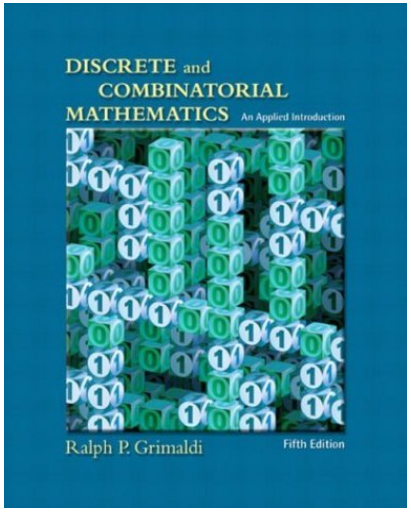
# Discrete Mathematics

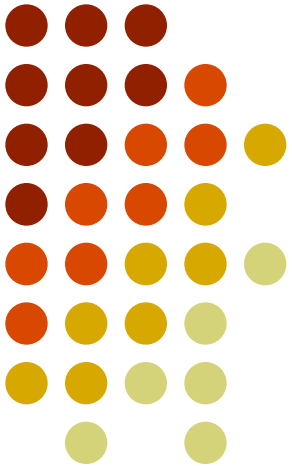
## *Syllabus & Introduction*



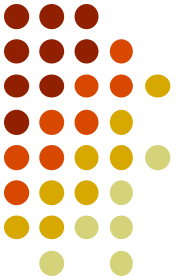


***Hung-Yu Kao***

***NCKU CSIE***



# Instructor Information

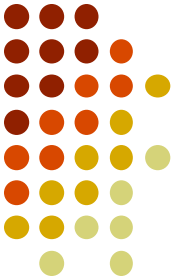


- Instructor: 高宏宇 (Hung-Yu Kao)


- Office Room: 資訊新系館12樓C11
- Tel: (06) 2757575 ext. 62546
- E-mail: hykao@mail.ncku.edu.tw
- Web Site: http://myweb.ncku.edu.tw/~hykao

- Office Hour: 每週三 2pm ~ 5pm

# Course Information

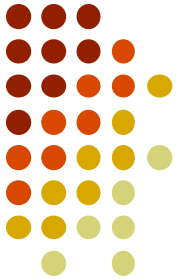


- Course Web URL
  - http://moodle.ncku.edu.tw/course/view.php?id=72781
  - Lecture notes : PDF files
- TA
  - 資訊新系館9樓903智慧型知識管理實驗室
  - E-mail: nckudm@gmail.com

