pf:	$F(s) = \int_{p}^{\infty}$	fle).e-stdx -	(1).	· , , , , , , , , , , , , , , , , , , ,
1	J's (11 dS =	$\int_{s}^{\infty} \int_{s}^{\infty} f(x) e^{-sx}$	t dt ds	
	= 5050	(t) e dsdt		

$$= \int_{0}^{\infty} f(x) \left[ -\frac{1}{k} e^{-sx} \right]_{s}^{\infty} dt$$

$$= \int_{0}^{\infty} f(x) \left[ 0 - \left( -\frac{1}{4} e^{-Sx} \right) \right] dx$$

$$= \int_{0}^{\infty} f(x) \cdot \frac{1}{4} e^{-Sx} dx = L \left[ \frac{1}{4} f(x) \right] \cdot x$$

$$\Rightarrow \lfloor \S + \sin t \rbrace = \int_{S}^{\infty} \frac{S}{S+1} dS$$

$$= \tan^{-1} S \Big|_{S}^{\infty} = \frac{TL}{2} - \tan^{-1} S \Big|_{X}^{\infty}$$

Review.  $ex. 3e^{2x} \xrightarrow{L}, \frac{3}{s-2} \qquad H(x) \xrightarrow{L}, \frac{1}{s}$ 

$$\frac{2s}{2\cos 3t} \stackrel{\perp}{\longrightarrow} \frac{2s}{s+9} \qquad \qquad \delta(t) \stackrel{\perp}{\longrightarrow} 1$$

 $3 \text{ sinzt} \xrightarrow{L} \frac{3 \times 2}{S^{2} + 4}$   $\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$   $S^{2}$ 

	novtice
Pro	perues

ex. 
$$e^{2t}\cos^{3t} \rightarrow \frac{S}{S^{2}+9}|_{S^{2}=S^{2}}$$

$$f(t) \to F(s)$$

$$e^{at}f(t) \to F(s-a)$$

$$=\frac{5-2}{(5-2)^2+9}$$

$$ex. F(s) = \frac{2}{s^2+4}e^{-2s}$$

$$f(t) \rightarrow F(s)$$

$$H(t-a)f(t-a) \rightarrow e^{-as}F(s)$$

4. 
$$f(k) \rightarrow F(s)$$

$$tf(t) \rightarrow -\frac{dF(s)}{ds}$$

$$t^2 f(t) \rightarrow -\frac{d}{ds} \left( -\frac{d F(s)}{ds} \right)$$

$$\frac{1}{t^2}f(t) \rightarrow \int_s^\infty \int_s^\infty F(s) ds ds.$$

我們知道 
$$sint \rightarrow \frac{1}{s+1}$$

$$\Rightarrow \frac{1}{t} \sin t \Rightarrow \int_{s}^{\infty} \frac{1}{s^{2}+1} ds = tom^{\frac{1}{s}} \left\{ s = \frac{\tau_{0}}{2} - tom^{\frac{1}{s}} \right\}$$

$$\lim_{s\to 0} \int_{0}^{\infty} \frac{\sin t}{t} e^{-st} dt = \lim_{s\to 0} \left[ \frac{1}{t} \sin t \right] = \lim_{s\to 0} \left( \frac{\tau_{0}}{2} - \tan^{2} s \right)$$

現在,結合性質 4, 5.
$$G(s) = \int_{s}^{s} F(s) ds F(s) \xrightarrow{dE(s)} G(s)$$

$$\downarrow \downarrow \downarrow^{-1} \qquad \downarrow \downarrow \downarrow^{-1}$$

$$g(t) = xt \qquad f(t) \qquad \downarrow t \qquad g(t)$$

$$\Rightarrow \qquad \sum \left\{ \left( \int_{S} \sum \left\{ g(t) \cdot t \right\} \cdot dS \right) \right\} = g(t)$$

ex. 
$$F(s) = \ln \frac{s+1}{s+2}$$
,  $f(t) = ?$ 

$$\frac{dF(s)}{ds} = \frac{d}{ds} \ln \frac{s+1}{s+2} = \frac{s+2}{s+1} \left( \frac{d}{ds} \frac{s+1}{s+2} \right) = \frac{-1}{(s+1)(s+2)}$$

$$= -\left(\frac{a}{s+1} + \frac{b}{s+2}\right) \Rightarrow a = -1, b = 1$$

$$\Rightarrow g(t) \xrightarrow{L} -\left(\frac{-1}{s+1} + \frac{1}{s+z}\right)$$

$$\Rightarrow g(t) = -(-e^{-t}t + e^{-t}) = e^{-t}t - e^{-t}$$

$$\Rightarrow f(t) = \frac{1}{t}(e^{-t}t - e^{-t}) + e^{-t}$$

\*再接下去做簸箅的話.

