Chapter 1
Introduction to Statistics and Data Analysis

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1.1 Overview

- Success of Statistical Methods
- Statistical Inference
  - Samples
  - Populations
  - Experimental Design
- Statistics and Data Analysis

Success of Statistical Methods

- Beginning in the 1980s
- The use of statistical methods in
  - Manufacturing
  - Development of food products
  - Computer software
  - Pharmaceuticals
  - Many other areas
- Examples
  - Improvement of quality in American industry
  - Japanese “industrial miracle”: <u>High-quality products</u>

Inferential Statistics

- Collecting scientific (commercial) data or information in a systematic way with planning. Data provide understanding of scientific phenomena.
- Information is gathered in the form of samples, or collections of observations. Samples are collected from populations.
- Scientists or engineers often focus only on certain properties of objects in the population, and seeks to learn about the population.
- Example
  - An engineer may need to study the effect of process conditions, temperature, humidity, amount of a particular ingredient, …

Statistics and Data Analysis

- Data analysis: center of location, variability, and general nature of the distribution of observations in the sample.
- Offspring of inferential statistics: a large of “Toolbox” of statistical methods which are used to <u>make scientific judgments in face of uncertainty and variation</u>.
- Modern statistical software packages allow for computation of means, medians, standard deviations.
- Graphical methods include histograms, stem and leaf plots, dot plots, and box plots.

Role of Probability

- Concepts in probability form a major component that supplements statistical methods and help estimate the strength of the statistical inference.
- Example1.1
  - In a manufacturing process, 100 items are sampled and 10 are found to be defective.
  - However, in the long run, the company can only tolerate 5% defective in the process.
  - Suppose we learn that if the process is acceptable, i.e., if it does produce items 5% of which are defective, there is a probability of 0.0282 of obtaining 10 or more defective items in a random sample of 100 items from the process.
  - The small probability suggests that the process indeed have a long-run defective exceeding 5%.
  - Probability aids in translation of sample information into conclusions.

Role of Probability

- Example1.2
  - Study the development of a relationship between the roots of trees and the action of a fungus (真菌).
  - Minerals are transferred from the fungus to the trees and sugars from the trees to the fungus.
  - Does the <u>use of nitrogen</u> influence stem weight?
  - Experimental Design: Two samples of 10 northern red oak seedlings (幼苗) are planted in a greenhouse, one containing seedlings treated <u>with nitrogen</u> and one containing <u>no nitrogen</u> .
  - All other environmental conditions are held constant.

Role of Probability

- Example1.2
  - The stem weights in grams were recorded after the end of 140 days.
  - 4 nitrogen observations are larger than any of the no-nitrogen observations.
  - Most of the no-nitrogen observations appear to be below the center of the data.
  - Would the data set indicate that nitrogen is effective?
  - How can this can be quantified or summarized in some sense ?

No nitrogen	Nitrogen
0.32	0.26
0.53	0.43
0.28	0.47
0.37	0.49
0.47	0.52
0.43	0.75
0.36	0.79
0.42	0.86
0.38	0.62
0.43	0.46

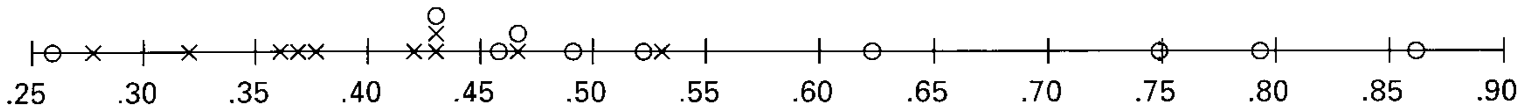

Figure 1.1 Stem weight data.

Role of Probability

- For a statistical problem, the sample along with inferential statistics allow us to draw conclusions about the population, with inferential statistics making clear use of elements of probability.

- Problems in probability allow us to draw conclusions about characteristics of hypothetical (假設的) data taken from the population based on known features of the population.

