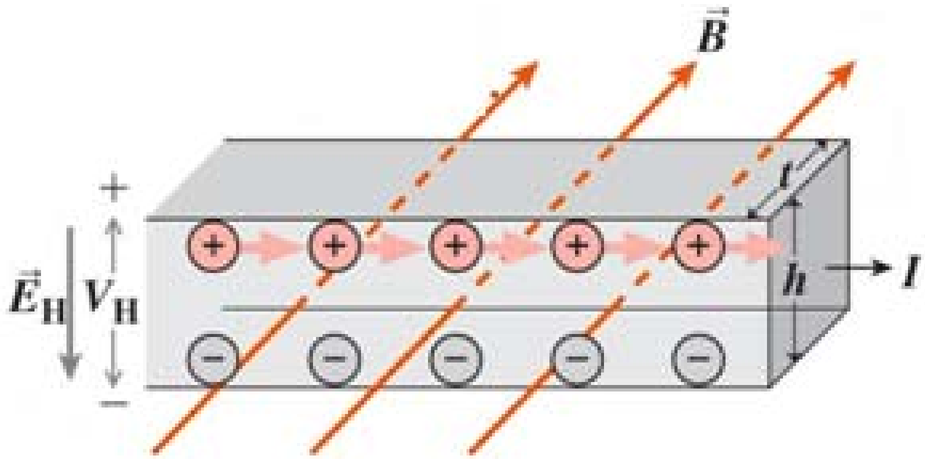
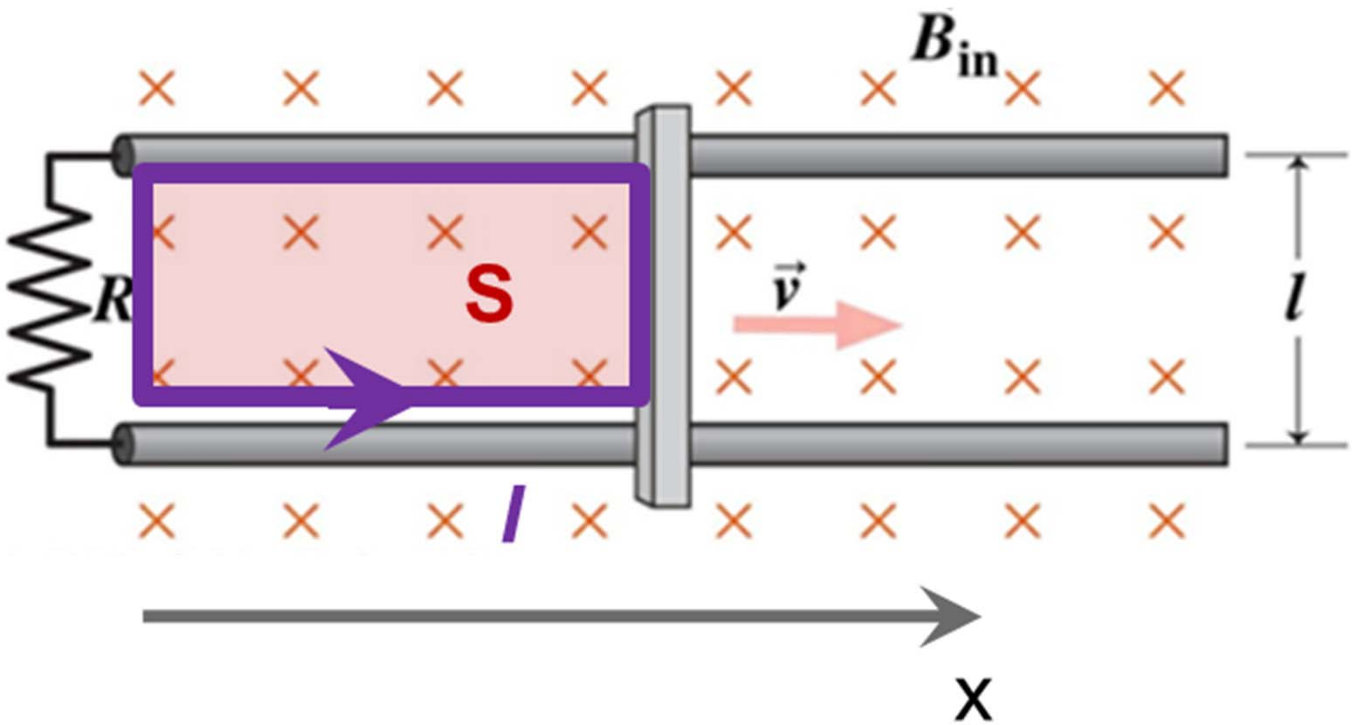
Magnetic Induction

