Electric Current & Resistance

Electric Current

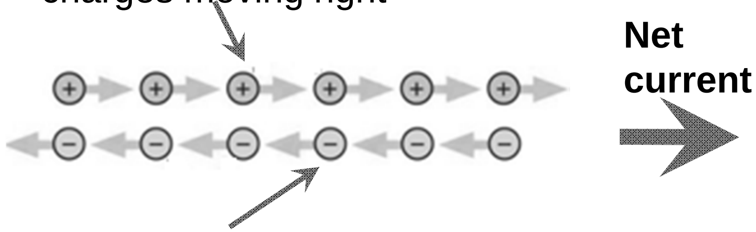
Current (I) = Net rate of (+) charge crossing an area.

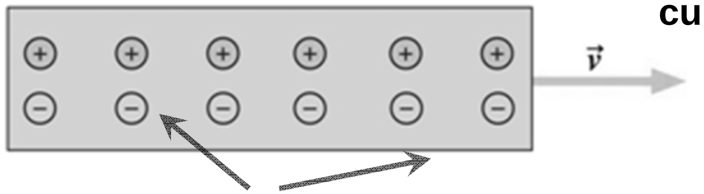
Electronics: $I \sim$ mA

Biomedics: $I \sim \mu$A

Steady current: $$I = \frac{\Delta Q}{\Delta t}$$

Instantaneous current: $$I = \frac{dQ}{dt}$$

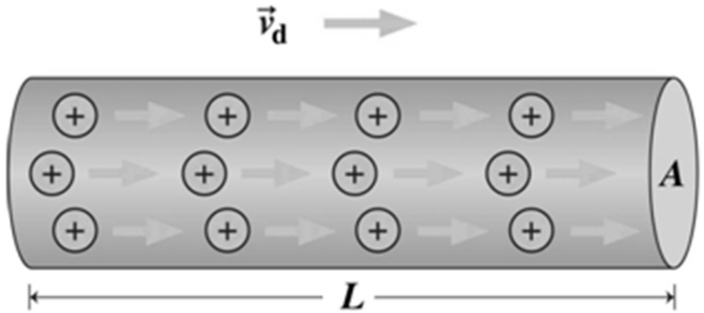




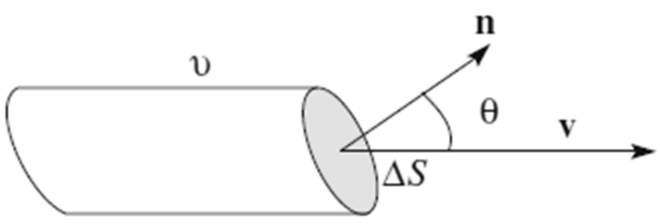
Current Density

Current Density J: the current intensity pre unit area perpendicular to the current flow.

Current Intensity $\Delta I = \vec{J} \cdot \Delta\vec{S} \Rightarrow I = \int \vec{J} \cdot d\vec{S}$



$$\vec{J} = q\, n_p \vec{v}$$

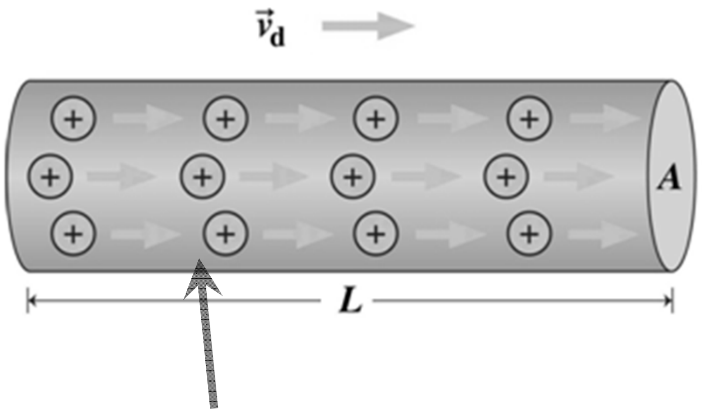


However v is not constant due thermal

motion ⟨ **v** ⟩ = 0

$$\vec{J} = q\, n_p < \vec{v} >$$

Curent: A Microscopic Look



Charge in this volume is $\Delta Q = n\,A\,L\,q.$

$$I = \frac{\Delta Q}{\Delta t} = \frac{n\,A\,L\,q}{L / v_d}$$

However v is not constant due thermal motion $\langle \mathbf{v} \rangle$

$$\vec{J} = q\, n_p < \vec{v} >$$

For $\mathbf{E} \neq 0$, $\mathbf{v_d} = \langle \mathbf{v} \rangle = 0.$

For $\mathbf{E} \neq 0$, $\mathbf{v_d} = \langle \mathbf{v} \rangle \neq 0.$ drift velocity

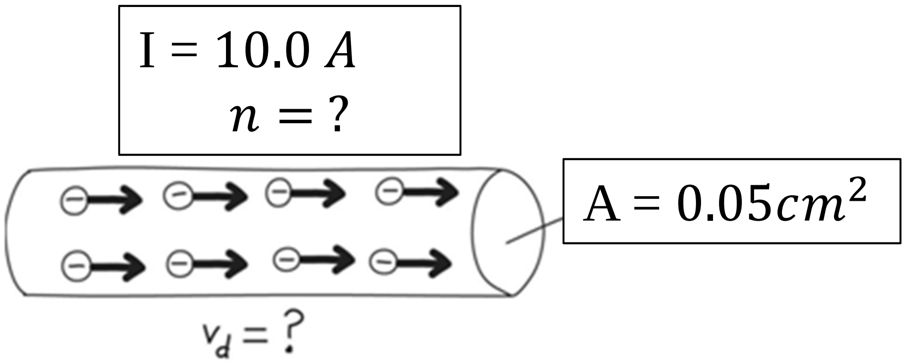
$n = n_p$ = number of carriers per unit volume

q = charge on each carrier

$$\boxed{I = n\,A\,q\,v_d}$$

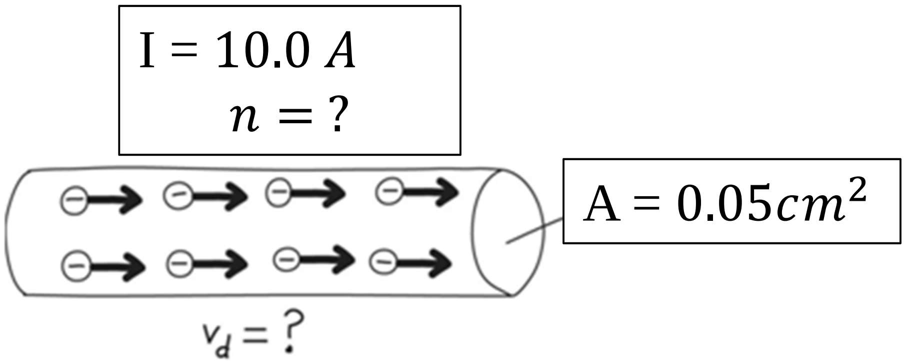
Example: A Copper Wire

A 10.0-A current flows in a copper wire with cross-sectional area 0.05 cm². Find the electron's drift speed.