1.2 Sampling Procedures

- The importance of proper sampling revolves around the degree of confidence with which the analyst is able to answer the questions being asked.
- Simple random sampling implies that any particular sample of a specified sample size has the same chance of being selected as any other sample of the same size.
- Biased sample: Simple random sampling is not always proper.
  - Example: A sample is chosen to answer certain questions regarding political preferences in a certain state in the U.S. Now, suppose that all or nearly all of the 1,000 sampling families chosen live in urban (vs. rural) areas.
- To characterize and quantify measures of variability is very important in inferential statistics.

1.3 Measures of Location: Sample Mean and Median

- Location measures in a data set are designed to provide some quantitative measure of where the data center is in a sample.
  - The observations in a sample are $x_1, x_2, \ldots, x_n$.
  - The sample mean is $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{x_1 + x_2 + \cdots + x_n}{n}$
  - The sample median is $\tilde{x} = \begin{cases} x_{(n+1)/2} & \text{if } n \text{ is odd.} \\ \dfrac{x_{n/2} + x_{n/2+1}}{2} & \text{if } n \text{ is even.} \end{cases}$
- Example: If the data set is the following: 1.7, 2.2, 3.11, 3.9, and 14.7, then $\bar{x} = 5.72$, $\tilde{x} = 3.11$
- The computation of $\bar{x}$ is the basis of an estimate of the population mean in statistical inference.

Other Measures of Locations

- A <u>trimmed mean</u> is computed by "trimming away" a certain percent of both the largest and smallest set of values.
- Example
  - The 10% trimmed mean is found by eliminating the largest 10% and smallest 10% and computing the average of the remaining values.
  - So, for the with nitrogen group the 10% <u>trimmed mean</u> is

No nitrogen	Nitrogen
0.32	~~0.26~~
0.53	0.43
0.28	0.47
0.37	0.49
0.47	0.52
0.43	0.75
0.36	0.79
0.42	~~0.86~~
0.38	0.62
0.43	0.46

$$\bar{x}_{tr(10)} = \frac{0.43 + 0.47 + 0.49 + 0 + 0.75 + 0.79 + 0.62 + 0.46}{8}$$

$$= 0.56625$$

1.4 Measure of Variability

- Process and product variability is a fact of life in engineering and scientific systems: the control or reduction of process variability is often a source of major difficulty.
- The sample range, $X_{max} - X_{min}$, has applications in the area of statistical quality control.
- The sample variance is donated by $s^2 = \sum_{i=1}^{n} \frac{(x_i - \overline{x})^2}{n-1}$
- The sample standard deviation is donated by

$$s = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \overline{x})^2}{n-1}}$$

Measure of Variability

- Example 1.4: An engineer is interested in testing the "bias" in a PH meter. Data are collected on the meter by measuring the PH of a neutral substance (H = 7.0). A sample of size 10 is taken with results given by

7.07	7.00	7.10	6.97	7.00
7.03	7.01	7.01	6.98	7.08

$$\bar{x} = \frac{7.07 + 7.00 + 7.10 + \ldots + 7.08}{10} = 7.0205$$

$$s^2 = \frac{(7.07 - 7.0205)^2 + (7.00 - 7.0205)^2 + (7.10 - 7.0205)^2 + \ldots + (7.08 - 7.0205)^2}{10 - 1}$$

$$= 0.001939$$

$$s = \sqrt{0.00193} = 0.0440$$

Measure of Variability

- In the context of statistical inference
  - Usually, focus on drawing conclusions about characteristics of populations, called population parameters.
  - Population mean and population variance are two important parameters.
  - The sample mean plays an explicit role to draw inferences about the population mean.
  - The sample variance (standard deviation) plays an explicit role to draw inferences about the population variance (standard deviation).

1.6 Statistical Modeling, Scientific Inspection, and Graphical Diagnostics

- The result of a statistical analysis is the estimation of parameters of a postulated (假設的) model.
- A statistical model is not deterministic but, rather, must entail (意味著) some probabilistic aspects.
- A model form is often the foundation of assumptions that are made by the analyst.
  - Example 1.2 scientists draw some distinction between "nitrogen" and "no-nitrogen" populations through the sample information.
  - The analysis may require a certain model for the data, e.g., normal (Gaussian) distributions (see Chapter 6).

