Homework 4

Due Date

2018/06/21(THURSDAY), 23:59 ◦ Late submission will not be accepted.

This homework typically requires much more time to test for proper simulation parameters. So please be advised to start this homework earlier. And keep in mind, you only have upload space for 20 files, so plan the program structure wisely.

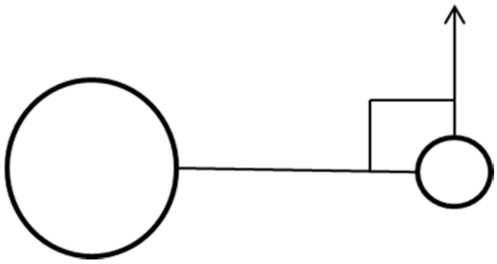
In this assignment, you should be able to accomplish an algorithm to solve differential equations numerically with given initial conditions. Please be advised that no grade will be given if any MATLAB's embedded function for solving differential equation, for solving equations, for finding derivatives or for integration is found.

Create a matlab script and change the filename to <your student id >_hw4.m. Link all the problem solving programs to this script. Make sure once type the filename' <your student id >_hw4', the required results of the following problems will pop-up automatically in order. Remember not to type any 'clear all ', 'close all' command in any of the codes.

Create a PDF file named "F7xxxxxxx_hw4.pdf" for the written report.

Problem1: Planetary motion - proof of Kepler's Law from Newtonian Dynamics

[F<your student id >_hw4_prob1.m]



Assume that a $2 \times 10^{30}\ kg$ Sun rests at the origin and a $6 \times 10^{24}\ kg$ planet is found at distance $r_0 = 1.5 \times 10^{11}\ m$ from the sun while the planet is moving at speed v toward the direction perpendicular to the segment connecting the center of the two objects. As the mass of the sun is much larger than the planet, the gravitational effect towing the Sun can be negligible. But you are encouraged to include the force imposed on the Sun to investigate its motion.

(a) Write down the equation of motion of the planet (and the sun if you do not feel like ignoring it). $(ex: \frac{d^2x_p}{dt^2} = \cdots, \frac{d^2y_p}{dt^2} = \cdots.)$

(b) What is the speed for the planet to move along a circular orbit? The speed is denoted by v_c for later use. (Note: if the motion of the sun is considered, v_c should be calculated according to the setting)

(c) [Fig 1-3] Use $v = v_c$ as the initial condition in addition to the distance provided to solve the differential equation of problem (a) numerically for at least one orbital period and generate three plots: (1) the trajectory (x-y), (2) the x-location (x-t) and (3) the y-location y(t) of the planet.

(d) Find the orbital period of the planet in the simulation (c) and explain how you find it.

(e) [Fig 4-6] If the initial condition changes to $|\vec{v}(0)| = 0.7\ v_c$, re-calculate the trajectory of the planet from the beginning for at least an orbital period and plot the results similar to those of (c).

(f) Verify that the trajectory of the planet in (e) is an ellipse and explain how to verify.

(g) Find the length of the semi-major axis in (e) and explain how you find it.

(h) Find the orbital period of the planet in (e) and explain how you find it.

(i) [Fig 7] You should have notice that the trajectory and the period of the planet changes with the initial conditions $|\vec{v}(0)| = kv_c$ and $r(0) = mr_0$, where k and m are positive numbers. Now change the value of k within a range (e.g. k = 1/5 to k = 1) OR m within a range (e.g. m = 0.1 to 2) and record the length of the semimajor axis (R) and the orbital period (T) of each condition and verify that $R^3 \propto T^2$(i.e. make a plot of $R^3 - T^2$ from the data and check if they can be fit on a line). In the text, specify what conditions are changed and how to make the changes in the program to get the data for verification.

Problem 2:The mission of the triplet Jay, Jake and Jim. (The blue birds)

[F<your student id >_hw4_prob2.m]

The triplet Jay, Jake and Jim have identical weight of 0.1 kgw. In a rescue mission, they were launched 20cm from ground zero at a speed of 20m/s and a shooting angle of 50^0 . They were ordered to split 2 seconds after taking off. Assume that they were in a line formation (Jay-Jake-Jim). During separation, they pushed each other horizontally with a constant force for 0.1 second. The force was 1.0N between Jay and Jake and 2.0N between Jake and Jim respectively. Start the simulation and answer the following questions:

(a) [Fig 8] Plot the trajectory (x-y) of the center of mass from launching to landing.

