DISCRETE MATHEMATICS – CH9 Homework9

9.2

2. (10 pts) Determine the sequence generated by each of the following generating functions.

c) $f(x) = x^3/(1-x^2)$

e) $f(x) = 1/(3-x)$

(c) $f(x) = x^3/(1-x^2) = x^3[1 + x^2 + x^4 + x^6 + \ldots] = x^3 + x^5 + x^7 + x^9 + \ldots$, so $f(x)$ generates the sequence $0, 0, 0, 1, 0, 1, 0, 1, 0, 1, \ldots$

(e) $f(x) = 1/(3-x) = (1/3)[1/(1-(x/3))] = (1/3)[1 + (x/3) + (x/3)^2 + (x/3)^3 + \ldots]$, so $f(x)$ generates the sequence $1/3, (1/3)^2, (1/3)^3, (1/3)^4, \ldots$

9.4

6. (10 pts) (a) Find the exponential generating function for the number of ways to arrange n letters, n ≥ 0, selected from each of the following words. ii) MISSISSIPPI (b) For section (ii) of part (a), what is the exponential generating function if the arrangement must contain at least two I's?

(a)(ii) $(1+x)(1+x+(x^2/2!))(1+x+(x^2/2!)+(x^3/3!)+(x^4/4!))^2$

(b) $(1+x)\cdot(1+x+(x^2/2!))\cdot(1+x+(x^2/2!)+(x^3/3!)+(x^4/4!))\cdot((x^2/2!)+(x^3/3!)+(x^4/4!))$.

Advanced assignment

1. (20 pts) Give some (a) strength and (b) weakness of generating function when it is applied on counting problems. (each correct answer get 5 pts)