ANSWER

3-5 17.

$\frac{d}{dx}\tan^{-1}(x^2y) = \frac{d}{dx}(x + xy^2) \Rightarrow \frac{1}{1+(x^2y)^2}(x^2y' + y \cdot 2x) = 1 + x \cdot 2yy' + y^2 \cdot 1$
$\Rightarrow \frac{x^2}{1+x^4y^2}y' - 2xyy' = 1 + y^2 - \frac{2xy}{1+x^4y^2} \Rightarrow y'\left(\frac{x^2}{1+x^4y^2} - 2xy\right) = 1 + y^2 - \frac{2xy}{1+x^4y^2}$
$\Rightarrow y' = \frac{1+y^2-\frac{2xy}{1+x^4y^2}}{\frac{x^2}{1+x^4y^2}-2xy}$ or $y' = \frac{1+x^4y^2+y^2+x^4y^4-2xy}{x^2-2xy-2x^5y^3}$

3-5 39.

If $x = 0$ in $xy + e^y = e$, then we get $0 + e^y = e$, so $y = 1$ and the point where $x = 0$ is $(0, 1)$. Differentiating implicitly with respect to x gives us $xy' + y \cdot 1 + e^y y' = 0$. Substituting 0 for x and 1 for y gives us $0 + 1 + ey' = 0 \Rightarrow ey' = -1 \Rightarrow y' = -1/e$. Differentiating $xy' + y + e^y y' = 0$ implicitly with respect to x gives us $xy'' + y' \cdot 1 + y' + e^y y'' + y' \cdot e^y y' = 0$. Now substitute 0 for x, 1 for y, and $-1/e$ for y'. $0 + \left(-\frac{1}{e}\right) + \left(-\frac{1}{e}\right) + ey'' + \left(-\frac{1}{e}\right)(e)\left(-\frac{1}{e}\right) = 0 \Rightarrow -\frac{2}{e} + ey'' + \frac{1}{e} = 0 \Rightarrow ey'' = \frac{1}{e} \Rightarrow y'' = \frac{1}{e^2}$.

3-5 69.

Since $A^2 < a^2$, we are assured that there are four points of intersection.

$(1)\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ \rightarrow\ \frac{2x}{a^2} + \frac{2yy'}{b^2} = 0 \ \rightarrow\ \frac{yy'}{b^2} = -\frac{x}{a^2} \ \rightarrow\ y' = m_1 = -\frac{xb^2}{ya^2}$

$(2)\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 \ \rightarrow\ \frac{2x}{A^2} + \frac{2yy'}{B^2} = 0 \ \rightarrow\ \frac{yy'}{B^2} = -\frac{x}{A^2} \ \rightarrow\ y' = m_2 = -\frac{xB^2}{yA^2}$

Now $m_1m_2 = -\frac{xb^2}{ya^2} \cdot -\frac{xB^2}{yA^2} = -\frac{b^2B^2}{a^2A^2} \cdot \frac{x^2}{y^2}$,(3). Subtracting equations, (1)-(2), gives us $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{x^2}{A^2} + \frac{y^2}{B^2} = 0 \ \rightarrow\ \frac{y^2}{b^2} + \frac{y^2}{B^2} = \frac{x^2}{A^2} - \frac{x^2}{a^2}$

$\rightarrow\ \frac{y^2B^2+y^2b^2}{b^2B^2} = \frac{x^2a^2+x^2A^2}{A^2a^2} \ \rightarrow\ \frac{y^2(B^2+b^2)}{b^2B^2} = \frac{x^2(a^2-A^2)}{A^2a^2}$

(4). Since $a^2 - b^2 = A^2 + B^2$, we have $a^2 - A^2 = b^2 + B^2$. Thus, equation(4) becomes $\frac{y^2}{b^2B^2} = \frac{x^2}{A^2a^2} \ \rightarrow\ \frac{x^2}{y^2} = \frac{A^2a^2}{b^2B^2}$, and substituting for $\frac{x^2}{y^2}$in equation (3) gives us $m_1m_2 = -\frac{B^2b^2}{a^2A^2} \cdot -\frac{A^2a^2}{B^2b^2} = -1$.

