DISCRETE MATHEMATICS – CH7 Homework7

7.1

10. If A = {*w*, *x*, *y*, *z*}, determine the number of relations on A that are (a) reflexive; (b) symmetric; (c) reflexive and symmetric; (d) reflexive and contain (*x*, *y*); (e) symmetric and contain (*x*, *y*); (f) antisymmetric; (g) antisymmetric and contain (*x*, *y*); (h) symmetric and antisymmetric; and (i) reflexive, symmetric, and antisymmetric. (10 pts)

a. Reflexive $2^{(n^2-n)} = 2^{(16-4)} = 2^{12}$
b. Symmetric $2^n 2^{(n^2-n)*1/2} = 2^4 2^6 = 2^{10}$
c. Reflexive and symmetric $2^{(n^2-n)*1/2} = 2^6$
d. Reflexive and contain (x,y) $2^{12-1} = 2^{11}$
e. Symmetric and contain (x,y) $2^n 2^{(n^2-n)*\frac{1}{2}-1} = 2^4 2^5 = 2^9$
f. Anti-symmetric $2^n 3^{(n^2-n)*1/2} = 2^4 3^6$
g. Anti-symmetric and contain (x,y) $2^n 3^{(n^2-n)*\frac{1}{2}-1} = 2^4 3^5$
h. Symmetric and anti-symmetric {(1,1),(2,2),(3,3),(4,4)}

Relation either include or exclude each of these pairs, so 2^4

i. Reflexive, symmetric and anti-symmetric : only 1

{(1,1),(2,2),(3,3),(4,4)}

7.3

18. Let $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7\}$, with $A = \mathcal{P}(\mathcal{U})$, and let $\mathcal{R}$ be the subset relation on A. For $B = \{\{1\}, \{2\}, \{2, 3\}\} \subseteq A$, determine each of the following.

a) The number of upper bounds of B that contain (i) three elements of $\mathcal{U}$; (ii) four elements of $\mathcal{U}$; (iii) five elements of $\mathcal{U}$

b) The number of upper bounds that exist for B

c) The lub for B

d) The number of lower bounds that exist for B

e) The glb for B

(10 pts)

(a) (i) Only one such upper bound – $\{1,2,3\}$. (ii) Here the upper bound has the form $\{1,2,3,x\}$ where $x \in \mathcal{U}$ and $4 \leq x \leq 7$. Hence there are four such upper bounds. (iii) There are $\binom{4}{2}$ upper bounds of B that contain five elements from $\mathcal{U}$.

(b) $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 2^4 = 16$

(c) $lub\, B = \{1,2,3\}$

(d) One – namely $\emptyset$

(e) $glb\, B = \emptyset$

7.4

-4

12. Let $A = \{w, x, y, z\}$. Determine the number of relations on A that are (a) reflexive and symmetric; (b) equivalence relations; (c) reflexive and symmetric but not transitive; (d) equivalence relations that determine exactly two equivalence classes; (e) equivalence relations where $w \in [x]$; (f) equivequivalence relations where $v, w \in [x]$; (g) equivalence relations where $w \in [x]$ and $y \in [z]$; and (h) equivalence relations where $w \in [x]$, $y \in [z]$, and $[x] \neq [z]$. (10 pts)

a. $2^6 = 64$

b. $\sum_{i=1}^{4} S(4,i) = 1 + 7 + 6 + 1 = 15$

c. $64 - 15 = 49$

d. $S(4,2) = 7$

e. $\sum_{i=1}^{3} S(3,i) = 1 + 3 + 1 = 5$

f. This problem is unclear. (v does not exist)

g. $\sum_{i=1}^{2} S(2,i) = 1 + 1 = 2$

h. $\left(\sum_{i=1}^{2} S(2,i)\right) - \left(\sum_{i=1}^{1} S(1,i)\right) = 2 - 1 = 1$

Advanced assignment (30 pts)

- Design a problem that can be solved by two different FSM with different number of states.
- Use the minimization process to reduce the bigger one.

Note:

- ✧ FSMs in textbook will be scored 0.
- ✧ The most similar FSM will be scored 0.

Ans

	下一個狀態		
	目前的輸入		
目前的狀態	0	1	輸出
0	3	1	1
1	4	1	0
2	3	0	1
3	2	3	0
4	1	0	1

{0,2,4} , {1,3} → {2,4} , {1,3} , {0}

A= {0} , B{2,4} , C = {1,3}

	下一個狀態		
	目前的輸入		
目前的狀態	0	1	輸出
A	*C*	*C*	1
B	*C*	*A*	1
C	*B*	*C*	0