DM final exam solution

<table>
<tr><td>1</td><td>
(a) TRUE
A = 10, B = 5
$$A \cup B = A + B - A \cap B = 1$$
$$A \cap B = 3$$
<u>所求:5 – 3 = 2#</u>
(b) FALSE
$$C_1^n D_1 \ = 一個不再位置上的可能$$
$$C_2^n D_2 \ = 兩個不再位置上的可能$$
…
…
而 n!為 n 個相異物亂序排的可能
<u>應該為$C_1^n D_1 + C_2^n D_2 + \cdots + C_k^n D_k + 1$才對#</u>
<u>((差了通通排正確的可能</u>
(c) TRUE
$11|3a+2b \rightarrow 11|-7(3a+2b)$ -------①
$11|22(a+b)$ ------ ②
①+②
$$\therefore 11|-7(3a+2b)+22(a+b)$$
$$\rightarrow 11|a(-21+22)+b(-14+22)$$
<u>$\rightarrow 11|a+8b$ #</u>
(d) FALSE
$H_{10}^5 = C_{10}^{14} = 1001$
$C_2^5 \times H_{10}^3 = 10 \times C_{10}^{12} = 660$
$\rightarrow \frac{660}{1001} = \frac{60}{91} > \frac{1}{4}$ #

答案正確但沒有解釋 -4 分
答案正確但解釋錯誤 -3 分
全錯 -5 分
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<tr><td>2</td><td>
(a)
$\frac{5!}{2!\,3!} \times 2^2 \times 1^3 + \frac{5!}{3!\,1!\,1!} \times 2^3 \times 3 \times 1$
=40+480
=520
(b)
$X^{50}(1+x^3+x^6+x^9+x^{12})^{10}$ ←寫到這裡 5 分
= $X^{50}[\frac{1-x^{15}}{1-x^3}]^{10}$
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	=$x^{50}(1-x^{15})^{10}(1-x^3)^{-10}$ = C(20,9)-10×C(15,9)+45×C(10,9) ←少一項扣 2 分
3	(a) (p↓q) (p↓q) (b) ((p↓p)↓q) ((p↓p)↓q) 因題目規定只能用↓，故如果使用 not 扣 3 分
4	S(7,2)+S(7,3)+S(7,4) ←算全對 =63+301+350=714 ←這邊算錯不扣分 少一項扣 3 分
5	(a) $4 \times 5 \times 4 \times 3 \times 2$, a 從{3,4,5,6}選 ， 剩下的各自分配 (b) 4! , {3,4,5,6}分配到{b,c,d,e} (c) 3×5^9 , {c,d,e}可當 identity, $\frac{4\times4-4}{2} + 4 - 1 = 9$ (d) S(4,3) + S(4,4) = 7 (e) $6! - 6 \times 5! + 11 \times 4! - 8 \times 3! + 2 \times 2! = 220$ 列出 Rook polynomial ←給 2 分
6.	(a) $C_i = x_i \geq 8$, $\mathrm{N}(C_1\ C_2\ C_3\ C_4) = S_0 - S_1 + S_2 - S_3 + S_4$ $= H_{18}^4 - C_1^4 H_{10}^4 + C_2^4 H_2^4 - 0 + 0$ $= C_{18}^{18+4-1} - 4 * C_{10}^{10+4-1} + 6 * C_2^{4+2-1}$ = 246 (b) $\mathrm{f(x)} = (1 + x + x^2 + x^3 + \cdots + x^7)^4 = \left(\frac{1-x^8}{1-x}\right)^4$ $= C_{18}^{-4}(-1)^{18} - C_1^4 C_{10}^{-4}(-1)^{10} + C_2^4 C_2^{-4}(-1)^2 = 246$ 計算錯誤扣 2 分
7.	(a) $\left(x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)^4 = (e^x - 1)^4$ 找 $\frac{x^{12}}{12!}$ 係數 $= e^{4x} - 4e^{3x} + 6e^{2x} - 4e^x + 1 = 4^{12} - C_1^4 3^{12} + C_2^4 2^{12} - C_3^4 1^{12}$ (b) by inclusion and exclusion： 全 －少一種顏色 －少兩種 －少三種 $S_0 - S_1 + S_2 - S_3 + S_4 = 4^{12} - C_1^4 3^{12} + C_2^4 2^{12} - C_3^4 1^{12}$
8	$a_n = 3 \times 2^n - 2 \times 3^n + \frac{17}{18} \times n \times 3^n + \frac{7}{8} \times n^2 \times 3^n$ $a_n^{(h)} = A \times 3^n + B \times n \times 3^n$ ←給 2 分 $a_n^{(p)} = C \times 2^n + D \times n^2 \times 3^n$ ←給 2 分 C = 3 , D = 7/18 ←給 3 分 A = -2 , B = 17/18 ←給 3 分

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9	(a) $a_1 = 1, a_2 = 2, a_3 = 4, a_4 = 7, \ldots\ldots a_7 = 44, a_8 = 81$ $\boldsymbol{a_n = a_{n-1} + a_{n-2} + a_{n-3}\ (when\ n \geq 4)}$ **#** $\boldsymbol{a_8 = 81}$**#** (b) $a_1 = 2\ , a_2 = 4\ , a_3 = 7, a_4 = 13, a_5 = 24\ \ldots\ldots..$ $(\text{when k} = 3) a_n = a_{n-1} + a_{n-2} + a_{n-3}$ $(\text{when k} = 3) a_n = a_{n-1} + a_{n-2} + a_{n-3} + a_{n-4}$ $\ldots.$ $\boldsymbol{(\textbf{when k} = \textbf{k}) a_n = a_{n-1} + a_{n-2} + a_{n-3} + a_{n-4} + \cdots + a_{n-k}}$ **#**