HW5 参考解答

4.23 (a) $E[g(X,Y)] = E(XY^2) = \sum_x \sum_y xy^2 f(x,y)$
$= (2)(1)^2(0.10) + (2)(3)^2(0.20) + (2)(5)^2(0.10) + (4)(1)^2(0.15) + (4)(3)^2(0.30)$
$+ (4)(5)^2(0.15) = 35.2.$

(b) $\mu_X = E(X) = (2)(0.40) + (4)(0.60) = 3.20,$
$\mu_Y = E(Y) = (1)(0.25) + (3)(0.50) + (5)(0.25) = 3.00.$

4.36 $\mu = (0)(0.4) + (1)(0.3) + (2)(0.2) + (3)(0.1) = 1.0,$
and $E(X^2) = (0)^2(0.4) + (1)^2(0.3) + (2)^2(0.2) + (3)^2(0.1) = 2.0.$
So, $\sigma^2 = 2.0 - 1.0^2 = 1.0.$

4.77 (a) $P(|X-10| \geq 3) = 1 - P(|X-10| < 3)$
$= 1 - P[10 - (3/2)(2) < X < 10 + (3/2)(2)] \leq 1 - \left[1 - \frac{1}{(3/2)^2}\right] = \frac{4}{9}.$

(b) $P(|X-10| < 3) = 1 - P(|X-10| \geq 3) \geq 1 - \frac{4}{9} = \frac{5}{9}.$

(c) $P(5 < X < 15) = P[10 - (5/2)(2) < X < 10 + (5/2)(2)] \geq 1 - \frac{1}{(5/2)^2} = \frac{21}{25}.$

(d) $P(|X-10| \geq c) \leq 0.04$ implies that $P(|X-10| < c) \geq 1 - 0.04 = 0.96.$
Solving $0.96 = 1 - \frac{1}{k^2}$ we obtain $k = 5$. So, $c = k\sigma = (5)(2) = 10.$

4.82 (a) $E(X) = \int_0^\infty \frac{x}{5} e^{-x/5}\, dx = 5.$

(b) $E(X^2) = \int_0^\infty \frac{x^2}{5} e^{-x/5}\, dx = 50$, so $Var(X) = 50 - 5^2 = 25$, and $\sigma = 5$.

(c) $E[(X+5)^2] = E\{[(X-5)+10]^2\} = E[(X-5)^2] + 10^2 + 20E(X-5)$
$= Var(X) + 100 = 125.$