Electric Charge, Force, and Field

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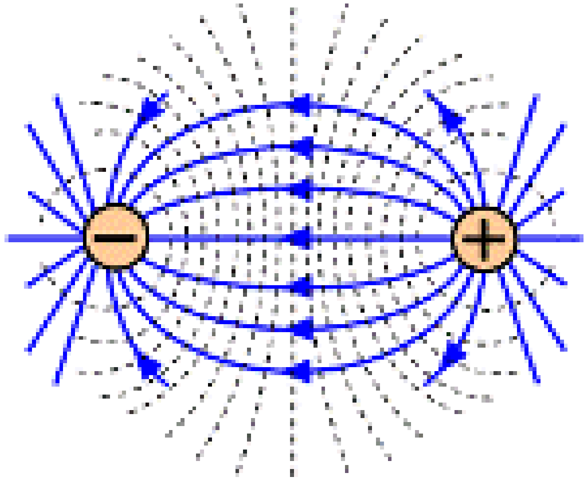
Electric Charge

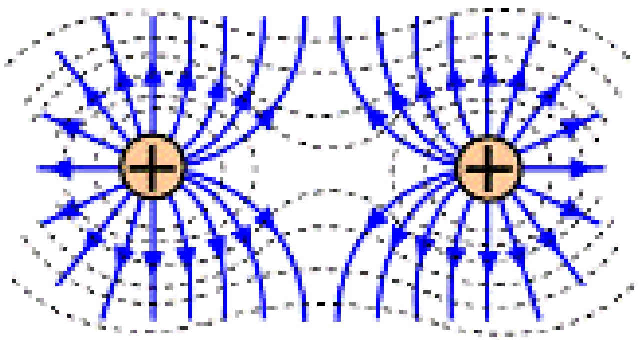
- 2 kinds of charges: + & –. , Scalar Quantity
- Total charge = algebraic sum of all charges.

$$Q = \sum_i q_i = \int d^3x \, \rho(\mathbf{r})$$

Conservation of charge: total charge in a closed region is always the same.

- Opposite charges attract. Like charges repel.





Electric Charge

- All electrons have charge –*e.*
- All protons have charge +*e*.

$$e = 1.60 \times 10^{-19}\,C$$ = elementary charge

1st measured by Millikan on oil drops.

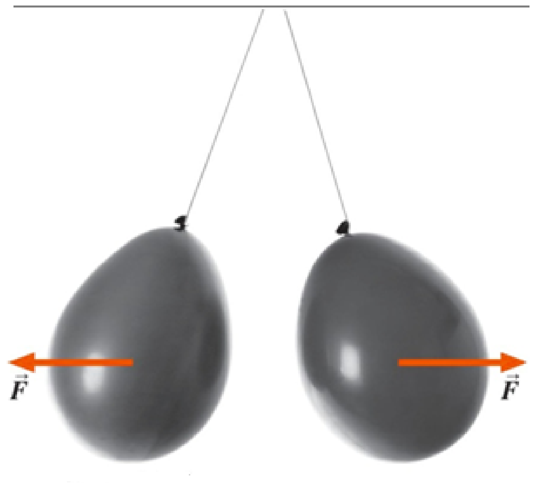
Theory (standard model) : basic unit of charge (carried by quark) = 1/3 *e*.

Quark confinement $\rightarrow$ no free quark can be observed.

$\therefore$ Smallest observable charge is *e*.

Electric Charge: Charging (Classical)

"Insulators" can be charged by rubbing.



Examples:

Rubbed balloon sticks to clothing.

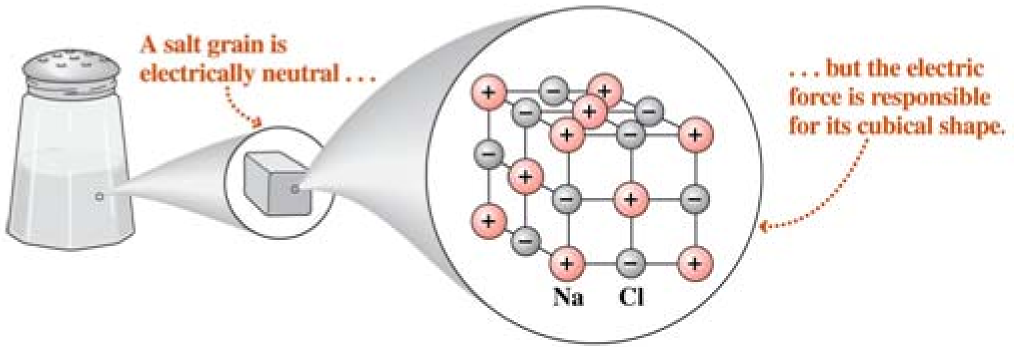
2 rubbed balloons repel each other.

Socks from dryer cling to clothings.

Bits of styrofoam cling to hand.

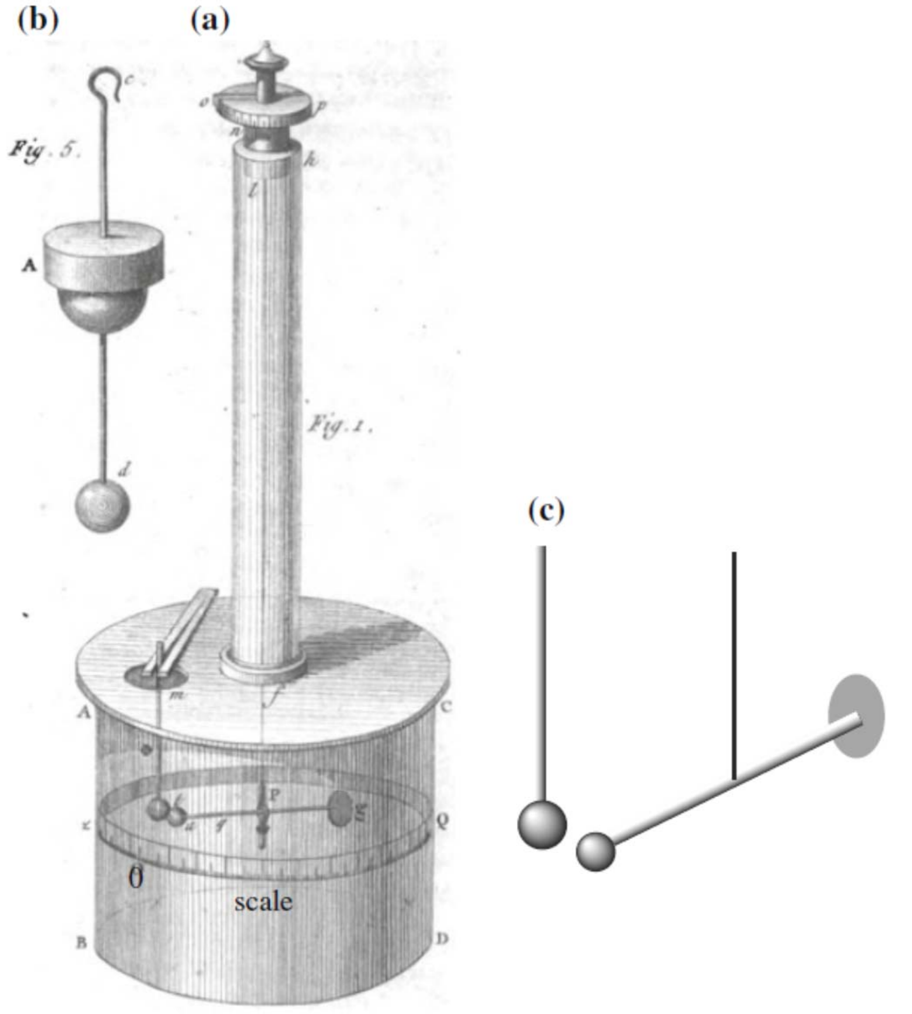
Walk across carpet & feel shock touching doorknob.