Example: A Copper Wire

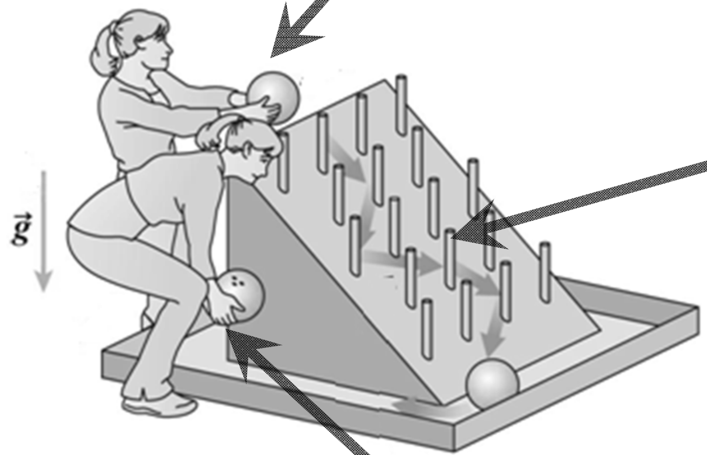
A 10.0-A current flows in a copper wire with cross-sectional area 0.05 cm^2. Find the electron's drift speed.



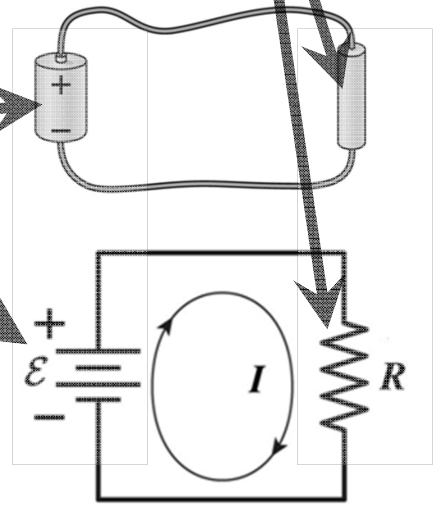
m ~ q

g ~ **E**

Collisions ~ resistance



Lifting ~ emf

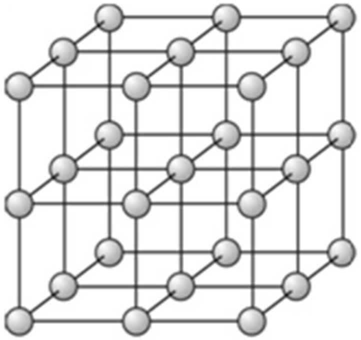


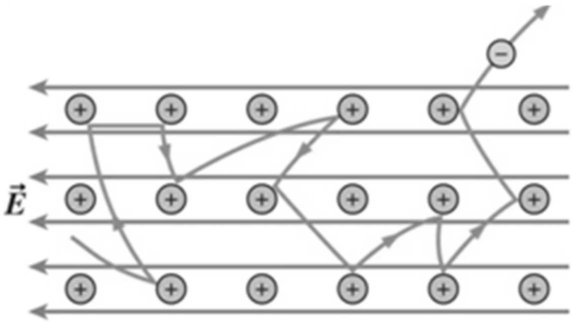
Ideal emf :
no internal energy loss.

Energy gained by charge transversing battery = $q\,\mathrm{E}$
(To be dissipated as heat in external R.)

Ohm's law: $I = \dfrac{E}{R}$

Conduction in Metals





Metal: $\rho \sim 10^{-8} - 10^{-6}\ \Omega\cdot m$

Matter waves

Atomic structure: polycrystalline.

Carriers: sea of "free" electrons, $v \sim 10^6$ m/s

E = 0: equal # of e moving ± directions → $\langle \mathbf{v} \rangle = 0$.

E ≠ 0: Collisions between e-e & e-ph → $\mathbf{v_d} \sim$ const.

Set q = -e as an elementary charge

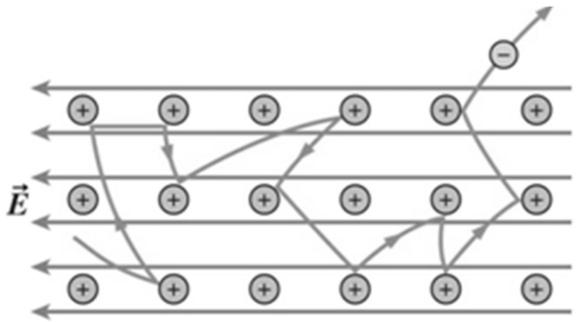
$$m\frac{dv}{dt} + \frac{m}{\tau}v = -eE$$

