QUIZ 1

NAME:____________________________ **NCKU id:**____________________

1. Let $f(x) = \sqrt{1-4x^2}$ with domain $0 \le x \le \frac{1}{2}$. Find the inverse function $f^{-1}(x)$.

Answer: Replace x by $y = f^{-1}(x)$ and y by x, we get,

$$1 - 4\left[f^{-1}(x)\right]^2 = x^2$$

Therefore,

$$\left[f^{-1}(x)\right]^2 = \frac{1-x^2}{4},$$

and hence

$$f^{-1}(x) = \frac{\sqrt{1-x^2}}{2}, 0 \le x \le 1$$

2. Let $f(x) = \frac{x^2+x-12}{|x-3|}$. Find the left limit $\lim_{x\to 3^-} f(x)$ and the right limit $\lim_{x\to 3^+} f(x)$.

Answer :If $x \to 3^-$, then $x-3<0$ and

$$\lim_{x\to 3^-} \frac{x^2+x-12}{|x-3|} = \lim_{x\to 3^-} \frac{(x+4)(x-3)}{3-x} = -7.$$

If $x \to 3^+$, then $x-3>0$ and

$$\lim_{x\to 3^+} \frac{x^2+x-12}{|x-3|} = \lim_{x\to 3^+} \frac{(x+4)(x-3)}{x-3} = 7.$$

Due to the fact that right and left limits of f at $x=3$ are not equal, $\lim_{x\to 3} f(x)$ does not exist.

Date: September 29th.

3. Consider the function $f(x)$ with $\mathrm{Dom}(f) = \mathbb{R}$ defined by

$$f(x) = \begin{cases} n, & \text{if } x = \frac{1}{n} \\ 0, & \text{otherwise} \end{cases}.$$

Does $\lim_{x\to 0} f(x)$ exist? Explain your reasoning.

Answer: For the sequence $a_n = \frac{1}{n}$, $\lim_{n\to\infty} a_n = 0$ but $\lim_{n\to\infty} f(a_n) = \infty$. On the other hand, we can find another sequence $b_n \to 0$ with $f(b_n) = 0$ for all n, and therefore $\lim_{n\to\infty} f(b_n) = 0$. Since there are two different sequences approaching $x = 0$ but with different behaviour of limit of $f(x)$, the limit $\lim_{x\to 0} f(x)$ does NOT exist.