Statistical Modeling, Scientific Inspection, and Graphical Diagnostics

- Some simple graphics (plots) can shed important light on the clear distinction between the samples, e.g., means and variability.
- Often, plots can illustrate information that sometimes are not retrieved from the formal analysis.

Cotton percentage	Tensile strength
15	7, 7, 9, 8, 10
20	19, 20, 21, 20, 22
25	21, 21, 17, 19, 20
30	8, 7, 8, 9, 10

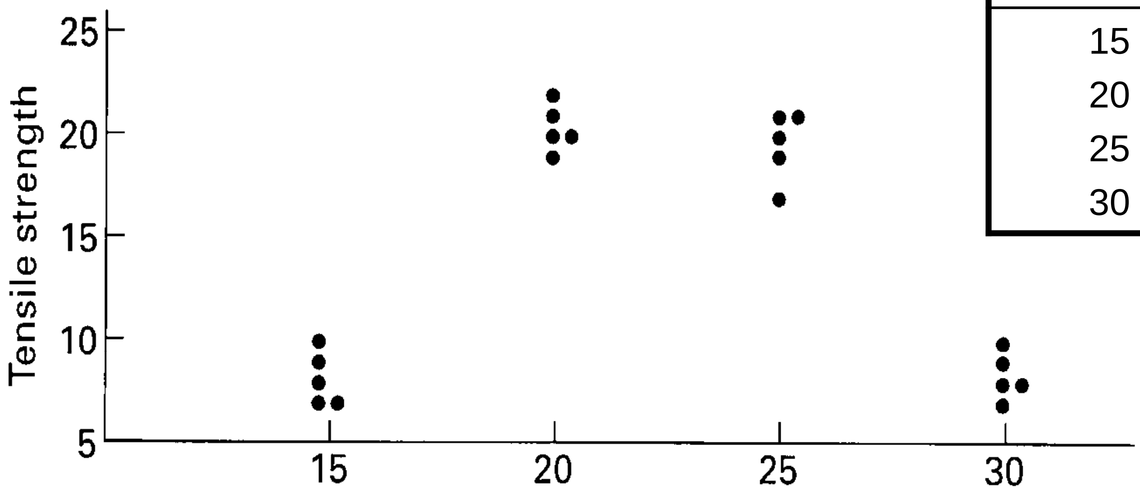

Figure 1.5 Plot of tensile strength and cotton percentages.

Statistical Modeling, Scientific Inspection, and Graphical Diagnostics

- It is likely that the scientist anticipates the existence of a <u>maximum population mean</u> tensile strength.
- Here the analysis of the data should revolve around a different type of model, whose structure relating the population mean tensile strength to the cotton concentration.
  - E.g., a regression model $\mu_{t,c} = \beta_0 + \beta_1 C + \beta_2 C^2$ where $\mu_{t,c}$ is the population mean of tensile strength, which varies with the amount of cotton in the product *C*.
  - The use of an empirical model is accompanied by estimation theory, where $\beta_0, \beta_1, \beta_2$ are estimated by the data.

Statistical Modeling, Scientific Inspection, and Graphical Diagnostics

- The type of model used to describe the data often depends on the goal of the experiment.
- The structure of the model should take advantage of nonstatistical scientific input.
- A selection of a model represents a fundamental assumption upon which the resulting statistical inference is based.
- Often, plots (graphics) can illustrate information that allows the results of the formal statistical inference to be better communicated to the scientist or engineer, and teach the analyst something not retrieved from the formal analysis.

