DISCRETE MATHEMATICS – CH3 Homework3

3.1

8. For $A = \{1, 2, 3, 4, 5, 6, 7\}$, determine the number of

a) subsets of A b) nonempty subsets of A

c) proper subsets of A d) nonempty proper subsets of A

e) subsets of A containing three elements f) subsets of A containing 1, 2

g) subsets of A containing five elements, including 1, 2

h) subsets of A with an even number of elements

i) subsets of A with an odd number of elements

a) 2^7 = 128

b) 2^7 -1 = 127

c) 2^7 – 1 = 127

d) 2^7 – 2 = 126

e) $\binom{7}{3} = 35$

f) 2^5 = 32

g) $\binom{5}{3} = 35$

h) $\binom{7}{0} + \binom{7}{2} + \binom{7}{4} + \binom{7}{6} = 64$

i) $\binom{7}{1} + \binom{7}{3} + \binom{7}{5} + \binom{7}{7} = 64$

20. a) Among the strictly increasing sequences of integers that start with 1 and end with 7 are: i) 1, 7 ii) 1, 3, 4, 7 iii) 1, 2, 4, 5, 6, 7
How many such strictly increasing sequences of integers start with 1 and end with 7?
b) How many strictly increasing sequences of integers start with 3 and end with 9?
c) How many strictly increasing sequences of integers start with 1 and end with 37? How many start with 62 and end with 98?
d) Generalize the results in parts (a) through (c).

a) 2^5 = 32

b) 2^5 = 32

c) 2^35, 2^35

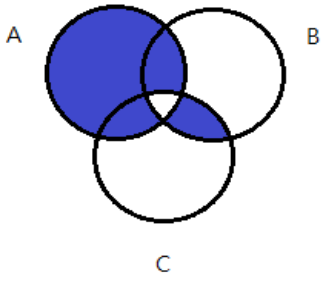
d) 2^(n-m-1), m < n

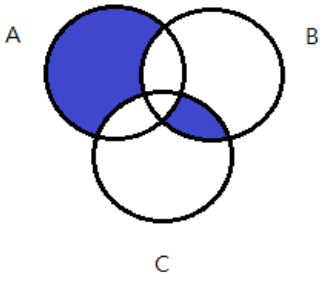
3.2

8. Using Venn diagrams, investigate the truth or falsity of each of the following, for sets $A, B, C \subseteq \mathcal{U}$.