Ground or low energy state of matter tends to be charge neutral.



Coulomb’s Law and the Electric Force

- In 1785, Charles Augustin de Coulomb.



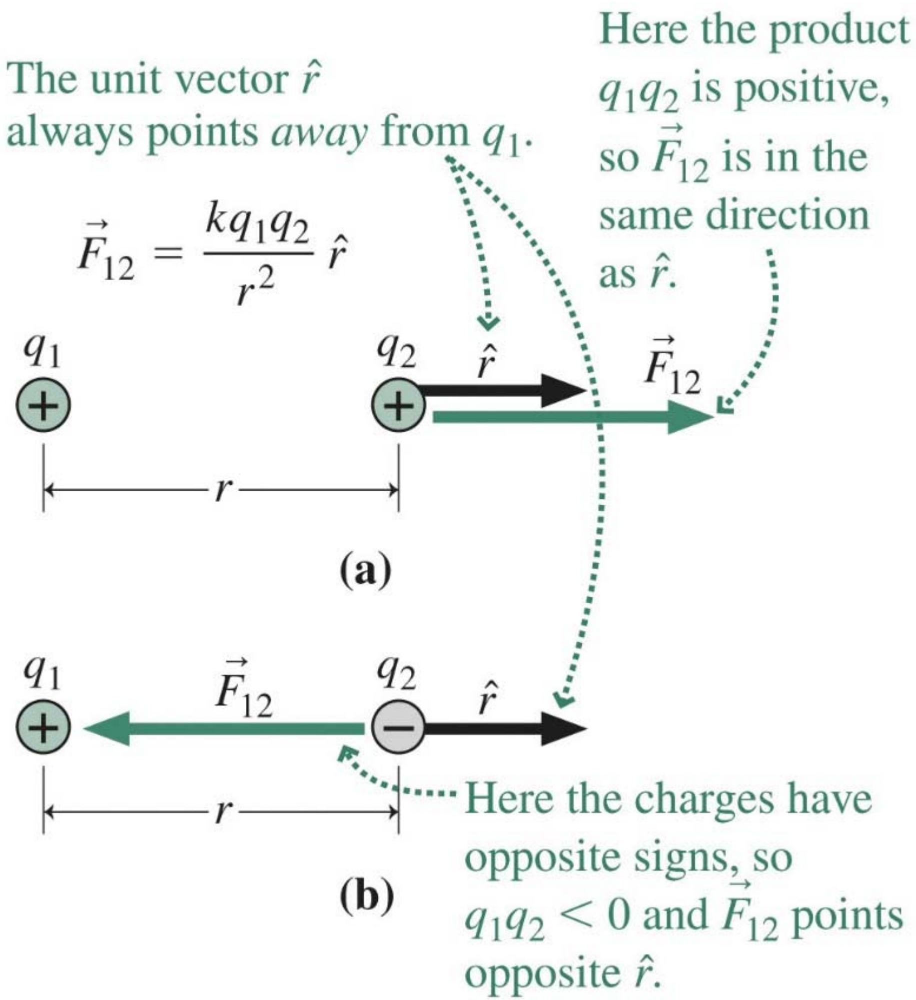
Coulomb's Law and the Electric Force

- In 1785, Charles Augustin de Coulomb.
- Electric Force is dependent on
  - The product of the two charges
  - The inverse square of the distance between them

$$F = k\frac{q_1 q_2}{r^2}$$

$$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N m}^2/\text{C}^2$$

Coulomb's Law and the Electric Force



$$\vec{F}_{1,2} = k\frac{q_1q_2}{|r_{1,2}|^2}\mathbf{u}_{1,2} = k\frac{q_1q_2}{|r_{1,2}|^2}\frac{\vec{r}_{1,2}}{|r_{1,2}|}$$

$$\vec{r}_{1,2} = \vec{r}_2 - \vec{r}_1$$

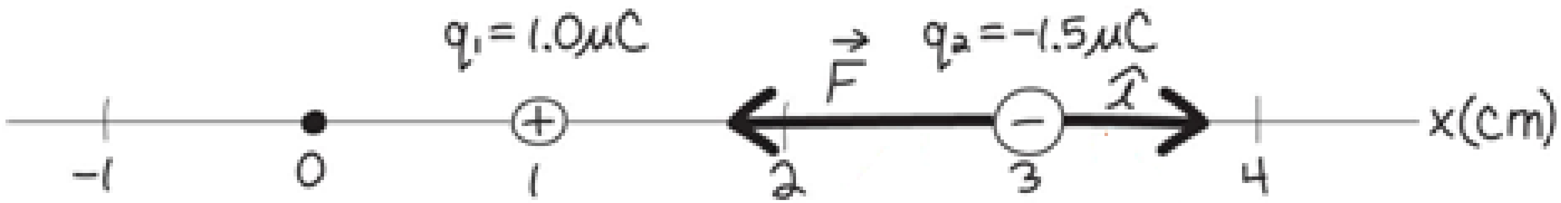
$$\hat{r} = \mathbf{u}_{1,2}$$

Example: Force Between Two Charges

A 1.0 μC charge is at x = 1.0 cm, & a $-$1.5 μC charge is at x = 3.0 cm.

What force does the positive charge exert on the negative one?

How would the force change if the distance between the charges tripled?



Example: Force Between Two Charges

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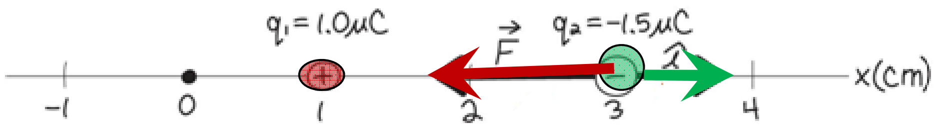


