Engineering Mathematics

Quiz 2, Fall 2014/10/20

請詳細列出計算過程，如用到公式,請列出公式的通式。請記得在答案卷上簽名。

A(1)B(3) Solve the given initial-value problem and give the largest interval I over which the solution is defined.

$$y'+(\cot x)y=\csc x \quad y(\frac{\pi}{2})=1$$

Ans:

$$\text{A solution of the initial-value problem is } y=\frac{x+1-\frac{\pi}{2}}{\sin x} \text{ for interval } I:\ 0<x<\pi$$

A(2)B(4) Solve the given differential equation by undetermined coefficients

$$y''-6y'+9y=6x^2+2-12e^{3x}$$

Ans:

$$\therefore \text{the general solution } y=y_h+y_{p_1}+y_{p_2}=c_1e^{3x}+c_2xe^{3x}+\frac{2}{3}x^2+\frac{8}{9}x+\frac{2}{3}-6x^2e^{3x}$$

A(3)B(1) Determine whether the given differential equation is exact. If it is exact, solve it

(1) $(1-\frac{3}{y}+x)\frac{dy}{dx}+y=\frac{3}{x}-1$

(2) $(\sin y-y\sin x)dx+(\cos x+x\cos y-y)dy=0$

Ans:

(1) $(1-\frac{3}{y}+x)\frac{dy}{dx}+y=\frac{3}{x}-1$

$u(x,y)=x+y+xy-3\ln|xy|=c$

(2) $(\sin y-y\sin x)dx+(\cos x+x\cos y-y)dy=0$

$u(x,y)=y\cos x+x\sin y-\frac{1}{2}y^2=c$

A(4)B(2) Solve the given differential equation by separation of variables

$\sin 3x dx+2y\cos^3 3x dy=0$

Ans:

$y^2=-\frac{1}{6}\sec^2 3x+c$