# Algorithm 2017 Spring

# Homework 1

範圍：**Chapter 1~ Chapter 4**

1. **(10pts) Illustrate the operation of merge sort on the array** $A =$ $\langle 57, 41, 226, 38, 57, 9, 49 \rangle$ .

2. **Answer "true" or "false" first, then explain the reason.**
   (a)(5pts) $2^{n+2} = O(2^n)$
   (b)(5pts) $2^{2n} = O(2^n)$

3. (10pts) Prove lg(n!) = $\Theta(n\lg n)$.

4. Give big-Theta estimates for the following functions.
   (a) (3pts) $5x^2 + 4x + 7$
   (b) (3pts) $1 + 2 + 3 + \ldots + n$
   (c) (4pts) $n*\lg(n!) + n^3$

5. Give tight asymptotic bounds for the following recurrences.

   (a). (3pts) $T(n) = 8T\left(\frac{n}{3}\right) + n^2$

   (b). (3pts) $T(n) = 4T\left(\frac{n}{4}\right) + n\lg n$

   (c). (4pts) $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\lg n}$

6. **Answer "true" or "false" first, then explain the reason or give a counterexample.**

   (a). (3pts) $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$

   (b). (3pts) $f(n) = \omega(f(n))$

   (c). (4pts) If $f(n) = O(g(n))$ then $2^{f(n)} = O(2^{g(n)})$

7. (10pts) Prove that $x^{\log_b y} = y^{\log_b x}$

8. (10pts) Assume that $a_k > 0$.

   Show that $p(n) = \sum_{i=0}^{k} a_i n^i$ is in $\theta(n^k)$.

**9. (10pts) Partition the following functions by their asymptotic order. (That is, $f$ and $g$ are in the same partition if, and only if, $f \in \theta(g)$.) Then list them from the lowest asymptotic order to highest asymptotic order.**

$$n, \ 2^n, \ n \lg n, \ n^3, \ n^2, \ 7n^5 - n^3 + n, \ n^2 + \lg n, \ e^n,$$
$$\sqrt{n}, \ 2^{n-1}, \ \lg\lg n, \ \lg n, \ \lg^2 n, \ n!, \ n^{1+\varepsilon}(0 < \varepsilon < 1).$$

**10.(10pts) Define a function** $g(x) = max\left\{p \,\middle|\, \log_2^{(p)}(x) \geq 1\right\}.$

**Compute a good upper bound for** $g(10^{100})$