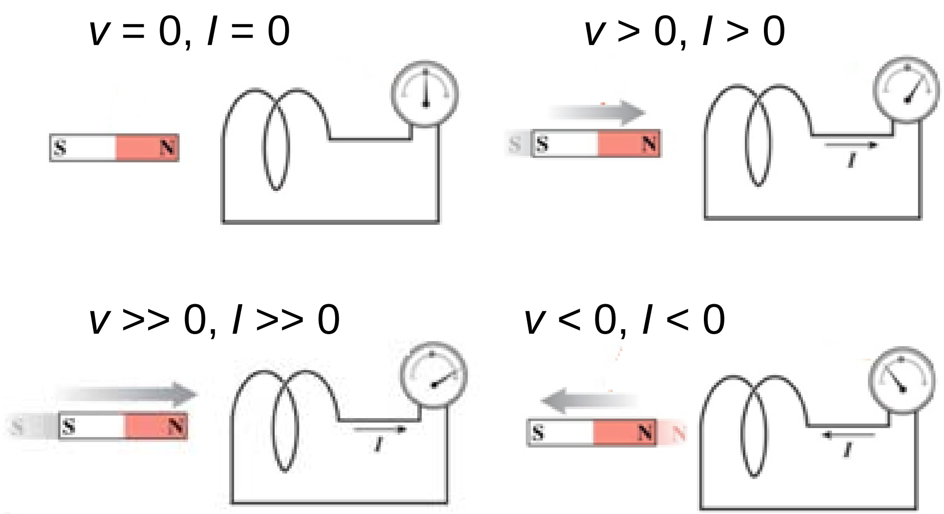
Electromagnetic induction



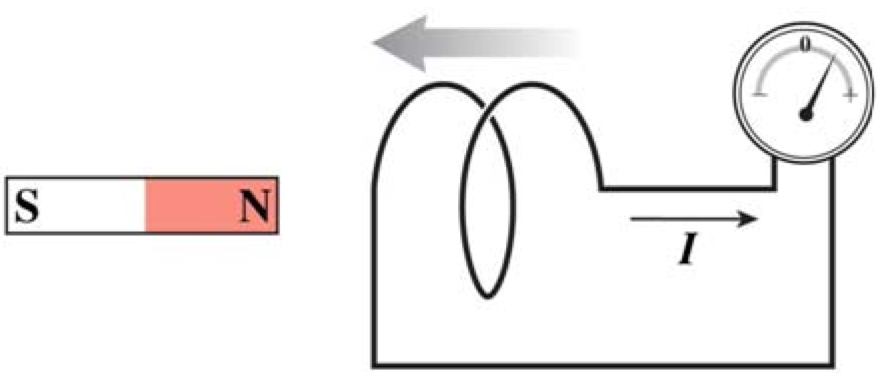


Electromagnetic induction

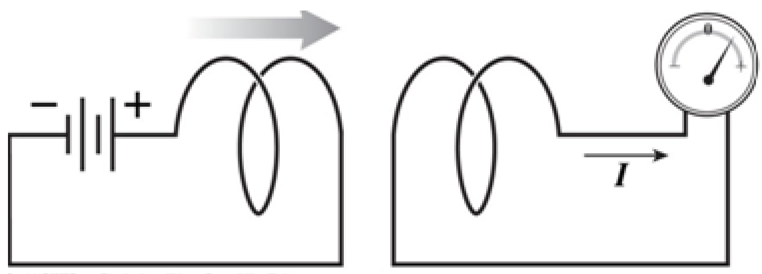
4 results from Faraday / Henry (1831)



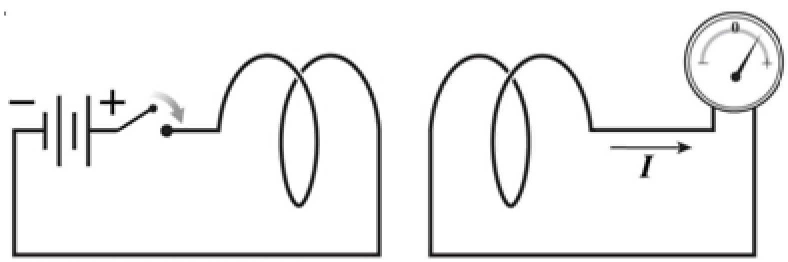
1. Current induced in coil by moving magnet bar.



2. Moving the coil instead of the magnet gives the same result.



3. An induced current also results when a current-carrying circuit replaces the magnet.

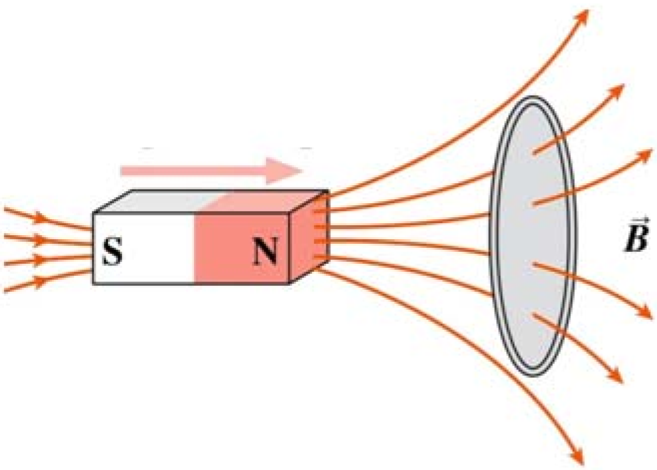


4. A current is also induced when the current in an adjacent circuit changes.

→ changing **B** induces currents (electromagnetic induction)

Magnetic flux

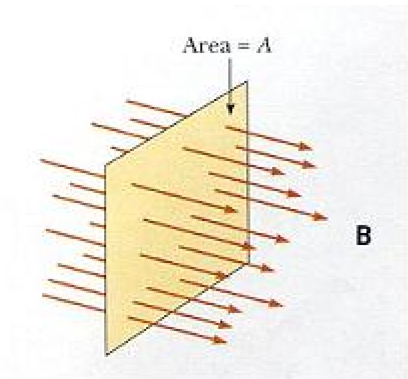
Magnetic flux: $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$



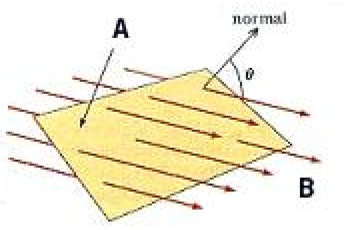
Move magnet right
$\rightarrow$ more lines thru loop

For a uniform **B** on a flat surface:

$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$$



Φ = BA



Φ = BA cos θ

Flux & Induced EMF

Faraday's law of induction:

The induced emf in a circuit is proportional to the rate of change of magnetic flux through any surface bounded by that circuit.

$$\mathcal{E} = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{A}$$

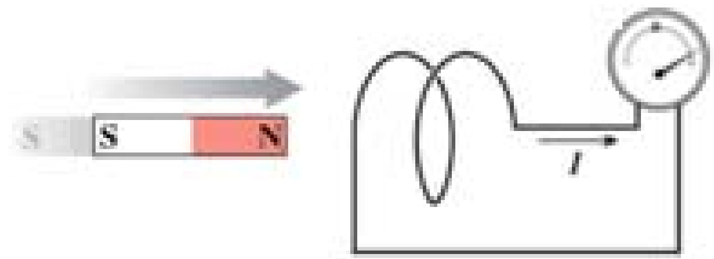
C is CCW about **S**.

Note: $d\Phi_B/dt$ can be due to

- changing **B** caused by
  - relative motion between circuit & magnet,
  - changing current in adjacent circuit,
- changing area of circuit,
- changing orientation between **B** & circuit.

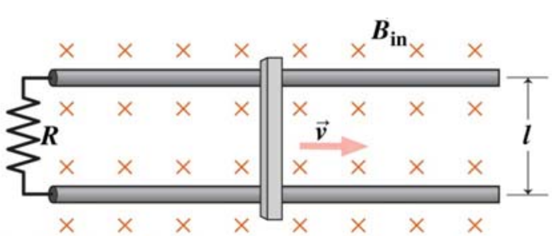
How to generate Induced Electrical Motive

- As long as the magnetic flux change or any of the charged carriers experience change of the magnetic field



- By changing the field strength

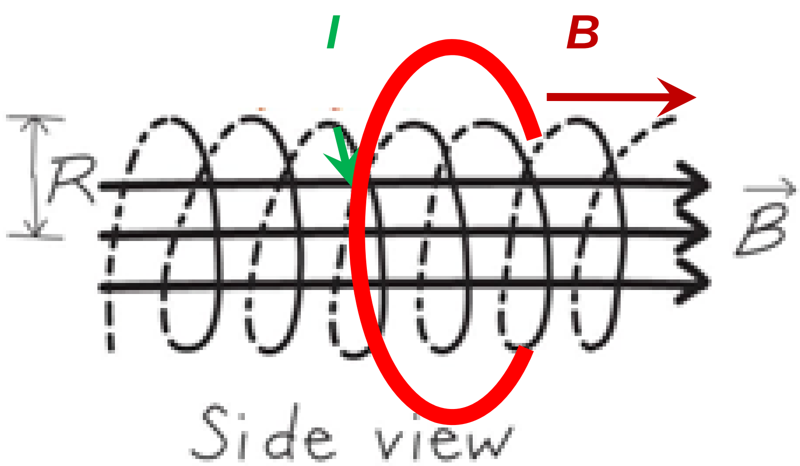
- By changing the surface for induction to generate induced field

