HW10 reference answers

7.2

From $y = x^2$, $x = 0, 1, 2, 3$, we obtain $x = \sqrt{y}$,

$$g(y) = f(\sqrt{y}) = \binom{3}{\sqrt{y}} \left(\frac{2}{5}\right)^{\sqrt{y}} \left(\frac{3}{5}\right)^{3-\sqrt{y}}, \qquad \text{for} y = 0, 1, 4, 9.$$

7.17

The moment-generating function of X is

$$M_X(t) = E(e^{tX}) = \frac{1}{k}\sum_{x=1}^{k} e^{tx} = \frac{e^t(1-e^{kt})}{k(1-e^t)},$$

by summing the geometric series of k terms.

7.19

The moment-generating function of a Poisson random variable is

$$M_X(t) = E(e^{tX}) = \sum_{x=0}^{\infty} \frac{e^{tx}e^{-\mu}\mu^x}{x!} = e^{-\mu}\sum_{x=0}^{\infty}\frac{(\mu e^t)^x}{x!} = e^{-\mu}e^{\mu e^t} = e^{\mu(e^t-1)}.$$

So,

$$\mu = M_X'(0) = \mu\, e^{\mu(e^t-1)+t}\Big|_{t=0} = \mu,$$
$$\mu_2' = M_X''(0) = \mu e^{\mu(e^t-1)+t}(\mu e^t + 1)\Big|_{t=0} = \mu(\mu+1),$$

and

$$\sigma^2 = \mu_2' - \mu^2 = \mu(\mu+1) - \mu^2 = \mu.$$