HOMEWORK 4

Section 3-9 13, 22, 38
Section 3-10 14, 16, 40
Problem Plus 7, 10, 11, 15, 27, 28
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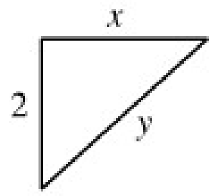
Section 4-3 2, 14, 26, 31, 43, 46, 47, 48, 54, 56, 72, 93

3-9 13.
(a.) Given: a plane ying horizontally at an altitude of 2 km and a speed of 800 km/h passes directly over a radar station. If we let t be time(in hours) and x be the horizontal distance traveled by the plane(in km), then we are given that $dx/dt = 800$ km/h.
(b.) Unknown: the rate at which the distance from the plane to the station is increasing when it is 3 km from the station. If we let y be the distance from the plane to the station, then we want to nd dy/dt when $y = 3$ km.
(c.)



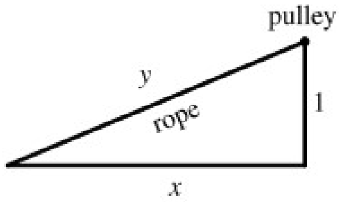
(d.) By the Pythagorean Theorem, $y^2 = x^2 + 2^2 \Rightarrow 2y(dy/dt) = 2x(dy/dt)$.
(e.) $\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{x}{y}(800)$. Since $y^2 = x^2 + 4^2$, when $y = 3$, $x = \sqrt{5}$,
so $\frac{dy}{dt} = \frac{\sqrt{5}}{3}(800) \approx 596$ km/h.

3-9 22. Given $\frac{dy}{dt} = 1$ m/s, find $\frac{dx}{dt}$ when $x = 8$ m.
$y^2 = x^2 + 1 \Rightarrow 2y\frac{dy}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{y}{x}\frac{dy}{dx} = -\frac{y}{x}$.
When $x = 8$, $y = \sqrt{65}$, so $\frac{dx}{dt} = -\frac{\sqrt{65}}{8}$.
Thus, the boat approaches the dock at $\frac{\sqrt{65}}{8} \approx 1.01$ m/s.

3-9 38. $PV^{1.4} = C \Rightarrow P \cdot 1.4V^{0.4}\frac{dV}{dt} + V^{1.4}\frac{dP}{dt} = 0 \Rightarrow \frac{dV}{dt} = -\frac{V^{1.4}}{P \cdot 1.4V^{0.4}}\frac{dP}{dt} = -\frac{V}{1.4P}\frac{dP}{dt}$.
When $V = 400$, $P = 80$ and $\frac{dP}{dt} = -10$, so we have $\frac{dV}{dt} = -\frac{400}{1.4(80)}(-10) = \frac{250}{7}$.
Thus, the volume is increasing at a rate of $\frac{250}{7} \approx 36$ cm^3/min.

3-10 14.

Date: November 21, 2017.

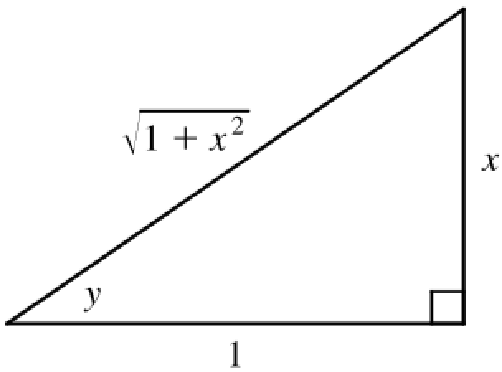


(a.) For $y= f(\theta)= \ln(\sin\theta)$, $f'(\theta)= \frac{1}{\sin\theta}\cos\theta= \cot\theta$, so $dy= \cot\theta d\theta$.
(b.) For $y= f(x)= \frac{e^x}{1-e^x}$, $f'(x)= \frac{(1-e^x)e^x-e^x(-e^x)}{(1-e^x)^2}= \frac{e^x[(1-e^x)-(-e^x)]}{(1-e^x)^2}= \frac{e^x}{(1-e^x)^2}$, so $dy= \frac{e^x}{(1-e^x)^2}dx$.

3-10 16.
(a) $y= \cos\pi x \Rightarrow dy= -\sin\pi x\cdot\pi dx= -\pi\sin\pi x dx$
(b) $x= \frac{1}{3}$ and $dx= -0.02 \Rightarrow dy= -\pi\sin\frac{\pi}{3}(-0.02)= \pi(\sqrt{3}/2)(0.02)= 0.01\pi\sqrt{3} \approx 0.054$.

