Higher Level Physics Internal Assessment
Session: May 2021

Upward force from air resistance in paperclip helicopters

Introduction

Helicopters have increased in importance in emergency response operations as well as military operations for the past 80 years. More recently another propeller utilising flying contraption has established itself in today's world and especially the future - a drone. The most basic principle in both is the same: a propeller rotates, creating a downward draft and lifting the machine into the air. Yet with companies such as Amazon planning on utilising drones for everyday delivery purposes[1] drones are more relevant than ever. Additionally drones can be used for personal entertainment and I, like many, own a small drone.

Being interested in propeller utilising flying machines I wanted to investigate something related to downward force from propellers. I recalled playing with a paper helicopter years ago and thought that it could make an interesting experiment to investigate the relation between a given variable in the propeller and the force it exerts. The variable which I could best isolate was area, since it allows other variables such as proportions to be completely fixed. It would make sense that, since a larger area allows for more air to hit the blades, the larger blades would enable greater downward force from air resistance.

In order to calculate the force, I needed a distance and a time and some formulas from the mechanics course which I will relay in the analysis. Hence I decided to drop paper helicopters with propellers of varying surface areas from a set height. The system is further explained in the methodology. I recorded the drops on my phone and proceeded to the analysis.

Research question

How does the area of the paper in a paper helicopter affect the upward force from air resistance from the paper?

[1] "Amazon Prime Air"

Variables

Independent variable

Area of paper	The area of the paper used for the rotors varied, with the shorter side varying in 0.03m, 0.04m. 0.05m, 0.06m and 0.07m (all ± 0.0005m). The longer side was always 1.5 times the length of the shorter side. Hence the area of the paper helicopter was $1.5X^2$, with X as the length of the shorter side.

Dependent variable

The upward force from paper propeller	The fall time of the helicopter was measured via analysing a video of all 25 falls, cut into separate pieces. This was done in LoggerPro. From this and the fall height the force from the rotating copter was calculated as Nm^{-2}.

Controlled variables

Fall height	The height of the fall was fixed to 3.22m ± 0.025m - this was done with the apparatus shown later. To keep the fall distance constant a piece of fishing line was added to the copters, also ensuring that the drop mechanism doesn't touch the copter.
Shape of paper	The angle of the wings of the helicopter was fixed for every helicopter and every fall. This was done with textile glue reinforcing the fold. The fold was kept constant by use of a mold visible in Appendix 3 and Diagram 1. The angle of the wings wasn't recorded, since being a fixed variable it doesn't affect the answer of the RQ.
Proportions of paper	The proportions of the paper were fixed for every copter to X as the shorter side and X*1.5 as the longer side.
Density of air	All the tests were done in the same airspace, meaning that air density remained approximately the same.
Acceleration of freefall	The acceleration of freefall was consistently approximately $9.81ms^{-2}$, as the experiment was performed on earth.

Risk assessment

Safety concerns

The drop mechanism had to be fastened steadily to the railing and the conductor of the experiment must be aware of any people walking in the drop zone, even if paper helicopters are unlikely to cause serious damage. Overall there were few serious safety concerns in this experiment perhaps with the exception of the relatively heavy drop mechanism.

Ethical concerns

There are no prominent ethical concerns in this experiment.

Environmental concerns

Due to environmental concerns the parts used for the experiment should be disposed of appropriately via recycling, when possible. For example the materials in the mold and the copters were recycled.

Methodology

The methodology starts with apparatus followed by four diagrams, which will be referred to in the procedure.

Apparatus

- Cardboard	2 x 20cm x 14cm + 24cm x 20cm
- Masking tape	100cm
- Fishing line	80cm
- A4 printer paper	x 1
- Textile glue	
- Thumbtacks	x 6
- Large(~1.5g) paper clip	x 1
- Scissors	x 2

- Table Clamp x 1
- Clamp x 1
- Electric Clamp x 1
- Metal Pipe Clamp x 1
- Metal pipe x 1

Diagrams

These diagrams are guidelines for setting up and crafting the necessary components of the experiment. They will be referred to in the procedure.