# 本課程核心能力

- 具備基礎專業數學及資訊理論知識之基本能力
- 具備理論推導及實驗數據分析歸納之能力
- 具備終身學習之能力
- 具備發掘、分析及解決資訊應用問題之能力

# Take a look …



$$1 * 8 + 1 = 9$$

$$12 * 8 + 2 = 98$$

$$123 * 8 + 3 = 987$$

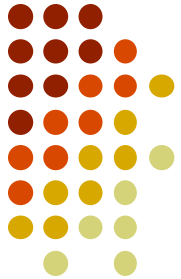
$$1234 * 8 + 4 = 9876$$

$$12345 * 8 + 5 = 98765$$

$$123456 * 8 + 6 = 987654$$

*Theorem is art, art is theorem.*

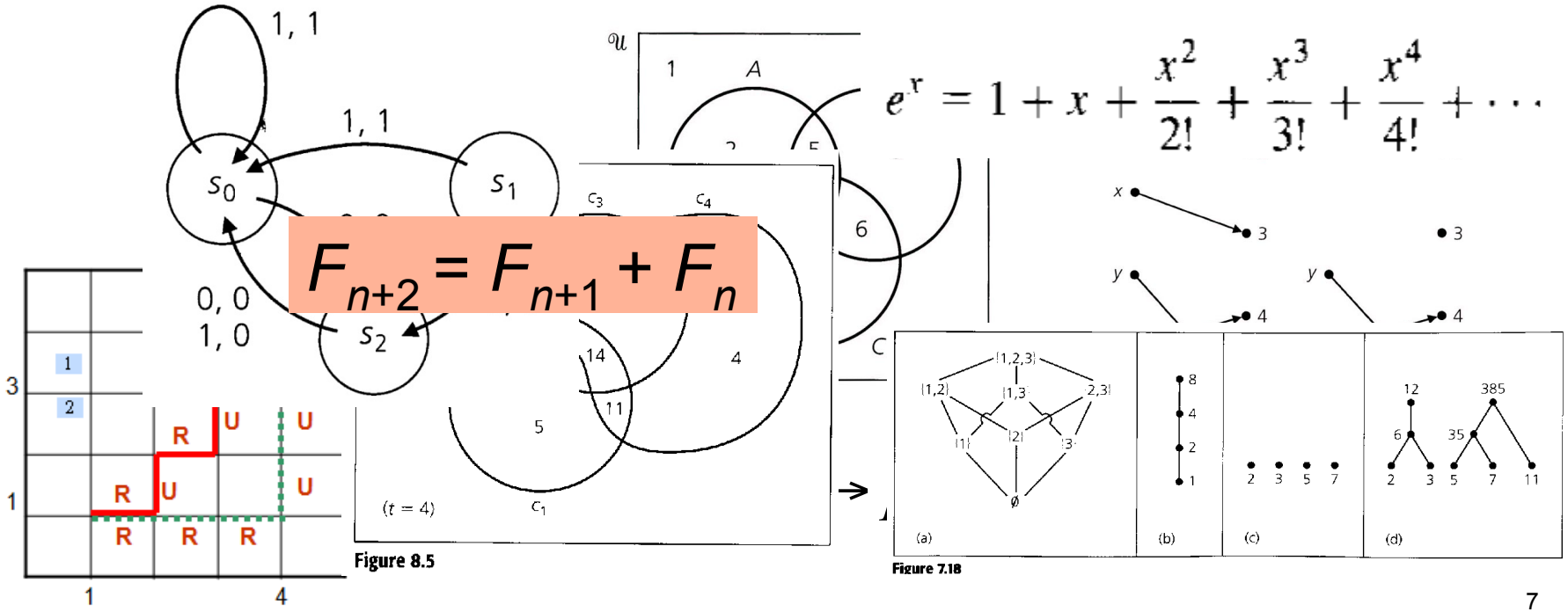
# What is Discrete Mathematics?



- 數學的幾個分支的總稱，以研究離散量的結構和相互間的關係為主要目標
  - 研究對象: 一般地是有限個或可數無窮個元素
  - 內容包含：數理邏輯、集合論、代數結構、圖論、組合學、數論等。(from Wikipedia)
- 研究有離散結構的系統的學科
  - e.g, 班級人數, 課程安排, ————
- 描述了電腦科學離散性的特點
- 基礎核心部分: 組合學（計數、排列、組合結構的分析）和圖論

# Why call “discrete”

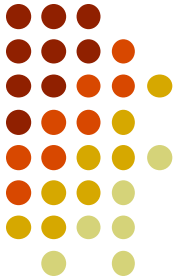
- discrete: 不連續的，分離的，抽象的
- v.s. continuous (e.g., calculus)



# Things to Expect

- 廣泛地介紹計算機使用之數學相關之理論及應用，其中包括
  - 離散方法
  - 代數學
  - 邏輯
  - 組合論
  - 圖論
  - 有限狀態機
  - 演算法分析
  - …

# Main Goal: Counting



- Capable of solving difficult problems.
  - Coding theory, probability and statistics, complexity analysis