Graphical Methods and Data Description

- Characterizing or summarizing the nature of collections of data is important.
- A summary of a collection of data via a graphical display can provide insight regarding the system from which the data were taken.
- Example

Table 1.4 Car Battery Life							
2.2	4.1	3.5	4.5	3.2	3.7	3.0	2.6
3.4	1.6	3.1	3.3	3.8	3.1	4.7	3.7
……							

Table 1.5 **Stem and Leaf Plot** of Battery Life		
Stem	Leaf	Frequency
1	69	2
2	25669	5
3	0011112223334445567778899	25
4	11234577	8

Table 1.6 **Double-Stem and Leaf Plot** of Battery Life		
Stem	Leaf	Frequency
1.	69	2
2*	2	1
2.	5669	4
3*	001111222333444	15
3.	5567778899	10
4*	11234	5
4.	577	3

Usually, we choose between 5 and 20 stems.

Table 1.7 **Relative Frequency Distribution** of Battery Life

Class interval	Class midpoint	Frequency, *f*	Relative Frequency
1.5-1.9	1.7	2	0.05
2.0-2.4	2.2	1	0.025
2.5-2.9	2.7	4	0.100
3.0-3.4	3.2	15	0.375
3.5-3.9	3.7	10	0.250
4.0-4.4	4.2	5	0.125
4.5-4.9	4.7	3	0.075

