## Engineering Mathematics      Homework 12 Solution

1. $y''+4y'+4y=3H(t-2), y(0)=y'(0)=0, \mathit{find}\ y(t)?$

Sol:

$$(s^2Y(s)-Y(0)-Y'(0))+4(sY(s)-Y(0))+4Y(s)=\frac{3}{s}e^{-2s}$$

$$(s^2Y(s)-0-0)+4(sY(s)-0)+4Y(s)=\frac{3}{s}e^{-2s}$$

$$(s^2+4s+4)Y(s)=\frac{3}{s}e^{-2s}$$

$$Y(s)=\frac{3}{s(s+2)^2}e^{-2s}=\left(\frac{\frac{3}{4}}{s}+\frac{-\frac{3}{4}}{s+2}+\frac{-\frac{3}{2}}{(s+2)^2}\right)e^{-2s}$$

$$y(t)=\left(\frac{3}{4}-\frac{3}{4}e^{-2(t-2)}-\frac{3}{2}(t-2)e^{-2(t-2)}\right)H(t-2)$$

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2. $y''+y=0$, *find* the Taylor *series solution at* $x=0$.

Sol:

$$x=0 \quad \text{(Ordinary Point)}$$

$$\therefore y(n)=\sum_{n=0}^{\infty} a_n x^n, |x-0|<L=\infty$$

$$y'(n)=\sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y''(x)=\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$y''+y=0$$

$$\Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}+\sum_{n=0}^{\infty} a_n x^n=0$$

$$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n+\sum_{n=0}^{\infty} a_n x^n=0$$

$$\Rightarrow \sum_{n=0}^{\infty} [(n+2)(n+1) a_{n+2}+a_n] x^n=0$$

$$\Rightarrow a_{n+2}=\frac{-a_n}{(n+2)(n+1)} \quad \text{(Recurrence Formula)}$$

$$a_2=\frac{-a_0}{2}, a_4=\frac{-a_2}{4\times 3}=\frac{a_0}{4!}, a_6=\frac{-a_4}{6\times 5}=\frac{-a_0}{6!}$$

$$a_3=\frac{-a_1}{3\times 2}, a_5=\frac{-a_3}{5\times 4}=\frac{a_1}{5!}, a_7=\frac{-a_5}{7\times 6}=\frac{-a_1}{7!}$$

$$a_{2n}=\frac{(-1)^n}{(2n)!} a_0$$

$$a_{2n+1}=\frac{(-1)^n}{(2n+1)!} a_1$$

$$\therefore y(x)=\sum_{n=0}^{\infty} a_n x^n$$

$$=\sum_{n=0}^{\infty} a_{2n} x^{2n}+\sum_{n=0}^{\infty} a_{2n+1} x^{2n+1}$$

$$=\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} a_0 x^{2n}+\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} a_1 x^{2n+1}$$

$$=a_0 \cos x+a_1 \sin x$$

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