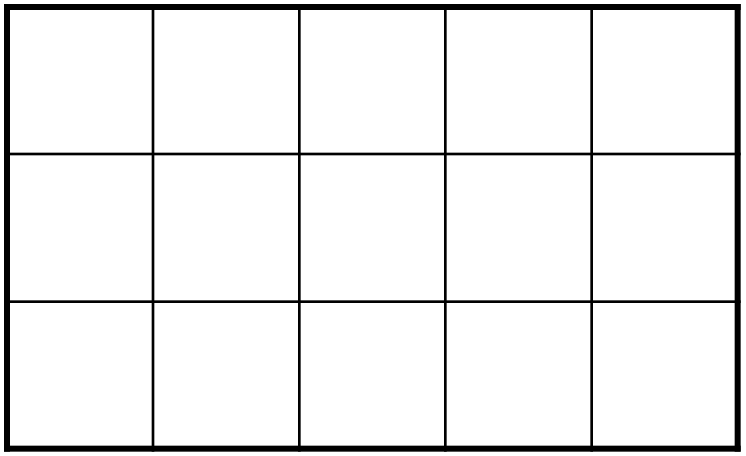
- Help the analysis and design of efficient algorithms.
- E.g.,

$$1+2+3=?$$

$$\frac{3\times 4}{2}=?$$

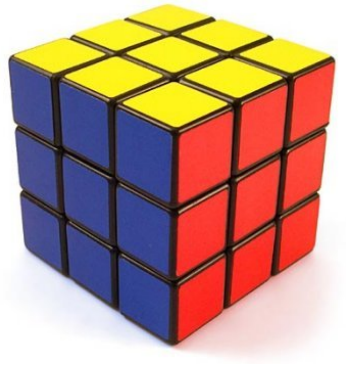
# Counting Example 1

- How many rectangles in this graph?

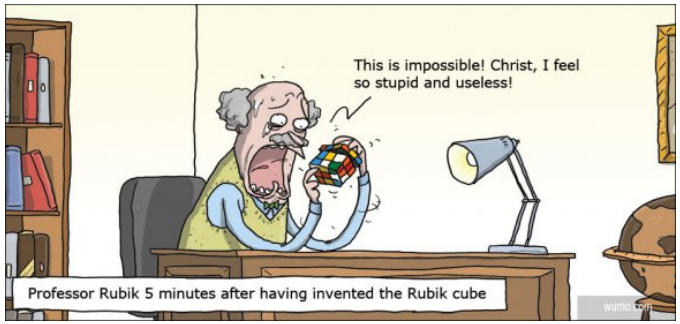


$$Ans = \binom{4}{2} * \binom{6}{2} - \#squares$$

# Counting Example 2

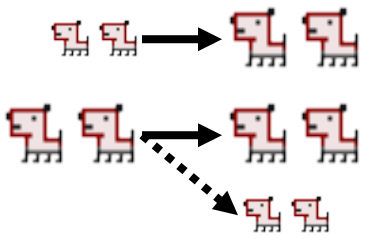


- 魔術方塊是1974年，由匈牙利布達佩斯的建築系教授 Erno Rubik發明的。
- 所有可能三階魔術方塊顏色分布數？
  - 4,325,003,274,489,856,000
  - $8!*12!*3^8*2^{12} / (2*2*3)$
- God's number = 20
- 35 CPU years is completed in several weeks by Google engineer John Dethridge



