DISCRETE MATHEMATICS – CH1 Homework1

Textbook assignment (60 pts)

1-2

22. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000? (if n > int("5"+last 6 digits of your student ID))

n > 5,430,035 (If your student id is P56430035)

(1) n > 5,000,000 = $\frac{6!}{2!} + \frac{6!}{2!2!} + \frac{6!}{2!2!} = 720$

(2) 5,000,000 < n < 5,400,000 = $\frac{5!}{2!} = 60$

(3) 5,400,000 < n < 5,430,000 = 0

(4) 5,430,000 < n < 5,430,030 = 0

(5) 5,430,030 < n < 5,430,035 = 0

Finally, the result = 720 – 60 = 660

34. How many distinct four-digit integers can one make from the digits 1, 3, 3, 7, 7, and 8?

Case1: 1,3,7,8 in □□□□

4! = 24

Case2: 2 the same numbers (3 and 7) in □□□□

$C(2,1) * C(3,2) * \frac{4!}{2!} = 72$

Case3: 3,3,7,7 in □□□□

$C(4,2) = \frac{4!}{2!2!} = 6$

Answer = 24 + 72 + 6 = 102

1-3

18. For the strings of length 10 in Example 1.24, how many have (a) four 0's, three 1's, and three 2's; (b) at least eight 1's; (c) weight 4?

(a) 10!/(4!*3!*3!)

(b) $\binom{10}{8}*2^2 + \binom{10}{9}*2 + 1$

(c) $\binom{10}{4} + \binom{10}{2} * \binom{8}{1} + \binom{10}{2}$

1-4

18. a) How many nonnegative integer solutions are there to the pair of equations

$$x_1 + x_2 + x_3 + \ldots + x_7 = 30,\ x_1 + x_2 + x_3 = 6?$$

b) How many solutions in part (a) have $x_1, x_2, x_3 > 0$?

(a) $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 30$

$6 + x_4 + x_5 + x_6 + x_7 = 30$

The first condition： $x_1 + x_2 + x_3 = 6$

The second condition： $x_4 + x_5 + x_6 + x_7 = 24$

$$C_{24}^{24+4-1} * C_6^{6+3-1} = \binom{27}{24} * \binom{8}{6}$$

(b) from (a)

The first condition：$x_1 + x_2 + x_3 = 6$, and $x_1, x_2, x_3 > 0$ (x_1, x_2, x_3 at least 1)

The second condition：$x_4 + x_5 + x_6 + x_7 = 24$

$$C_3^{3+3-1} * C_{24}^{24+4-1} = \binom{5}{3} * \binom{27}{24}$$

25. Consider the 2^{19} compositions of 20. (a) How many have each summand even? (b) How many have each summand a multiple of 4?

(a) $20 = 2 + 4 + 12 + 2 = 2(1 + 2 + 6 + 1) = 2*10$

The number of composition of 10 – namely , $2^{10-1} = 2^9$

(b) multiple of 4 = {4 , 8 , 12 , 16 , 20}

The number of composition of 5 – namely , $2^{5-1} = 2^4$

Supplementary

26. a) In how many ways can 17 be written as a sum of 2's and 3's if the order of the summands is (i) not relevant? (ii) relevant?

b) Answer part (a) for 18 in place of 17.

(學號偶數(a),奇數(b))

(a) (i) 1(one 3) + 1(three 3's) + 1(five 3's) = 3

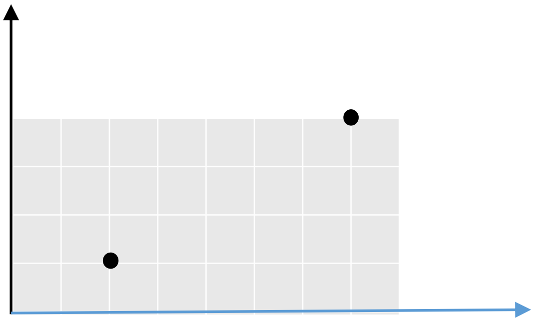
(ii) C_1^8(one 3) + C_3^7(three 3's) + C_5^6(five 3's)

(b) (i)1(no 3) + 1(two 3's) + 1(four 3's) + 1(six 3's) = 4

(ii)C_0^9(no 3) + C_2^8(two 3's) + C_4^7(four 3's) + C_6^6(six 3's)

Bonus assignment (40 pts)

- In the example 14, if we have one new step R-, that means a backward walking $x = x$-1, think about how to calculate the number of paths from (2, 1) to (7, 4). Note that R- can't follow by a step R.



As long as walker can take one step toward left, he can return from any (8,y) position to (7, y+1) position after one U step. The possible number of step is now 10. As long as it says R-(left) can't be followed by R, there must be U after R-.

a. So R-U combination is fixed. The number of possible position of R-U is 9. For the remaining 8 positions, 2 U and 6 R should be arranged. Therefore,

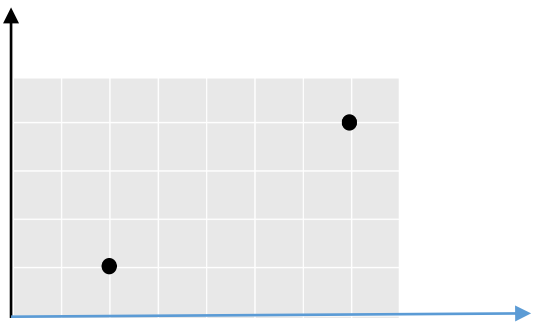
$$9 * \frac{8!}{2! * 6!} = 9 * \frac{8 * 7 * 6!}{2 * 1 * 6!} = 9 * 4 * 7 = 252$$

b. In the case that R- is the last element, the element in the second position from tail is fixed U. So the number of combination is

$$\frac{8!}{2! * 6!} = \frac{8 * 7 * 6!}{2 * 1 * 6!} = 4 * 7 = 28$$

Total possible route is 252+28=280

- Also, if we have U-



In the case there are one U- and one R-, the number of possible step would be 12. U- should be followed by either R or R-. R- should be followed by either U or U-.

So 4 of 12 position are fixed.

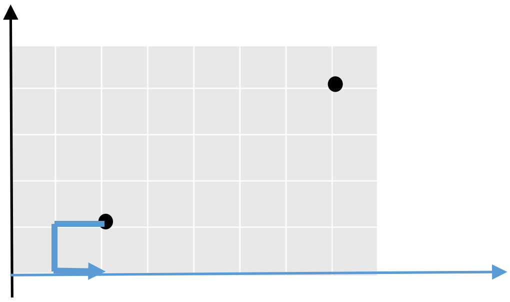
a. U-R and

i. R-U

$$\frac{8!}{3!\,5!} = \frac{8*7*6*5!}{5!*3*2*1} = 56$$

ii. R-U-

$$\frac{9!}{5!*4!} = \frac{9*8*7*6*5!}{5!*4*3*2*1} = \frac{126}{1} = 126$$



b. U-R-

i. R-U

$$\frac{9!}{6!*3!} = \frac{9*8*7*6!}{6!*3*2*1} = 84$$

Totally, 56+126+84=266

- Give an example problem (programming) this assignment related to