NCKU CSIE Discrete Mathematics (2014 Spring) Midterm I *(total 105 pts)*

1. (*30 pts*) For each of the following statements, **determine** and **explain** whether it is correct or not.
   (1). $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ *and* $\{\emptyset\} \in \{\emptyset, \{\emptyset\}\}$
   (2). There are two sets A and B, where A-B={1, 3, 7, 11}, B-A={2, 6, 8}, and A∩*B*={4, 9}. The number of members in the set $A \cup B$ is 9.
   (3). If 17 | 2a+3b then 17 | 9a+5b.
   (4). $2\binom{n}{0} + \binom{n}{1} + 2\binom{n}{2} + \binom{n}{3} + 2\binom{n}{4} + \binom{n}{5} + \cdots + 2\binom{n}{n-2} + \binom{n}{n-1} + 2\binom{n}{n} = 2^{n-1} + 2^{n-2}$
   (5). $f: \boldsymbol{R} \to \boldsymbol{R}, f(x) = \sqrt{x}$ is a function.
   (6). $f: \boldsymbol{R} \to \boldsymbol{R^2}, f(x) = (x^2, -x^2)$ is an one-to-one function.

2. (*10:2,2,3,3 pts*) Determine the following sets: (a) $\emptyset \cup \{\emptyset\}$ (b) $\emptyset \cap \{\emptyset\}$ (c) $\emptyset \oplus \{a, \emptyset, \{\emptyset\}\}$ (d) $\{\emptyset\} \oplus \{a, \emptyset, \{\emptyset\}\}$

3. (*10 pts*) Solve the equation $x_1+x_2+x_3+x_4 < 10$ and find the integer solutions where $x_1, x_2 > 0, x_3 > 1, x_4 > -2$.

4. (*15 pts*) Show that postage of 24 cents or more can be achieved by using only 5-cent and 7-cent stamps.

5. (*10:2,2,2,4 pts*) For the complete expansion of $(2x - y + 3z^{-1} + 1)^6$, determine the following value (a) the coefficient of x^2yz^{-2} (b) the number of the distinct terms (c) the sum of all coefficients, and (d) if we change the constant term '1' to '$1+x^2$',what's the coefficient of x^2yz^{-2}.

6. (*10 pts*) Validate the argument $((p \wedge q) \wedge (p \to (r \wedge q)) \wedge (r \to (s \vee t)) \wedge \neg s) \to t$

7. (*10:3,3,4 pts*) (a) How many times is the *printf* statement executed for the following program segments if *p*=24? (b) How many distinct numbers printed by this program? (c) discuss the result of (a) when *p*=12.

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for (i=1; i <= p; i++)
    for (j=i; j <= 24; j++)
        for (k=j; k <= 24; k++)
            printf("%d\n", i+j+k);
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8. (*10 pts*) Simplify the following expressions. (a) $\neg[(p \wedge \neg q) \vee \neg(r \wedge q)]$ (b) $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)]$