Polynomial chaos expansions: Exercises

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Traveling with constant acceleration

Example code: https://github.com/hplgit/chaospy/blob/master/example2.py

Distance when traveling with constant acceleration is

$$s(t) = v_0 t + \frac{1}{2} a t^2$$

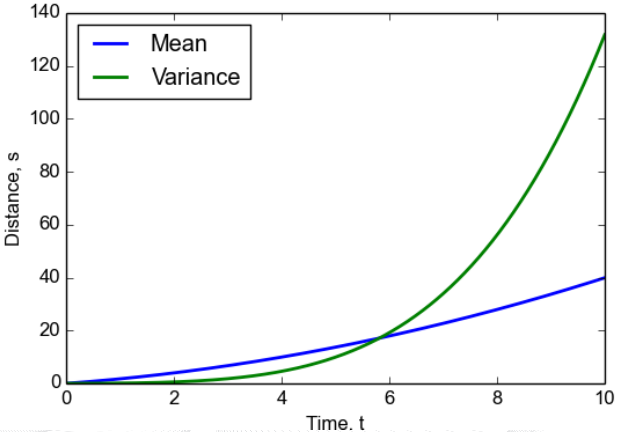
The initial velocity and acceleration is measured to

$$v_0 \sim \text{Uniform}(1,\ 2) \qquad a \sim \text{Beta}(2,\ 2)$$

Task: Find the expectation value and variance in the time interval $t = [0, 10]$ and plot them. Use:

- Classical Monte Carlo integration.
- Quasi-Monte Carlo using Sobol sequence.
- Pseudo-spectral projection with full tensor grid Gaussian quadrature.
- Pseudo-spectral projection with Clenshaw-Curtis and Smolyak sparse grid.
- Point collocation with random samples and least squares minimization.
- Point collocation with Hammersley samples and Tikhonov regularization.

A Monte Carlo solution gives



A different differential equation

Example code:
https://github.com/hplgit/chaospy/blob/master/example3.py

We have the differential equation

$$\frac{du(x)}{dx} = u(x) + a \qquad u(0) = I,$$

where a and I is uncertain and given by

$$a \sim \text{Normal}(4,\ 1) \qquad I \sim \text{Uniform}(2,\ 6)$$

Task: Find the expectation value and variance in the time interval $t = [0, 1]$ and plot them. Use:

- A pseudo spectral method with a Rosenblat transformation, map against a Normal(0, 1) and Uniform(-1, 1) distribution.
- The intrusive Galerkin method.

A Monte Carlo solution gives