τ = relaxation time

Steady state: $\dfrac{dv}{dt} = 0 \quad \rightarrow \quad v_d = -\dfrac{eE\tau}{m}$

$$J = n(-e)v_d = \frac{ne^2\tau}{m}E = g\vec{E}$$

Ohm's law

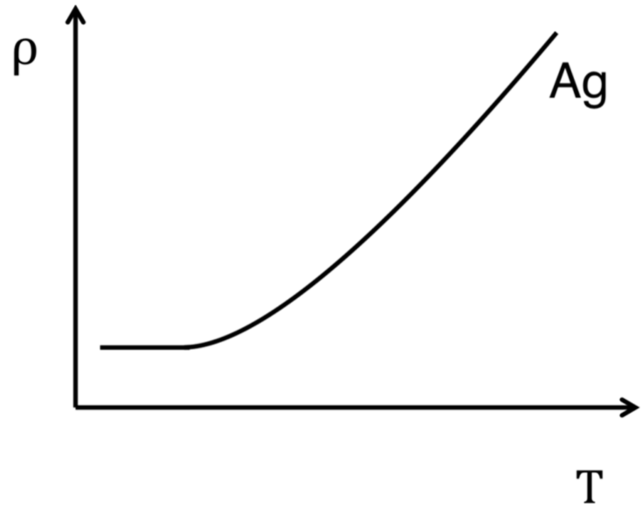


$$m\frac{dv}{dt}+\frac{m}{\tau}v=-eE$$

Steady state: $\frac{dv}{dt}=0 \rightarrow$ $v_d=-\frac{eE\tau}{m}$

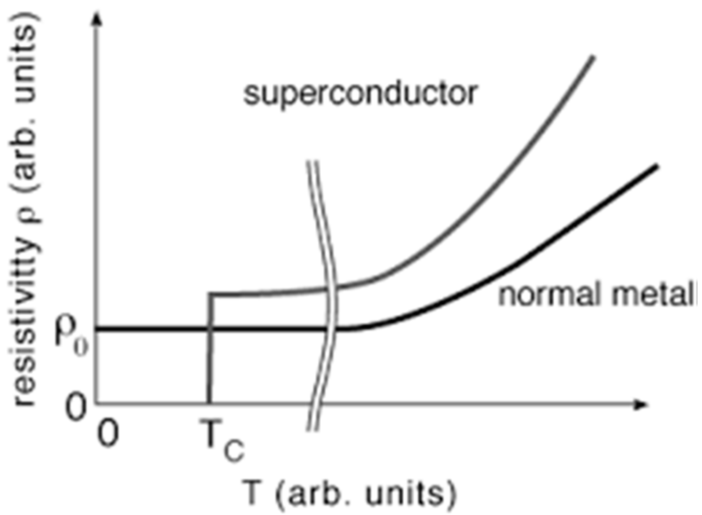
$$J=n(-e)v_d=\frac{ne^2\tau}{m}E=g\vec{E}$$ Ohm's law

$$\vec{E}=\rho\vec{J}$$



Due to Fermi statistics.

c.f., $v_{th} \propto \sqrt{T}$

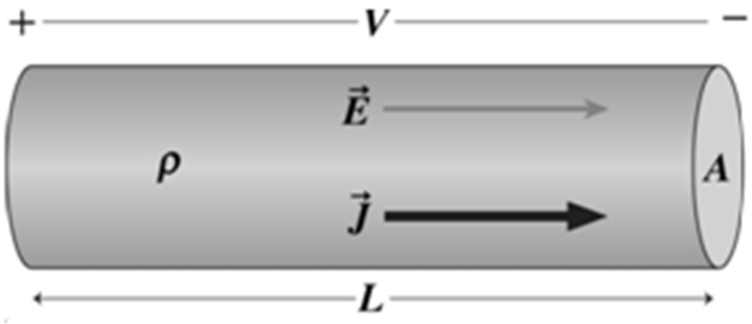


Resistance & Ohm's Law

Ohm's Law $\quad I = \dfrac{V}{R} \quad$ macrscopic version

Open circuit: $\quad R \to \infty \quad \therefore \quad I = 0 \quad \forall\, V$

Short circuit: $\quad R = 0 \quad \therefore \quad I \to \infty \quad \forall\, V$



$$\mathbf{J} = \frac{\mathbf{E}}{\rho} \quad \to \quad I = J\,A = \frac{E}{\rho}A = \frac{V}{\rho\, L}A$$

$$\to \quad \boxed{R = \rho\frac{L}{A}}$$

Resistor: piece of conductor made to have specific resistance.

All heating elements are resistors.

So are incandescent lightbulbs.

Starting Your Car

A copper wire 0.50 cm in diameter & 70 cm long

connects your car's battery to the starter motor.

What's the wire's resistance?

If the starter motor draws a current of 170A,

what's the potential difference across the wire?

$$R = \rho \frac{L}{A} = \left(1.68 \times 10^{-8} \Omega \cdot m\right) \frac{0.70\, m}{\pi \left(0.25 \times 10^{-2} m\right)^2} = 0.60\, m\,\Omega$$

$$V = I\,R = \left(170\, A\right)\left(0.60\, m\,\Omega\right) = 0.10\, V$$

Electric Power

Electric Power : $P = \frac{d}{dt}(qV) \quad = IV$ for time independent V

$$\boxed{P = IV} = I^2 R = \frac{V^2}{R} \qquad V = I\,R$$

Power increase with R
(for fixed *I*)

Power decrease with R
(for fixed *V*)

No contradiction

Making the Connection

What is the current in a typical 120 V, 100 W lightbulb?

What's the bulb's resistance?

$$I = \frac{P}{V} = \frac{100\,W}{120\,V} = 0.833\ A$$

$$R = \frac{V}{I} = \frac{120\,V}{0.833\ A} = 144\,\Omega$$

Electrical Safety

TABLE 24.3. Effects of Externally Applied Current on Humans

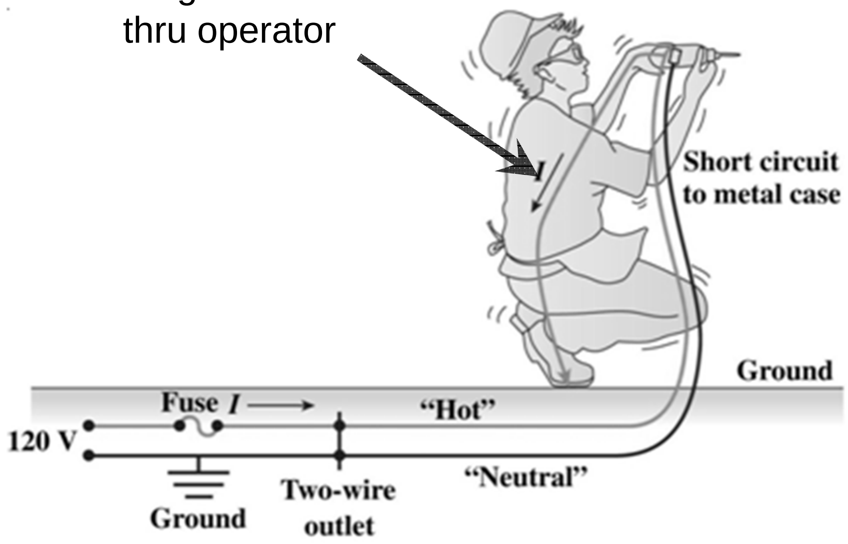
Current Range	Effect
0.5 – 2 mA	Threshold of sensation
10 – 15 mA	Involuntary muscle contractions; can't let go
15 – 100 mA	Severe shock; muscle control lost; breathing difficult
100 – 200 mA	Fibrillation of heart; death within minutes
> 200 mA	Cardiac arrest; breathing stops; severe burns

Typical human resistance ~ 10^5 Ω.