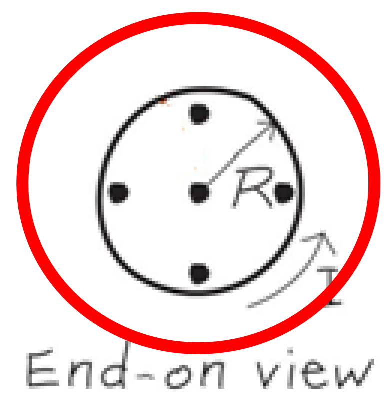


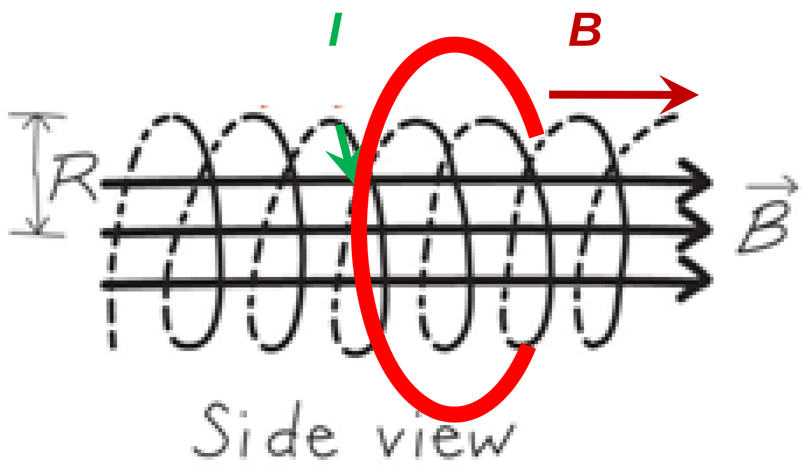
- By changing a certain part of the conducted loop for the induced field

Solenoid

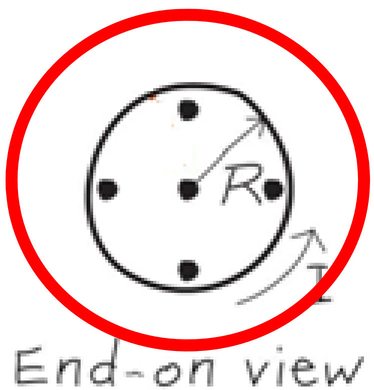
An infinite solenoid has 10 turns/cm and a radius of 2cm. A flat circular coil of radius 4cm and 15 turns is placed around the solenoid with its plane perpendicular to the axis of the solenoid. If the current in the solenoid drops steadily from 3A to 2A in 0.05s, what is the emf induced in the coil.







I
B
R
B
Side view

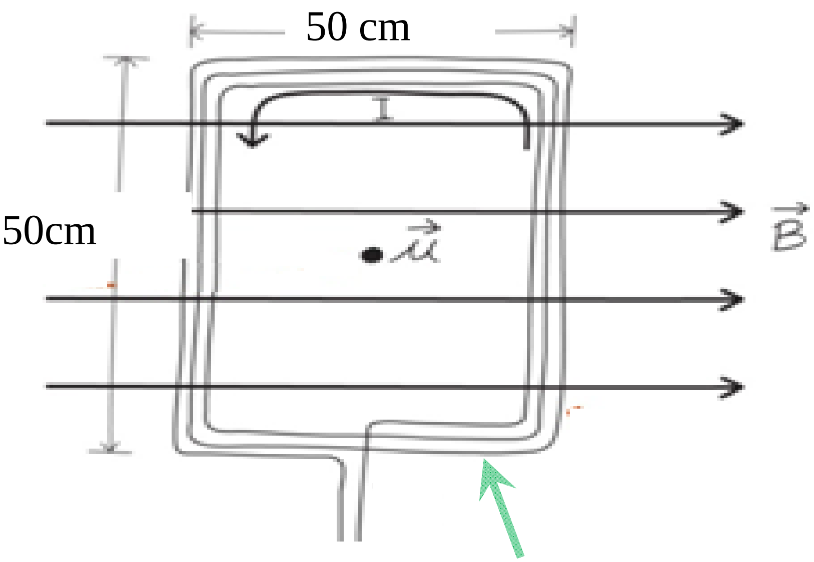


R
I
End-on view

Designing a Generator

A square coil has 25 turns and sides of length 50cm. It rotates at 120 rpm in a field of 400G. At t = 0, the plance of the coil is normal to the lines. Find: (a) the peak value of the emf; (b) the emf at t = 1/24 s

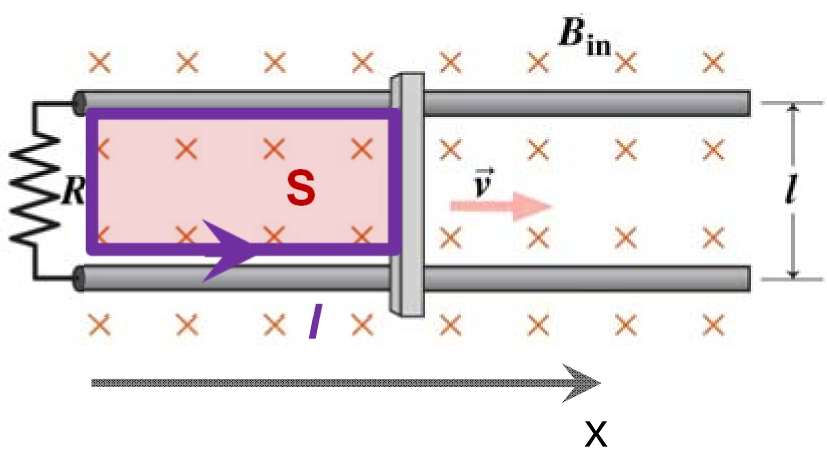
Loop
rotation



25turn coil

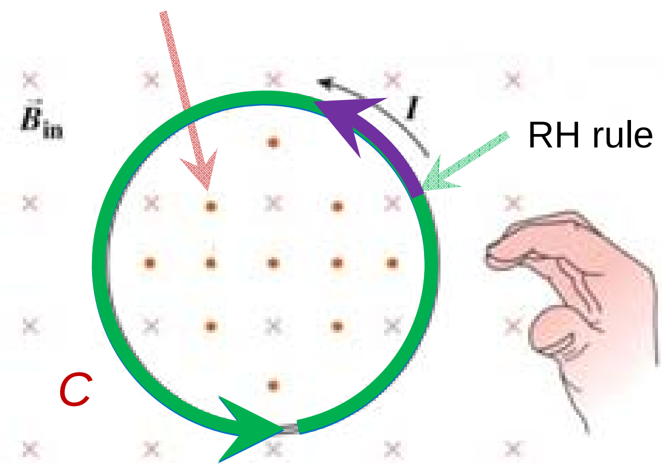
Changing Area

A metal rod of length l slides at constant velocity v on conduction rails that terminate in a resister R. There is a uniform and constant magnetic field perpendicular to the plane of the rails. Find (a) the current in the resistor, (b) the power dissipated to the resistor, (c) the mechanical power needed to pull the rod.

