NCKU CSIE Discrete Mathematics (2014 Spring) Midterm I *(total 105 pts)*

1. *(30 pts)* For each of the following statements, **determine** and **explain** whether it is correct or not.

(a) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ *and* $\{\emptyset\} \in \{\emptyset, \{\emptyset\}\}$

(b) There are two sets A and B, where A-B={1, 3, 7, 11}, B-A={2, 6, 8}, and A$\cap B$={4, 9}. The number of members in the set $A \cup B$ is 9.

(c) If 17 | 2a+3b then 17 | 9a+5b.

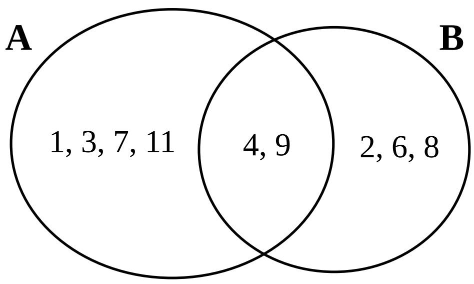
(d) $2\binom{n}{0} + \binom{n}{1} + 2\binom{n}{2} + \binom{n}{3} + 2\binom{n}{4} + \binom{n}{5} + \cdots + 2\binom{n}{n-2} + \binom{n}{n-1} + 2\binom{n}{n} = 2^{n-1} + 2^{n-2}$

(e) $f: \boldsymbol{R} \to \boldsymbol{R}, f(x) = \sqrt{x}$ is a function.

(f) $f: \boldsymbol{R} \to \boldsymbol{R^2}, f(x) = (x^2, -x^2)$ is an one-to-one function.

(1) T $\{\emptyset\}$ is both subset and element of $\{\emptyset, \{\emptyset\}\}$

(2) T



(3) T

$$\begin{cases} 17 \mid 2a+3b \\ 17 \mid 17a+17b \end{cases} \Rightarrow m(2a+3b)+n(17a+17b)=9a+5b$$

$$\Rightarrow \begin{cases} 2m+17n=9 \\ 3m+17n=5 \end{cases} \Rightarrow m=-4, n=1$$

$$\therefore 17 \mid (17a+17b)-4(2a+3b) \Rightarrow 17 \mid 9a+5b$$

(4) F

$$\text{Rt} = 2\times\left[\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\cdots+\binom{n}{n}\right]+\left[\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\cdots+\binom{n}{n-1}\right]$$

$$= 2\times 2^{n-1} + 2^{n-1}$$

$$= 2^n + 2^{n-1}$$

(5) F $x=-1,\ f(-1)=\sqrt{-1}=i \notin \mathbf{R}$, so $f(x)$ is not a function for $\mathbf{R} \to \mathbf{R}$

(6) F $f(1)=(1, -1) = f(-1)$

2. (*10:2,2,3,3 pts*) Determine the following sets: (a) $\emptyset \cup \{\emptyset\}$ (b) $\emptyset \cap \{\emptyset\}$ (c) $\emptyset \oplus \{a, \emptyset, \{\emptyset\}\}$ (d) $\{\emptyset\} \oplus \{a, \emptyset, \{\emptyset\}\}$

(a) $\{\phi\}$
(b) ϕ
(c) $(\phi \cup \{a, \phi, \{\phi\}\}) - (\phi \cap \{a, \phi, \{\phi\}\}) = \{a, \phi, \{\phi\}\}$
(d) $\{a, \{\phi\}\}$

3. (*10 pts*) Solve the equation $x_1+x_2+x_3+x_4<10$ and find the integer solutions where $x_1, x_2 > 0$, $x_3 > 1$, $x_4 > -2$.

Let $x_1' = x_1 - 1 \geqq 0$
$x_2' = x_2 - 1 \geqq 0$
$x_3' = x_3 - 2 \geqq 0$
$x_4' = x_4 + 1 \geqq 0$
Then $0 \leqq x_1' + x_2' + x_3' + x_4' = x_1 + x_2 + x_3 + x_4 - 3 \leqq 6$
⇨ 所求 $= \binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \ldots + \binom{9}{3}$
$= 1 + 4 + 10 + 20 + \ldots + 84 = \binom{10}{4}$
$= 210$

4. (*15 pts*) Show that postage of 24 cents or more can be achieved by using only 5-cent and 7-cent stamps.