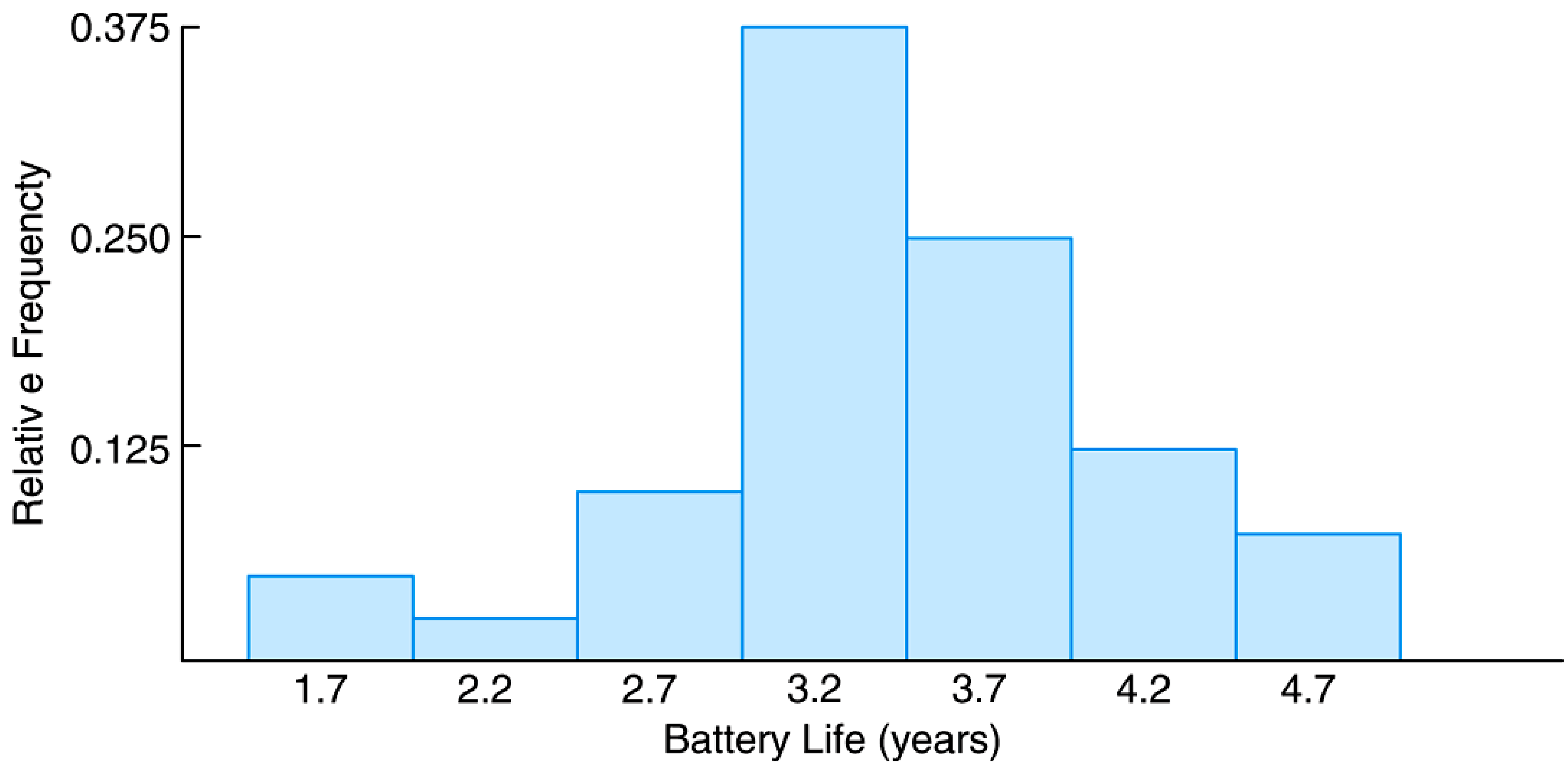

Figure 1.6 Relative frequency histogram

- Continuous frequency distribution in Figure 1.7:
  Bell-shaped curve
- Distribution (probability distribution) is a property of the population (Chapters 5 & 6)

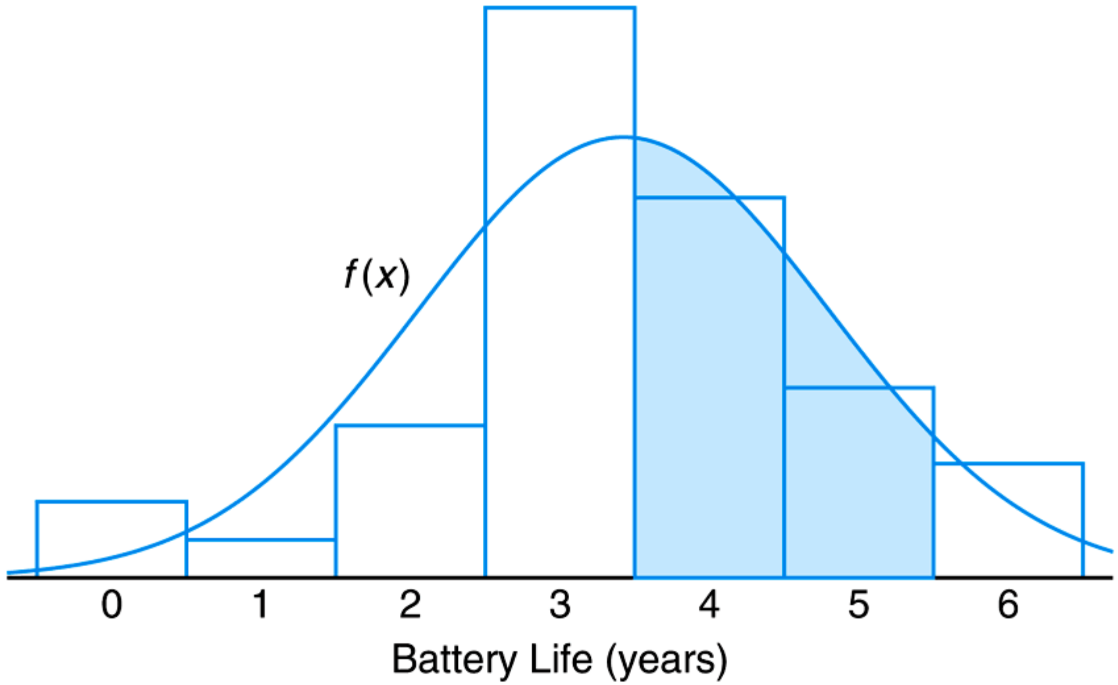

Figure 1.7 Estimating frequency distribution

- A distribution is symmetric if it can be folded along a vertical axis so that the two sides coincide, otherwise skewed.
- By rotating a stem and leaf plot counterclockwise through an angle of 90º, the resulting columns of leaves form a picture that is similar to a histogram.