Fatal current ~ 100 mA = 0.1 A. $\rightarrow$ $V = (0.1A)(10^5\Omega) = 10,000\,V$

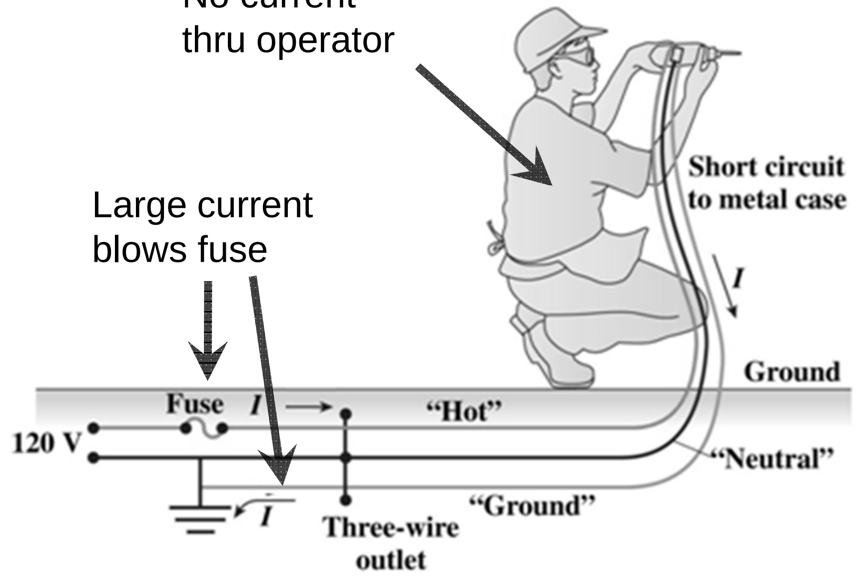
A wet person can be electrocuted by 120V.

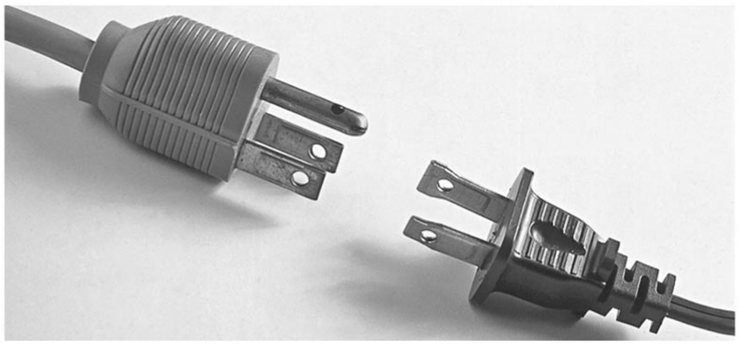
Large current
thru operator



No current
thru operator

Large current
blows fuse

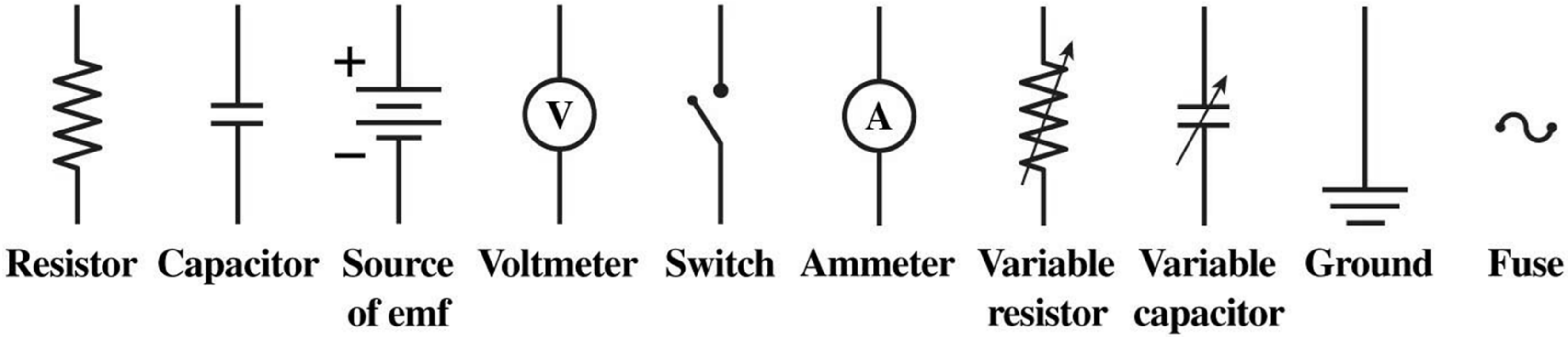




Ground fault interupter

Circuits, Symbols, & Electromotive Force

Common circuit symbols



All wires ~ perfect conductors → V = const on wire

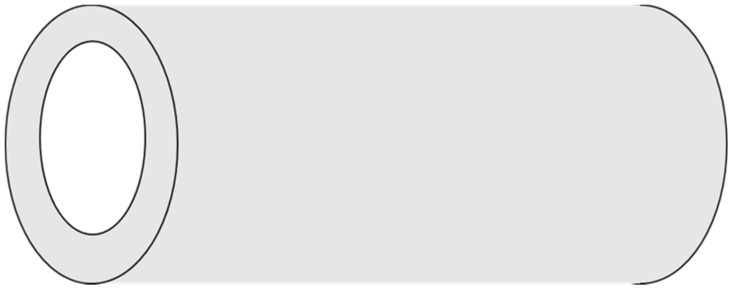
Electromotive force (emf) = device that maintains fixed ΔV across its terminals.