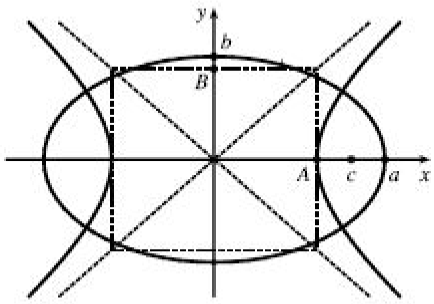
Hence, the ellipse and hyperbola are orthogonal trajectories.

3-5 70.

$y = (x+c)^{-1} \ \rightarrow\ y' = -(x+c)^{-2}$ and $y = a(x+k)^{1/3} \ \rightarrow\ y' = \frac{1}{3}a(x+k)^{-2/3}$,so the curves are othogonal if the product of the slopes is -1,
that is ,$-\frac{1}{(x+c)^2} \cdot \frac{a}{3(x+k)^{2/3}} = -1 \ \rightarrow\ a = 3(x+c)^2(x+k)^{2/3}$

$a = 3(\frac{1}{y})^2(\frac{y}{a})^2$ [since $y^2 = (x+c)^2$ and $y^2 = a^2(x+k)^{2/3}$] $\rightarrow\ a = 3\frac{1}{a^2} \ \rightarrow\ a^3 = 3 \ \rightarrow\ a = \sqrt[3]{3}$

Date: October 26, 2017.



3-5 77.

(a) If $y = f^{-1}(x)$, then $f(y) = x$. Differentiating implicitly with respect to x and remembering that y is a function of x, we get $f'(y)\frac{dy}{dx} = 1$, so$\frac{dy}{dx} = \frac{1}{f'(y)} \ \rightarrow \ (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$

(b) $f(4) = 5 \ \rightarrow \ f^{-1}(5) = 4$. By part(a) ,$(f^{-1})'(5) = \frac{1}{f'(f^{-1})(5)} = \frac{1}{f'(4)} = \frac{3}{2}$

3-6 32. $f(x) = \cos(\ln x^2) \Rightarrow f'(x) = -\sin(\ln x^2)\frac{d}{dx}\ln x^2 = -\sin(\ln x^2)\frac{1}{x^2}(2x) = -\frac{2\sin(\ln x^2)}{x}$. Substitute 1 for x to get $f'(1) = -\frac{2\sin(\ln 1^2)}{1} = -2\sin 0 = 0$.

3-6 39. $y = (2x+1)^5(x^4-3)^6 \Rightarrow \ln y = \ln((2x+1)^5(x^4-3)^6)$
$\Rightarrow \ln y = 5\ln(2x+1) + 6\ln(x^4-3) \Rightarrow \frac{1}{y}y' = 5 \cdot \frac{1}{2x+1} \cdot 2 + 6 \cdot \frac{1}{x^4-3} \cdot 4x^3$
$\Rightarrow y' = y(\frac{10}{2x+1} + \frac{24x^3}{x^4-3}) = (2x+1)^5(x^4-3)^6(\frac{10}{2x+1} + \frac{24x^3}{x^4-3})$.

3-6 53.
$f(x) = \ln(x-1) \Rightarrow f'(x) = \frac{1}{x-1} = (x-1)^{-1} \Rightarrow f''(x) = -(x-1)^{-2} \Rightarrow f'''(x) = 2(x-1)^{-3} \Rightarrow$
$f^{(4)} = -2 \cdot 3(x-1)^{-4} \Rightarrow \cdots \Rightarrow f^n(x) = (-1)^{n-1} \cdot 2 \cdot 3 \cdot 4 \cdots (n-1)(x-1)^{-n} = (-1)^{n-1}\frac{(n-1)!}{(x-1)^n}$