

# Homework 3

## Due Date

2018/06/13 (Wednesday), 23:59 • Late submission will not be accepted.

In this assignment, you should be able to accomplish an algorithm to solve differential equation numerically with specified initial conditions. Please be advised that no grade will be given if any MATLAB's embedded function for solving differential equation is found.

Create a matlab script and change the filename to F7xxxxxxx\_hw3.m. Link all the programs solving following problems to this script. Make sure once type the filename 'F7xxxxxxx\_hw', the required results of the following problems will pop-up automatically in order. Remember not to type any 'clear all', 'close all' command in any of the codes.

Create a PDF file named "F7xxxxxxx\_hw3.pdf" for the written report.

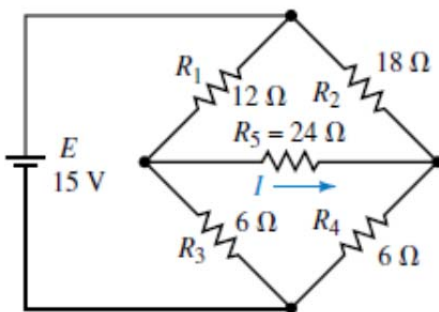
## Problem1: Circuit analysis

[F<your student id >\_hw3\_prob1.m ]

Given the following circuit and assume that  $I_j$  is the current in the resistor  $R_j$

- (1) Write down the equations of the circuit analysis
- (2) Solve the equation to find  $I_5$  using Gauss elimination or Jacobi Method. Specify which method is used to solve this problem. If Jacobi method is used, the answer should be presented with proper precision.

[Print on the console: " Problem 1: I5 = x.xxx A"]



## Problem2: Numerical Solution for a Damped Harmonic Oscillator

[ F<your student id >\_hw3\_prob2.m ]

A shock absorber consists of a piston oscillating in a chamber filled with oil, which can be modelled as the following system. Suppose the mass of the piston is  $1.200 \times 10^3 \text{ kg}$  mounted on a spring of constant  $k = 58.00 \times 10^3 \text{ N/m}$ . The piston is submerged into the oil chamber. The dynamics of the damped harmonic oscillator can be described as:

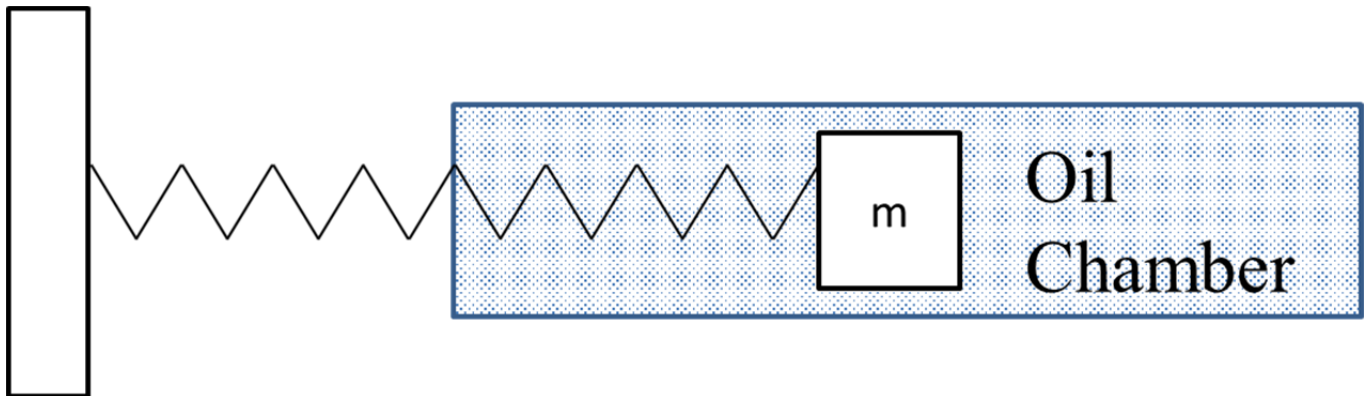
$$m \frac{d^2}{dt^2} \vec{x}(t) + \beta \frac{d}{dt} \vec{x}(t) + k \vec{x}(t) = \vec{F}(t)$$

This equation is a general form where  $F(t)$  typically represents additional external forces.

The damping constant ( $\beta$ ) for the oil is  $4.000 \times 10^3 \text{ kg/s}$ . Let  $x = 0$  refer to the equilibrium location of the brick and ignore buoyancy.

- (1) [Fig.1] At  $t = 0$ , the system is at rest and is compressed by 20.00 cm from its equilibrium. Plot the temporal change of brick's displacement from the equilibrium.
- (2) Based on the result of (1), find the oscillation frequency of the system.