3-10 40. $F= kR^4 \Rightarrow dF= 4kR^3dR \Rightarrow \frac{dF}{F}= \frac{4kR^3dR}{kR^4}= 4(\frac{dR}{R})$. Thus, the relative change in F is about 4 times the relative change in R. So a 5% increase in the radius corresponds to a 20% increase in blood flow.

plus.7
Let $y = \tan^{-1}x$. Then $\tan y = x$, so from the triangle we see that $\sin(\tan^{-1}x) = \sin y = \frac{x}{\sqrt{1+x^2}}$. Using this fact we have that $\sin(\tan^{-1}(\sinh x)) = \frac{\sinh x}{\sqrt{1+\sinh^2 x}} = \frac{\sinh x}{\cosh x} = \tanh x$. Hence, $\sin^{-1}(\tanh x) = \sin^{-1}(\sin(\tan^{-1}(\sinh x))) = \tan^{-1}(\sinh x)$.



plus.10
$$\lim_{x\to a}\frac{f(x)-f(a)}{\sqrt{x}-\sqrt{a}} = \lim_{x\to a}[\frac{f(x)-f(a)}{\sqrt{x}-\sqrt{a}}\cdot\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}] = \lim_{x\to a}[\frac{f(x)-f(a)}{\sqrt{x}-\sqrt{a}}\cdot$$
$$(\sqrt{x}+\sqrt{a})] = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}\cdot\lim_{x\to a}(\sqrt{x}+\sqrt{a}) = f'(a)\cdot(\sqrt{a}+\sqrt{a}) = 2\sqrt{a}f'(a)$$

plus.11

We must find a value x_0 such that the normal lines to the parabola $y = x^2$ at $x = \pm x_0$ intersect at a point one unit from the point $(\pm x_0, x_0^2)$. The normals to $y = x^2$ at $x = \pm x_0$ have slopes $-\frac{1}{\pm 2x_0}$ and pass through $(\pm x_0, x_0^2)$ respectively, so the normals have the equations $y - x_0^2 = -\frac{1}{2x_0}(x - x_0)$ and $y - x_0^2 = \frac{1}{2x_0}(x + x_0)$. The common y-intercept is $x_0^2 + \frac{1}{2}$.We want to find the value of x_0 for which the distance from $(0, x_0^2 + \frac{1}{2})$ to (x_0, x_0^2) equals 1. The square of the distance is $(x_0 - 0)^2 + [x_0^2 - (x_0^2 + \frac{1}{2})]^2 = x_0^2 + \frac{1}{4} = 1 \Leftrightarrow x_0 = \pm\frac{\sqrt{3}}{2}$. For these values of x_0, the y-intercept is $x_0^2 + \frac{1}{2} = \frac{5}{4}$, so the center of the circle is at $(0, \frac{5}{4})$.
Another solution : Let the center of the circle be $(0, a)$. Then the equation of the circle is $x^2 + (y - a)^2 = 1$. Solving with the equation of the parabola, $y = x^2$, we get $x^2 + (x^2 - a)^2 = 1 \Leftrightarrow x^2 + x^4 - 2ax^2 + a^2 = 1 \Leftrightarrow x^4 + (1 - 2a)x^2 + a^2 - 1 = 0$. The parabola and the cirlcle will be tangent to each other when this quadratic in x^2 has equal roots ; that is, when the discriminant is 0. Thus, $(1 - 2a^2) - 4(a^2 - 1) = 0 \Leftrightarrow 1 - 4a + 4a^2 - 4a^2 + 4 = 0 \Leftrightarrow 4a = 5$, so $a = \frac{5}{4}$. The center of the circle is $(0, \frac{5}{4})$.

plus.15

We can assume without loss of generality that $\theta = 0$ at time $t = 0$, so that $\theta = 12\pi t$ rad. [The angular velocity of the wheel is 360 rpm $= 360 \cdot (2\pi \text{ rad})/(60s) = 12\pi$ rad$/s$.] Then the position of A as a function of time is $A = (40\cos\theta, 4\sin\theta = (40\cos 12\pi t, 40\sin 12\pi t), so \sin\alpha = \frac{y}{1.2m} = \frac{40\sin\theta}{120} = \frac{\sin\theta}{3} = \frac{1}{3}\sin 12\pi t$.

(a) Differentiating the expression for $\sin\alpha$, we get $\cos\alpha \cdot \frac{d\alpha}{dt} = \frac{1}{3} \cdot 12\pi \cdot \cos 12\pi t = 4\pi\cos\theta$. When $\theta = \frac{\pi}{3}$, we have $\sin\alpha = \frac{1}{3}\sin\theta = \frac{\sqrt{3}}{6}$, so $\cos\alpha = \sqrt{1 - (\frac{\sqrt{3}}{6})^2} = \sqrt{\frac{11}{12}}$ and $\frac{d\alpha}{dt} = \frac{4\pi\cos\frac{\pi}{3}}{\cos\alpha} = \frac{2\pi}{\sqrt{11/12}} = \frac{4\pi\sqrt{3}}{\sqrt{11}} \approx 6.56$ rad/s.