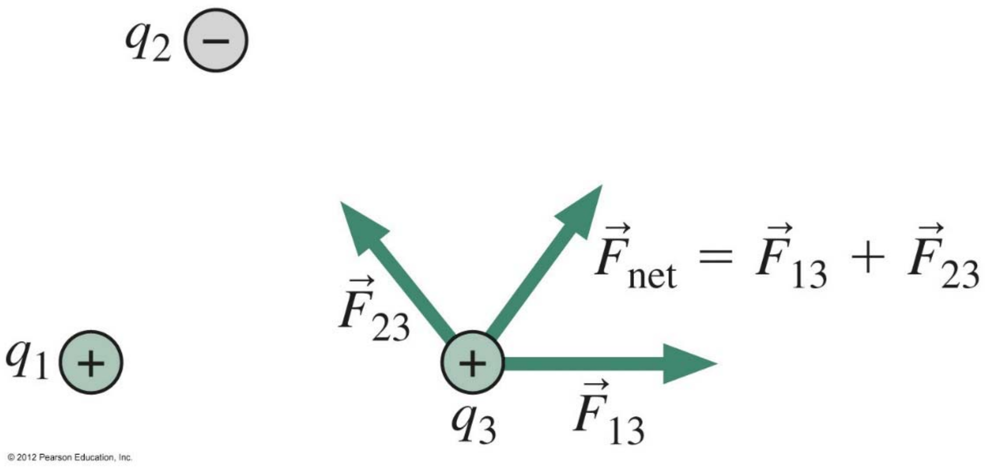
$$\mathbf{F}_{12} = \frac{k\, q_1\, q_2}{r^2}\hat{\mathbf{r}} \quad = \frac{\left(9.0\times10^{9}\ Nm^2/C^2\right)\left(1.0\times10^{-6}C\right)\left(-1.5\times10^{-6}C\right)}{\left(0.020\ m\right)^2}\hat{\mathbf{i}} \quad = -34\,\hat{\mathbf{i}}\ N$$

Distance tripled $\rightarrow$ force drops by $1/3^2$.

$$\mathbf{F}_{12} = -\frac{34}{9}\hat{\mathbf{i}}\ N = -3.8\,\hat{\mathbf{i}}\ N$$

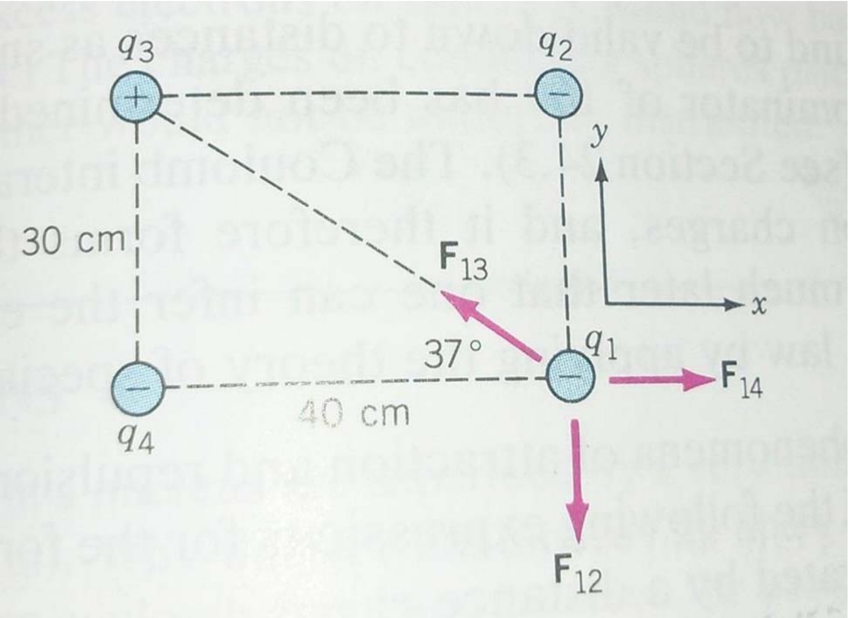
The Superposition Principle

- The electric force obeys the **superposition principle.**
  - That means the force two charges exert on a third force is just the vector sum of the forces from the two charges, each treated without regard to the other charge.
  - The superposition principle makes it mathematically straightforward to calculate the electric forces exerted by distributions of electric charge.
    - The net electric force is the sum of the individual forces.



Example

Find the net force acting on q1. $q_1 = -5.0\mu C,\ q_2 = -8.0\mu C,\ q_3 = 15\mu C, and\ q_4 = -16\mu C$

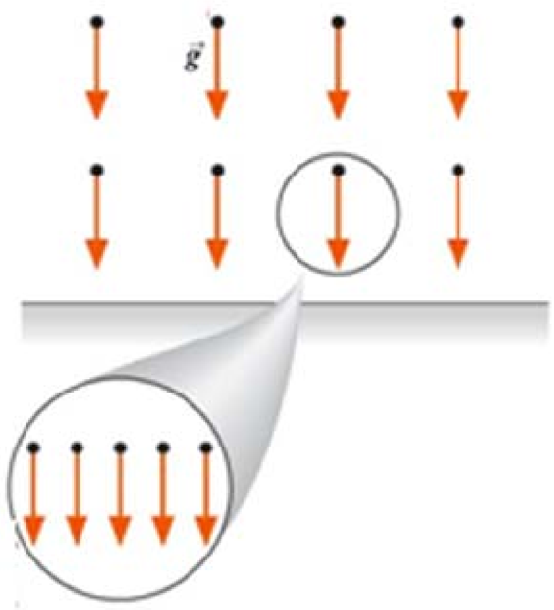


The Electric Field

Electric field **E** at **r** = Electric force on unit point charge at **r**.

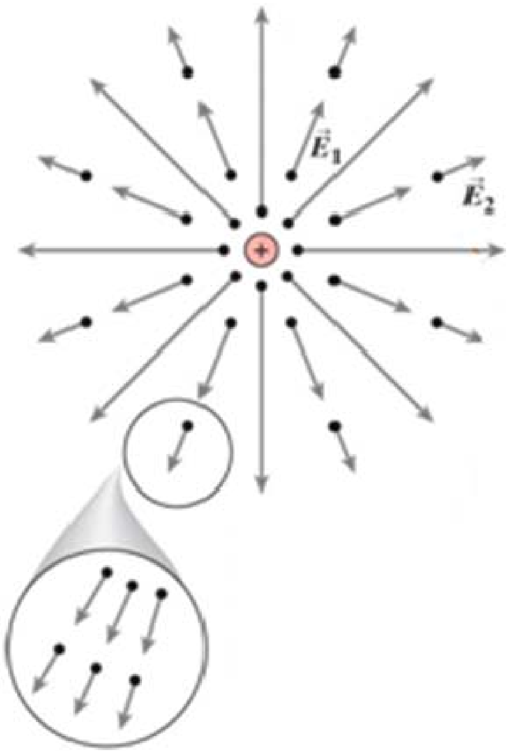
F = electric force on point charge *q*.

g = F/m



Gravitational field

E = F/q



Electric field

[*E*] = N / C
= V / m

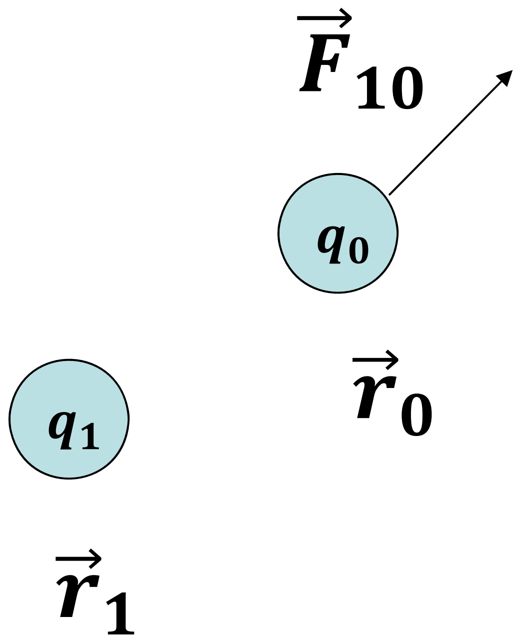
V = Volt

Implicit assumption:
q doesn't disturb **E**.

