Discrete Mathematics (2012 Spring) Final

1. (***10 points***) For each of the following statements, determine whether it is correct or not.
   (1) $\phi \subset \phi$ (2) $\phi \subseteq \{\phi\}$ (3) $P(A \cup B) = P(A) \cup P(B)$
   (4) Let A ={1, 2, 3, 4, 5, 6}, there is an equivalence relation R on A with |R|=8.
   (5) Let (A, **R**) be a poset. If (A, **R**) is a total order, then it is a lattice.

2. (***8 points***) In how many ways can 30030 (2*3*5*7*11*13) be factored into three or more factors, each greater than 1 and the order of the factors is relevant?

3. (***12 points***) Find (a) the generating function for the number of solutions of 2w + 3x + 5y + 7z = n, $0 \le w$, $2 \le x, y$, $3 \le z$, (b) the exponential generating function for the sequence 0!, 1!, 2!, 3!, …, (c) the exponential generating function for the number of ways to arrange n letters, $n \geqq 0$, selected from the word "MISSISSIPPI" and the arrangement must contain at least two S's and one I.

4. (***5 points***) For n distinct objects, let $a(n, r)$ denote the number of ways we can select, *without repetition*, r of the n objects when $0 \le r \le n$. Here $a(n, r) = 0$ when $r > n$. Please describe $a(n, r)$ in a recurrence relation form.

5. (***10 points***) Suppose the symbols of legal arithmetic expressions include 0, 1,…, 9, +, *, /. Let a_n be the number of legal arithmetic expressions that are made up of n symbols. Solve a_n. a_1=10 and a_2=100.

6. (***10 points***) Please determine how many integer solutions for $x_1 + x_2 + x_3 = 10$, $1 \le x_1 \le 4$, $0 \le x_2 \le 6$, $2 \le x_3 \le 7$. (exhaustively list all answers is not allowed.)

7. (***8 points***) Determine (a) the sequence generated by $f(x)=1/(2+3x)$, (b) the coefficient of x^{15} in $(x^3-5x)/(1-x)^3$.

8. (***10 points***) Find the number of permutations of 0, 1, 2, 3, …, 8, 9 in which none of the patterns '1234', '76', '23', '891' occurs.

9. (***8 points***) (10%) Determine the value of $c \in Z^+, 10 \le c \le 15$ such that equation $84x + 990y = c$ has integer solutions. Determine the solutions for this c value.

10. (***9 points***) If A = {v, w, x, y, z}, determine the number of relations on A that are (a) antisymmetric and contain (x,y), (b) equivalence relations that determine more than three (include three) equivalence classes, (c) reflexive and symmetric but not transitive.

11. (***10 points***) For A={1, 2, 3, 4, 5} and B={u, v, w, x, y, z}, determine the number of one-to-one functions f: A→B where $f(1) \neq$ v, w, $f(2) \neq$ u, w, $f(3) \neq$ y and $f(4) \neq$ x, $f(5)$=z.

12. (***5 points***) Please list 2 examples/methods/strategies to improve your (or others') learning motivation/performance.

	$S(m, n)$					
m \ n	1	2	3	4	5	6
1	1					
2	1	1				
3	1	3	1			
4	1	7	6	1		
5	1	15	25	10	1	
6	1	31	90	65	15	1

Table of Stirling number of the second kind