(b) By the law of Cosines, $|AP|^2 = |OA|^2 + |OP|^2 - 2|OA||OP|\cos\theta \Rightarrow$
$120^2 = 40^2 + |OP|^2 - 2 \cdot 40|OP|\cos\theta \Rightarrow |OP|^2 - (80\cos\theta)|OP| - 12800 = 0 \Rightarrow$
$|OP| = \frac{1}{2}(80\cos\theta \pm \sqrt{6400\cos^2\theta + 51,200}) = 40\cos\theta \pm 40\sqrt{\cos^2\theta + 8} = 40(\cos\theta + \sqrt{8 + \cos^2\theta})$ cm [since $|OP| > 0$.] As a check, note that $|OP| = 160$ cm when $\theta = 0$ and$|OP| = 80\sqrt{2}$ cm when $\theta = \frac{\pi}{2}$.

(c) By part (b), the x-coordinate of P is given by $x = 40(\cos\theta + \sqrt{8 + \cos^2\theta})$, so $\frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = 40(-\sin\theta - \frac{2\cos\theta\sin\theta}{2\sqrt{8+\cos^2\theta}}) \cdot 12\pi = -480\pi\sin\theta(1 + \frac{\cos\theta}{\sqrt{8+\cos^2\theta}})$ cm/s. In particular, $dx/dt = 0$ cm/s when $\theta = 0$ and $dx/dt = -480\pi$ cm/s when $\theta = \frac{\pi}{2}$.

plus.27

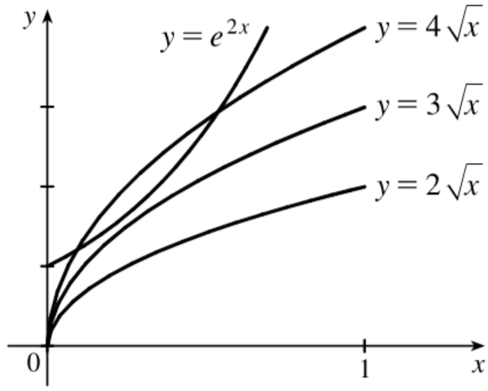
Let $f(x) = e^{2x}$ and $g(x) = k\sqrt{x}[k > 0]$. From the graphs of f and g, we see that f will intersect g exactly once when f and g share a tangent line. Thus, we must have $f = g$ and $f' = g'$ at $x = a$.

$$f(a) = g(a) \Rightarrow e^{2a} = 5\sqrt{a} - (*)$$

and

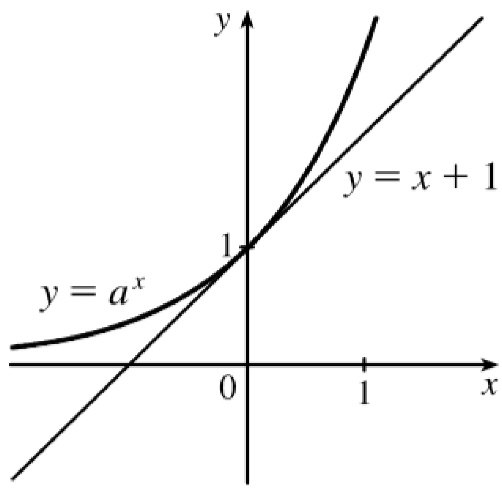
$$f'(a) = g'(a) \Rightarrow 2e^{2a} = \frac{k}{2\sqrt{a}} \Rightarrow e^{2a} = \frac{k}{4\sqrt{a}}.$$

So we must have $k\sqrt{a} = \frac{k}{4\sqrt{a}} \Rightarrow (\sqrt{a})^2 = \frac{k}{4k} \Rightarrow a = \frac{1}{4}$.
From $(*)$, $e^{2(1/4)} = k\sqrt{1/4} \Rightarrow k = 2e^{1/2} = 2\sqrt{e} \approx 3.297$.



plus.28

We see that at $x = 0, f(x) = a^x = 1 + x = 1$, so if $y = a^x$ is to lie above $y = 1 + x$, the two curves must just touch at $(0, 1)$, that is, we must have $f'(0) = 1$. [To see this analytically, note that $a^x \geq 1 + x \Rightarrow a^x - 1 \geq x \Rightarrow \frac{a^x - 1}{x} \geq a$ for $x > 0$, so $f'(0) = \lim\limits_{x \to 0^+} \dfrac{a^x - 1}{x} \geq 1$.
Similarly, for $x < 0, a^x - 1 \geq x \Rightarrow \frac{a^x - 1}{x} \leq 1$, so $f'(0) = \lim\limits_{x \to 0^-} \dfrac{a^x - 1}{x} \leq 1$. Since $1 \leq f'(0) \leq 1$, we must have $f'(0) = 1$.]
But $f'(x) = a^x lna \Rightarrow f'(0) = lna$, so we must have $lna = 1 \Leftrightarrow a = e$.
Another method : The inequality certainly holds for $x \leq -1$, so consider $x > -1, x \neq 0$.
Then $a^x \geq 1 + x \Rightarrow a \geq (1 + x)^{1/x}$ for $x > 0 \Rightarrow$
$a \geq \lim\limits_{x \to 0^+} (1 + x)^{1/x} = e$, by Equation 3.6.5. Also, $a^x \geq 1 + x \Rightarrow$
$a \leq (1 + x)^{1/x}$ for $x < 0 \Rightarrow a \leq \lim\limits_{x \to 0^-} (1 + x)^{1/x} = e$.
So since $e \leq a \leq e$, we must have $a = e$.



