Practice exercise 参考解答

1.18

(a) A stem-and-leaf plot is shown below.

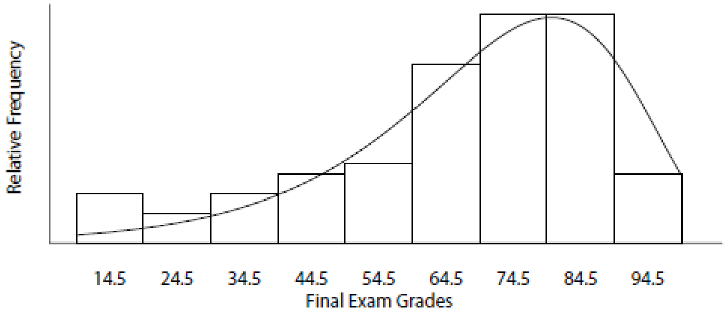
Stem	Leaf	Frequency
1	057	3
2	35	2
3	246	3
4	1138	4
5	22457	5
6	00123445779	11
7	01244456678899	14
8	00011223445589	14
9	0258	4

(b) The following is the relative frequency distribution table.

Relative Frequency Distribution of Grades

Class Interval	Class Midpoint	Frequency, f	Relative Frequency
10 – 19	14.5	3	0.05
20 – 29	24.5	2	0.03
30 – 39	34.5	3	0.05
40 – 49	44.5	4	0.07
50 – 59	54.5	5	0.08
60 – 69	64.5	11	0.18
70 – 79	74.5	14	0.23
80 – 89	84.5	14	0.23
90 – 99	94.5	4	0.07

(c) A histogram plot is given below.



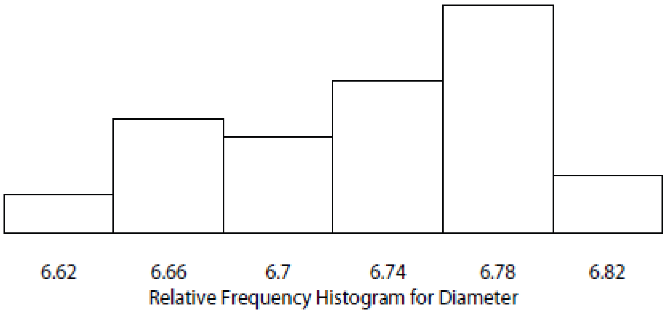
The distribution skews to the left.

(d) $\bar{X} = 65.48$, $\tilde{X} = 71.50$ and $s = 21.13$.

1.22

(a) $\bar{X} = 6.7261$ and $\tilde{X} = 0.0536$.

(b) A histogram plot is shown next.



(c) The data appear to be skewed to the left.

2.38

(a) $8! = 40320$.

(b) There are $4!$ ways to seat 4 couples and then each member of a couple can be interchanged resulting in $2^4(4!) = 384$ ways.

(c) By Theorem 2.3, the members of each gender can be seated in $4!$ ways. Then using Theorem 2.1, both men and women can be seated in $(4!)(4!) = 576$ ways.

2.63

(a) 0.32;

(b) 0.68;

(c) office or den.

2.72

$P(A' \cap B') = 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(A \cap B) = 1 + P(A \cap B) - P(A) - P(B)$.

2.81

Consider the events:
H: husband watches a certain show,
W: wife watches the same show.

(a) $P(W \cap H) = P(W)P(H \mid W) = (0.5)(0.7) = 0.35$.

(b) $P(W \mid H) = \frac{P(W \cap H)}{P(H)} = \frac{0.35}{0.4} = 0.875$.

(c) $P(W \cup H) = P(W) + P(H) - P(W \cap H) = 0.5 + 0.4 - 0.35 = 0.55$.

2.93

This is a parallel system of two series subsystems.

(a) $P = 1 - [1 - (0.7)(0.7)][1 - (0.8)(0.8)(0.8)] = 0.75112$.

(b) $P = \frac{P(A' \cap C \cap D \cap E)}{P\text{system works}} = \frac{(0.3)(0.8)(0.8)(0.8)}{0.75112} = 0.2045$.

