Algorithm 2015 fall Homework 3

1. What are the two key ingredients that an optimization problem must have in order for dynamic programming to be applicable and explain why memorization is ineffective in speed up a good divide-and-conquer algorithm such as MERGE_SORT ?
2. Consider the knapsack problem consists of 3 items, and the capacity of the knapsack is equal to 8. The profits and weights of the three items are (p1, p2, p3) = (8, 6, 3) and (w1, w2, w3) = (6, 5, 3), respectively.
   (a) Assume that you are allowed to put in a fraction of an item. Use the greedy method to solve for the maximum profit and show the items to be included in the knapsack.
   (b) Now suppose that you must take each item as a whole (i.e 0/1 knapsack problem). Show how you can use dynamic programming to solve the problem. What are the total profit and the list of items to be included in the knapsack?
3. In the Knapsack problem, if the size of each object is arbitrary real number, does the dynamic programming method still work? Explain your answer.
4. Determine the smallest expected search cost and structure of an optimal binary search tree for a sequence K = hx1, x2, x3, x4, x5i of 5 distinct keys with the probabilities: p1 = 0.15, p2 = 0.2, p3 = 0.15, p4 = 0.2, p5 = 0.3.
5. Give a dynamic-programming solution to the 0-1 knapsack problem that runs in O(n W) time, where n is the number of items and W is the maximum weight of items that the thief can put in his knapsack.
6. Determine an LCS of ⟨1,0,0,1,0,1,0,1⟩ and ⟨0,1,0,1,1,0,1,1,0⟩.
7. Determine which one of the **0-1 knapsack problem** and the **fractional knapsack problem** cannot be solved using the greedy strategy? Give an example to explain that.
8. Determine the cost and structure of an optimal binary search tree for a set of n = 5 keys with the following probabilities:

i	1	2	3	4	5
P_i	0.25	0.15	0.2	0.35	0.05

9. Given a chain <A1, A2, A3, A4> of 4 matrices and their matrix dimensions: A1: 3×5, A2: 5×2, A3: 2×6, A4: 6×4. Please compute the minimum number of scalar multiplications to multiply them.

10. The matrix-chain multiplication problem can be stated as follows: Given a chain $\langle A_1, A_2, \ldots, A_n \rangle$ of n matrices, where for $i=1,2,\ldots,n$, matrix A_i has dimension $p_{i-1} \times p_i$, fully parenthesize the product $A_1 A_2 \cdots A_n$ in a way that minimizes the number of scalar multiplications. Give a dynamic-programming algorithm to solve the problem and analyze its time complexity