(a) $A \Delta (B \cap C) = (A \Delta B) \cap (A \Delta C)$

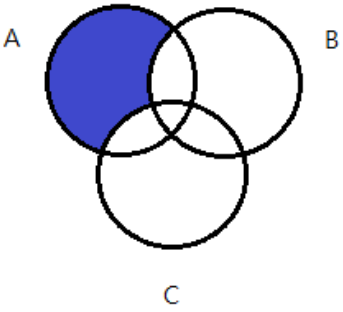
false

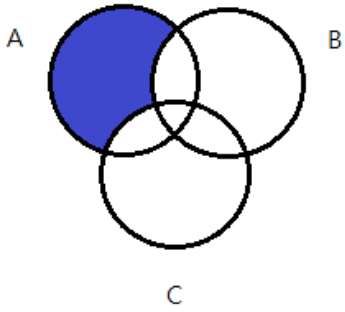




(b)$A-(B \cup C) = (A-B) \cap (A-C)$

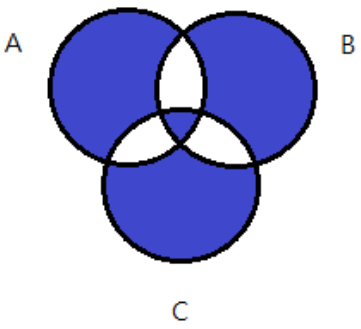
True

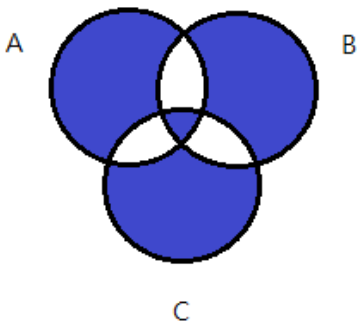




(c) $A \Delta (B \Delta C) = (A \Delta B) \Delta C$

TRUE

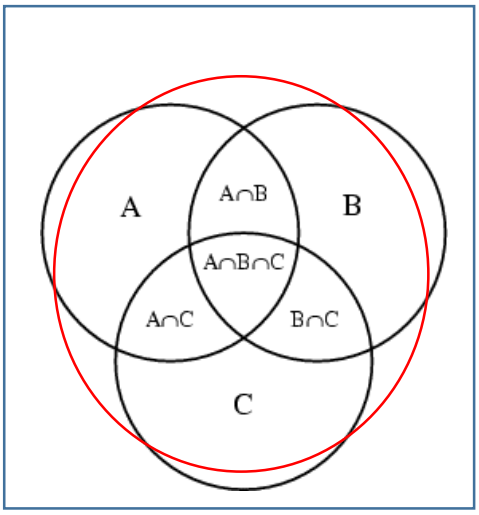




3.3

9. How many arrangements of the letters in AAATTCCGGG / AATTCCCCGG have no pair of consecutive identical letters?

- AAATTCCGGG



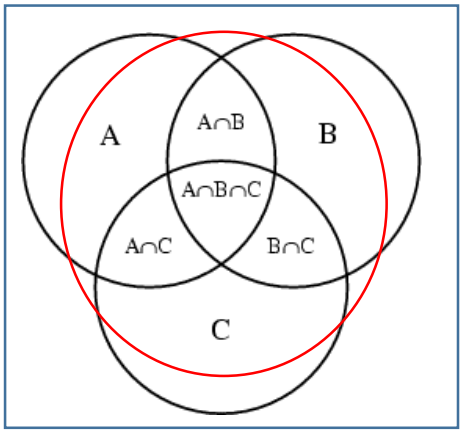
AAA+CC+TT+GGG-AAACC-AAATT-GGGAAA-GGGTT-GGGCC-CCTT +AAACCTT +GGGAACC+GGGAAATT+GGGCCTT-2*AAACCGGGTT= 8!/(2!2!3!)+9!/(3!3!2!)+9!/(3!3!2!)+ 8!/(2!2!3!)-7!/(2!3!)- 7!/(2!3!)-6!/(2!2!)- 7!/(2!3!)- 7!/(2!3!)-8!(3!3!)+6!/(3!)+5!/(2!)+5!/(2!)+6!/3!-2*4!

Non consecutive: 10!/(2!2!3!3!)-[8!/(2!2!3!)+9!/(3!3!2!)+9!/(3!3!2!)+ 8!/(2!2!3!)-7!/(2!3!)-7!/(2!3!)-6!/(2!2!)- 7!/(2!3!)- 7!/(2!3!) -8!(3!3!)+6!/(3!)+5!/(2!)+5!/(2!)+6!/3!-2*4!]

- AATTCCCCGG

AA+CCCC+TT+GG-AACCCC-AATT-GGAA-GGTT-GGCCCC -CCCCTT+AACCCCTT +GGAACCCC+GGAATT+GGCCCCTT-2*AACCCCGGTT=9!(4!2!2!)+7!/(2!2!2!)+ 9!(4!2!2!)+ 9!(4!2!2!)-6!/(2!2!)-8!/(4!2!)- 8!/(4!2!)- 8!/(4!2!)- 6!/(2!2!)- 6!/(2!2!)+5!/2!+5!/2!+7!/4!+5!/2!-2*4!

Non consecutive: 10!/(2!2!4!2!)-[9!(4!2!2!)+7!/(2!2!2!)+ 9!(4!2!2!)+ 9!(4!2!2!)-6!/(2!2!)-8!/(4!2!)-8!/(4!2!)- 8!/(4!2!)- 6!/(2!2!)- 6!/(2!2!)+5!/2!+5!/2!+7!/4!+5!/2!-2*4!]



10. How many arrangements of the letters in CHEMIST have H before E, or E before T, or T before M? (Here "before" means anywhere before, not just immediately before.)

The number of arrangements with either H before E, or E before T, or T before M equals the total number of arrangements (i.e., 7!) minus the number of arrangements where E is before H, and T is before E, and M is before T. There are 3! ways to arrange C, I, S. For each arrangement there are four locations (one at the start, two between pairs of letters, and one at the end) to select from, with repetition, to place M, T, E, H in this prescribed order. Hence there are $(3!)\binom{4+4-1}{4} = (3!)\binom{7}{4}$ arrangements where M is before T, T before E, and E before H. Consequently, there are $7! - (3!)\binom{7}{4}$ arrangements with either H before E, or E before T, or T before M.

Advanced assignment

- Read Example 3.9 (page138, page128 in old textbook) and write your comments.
  - What does this example say?
    gray code 的產生方式，相鄰的 gray code 差 1 個 bit。
  - What are its extension and application?
    Genetic algorithm、河內塔、電路設計、…。
  - What does you get after this reading? (自行發揮)