2.100

Consider the events
E: a malfunction by other human errors,
A: station A, B: station B, and C: station C.
$P(C \mid E) = \frac{P(E \mid C)P(C)}{P(E \mid A)P(A)+P(E \mid B)P(B)+P(E \mid C)P(C)} = \frac{(5/10)(10/43)}{(7/18)(18/43)+(7/15)(15/43)+(5/10)(10/43)} = \frac{0.1163}{0.4419} = 0.2632.$

3.14

(a) $P(X < 0.2) = F(0.2) = 1 - e^{-1.6} = 0.7981;$

(b) $f(x) = F'(x) = 8e^{-8x}$. Therefore, $P(X < 0.2) = 8\int_0^{0.2} e^{-8x}\,dx = -e^{-8x}\big|_0^{0.2} = 0.7981.$

3.49

(a)

x	1	2	3
$g(x)$	0.10	0.35	0.55

(b)

y	1	2	3
$h(y)$	0.20	0.50	0.30

(c) $P(Y = 3 \mid X = 2) = \frac{0.1}{0.05+0.10+0.20} = 0.2857.$

3.63

(a) $g(x) = \int_0^\infty ye^{-y(1+x)}\,dy = -\frac{1}{1+x}ye^{-y(1+x)}\Big|_0^\infty + \frac{1}{1+x}\int_0^\infty e^{-y(1+x)}\,dy$
$= -\frac{1}{(1+x)^2}\,e^{-y(1+x)}\big|_0^\infty$
$= \frac{1}{(1+x)^2}$, for $x > 0$.
$h(y) = ye^{-y}\int_0^\infty e^{-yx}\,dx = -e^{-y}\,e^{-yx}\big|_0^\infty = e^{-y}$, for $y > 0$.

(b) $P(X \geq 2, Y \geq 2) = \int_2^\infty \int_2^\infty ye^{-y(1+x)}\,dx\,dy = -\int_2^\infty e^{-y}\,e^{-yx}\big|_2^\infty\,dy = \int_2^\infty e^{-3y}dy$
$= -\frac{1}{3}e^{-3y}\big|_2^\infty = \frac{1}{3e^6}.$

4.23

(a) $E[g(X,Y)] = E(XY^2) = \sum_x \sum_y xy^2 f(x,y)$
$= (2)(1)^2(0.10) + (2)(3)^2(0.20) + (2)(5)^2(0.10) + (4)(1)^2(0.15) + (4)(3)^2(0.30)$
$+ (4)(5)^2(0.15) = 35.2.$

(b) $\mu_X = E(X) = (2)(0.40) + (4)(0.60) = 3.20,$
$\mu_Y = E(Y) = (1)(0.25) + (3)(0.50) + (5)(0.25) = 3.00.$

4.36

$\mu = (0)(0.4) + (1)(0.3) + (2)(0.2) + (3)(0.1) = 1.0,$
and $E(X^2) = (0)^2(0.4) + (1)^2(0.3) + (2)^2(0.2) + (3)^2(0.1) = 2.0.$
So, $\sigma^2 = 2.0 - 1.0^2 = 1.0.$

4.77

(a) $P(|X-10| \geq 3) = 1 - P(|X-10| < 3)$
$= 1 - P[10 - (3/2)(2) < X < 10 + (3/2)(2)] \leq 1 - \left[1 - \frac{1}{(3/2)^2}\right] = \frac{4}{9}$.

(b) $P(|X-10| < 3) = 1 - P(|X-10| \geq 3) \geq 1 - \frac{4}{9} = \frac{5}{9}$.

(c) $P(5 < X < 15) = P[10 - (5/2)(2) < X < 10 + (5/2)(2)] \geq 1 - \frac{1}{(5/2)^2} = \frac{21}{25}$.

(d) $P(|X-10| \geq c) \leq 0.04$ implies that $P(|X-10| < c) \geq 1 - 0.04 = 0.96$.
Solving $0.96 = 1 - \frac{1}{k^2}$ we obtain $k = 5$. So, $c = k\sigma = (5)(2) = 10$.

4.82

(a) $E(X) = \int_0^\infty \frac{x}{5} e^{-x/5}\, dx = 5$.

(b) $E(X^2) = \int_0^\infty \frac{x^2}{5} e^{-x/5}\, dx = 50$, so $Var(X) = 50 - 5^2 = 25$, and $\sigma = 5$.

(c) $E[(X+5)^2] = E\{[(X-5)+10]^2\} = E[(X-5)^2] + 10^2 + 20E(X-5)$
$= Var(X) + 100 = 125$.

5.9

For $n = 15$ and $p = 0.25$, we have

(a) $P(3 \leq X \leq 6) = P(X \leq 6) - P(X \leq 2) = 0.9434 - 0.2361 = 0.7073$.

(b) $P(X < 4) = P(X \leq 3) = 0.4613$.

(c) $P(X > 5) = 1 - P(X \leq 5) = 1 - 0.8516 = 0.1484$.

