Discrete Mathematics (2010 Spring) Final

(total:110 points, max:100 points)

1. **(20 points)** For each of the following statements, determine and explain (required) whether it is correct or not.
   (1). The number of integer solutions for $c_1 + c_2 + c_3 + c_4 + c_5 = 30$, $1 \le c_i$ for all i, with c_2 even and c_3 odd is equal to the coefficient of x^{24} in $(1+x+x^2+x^3+\ldots)^3(1+x^2+x^4+\ldots)^2$.
   (2). The coefficient of x^{48} in $(x^6+x^7+x^8+\ldots)^7$ is $\binom{11}{5}$.
   (3). If f is the generating function for the sequence 1,0,1,0,…, the function f' generates the sequence 0,1,0,1,… .
   (4). the generating function $f(x) = \dfrac{1}{(3-2x)}$ generates sequence $\dfrac{1}{3}, \dfrac{2}{9}, \dfrac{4}{27}, \dfrac{8}{81}, \ldots$

2. **(10 points)** In how many ways can 30030 be factored into 3 factors? each factor is greater than 1 and the order of the factors is not relevant. [S(4, 2)=7, S(4, 3)=6, S(5, 2)=15.]

3. **(10 points)** For the complete expansion of $(x^2 - 2y + 3z^{-1} - 4)^4$, determine the following value (a) the coefficient of x^2yz^{-2}, (b) the number of distinct terms.

4. **(10 points)** Let A={a, b, c, d} and B ={1, 2, 3, 4, **5**}, please determine the following value. (a) The number of closed binary operations on A that have an identity. (b) The number of relations from A to B. (c) The number of one-to-one functions from A to B. (d) The number of onto functions from A to B. (e) The closed binary operations on B that are commutative.

5. **(15 points)** What is the number of integer solutions for $x_1+x_2+x_3 = Z$, if (a) $0 \le x_1, x_2, x_3$, $Z=8$, (b) $x_1, x_2 > 0$, $x_3 > 1$, $Z < 8$, (c) $0 \le x_1 \le 4$, $0 \le x_2 \le 5$, $0 \le x_3$, $Z=8$. *(an exhaustive list get 0 point.)*

6. **(10 points)** For A={1, 2, 3, 4} and B={u, w, x, y, z}, determine the number of one-to-one functions f: A→B where $f(1) \ne$ x, y, $f(2) \ne$ w, $f(3) \ne$ x, y and $f(4) \ne$ z.

7. **(5+10 points)** Solve the recurrence relation $a_{n+2} - 2a_{n+1} + a_n = 2^n$. $n \ge 0$, $a_0=1$, $a_1=2$ (1) by *characteristic equation*, (2) by *generating functions*.

8. **(15 points)** Put n balls with ID from 0 to n-1 into n buckets with ID from 1 to n (one bucket contains one ball). Please (1) show all derangements (the ball with ID i is not in the i-th bucket) when n=3, (2) count the derangement cases when n=5. (3) If d_n denotes the number of derangements of $\{1,2,3,\ldots,n\}$ as described in Chapter 8.3, show that d_n satisfies the recurrence relation $d_n = (n-1)(d_{n-1} + d_{n-2})$, when $n > 2$. (*Hint: discuss two disjoint cases when 1 is placed in position i, $2 \le i \le n$.*)

9. **(5 points)** Please list 2 examples/methods/strategies to improve your learning motivation/performance.