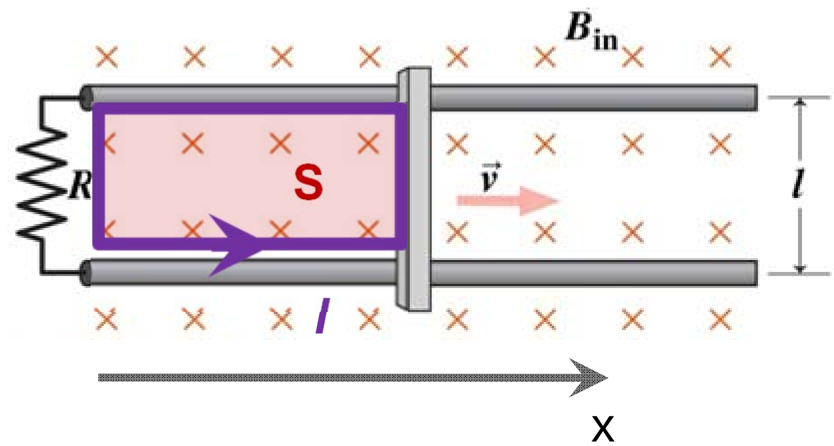


The origin of the induced EMF

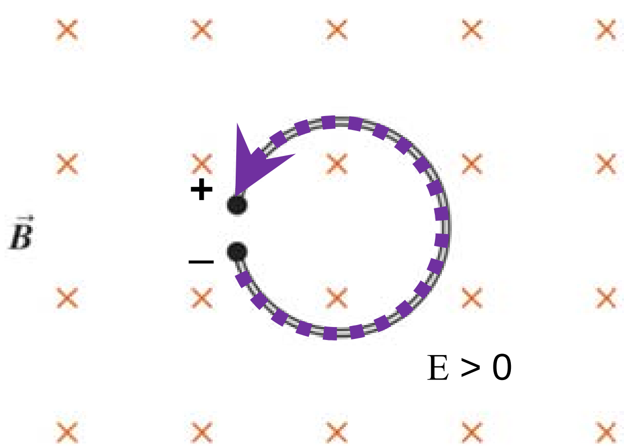
B of induced *I* points out of page



RH rule gives CCW *I*



$B_{in} \uparrow$



$B \uparrow$

$$F = q\left(E + \vec{v} \times \vec{B}\right)$$

$$EMF = \oint \vec{E} + \vec{v} \times \vec{B}\, d\,\vec{l}$$

Induced EMF, Induced electric field

- Macroscopic Point of View:
  Change of magnetic flux induces “Induced Electrical Field”

- The “Induced Electrical Field” is not a static “ELECTRIC FIELD”.
  - The induced field has no reason to follow the rules of electrostatics
  - The induced field is not a conservative field

- Microscopic Point of View:
  Charged carriers that experience relative “Change” of the magnetic field will experience the “Induced Electrical Field”
  - This change includes motion of the carrier and change of the B field.

Flux & Induced EMF

Faraday's law of induction:

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- changing area of circuit,
- changing orientation between **B** & circuit.

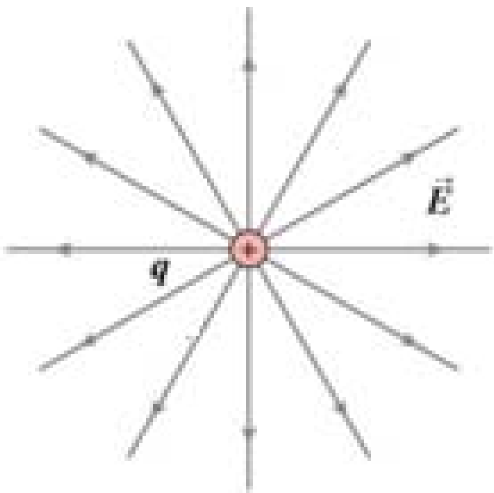
Induced Electric Fields

EMF acts to separate charges:

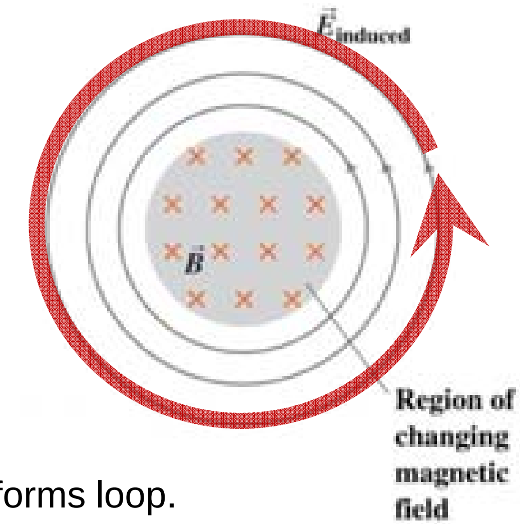
Battery:	chemical reaction
Motional emf:	**F** = **v** × **B**.
Stationary loop in changing **B** :	induced **E**

$$\oint_S \vec{E} \cdot d\vec{r} = -\frac{d}{dt}\Phi_B \quad = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{A}$$

Faraday's law

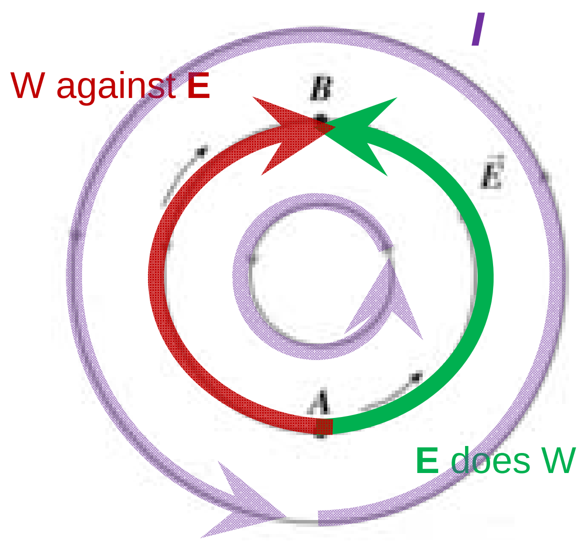


Static **E** begins / ends on charge.



