Discrete Mathematics (2010 Spring) Midterm II

1. **(9 points)** For each of the following statements, **determine** and **explain** whether it is correct or not.
   (1). $1^2 + 2^2 + \cdots + n^2 = O(n^2)$.
   (2). R is anti-symmetric if and only if $M \cup M^{tr} \geq M$. Let M denote the relation matrix for R.
   (3). If $v(s_1, x) = v(s_2, x)$ for all $x \in I^k$, then states s_1 and s_2 are k-equivalent.

2. **(10 points)** (a) In how many ways can 2310 be factored into 3 factors? (b) In how many ways can 2310 be factored into two or more factors? For (a) and (b), each factor is greater than 1 and the order of the factors is not relevant. [S(4, 2)=7, S(4, 3)=6, S(5, 2)=15.]

3. **(15 points)** Let A={a, b, c, d} and B ={1, 2, 3, 4}, please determine the following value. (a) The number of closed binary operations on A that have an identity. (b) The number of relations from A to B. (c) The number of one-to-one functions from A to B. (d) The number of onto functions from A to B. (e) The closed binary operations on A that are commutative.

4. **(10 points)** Suppose R is an equivalence class relation on {1, 2, 3, 4, 5, 6, 7} and the equivalence class induced by R are {1, 5, 6}, {2, 4}, {3, 7}. What is the value of |R|?

5. **(14 points, 5,3,3,3)** Let p, q, r be three distinct primes. We denote relation $x\mathbf{R}y$ if x divides y. Under this relation **R**, please determine (a) the Hasse diagram of all positive divisors of p^2q, (b) the maximal element, (c) the greatest element, (d) glb{pq, p^2}.

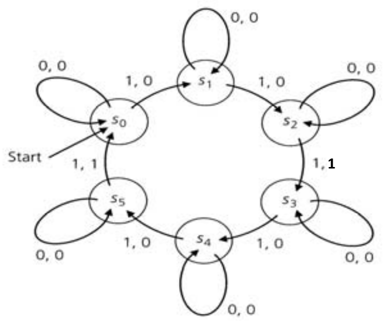
6. **(10 points)** Please fill in the blank (a)~(d) in the following table.

Objects Are Distinct	Containers Are Distinct	Some Container(s) May Be Empty	Number of Distributions
Yes	Yes	Yes	(a)
Yes	Yes	No	(b)
Yes	No	Yes	(c)
Yes	No	No	(d)

7. **(10 points)** Let S = {0, 4, 8, 12, 16, … , 92, 96, 100}. How many elements must we select from S to insure that there will be at least two whose sum is 108.

8. **(10 points)** We use $s(m, n)$ to denote the number of ways to seat m people at n circular tables with at least one person at each table. The arrangements at any one table are not distinguished if one can be rotated into another. The ordering of the tables is ***not*** taken into account. (1) For m ≥ 1, what are $s(m, m)$ and $s(m, 1)$? (2) Prove that $s(m, n) = (m-1)s(m-1, n) + s(m-1, n-1)$ for $m \geq n \geq 1$.

9. **(12 points)** Let M be the finite state machine in the figure. (1) Design a finite state machine different with M that can recognize the number of 1s in the input is 3k, k>=1. (2) if s_3 and s_4 are k-equivalent, what is the maximal k? (3) What is the minimal distinguishing string for s_3 and s_4?



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