4-1 42.

$h(t) = 3t = \arcsin t \Rightarrow h'(t) = 3 - \frac{1}{\sqrt{1-t^2}}$. $h'(t) = 0 \Rightarrow 3 = \frac{1}{\sqrt{1-t^2}} \Rightarrow \sqrt{1-t^2} = \frac{1}{3} \Rightarrow 1 - t^2 = \frac{1}{9} \Rightarrow t^2 = \frac{8}{9} \Rightarrow t = \pm\frac{2}{3}\sqrt{2} \approx \pm 0.94$, both in the domain of h, which is $[-1, 1]$.

4-1 59.

$f(x) = x^{-2}\ln x$, $[\frac{1}{2}, 4]$.
$f'(x) = x^{-2} \cdot \frac{1}{x} + (\ln x)(-2x^{-3}) = x^{-3} - 2x^{-3}\ln x = x^{-3}(1 - 2\ln x) = \dfrac{1 - 2\ln x}{x^3}$.
$f'(x) = 0 \Leftrightarrow 1 - 2\ln x = 0 \Leftrightarrow 2\ln x = \frac{1}{2} \Leftrightarrow x = e^{1/2} \approx 1.65$.
$f'(x)$ does not exist when $x = 0$, which is not in the given interval, $[\frac{1}{2}, 4]$.
$f(\frac{1}{2}) = \frac{\ln 1/2}{(1/2)^2} = \frac{\ln 1 - \ln 2}{1/4} = -4\ln 2 \approx -2.773$,
$f(e^{1/2}) = \frac{\ln e^{1/2}}{(e^{1/2})^2} = \frac{1/2}{e} = \frac{1}{2e} \approx 0.184$, and $f(4) = \frac{\ln 4}{4^2} = \frac{\ln 4}{16} \approx 0.087$.
So $f(e^{1/2}) = \frac{1}{2e}$ is the absolute maximum value and $f(\frac{1}{2}) = -4\ln 2$ is the absolute minimum value.

4-3 2.

(a) f is increasing on $(0, 1)$ and $(3, 7)$.
(b) f is decreasing on $(1, 3)$.
(c) f is concave upward on $(2, 4)$ and $(5, 7)$.
(d) f is concave downward on $(0, 2)$ and $(4, 5)$.
(e) The points of inflection are $(2, 2), (4, 3)$ and $(5, 4)$.

4-3 14.

(a) $f(x) = \cos^2 x - 2\sin x, 0 \le x \le 2\pi$. $f'(x) = -2\cos x \sin x - 2\cos x = -2\cos x(1 + \sin x)$. Note that $1 + \sin x \ge 0$ [since $\sin x \ge -1$], with equality $\Leftrightarrow \sin x = -1 \Leftrightarrow x = \frac{3\pi}{2}$ [since $0 \le x \le 2\pi$] $\cos x = 0$. Thus, $f'(x) > 0 \Leftrightarrow \cos x < 0 \Leftrightarrow \frac{\pi}{2} < x < \frac{3\pi}{2}$ and

$f'(x) < 0 \Leftrightarrow \cos x > 0 \Leftrightarrow 0 < x < \frac{\pi}{2}$ or $\frac{3\pi}{2} < x < 2\pi$. Thus, f is increasing on $(\frac{\pi}{2}, \frac{3\pi}{2})$ and f is decreasing on $(0, \frac{\pi}{2})$ and $(\frac{3\pi}{2}, 2\pi)$.

(b) f changes from decreasing to increasing at $x = \frac{\pi}{2}$ and from increasing to decreasing at $x = \frac{3\pi}{2}$. Thus, $f(\frac{\pi}{2}) = -2$ is a local minimum value and $f(\frac{3\pi}{2}) = 2$ is a local maximum value.

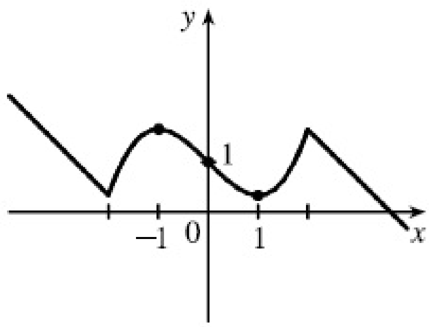
(c)

$$\begin{aligned} f''(x) &= 2\sin x(1+\sin x) - 2\cos^2 x = 2\sin x + 2\sin^2 x - 2(1-\sin^2 x) \\ &= 4\sin^2 x + 2\sin x - 2 = 2(2\sin x - 1)(\sin x + 1) \end{aligned}$$