Rigorous definition:

$$\mathbf{E} = \lim_{q \to 0} \frac{1}{q} \mathbf{F}$$

The Electric Field



Let us consider the simplest case:

(1) a single, point charge q1 at rest in the position r1 as the source of the field.

(2) Its field is a mean for describing its action on other charges.
Let q0 be such a charge,
r0 its position

$$\mathbf{F}_{10} = q_0 \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{10}^2} \mathbf{u}_{10}$$

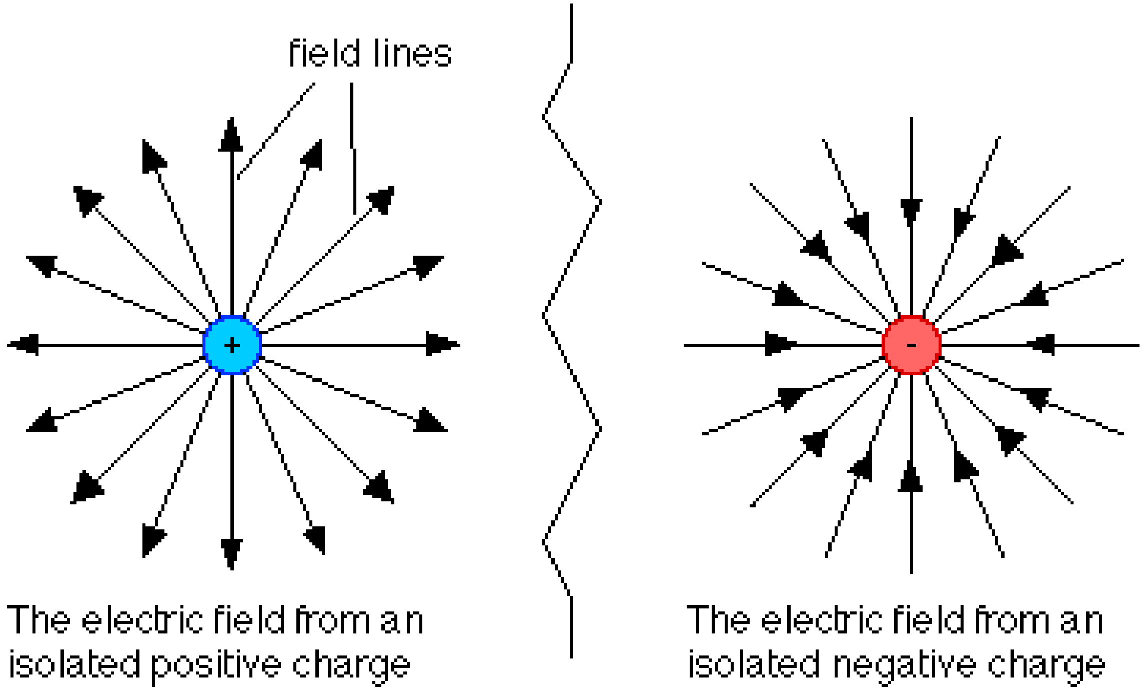
$$\mathbf{E}(\mathbf{r}_0) = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{10}^2} \mathbf{u}_{10}$$

$$\mathbf{F}_{10} = q_0 \mathbf{E}(\mathbf{r}_0)$$

Fields of Point Charges and Charge Distributions

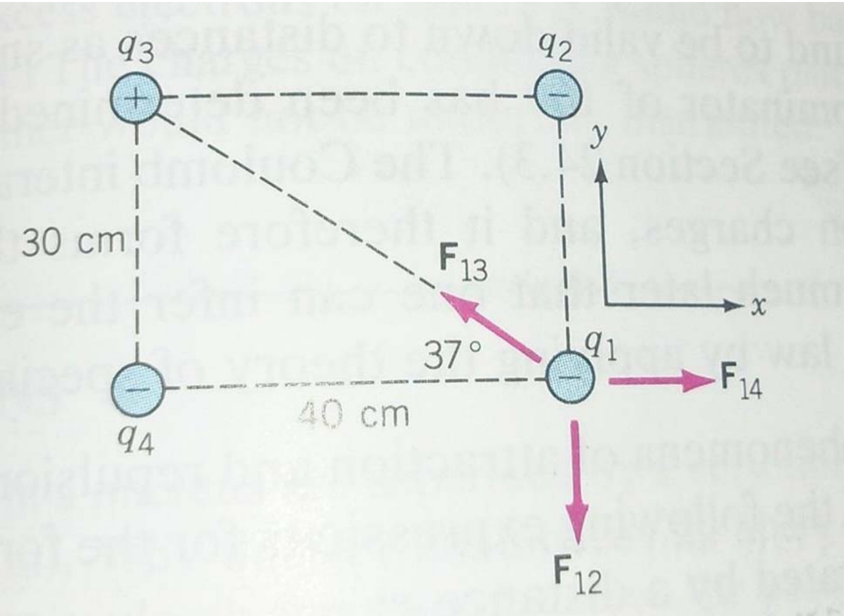
- The field of a point charge is radial, outward for a positive charge and inward for a negative charge.

$$\vec{E}_{\text{point charge}} = \frac{kq}{r^2}\hat{r}$$



Fields of Point Charges and Charge Distributions

- Superposition principle
  - the field due to a charge distribution is the vector sum of the fields of the individual charges.

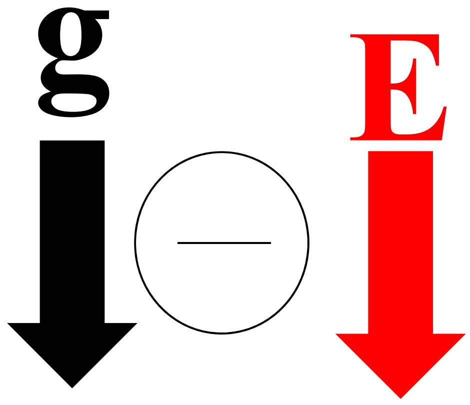


$$\mathbf{E}(\mathbf{r}_0) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{i0}^2}\mathbf{u}_{i0}$$

$$E_x(x_0, y_0, z_0) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i(x_0 - x_i)}{\left[(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2\right]^{3/2}}$$

Example

There is an electric field of approximate 100N/C directed vertically down at earth's surface. Compare the electrical and gravitational forces on an electron.



Gravity & Electric Force

The electric force is far stronger than the gravitational force,
yet gravity is much more obvious in everyday life.

Why?