Diagram 1 - Setup

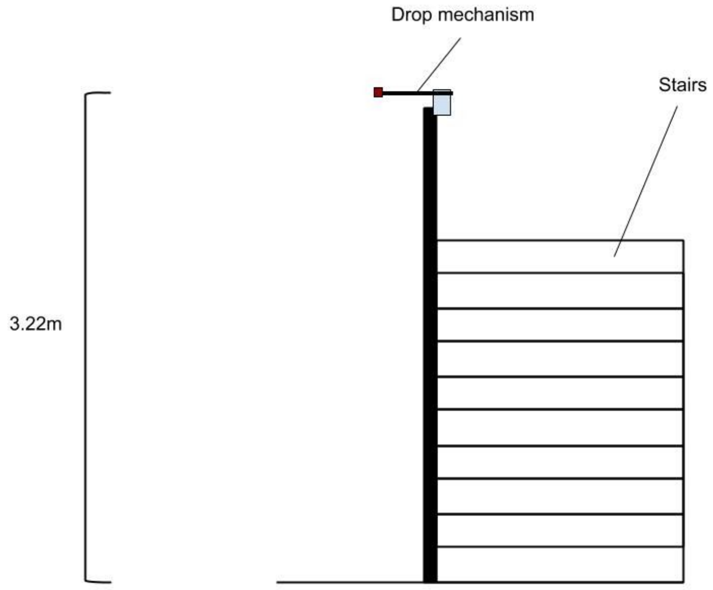


Diagram 2 - Drop mechanism

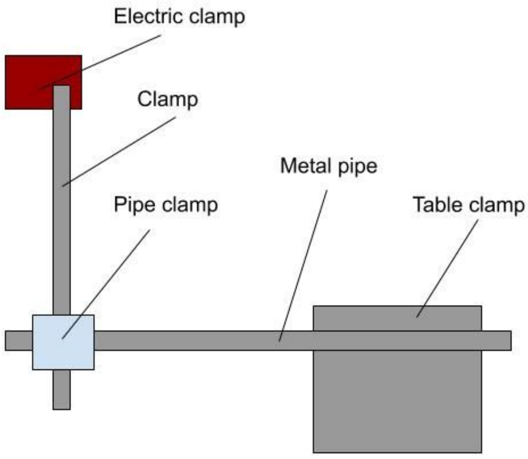


Diagram 3 - Mold

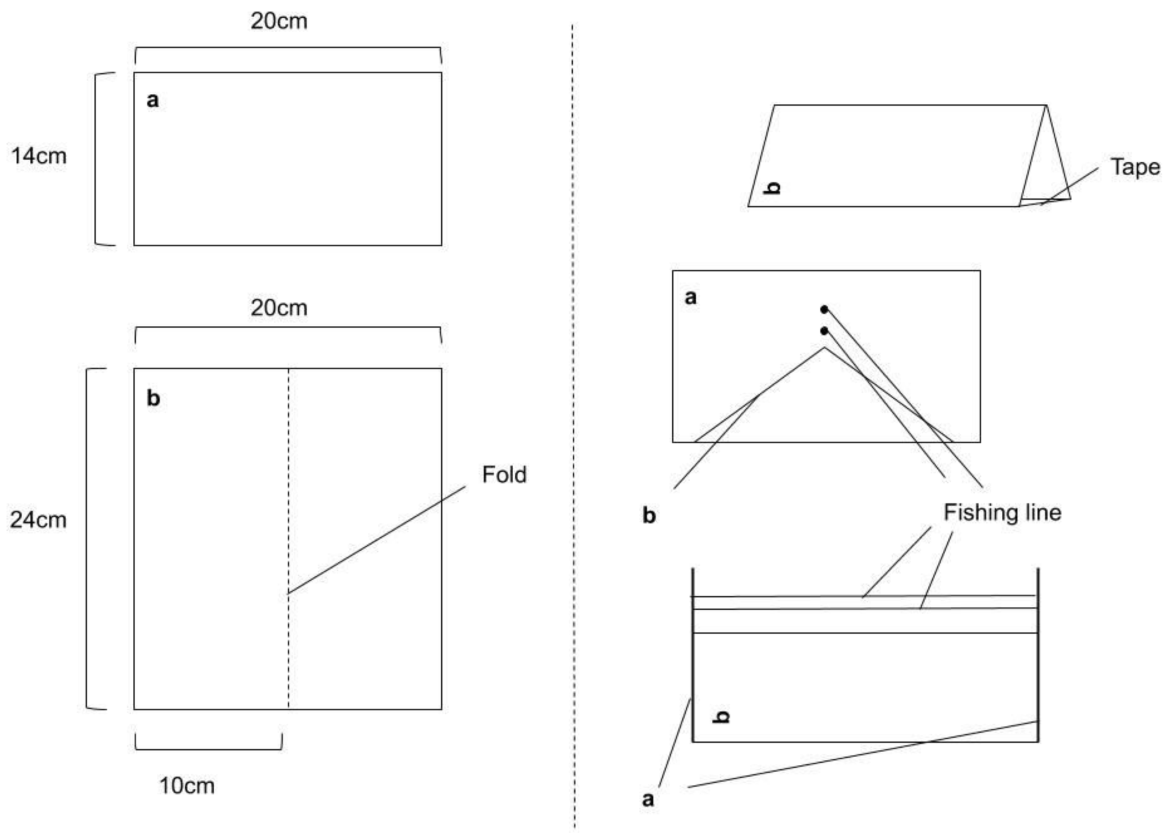
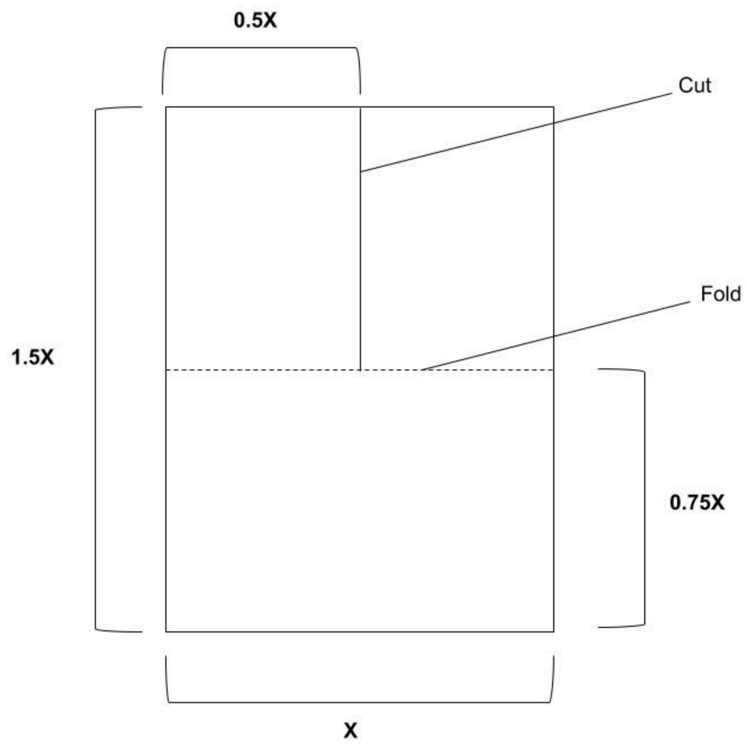


Diagram 4 - Shape of copter



Procedure

1. Making the copters
    a. Cut the pieces for the mold and glue / tape them together as per diagram 3. Here the priority is consistency, the mold should remain the same for each copter.
        i. The angle of the fold should be kept constant with marker's tape - the cardboard should push against the tape.
        ii. Placing paper on the central angle is recommended in order to sharpen the angle and make following steps easier.
        iii. While the glue dries the mold should be on it's side with a weight on the other side.
    b. Cutting and folding the paper in five different sizes according to diagram 4.
    c. Placing the copters two and three at a time in the mold and reinforcing the folds with glue. Held in place with the thumb tacks until dry.
    d. Add a piece of fishing line to ensure constant dropping distance and remove any possibility of the drop mechanism interfering with the copter.
2. Setting up the apparatus according to diagrams 1 and 2
3. Conducting the experiment
    a. Setting up a video recording device to film the experiment.
    b. Dropping each of the 5 copters 5 times to reduce random uncertainty.
4. Analysing the video. LoggerPro is recommended but any frame by frame video analysis software fulfills the requirements. From the video you see the flight time.
5. Analyse the data, convert the fall time to a force.

