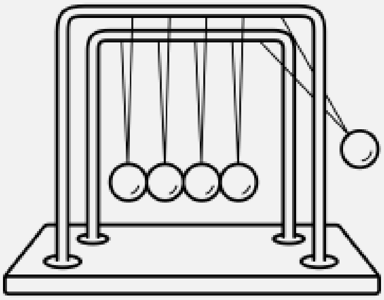
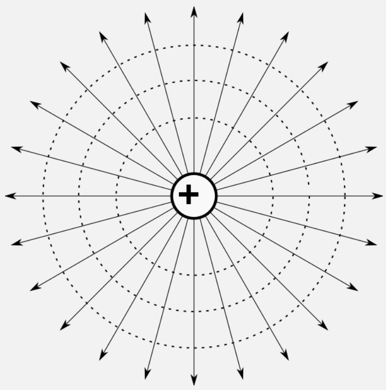
Calculus-Based Mechanics & Electromagnetics

Summer of Making

HackClub





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Preface

Special thanks to Mr. H and Mr. G who have introduced me to the big wide world of physics. This book's philosophy is based off their teachings. While fundamental theorems will be listed in the following chapters, the ultimate goal is to provide the intuition for physics through mathematical reasoning. Emphasis, therefore, will be placed on understanding that builds upon previous concepts.

Contents

Chapter 1

Vectors and Math

1.1 What is a vector?

A vector is a quantity with both magnitude and direction. We represent them as arrows wherein:

- Length indicates magnitude
- Orientation indicates direction

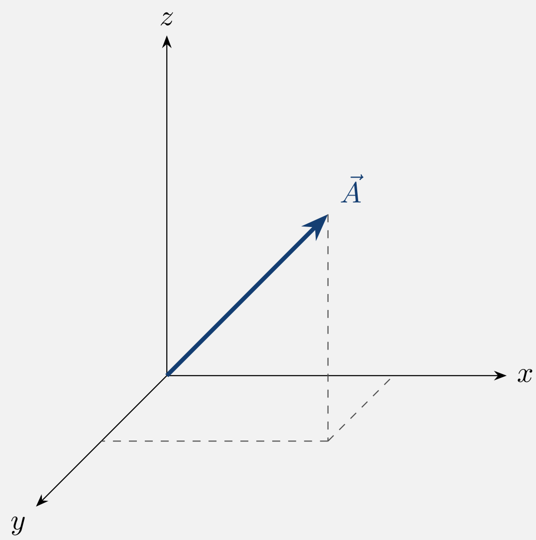

Figure 1.1: Vector $\vec{A}$ with components A_x, A_y, and A_z in 3D space

The length of a vector is traditionally denoted with double vertical bars, $|| \; ||$, and can be determined either graphically or analytically. This leads us to our first equation.

$$||\vec{A}|| = \sqrt{A_x^2 + A_y^2 + A_z^2} \tag{1.1}$$

Equation 1.1 is a representation of the Pythagorean Theorem.

The direction of a vector is given by θ or ϕ and can be found physically such as with a protractor or analytically.

$$\phi = \arctan\left(\frac{A_y}{A_x}\right) \tag{1.2}$$

Angles in 2D space are often measured relative to the positive x-axis.

1.1.1 Vector Operations

Addition

Vector addition is commutative and follows the **parallelogram rule**:

$$\vec{A} + \vec{B} = \vec{C}$$

$$\vec{C} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

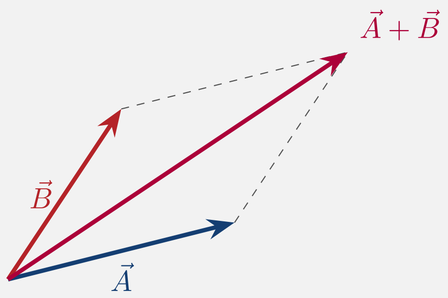

Figure 1.2: The sum of $\vec{A}$ and $\vec{B}$ using the parallelogram rule.

The "tip-to-tail" method can also be used by placing the tip of one vector at the tail of the other vector and forming the resultant triangle. Try it out for yourself!

Subtraction

To perform vector subtraction, it is useful to consider the following example:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

We treat the operation as if it were addition but flip the sign of $\vec{B}$. Graphically, this means rotating the vector 180°.

Multiplication

There are two types of vector multiplication.

- Dot Product: The dot product finds the component of $\vec{A}$ in the direction of $\vec{B}$.

$$\vec{A} \cdot \vec{B} = |A||B| \cos\theta \tag{1.3}$$

- Cross Product: The cross product finds the component of $\vec{A}$ orthogonal to $\vec{B}$.

$$\vec{A} \times \vec{B} = |A||B| \sin\theta \tag{1.4}$$

Note that vector division is not defined.

1.2 Coordinate Systems

It is useful and even necessary at times to establish a consistent coordinate system. An example is the right-hand coordinate system. Extend your hand in the direction of the positive x-axis. Now curl your fingers toward the positive y-axis. Finally, stick out your thumb. It should point toward the positive z-axis.