so $f''(x) > 0 \Leftrightarrow \sin x > \frac{1}{2} \Leftrightarrow \frac{\pi}{6} < x < \frac{5\pi}{6}$,
and $f''(x) < 0 \Leftrightarrow \sin x < \frac{1}{2}$ and $\sin x \neq -1 \Leftrightarrow 0 < x < \frac{\pi}{6}$ or $\frac{5\pi}{6} < x < \frac{3\pi}{2}$ or $\frac{3\pi}{2} < x < 2\pi$. Thus, f is concave upward on $(\frac{\pi}{6}, \frac{5\pi}{6})$ and concave downward on $(0, \frac{\pi}{6}), (\frac{5\pi}{6}, \frac{3\pi}{2})$ and $(\frac{3\pi}{2}, 2\pi)$. There are inflection points at $(\frac{\pi}{6}, -\frac{1}{4})$ and $(\frac{5\pi}{6}, -\frac{1}{4})$

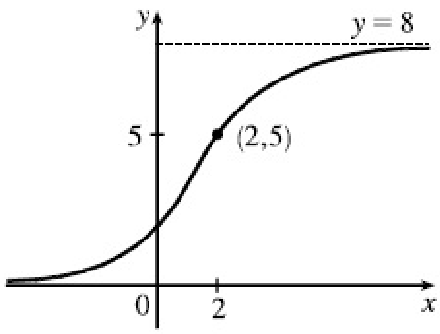
4-3 26.

$f'(1) = f'(-1) = 0 \Rightarrow$ horizontal tangents at $x = \pm 1$.
$f'(x) < 0$ if $|x| < 1 \Rightarrow f$ is decreasing on $(-1, 1)$.
$f'(x) > 0$ if $1 < |x| < 2 \Rightarrow f$ is increasing on $(-2, 1)$ and $(1, 2)$.
$f'(x) = -1$ if $|x| > 2 \Rightarrow$ the graph of f has constant slope -1 on $(-\infty, -2)$ and $(2, infty)$.
$f''(x) < 0$ if $-2 < x < 0 \Rightarrow f$ is concave downward on $(-2, 0)$. The point $(0, 1)$ is an inflection point.



4-3 31.

$f'(x) > 0$ if $x \neq 2 \Rightarrow f$ is increasing on $(-\infty, 2)$ and $(2, \infty)$.
$f''(x) > 0$ if $x < 2 \Rightarrow f$ is concave upward on $(-\infty, 2)$.
$f''(x) < 0$ if $x > 2 \Rightarrow f$ is concave downward on $(2, \infty)$.
f has inflection point $(2, 5) \Rightarrow f$ changes concavity at the point $(2, 5)$.
$\lim_{x\to\infty} f(x) = 8 \Rightarrow f$ has a horizontal asymptote of $y = 8$ as $x \to \infty$.
$\lim_{x\to-\infty} f(x) = 0 \Rightarrow f$ has a horizontal asymptote of $y = 0$ as $x \to -\infty$.



4-3 43.

(a)

$$F'(x) = x \cdot \frac{1}{2}(6-x)^{-\frac{1}{2}}(-1) + (6-x)^{-\frac{1}{2}}(1) = \frac{1}{2}(6-x)^{-\frac{1}{2}}[-x + 2(6-x)]$$
$$= \frac{-3x+12}{2\sqrt{6-x}}$$

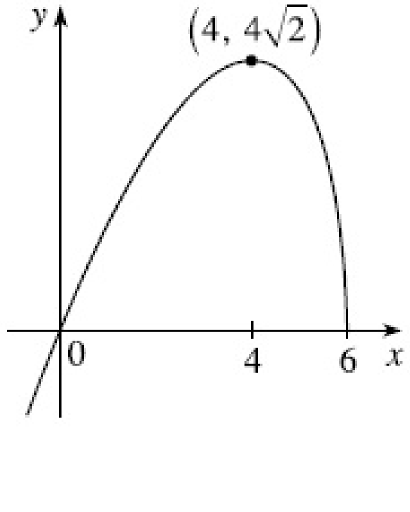
$F'(x) > 0 \Leftrightarrow -3x + 12 > 0 \Leftrightarrow x < 4$ and $F'(x) < 0 \Leftrightarrow 4 < x < 6$.
So F is increasing on $(-\infty, 4)$ and F is decreasing on $(4, 6)$.

(b) F changes from increasing to decreasing at $x = 4$, so
$F(4) = 4\sqrt{2}$ is a local maximum value. There is no local minimum value.

(c) $F'(x) = -\frac{3}{2}(x-4)(6-x)^{-\frac{1}{2}}$

$$F''(x) = -\frac{3}{2}\left[(x-4)\left(-\frac{1}{2}(6-x)^{-\frac{3}{2}}(-1)\right) + (6-x)^{-\frac{1}{2}}(1)\right]$$
$$= -\frac{3}{2} \cdot \frac{1}{2}(6-x)^{-\frac{3}{2}}[(x-4) + 2(6-x)] = \frac{3(x-8)}{4(6-x)^{\frac{3}{2}}}$$

$F''(x) < 0$ on $(-\infty, 6)$, so F is CD on $(-\infty, 6)$. There is no inflection point.



