## Discrete Mathematics (2010 Spring) Midterm I

1. (40%) For each of the following statements, **determine** and **explain** whether it is correct or not.
   (1). $\phi \subset \phi$
   (2). $\phi \subseteq \{\phi\}$
   (3). $Q^* \cap Z = Z$
   (4). $R^+ \cap C = R^+$
   (5). $[(p \vee (p \wedge q) \vee (p \wedge q \wedge r)] \wedge [(p \wedge r \wedge t) \vee t] \Leftrightarrow p \wedge t$
   (6). $\neg(p \leftrightarrow q) \Leftrightarrow (p \wedge q) \vee (\neg p \wedge \neg q)$
   (7). $A\Delta(B \cap C) = (A\Delta B) \cap (A\Delta C)$
   (8). $(p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology.
   (9). $A = \{2n \mid n \in Z\}$, $B = \{6n \mid n \in Z\}$, *then* $\overline{B} \subseteq \overline{A}$.
   (10). If $n$ is formed by using the digits 3, 3, 4, 5, 5, 6, 7. The number of positive integers $n$ that exceed 4,000,000 is 720.

2. (10%, 2,2,3,3) For the complete expansion of $(x - 2y + 3z^{-1}+4)^4$, determine the following value (a) the coefficient of $yz^{-2}$, (b) the coefficient of $xyz^{-2}$, (c) the number of distinct terms, (d) the sum of all coefficients.

3. (10%) What is the number of integer solutions for $x_1+x_2+x_3 = Z$, if (a) $x_1, x_2, x_3 > 0$, Z=8, (b) $x_1, x_2 > 0, x_3 > 1, Z < 8$.

4. (10%) What is the probability of each summand even in all compositions of 20?

5. (10%) Validate the argument

$$p \wedge q$$
$$p \rightarrow (r \wedge q)$$
$$r \rightarrow (s \vee t)$$
$$\underline{\neg s}$$
$$\therefore t$$

6. (10 points) Prove that for every $n \in Z^+$ *where* $n \geq 14$, n can be written as a sum of 3's and / or 8's.

7. (10 points) One rock-n-roll music CD costs $29 and one classic music CD costs $33. How many CDs of these two kinds you should buy if you pay $500 for them?