5.26

$n = 8$ and $p = 0.60$;

(a) $P(X = 6) = \binom{8}{6}(0.6)^6(0.4)^2 = 0.2090$.

(b) $P(X = 6) = P(X \leq 6) - P(X \leq 5) = 0.8936 - 0.6846 = 0.2090$.

5.41

Using the binomial approximation of the hypergeometric distribution with 0.7, the probability is $1 - \sum_{x=10}^{13} b(x; 18, 0.7) = 0.6077$.

5.49

Using the negative binomial distribution, the required probability is

$$b^*(10; 5, 0.3) = \binom{9}{4}(0.3)^5(0.7)^5 = 0.0515.$$

5.55

Using the geometric distribution

(a) $P(X = 3) = g(3; 0.7) = (0.7)(0.3)^2 = 0.0630$.

(b) $P(X < 4) = \sum_{x=1}^{3} g(x; 0.7) = \sum_{x=1}^{3} (0.7)(0.3)^{x-1} = 0.9730$.

5.65

(a) $P(X \leq 3|\lambda t = 5) = 0.2650$.

(b) $P(X > 1|\lambda t = 5) = 1 - 0.0404 = 0.9596$.

6.9

(a) $z = (15 - 18)/2.5 = -1.2$; $P(X < 15) = P(Z < -1.2) = 0.1151$.

(b) $z = -0.76$, $k = (2.5)(-0.76) + 18 = 16.1$.

(c) $z = 0.91$, $k = (2.5)(0.91) + 18 = 20.275$.

(d) $z_1 = (17 - 18)/2.5 = -0.4$, $z_2 = (21 - 18)/2.5 = 1.2$;
$P(17 < X < 21) = P(-0.4 < Z < 1.2) = 0.8849 - 0.3446 = 0.5403$.

6.13

(a) $z = (32 - 40)/6.3 = -1.27$; $P(X > 32) = P(Z > -1.27) = 1 - 0.1020 = 0.8980$.

(b) $z = (28 - 40)/6.3 = -1.90$, $P(X < 28) = P(Z < -1.90) = 0.0287$.

(c) $z_1 = (37 - 40)/6.3 = -0.48$, $z_2 = (49 - 40)/6.3 = 1.43$;
So, $P(37 < X < 49) = P(-0.48 < Z < 1.43) = 0.9236 - 0.3156 = 0.6080$.

6.19

$\mu = \$15.90$ and $\sigma = \$1.50$.

(a) 51%, since $P(13.75 < X < 16.22) = P\left(\frac{13.745-15.9}{1.5} < Z < \frac{16.225-15.9}{1.5}\right)$
$= P(-1.437 < Z < 0.217) = 0.5871 - 0.0749 = 0.5122$.

(b) \$18.36, since $P(Z > 1.645) = 0.05$; $x = (1.645)(1.50) + 15.90 + 0.005 = 18.37$.

6.25

$n = 100$.

(a) $p = 0.01$ with $\mu = (100)(0.01) = 1$ and $\sigma = \sqrt{(100)(0.01)(0.99)} = 0.995$.
So, $z = (0.5 - 1)/0.995 = -0.503$. $P(X \leq 0) \approx P(Z \leq -0.503) = 0.3085$.

(b) $p = 0.05$ with $\mu = (100)(0.05) = 5$ and $\sigma = \sqrt{(100)(0.05)(0.95)} = 2.1794$.
So, $z = (0.5 - 5)/2.1794 = -2.06$. $P(X \leq 0) \approx P(X \leq -2.06) = 0.0197$.

6.53

$\alpha = 5$; $\beta = 10$;

(a) $\alpha\beta = 50$.

(b) $\sigma^2 = \alpha\beta^2 = 500$; so $\sigma = \sqrt{500} = 22.36$.

(c) $P(X > 30) = \frac{1}{\beta^\alpha\Gamma(\alpha)}\int_{30}^{\infty} x^{\alpha-1}e^{-x/\beta}\, dx$. Using the incomplete gamma with $y = x/\beta$, then

$$1 - P(X \leq 30) = 1 - P(Y \leq 3) = 1 - \int_0^3 \frac{y^4 e^{-y}}{\Gamma(5)}\, dy = 1 - 0.185 = 0.815.$$

6.58

$\beta = 1/5$ and $\alpha = 10$.

(a) $P(X > 10) = 1 - P(X \leq 10) = 1 - 0.9863 = 0.0137$.

