Engineering Mathematics

(Solutions)

Final Exam 2018/01/15 .

1.

(a)

$$\mathscr{L}^{-1}\left\{\frac{s+1}{s^2-4s}\right\}=\mathscr{L}^{-1}\left\{-\frac{1}{4}\cdot\frac{1}{s}+\frac{5}{4}\cdot\frac{1}{s-4}\right\}=-\frac{1}{4}+\frac{5}{4}e^{4t}$$

(b)

$$\mathscr{L}^{-1}\left\{\frac{s}{(s^2+4)(s+2)}\right\}=\mathscr{L}^{-1}\left\{\frac{1}{4}\cdot\frac{s}{s^2+4}+\frac{1}{4}\cdot\frac{2}{s^2+4}-\frac{1}{4}\cdot\frac{1}{s+2}\right\}$$
$$=\frac{1}{4}\cos 2t+\frac{1}{4}\sin 2t-\frac{1}{4}e^{-2t}$$

2.

(a)

The Laplace transform of the initial-value problem is

$$s^2\mathscr{L}\{y\}-sy(0)-y'(0)-4\left[s\mathscr{L}\{y\}-y(0)\right]=\frac{6}{s-3}-\frac{3}{s+1}.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\}=\frac{6}{(s-3)(s^2-4s)}-\frac{3}{(s+1)(s^2-4s)}+\frac{s-5}{s^2-4s}$$
$$=\frac{5}{2}\cdot\frac{1}{s}-\frac{2}{s-3}-\frac{3}{5}\cdot\frac{1}{s+1}+\frac{11}{10}\cdot\frac{1}{s-4}.$$

Thus

$$y=\frac{5}{2}-2e^{3t}-\frac{3}{5}e^{-t}+\frac{11}{10}e^{4t}.$$

(b)

The Laplace transform of the initial-value problem is

$$s^2\mathscr{L}\{y\}+9\mathscr{L}\{y\}=\frac{1}{s-1}.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\}=\frac{1}{(s-1)(s^2+9)}=\frac{1}{10}\cdot\frac{1}{s-1}-\frac{1}{10}\cdot\frac{1}{s^2+9}-\frac{1}{10}\cdot\frac{s}{s^2+9}.$$

Thus

$$y=\frac{1}{10}e^{t}-\frac{1}{30}\sin 3t-\frac{1}{10}\cos 3t.$$

3.

(a)

$$\mathscr{L}^{-1}\left\{\frac{5s}{(s-2)^2}\right\} = \mathscr{L}^{-1}\left\{\frac{5(s-2)+10}{(s-2)^2}\right\} = \mathscr{L}^{-1}\left\{\frac{5}{s-2}+\frac{10}{(s-2)^2}\right\} = 5e^{2t}+10te^{2t}$$

(b)

$$L^{-1}\left\{\frac{(s+1)^2}{(s+2)^2}\right\}=L^{-1}\left\{\frac{(s+2)^2-2s-3}{(s+2)^2}\right\}=L^{-1}\left\{1-\frac{2(s+2)+1}{(s+2)^2}\right\}=L^{-1}\left\{1-\frac{1}{(s+2)}+\frac{1}{(s+2)^2}\right\}$$

Ans: $\delta(t)-2e^{-2t}+te^{-2t}$

4.

The Laplace transform of the given equation is

$$\mathscr{L}\{f\}+2\mathscr{L}\{\cos t\}\mathscr{L}\{f\}=4\mathscr{L}\left\{e^{-t}\right\}+\mathscr{L}\{\sin t\}.$$

Solving for $\mathscr{L}\{f\}$ we obtain

$$\mathscr{L}\{f\}=\frac{4s^2+s+5}{(s+1)^3}=\frac{4}{s+1}-\frac{7}{(s+1)^2}+4\frac{2}{(s+1)^3}.$$

Thus

$$f(t)=4e^{-t}-7te^{-t}+4t^2e^{-t}.$$

5.

The Laplace transform of the given equation is

$$\mathscr{L}\{t\}-2\mathscr{L}\{f\}=\mathscr{L}\left\{e^t-e^{-t}\right\}\mathscr{L}\{f\}.$$

Solving for $\mathscr{L}\{f\}$ we obtain

$$\mathscr{L}\{f\}=\frac{s^2-1}{2s^4}=\frac{1}{2}\frac{1}{s^2}-\frac{1}{12}\frac{3!}{s^4}.$$

Thus

$$f(t)=\frac{1}{2}t-\frac{1}{12}t^3.$$

6.

Take the transform of both sides of the equation to get

$$\mathscr{L}\{f(t)\} = \mathscr{L}\{e^t\} + \mathscr{L}\left\{e^t \cdot \int_0^t f(\tau)e^{-\tau}\, d\tau\right\}$$

$$F(s) = \frac{1}{s-1} + \left[\mathscr{L}\{f(t)e^{-t}\}\,\mathscr{L}\{1\}\right]_{s \to s-1}$$

$$F(s) = \frac{1}{s-1} + \left[\mathscr{L}\{f(t)e^{-t}\}\right]_{s \to s-1} \cdot \left[\mathscr{L}\{1\}\right]_{s \to s-1}$$

$$F(s) = frac1s - 1 + [F(s+1)]_{s \to s-1} \cdot \left[\frac{1}{s}\right]_{s \to s-1}$$

$$F(s) = \frac{1}{s-1} + F(s) \cdot \frac{1}{s-1}$$

Solve the last equation for $F(s)$ to get $F(s) = 1/(s-2)$ therefore $f(t) = e^{2t}$.

7.

Taking the Laplace transform of the differential equation we obtain

$$\begin{aligned}\mathscr{L}\{y\} &= \frac{s^3+2}{s^3(s-5)} - \frac{2+2s+s^2}{s^3(s-5)}e^{-s} \\ &= -\frac{2}{125}\frac{1}{s} - \frac{2}{25}\frac{1}{s^2} - \frac{1}{5}\frac{2}{s^3} + \frac{127}{125}\frac{1}{s-5} - \left[-\frac{37}{125}\frac{1}{s} - \frac{12}{25}\frac{1}{s^2} - \frac{1}{5}\frac{2}{s^3} + \frac{37}{125}\frac{1}{s-5}\right]e^{-s}\end{aligned}$$

so that

$$y = -\frac{2}{125} - \frac{2}{25}t - \frac{1}{5}t^2 + \frac{127}{125}e^{5t} - \left[-\frac{37}{125} - \frac{12}{25}(t-1) - \frac{1}{5}(t-1)^2 + \frac{37}{125}e^{5(t-1)}\right]\mathscr{U}(t-1).$$