Probability and Statistics – Quiz 2 Solution

1.

(a)

1. The mode, which is the point on the horizontal axis where the curve is a maximum, occurs at $x = \mu$.
2. The curve is symmetric about a vertical axis through the mean μ.
3. The curve has its points of inflection at $x = \mu \pm \sigma$; it is concave downward if $\mu - \sigma < X < \mu + \sigma$ and is concave upward otherwise.
4. The normal curve approaches the horizontal axis asymptotically as we proceed in either direction away from the mean.
5. The total area under the curve and above the horizontal axis is equal to 1.

(b)

Using Normal Curve From Table A.3

$z_1 = (171.25 - 174.5)/6.9 = -0.47$, $z_2 = (182.25 - 174.5)/6.9 = 1.12$.
$P(171.25 < X < 182.25) = P(-0.47 < Z < 1.12) = 0.8686 - 0.3192 = 0.5494$.
Therefore, $(1000)(0.5494) = 549$ students.

2.

(a)

$$M_X(t) = \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x q^{n-x} = \sum_{x=0}^{n} \binom{n}{x} (pe^t)^x q^{n-x}.$$

Recognizing this last sum as the binomial expansion of $(pe^t + q)^n$, we obtain

$$M_X(t) = (pe^t + q)^n.$$

(b)

Now

$$\frac{dM_X(t)}{dt} = n(pe^t + q)^{n-1}pe^t$$

and

$$\frac{d^2M_X(t)}{dt^2} = np[e^t(n-1)(pe^t + q)^{n-2}pe^t + (pe^t + q)^{n-1}e^t].$$

Setting $t = 0$, we get

$$\mu_1' = np \text{ and } \mu_2' = np[(n-1)p + 1].$$

Therefore,

$$\mu = \mu_1' = np \text{ and } \sigma^2 = \mu_2' - \mu^2 = np(1-p) = npq,$$

3.

According to Lecture07 slide(page.17)-Approximation of Binomial Distribution by a Poisson Distribution

Theorem 5.5: Let X be a binomial random variable with probability distribution $b(x; n, p)$. When $n \to \infty$, $p \to 0$, and $\mu = np$ remains constant. $b(x;n,p) \to p(x;\mu)$.

Therefore, Using Poisson distribution from Table A.2

μ = 10.

we have P(X<=5) = 0.0671(r=5) , P(X<=8) = 0.3328(r=8).

$\mu = np = (10000)(0.001) = 10$, so

$$P(6 \le X \le 8) = P(X \le 8) - P(X \le 5) \approx \sum_{x=0}^{8} p(x;10) - \sum_{x=0}^{5} p(x;10) = 0.2657.$$