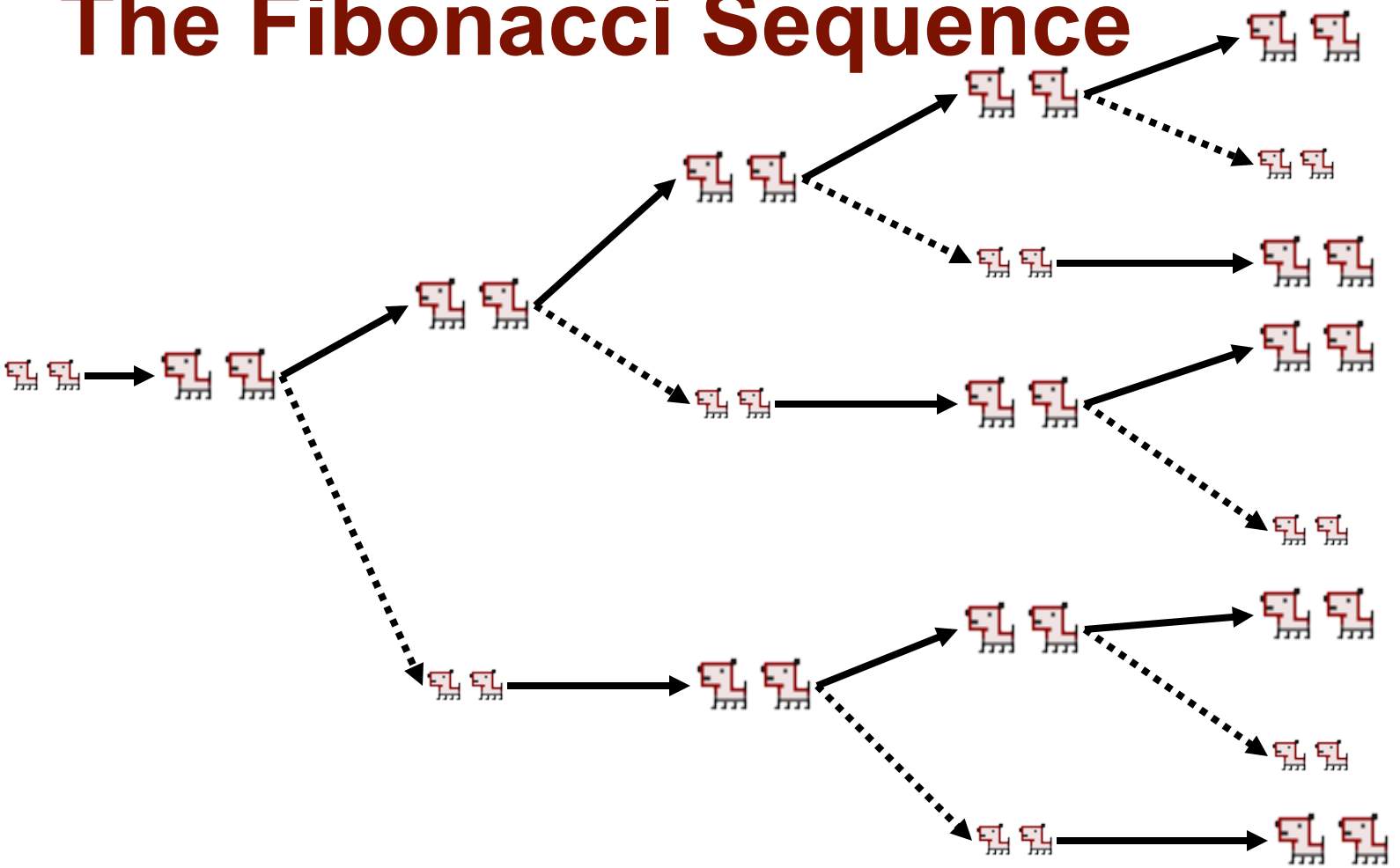
# Counting Example 3

- Fibonacci Sequence: 1, 1, 2, 3, 5, 8, 13, …
  - By Fibonacci (Leonardo of Pisa, or Filius Bonaccii "son of Bonaccio", p442), an mathematician in the 13th century
  - Counting rabbits
  - rules:



*How many rabbits after some period of time ?*

# The Fibonacci Sequence



1 pair $F_1$ | 1 pair $F_2$ | 2 pairs $F_3$ | 3 pairs $F_4$ | 5 pairs $F_5$ | 8 pairs $F_6$

# The Fibonacci Sequence

- $F_n$ represents the number of rabbits in period $n$
- $F_n = F_{n-1} + F_{n-2} = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$ 用無理數算出自然數!
  - $F_{n-1}$ : *the number of* ***adult rabbits*** *at time period n = the number of rabbits (adult and baby) in the previous time period*
  - $F_{n-2}$ : *the number of* ***baby rabbits*** *at time period n = the number of adult rabbits in* $F_{n-1}$, *which is* $F_{n-2}$

$$\text{Golden ratio} = \lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \frac{1+\sqrt{5}}{2}$$

