2016 Fall Algorithm Homework 2

1. Prove that COUNTING-SORT is stable.

2. Illustrate the operation of RADIX-SORT on the following list of English words: COW, DOG, SEA,RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX.

3. Please show the tables produced by LCS-LENGTH on the sequences X = {A, B, C, B, D, A, B} and Y = {B, D, C, A, B, A}

4. Determine the (a) cost and (b) structure of an optimal binary search tree for a set of n = 5 keys with the following probabilities:

i	1	2	3	4	5
P_i	0.25	0.15	0.2	0.35	0.05

5. Suppose that you have 6 matrices: A_1 has dimension 30×35, A_2 has dimension 35×15, A_3 has dimension 15×5, A_4 has dimension 5×10, A_5 has dimension 10×20, A_6 has dimension 20×25. Please use matrix-chain multiplication to calculate the minimum number of scalar multiplications.

6. Given a chain$\langle A_1, A_2, A_3, A_4 \rangle$ of 4 matrices and their matrix dimensions:

   $A_1 : 3 \times 5$, $A_2 : 5 \times 2$, $A_3 : 2 \times 6$, $A_4 : 6 \times 4$. Please compute the

   minimum number of scalar multiplications to multiply them.

7. What are the two key ingredients that an optimization problem must have in

   order for dynamic programming to be applicable?

8. In the Knapsack problem, if the size of each object is arbitrary real number,

   does the dynamic programming method still work? Explain your answer

9. Determine an LCS of $\langle 1,0,0,1,0,1,0,1 \rangle$ and $\langle 0,1,0,1,1,0,1,1,0 \rangle$.

10. Consider the knapsack problem consists of 3 items, and the capacity of the

    knapsack is equal to 8. The profits and weights of the three items are (p1, p2,

    p3) = (8, 6, 3) and (w1, w2, w3) = (6, 5, 3), respectively.

    (a) Assume that you are allowed to put in a fraction of an item. Use the

    greedy method to solve for the maximum profit and show the items to be

    included in the knapsack.

    (b) Now suppose that you must take each item as a whole (i.e 0/1

    knapsack problem). Show how you can use dynamic programming to solve

    the problem. What are the total profit and the list of items to be included in

    the knapsack?