Discrete Mathematics (2011 Spring) Midterm II

1. (20 points) For each of the following statements, determine and explain (required) whether it is correct or not.
   (1). R is an equivalence relation on Z where xRy if x-y is even.
   (2). String 00110 is in the language $\{00\}^*\{1\}^*\{0\}^+$ and is also in the language $\{01\}^*\{0\}^*\{11\}^*\{1,0\}^*$.
   (3). Any subset of size 15 from the set S = {3, 7, 11, 15, 19, …, 95, 99, 103} must contain two elements whose sum is 114.
   (4). The number of the equivalence relations on A = {*a*, *b*, *c*, *d*, *e*, *f*, *g*} that have exactly one equivalence class of size 4 is 5*C(6, 4).

2. (20 points, 2,3,3,6,6) If A = {*v*, *w*, *x*, *y*, *z*}, determine the number of relations on A that are (a) symmetric, (b) antisymmetric, (c) antisymmetric and contain (x,y), (d) equivalence relations that determine exactly three equivalence classes, (e) reflexive and symmetric but not transitive.
   [Note: S(4,2)=7, S(4,3)=6, S(5,2)=15]

3. (10 points) Let *p*, *q* be distinct primes. How many edges are there in the Hasse diagram of all positive divisors of p^3q^3.

4. (10 points) For *I* = *O* = {0, 1}, construct a finite state machine *M* that recognizes all strings in the language $\{0, 1\}^*\{00\} \cup \{0, 1\}^*\{11\}$.

5. (10 points) (a) How many ways can 81,345 be factored into two or more factors where each factor is greater than 1 and the order of the factors is not relevant? (b) Answer (a), assuming the order of the factors is taken into consideration. [Note: 81,345=3*5*11*17*29].

6. (15 points) Let A = {*a*, *b*, *c*, *d*}, and B = {*u*, *v*, *w*, *x*, *y*, *z*}. (a) If *f* : A→B is randomly generated, what is the probability that it is one-to-one? (b) How many closed binary operations on A that have c as the identity? (c) How many closed binary operations on A that are commutative and have an identity?

7. (10 points) Let *U*={1, 2, 3, 4, 5, 6}, with A=*P*(*U*), and let *R* be the *subset relation* on A. For B={{1}, {2}, {2, 3}} ⊆ A, determine each of the following. (a) The number of upper bounds that exist for B. (b) The lub for B.

8. (15 points, 10,5) Let M be the finite state machine given in the state diagram shown in the following figure. (a) Minimize machine M. (b) find a distinguishing string for s_3 and s_6.