$n = 24$, $24 = 5*2 + 7*2$ 命題成立
$n = 25$, $25 = 5*5$
$n = 26$, $26 = 5*1 + 7*3$
$n = 27$, $27 = 5*4 + 7*1$
$n = 28$, $28 = 7*4$
$k = n$, $n = 29$, 可寫成 $k = (n - 5) + 5$, 因為 $n - 5 = 24$ 成立, 所以 k 亦成立
$k+1 = (n - 4) + 5$, 因為 $n-4 = 25$ 成立, 所以 $k + 1$ 成立, 由數學歸納法得知, 當 $k>=24$, 皆可寫成 5 和 7 的倍數加總

5. (*10:2,2,2,4 pts*) For the complete expansion of $(2x - y + 3z^{-1} + 1)^6$, determine the following value (a) the coefficient of x^2yz^{-2} (b) the number of the distinct terms (c) the sum of all coefficients, and (d) if we change the constant term '1' to '$1+x^2$', what's the coefficient of x^2yz^{-2}.

(a) $\frac{6!}{2!2!} \times 2 \times 2 \times (-1) \times 3 \times 3 = -6480$
(b) $H_6^4 = C_6^{4+6-1} = 84$
(c) 把 x、y、z 都代入 1, $(2 - 1 + 3 + 1)^6 = 15625$
(d) 1 用 $1 + x^2$ 代換後，式子變成 $(x^2 + 2x - y + 3z + 1)^6$

用兩個 2x 組成的 x^2yz^{-2} => $\frac{6!}{2!2!} \times 2 \times 2 \times (-1) \times 3 \times 3 = -6480$

用一個 x^2 組成的 x^2yz^{-2} => $\frac{6!}{2!2!} \times (-1) \times 3 \times 3 = -1620$

$(-6480) + (-1620) = -8100$

6. (*10 pts*) Validate the argument $((p \wedge q) \wedge (p \rightarrow (r \wedge q)) \wedge (r \rightarrow (s \vee t)) \wedge \neg s) \rightarrow t$

$(p \wedge q) \rightarrow p$	by Rule of Conjunctive simplification
$(r \wedge q) \rightarrow r$	by Rule of Conjunctive simplification
$p \wedge (p \rightarrow r) \rightarrow r$	by Rule of Detachment(Modus Ponens)
$r \wedge (r \rightarrow (s \vee t)) \rightarrow (s \vee t)$	by Rule of Detachment(Modus Ponens)
$((s \vee t) \wedge \neg s) \rightarrow t$	by Rule of Disjunctive syllogism
$\therefore t$	

7. (*10:3,3,4 pts*) (a) How many times is the *printf* statement executed for the following program segments if *p*=24? (b) How many distinct numbers printed by this program? (c) discuss the result of (a) when *p*=12.

```
for (i=1; i <= p; i++)
    for (j=i; j <= 24; j++)
        for (k=j; k <= 24; k++)
            printf("%d\n", i+j+k);
```

a. $\binom{24+3-1}{3} = \binom{26}{3} = 13 * 52 * 8 = 2600$

b. 70 distinct words: 72-3+1=70

{3=1, 4=1, 5=2, 6=3, 7=4, 8=5, 9=7, 10=8, 11=10, 12=12, 13=14, 14=16, 15=19, 17=24, 16=21, 19=30, 18=27, 21=37, 20=33, 23=44, 22=40, 25=52, 24=48, 27=60, 26=56, 29=66, 28=63, 31=71, 30=69, 34=76, 35=77, 32=73, 33=75, 38=78, 39=78, 36=78, 37=78, 42=75, 43=73, 40=77, 41=76, 46=66, 47=63, 44=71, 45=69, 51=48, 50=52, 49=56, 48=60, 55=33, 54=37, 53=40, 52=44, 59=21, 58=24, 57=27, 56=30, 63=12, 62=14, 61=16, 60=19, 68=4, 69=3, 70=2, 71=1, 64=10, 65=8, 66=7, 67=5, 72=1}

c. 2236 times

8. (*10 pts*) Simplify the following expressions. (a) $\neg[(p \wedge \neg q) \vee \neg(r \wedge q)]$ (b) $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)]$

(a) $\neg[(p \wedge \neg q) \vee \neg(r \wedge q)] \Leftrightarrow \neg[(p \wedge \neg q) \vee \neg r \vee \neg q] \Leftrightarrow \neg[(p \wedge \neg q) \vee \neg q \vee \neg r] \Leftrightarrow \neg[\neg q \vee \neg r] \Leftrightarrow q \wedge r$

(b) $(p \rightarrow q) \wedge [\neg p \wedge (r \vee \neg q)] \Leftrightarrow (\neg p \vee q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow (\neg p \vee q) \wedge \neg q \Leftrightarrow (\neg q \wedge \neg p) \vee (\neg q \wedge q) \Leftrightarrow (\neg q \wedge \neg p) \vee F_0 \Leftrightarrow \neg q \wedge \neg p$