DISCRETE MATHEMATICS – CH5 Homework5

5.3

2. For each of the following functions $f: \mathbf{Z} \to \mathbf{Z}$, determine whether the function is one-to-one and whether it is onto. If the function is not onto, determine the range $f(\mathbf{Z})$. (10 pts)

(b) $f(x) = 2x - 3$

here the range of $f = \{\ldots,-13, -11,-9,-7,-5,-3,-1,1,3,5,7,\ldots\} \subset Z$, so f is not an onto function. It is one-to-one function

(e) $f(x) = x^2 + x$

here the range of $f = \{0,2,6,12,20,30,\ldots\ldots\} \subset Z$, so f is not an onto function. It is not one-to-one since $f(-6)=f(5)$……

(f) $f(x) = x^3$

the range of $f = \{\ldots,-125,-64,-27,-8,-1,0,1,8,27,64,127,\ldots\ldots\} \subset Z$, so f is not an onto function. It is one-to-one function.

4. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. (10 pts)

(a) How many functions are there from <u>A to B</u>? How many of these are one-to-one? How many are onto?

functions: $6^4 = 1296$

one-to-one: $P_4^6 = 6 \times 5 \times 4 \times 3 = 360$

onto: 0 ($\because |A| < |B| \quad \therefore$ There is no onto function from A to B)

(b) How many functions are there from <u>B to A</u>? How many of these are onto? How many are one-to-one?

functions: $4^6 = 4096$

one-to-one: 0 ($\because |A| < |B|$)

$$\text{onto:} \quad 4! \cdot S(6,4) = \binom{4}{0}0^6 - \binom{4}{1}1^6 + \binom{4}{2}2^6 - \binom{4}{3}3^6 + \binom{4}{4}4^6$$

$$= 0 - 4 + 6 \times 64 - 4 \times 729 + 1 \times 4096 = 1560$$

12. (a) In how many ways can 31,100,905 be factored into three factors, each greater than 1, if the order of the factors is not relevant?

(b) Answer part (a), assuming the order of the three factors is relevant.

(c) In how many ways can one factor 31,100,905 into two or more factors where each factor is greater than 1 and no regard is paid to the order of the factors?

(d) Answer part (c), assuming the order of the factors is to be taken into consideration. (10 pts)

$31{,}100{,}905 = 5 \times 11 \times 17 \times 29 \times 31 \times 37$

(a) $S(6, 3) = 90$

(b) $3! \times S(6, 3) = 540$

(c) $\sum_{i=2}^{6} S(6, i) = 31 + 90 + 65 + 15 + 1 = 202$

(d) $\sum_{i=2}^{6} (i\,!)S(6, i) = 31 + 90 + 65 + 15 + 1 = 4682$

5.4

6. Let $A = \{x, a, b, c, d\}$. (10 pts)

(a) How many closed binary operations f on A satisfy $f(a, b) = c$?

(b) How many of the functions f in part (a) have x as an identity?

(c) How many of the functions f in part (a) have an identity?

(d) How many of the functions f in part (c) are commutative?

(a) 5^{24} (b) 5^{15}

(c) $3 \cdot 5^{15}$, because neither a nor b can be an identity.

(d) $3 \cdot 5^{9}$

5.5

4. Let $\boldsymbol{S} = \{3, 7, 11, 15, 19, \ldots, 95, 99, 103\}$. How many elements must we select from $\boldsymbol{S}$ to insure that there will be at least two whose sum is 110? (10 pts)

{3}, {7,103}, {11,99}, {15,95}, {19, 91}, {23,87}, {27,83}, {31,79}, {35,75}, {39,71}, {43,67}, {47,63}, {51,59},{55}

By the Pigeonhole Principle => 15 element

5.6

20. (a) Give an example of a function $f : \mathbf{Z} \to \mathbf{Z}$ where (i) f is one-to-one but not onto; and (ii) f is onto but not one-to-one.

(b) Do the examples in part (a) contradict Theorem 5.11? (10 pts)

(i) $f(x) = 2x$

(ii) $f(x) = x^2$

(iii) No, the set **Z** is not finite.

Advanced assignment (30 pts)

(1) Answer 5.4-6(a)~(d) again, if we know $f(a, c) \neq b$ and $f(a, d) \neq b$

(a) 5^(25 – 3) * 4 * 4 = 16 * 5^22

(b) 5^(25 – 3 – 9) * 4 * 4 = 16 * 5^13

(c) x 是 identity => 16 * 5^13

c 是 identity => 4 * 5^14

d 是 identity => 4 * 5^14

= 16 * 5^13 + 2 * 4 * 5^14

(d) x: 4 * 4 * 5^(13 – $\binom{4}{2}$) = 16 * 5^7

c: 4 * 5^*(14 - $\binom{4}{2}$) = 4 * 5^8

d: 4 * 5^*(14 - $\binom{4}{2}$) = 4 * 5^8

= 16 * 5^7 + 2 * 4 * 5^8

(2) In (1), if $f(a, c) \neq b$ or $f(a, d) \neq b$

(a) 5^(25 – 3) * (5^2 – 1) = 24 * 5^22

(b) 5^(25 – 3 – 9) * (5^2 – 1) = 24 * 5^13

(c) x: 24 * 5^13

c: 1 * 5 * 5^14

d: 5 * 1 * 5^14

= 24 * 5^13 + 2 * 5^15

(d) x: 24 * 5^(13 – $\binom{4}{2}$) = 24 * 5^7

c: 1 * 5 * 5^(14 – $\binom{4}{2}$) = 5^9

d: 5 * 1 * 5^(14 – $\binom{4}{2}$) = 5^9