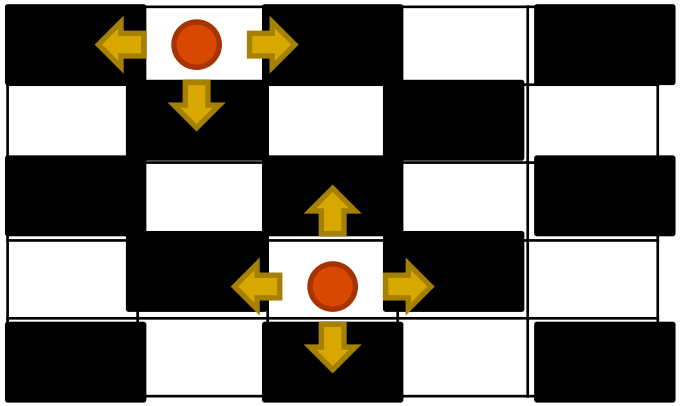
***Q: can you model the rabbit number if the adult rabbit will die after m periods when they create a new pair of rabbits ?***

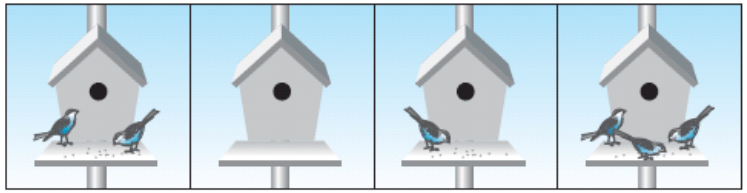
***e.g., Fn = Fn-1 when m=1***

# Counting Example 4

- 班上有25個人，坐在教室5x5的座位上。新學期開始要重新編排座位。**能不能**使得每個人換完座位後都做在原來位置的旁邊(前後左右皆可)?

*From 國際數學奧林匹亞競賽, International Mathematical Olympiad (IMO)*





# Counting Example 5

- 丟 $2n$ 次銅板，正面恰好出現 $n$ 次的機率有多大？
  - Ans: $P_{2n}=C(2n, n)/2^{2n} =(2n)!/n!n!\ 2^{2n}$
  - P=? 當n無窮大
    - ½? 1? **0**?
  - First, you should know "Stirling formula" and prove this $\lim_{n\to\infty} \frac{n!}{\sqrt{2\pi n}n^n e^{-n}} = 1$ *(Stirling 公式，1730年)*

*Ref: http://episte.math.ntu.edu.tw/articles/mm/mm_17_2_05/index.html*

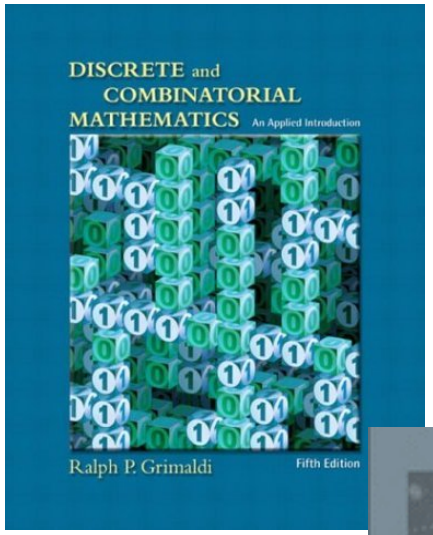
# Counting Example 6

- 假設有*n*個人。從中挑選若干人（任意數量）出來。將挑選出來的人分成若干（任意數量）組。將任意一個這樣分出來的組稱之為一個“系統”。那麼一共可以有多少個兩兩不同的系統呢?

# Textbook

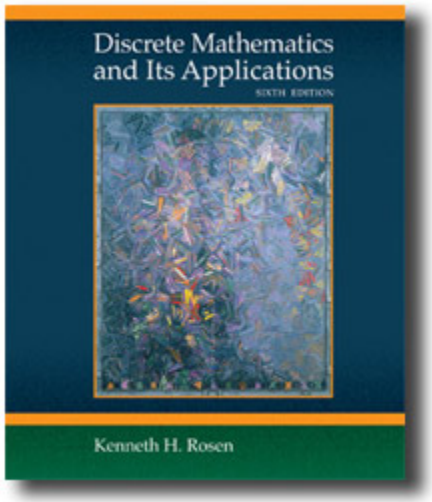
- Ralph P. Grimaldi, "**Discrete and Combinatorial Mathematics:** An Applied Introduction," 5/e, Pearson Education, 2004.