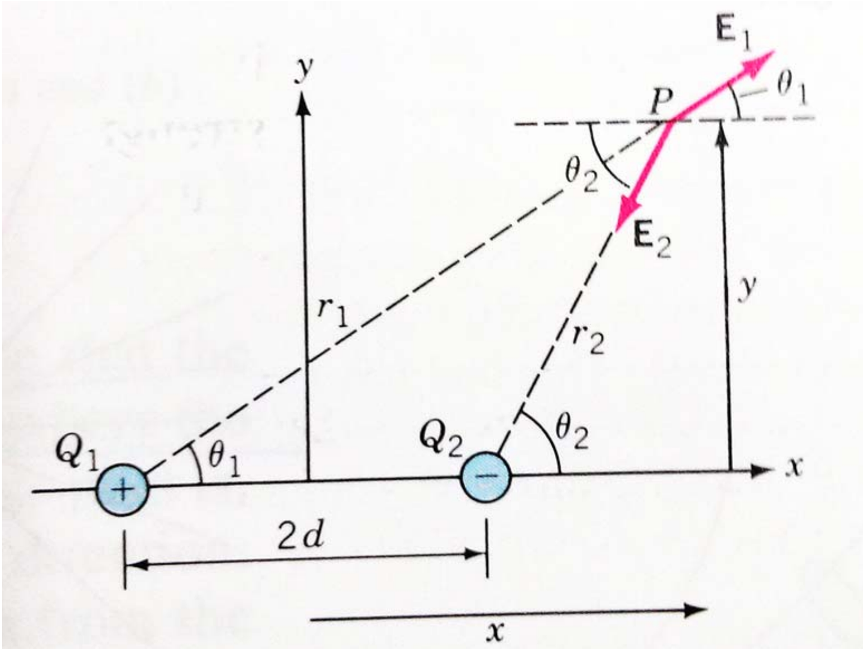
Only 1 kind of gravitational "charge"
$\rightarrow$ forces from different parts of a source tend to reinforce.

2 kinds of electric charges
$\rightarrow$ forces from different parts of a neutral source tend to cancel out.

Example

Let Q1 = 20 μC , Q2 = $-10\mu C$ and d = 1.0 m.

Find the field strength at P.

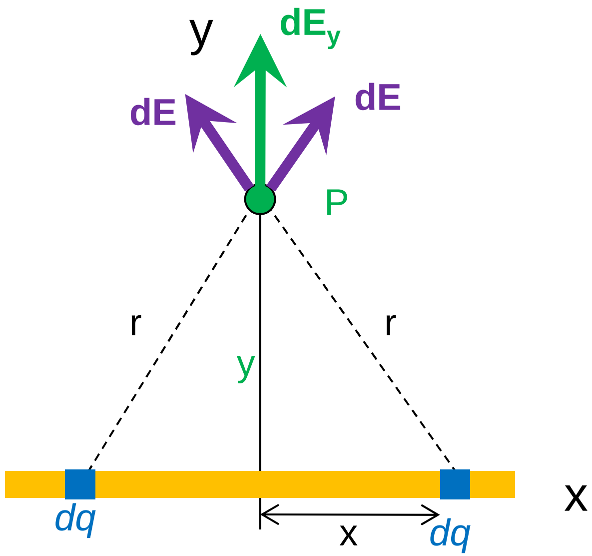


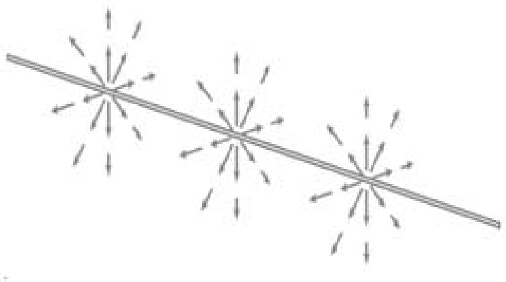
Example: Linear Charged Distribution

A long electric power line running along the *x*-axis

carries a uniform charge density λ [C/m].

Find **E** on the *y*-axis, assuming the wire to be infinitely long.





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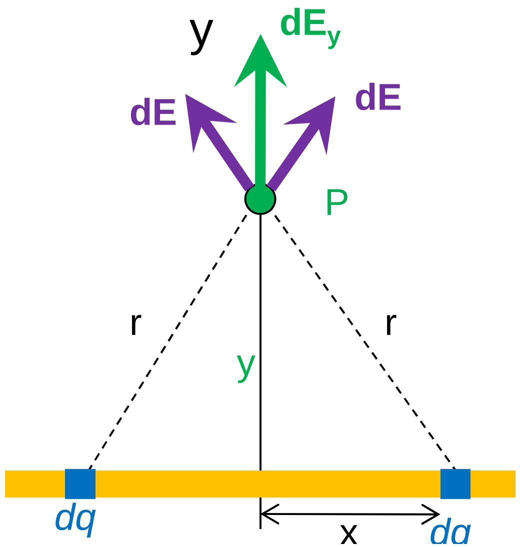

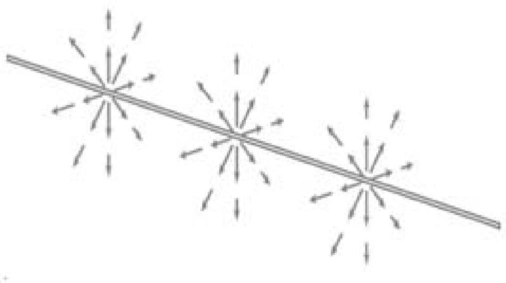

By symmetry, **E** has only *y*- component.

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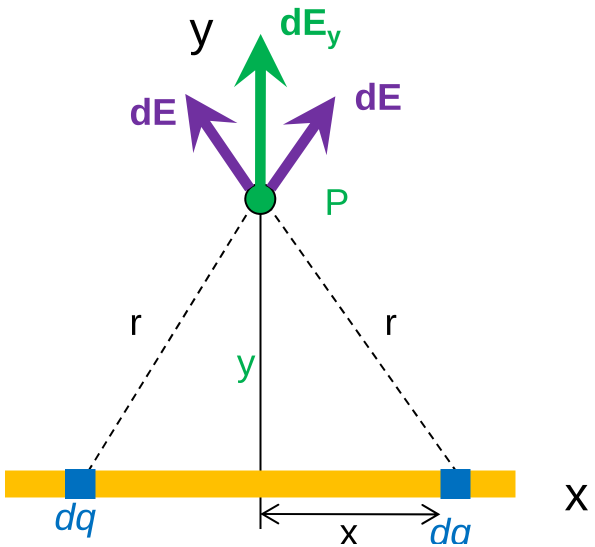
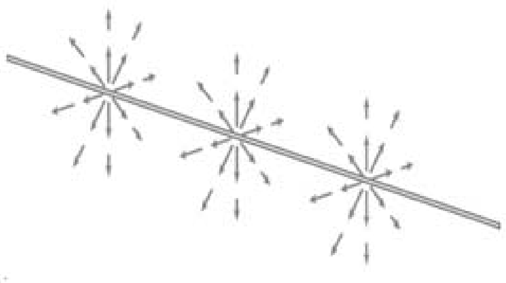




By symmetry, **E** has only *y*- component.

$$E_y = \int_{Line} dE_y = \int_{Line} \frac{k\,dq}{r^2}\left(\frac{y}{r}\right) = \int_{Line} \frac{k\,\lambda\,dx}{r^2}\left(\frac{y}{r}\right)$$

$$= k\,y\,\lambda \int_{-\infty}^{\infty} \frac{dx}{\left(y^2+x^2\right)^{3/2}} = k\,y\,\lambda\left[\frac{x}{y^2\sqrt{y^2+x^2}}\right]_{-\infty}^{\infty}$$

$$= k\,y\,\lambda\left[\frac{1}{y^2} - \left(-\frac{1}{y^2}\right)\right] = \frac{2\,k\,\lambda}{y}$$

