

Discrete Mathematics (2013 Spring) Midterm I

1. (24%) For each of the following statements, **determine** and **explain** whether it is correct or not.
 - (1). $\phi \subset \phi$
 - (2). $\phi \subseteq \{\phi\}$
 - (3). $A = \{2n \mid n \in \mathbb{Z}\}, B = \{6n \mid n \in \mathbb{Z}\}, \text{ then } \bar{B} \subseteq \bar{A}.$
 - (4). $[(p \vee (p \wedge q) \vee (p \wedge q \wedge r))] \wedge [(p \wedge r \wedge t) \vee t] \Leftrightarrow p \wedge t$
 - (5). $\neg(p \Leftrightarrow q) \Leftrightarrow (p \wedge q) \vee (\neg p \wedge \neg q)$
 - (6). $A \Delta (B \cap C) = (A \Delta B) \cap (A \Delta C)$
 - (7). $P(A \cup B) = P(A) \cup P(B)$
 - (8). $P(A \cap B) = P(A) \cap P(B)$
2. (10%) Solve the equation $x_1 + x_2 + x_3 + x_4 < 10$ and find the integer solutions where $x_1, x_2 > 0, x_3 > 1, x_4 > 2$.
3. (10%) For the strings of length 10, how many have (a) three 0's, four 1's, and three 2's; (b) sum of digits is 4?
4. (16%) For the complete expansion of $(2x - 2y + 3z^{-1} + 1)^5$, determine the following value (a) the coefficient of yz^{-2} (b) the number of the distinct terms (c) the sum of all coefficients, and (d) if we change the constant term '1' to x^2 , what's the coefficient of x^2yz^{-2} .
5. (10%) What is the probability of each summand even in all compositions of 18?
6. (15%) One rock-n-roll music CD costs \$33 and one classic music CD costs \$29. How many CDs of these two kinds you should buy if you pay \$2490 for them?
7. (15%) Define the connective "Nor" by $(p \downarrow q) \Leftrightarrow \neg(p \vee q)$, for any statements p, q . Represent the following using only this connective. (a) $\neg p$ (b) $p \wedge q$, (c) $p \rightarrow q$.