Algorithms Quiz 2013/12/25

1.(20%) (1) (10%) The matrix-chain multiplication problem can be stated as follows: Given a chain $\langle A_1, A_2, \ldots, A_n \rangle$ of n matrices, where for $i=1,2,\ldots,n$, matrix A_i has dimension $p_{i-1} \times p_i$, fully parenthesize the product $A_1 A_2 \cdots A_n$ in a way that minimizes the number of scalar multiplications. Give a dynamic-programming algorithm to solve the problem and analyze its time complexity.

(2) (10%) Suppose that you have 6 matrices: A_1 has dimension 30×35, A_2 has dimension 35×15, A_3 has dimension 15×5, A_4 has dimension 5×10, A_5 has dimension 10×20, A_6 has dimension 20×25. Please use your algorithm to calculate the minimum number of scalar multiplications.

2.(10%) Describe two key ingredients that an optimization problem must have in order for dynamic-programming to be applicable.

3.(20%) (1) (10%) Determine which one of the **unweighted shortest path problem** and the **unweighted longest simple path problem** does not satisfy the optimal substructure? (2) 10%Give an example to explain that.

4.(20%) Give an $O(n^2)$-time algorithm to find the longest monotonically increasing subsequence of a sequence of n numbers.

5.(20%) (1) (10%) Determine which one of the **0-1 knapsack problem** and the **fractional knapsack problem** cannot be solved using the greedy strategy? (2) (10%) Give an example to explain that.

6.(10%) Determine an LCS of ⟨1,0,0,1,0,1,0,1⟩ and ⟨0,1,0,1,1,0,1,1,0⟩.