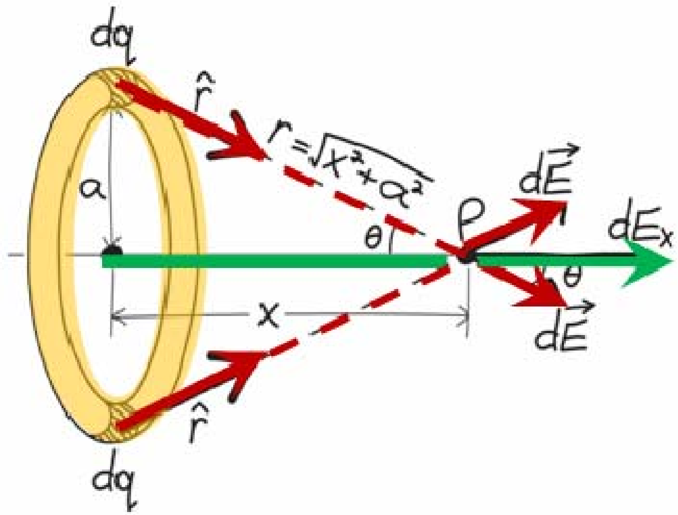
$$\boxed{\mathbf{E} = \frac{2\,k\,\lambda}{\rho}\;\hat{\boldsymbol{\rho}}}$$

Perpendicular to an infinite wire

Example: Charged Ring

A ring of radius *a* carries a uniformly distributed charge *Q*.

Find **E** at any point on the axis of the ring.



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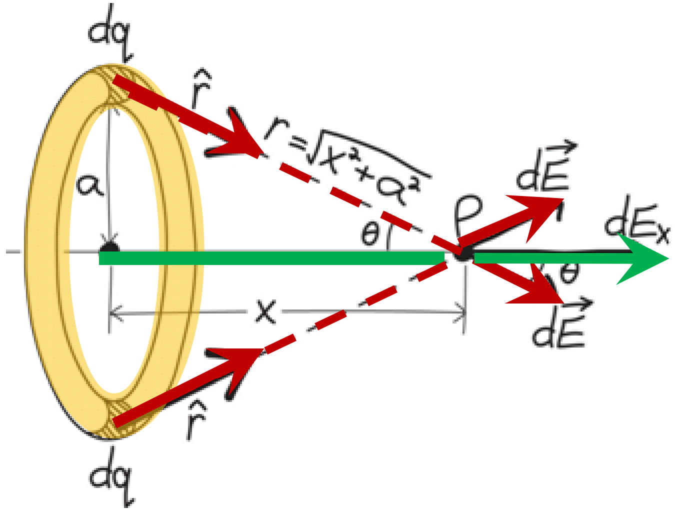


By symmetry, **E** has only axial (x-) component.

$$E_x = \int_{Ring} dE_x = \int_{Ring} \frac{k\,dq}{r^2}\left(\frac{x}{r}\right)$$

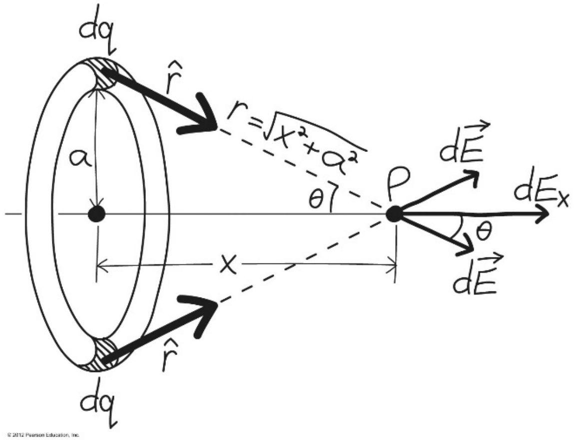
$$= \frac{k\,x}{\left(a^2+x^2\right)^{3/2}}\int_{Ring} dq = \frac{k\,Q\,x}{\left(a^2+x^2\right)^{3/2}}$$

$$\boxed{\mathbf{E} = \frac{k\,Q\,x}{\left(a^2+x^2\right)^{3/2}}\,\hat{\mathbf{i}}}$$

On axis of uniformly charged ring

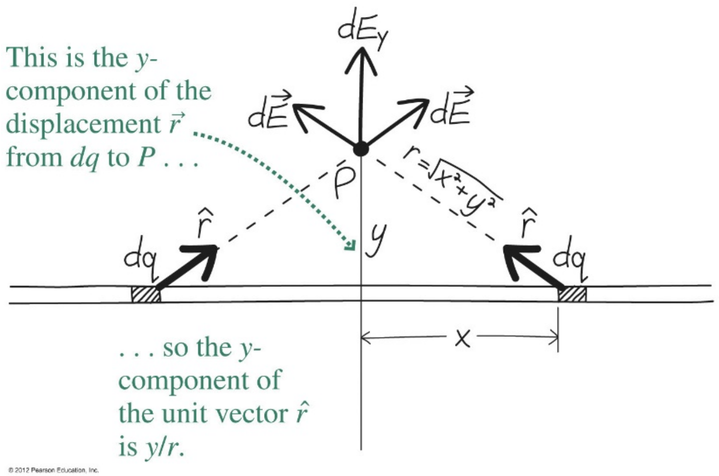
Two Examples

- The electric field on the axis of a charged ring:

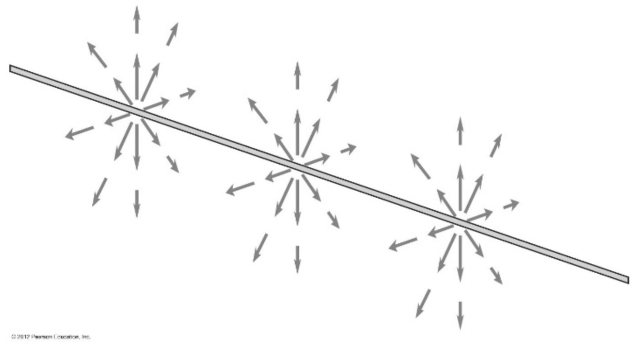


$$\vec{E}_{\text{on axis}} = \frac{kQx}{\left(x^2 + a^2\right)^{3/2}} \hat{i}$$

- The electric field of an infinite line of charge:
  – The line carries charge density λ C/m:

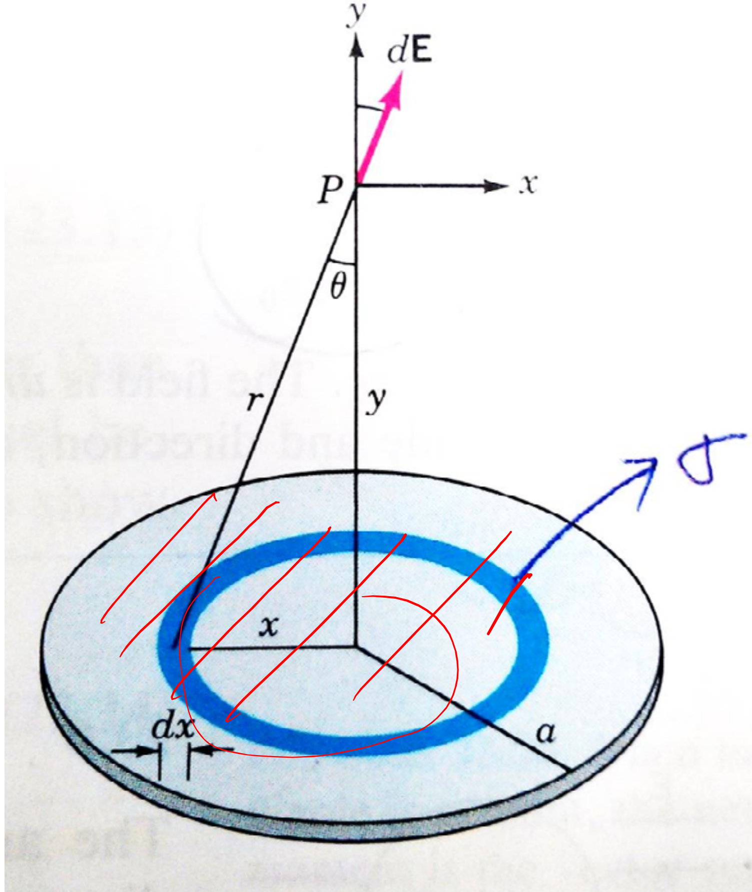


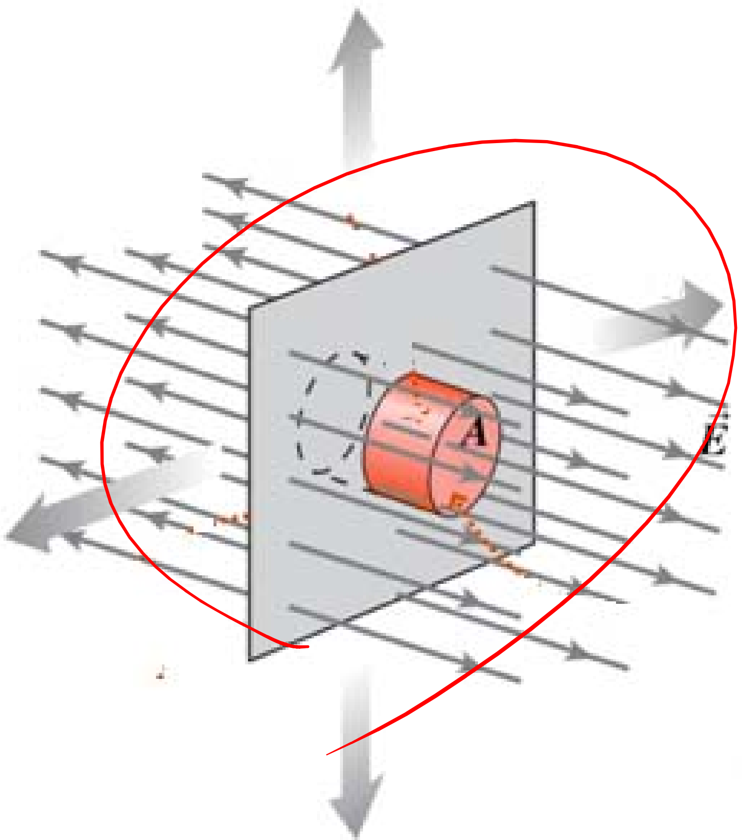
$$E = \frac{2k\lambda}{y}$$



direction radially outward for + charge; inward for ? charge

Example 23.8 and 23.9



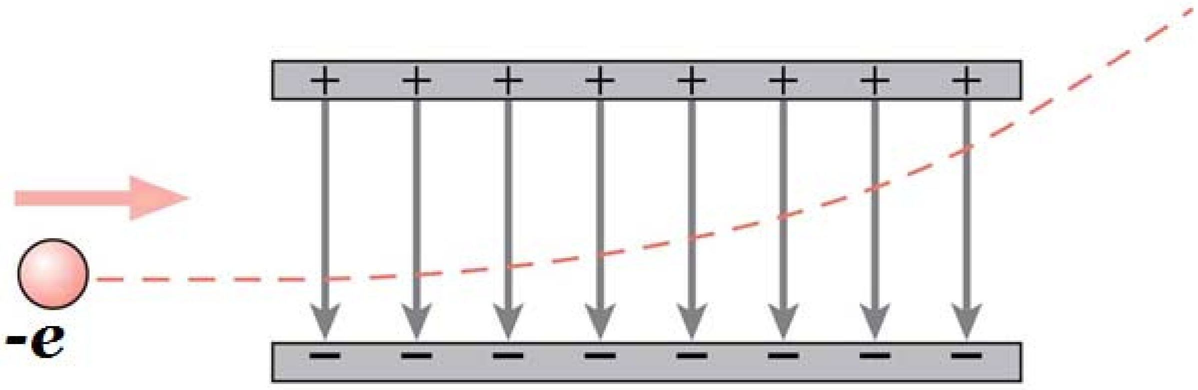


Matter in Electric Fields

- For a point charge q in an electric field $\vec{E}$
- Newton's law and the electric force combine to give acceleration:

$$\vec{a} = \frac{q\vec{E}}{m} = \frac{q}{m}\vec{E}$$

$\therefore$ **Trajectory determined by charge-to-mass ratio *q/m*.**



Constant E
$\rightarrow$ constant a.

E.g., CRT, inkjet printer, ….

Uniform field between charged plates (capacitors).

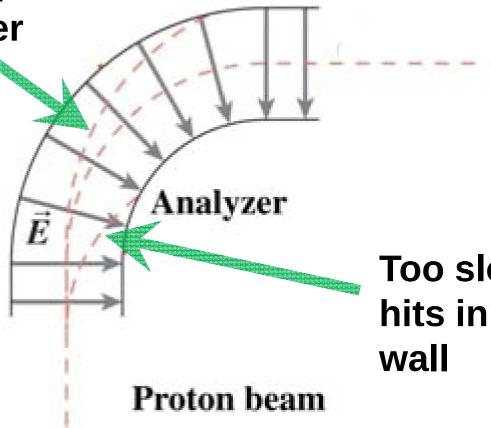
Example Electrostatic Analyzer

Two curved metal plates establish a field of strength $E = E_0\ (b/r)$, where E_0 & b are constants.

E points toward the center of curvature, & r is the distance to the center.

Find speed v with which a proton entering vertically from below will leave the device moving horizontally.

Too fast,
hits outer
wall

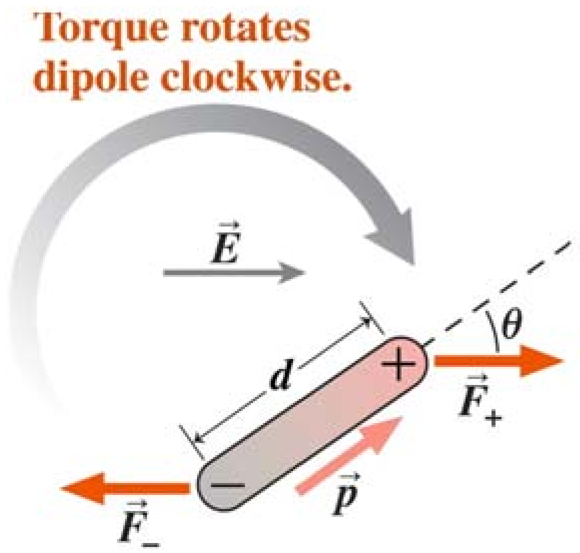


Too slow,
hits inner
wall

For a uniform circular motion:

$$m\frac{v^2}{r} = e\,E_0\,\frac{b}{r} \qquad \rightarrow \qquad v = \sqrt{\frac{e}{m}E_0\,b}$$

