2016 Algorithm HW4 Solutions

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Question 1(10pts)

Solution:

- The presence of a negative-weight cycle can be determined by looking at the diagonal of the matrix $L^{(n-1)}$ computed by an all-pairs shortest-path algorithm. If the diagonal contains any negative number there must be a negative-weight cycle.

Question 2(10pts)

Solution:

- The identity matrix for “multiplication” should look as the one given in the exercise since 0 is the identity for + and ∞ is the identity for min.

Question 3(10pts)

Solution:

- We wish to compute the transitive closure of a directed graph $G = (V, E)$. Construct a new graph $G^* = (V, E^*)$ where E^* is initially empty. For each vertex v traverse the graph G adding edges for every node encountered in E^* . This takes $O(VE)$ time.

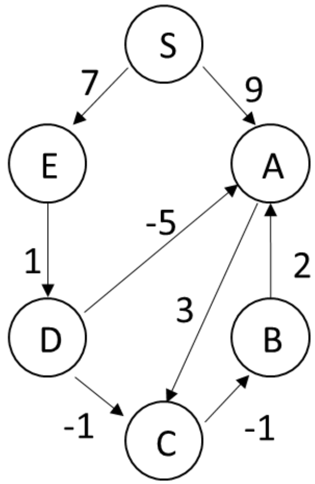
Question 4(10pts)

Solution:

- PPT CH24 P8.9

Question 5(10pts)

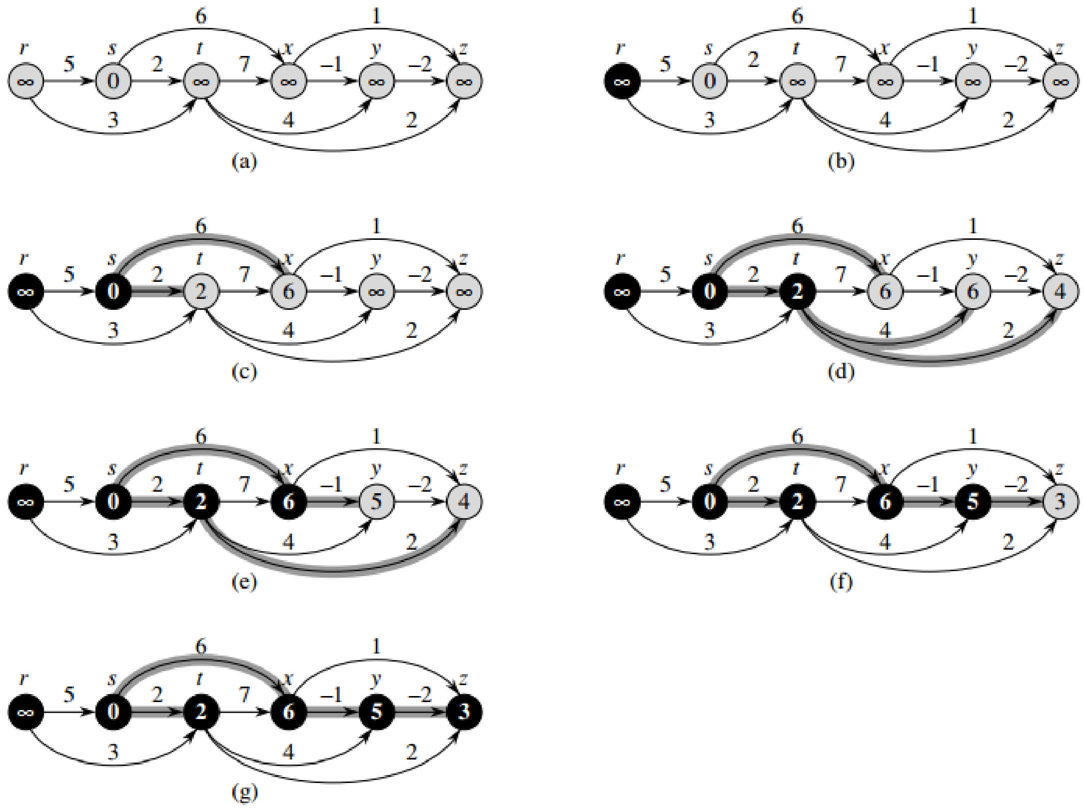
Solution:



	S	A	B	C	D	E
i=1	0	9	11	12	8	7
i=2	0	3	11	7	8	7
i=3	0	3	5	6	8	7
i=4	0	3	5	6	8	7

Question 6(10pts)

Solution:



Question 7(10pts)

Solution:

- After forming the augmented constraint graph and seeking the shortest path from node 0 to all other nodes, using an algorithm with negative length cycle detection, one finds there is a negative length cycle (2, 3, 5, 4, 2) with length $1 - 7 + 10 - 6 = -2$. Thus the system is infeasible.

Question 8(10pts)

Solution:

- Since there is an arc of length 0 from node 0 to every other node, the label on every node (representing the length of the shortest path found so far from node 0 to that node) is set to 0 in the first step. Since it is only modified if a shorter path is found, of necessity such a path must have length less than 0, and so cannot be positive; the answer is “no”.

Question 9 (10pts)

Solution:

- Slow-All-Pairs-Shortest-Paths (5%)
- L^1(1%)

$$\begin{Bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & \infty & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & 3 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{Bmatrix}$$

L^2(1%)

$$\begin{Bmatrix} 0 & 6 & \infty & \infty & -1 & \infty \\ -2 & 0 & \infty & 2 & 0 & \infty \\ 3 & -3 & 0 & 4 & \infty & -8 \\ -4 & 10 & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & \infty & 0 \end{Bmatrix}$$

- L^3 (1%)

$$\begin{Bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ -2 & -3 & 0 & -1 & 2 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 5 & 0 \end{Bmatrix}$$

L^4(1%)

$$\begin{Bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ -5 & -3 & 0 & -1 & -3 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{Bmatrix}$$

- L^5(1%)

$$\begin{Bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ -5 & -3 & 0 & -1 & -6 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{Bmatrix}$$

Question 9(10pts)

Solution:

- Faster-All-Pairs-Shortest-Paths(5%)
- L^2 (1%)

$$\begin{pmatrix} 0 & 6 & \infty & \infty & -1 & \infty \\ -2 & 0 & \infty & 2 & 0 & \infty \\ 3 & -3 & 0 & 4 & \infty & -8 \\ -4 & 10 & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & \infty & 0 \end{pmatrix}$$

L^4 (2%)

$$\begin{pmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ -5 & -3 & 0 & -1 & -3 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{pmatrix}$$

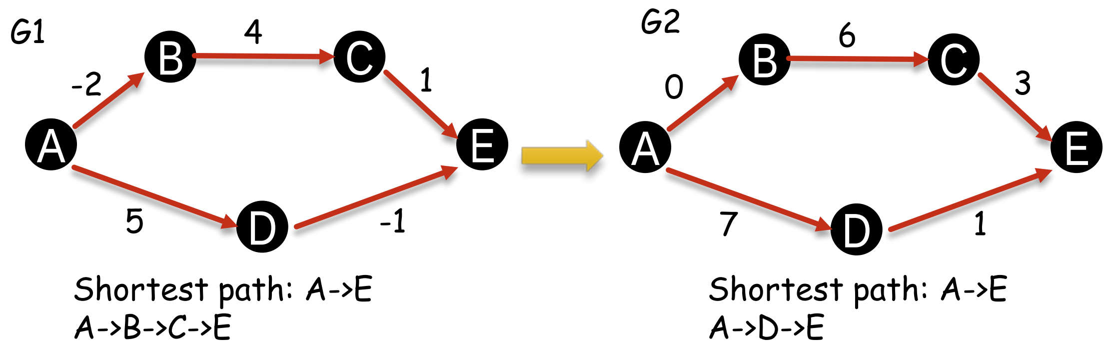
- L^8 (2%)

$$\begin{pmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ -5 & -3 & 0 & -1 & -6 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{pmatrix}$$

Question 10(10pts)

Solution:

- (I) True or False (3%)
  - Running time = $\Theta(n^3 \lg n)$
  - Space requirement = $\Theta(n^2 \lg n)$ or $\Theta(n^2)$
- (II) False (3%)
  - Counter example



- (III) True (2%)
- (IV) True (2%)