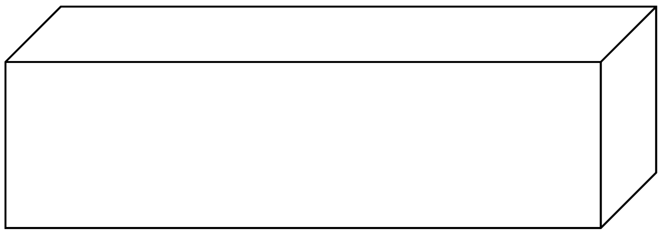
E.g., batteries (chemical),

generators (mechanical),

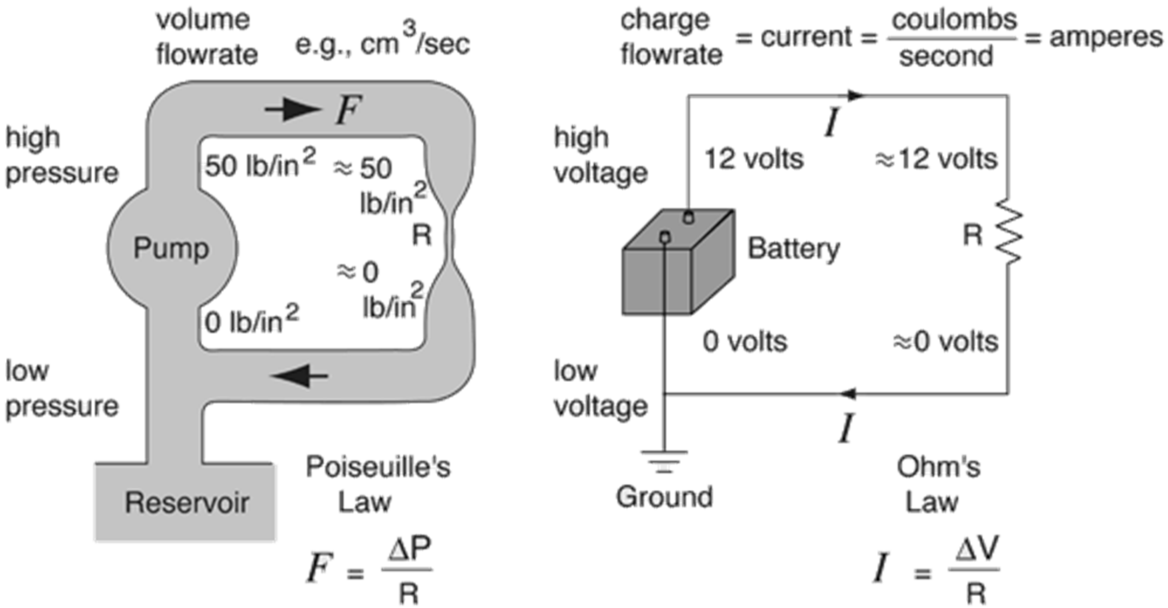
photovoltaic cells (light),

cell membranes (ions).

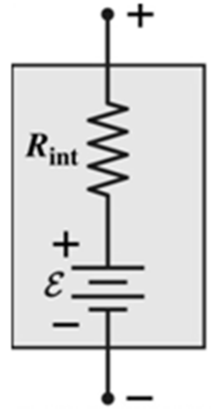




Electric Motive Force & Circuit

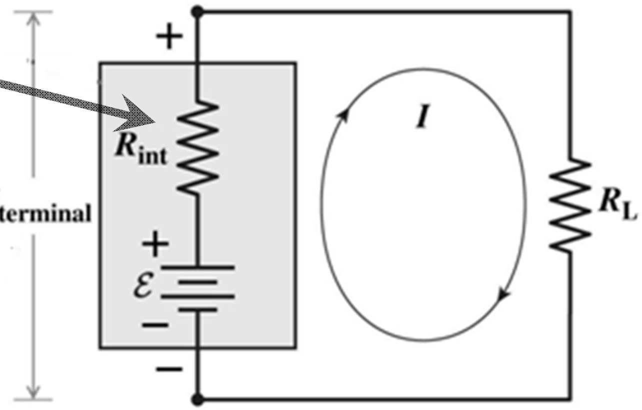


Real Batteries



Model of real battery = ideal emf E in series with internal resistance R_{int} .

I means *V* drop $I\ R_{int}$



$\therefore\ V_{terminal} < \mathrm{E}$

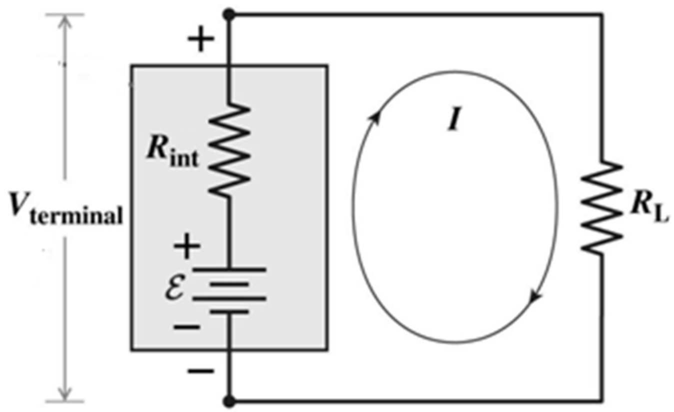
$$E - I\ R_{\text{int}} = I\ R_L$$

$$I = \frac{E}{R_{\text{int}} + R_L} \quad < \quad \frac{E}{R_L}$$

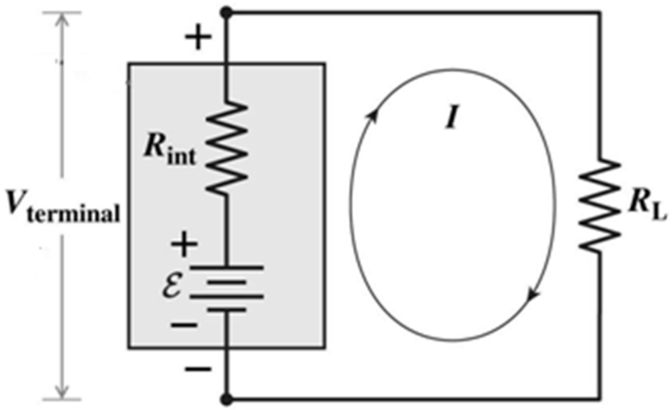
$$V_{R_L} = \frac{R_L}{R_{\text{int}} + R_L} E$$