Induced **E** forms loop.

Conservative & Nonconservative Electric



For stationary charges (electrostatics) :

$$\boxed{\oint_S \vec{E}\cdot d\vec{r} = 0}$$

E is conservative

Induced fields (electromagnetics) :

$$\oint_S \vec{E}\cdot d\vec{r} = -\frac{d}{dt}\Phi_B$$

E is non-conservative

Induced Electric field from a Solenoid current distribution

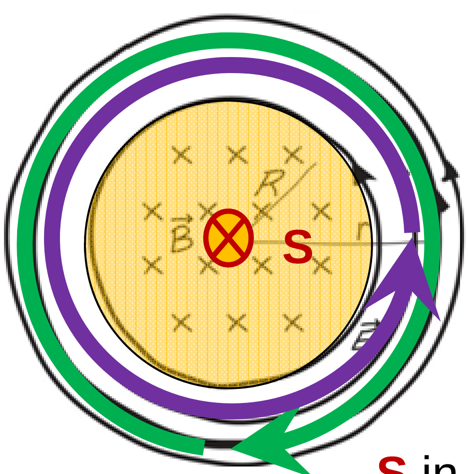
A long solenoid has circular cross section of radius *R*.

The solenoid current is increasing, & as a result so is **B** in solenoid.

The field strength is given by *B* = *b t*, where *b* is a constant.

Find the induced **E** (a) Inside and (b) outside the solenoid, a distance *r* from the axis.

Loop for Faraday's law



S in $\rightarrow$ C cw

Symmetry $\rightarrow$ **E** lines are circles.

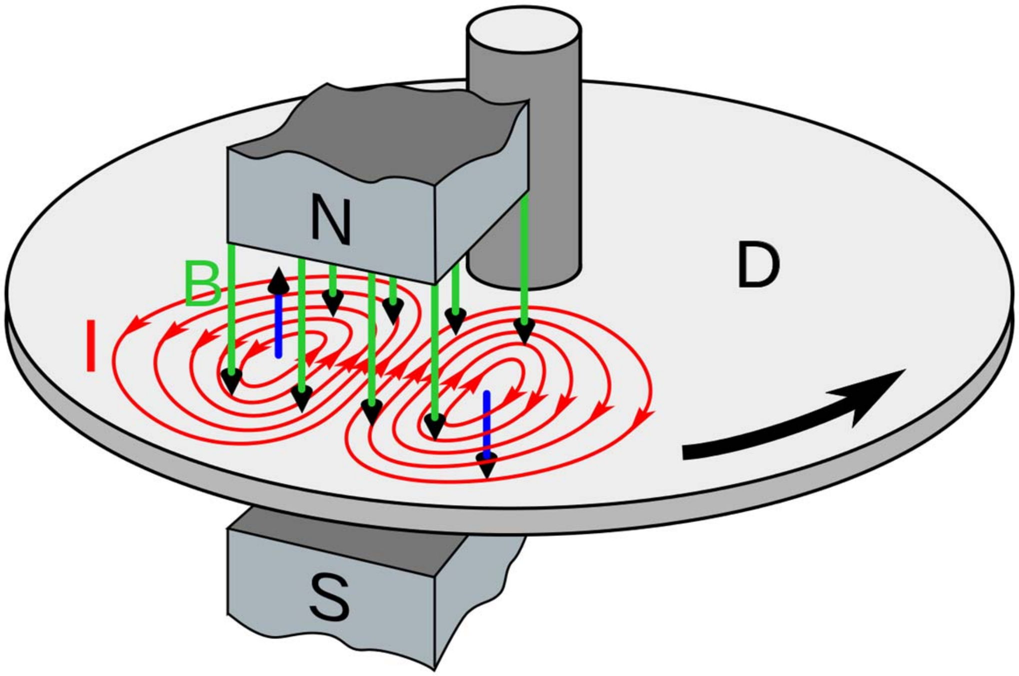
$$\oint_S \vec{E} \cdot d\vec{r} = 2\pi r \cdot E$$

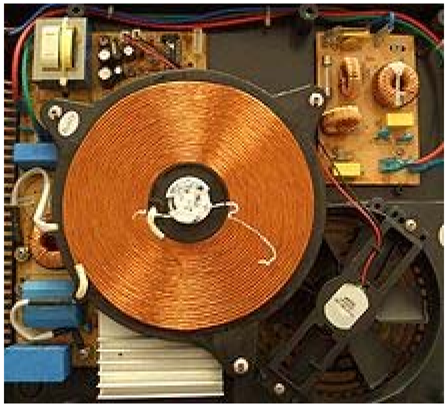
$$= -\frac{d\,\Phi_B}{d\,t} = -\frac{d}{dt}\left(b\,t\,\pi\,R^2\right) = -b\,\pi\,R^2$$

$$E = -\frac{b\,R^2}{2\,r}$$

CCW

Eddy Current





Wtshymanski at en.wikipedia

Law of Electro-Magnetic in vacuum

$$Gauss's\ Law: \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \quad A\ fundamental\ Law\ for\ Electrostatics$$

$$Gauss's\ Law: \oint \vec{B} \cdot d\vec{A} = 0 \quad A\ fundamental\ Law\ for\ Magnetostatics$$

$$Ampère's\ Law: \oint_S \vec{B} \cdot d\vec{r} = \mu_0 I_{enc} = \mu_0 \int_{area\ enclosed\ by\ S} \vec{J} \cdot d\vec{A}$$

$$A\ fundamental\ Law\ for\ Magnetostatics\ @\ steady\ current$$

$$Faraday's\ Law: \oint_S \vec{E} \cdot d\vec{r} = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}\int_{area\ enclosed\ by\ S} \vec{B}(t) \cdot d\vec{A}(t)$$

$$A\ fundamental\ Law\ for\ Magnetodynamics$$

Law of Electro-Magnetic in vacuum

$$Gauss's\ Law: \oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \quad Charge\ is\ the\ source\ of\ the\ electric\ field$$

$$Gauss's\ Law: \oint \vec{B}\cdot d\vec{A} = 0 \quad There\ is\ no\ magnetic\ monopole$$

