Engineering Mathematics Homework 2 - Solution

1. $y(x) = c_1 e^{5x} + c_2 e^{3x}$, 求原 *Differential Equation* 為何?

Sol:

$$令 \quad y(x) = c_1 e^{5x} + c_2 e^{3x} \qquad -(a)$$

$$\frac{dy(x)}{dx} = 5c_1 e^{5x} + 3c_2 e^{3x} \qquad -(b)$$

$$\frac{d^2 y(x)}{dx^2} = 25c_1 e^{5x} + 9c_2 e^{3x} \qquad -(c)$$

$$(a) \times 15 + (b) \times (-8) + (c)$$

$$\Rightarrow y''(x) - 8y'(x) + 15y(x) = 0$$

2. 下列何者具唯一解?

　1. $y' = e^{x^2 y}, y(0) = 1$

　2. $y' = \sqrt{2-y}, y(0) = 0$

　3. $y' = \sqrt{2-y}, y(0) = 2$

　4. $y' = \sqrt{4-y^2}, y(0) = 2$

Sol:

1.

$$f(x, y) = e^{x^2 y} \qquad (0,1)$$

$$\frac{\partial f(x, y)}{\partial y} = x^2 e^{x^2 y} \qquad (0,1) \quad \Rightarrow 具唯一解$$

2.

$$f(x, y) = \sqrt{2 - y} \qquad (0,0)$$

$$\frac{\partial f(x, y)}{\partial y} = \frac{1}{2\sqrt{2 - y}} \qquad (0,0) \quad \Rightarrow 具唯一解$$

3.

$$f(x, y) = \sqrt{2 - y} \qquad (0,2)$$

$$\frac{\partial f(x, y)}{\partial y} = \frac{1}{2\sqrt{2 - y}} \qquad (0,2) \Rightarrow 不具唯一解$$

4.

$$f(x, y) = \sqrt{4 - y^2} \qquad (0,2)$$

$$\frac{\partial f(x, y)}{\partial y} = \frac{-2y}{2\sqrt{4 - y^2}} \qquad (0,2) \quad \Rightarrow 不具唯一解$$

3. $Solve: 3x^2ydx + (x^3 - 5)dy = 0$

Sol:

$M = 3x^2y, N = x^3 - 5$

$\dfrac{\partial M}{\partial y} = 3x^2 = \dfrac{\partial N}{\partial y}$ $\qquad \Rightarrow$ 正合

$\dfrac{\partial u(x,y)}{\partial x} = 3x^2y$ $\qquad \int \partial u(x,y) = \int 3x^2y\,dx + f(y)$

$\dfrac{\partial u(x,y)}{\partial y} = x^3 - 5$ $\qquad \int \partial u(x,y) = \int (x^3 - 5)dy + g(x)$

$u = \begin{cases} x^3y + f(y) \cdots\cdots\cdots (1) \\ x^3y - 5y + g(x) \cdots\cdots (2) \end{cases}$

$f(y) = -5y$

$g(x) = 0$

$u(x,y) = x^3y - 5y = C$