Example

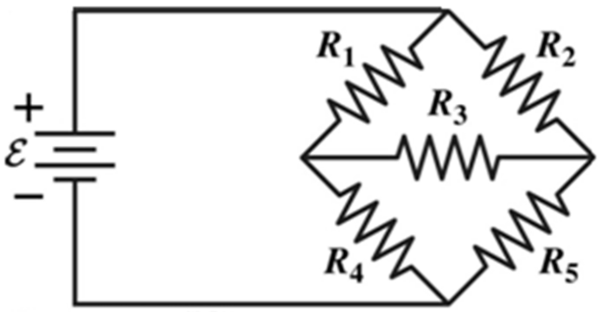
- A load resister R_L is connected to a source of emf whose internal resistance is R_{int}. For what value of R_L will the power supplied to the load be a maximum.



Example



Kirchhoff’s Laws & Multiloop Circuits



This circuit can’t be analyzed using series and parallel combinations.

Kirchhoff’s loop law:

$\sum V = 0$ around any closed loop.

(energy is conserved)

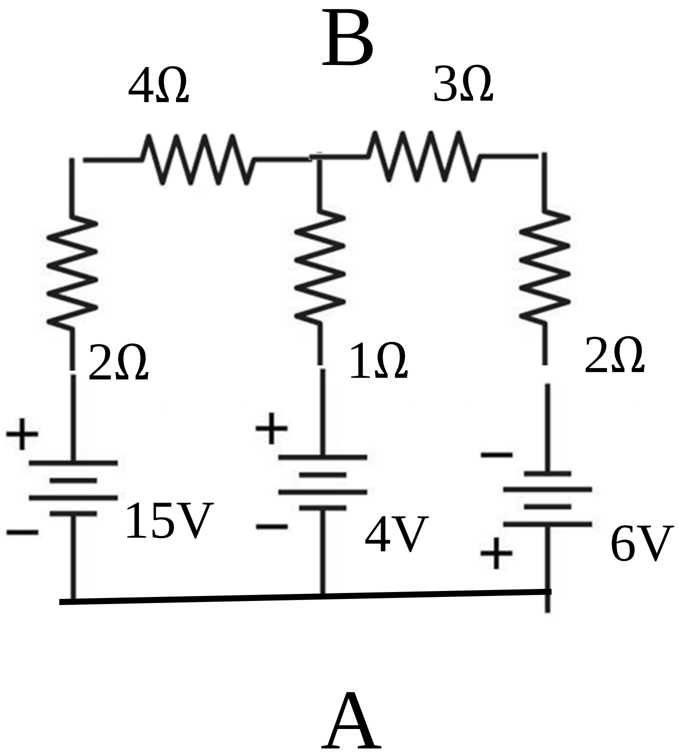
Kirchhoff’s node law:

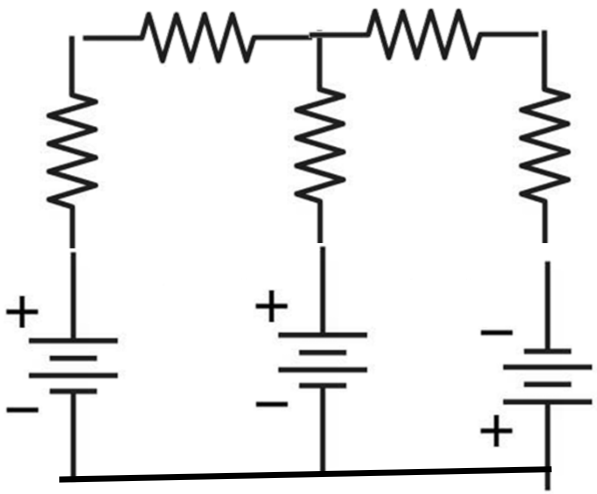
$\sum I = 0$ at any node.

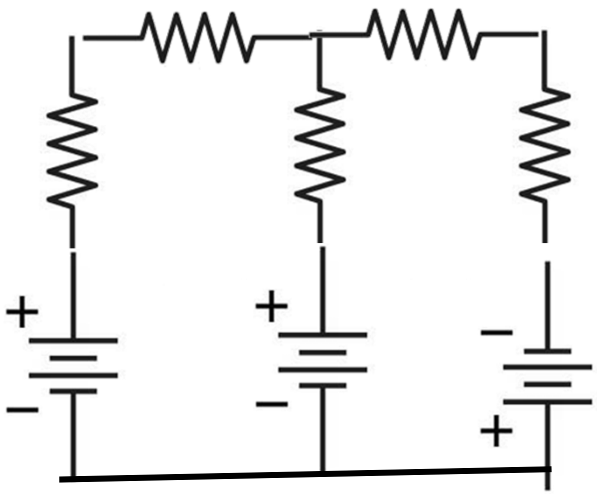
(charge is conserved)

Example

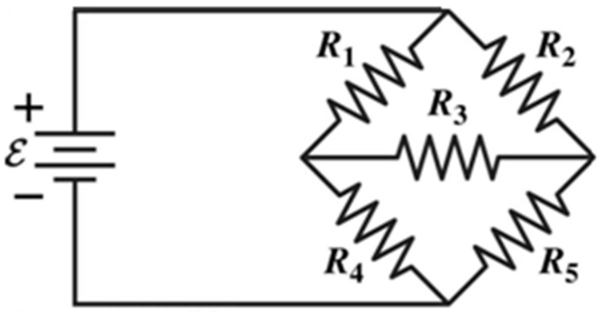
- The circuit has two loops and three sources of emf. Determine the current of the two loops and what is the change in potentia $V_A - V_B$





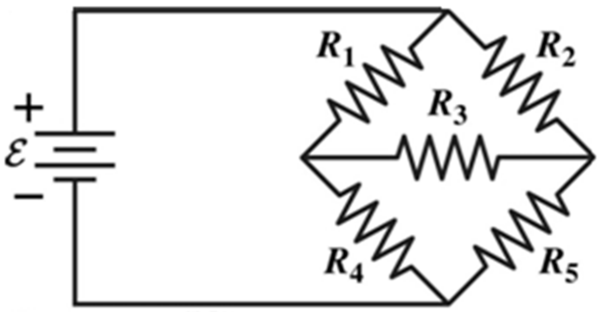


Example



This circuit can’t be analyzed using series and parallel combinations.