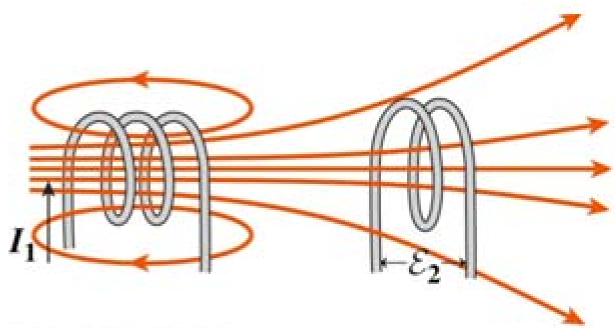
$$Ampère's\ Law: \oint_S \vec{B}\cdot d\vec{r} = \mu_0 I_{enc} = \mu_0 \int_{area\ enclosed\ by\ S} \vec{J}\cdot d\vec{A}$$

$$The\ source\ of\ magnetic\ field\ comes\ from\ electric\ current?!$$

$$Faraday's\ Law: \oint_S \vec{E}\cdot d\vec{r} = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}\int_{area\ enclosed\ by\ S} \vec{B}(t)\cdot d\vec{A}(t)$$

$$Change\ in\ magnetic\ field\ comes\ with\ a\ non-conservative\ induced\ electric\ field$$

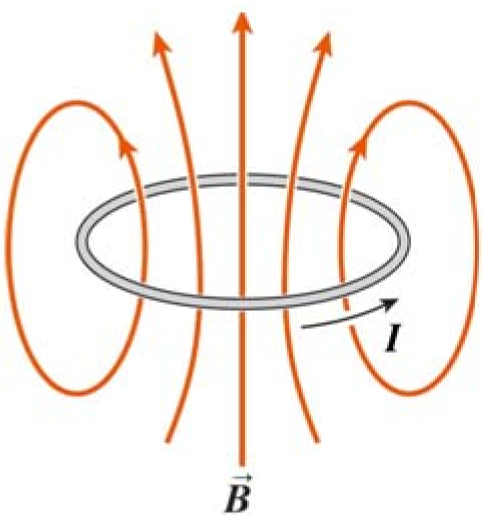
Inductance



Mutual Inductance:
Changing current in one circuit induces an emf in the other.

Large inductance:
two coils are wound on same iron core.

Applications:
Transformers, ignition coil, battery chargers, …



Self-Inductance:
Changing current induces emf in own circuit & opposes further changes.

Applications:
Inductors → frequency generator / detector …

$$\Phi_B = L_{self}\ I$$

[L] = T · m^2 / A = Henry

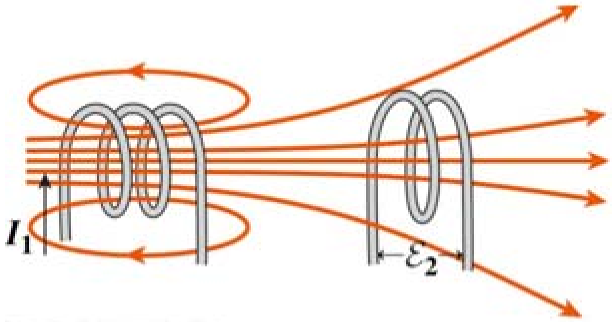
Inductance

$$\Phi_B = \vec{B} \cdot \vec{A} = |B||A|cos\theta$$

For a single coil

$$\Phi_{total} = N\Phi_B = N\vec{B} \cdot \vec{A}$$

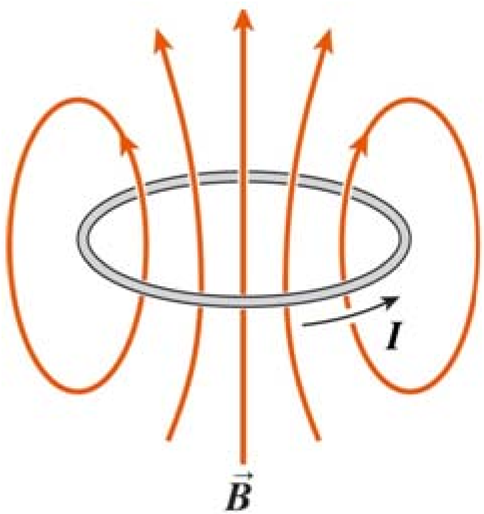
For a coil series or a solenoid



$$\Phi_{coil2} = N_{coil2}(\vec{B}_1(@coil2) + \vec{B}_2) \cdot \vec{A}$$
$$= \Phi_{21} + \Phi_{22}$$

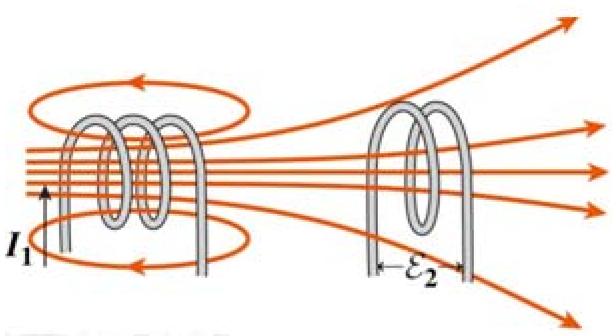
M flux through coil2 from coil 1

$$\Phi_{21} \propto I_1$$



$$\Phi = N\vec{B} \cdot \vec{A} \propto I$$

Inductance

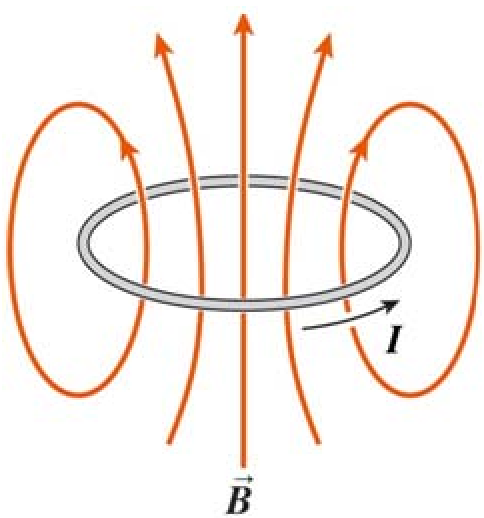


M flux through coil2 from coil 1

$$\Phi_{21} \propto I_1$$

Mutual Inductance:

$$\mathrm{M}_{21} \equiv \frac{\Phi_{21}}{\mathrm{I}_1}$$

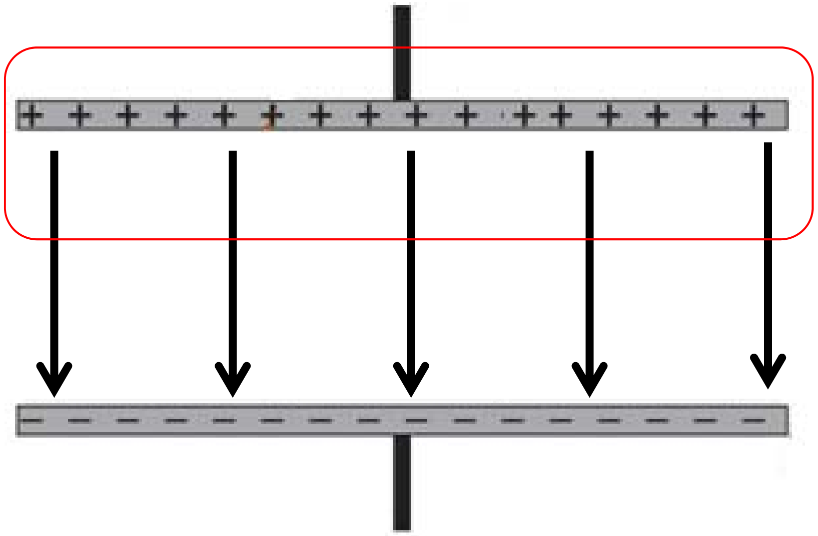


$$\Phi = N\vec{B} \cdot \vec{A} \propto I$$

Self Inductance:

