NCKU CSIE Discrete Mathematics (2017 Spring) Midterm I

1. (*35 pts*) For each of the following statements, **determine** (*2 pts*) and **explain** (*3 pts*) whether it is correct or not.
   (1). $(p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology.
   (2). If 17 | (3a+5b) then 17 | (4a+b), if $a, b \in N$.
   (3). Let |A|=4 and |B|=6, the probability of all functions from A to B that are one-to-one is smaller than 1/3.
   (4). The number of compositions of 20 that have all even summands is 2^{10}.
   (5). $f: \boldsymbol{Z}^+ \rightarrow \boldsymbol{Z}, f(x) = x^2 + 1$ is an one-to-one function.
   (6). If a collection of subsets of {1,2,…, n} has the property that each pair of subsets has at least one element in common, then there are at most 2^{n-1} subsets in the collection.
   (7). If $n \in Z^+$, then 43 divides $6^{n+2} + 7^{2n+1}$.

2. (*15:2,2,3,4,4 pts*) For the complete expansion of $(2x^2 - y + 3z^{-1} + 4)^5$, determine the following value.
   (a) the coefficient of x^2yz^{-2} , (b) the number of the distinct terms , (c) the sum of all coefficients, (d) if we change the constant term '*y*' to '*x*', what's the coefficient of x^4z^{-1}, (e) in (d), the number of the distinct terms is the same as that in (b)? why?

3. (1*0 pts*) Define the connective "Nor" by $(p \downarrow q) \Leftrightarrow \neg(p \vee q)$, for any statements *p*, *q*. Represent the following using only this connective. (a) $p \wedge q$ (b) $p \rightarrow q$.

4. (*10: 2,2,3,3 pts*) Determine the following sets: (a) $\emptyset \cup \{\emptyset\}$, (b) $\emptyset \Delta \{a, \emptyset, \{\emptyset\}\}$ (c) $\{\emptyset\} \Delta \{a, \emptyset, \{\emptyset\}\}$ (d) $power\ set\ of\ \{\emptyset, a, \{a\}\}$.

5. (*10 pts*) What is the number of integer solutions for $x_1+x_2+x_3 = Z$, if (a) $x_1, x_2, x_3 > 0$, Z=8, (b) $x_1, x_2 > 0$, $x_3 > 2$, $Z < 8$.

6. (*10:5, 5 pts*) (a) Assume that there are 100 students of different heights, from which two groups of 10 students each are selected. In how many ways can the selection be made so that the tallest student in the first group is shorter than the shortest student in the second group? (b) Use a combinatorial argument to show that $\binom{3n}{2} = 3\binom{n}{2} + 3n^2$.

7. (*10 pts*) One rock-n-roll music CD costs $29 and one classic music CD costs $33. How many CDs of these two kinds you should buy if you pay $500 for them? (exhaustively listing all answers gets 0 pts.)