2009/6/22

Discrete Mathematics (2009 Spring) Final

(total:110 points, max:100 points)

1. (30 points) For each of the following statements, **determine** and **explain** (required) whether it is correct or not.
   (1). The number of the distinct terms in the complete expansion of $(2x + 3y^{-1}+1)^4$ is 20.
   (2). The sequence generated by the generating function $f(x) = \dfrac{1}{(3-x)} - (3-x)$ is -1/3, 10/9, $(1/3)^3$, $(1/3)^4$, … .
   (3). For sets A, B, C $\subseteq$ U, $A - (B \cup C) = (A - B) \cap (A - C)$.
   (4). Let A={a, b, c, d}. the number of closed binary operations *f* on A satisfy that *f*(a, b)=c and *f* have an identity is $3*4^8$.
   (5). If A={1, 2, 3} and there are 504 injective functions f: A→B, then |B|=9.
   (6). For |A|=6, the number of symmetric relations is 2^{21}.

2. (5+5+5 points) Determine how many integer solutions there are to $x_1 + x_2 + x_3 + x_4 = 18$, if (1) $1 \le x_i$ for all i, (2) $x_1 + x_2 = 8$, $0 \le x_i$ for all i, (3) $1 \le x_i \le 6$ for all i.

3. (12 points) For A={1, 2, 3, 4} and B={u, v, x, y}, determine the number of one-to-one functions f : A→B where $f(1)\neq$v, $f(2)\neq$u, $f(3)\neq$x, y and $f(4)\neq$x, y.

4. (10 points) How many three-element subsets of S = {1, 2,…,10} contains no consecutive integers?

5. (10 points) Solve the recurrence relation $a_{n+2}+ 4a_{n+1} + 4a_n = (-2)^n$. $n \ge 0$, $a_0=1$, $a_1=2$.

6. (8 points) In the alphabet {0, 1, 2}, let a_n to be the number of strings of length n in which there is never a 2 anywhere to the right of a 0. Please describe and explain a_n in a recurrence relation form.

7. (10 points) Find the number of permutations of 26 letters of the alphabet in which none of the patterns *start*, *fist*, *love*, or *ten* occurs.

8. (4+6 points) For n distinct objects, let $a(n, r)$ denote the number of ways we can select, *without repetition*, r of the n objects when $0 \le r \le n$. Here $a(n, r) = 0$ when $r > n$. (1) Describe $a(n, r)$ in a recurrence relation form, (2) and show that $f(x)=(1+x)^n$ generates $a(n, r)$, $r \ge 0$.

9. (5 points) Please list 2 examples/methods/strategies to improve your learning motivation.