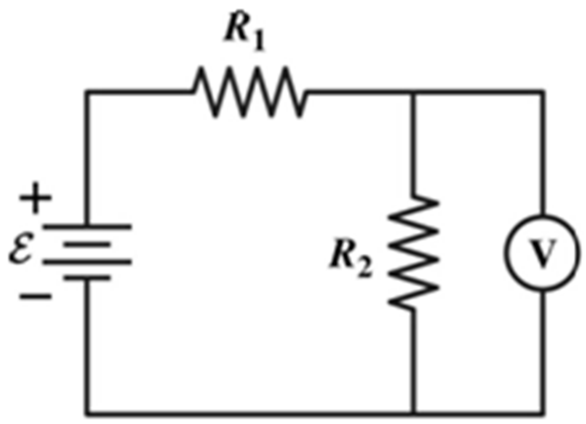
Example



This circuit can’t be analyzed using series and parallel combinations.

Conceptual Example   Measuring Voltage

What should be the electrical resistance of an ideal voltmeter?



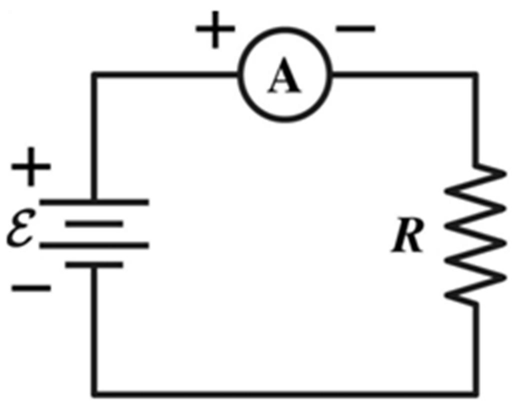
An ideal voltmeter should not change the voltage across R_2 after it is attached to the circuit.

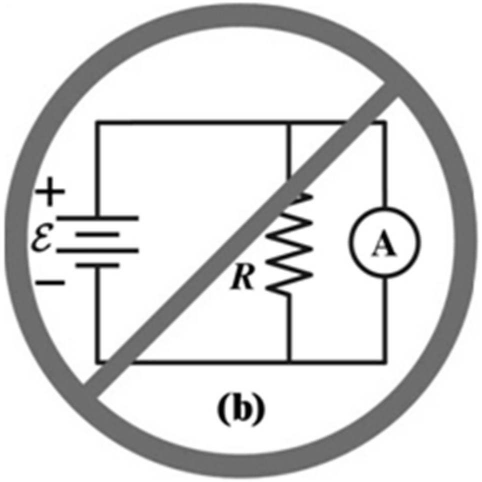
The voltmeter is in parallel with R_2.

In order to leave the combined resistance, and hence the voltage across R_2 unchanged, R_V must be ∞.

Ammeters

An ammeter measures the current flowing through itself.

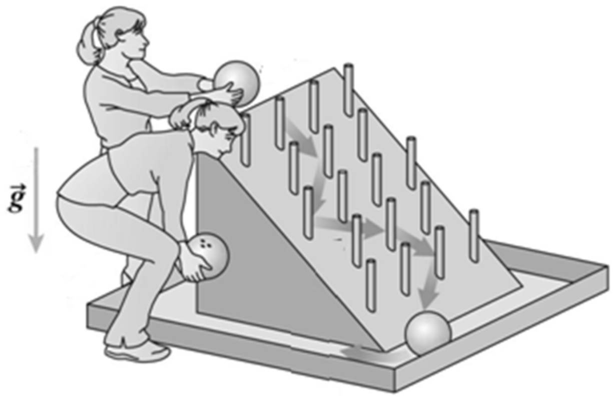




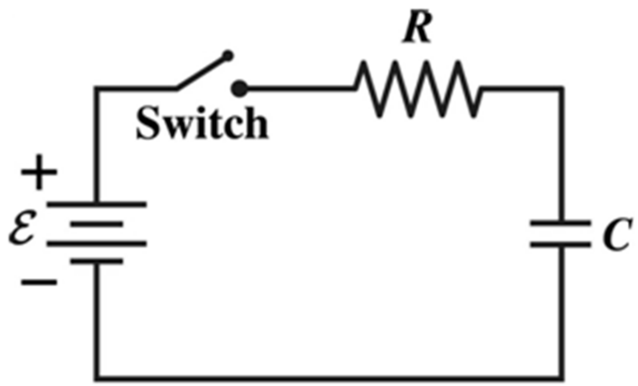
Ideal voltmeter: no voltage drop across it $\rightarrow$ $R_m = 0$

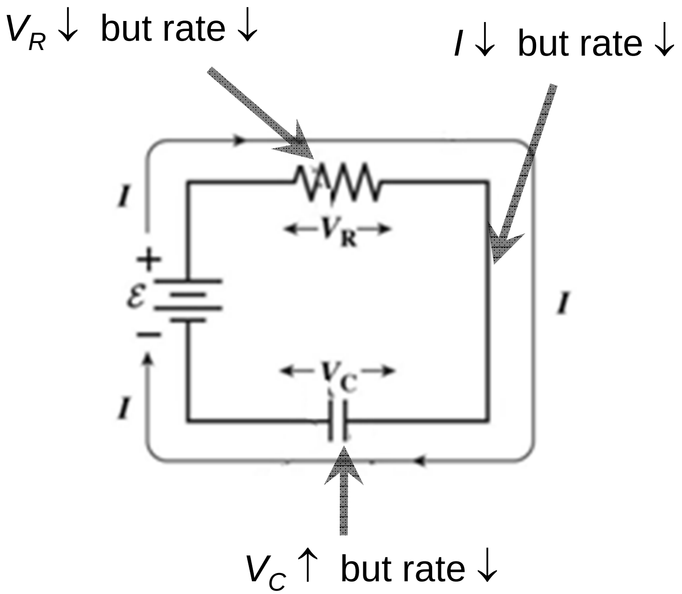
Capacitors in Circuits

Voltage across a capacitor cannot change instantaneously.