$$\mathrm{L} \equiv \frac{\Phi}{I}$$

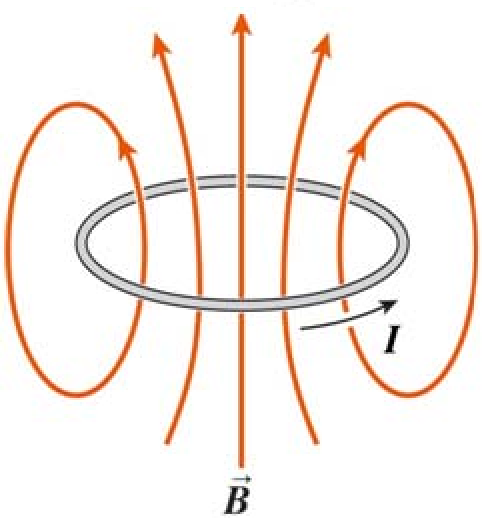
Inductor and Capacitor



$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \epsilon Q \propto V$$

capacitance:

$$C \equiv \frac{Q}{V}$$



$$\Phi = N\vec{B} \cdot \vec{A} \propto I$$

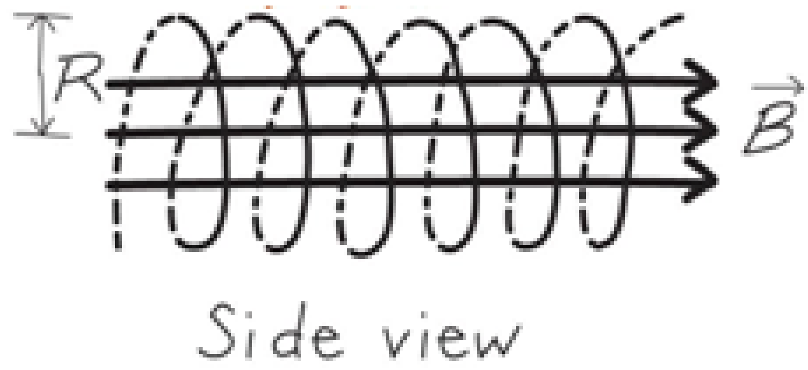
Self Inductance:

$$L \equiv \frac{\Phi}{I}$$

Inductance of a solenoid

A long solenoid of cross section area *A* and length *l* has *n* turns per unit length. Find its self-inductance.

B of solenoid:



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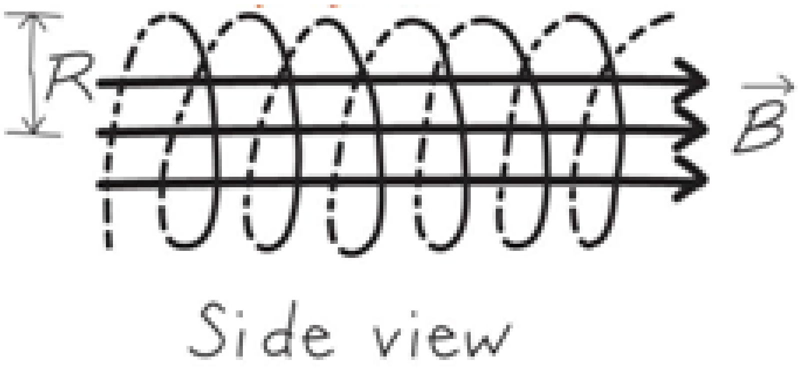


B of solenoid: $B = \mu_0\, n\, I$

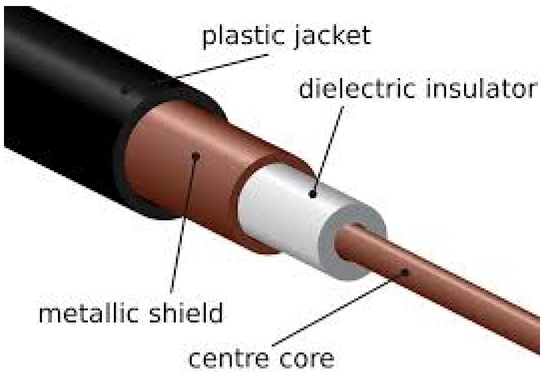
$$\Phi_{1\,turn} = BA = \mu_0\, n\, I\, A$$

$$\Phi = n\, l\, \Phi_{1\,turn} = \mu_0\, n^2 l\, I\, A$$

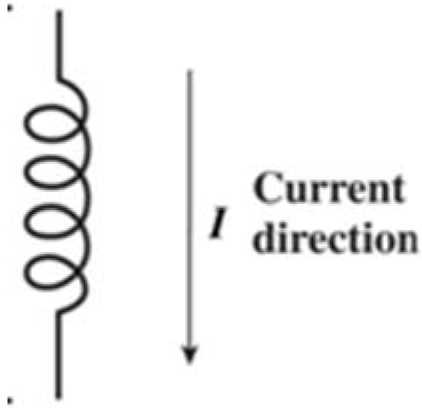
$$L = \frac{\Phi}{I} = \mu_0\, n^2 l\, A$$

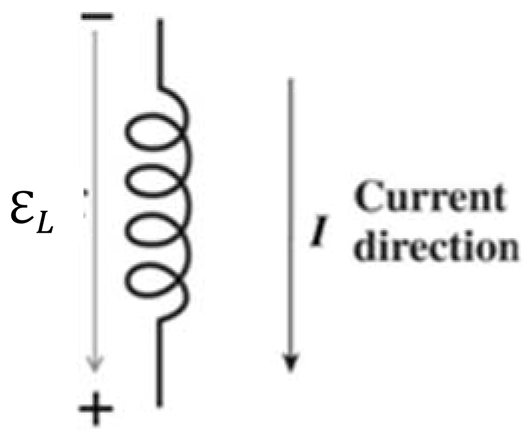
Inductance of a coaxial cable

A Coaxial Cable is often used to carry electrical signals. A cable consisting of an inner wire of radius a that carries current I and an outer cylindrical conductor b carries the same current flowing to the opposite direction. Find the self-inductance of this type of coaxial cable of length l. Ignore the effect of the dielectric insulator and the magnetic flux within the inner wire.



