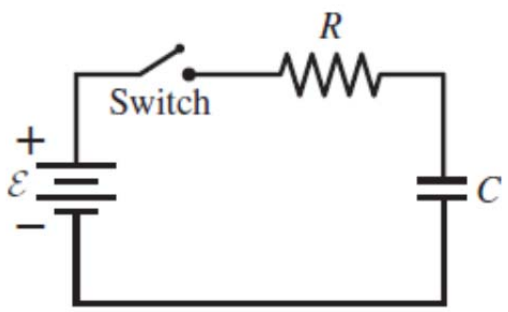
Quiz (VII)

Finished by 18:30 on 6/8

Create a matlab script and change the filename to F7xxxxxxx_quiz7.m. Link all the programs to solve following problems to this script. Make sure once type the filename' F7xxxxxxx_quiz7', the results of the following problems will pop-up automatically in order. Remember not to type any 'clear all ', 'close all' command in any of the codes.

1. [F7xxxxxxx_quiz7_prob1.m] The circuit shown in the figure consists of a DC power source (5V), a capacitor ($20\mu F$) and a resistor ($50k\Omega$). The loop equation of this circuit can be described as $\mathcal{E} - CR\frac{dV_c(t)}{dt} - V_c(t) = 0$. Suppose the voltage of the capacitor before t = 0 is 0.5V.

   (1) Plot the temporal change of the voltage of the capacitor. Which method do you use to solve V_c(t)?
   (2) What is the capacitor's voltage at t = 0.5 sec? What is the precision of the answer and what step size do you use? (The precision should be at least 3 significant digits)
   (3) How long does it take for the capacitor's voltage to charge up to 3.5V? (The precision should be at least 3 significant digits)



2. [F7xxxxxxx_quiz7_prob2.m] The vertical motion of any object near the Earth's surface can be expressed by the equation of motion : $\frac{d^2}{dt^2}\,y(t) = -g = -9.8\,m/s^2$. If an object was thrown up from y(0) = 1.5m with the velocity $y'(t) = 20m/s$, plot its trajectory from t = 0 to t = 3s by solving the differential equation numerically. Record y(3), y'(3) and the step size.

3. {Bonus} [F7xxxxxxx_quiz7_prob3.m]

Suppose in a closed eco-system, where no migration is allowed to get in or out, there are two species of animals: the wolf and the rabbit. They form a simple food chain where the wolf hunts the rabbit, while the rabbit is grass-fed. The size of the two populations can bed described by a simple system of two non-linear 1st order differential equation:

$$\frac{d}{dt}\,r(t) = (a\,r(t) - br(t)^2) - \alpha\,r(t)w(t);$$

$$\frac{d}{dt}\,w(t) \;=\; -c\,w(t) + \gamma\,r(t)w(t);$$

, where w(t) and r(t) denotes the population of the wolf and the rabbit respectively. The other parameters reflect the birth rate, death rate and the food-chain dependency of the two species.

In the following simulation, set a = 0.5, b= 0.001, c = 0.4, α = 0.02, and γ = 0.004, then run the simulation with the specific initial conditions

(a) At the beginning, there are only 100 rabbits. Plot growing curve of the population and explain the phenomena.
(b) At the beginning, there are only 100 wolfs. Plot growing curve of the population and explain the phenomena.
(c) At the beginning, there are 100 rabbits and 6 wolfs. Plot growing curve of the population and explain the phenomena. Will the system become stable?
(d) At the beginning, there are 100 rabbits and 600 wolfs. Plot growing curve of the population and explain the phenomena.