- 2014 International Ed. of 5th revised ed., 歐亞書局代理
  - ISBN: 9781292022796
  - http://www.eurasia.com.tw/product/books_detail.asp?bokno=1279250P119

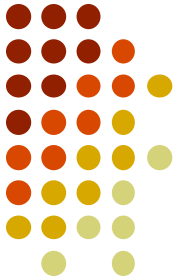



# Reference

- "Discrete Mathematics and Its Applications", Kenneth H. Rosen, 6th Edition
- http://www.mhhe.com/math/advmath/rosenindex.mhtml

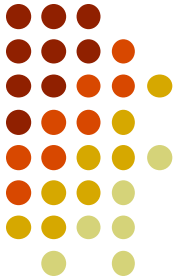


# Something about Textbook



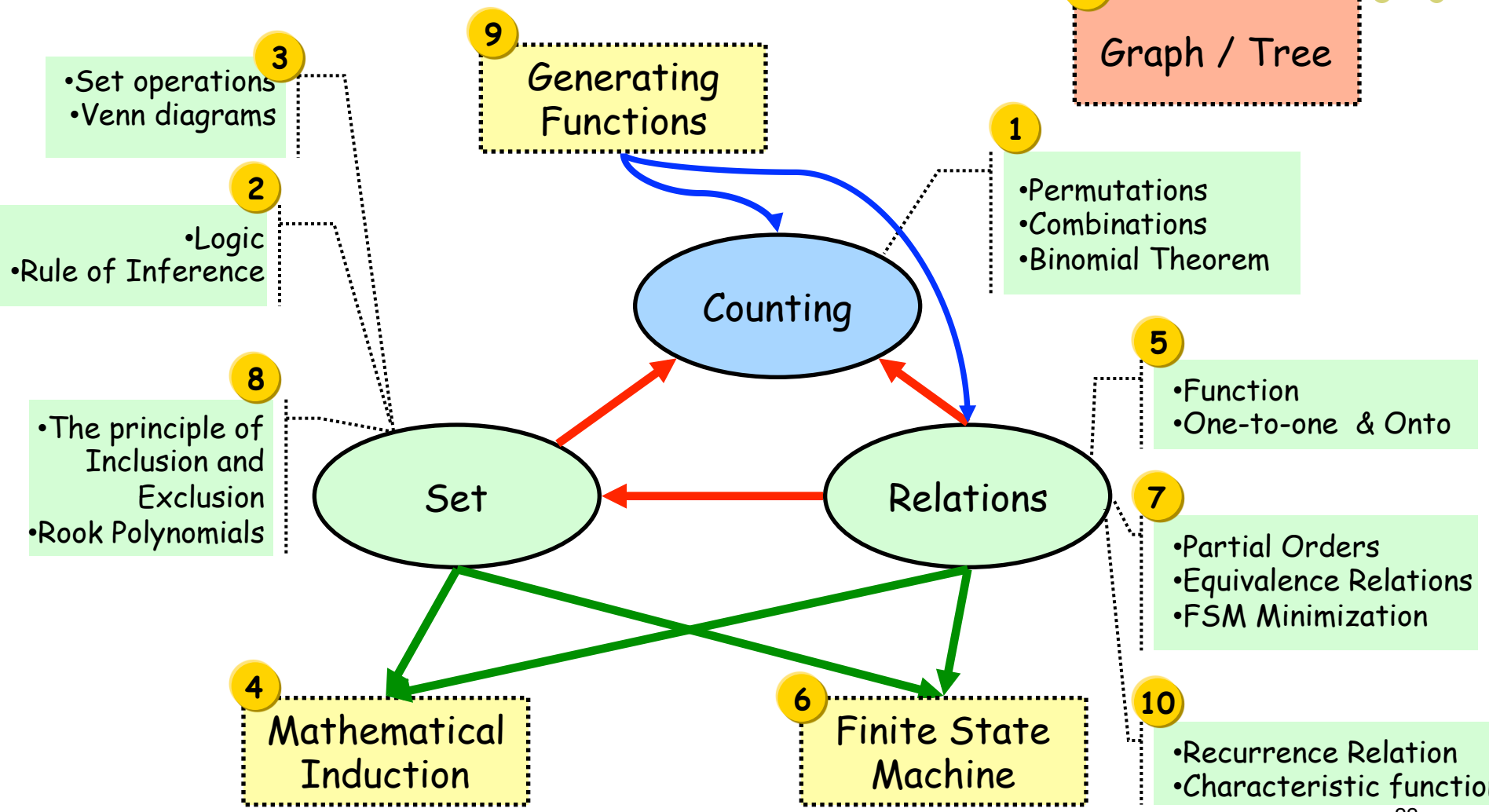
- 4 major areas
  - Chapter **1-7**: the fundamental of discrete mathematics
  - Chapter **8-10**: combinatorics analysis
  - Chapter **11, 12, 13**: the theory and applications of finite graphs
  - Chapter 14-17: the applied algebra
- More than 1900 exercises in 17 chapters