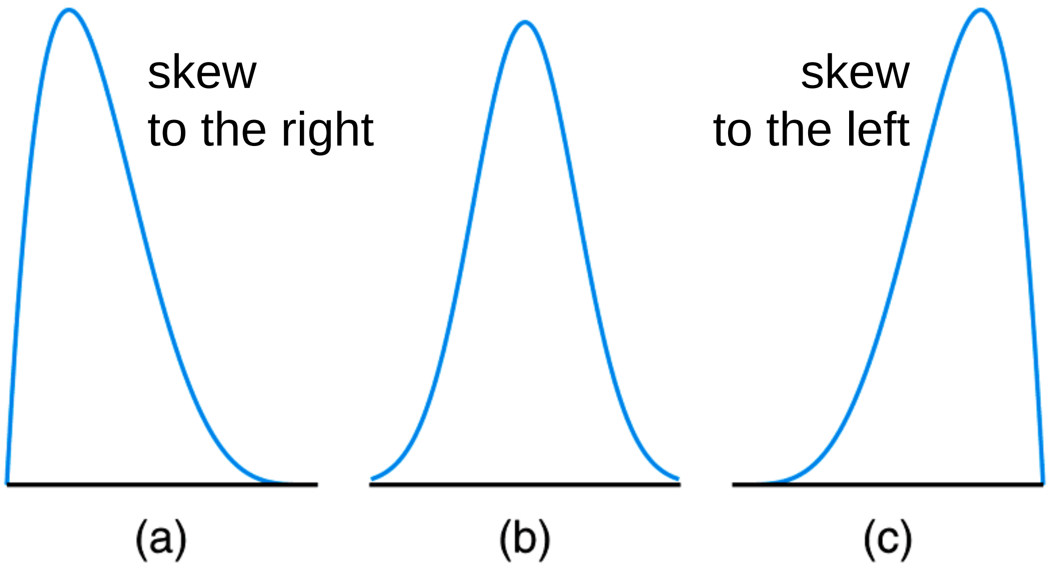

Figure 1.8 Skewness of data

1.09	1.92	2.31	1.79	2.28	1.74	1.47	1.97
0.85	1.24	1.58	2.03	1.70	2.17	2.55	2.11
1.86	1.90	1.68	1.51	1.64	0.72	1.69	1.85
1.82	1.79	2.46	1.88	2.08	1.67	1.37	1.93
1.40	1.64	2.09	1.75	1.63	2.37	1.75	1.69