(d)

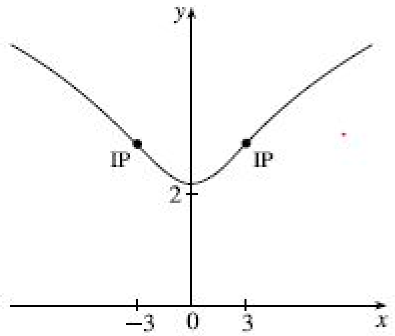
4-3 46.

(a) $f(x) = \ln(x^2+9) \Rightarrow f'(x) = \frac{1}{x^2+9}\dot{2}x = \frac{2x}{x^2+9}$. $f'(x) > 0 \Leftrightarrow 2x > 0 \Leftrightarrow x > 0$ and $f'(x) < 0 \Leftrightarrow x < 0$. So f is increasing on $(0\ ,\ \infty)$ and f is decreasing on $(-\infty\ ,\ 0)$.

(b) f changes from decreasing to increasing at $x = 0$, so $f(0) = \ln 9$ is a local minimum value. There is no local maxinum value.

(c) $f''(x) = \frac{(x^2+9)\dot{-}2x(2x)}{(x^2+9)^2} = \frac{18-2x^2}{(x^2+9)^2} = \frac{-2(x+3)(x-3)}{(x^2+9)^2}$. $f''(x) = 0 \Leftrightarrow x = \pm 3$.
$f''(x) > 0$ on $(-3,3)$ and f is CD on (-∞,-3) and (3,∞). There are inflection points at ($\pm$3,$\ln 18$)

(d)



4-3 47.

(a) $f(\theta) = 2\cos\theta + \cos^2\theta, 0 \le \theta \le 2\pi \Rightarrow f'(\theta) = -2\sin\theta + 2\cos\theta(-\sin\theta)$
$= -2\sin\theta(1+\cos\theta)$.

$f'(\theta) = 0 \Leftrightarrow \theta = 0, \pi$and $2\pi . f'(\theta) > 0 \Rightarrow \pi < \theta < 2\pi$ and $f'(\theta) < 0 \Leftrightarrow 0 < \theta < \pi$. So f is increasing on $(\pi, 2\pi)$ and f is decreasing on (0,π).

(b) $f(\pi)$ = -1 is a local minimum value.

(c) $f'(\theta) = -2\sin\theta(1+\cos\theta) \Rightarrow$
$f''(\theta) = -2\sin\theta(-sin\theta) + (1+\cos\theta)(-2\cos\theta) = 2\sin^2\theta - 2\cos\theta - 2\cos^2\theta$
$= 2(1-\cos^2\theta) - 2\cos\theta - 2\cos^2\theta = -4\cos^2\theta - 2\cos\theta + 2$
$= -2(2\cos^2\theta + \cos\theta - 1) = -2(\cos\theta - 1)(\cos\theta + 1)$

since $-2(\cos\theta + 1) < 0[for\theta \neq \pi] . f''(\theta) > 0 \Rightarrow 2\cos\theta - 1 < 0 \Rightarrow \cos\theta < \frac{1}{2} \Rightarrow \frac{\pi}{3} < \theta < \frac{5\pi}{3}$ and $f''(\theta) < 0 \Rightarrow \cos\theta > \frac{1}{2} \Rightarrow 0 < \theta < 2\pi$. So f is CU on $(\frac{\pi}{3}, \frac{5\pi}{3})$ and f is CD on $(0, \frac{\pi}{3})$ and $(\frac{5\pi}{3}, 2\pi)$. There are points of inflection at $(\frac{\pi}{3}, f(\frac{\pi}{3})) = (\frac{\pi}{3}, \frac{5}{4})$ and $(\frac{5\pi}{3}, f(\frac{5\pi}{3}))$ =$(\frac{5\pi}{3}, \frac{5}{4})$

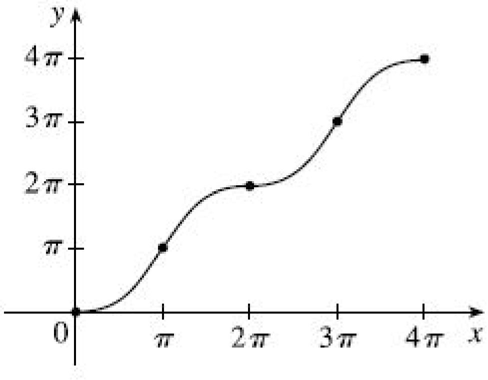
4-3 48.

