Fys4150 Project 1

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Abstract

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1 Introduction

2 Theory

Vectorized second derivative

If we have a 1d data-set on the form:

$$V(\vec{x}) = [v_0,\ v_1,\ \cdots,\ v_{n-1},\ v_n,\ v_{n-1},\ \cdots]$$

Then we can write the second derivative of the data-set as:

$$f_n = -\frac{v_{n+1} + v_{n-1} - 2v_n}{\Delta x^2}$$

Where Δx is the change in variable we are derivating based on; usually time.

Rather than calculating the second derivatives of this data-set individually, we can instead calculate them all at the same time using linear algebra. This can be done by finding a matrix A such that

$$A\vec{V} = \vec{f}$$

Where:

$$\vec{f} = \begin{bmatrix} 2v_i - v_2 \\ -v_1 + 2v_2 - v_3 \\ \vdots \\ -v_{n-1} + 2v_n - v_{n+1} \\ -v_n + 2v_{n+1} - v_{n+2} \\ \vdots \end{bmatrix}$$

As multiplying $\vec{f}$ with $\frac{1}{\Delta x^2}$ would give us an array containing all the second derivatives. We can see that $\mathbf{A}$ must be:

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 & 0 & \cdots \\ -1 & 2 & -1 & 0 & \cdots \\ 0 & -1 & 2 & -1 & \cdots \\ \vdots & 0 & -1 & 2 & -1 \end{bmatrix}$$

3 Method

4 Results

5 Discussion

6 Conclusion