Table 1.8 Nicotine Data for Example 1.5

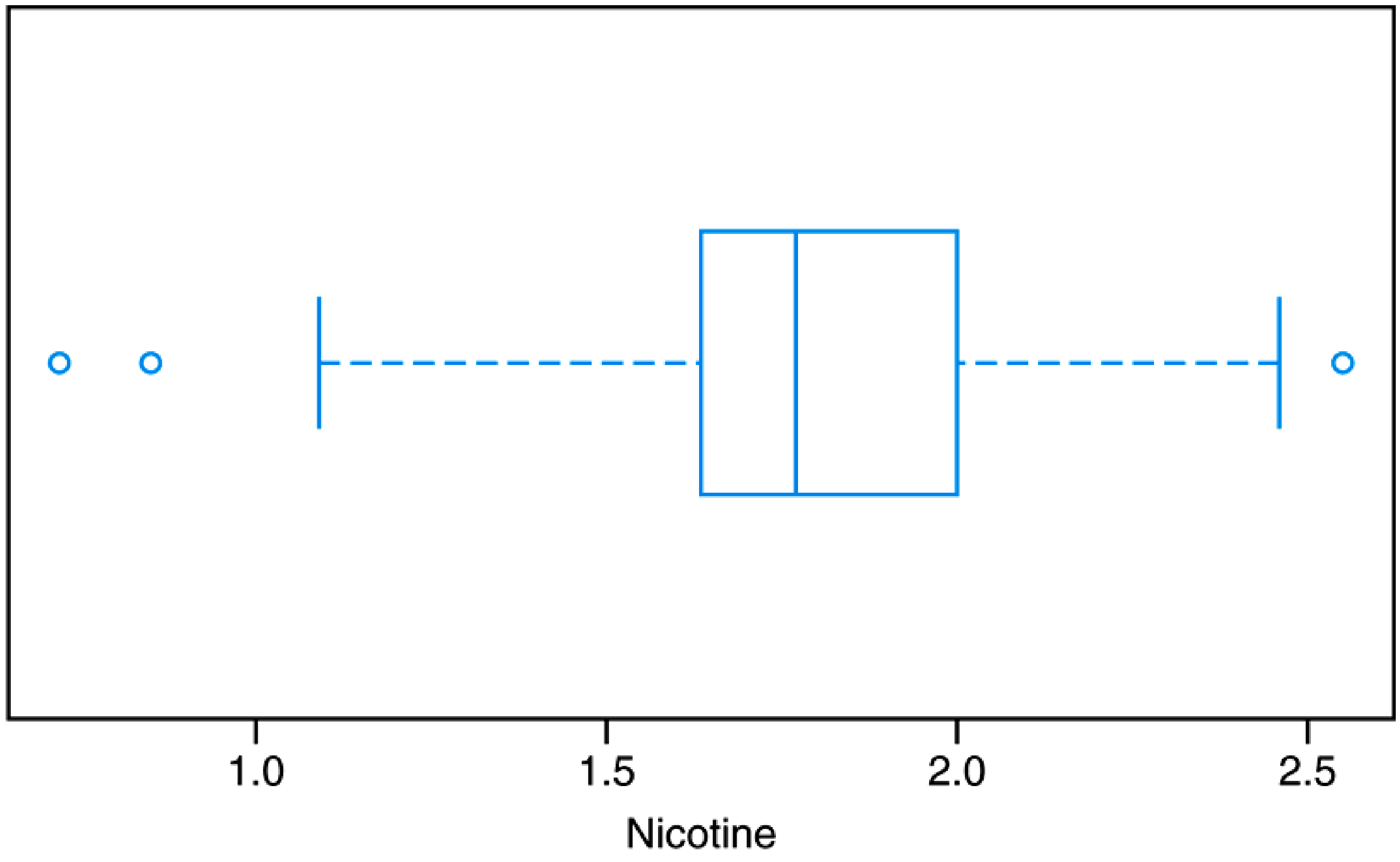

Figure 1.9 Box-and-whisker plot for Example 1.5

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The decimal point is 1 digit(s) to the left of the |
 7 | 2
 8 | 5
 9 |
10 | 9
11 |
12 | 4
13 | 7
14 | 07
15 | 18
16 | 3447899
17 | 045599
18 | 2568
19 | 0237
20 | 389
21 | 17
22 | 8
23 | 17
24 | 6
25 | 5
```

Figure 1.10 Stem-and-Leaf plot for the nicotine data

Sample	Measurements	Sample	Measurements
1	29 36 39 34 34	16	35 30 35 29 37
2	29 29 28 32 31	17	40 31 38 35 31
3	34 34 39 38 37	18	35 36 30 33 32