# Syllabus



- Fundamental of Discrete Mathematics
  - Ch1: Counting (week 1-2) [35 slides]
  - Ch2: Logic (week 2-3) [61 slides]
  - Ch3: Set (week 4-5) [29 slides]
  - Ch4: Mathematical Induction (week 6-8) (Midterm1) [62 slides]
  - Ch5: Relations and Functions (week 8-9) [79 slides]
  - Ch6: Finite State Machines (week 10-11) [32 slides]
  - Ch7: Relations: The Second Time Around (week 12-13) (Midterm2) [61 slides]
- Enumeration
  - Ch8: Inclusion and Exclusion (week 14-15) [39 slides]
  - Ch9: Generating Functions (week 16) [44 slides]
  - Ch10: Recurrence Relations (week 17) [69 slides]
- Graph
  - Graph Theory (week 18)
  - Trees (week18)

# Chapter interconnection

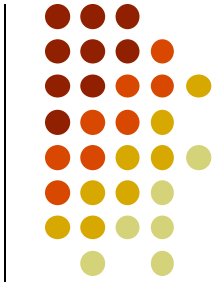


# Grading / Schedule

- **1 holiday** (4/1[week6])

- **Homework 20%**
- **2 Middle Exams**: **50%** (4/8[week7], 5/20[week13])
- **Final Exam**: **30%** (6/24[week18])
- All scores are relative, not absolute.
  - progress bonus
- About考試請假

# Homework

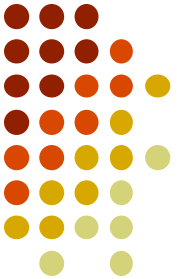


- Content
  - Textbook (50%)
  - Advance / new (30%)
  - Your originality (20%)
- 抄襲?

# For your information

| Year | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|---|---|---|
| Total students | 129 | 131 | 124 | 120 | 126 | 126 | 125 |
| >90 | 12 | 9 | 20 | 17 | 5 | 5 | 2 |
| <60 | 9 | 7 | 10 | 15 | 11 | 19 | 15 |
| Pass% | 93.0% | 94.7% | 93.5% | 87.5% | 91.3% | 84.9% | 88.0% |

# How to get a high score in this course?



- 讀原文書
- 課前預習
  - 大略式讀法
- 上課記下心得與聽不懂的地方
  - 還有精彩的地方
- 課後複習
  - 讀課本, 講義只是備用
  - 例題看完自己從頭演練一遍
  - 讀完立刻練習 (作習題)
  - 善用課堂錄音, 助教, 老師

# 教學改善

- 老師的office hour為星期三 2pm-5pm
- 期中學生問卷
- 期末學生問卷

- 有任何教學和學習上的問題和建議, 請與老師討論和反映
  - 有問題就問
  - 沒問題也可以問
  - 課堂上, 老師辦公室, email, …