Discrete Mathematics (2009 Spring) Midterm II

1. **(30 points)** For each of the following statements, **determine** and **explain** whether it is correct or not.
   (1). The string 00010 is in the language {00}{0}*{10} also in {000}*{1}*{0}.
   (2). Let (A, **R**) be a poset. If (A, **R**) is a lattice, then it is a total order.
   (3). If A={1, 2, 3, 4} and there are 1680 injective functions f: A→B, then |B|=8.
   (4). The maximal number of rolling a single die to get the same score at least 4 times is 25.
   (5). Function f denotes a closed binary operations on $\mathbf{P}(\mathbf{Z}^+)$. For A, B$\subseteq\mathbf{Z}^+$, f(A, B)=A$\cap$B then f is one-to-one.
   (6). Two states are not 2-equivalent if and only if they are not 3-equivalent.

2. **(20 points, 5+5+5+5)** Let A={a, b, c, d}. Determine the following values (a) the number of functions f: A x A→A, (b) the number of closed binary operations f on A satisfy f(a, b)=b and have an identity, (c) the number of closed binary operations in (b) are commutative, (d) the number of relations on A that are reflexive and symmetric but not transitive.

3. **(10 points)** Suppose R is an equivalence class relation on {1, 2, 3, 4, 5, 6} and the equivalence class induced by R are {1, 5, 6}, {2, 4}, {3}. What is the value of |R|?

4. **(15 points)** Let **I** = **O** = {0, 1}. Construct a state diagram for a finite state machine that reverses (from 0 to 1 or from 1 to 0) the symbols appearing in the *4kth* and *(4k+1)th* positions of an input string x$\in\mathbf{I}^+$, where k≥1. For example, if s_0 is the starting state, then $w(s_0, 00010111)=00001110$.

5. **(15 points, 5+10)** Let p, q, r be three distinct primes. We denote relation $x\mathbf{R}y$ if x divides y. Under this relation **R**, (1) please draw the Hasse diagram of all positive divisors of p^3q^2; (2) and answer how many edges are there in the Hasse diagram of all positive divisors of $p^mq^nr^k$, m, n, k $\in$Z+.

6. **(5 points)** Considering the Stirling number of the second kind S(m, n), we have S(7, 4)=350, S(6, 4)=65, S(6,5)=15. What is S(8, 5) ?

7. **(15 points, 10+5)** (1) Apply the minimization process to the finite state machine in the following state table. (2) What is the minimal distinguishing string for s_1 and s_5?

	v		ω	
	0	1	0	1
s_1	s_6	s_3	0	0
s_2	s_3	s_1	0	0
s_3	s_2	s_4	0	0
s_4	s_7	s_4	0	0
s_5	s_6	s_7	0	0
s_6	s_5	s_2	1	0
s_7	s_4	s_1	0	0