(a) $S(x) = x - \sin x, 0 \le x \le 4\pi \Rightarrow S'(x) = 1 - \cos x . S'(x) = 0 \Leftrightarrow \cos x = 1 \Leftrightarrow x = 0\ .2\pi$, and$4\pi$.

$S'(x) > 0 \Leftrightarrow \cos x < 1$, which is true for all x except integer multiples of 2π, so S is increasing on (0,4π) since $S'(2\pi) = 0$

(b) There is no local maximum or minimum

(c)



(d) $S''(x) = \sin x . S''(x) > 0$ if $0 < x < \pi$ or $2\pi < x < 3\pi$, and $S''(x) < 0$ if $\pi < x < 2\pi$ or $3\pi < x < 4\pi$. So S is CU on $(0,\pi)$ and $(2\pi, 3\pi)$, and S is CD on $(\pi, 2\pi)$ and $(3\pi, 4\pi)$. There are inflection points at (π, π),$(2\pi, 2\pi)$, and $(3\pi, 3\pi)$.

4-3 54.

$f(x) = x - \frac{1}{6}x^2 - \frac{2}{3}\ln x$ has domain $(0,\infty)$.

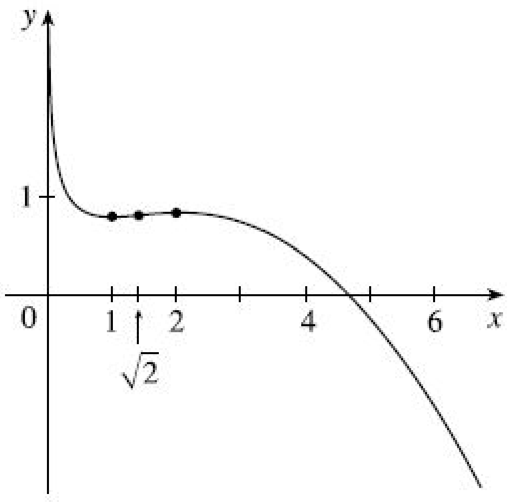
(a) $\lim\limits_{x\to 0^+}(x - \dfrac{1}{6}x^2 - \dfrac{2}{3}\ln x) = \infty$ since $\ln x \to -\infty$ as $x \to 0^+$, so x =0 is a VA. There is no HA.

$(b) f'(x) = 1 - \frac{1}{3}x - \frac{2}{3x} = \frac{3x-x^2-2}{3x} = -\frac{(x-1)(x-2)}{3x}$.$f'(x) > 0 \Leftrightarrow (x-1)(x-2) < 0 \Leftrightarrow 1 < x < 2$ and $f'(x) < 0 \Leftrightarrow 0 < x < 1$or $x > 2$. So f is increasing on (1,2) and f is decreasing on (0,1) and $(2,\infty)$.

(c) f changes from defreasing to increasing at $x = 1$, so$f(1) = \frac{5}{6}$ is a local minimum value. f changes from increasing to decreasing at $x = 2$, so $f(2) = \frac{4}{3} - \frac{2}{3}\ln 2 \approx 0.87$ is a local maximum value.

(d) $f''(x) = -\frac{1}{3} + \frac{2}{3x^2} = \frac{2-x^2}{3x^2}$.$f''(x) > 0 \Leftrightarrow 0 < x < \sqrt{2}$ and $f''(x) < 0 \Leftrightarrow x > \sqrt{2}$. So f is CU on $(0,\sqrt{2})$ and f is CD on $(\sqrt{2},\infty)$. There is an inflection point at $(\sqrt{2}, \sqrt{2} - \frac{1}{3} - \frac{1}{3}\ln 2)$.

(e)



4-3 56.

(a) $\lim\limits_{x\to\infty} arc\tan x = \dfrac{\pi}{2}$, so $\lim\limits_{x\to\infty} e^{arc\tan x} = e^{\frac{\pi}{2}}[\approx 4.81]$, so $y = e^{\frac{\pi}{2}}$ is a HA.

$\lim\limits_{x\to-\infty} e^{arc\tan x} = e^{\frac{-\pi}{2}}[\approx 0.21]$, so $y = e^{\frac{-\pi}{2}}$ is a HA. No VA.

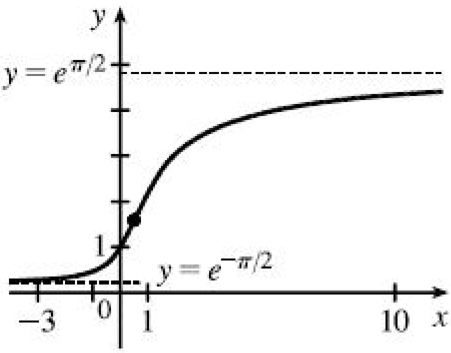
(b) $f(x) = e^{arc\tan x} \Rightarrow f'(x) = e^{arc\tan x}.\ \frac{1}{1+x^2} > 0$ for all x. Thus, f is increasing on $\mathbb{R}$.

(c) There is no local maximum or minimum .

