2008/5/26

Discrete Mathematics (2008 Spring) Midterm II

1. (12 points) For each of the following statements, determine and explain (required) whether it is correct or not.

   (1). $f: R \to R\ f(x) = \dfrac{1}{(x^2-2)}$ is a function.

   (2). $\sum_{k=0}^{n} (-1)^k \binom{n}{n-k} (n-k)^m = 0,\ if\ \ m \le n.$

   (3). $1^2 + 2^2 + \cdots + n^2 = O(n^2)$

2. (20 points, 2,2,3,4,4,5) Let A={a, b, c, d, e}, *f*: A→A and *g*:AxA→A. (1) How many relations from A to A? (2) How many one-to-one functions in *f*? (3) How many one-to-one correspondence functions in *f*? (4) How many commutative functions in *g*? (5) How many closed binary operations on A have c as the identity? (6) How many functions in part (5) are commutative?

3. (20 points, 4, 6, 10 respectively) We use *s(m, n)* to denote the number of ways to seat *m* people at *n* circular tables with at least one person at each table. The arrangements at any one table are not distinguished if one can be rotated into another. The ordering of the tables is ***not*** taken into account. (1) For m ≥ 1, what are *s(m, m)* and *s(m, 1)*? (2) Show that for m ≥ 3, *s(m, m-2)* = $(\frac{1}{24})m(m-1)(m-2)(3m-1)$. (3) Prove that *s*(*m*, *n*) = (*m-1*)*s*(*m-1, n*) + *s*(*m-1, n-1*) for *m* ≥ *n* ≥ 1.

4. (5 points) Considering the Stirling number of the second kind *S*(m, n), we have *S*(7, 4)=350, *S*(7,5)=140, *S*(8,4)=1701. What is *S*(8, 5) ?

5. (12 points) Let *M* be the finite state machine in Figure 1. For states S_i, S_j , where $0 \le i,\ j \le 2$, let $\boldsymbol{O_{ij}}$ denote the set of all nonempty output strings that *M* can produce as it goes from state S_i to state S_j, e.g., $\boldsymbol{O_{20}} = \{0\}\{1, 00\}^*$. Find $\boldsymbol{O_{22}}$, $\boldsymbol{O_{11}}$, and $\boldsymbol{O_{10}}$.

6. (14 points, 4,6,4) Let *M* be the finite state machine in Figure 2. (1) Describe in words what this finite state machine does. (2) Applying the minimization process on M. (3) What is the minimal distinguishing string for s_2 and s_4?

7. (5 points) How many 6 x 6 (0, 1)-matrices *A* are there with $A=A^{tr}$?

8. (12 points) Let *U*={1, 2, 3, 4, 5, 6, 7}, with A=*P*(*U*), and let *R* be the subset relation on A. For B={{1}, {2}, {2, 3}}$\subseteq$A, determine each of the following. (1) The number of upper bounds that exist for B. (2) The lub for B. (3) The number of lower bounds that exist for B. (4) The glb for B.

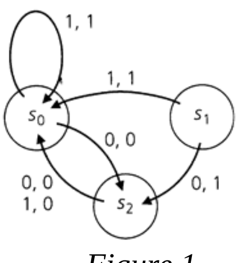

Figure 1

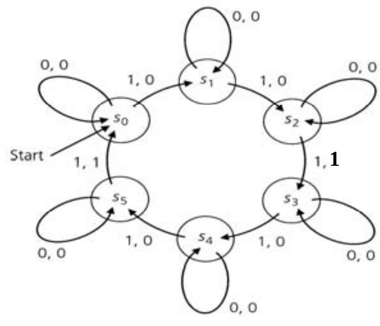

Figure 2