(b) $P(X > 2)$ before 10 cars arrive.

$$P(X \leq 2) = \int_0^2 \frac{1}{\beta^\alpha} \frac{x^{\alpha-1} e^{-x/\beta}}{\Gamma(\alpha)}\, dx.$$

Given $y = x/\beta$, then

$$P(X \leq 2) = P(Y \leq 10) = \int_0^{10} \frac{y^{\alpha-1} e^{-y}}{\Gamma(\alpha)}\, dy = \int_0^{10} \frac{y^{10-1} e^{-y}}{\Gamma(10)}\, dy = 0.542,$$

with $P(X > 2) = 1 - P(X \leq 2) = 1 - 0.542 = 0.458$.

7.2

From $y = x^2$, $x = 0, 1, 2, 3$, we obtain $x = \sqrt{y}$,

$$g(y) = f(\sqrt{y}) = \binom{3}{\sqrt{y}} \left(\frac{2}{5}\right)^{\sqrt{y}} \left(\frac{3}{5}\right)^{3-\sqrt{y}}, \qquad \text{for} y = 0, 1, 4, 9.$$

7.17

The moment-generating function of X is

$$M_X(t) = E(e^{tX}) = \frac{1}{k} \sum_{x=1}^{k} e^{tx} = \frac{e^t(1 - e^{kt})}{k(1 - e^t)},$$

by summing the geometric series of k terms.

7.19

The moment-generating function of a Poisson random variable is

$$M_X(t) = E(e^{tX}) = \sum_{x=0}^{\infty} \frac{e^{tx} e^{-\mu} \mu^x}{x!} = e^{-\mu} \sum_{x=0}^{\infty} \frac{(\mu e^t)^x}{x!} = e^{-\mu} e^{\mu e^t} = e^{\mu(e^t - 1)}.$$

So,

$$\mu = M_X'(0) = \mu\, e^{\mu(e^t-1)+t}\Big|_{t=0} = \mu,$$
$$\mu_2' = M_X''(0) = \mu e^{\mu(e^t-1)+t}(\mu e^t + 1)\Big|_{t=0} = \mu(\mu + 1),$$

and

$$\sigma^2 = \mu_2' - \mu^2 = \mu(\mu + 1) - \mu^2 = \mu.$$

8.1

(a) Responses of all people in Richmond who have telephones.

(b) Outcomes for a large or infinite number of tosses of a coin.

(c) Length of life of such tennis shoes when worn on the professional tour.

(d) All possible time intervals for this lawyer to drive from her home to her office.

8.14

(a) Replace X_i in S^2 by $X_i + c$ for $i = 1, 2, \ldots, n$. Then $\bar{X}$ becomes $\bar{X} + c$ and

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}[(X_i + c) - (\bar{X} + c)]^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2.$$

(b) Replace X_i by cX_i in S^2 for $i = 1, 2, \ldots, n$. Then $\bar{X}$ becomes $c\bar{X}$ and

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(cX_i - c\bar{X})^2 = \frac{c^2}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2.$$

8.17

$z_1 = -1.9$, $z_2 = -0.4$. Hence,

$P(\mu_{\bar{X}} - 1.9\sigma_{\bar{X}} < \bar{X} < \mu_{\bar{X}} - 0.4\sigma_{\bar{X}}) = P(-1.9 < Z < -0.4) = 0.3446 - 0.0287 = 0.3159.$

8.29

$\mu_{\bar{X}_1-\bar{X}_2} = 72 - 28 = 44$, $\sigma_{\bar{X}_1-\bar{X}_2} = \sqrt{100/64 + 25/100} = 1.346$ and $z = (44.2 - 44)/1.346 = 0.15$. So, $P(\bar{X}_1 - \bar{X}_2 < 44.2) = P(Z < 0.15) = 0.5596$.

8.39

(a) $\chi^2_\alpha = \chi^2_{0.99} = 0.297$.

(b) $\chi^2_\alpha = \chi^2_{0.025} = 32.852$.

(c) $\chi^2_{0.05} = 37.652$. Therefore, $\alpha = 0.05 - 0.045 = 0.005$. Hence, $\chi^2_\alpha = \chi^2_{0.005} = 46.928$.

8.49

$t = (24 - 20)/(4.1/3) = 2.927$, $t_{0.01} = 2.896$ with 8 degrees of freedom. Conclusion: no, $\mu > 20$.

8.59

$P\left(\frac{S_1^2}{S_2^2} < 4.89\right) = P\left(\frac{S_1^2/\sigma^2}{S_2^2/\sigma^2} < 4.89\right) = P(F < 4.89) = 0.99$, where F has 7 and 11 degrees of freedom.