Analysis

Raw data

Table 1 - Flight time for C1

Trial	1	2	3	4	5
Time t (s)	1.533	1.500	1.537	1.500	1.500
Max time t_{max} (s)	1.537	Min time t_{min} (s)	1.500	Uncertainty (s)	0.0185
Average Flight Time t_{avg} (s)		**1.514**			
Percentage Uncertainty $\frac{\Delta t}{t}$ (%)		**1.22**			

Here $\Delta t = \frac{t_{max} - t_{min}}{2}$ and $\frac{\Delta t}{t} = \frac{\Delta t}{t_{avg}}$

This experiment was conducted with quantitative data. The data analysis process, an example of which can be seen in Table 1, takes the flight times and finds the values we need via data analysis in Google Sheets. These values are the average flight time and the percentage uncertainty of said time. The same process is repeated for each value, as seen from Table 4[2] in the appendix. The processed data from this analysis can be seen in Table 2 underneath.

Table 2 - Flight time and uncertainty

Copter / C	A (m^2)	t (s)	$\frac{\Delta t}{t}$ (%)
1	0.00735	1.514	1.22
2	0.0054	1.338	1.68
3	0.00375	1.140	2.94
4	0.0024	1.079	5.19
5	0.00135	0.927	3.61

Table 3 - Gradients and Gradient uncertainty

Gradient	Slope (sm^{-2})
Best Fit Gradient	95.43
Minimum Gradient	102.1
Maximum Gradient	91.16
Gradient Δt	5.47
Gradient $\frac{\Delta t}{t}$	5.73%

