Discrete Mathematics (2009 Spring) Midterm I

1. (25%) For each of the following statements, **determine** and **explain** whether it is correct or not.
   (1). $\phi \subset \phi$
   (2). $\phi \subseteq \{\phi\}$
   (3). $\neg[(p \wedge q) \rightarrow r] \Leftrightarrow (p \wedge q) \vee \neg r$
   (4). $A = \{2n \mid n \in Z\}$, $B = \{6n \mid n \in Z\}$, *then* $\overline{B} \subseteq \overline{A}$.
   (5). The number of integer solutions for $x_1+x_2+x_3 = 6$ and $x_1, x_2, x_3 > 0$ is 10.

2. (15%) For the complete expansion of $(2x - 2y + 3z^{-1}+1)^4$, determine the following value (a) the coefficient of yz^{-2} (b) the number of the distinct terms (c) the sum of all coefficients.

3. (15%) What is the probability of each summand even in all compositions of 20?

4. (15%) Using Venn diagrams to prove the truth or falsity of $A\Delta(B \cap C) = (A\Delta B) \cap (A\Delta C)$, for sets $A, B, C \subseteq U$.

5. (20%) For primitive statements *p*, *q*, *r*, and *s*, simplify the compound statement $[[[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r]] \vee \neg q] \rightarrow s$.

6. (20%) If *a*, *b* are relatively prime and $a > b$, prove that $gcd(a-b, a+b)=1$ or 2.