(b) Estimate how far from the catapult would Jim be when it was landed.

(c) Estimate the speed of Jake right before landing.

(d) Estimate the kinetic energy of the triplet when landing.

Problem 3: Simple Molecular Dynamics

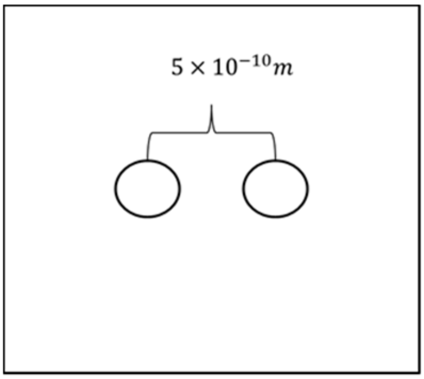
[F<your student id >_hw4_prob3.m]

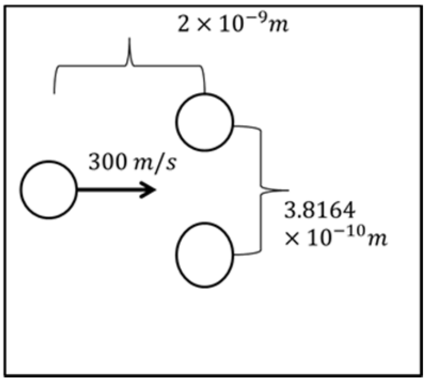
Assume that there are argon(^{40}Ar) atoms in a square cubic box. The length of each side of the box is $4.0\ nm$. Assume the center of the two argons as well as the box is at the origin of a coordinate. And the two atoms are placed on the x-axis. The parameters of the Lennard-Jones Potential (Energy) for the argon atom are as follows: $\epsilon = 1.66 \times 10^{-21} J\ and\ \sigma = 3.4 \times 10^{-10} m$. Based on the facts, a system containing argon atoms can be simulated

Use the given initial conditions to simulate the dynamic system for $0 \le t < 10 \times 10^{-10}\ sec$. For each initial condition, you should (1) specify the time step Δt for the simulation in the text, (2) plot the total energy versus time of the system and choose the data for the plot properly, and (3) record the locations of the atoms and make an animation to show how the atoms move. Save the animation in gif format. You may need the functions "imwrite" to accomplish the task. Choose the frame rate wisely to demonstrate the particles' motion within reasonable show time.

(a) [Fig 9] Suppose two atoms are found rest and separated by 5×10^{-10}m relative to the center of the box at t = 0. In addition to the three results to present, which type of phase (gas, liquid and solid) do you think this state is if more than two atoms of the same state are put together? (Fig. 9 is the energy plot with time for this simulation)

(b) [Fig 10] Suppose at t = 0, two atoms are found rest and separated by 3.8164×10^{-10}m relative to the center. Another one is flying toward the center of these two atoms at a speed of $300\ m/s$ from a distance of 2×10^{-9}m.





Contents to submit: (IMPORTANT)

1. All the m-files you compose for the assignment.
2. All the m-files should include proper COMMENTS.
   (No comment, no score)
3. The PDF file must contain Your Name, Your Student ID Number.
   In the document, you will need to include all the required answers including those shown in the console, plots, and the following contents: (1) Which method do you use to solve the differential problems? (2) The step length you choose for each problem. (3) How long does it take roughly for the matlab to finish all problems after executing the main script?
4. The gif files to demonstrate the animation of problem 3.

Notice:

1. No point will be given if the execution time is longer than 40 minutes or if the code requires more than 4.0 Giga Bytes memory space.
2. DO NOT PLAGIARIZE. You are encouraged to ask and to discuss the homework content with your fellow classmates, the TAs and the instructor. But identical core program wording is NEVER ACCEPTABLE.
3. Upload all the files without archiving (No zip, No rar, No tar, No 7zip). Do not upload files that don't work well. Any missing file or function that leads to execution failure will be regarded as a program that never works.