Discrete Mathematics (2013 Spring) Final

1. **(18 points)** For each of the following statements, determine and explain (required) whether it is correct or not.
   (1). $P(A \cap B) = P(A) \cap P(B)$
   (2). The number of the distinct terms of $(x - y + z^{-1}+1)^4$ is 35.
   (3). The number of different relations on {0, 1, 2} contain the pair (0, 1) is 128.
   (4). Let f: A→B and g: B→C, f and g are one-to-one if $g \circ f$ is one-to-one.
   (5). Let A={1, 2, 3, 4}, and B={1, 2, 3, 4, 5, 6}. The number of function f : A → B satisfy $|f(A)|= 4$ is $\binom{6}{4} * 4! * 65$.
   (6). The number of derangements of 1, 2, 3, 4, 5, 6, 7, 8 start with 1, 2, 3 in some order is 88.

2. **(10 points)** Find the integer solutions of the equation (a) $x_1+x_2+x_3+x_4 <10$ where $x_1, x_2 > 0$, $x_3 > 1, x_4 > 2$, (b) $x_1+x_2+x_3+x_4 =10$ where $x_1 < 3, x_3 > 1, x_4 < 2$. (exhaustively list all answers is not allowed.)

3. **(8 points)** Let A={a, b, c, d, e}, how many closed binary operations f have an identity and f(a, b)≠c and f(a, b) ≠d?

4. **(8 points)** If A = {a, b, c, d, e, f}, determine the number of relations on A that are (a) reflexive and symmetric but not transitive, (b) equivalence relations that determine more than 2 (include 2) equivalence classes.

5. **(10 points)** For A={1, 2, 3, 4} and B={u, v, x, y}, determine the number of one-to-one functions f : A→B where $f(1)\neq$v, $f(2)\neq$u, $f(3)\neq$x, y and $f(4)\neq$x, y.

6. **(10 points)** Find the exponential generating function for (a) the sequence 0!, 1!, 2!, 3!, …, (b) the number of ways to arrange n letters, $n \geqq 0$, selected from the word “MISSISSIPPI” and the arrangement must contain at least two I’s.

7. **(10 points)** (a) Find a recurrence relation for the number of **4**-ary sequences (e.g., 0213, 01132) of length n that have no consecutive 0’s. (b) Solve the recurrence relation in (a).

8. **(10 points)** (a) For S={0, 1}, let $A \subseteq S^*$, where A={00, 1}. For $n \geqq 1$, let a_n count the number of strings in A^* of length n. Find a recurrence relation for a_n. (b) In the alphabet {0, 1, 2}, let a_n to be the number of strings of length n in which there is never a 2 anywhere to the right of a 0. Please describe and explain a_n in a recurrence relation form.

9. **(16 points)** Determine (a) the sequence generated by the generating function $f(x) = \frac{2}{3-2x}$, (b) the generating function for the sequences 0, 1, 3, 6, 10, …, (c) the coefficient of x^{48} in $(x^5+x^6+x^7+x^8+\ldots)^7$, (d) the coefficient of x^{15} in $(1+x)^4/(1-x)^4$

10. **(8 points)** Please answer (a) more quizzes is good for you? (b) a roll call at the class is good for you? (c) 2 examples/methods/strategies to improve your (or others’) learning motivation/performance.

m \ n	1	2	3	4	5	6
1	1					
2	1	1				
3	1	3	1			
4	1	7	6	1		
5	1	15	25	10	1	
6	1	31	90	65	15	1

Table of Stirling number of the second kind $S(m, n)$