Dipole in a uniform electric field



Uniform E:

Total force: $\mathbf{F} = q\,\mathbf{E} + (-q)\,\mathbf{E} \;= \mathbf{0}$

Torque about center of dipole:

$$\boldsymbol{\tau} = \frac{d}{2}\,\hat{\mathbf{p}} \times (q\,\mathbf{E}) + \left(-\frac{d}{2}\right)\hat{\mathbf{p}} \times (-q\,\mathbf{E}) \;= d\,q\,\hat{\mathbf{p}} \times \mathbf{E}$$

$\boxed{\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}}$ $\quad \mathbf{p} = q\,d\,\hat{\mathbf{p}}$ **= dipole moment**

Work done by E to rotate dipole : $W = \int_{\theta_i}^{\theta_f} \mathbf{F} \cdot \hat{\mathbf{t}}\, r\, d\theta$ **t // tangent**

$$W = \int_{\theta_i}^{\theta_f} \left(-qE\sin\theta - qE\sin\theta\right)\frac{d}{2}\,d\theta \;= -p\,E \int_{\theta_i}^{\theta_f} \sin\theta\, d\theta \;= p\,E\left(\cos\theta_f - \cos\theta_i\right)$$

Potential energy of dipole in E ($\theta_i = \pi/2$) $\quad U = W = -p\,E\cos\theta_f \;= -\mathbf{p} \cdot \mathbf{E}$ $\quad$ **(U = 0 for p ⊥ E)**

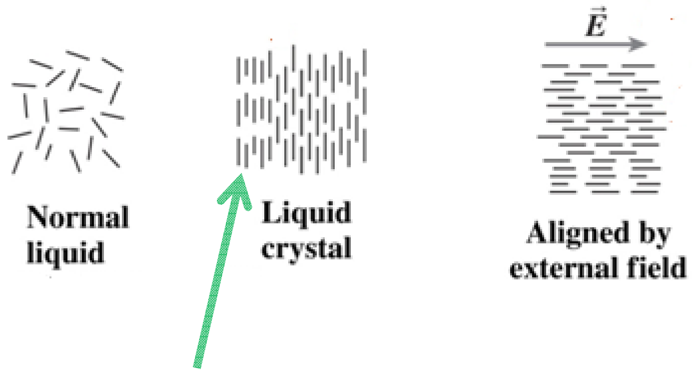
Application: Microwave Cooking & Liquid

Microwave oven:

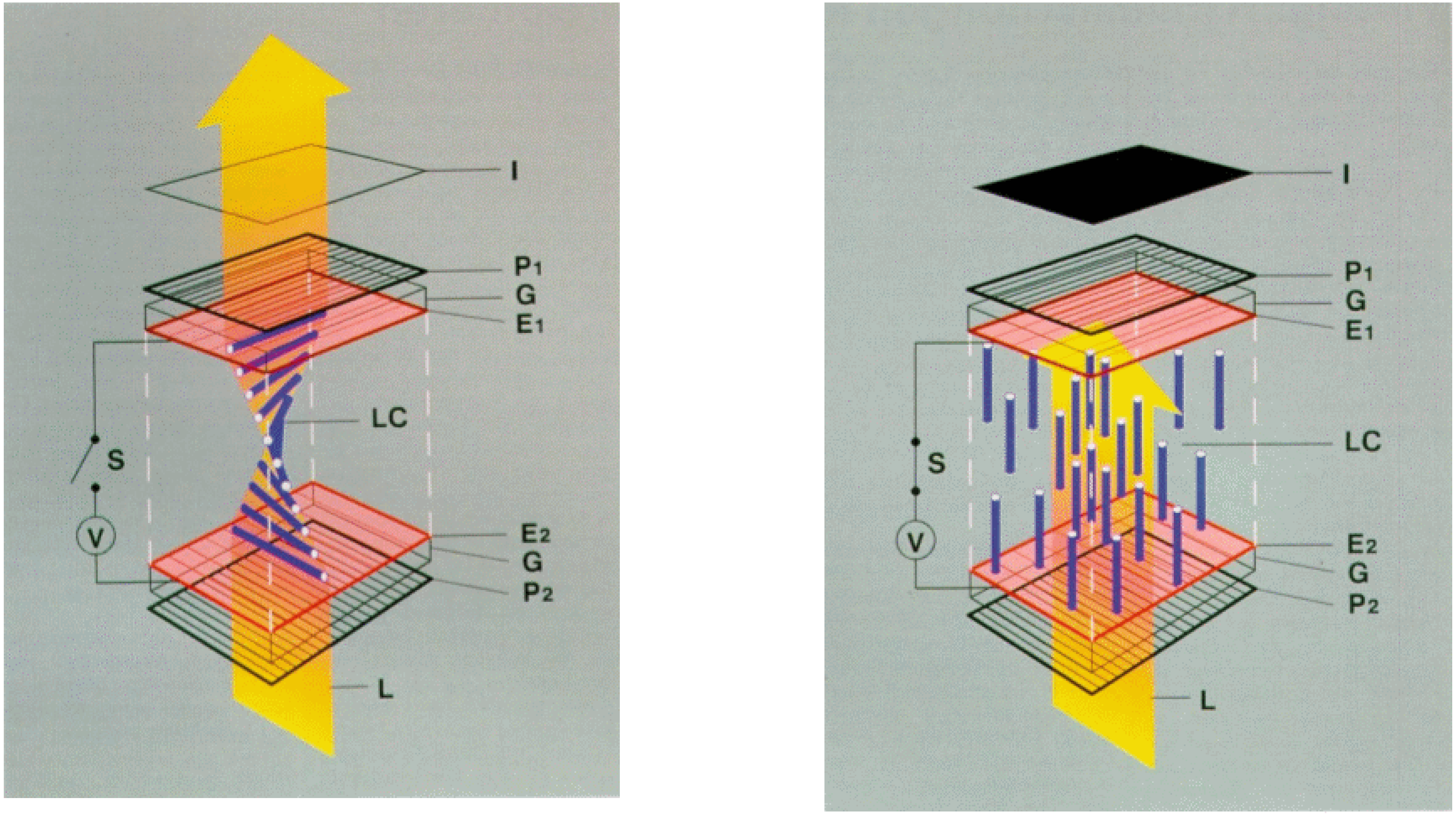
GHz EM field vibrates (dipolar) H_2O molecules in food

→ heats up.

Liquid Crystal Display (LCD)



dipolar molecules aligned but positions irregular



Exploded view of a TN (Twisted Nematic) liquid crystal cell showing the states in an OFF state (left), and an ON state with voltage applied (right)

Conductors, Insulators, and Dielectrics

- Materials in which charge is free to move are **conductors.**
- Materials in which charge isn't free to move are **insulators.**
  - Insulators generally contain molecular dipoles, which experience torques and forces in electric fields.
    - Such materials are called **dielectrics.**
  - Even if molecules aren't intrinsically dipoles, they acquire **induced dipole moments** as a result of electric forces stretching the molecule.
  - Alignment of molecular dipoles reduces an externally applied field.