Quiz (I)

Finished by 18:30 on 5/3

1. Create an m file and change the filename to F7xxxxxxx_quiz1.m. Type the main code in the m-file to accomplish the later tasks.

2. Try to solve the roots of each quadratic equation using the two forms:
   (a) original form [2 roots] and (b) the numerator-rationalized form [2 roots].
   Print out the 4 roots for at least up to 16 decimal digit, and explain how you will choose the algorithm to solve the quadratic equations.
   (1) $x^2 - (500 + k)x + 500k = 0 \ \ for\ k = 10^{-2}, 10^{-4}, \ldots, 10^{-10}$

   (2) $0.0001x^2 + 30.0003x + 90 = 0$

   (3) $x^2 + 300003\ \text{x} + 900000 = 0$

   (4) $x^2 - 302 \times 10^{152}x + 6 \times 10^{306} = 0$

   Output Format:
   Prob(1), The roots are:
   (a-1) xxxx.xxxxxxxx
   (a-2) xxxx.xxxxxxx
   (b-1) xxxx.xxxxxxxx
   (b-2) xxxx.xxxxxxx
   Prob(2), The roots are:
   (a-1) xxxx.xxxxxxxx
   (a-2) xxxx.xxxxxxx
   (b-1) xxxx.xxxxxxxx
   (b-2) xxxx.xxxxxxx

3. Write a function using partial sums to calculate $e^x$ based on Taylor's Expansion:

$$\text{My\_Exp}(N, x) = \Sigma_{k=0}^{N} \left(\frac{x^k}{k!}\right) \cong e^x \quad \text{for } x \geq 0$$

(1) Estimate $e^1$ using different N (=0,1,2,…), the order of the expansion. e.g. $e^1 \cong My\_Exp(20,1)$. Compare the result with Matlab's function exp(x),

i. How many terms (N) does it require to have $My\_Exp(N,1)$ ) reach an accuracy of 10 significant digits?

ii. How many terms does it require for $My\_Exp(N,-1)$ ) reach an accuracy of 10 significant digits?

(2) Repeat (1) to calculate $e^{30}$ using $My\_Exp(N,30)$ and $e^{-30}$ using $My\_Exp(N,-30)$

i. How many terms (N) does it require to have $A(N,30)$ reach an accuracy of 10 significant digits?

ii. How many terms (N) does it require to have $A(N,-30)$ reach an accuracy of 10 significant digits?

iii. How do you propose to approximate $e^{-30}$ with your function?

==========Bonus==========

iv. What do you propose to write an exponential function with good accuracy to cover at least from $e^{-50}$ to $e^{50}$ ?

v. What do you propose to write an exponential function with good accuracy to cover the range of floating point numbers?