$$\int_{\infty}^{\infty} \frac{1}{k} (e^{2k} - e^{-k}) dt$$

$$= \int_0^\infty \lim_{s \to 0} \frac{1}{t} \left( e^{-\lambda t} - e^{-\lambda t} \right) e^{-st} dt$$

$$= \lim_{s \to 0} \int_{2}^{\infty} \frac{1}{t} (e^{-2t} - e^{-t}) e^{-st} dt$$

$$\frac{1}{L} \left\{ \frac{1}{t} \left( e^{-t} - e^{-t} \right) \right\}$$

$$= \lim_{s \to 0} \ln \frac{s+1}{s+2} = -\ln 2 = -0.693$$

ex. 
$$F(s) = \frac{2s}{(s^2+4)^2}$$
,  $f(k) = ?$ 

$$\int_{S}^{\infty} F(s) ds = \int_{S^{2}+4}^{\infty} \frac{1}{t^{2}} dt = -\frac{1}{t} |_{S^{2}+4}^{\infty}$$

$$= 0 - \left(-\frac{1}{s^{2}+4}\right) = \frac{1}{s^{2}+4}$$

$$\Rightarrow g(t) = \frac{1}{s^{2}+4} \Rightarrow f(t) = \frac{1}{2}t \sin 2t.$$

$$4. \int_{0}^{\infty} \frac{1}{z} t \sin t dt = 0$$

property 
$$6$$
.
$$f(t) \stackrel{L}{\longrightarrow} F(s)$$

$$ex \cdot 3' + 23 = e^{t}$$
  $3(0) = 0$ 

$$\Rightarrow SY(s) - g(o) + 2Y(s) = \frac{1}{s-1}$$

$$\Rightarrow Y(s) = \frac{-3}{5+2} + \frac{3}{5-1}$$

$$\Rightarrow 3(t) = -\frac{1}{3}e^{-2t} + \frac{1}{3}e^{t}$$

$$pf: L[f(t)] = \int_{0}^{\infty} f(t) e^{-st} dt = \int_{0}^{\infty} e^{-st} df(t)$$

$$= e^{-st} f(x) \Big|_{s=-st}^{\infty} f(x) \left(-se^{-st}\right) dt$$

$$= (o) - f(o) + s \int_{0}^{\infty} f(t) e^{-st} dt = sF(s) - f(o)$$
与在f(t)

為指軟階時才成立

<del></del>	
6.1	L 1 f (t) }
	= [ (f(t)) ]
	= L { g(t) }
	= S L {g(t)} - g(0)
	= s(SFS-f(0)) - f(0) = s2F(5) - sf(0) - f(0) #

6.2. 
$$\lfloor f^{(n)}(k) \rfloor$$
  
=  $s^n F(s) - s^{n-1} f(o) - s^{n-2} f(o) \cdots - f^{(n-1)}(o)$ 

ex. 
$$g'' + 3g' + 2g = e^{x}$$
,  $g(s) = 0$ ,  $g'(s) = 0$ .  
 $[fg''] = S^{2}Y(s) - Sg(s) - g'(s)$   
 $[fg'] = SY(s) - g(s)$   
 $[fg'] = SY(s)$   
 $[fg'] = SY(s$ 

$$a = \frac{1}{6}$$
,  $b = -\frac{1}{2}$ ,  $c = \frac{1}{3}$ .

