Physics-Informed Neural Networks on Bending Moment of Laterally Loaded Piles

ENHANCE BIP

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1 Introduction

The behaviour and structural design of underground structures is governed by the distribution of internal forces. Out of these internal forces, bending moments are most critical for structures supporting bending forces, such as laterally loaded piles. However, despite the importance of these internal forces, traditional monitoring techniques of these structures concentrate on measuring total or relative deformations to verify design assumptions rather than enabling direct conclusions about the governing internal forces of the structure itself (Fuentes [2015]). With regards to bending moments, the available solutions to obtain bending moments and axial loads from displacements involve back-calculation and iterative processes using models that are successful in forward prediction – e.g., continuum models, or finite element analysis.

2 Objective

In this case, we want to use physics Informed Neural Networks to predict bending moments of laterally loaded piles. The presented governing equations are based on the principle of virtual work, and enables calculating the internal force distributions of laterally loaded piles when the displacements of the structure are known, without the need of any boundary conditions. The following general assumptions are made: linear elastic material behaviour applies; cross sections that are plane before deformation remain plane; and only small deformations are applied to the structure. Since the focus for piles is on calculating bending moments, only the displacements perpendicular to the pile/retaining wall longitudinal axis need to be considered. Laterally loaded piles treated as a cantilever beam (Fig. 1b).

3 Governing Equations

In General, the following Euler–Bernoulli equation describing the relationship between the beam's deflection and the applied load should be satisfied for the laterally loaded piles:

$$EI\frac{d^4y}{dz^4} = p(z) \quad \text{where} \quad p(z) = -ky \tag{1}$$

$$EI\frac{d^2y}{dz^2} = -M(z) \quad \text{and} \quad -p(z) = \frac{d^2M}{dz^2} \tag{2}$$

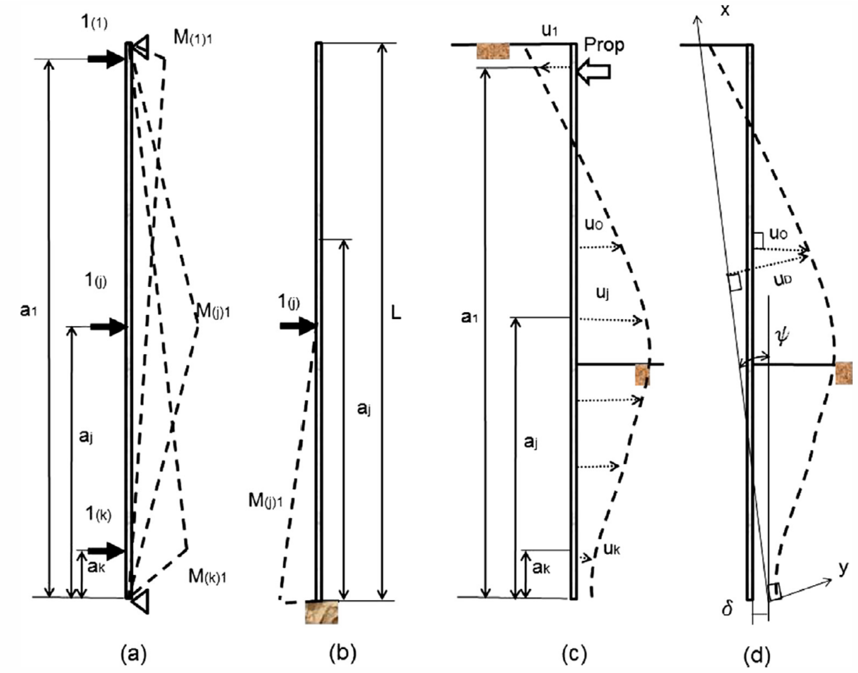

Figure 1: Laterally loaded piles.

where M is the bending moment, y(z) describes the deflection of the beam, E is the elastic modulus and I is the second moment of area of the beam's cross section.

4 Tasks

1. **Non-dimensionalize the governing equation**
   Reformulate the equations into nondimensional form to improve numerical stability and generality of the PINN models.

2. **Develop a forward PINN model**
   Train a forward Physics-Informed Neural Network (PINN) model that estimates the bending moment distribution using the least amount of displacement data.

3. **Develop an inverse PINN model**
   Train an inverse PINN using displacement data from Reese case. In this model, treat the bending stiffness parameter EI as a learnable quantity within the neural network.

4. **Build a parametric PINN model with variable EI**
   Extend the PINN framework to include the stiffness parameter EI as an input variable. Evaluate and compare the results.

5. **Prepare a Presentation**
   Create a concise presentation summarizing the methodology, implementation details, model comparisons, and final results of all PINN experiments.

References

Raul Fuentes. *Internal forces of underground structures from observed displacements. Tunnelling and underground space technology*, 49:50–66, 2015.