If I is a constant current





d I /d t < 0

$$\Phi_B = L\ I$$

$$-V_L = \mathcal{E}_L = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

back emf

Rapid switching of inductive devices
can destroy delicate electronic devices.

Magnetic Energy in an Inductor

$$\mathcal{E}_0 - V_R + \mathcal{E}_L = 0$$

RL circuit: $\mathcal{E}_0 - IR + \mathcal{E}_L = 0 \quad \rightarrow \quad I\mathcal{E}_0 - I^2R + I\mathcal{E}_L = 0$

$$I\mathcal{E}_0 - I^2R - IL\frac{dI}{dt} = 0$$

$$P_L = L\,I\,\frac{dI}{dt}$$

Power from battery

Power dissipated

Power taken by inductor

Energy stored in inductor:

$$U = \int P\,dt = \int L\,I\,\frac{dI}{dt}\,dt = \frac{1}{2}L\,I^2 \qquad U(I=0) = 0$$

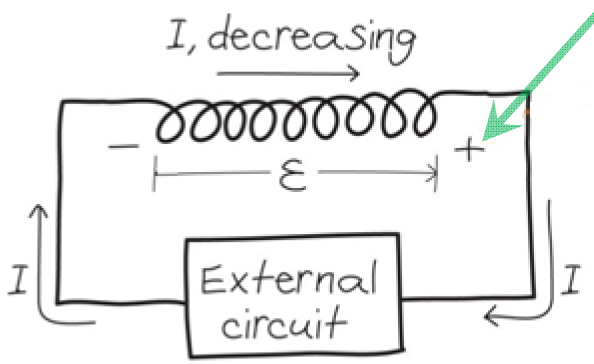
Example Dangerous Inductor

A 5.0-A current is flowing in a 2.0-H inductor.

The current is then reduced steadily to zero over 1.0 ms.

Find the magnitude & direction of the inductor emf during this time.

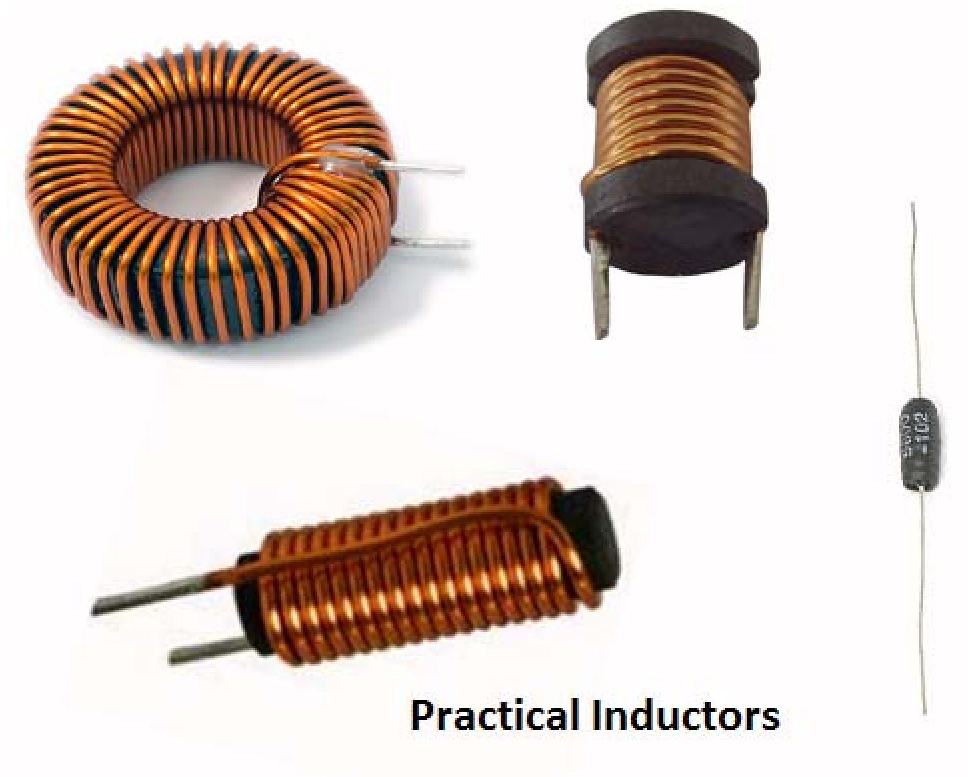
+ here ($\mathcal{E} > 0$) helps keep *I* flowing



$$\mathcal{E} = -L\frac{dI}{dt} > 0$$

$$\mathcal{E} = -(2.0\,H)\left(-\frac{5.0\,A}{1.0\times 10^{-3}s}\right)$$

$$= 10000\,V$$



Practical Inductors

Magnetic Energy Density

Solenoid with length *l* & cross-section area *A* : $L = \mu_0 n^2 A l$ (Eg. 27.6)

$$U = \frac{1}{2} L I^2 = \frac{1}{2} \mu_0 n^2 A l I^2 = \frac{1}{2 \mu_0} B^2 A l \qquad B = \mu_0 n I$$

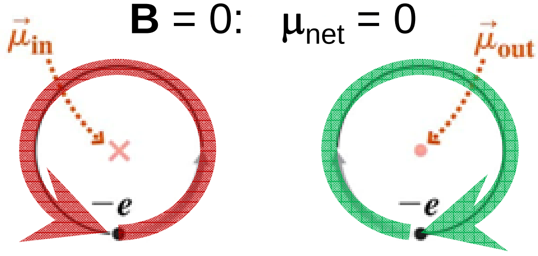
Magnetic Energy Density : $\boxed{u_B = \frac{1}{2 \mu_0} B^2}$

c.f. electric energy density : $u_E = \frac{1}{2} \varepsilon_0 E^2$

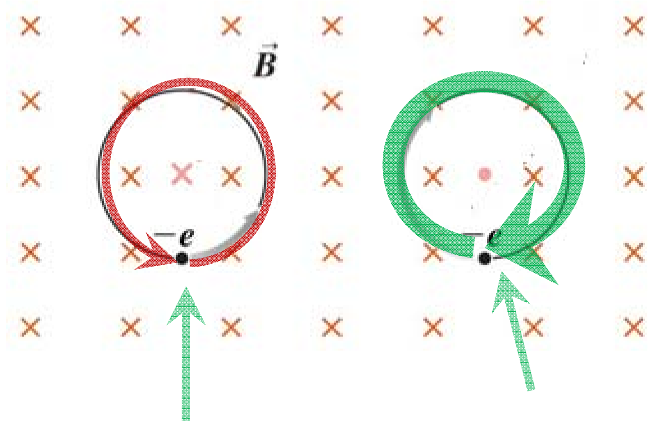
Diamagnetism

Classical model of diamagnetism (not quite right)

B = 0: $\mu_{net} = 0$



B $\neq$ 0 $\mu_{net} \neq 0$



This e slows down. This e speeds up.

Superconductor is a perfect diamagnet (Meissner effect).