NCKU CSIE Discrete Mathematics (2018 Spring) Midterm I

1. (*30 pts*) For each of the following statements, **determine** (*2 pts*) and **explain** (*3 pts*) whether it is correct or not.
   (1). $(p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology.
   (2). The number of positive divisors of $2^5 5^8 7^3$ is 216.
   (3). The number of compositions of 20 that have all even summands is 2^{10}.
   (4). For any set $A, B, C \subseteq U, (A - B) - C = (A - C) - (B - C)$.
   (5). Someone says: "*I am lying*." This sentence can't be either true or false.
   (6). If $n \in Z^+$, then 43 divides $6^{n+2} + 7^{2n+1}$.

2. (*10:3,3,4 pts*) For the complete expansion of $(2x^2 - y + 3z^{-1} + 4)^6$, determine the following value.
   (a) the coefficient of xyz^{-2} , (b) the number of the distinct terms , (c) if we change the constant term '*y*' to '*x*', what's the coefficient of x^4z^{-1}.

3. (8: 2,3,3 *pts*) When does a positive integer *n* have exactly (a) two, (b) three, (c) four positive divisors?

4. (*12 pts*) (a) Prove that *gcd*(*n*, *n*+2)=1 or 2, (b) what possible values can *gcd*(*n*, *n*+*k*) have? (c) Show that 17 | (2a+3b) then 17 | (9a+5b), if $a, b \in N$.

5. (*10 pts*) For all $n \in Z^+$, show that if $n \geq 64$, then *n* can be written as a sum of 5's and/or 17's.

6. (1*0 pts*) Define the connective "Nor" by $(p \downarrow q) \Leftrightarrow \neg(p \vee q)$, for any statements *p*, *q*. Represent the following using only this connective. (a) $p \wedge q$ (b) $p \rightarrow q$.

7. (*10 pts*) What is the number of integer solutions for $x_1+x_2+x_3 = Z$, if (a) $x_1, x_2, x_3 > 0$, Z=9, (b) $x_1, x_2 > 0, x_3 > 2, Z < 9$.

8. (*10 pts*) Use a combinatorial argument to show that $n\binom{n-1}{r} = (r+1)\binom{n}{r+1}$.