1. A globally optimal solution can be arrived at by making locally optimal (greedy) choice.
2.

$$BB^T(i, j) = \sum_{e \in E} b_{ie} b^T_{ej} = \sum_{e \in E} b_{ie} b_{je}$$

- If $i = j$, then $b_{ie}b_{je} = 1$ (it is $1 \cdot 1$ or $(-1) \cdot (-1)$) whenever e enters or leaves vertex i, and 0 otherwise.
- If $i \neq j$, then $b_{ie}b_{je} = -1$ when $e = (i, j)$ or $e = (j, i)$, and 0 otherwise.

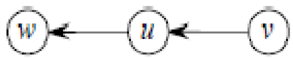
Thus,

$$BB^T(i, j) = \begin{cases} \text{degree of } i = \text{in-degree} + \text{out-degree} & \text{if } i = j, \\ -(\text{\# of edges connecting } i \text{ and } j) & \text{if } i \neq j. \end{cases}$$

3.

Let us consider the example graph and depth-first search below.

	d	f
w	1	2
u	3	4
v	5	6



Clearly u has both incoming and outgoing edges in G but a depth-first search of G produced a depth-first forest where u is in a tree by itself.

4.

An undirected graph is acyclic (i.e., a forest) if and only if a DFS yields no back edges.

- If there's a back edge, there's a cycle.
- If there's no back edge, then by Theorem 22.10, there are only tree edges. Hence, the graph is acyclic.

Thus, we can run DFS: if we find a back edge, there's a cycle.

- Time: $O(V)$. (Not $O(V + E)$!)
  If we ever see $|V|$ distinct edges, we must have seen a back edge because (by Theorem B.2 on p. 1085) in an acyclic (undirected) forest, $|E| \leq |V| - 1$.

5.

Theorem

Let A be a subset of some MST, $(S, V - S)$ be a cut that respects A, and (u, v) be a light edge crossing $(S, V - S)$. Then (u, v) is safe for A.

Let A be the empty set and S be any set containing u but not v. So (u, v) *belongs to some MST of G.*

6. A triangle whose edge weights are all equal is a graph in which every edge is a light edge crossing some cut. But the triangle is cyclic, so it is not a minimum spanning tree.
7. Observe that after the first pass, all *d* values are at most 0, and that relaxing edges (v_0, v_i) will never again change a *d* value. Therefore, we can eliminate v_0 by running the Bellman-Ford algorithm on the constraint graph without the v_0 node but initializing all shortest path estimates to 0 instead of ∞.