[2] Appendix 1

Graph 1 - Flight time vs Area

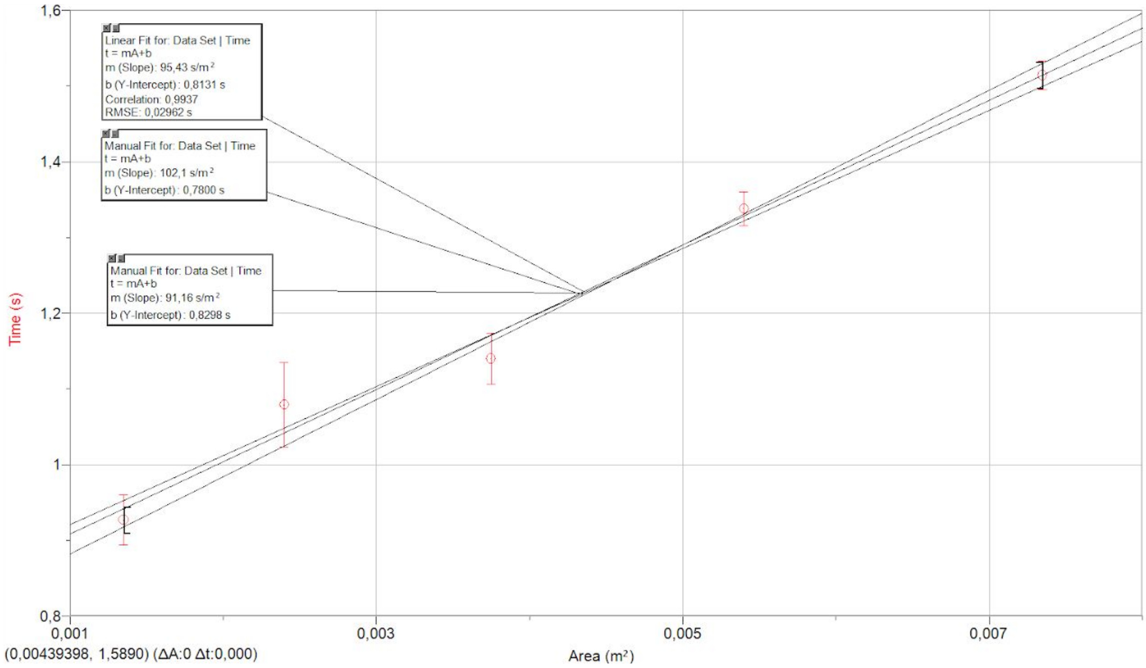


Table 3 gives us the relation between flight time and area of paper in the form of a graph. The linear graph implies a directly proportional relation between area of paper and flight time. In order to analyse the exerted force we must take the following steps. Copter 0, with a time of t_0, represents the paper clip being dropped without a paper attached. The time for this is estimated as the y intercept in Graph 1, with the gradient uncertainty as the uncertainty for the measurement of copter 0. This led to the following calculations, the results of which can be seen in Table's 4 and 5. $\Delta a = a_0 - a$, where a_0 is the acceleration of the clip on its own, in order to investigate the actual effect of the paper only. Using the course book[3], we find:

$$a = \frac{2 \cdot h}{t^2} \quad from \quad h = u \cdot t + \frac{1}{2} \cdot a \cdot t^2 = \frac{1}{2} \cdot a \cdot t^2$$

$$t_0 = 0.813ms^{-1} \pm 5.73\%$$

$$h = 3.22m \pm 0.005m = 3.22m \pm 5 \cdot 10^{-3}m$$

$$m = m \pm 0.0000005kg = m \pm 5 \cdot 10^{-7}kg$$

$$F = m \cdot \Delta a = m \cdot (a_0 - a) = m \cdot \left(\frac{2 \cdot h}{t_0^2} - \frac{2 \cdot h}{t^2} \right) = 2 \cdot h \cdot m \cdot \left(\frac{1}{t_0^2} - \frac{1}{t^2} \right)$$

$$\frac{\Delta h}{h} = \frac{5 \cdot 10^{-3}m}{3.22m} \cdot 100\% = 0.15...\%$$

$$\frac{\Delta m}{m} = \frac{5 \cdot 10^{-7}kg}{m} \cdot 100\% \quad where\ m\ is\ particular\ to\ the\ copter\ in\ question$$

$$\frac{\Delta F}{F} = \frac{\Delta h}{h} + 2 \cdot \frac{\Delta t}{t} + 2 \cdot \frac{\Delta t_0}{t_0} + \frac{\Delta m}{m}$$

[3] Tsokos, 2014

Table 4 - Calculating the forces

C #	t 0 (s)	h (m)	m (kg)	t (s)	F (N)
1	0.813	3.22	0.002230	1.51	0.0155
2	0.813	3.22	0.001959	1.34	0.0120
3	0.813	3.22	0.001832	1.14	0.0088
4	0.813	3.22	0.001720	1.08	0.0072
5	0.813	3.22	0.001638	0.93	0.0037

Table 5 - Calculating the percentage uncertainties

C #	$\frac{\Delta t_0}{t_0}$ (%)	$\frac{\Delta t}{t}$ (%)	$\frac{\Delta h}{h}$ (%)	$\frac{\Delta m}{m}$ (%)	$\frac{\Delta F}{F}$ (%)
1	5.73	1.22	0.15528	0.003203	14.062340
2	5.73	1.68	0.15528	0.004109	14.983120
3	5.73	2.94	0.15528	0.005622	17.498094
4	5.73	5.19	0.15528	0.006794	22.002055
5	5.73	3.61	0.15528	0.013200	18.857655

Next the values were compiled to Table 6 and graphed.

Table 6 - Values graphed and absolute uncertainty calculated

C #	A (m^2)	F (N)	$\frac{\Delta F}{F}$ (%)	$\frac{\Delta A}{A}$ (%)
1	0.0074	0.0155	14.062	0.00217
2	0.0054	0.0120	14.983	0.00180
3	0.0038	0.0088	17.498	0.00153
4	0.0024	0.0072	22.002	0.00159
5	0.0014	0.0037	18.858	0.00069

