DISCRETE MATHEMATICS – CH2 Homework2

Textbook assignment (30 pts)

2-2

6. Negate each of the following and simplify the resulting statement.

a) $p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)$

b) $(p \wedge q) \rightarrow r$

c) $p \rightarrow (\neg q \wedge r)$

d) $p \vee q \vee (\neg p \wedge \neg q \wedge r)$

a) $\neg(p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r))$
$\leftrightarrow \neg p \vee (\neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r)$
$\leftrightarrow (\neg q \wedge \neg r) \vee (T_0 \wedge (\neg p \vee (q \wedge \neg r))$
$\leftrightarrow \neg p \vee (\neg q \wedge \neg r) \vee (q \wedge \neg r)$
$\leftrightarrow \boldsymbol{\neg p \vee \neg r}$

b) $\neg((p \wedge q) \rightarrow r)$
$\leftrightarrow \neg(\neg (p \wedge q) \vee r)$
$\leftrightarrow \boldsymbol{(p \wedge q) \wedge \neg r}$

c) $\neg(p \rightarrow (\neg q \wedge r))$
$\leftrightarrow \neg(\neg p \vee (\neg q \wedge r))$
$\leftrightarrow \boldsymbol{p \wedge (q \vee \neg r)}$

d) $\neg(p \vee q \vee (\neg p \wedge \neg q \wedge r))$
$\leftrightarrow \neg(p \vee q) \wedge \neg((\neg p \wedge \neg q) \wedge r))$
$\leftrightarrow \neg(p \vee q) \wedge ((p \vee q) \vee \neg r))$
$\leftrightarrow F_0 \vee (\neg(p \vee q) \wedge \neg r))$
$\leftrightarrow \boldsymbol{\neg p \wedge \neg q \wedge \neg r}$

2-3

10. Establish the validity of the following arguments.

c)
$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \neg r \\ \hline \therefore \neg(p \vee r) \end{array}$$

d)
$$\begin{array}{l} p \rightarrow q \\ r \rightarrow \neg q \\ r \\ \hline \therefore \neg p \end{array}$$

e) $\begin{array}{l} p \rightarrow (q \rightarrow r) \\ \neg q \rightarrow \neg p \\ \underline{p \qquad\qquad} \\ \therefore r \end{array}$

h) $\begin{array}{l} p \vee q \\ \neg p \vee r \\ \underline{\neg r \qquad} \\ \therefore q \end{array}$

***(c)(e) for students with odd ID, (d)(h) for even ID.**

(c)

(1) $p \rightarrow q$, $\neg q$	Premises
(2) $\neg p$	Step(1) and Modus Tollens
(3) $\neg r$	Premise
(4) $\neg p \wedge \neg r$	Step(2), (3)and the Rule of Conjunction
(5) $\therefore \neg (p \vee r)$	Step(4) and DeMorgan's Laws

(d)

(1) r , $r \rightarrow \neg q$	Premises
(2) $\neg q$	Step(1) and the Rule of Detachment
(3) $p \rightarrow q$	Premise
(4) $\therefore \neg p$	Steps(2), (3) and Modus Tollens

(e)

(1) p	Premise
(2) $\neg q \rightarrow \neg p$	Premise
(3) $p \rightarrow q$	Step(2) and $(p \rightarrow q) \Leftrightarrow (\neg q \rightarrow \neg p)$
(4) q	Steps(1), (3) and the Rule of Detachment
(5) $p \wedge q$	Steps(1), (4) and the Rule of Conjunction
(6) $p \rightarrow (q \rightarrow r)$	Premise
(7) $(p \wedge q) \rightarrow r$	Step(6), and $[p \rightarrow (q \rightarrow r)] \Leftrightarrow [(p \wedge q) \rightarrow r]$
(8) $\therefore r$	Steps(5), (7) and the Rule of Detachment

(h)

(1) $\neg p \vee r$	Premise
(2) $p \rightarrow r$	Step(1) and $(p \rightarrow r) \Leftrightarrow (\neg p \vee r)$
(3) $\neg r$	Premise
(4) $\neg p$	Steps(2), (3) and Modus Tollens
(5) $p \vee q$	Premise
(6) $\neg p \rightarrow q$	Steps(5) and $(p \vee q) \Leftrightarrow (\neg \neg p \vee q) \Leftrightarrow (\neg p \rightarrow q)$
(7) $\therefore q$	Step(4),(6) and Modus Ponens

2-4

10. For the following program segment, m and n are integer variables. The variable A is a two-dimensional array A[1, 1], A[1, 2],...,A [1, 20],...,A [10, 1],...,A [10, 20],with10 rows (indexed from 1 to 10) and 20 columns (indexed from 1 to 20).

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for m := 1 to 10 do
        for n := 1 to 20 do
                A[m,n] :=m +3*n
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Write the following statement in symbolic form.(The universe for the variable m contains only the integers from 1 to 10 inclusive; for n the universe consists of the integers from 1 to 20 inclusive.)

d) The entries in each row of A are sorted into (strictly) ascending order.

e) The entries in each column of A are sorted into (strictly) ascending order.

f) The entries in the first three rows of A are distinct.

(d) $\forall m \;\; [(1 \le n < 19) \rightarrow (A[m,n] < A[m,n+1])]$

(e) $\forall n \;\; [(1 \le m < 9) \rightarrow (A[m,n] < A[m+1,n])]$

(f) $\forall \;\; 1 \le m, i \le 3 \;\; \forall \;\; 1 \le n, j \le 20 \; [((m,n) \ne (i,j)) \rightarrow (A[m,n] \ne A[i,j])]$

Advanced assignment (20+5 pts)(5 for your novelty)

- Write an argument (include statement description) and prove it is valid. (More complicated argument gets a higher score)
  - A 3pts-example:
    - p: It rains. q: absent from discrete mathematics
    - Argument $((p \rightarrow q) \wedge \neg p) \rightarrow \neg p$
      - If it rains, I will be absent from discrete mathematics.
      - I join the discrete mathematics today
      - Conclusion: Today is not rainy