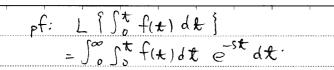
$$\Rightarrow f(t) = \frac{1}{6}e^{t} - \frac{1}{2}e^{-t} + \frac{1}{3}e^{-t} + \frac{1}{3}e^{-t}$$

property 7.
$$f(t) \stackrel{\downarrow}{\Rightarrow} F(s)$$

$$\int_{0}^{t} f(t) dt \stackrel{\downarrow}{\Rightarrow} \frac{1}{s} F(s)$$

$$\int_{0}^{t} 1 dt \stackrel{\downarrow}{\Rightarrow} \frac{1}{s} \frac{1}{s} \frac{1}{s^{2}}$$

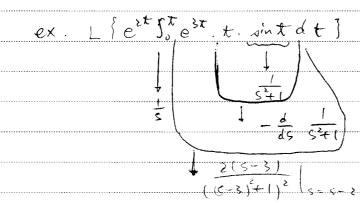
推廣. 
$$\sum_{s \in S} \int_{s \in S} \int_{ks} \int_{ks} \int_{ks} \int_{ks} \int_{s \in S} \int$$



$$= \int_{0}^{\infty} \int_{0}^{t} f(x) dx e^{-st} dt = uv - \int v du$$

$$= \left( \int_{0}^{t} f(x) dx \right) \left( -\frac{1}{3} e^{-st} \right) \left| -\int_{0}^{\infty} -\frac{1}{3} e^{-st} \cdot f(x) \cdot dx \right|$$

= 0-0 + 
$$\frac{1}{5}$$
  $\int f(x) e^{-5x} dx = \frac{1}{5} F(5) + \frac{1}{5} f(5) = \frac{1}{5} f(5) + \frac{1}{5} f(5) = \frac{1}{5} f(5$ 



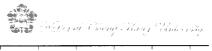
1. 
$$f(t) + g(t) \triangleq \int_{0}^{t} f(\lambda)g(t-\lambda)d\lambda$$

ex. 
$$f(t) = e^{2t}$$
,  $g(t) = t$   

$$f(t) * g(t) = \int_{0}^{t} f(\lambda)g(t-\lambda)d\lambda$$

$$= \int_{0}^{t} e^{2\lambda}(t-\lambda)d\lambda$$

## $f(t)^{*}g(t) \triangleq \int_{0}^{t} f(\lambda)g(t-\lambda)d\lambda$ 令 x= t-ス, dx =-dス $=\int_{x}^{\infty}f(t-x)g(x)-dx$ $= \int_0^x f(x-x) g(x) dx$ $\Rightarrow \int_0^t g(\lambda) \cdot f(t-\lambda) d\lambda = g(t) * f(t)$ ex. 10002 e2(t-2) d2 convolution thm = coxt \* e2\* $f(\star) \rightarrow F(s)$ $g(t) \rightarrow G(s)$ $f(t) * g(t) \rightarrow F(s) \cdot G(s)$ $L\{\int_0^{\pi} e^{2\lambda} (t-\lambda) d\lambda\} = \frac{1}{S-2} \cdot \frac{1}{S^2}$ ex. $f(t) = e^{\lambda t}$ , $g(t) = e^{\lambda t}$ $f(t) * g(t) = \int_{0}^{t} e^{\lambda t} \cdot e^{\lambda(t-\lambda)} d\lambda$ $= e^{\lambda t} \int_{0}^{t} e^{-\lambda} d\lambda$ $= e^{3t} \cdot (-e^{-1})^{t} = e^{3t} \cdot (-e^{-t} - (-1))$ $= e^{3t} - e^{3t}$ $= 1 \cdot 3e^{3t} - e^{2t} = \frac{1}{5-3} - \frac{1}{5-2}$ pf: L { f(t) \* g(t) }. = 50 [f(x) \* g(t)] · e st dt. $= \int_0^\infty \int_0^x f(x) q(x - x) dx e^{-st} dt$ = $\int_0^\infty \int_0^x f(\eta) g(x-\eta) e^{-st} dt d\eta$



若想対調 da. dt. 则横分区域 客相同.
$=\int_{0}^{\infty}\int_{\lambda}^{\infty}f(\lambda)g(\lambda-\lambda)e^{-st}d\lambda d\lambda$
$= \int_{0}^{\infty} f(\lambda) \int_{0}^{\infty} g(t-\lambda)e^{-st}dt d\lambda$
$\xi x = x - \lambda$ , $dx = dx$
$= \int_{0}^{\infty} f(\lambda) \int_{0}^{\infty} g(x) e^{-s(x+\lambda)} dx d\lambda.$
= for f(x) e sx for g(x) e sx dx, dx.
= \int f(\lambda) e \int \int g(x) e \int dx, dx.  L. G(s)
= G(5) 5° f(2) e-57 d2 = G(5) F(5) #
2 0(1) 7 (1)