Graph 2 - Resisting force from paper

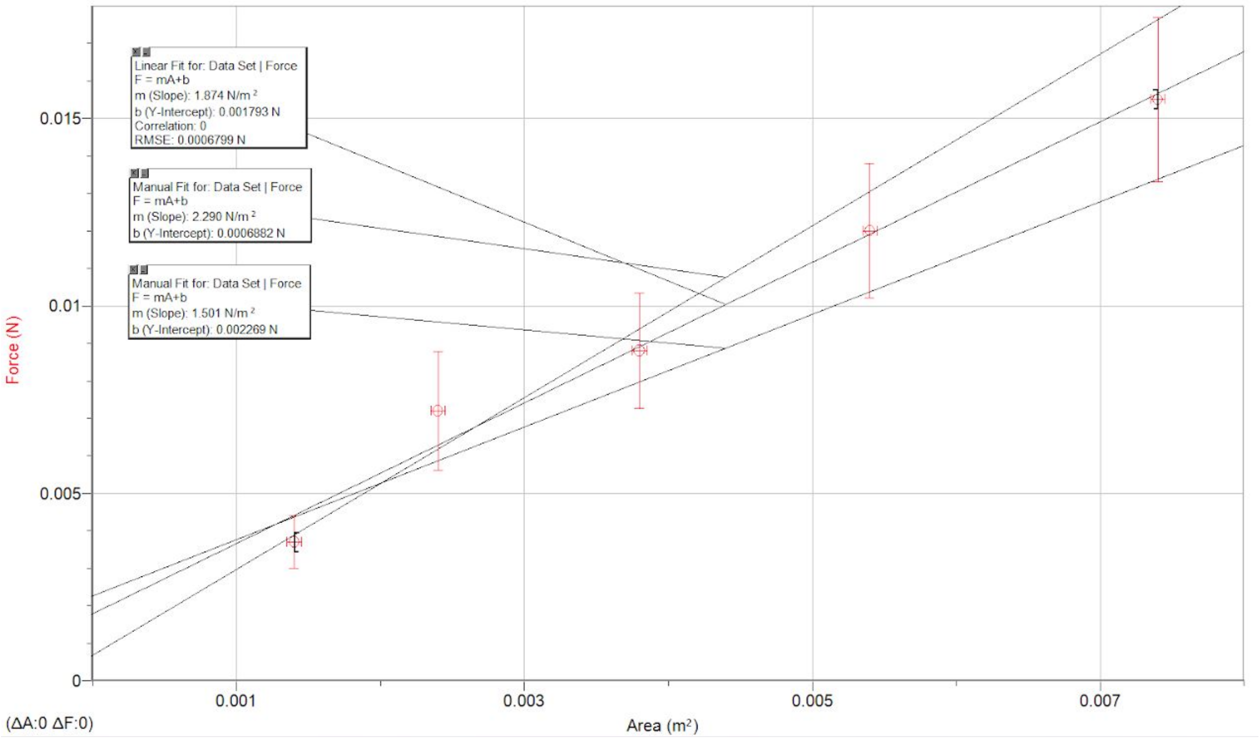


Table 7 - values from Graph 2

Best fit gradient	1.874 Nm^{-2}
Maximum gradient	2.290 Nm^{-2}
Minimum gradient	1.501 Nm^{-2}
Gradient uncertainty	0.395 Nm^{-2}
Gradient uncertainty (%)	21.05%
Force per area	1.874 Nm^{-2} ± 0.4Nm^{-2}

<u>Conclusion</u>

In conclusion the force exerted by the propeller is directly proportional to the surface area of the propeller.

The upward force from the paper was 1.874 Nm^{-2} ± 0.4 Nm^{-2}. These results may seem large at first but having a paper helicopter actually exert 2N of force is largely quite improbable without additional supporting structures due to the flimsy structure of paper.

The large uncertainty is due to the fairly extensive calculation, which made such an impact due to the uncertainty of graph 1, the time measurements and the experiment lacking the dropping of the paperclip on its own. This will be addressed further in the “Evaluation & Improvements” section. The result is consistent with the hypothesis.

Evaluation & Improvements

The experiment was fit for producing a range of results that led to a proper conclusion. The data was consistent and a linear relation was apparent, as expected.

Regarding the large uncertainty of the result the following improvements could be made.

