2017/5/24

Discrete Mathematics (2017 Spring) Midterm II

1. (**25** points) For each of the following statements, determine and explain (required) whether it is correct or not.
   (a). Let R and S be relations on X. If R and S are transitive, then $R \cap S$ is transitive.
   (b). String 01011 is in the language $\{00\}^*\{01\}^+\{1\}^*$ and is also in the language $\{01\}^*\{0\}^*\{11\}^*\{1,0\}^+$.
   (c). The number of different set $A = \{a, b, c\} \subseteq Z^+, where\ a, b, c \geq 1$, satisfy the property $a \cdot b \cdot c = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$ is 40.
   (d). Let f: A→B and g: B→C, $g \circ f$ is one-to-one if and only if f and g are one-to-one.
   (e). A={1,2,3,4,5}x{1,2,3,4,5}, and define $\boldsymbol{R}$ on A by $(x_1, y_1)\boldsymbol{R}(x_2, y_2), if\ x_1 + y_1 = x_2 + y_2$. $\boldsymbol{R}$ is an equivalence relation on A.

2. (**12** points)
   (a) Let S be a set of seven positive integers whose maximum is at most 24. Show that the sums of the elements in all the nonempty subsets of S cannot all be distinct.
   (b) A wheel of fortune has the integers from 1 to 25 places on it in a random manner. Show that regardless of how the numbers are positioned on the wheel, there are three adjacent numbers whose sum is at least 39.
   (c) A course gives a single choice quiz that has 4 questions, each with 4 possible responses. What is the minimum number of students to guarantee that at least 4 answer sheets must be identical?

3. (**8** points) If A = $\{a, b, c, d, e, f\}$, determine the number of relations on A that are
   (a) reflexive and symmetric but not transitive,
   (b) antisymmetric but not reflexive

4. (**5, 2, 3** points) Let p, q be two distinct primes. We denote relation $x\mathbf{R}y$ if x divides y. Under this relation **R**, (a) please draw the Hasse diagram of all positive divisors of p^3q^2 that are smaller than p^3q^2. (b) Please answer the maximum element(s), the greatest element(s), and (c) glb $\{p^2, p^2q\}$ and lub $\{pq, p^2, p^2q\}$.

5. (**3, 4, 3** points) Let A={a, b, c, d, e, f} (a) how many closed binary operations f on A satisfy f(a, b) ≠c? (b) How many closed binary operations f on A have an identity and f(a, b)=c? (c) How many f in (b) are commutative?

6. (**15** points) Design a problem that can be solved by two different FSMs that their different of the number of states is 2. Use and show the minimization process to reduce the bigger one.

7. (**10** points) Suppose that R is a relation on X that is symmetric and transitive but not reflexive. Support also that $|X| \geq 2$. Define the relation $\bar{R}$ on X by $\bar{R} = X \times X - R$. (a) $\bar{R}$ is reflexive? (b) $\bar{R}$ is symmetric? (c) $\bar{R}$ is not antisymmetric? (d) $\bar{R}$ is transitive? (e) $\bar{R}$ is a partial order?

8. (**3, 3, 4** points) Let A be a set with |A|=n, and let R be a relation on A. (a) If R is antisymmetric. What is the maximum value for $|R|$? (b) If n=30 and R is an equivalent relation and partition A into disjoint equivalence classes A_1, A_2, A_3, where $|A_1|=|A_2|=|A_3|$. What is $|R|$? (c) IF n=8, how many equivalence relations on A that have exactly one equivalence class of size 4?

(Stirling number of the second kind: S(4,2)=7, S(4, 3)=6, S(5, 2)=15, S(5, 3)=25, S(5, 4)=10, S(6, 2)=31, S(6, 3)=90, S(6, 4)=65, S(6,5)=15)