[Print on the console: " Problem 2: The oscillation frequency is x.xxx Hz"]



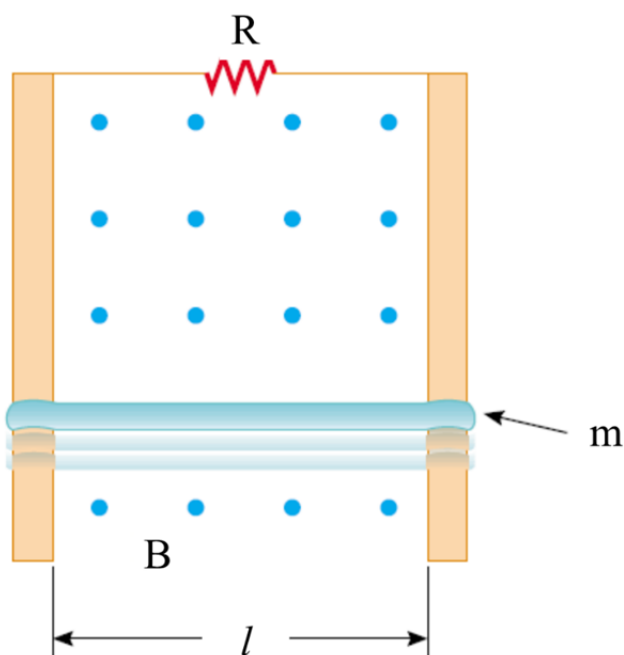
### Problem 3: falling metal rod in a magnetic field

[F<your student id >\_hw3\_prob3.m ]

A horizontal wire is free to slide on the vertical rails of a conducting frame, as shown in the figure. The wire has mass  $m = 0.5\text{kg}$  and length  $l = 0.5\text{m}$ , and the resistance of the circuit is  $R = 1\Omega$ . If uniform magnetic field of  $1\text{T}$  is directed perpendicular to the frame and the wire falls under the force of gravity

- Write down the equation of motion in terms of  $v(t)$ ,  $R$ ,  $m$ ,  $B$ ,  $g$ , and  $l$ .
- Assume that  $v(0) = 0$  and the rail is very long, solve the equation of motion for  $v(t)$  and plot  $v(t)$  until it reaches around 99% of the terminal speed.
- What is the terminal speed? Explain how to estimate the terminal speed from the solved  $v(t)$ .

[Print on the console: “ Problem 3: The terminal speed is x.xxx m/s”]

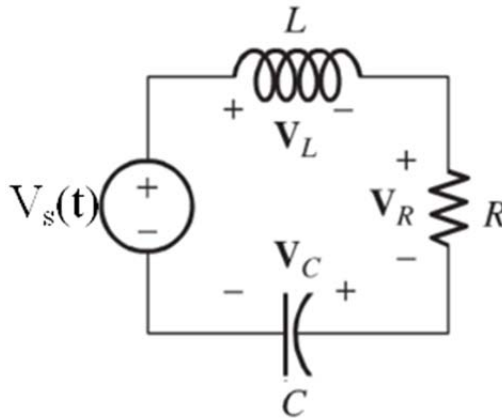


**Problem4: an RLC circuit with a source to look for resonant frequency.**

[ Principal m-file name: F<your student id>\_hw3\_prob4.m ]

In the following figure, there is an RLC circuit with elements in series. Assume that the voltage provided is a function of time  $V_s(t)$  with  $L = 2mH$ ,  $R = 8\Omega$ , and  $C = 5\mu F$ . Before the voltage source is turned on, there is no charge stored in the capacitor and there is no current flowing on the circuit.

- (a) Show that the current of the loop,  $I(t)$ , can be described by  $\frac{d}{dt}V_s(t) = L \frac{d^2}{dt^2}I(t) + R \frac{d}{dt}I(t) + \frac{1}{C} I(t)$
- (b) [Fig. 5] If  $V_s(t) = \cos(6000 t)$ , solve the differential equation numerically and plot  $V_R(t)$  and  $V_s(t)$  on the same figure, for  $0 \leq t \leq 10ms$
- (c) [Fig. 6] If  $V_s(t) = \cos(10000 t)$ , solve the differential equation numerically and plot  $V_R(t)$  and  $V_s(t)$  on the same figure, for  $0 \leq t \leq 10ms$
- (d) [Fig. 7] If  $V_s(t) = \cos(20000 t)$ , solve the differential equation numerically and plot  $V_R(t)$  and  $V_s(t)$  on the same figure, for  $0 \leq t \leq 10ms$



**Contents to submit: (IMPORTANT)**

1. All the m-files you compose for the assignment.
2. All the m-files should include proper COMMENTS.

(No comment, no score)

3. The PDF file must contain Your Name, Your Student ID Number.

In the document, you will need to include all the answers including those shown in the console, plots, and the following contents: (1) Which method do you use to solve the differential problems? (2) The step length you choose for each problem. (3) Which coordinate do you use to analyze the system ( Which direction is the “POSITIVE” direction )? (4) How long does it take roughly for the matlab to finish all problems after executing the main script?

**Notice:**

1. No point will be given if the execution time is longer than 30 minutes or if the code requires more than 2.0 Giga Bytes memory space.
2. DO NOT PLAGIARIZE. You are encouraged to ask and to discuss the homework content with your fellow classmates, the TAs and the instructor. But identical core program wording is NEVER ACCEPTABLE.
3. Upload all the files without archiving (No zip, No rar, No tar, No 7zip). Do not upload files that don't work well. Any missing file or function that leads to execution failure will be regarded as a program that never works.