The RC Circuit: Charging





C initially uncharged $\rightarrow$ $V_C = 0$

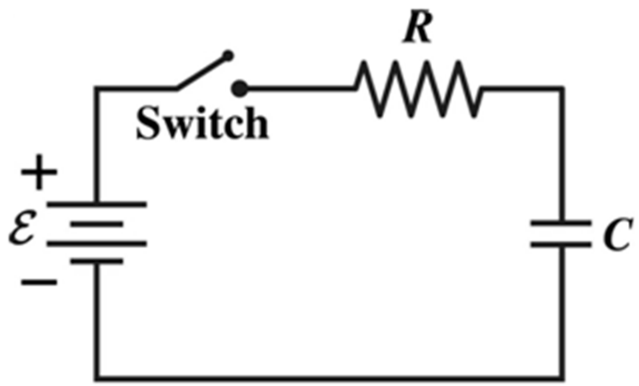
Switch closes at $t = 0$.

$V_R\,(t = 0) = \mathcal{E}$

$\rightarrow$ $I\,(t = 0) = \mathcal{E} / R$

C charging: $V_C \uparrow \;\rightarrow\; V_R \downarrow \;\rightarrow\; I \downarrow$

Charging stops when $I = 0$.


R
Switch
+
ℰ
−
C

Example Camera Flash

A camera flash gets its energy from a 150-μF capacitor & requires 170 V to fire.

If the capacitor is charged by a 200-V source through an 18-kΩ resistor,

how long must the photographer wait between flashes?

Assume the capacitor is fully charged at each flash.

$$t = -RC\ln\left(1 - \frac{V_C}{E}\right)$$

$$= -\left(18\times10^3\ \Omega\right)\left(150\times10^{-6}\ F\right)\ln\left(1 - \frac{170\,V}{200\,V}\right)$$

$$= 5.1\ s$$

RC Circuits: Long- & Short- Term Behavior

For $\Delta t << RC$: $V_C \approx$ const,

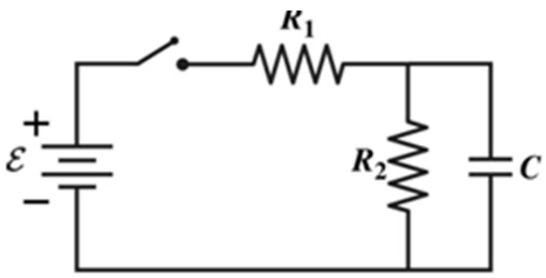
$\rightarrow$ C replaced by short circuit if uncharged.

$\rightarrow$ C replaced by battery if charged.

For $\Delta t >> RC$: $I_C \approx 0$,

$\rightarrow$ C replaced by open circuit.

Long & Short Times

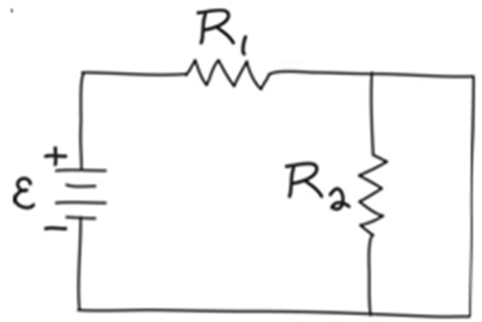


The capacitor in figure is initially uncharged.

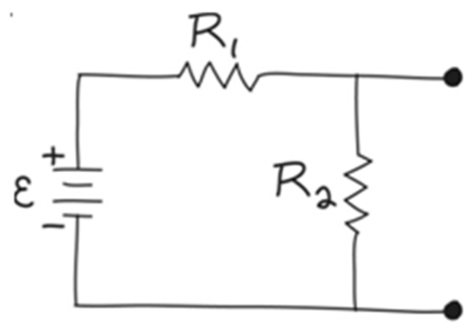
Find the current through R_1

(a) the instant the switch is closed and

(b) a long time after the switch is closed.



(a) $$I_1 = \frac{E}{R_1}$$



(b) $$I_1 = \frac{E}{R_1 + R_2}$$

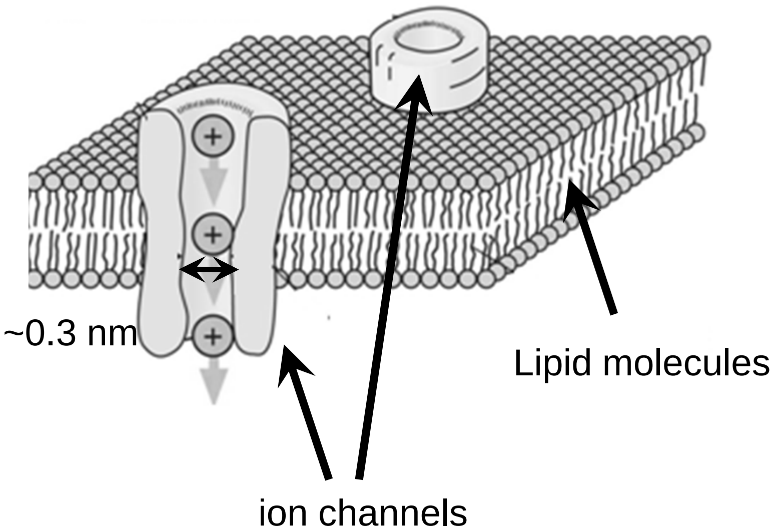
Cell Membrane

Ion channels are narrow pores that allow ions to pass through cell membranes.

A particular channel has a circular cross section 0.15 nm in radius;

it opens for 1 ms and passes 1.1×10^4 singly ionized potassium ions.

Find both the current & the current density in the channel.



$$I = \frac{\Delta Q}{\Delta t} = \frac{\left(1.1\times10^4\right)\left(1.6\times10^{-19}\ C\right)}{1\times10^{-3}\ s}$$

$$= 1.8\times10^{-12}\ A \quad = 1.8\ pA$$

$$J = \frac{I}{A} = \frac{1.8\times10^{-12}\ A}{\pi\left(0.15\times10^{-9}\ m\right)^2}$$

$$= 25\times10^6\ A/m^2 \quad = 25\ MA/m^2$$

AWG 10 : $J = \frac{30A}{5.26\ mm^2} = 5.7MA/m^2$

~ 4 times max. safe current density in household wirings

Application: Cell Membrane

Hodgkin-Huxley (1952) circuit model of cell membrane (Nobel prize, 1963):