

## Engineering Mathematics Homework 11 Solution

1.  $F(s) = \frac{s^2 + 6s + 9}{(s-1)(s-2)(s+4)}$ , find  $f(t) = ?$

Sol:

$$F(s) = \frac{s^2 + 6s + 9}{(s-1)(s-2)(s+4)} = \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s+4}$$

$$A = \frac{1^2 + 1 \times 6 + 9}{(1-2)(1+4)} = -\frac{16}{5}, B = \frac{2^2 + 2 \times 6 + 9}{(2-1)(2+4)} = \frac{25}{6}$$

$$C = \frac{4^2 - 4 \times 6 + 9}{(-4-1)(-4-2)} = \frac{1}{30}$$

$$\begin{aligned} f(t) &= \mathcal{L}^{-1} \left\{ \frac{s^2 + 6s + 9}{(s-1)(s-2)(s+4)} \right\} \\ &= -\frac{16}{5} \mathcal{L}^{-1} \left\{ \frac{1}{s-1} \right\} + \frac{25}{6} \mathcal{L}^{-1} \left\{ \frac{1}{s-2} \right\} + \frac{1}{30} \mathcal{L}^{-1} \left\{ \frac{1}{s+4} \right\} \\ &= -\frac{16}{5} e^t + \frac{25}{6} e^{2t} + \frac{1}{30} e^{-4t} \end{aligned}$$

2. Solve:  $f(t) = 3t^5 + \int_0^t f(t-\tau) e^{-\tau} d\tau$

Sol:

$$\begin{aligned} F(s) &= F(s) \frac{1}{s+1} + 3 \frac{5!}{s^{5+1}} \\ \Rightarrow \frac{s}{s+1} F(s) &= 3 \frac{5!}{s^{5+1}} \end{aligned}$$

$$F(s) = \frac{3 \times 5! \times (s+1)}{s \times s^6} = \frac{3 \times 5!}{s^6} + \frac{3 \times 5! \times \frac{6!}{6!}}{s \times s^6}$$

$$f(t) = 3t^5 + \frac{1}{2} t^6$$