(d) $f''(x) = e^{arc\tan x}[\frac{-2x}{(1+x^2)^2}] + \frac{1}{1+x^2} \cdot e^{arc\tan x} \cdot \frac{1}{1+x^2} = \frac{e^{arc\tan x}}{(1+x^2)^2}(-2x+1)$

$f''(x) > 0 \Leftrightarrow -2x + 1 > 0 \Leftrightarrow x < \frac{1}{2}$ and$f''(x) < 0 \Leftrightarrow x > \frac{1}{2}$, so f is CU on $(-\infty, \frac{1}{2})$ and f is CD on $(\frac{1}{2},\infty)$. There is an inflection point at $(\frac{1}{2}, f(\frac{1}{2})) \approx (\frac{1}{2}, 1.59)$.

(e)

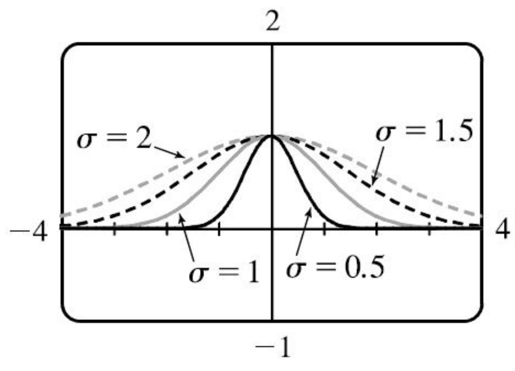


4-3 72.

(a) As $|x| \to \infty, t = -\frac{x^2}{2\sigma^2} \to -\infty$, and$e^t \to 0$. The HA is y=0. Since t takes on its maximum value at x =0, so does e^t. Showing this result using derivatives, we have $f(x) = e^{\frac{-x^2}{2\sigma^2}} \Rightarrow$
$f'(x) = e^{\frac{-x^2}{2\sigma^2}}(\frac{-x}{\sigma^2})$.
$f'(x) = 0 \Leftrightarrow x = 0$. Because f' changes from positive to negative at $x = 0, f(0) = 1$ is a local maximum. For inflection points, we find $f''(x) = \frac{1}{\sigma^2}[e^{\frac{-x^2}{2\sigma^2}}] = \frac{-1}{\sigma^2}e^{\frac{-x^2}{2\sigma^2}}(\frac{-x}{\sigma^2})$.
$f''(x) = 0 \Leftrightarrow x^2 = \sigma^2 \Leftrightarrow x = \pm\sigma$. $f''(x) < 0 \Leftrightarrow x^2 < \sigma^2 \Leftrightarrow -\sigma < x < \sigma$.
So f is CD on $(-\sigma, \sigma)$ and CU on $(-\infty, -\sigma)$ and (σ, ∞). IP at $(\pm\sigma, e^{-1/2})$.

(b) Since we have IP at $x = \pm\sigma$, the inflection points move away from the y-axis as σ increases.

(c)



From the graph, we see that as σ increases, the graph tends to spread out and there is more area between the curve and the x-axis.

4-3 93.

(a) $f(x) = \frac{x^4}{sin\frac{1}{x}} \Rightarrow f'(x) = x^4 \cos\frac{1}{x} - \frac{1}{x}^2 + sin\frac{1}{x}(4x^3) = 4x^3 \sin\frac{1}{x} - x^2 \cos\frac{1}{x}$.
$g(x) = x^4(2 + \sin\frac{1}{x}) = 2x^4 + f(x) \Rightarrow g'(x) = 8x^3 + f'(x)$
$h(x) = x^4(-2 + \sin\frac{1}{x}) = 2x^4 + f(x) \Rightarrow g'(x) = -8x^3 + f'(x)$

It is given that $f(0) = 0$, so $f'(0) = \lim_{x\to 0} \dfrac{f(x) - f(0)}{x - 0} = \lim_{x\to 0} \dfrac{x^4 \sin\frac{1}{x} - 0}{x} = \lim_{x\to 0} x^3 \dfrac{1}{x}$. Since$-|x^3| \leq x^3 \sin 1x^3 \leq |x^3| and \lim_{x\to 0}|x^3| = 0$, we see that $f'(0) = 0$ by the squeeze Theorem. Also, 0 is a critical number of $f, g,$and h.

(b) $f(0) = 0$and since $\sin\frac{1}{x}$ and hence$x^4 \sin\frac{1}{x}$is both positive and negative infinitely often on both sides of 0. and arbitrarily close to 0, f has neither a local maximum nor a loca minimum at 0.

Since $2 + \sin\frac{1}{x} \geq 1, g(x) = x^4(2 + \sin\frac{1}{x}) > 0$ for $x \neq 0$, so $g(0) = 0$ is a local minimum.

Since $-2 + \sin\frac{1}{x} \leq -1, h(x) = x^4(-2 + \sin\frac{1}{x}) > 0$ for $x \neq 0$, so $h(0) = 0$ is a local maximum.