Discrete Mathematics (2013 Spring) Midterm II

1. (20 points) For each of the following statements, determine and explain (required) whether it is correct or not.
   (1). The number of different relations on {0, 1} contain the pair (0, 1) is 4.
   (2). The poset ({2, 4, 5, 10, 12, 20, 25}, |) has 3 maximal elements and 2 minimal elements.
   (3). The least upper bound of {1, 2, 4, 5, 10} in the poset(Z^+, |) does not exist.
   (4). Let (A, **R**) be a poset. If (A, **R**) is a lattice, then it is a total order.
   (5). The subset relation is a partial order relation.
   (6). The proper subset relation is a partial order relation.
   (7). Let A ={1, 2, 3, 4, 5, 6, 7}, there is an equivalence relation R on A with |R|=8.
   (8). String 01011 is in the language $\{00\}^*\{01\}^+\{1\}^*$ and is also in the language $\{01\}^*\{0\}^*\{11\}^*\{1,0\}^*$.

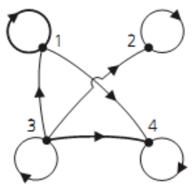
2. (10 points) (a) How many two-factor ordered factorizations, where each factor is greater than 1, are there for 156,009 (3*7*17*19*23)? (b) In how many ways can 156,009 be factored into two or more factors, each greater than 1 and the order of the factors is relevant?

3. Is (S, R) a poset if S is the set of all people in the world and (a, b)

4. (10 points) List the ordered pairs in the equivalence relations produced by these partitions of {0, 1, 2, 3, 4, 5} (a) {0, 1, 2}, {3, 4, 5} (b) {0, 1}, {2, 3}, {4, 5} (c) {0}, {1}, {2}, {3}, {4}, {5}

5. (10 points) If A = {*a*, *b*, *c*, *d*, *e*}, determine the number of relations on A that are (a) reflexive and symmetric, (b) antisymmetric and contain (x,y), (c) symmetric and antisymmetric, (d) equivalence relations that determine more than three (include three) equivalence classes, (e) reflexive and symmetric but not transitive.

6. (15 points) The directed graph G for a relation R on set A = {1, 2, 3, 4} is shown in the following graph. (a) Please verify that (A, R) is a poset. (b) Draw its Hasse diagram. (c) Topologically sort (A, R). (d) How many more directed edges are needed in the following graph to extend (A, R) to a total order? (e) How many more directed edges are needed in the following graph to extend R to an equivalence relation?



7. (10 points) Let S = {3, 7, 11, 15, 19,..., 95, 99, 103}. How many elements must we select from S to insure that there will be at least two whose sum is 110?

8. (10 points) Let A = {*a*, *b*, *c*}, and B = {*u*, *v*, *w*, *x*, *y*}. (a) If *f* : A→B is randomly generated, what is the probability that it is one-to-one? (b) How many closed binary operations on A that are commutative and have an identity?

9. (10 points) Let *U*={1, 2, 3, 4}, with A be the proper subsets of *U*, and let *R* be the *subset relation* on A. For B={{1}, {2}, {2, 3}}$\subseteq$A, determine each of the following. (a) The maximal element of A, (b) The minimal element of A, (c) The greatest element of A, (d) The number of upper bounds that exist for B. (e) The lub for B.

10. (15 points) (a) Please design a 1-unit delay FSM. (b) Can the number of states in (a) be minimized? Please verify and minimize it (if needed). (c) Construct a state diagram for a FSM

with $\boldsymbol{I} = \boldsymbol{O} = \{0, 1\}$ that recognizes strings 1xx00, where x={0, 1}.

m \ n	1	2	3	4	5	6
	$S(m, n)$					
1	1					
2	1	1				
3	1	3	1			
4	1	7	6	1		
5	1	15	25	10	1	
6	1	31	90	65	15	1

Table of Stirling number of the second kind