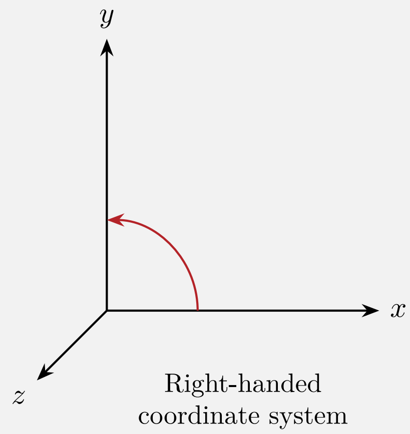

Figure 1.3: Right-Hand Coordinate System

Other coordinate systems will be discussed in separate chapters, but this system can represent most systems in undergraduate mechanics and electromagnetics.

1.3 Math

The following section will list important mathematical foundations used in basic physics problems.

- **Derivative Definition**

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- **Product Rule**

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

- **Chain Rule**

$$\frac{df(g(x))}{dx} = f'(g(x))g'(x)$$

- **Quotient Rule**

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

- **Taylor Series**

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

- **Power Rule**

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

- **Derivative of e^{ax}**

$$\frac{d}{dx}e^{ax} = ae^{ax}$$

- **Derivative of $\ln(ax)$**

$$\frac{d}{dx}\ln(ax) = \frac{1}{x}$$

- **Derivative of sin**

$$\frac{d}{dx}\sin(ax) = a\cos(ax)$$

- **Derivative of cos**

$$\frac{d}{dx}\cos(ax) = -a\sin(ax)$$

- **Integral of x^n, n $\neq$ -1**

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

- **Integral of e^{ax}**

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

- **Integral of $\ln(ax)$**

$$\int \frac{1}{x+a}\,dx = \ln|x+a| + C$$

- **Integral of sin**

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + C$$

- **Integral of cos**

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$$

- **Circumference of Circle**

$$C = 2\pi r$$

- **Arc Length of Circle**

$$s = r\theta$$

- **Volume of Cylinder**

$$\pi r^2 l$$

- **Volume of Sphere**

$$\frac{4}{3}\pi r^3$$

- **Surface Area of Cylinder**

$$2\pi r l$$

- **Surface Area of Sphere**

$$4\pi r^2$$

- **Double Angle Identity**

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

- **Sin and Cos Identity**

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

- **Log Identity**

$$\log(a \cdot b^x) = \log a + x\log b$$

Chapter 2

Kinematics

2.1 Describing Motion

Kinematics is the study of motion. Generally, we discuss objects, otherwise known as projectiles, moving at constant **velocity** or **acceleration**. These are vectors that can indicate how position changes with time. In other words:

$$\vec{v} = \frac{dx}{dt} \tag{2.1}$$

$$\vec{a} = \frac{d^2x}{dt^2} \tag{2.2}$$

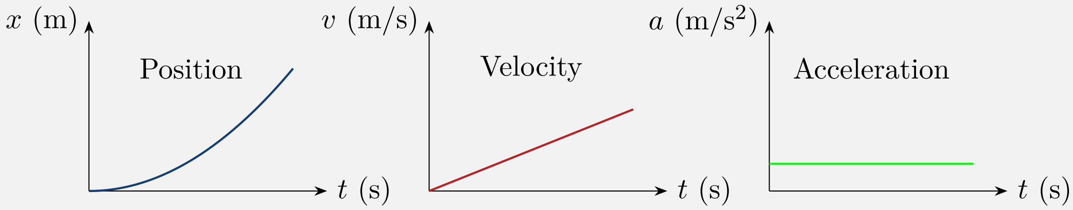

Figure 2.1: Side by Side Position, Velocity, and Acceleration Graphs

x(t) exhibits quadratic growth, v(t) linear, a(t) acceleration constant.

2.2 Constant Acceleration Kinematics

From the fundamental definitions:

$$\begin{aligned} v &= v_0 + at \\ x &= x_0 + v_0 t + \frac{1}{2}at^2 \\ v^2 &= v_0^2 + 2a\Delta x \end{aligned}$$

2.3 Graphical Analysis

2.4 Projectile Motion

Decomposed into independent horizontal/vertical motions:

$$x(t) = x_0 + v_{x0}t$$
$$y(t) = y_0 + v_{y0}t - \frac{1}{2}gt^2$$
$$\text{Range} = \frac{v_0^2 \sin 2\theta}{g} \quad (\text{for } y_0 = 0)$$

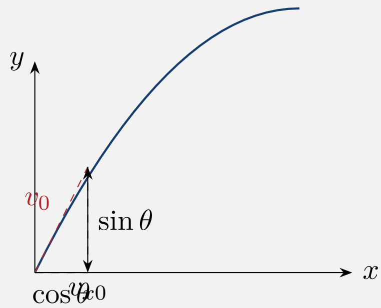

Figure 2.2: Projectile motion components showing velocity decomposition