4	35 37 33 38 41	19	35 34 35 30 36
5	30 29 31 38 29	20	35 35 31 38 36
6	34 31 37 39 36	21	32 36 36 32 36
7	30 35 33 40 36	22	36 37 32 34 34
8	28 28 31 34 30	23	29 34 33 37 35
9	32 36 38 38 35	24	36 36 35 37 37
10	35 30 37 35 31	25	36 30 35 33 31
11	35 30 35 38 35	26	35 30 29 38 35
12	38 34 35 35 31	27	35 36 30 34 36
13	34 35 33 30 34	28	35 30 36 29 35
14	40 35 34 33 35	29	38 36 35 31 31
15	34 35 38 35 30	30	30 34 40 28 30

Table 1.9 Data for Example 1.6

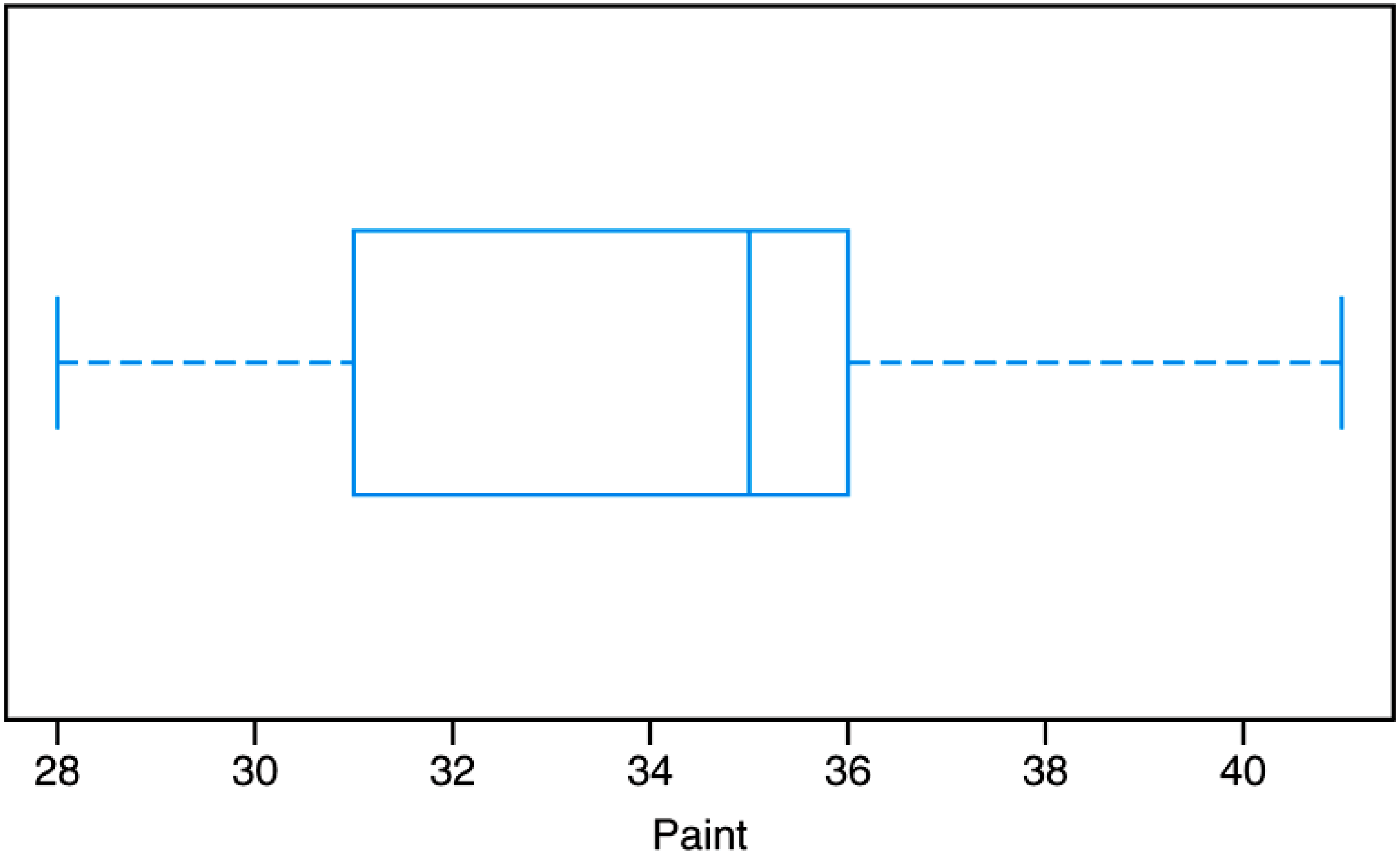

Figure 1.11 Box-and-whisker plot for thickness of paint can "ears"