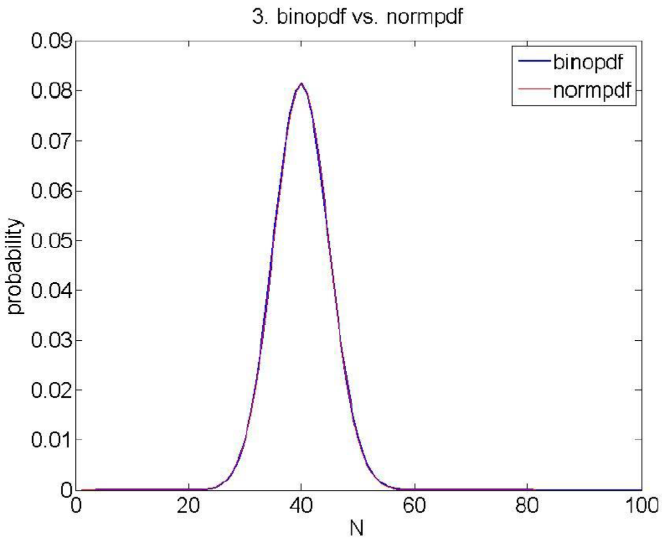
with $P(X > 2) = 1 - P(X \le 2) = 1 - 0.542 = 0.458$.

Matlab

```
% HW09 matlab question

% 1. plot the binomial disrbution of X=1:100
x=1:100;
y=binopdf(x,100,0.4);
figure;
plot(x,y,'-');title('1. binopdf');xlabel('N');ylabel('probability');

% 2. plot the normal distribution that approximates the previous
% distribution
mean=100*0.4;
sd=sqrt(100*0.4*0.6);     %sd=Standard Deviation
z=normpdf(x,mean,sd);
figure;plot(x,z,'r');title('2. normpdf');xlabel('N');ylabel('probability');

% 3. compare the two pdf
figure;plot(x,y,'b',x,z,'r');title('Comparison');title('3. binopdf vs. normpdf');xlabel('N');
ylabel('probability'); legend('binopdf','normpdf');
```







In this case, the result shows that normal distribution can provide a good approximation to binomial distribution. However, there are still some tiny differences at some points.