Weakness	Improvement
The uncertainty of Graph 1 was 5.73%, which spilled over to the accuracy of the final calculation, since it's uncertainty was added twice to each force calculation. In practice the problem is the inaccuracy of the time measurements.	A better camera, a standard mobile phone records at about 60 frames per second, which results in greater systematic error. Alternatively decreasing the random error by dropping each copter more times decreases uncertainty.
Inaccuracy of t_0: the inaccuracy of Graph 1 spilled into the final result most severely in the calculation, since the uncertainty of t_0, which was the gradient uncertainty, had to be added to the total uncertainty twice each time.	Dropping the pin as part of the experiment to get t_0. The measurements' average uncertainty was 2.93% - 2.3% less than the gradient uncertainty of Graph 1 - hence dropping the pin would've yielded approximately 4.6% less error into each individual force calculation, resulting in less gradient uncertainty.

Were the experiment done again, dropping the pin on its own is an improvement which lessens both of the most significant weaknesses the experiment had.

Bibliography

"Amazon Prime Air." Amazon.com: Prime Air, Amazon, amazon.com/b?node=8037720011.

Tsokos, K. A. "Physics for the IB Diploma", 6th ed., Cambridge University Press, 2014, pp. 7-108.

Appendices

Table 7 - Raw Data and Processing

Flight time for C1					
Trial	1	2	3	4	5
Time	1.533	1.500	1.537	1.500	1.500
Max time	1.537	Min time	1.500	Uncertainty	0.0185
Average Flight Time		**1.514**			
Percentage Uncertainty		**1.22%**			

Flight time for C2					
Trial	1	2	3	4	5
Time	1.333	1.333	1.334	1.367	1.322
Max time	1.367	Min time	1.322	Uncertainty	0.0225
Average Flight Time		**1.338**			
Percentage Uncertainty		**1.68%**			

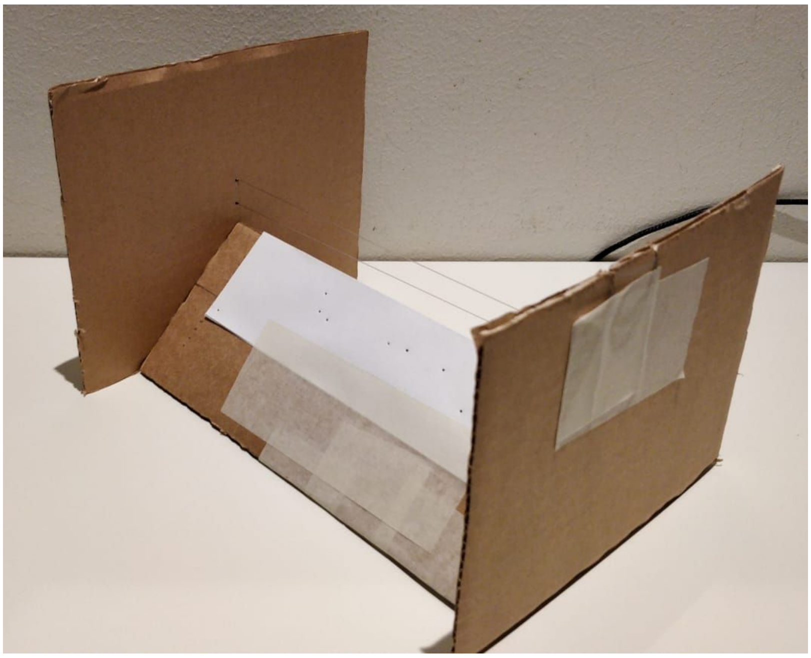
Flight time for C3					
Trial	1	2	3	4	5
Time	1.166	1.340	1.330	1.100	1.167
Max time	1.340	Min time	1.100	Uncertainty	0.12
Average Flight Time		**1.221**			
Percentage Uncertainty		**9.83%**			

Flight time for C4					
Trial	1	2	3	4	5
Time	1.100	-	1.060	1.022	1.134
Max time	1.134	Min time	1.022	Uncertainty	0.056
Average Flight Time		**1.079**			
Percentage Uncertainty		**5.19%**			

Flight time for C5					
Trial	1	2	3	4	5
Time	0.934	0.900	0.933	0.900	0.967
Max time	0.967	Min time	0.900	Uncertainty	0.0335
Average Flight Time		**0.927**			
Percentage Uncertainty		**3.61%**			

Appendix 2 - Picture of the system

Appendix 3 - The mold



Appendix 4 - Tools and materials