

DISCRETE MATHEMATICS – CH7 Homework7

7.1

10. If $A = \{w, x, y, z\}$, determine the number of relations on A that are (a) reflexive; (b) symmetric; (c) reflexive and symmetric; (d) reflexive and contain (x, y) ; (e) symmetric and contain (x, y) ; (f) antisymmetric; (g) antisymmetric and contain (x, y) ; (h) symmetric and antisymmetric; and (i) reflexive, symmetric, and antisymmetric. (10 pts)

- a. Reflexive $2^{(n^2-n)} = 2^{(16-4)} = 2^{12}$
- b. Symmetric $2^n 2^{(n^2-n)*1/2} = 2^4 2^6 = 2^{10}$
- c. Reflexive and symmetric $2^{(n^2-n)*1/2} = 2^6$
- d. Reflexive and contain (x,y) $2^{12-1} = 2^{11}$
- e. Symmetric and contain (x,y) $2^n 2^{(n^2-n)*\frac{1}{2}-1} = 2^4 2^5 = 2^9$
- f. Anti-symmetric $2^n 3^{(n^2-n)*1/2} = 2^4 3^6$
- g. Anti-symmetric and contain (x,y) $2^n 3^{(n^2-n)*\frac{1}{2}-1} = 2^4 3^5$
- h. Symmetric and anti-symmetric $\{(1,1),(2,2),(3,3),(4,4)\}$

Relation either include or exclude each of these pairs, so 2^4

- i. Reflexive, symmetric and anti-symmetric : only 1
 $\{(1,1),(2,2),(3,3),(4,4)\}$

7.3

18. Let $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7\}$, with $A = \mathcal{P}(\mathcal{U})$, and let \mathcal{R} be the subset relation on A . For $B = \{\{1\}, \{2\}, \{2, 3\}\} \subseteq A$, determine each of the following.

- a) The number of upper bounds of B that contain (i) three elements of \mathcal{U} ; (ii) four elements of \mathcal{U} ; (iii) five elements of \mathcal{U}
- b) The number of upper bounds that exist for B
- c) The lub for B
- d) The number of lower bounds that exist for B
- e) The glb for B

(10 pts)

(a) (i) Only one such upper bound – $\{1,2,3\}$. (ii) Here the upper bound has the form $\{1, 2, 3, x\}$ where $x \in \mathcal{U}$ and $4 \leq x \leq 7$. Hence there are four such upper bounds. (iii) There are $\binom{4}{2}$ upper bounds of B that contain five elements from \mathcal{U} .

(b) $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 2^4 = 16$

(c) $\text{lub } B = \{1, 2, 3\}$

(d) One – namely \emptyset

(e) $\text{glb } B = \emptyset$

7.4

12. Let $A = \{v, w, x, y, z\}$. Determine the number of relations on A that are (a) reflexive and symmetric; (b) equivalence relations; (c) reflexive and symmetric but not transitive; (d) equivalence relations that determine exactly two equivalence classes; (e) equivalence relations where $w \in [x]$; (f) equivalence relations where $v, w \in [x]$; (g) equivalence relations where $w \in [x]$ and $y \in [z]$; and (h) equivalence relations where $w \in [x]$, $y \in [z]$, and $[x] \neq [z]$. (10 pts)

a. $2^{10} = 1024$

b. $\sum_{i=1}^5 S(5, i) = 1 + 15 + 25 + 10 + 1 = 52$

c. $1024 - 52 = 972$

d. $S(5, 2) = 15$

e. $\sum_{i=1}^4 S(4, i) = 1 + 7 + 6 + 1 = 15$

f. $\sum_{i=1}^3 S(3, i) = 1 + 3 + 1 = 5$

g. $\sum_{i=1}^3 S(3, i) = 1 + 3 + 1 = 5$

h. $(\sum_{i=1}^3 S(3, i)) - (\sum_{i=1}^2 S(2, i)) = 3$

Advanced assignment (30 pts)

- Design a problem that can be solved by two different FSM with different number of states.
- Use the minimization process to reduce the bigger one.

Note:

- ✧ FSMs in textbook will be scored 0.
- ✧ The most similar FSM will be scored 0.

Ans

		下一個狀態	
		目前的輸入	
目前的狀態	0	1	輸出
0	3	1	1
1	4	1	0
2	3	0	1
3	2	3	0
4	1	0	1

$\{0,2,4\}, \{1,3\} \rightarrow \{2,4\}, \{1,3\}, \{0\}$

$A = \{0\}, B = \{2,4\}, C = \{1,3\}$

		下一個狀態	
		目前的輸入	
目前的狀態	0	1	輸出
<i>A</i>	<i>C</i>	<i>C</i>	1
<i>B</i>	<i>C</i>	<i>A</i>	1
<i>C</i>	<i>B</i>	<i>C</i>	0