Week 8 : Link Analysis (PART2)

1. Observe the graph shown in Figure 1. According to the principle of repeated improvement, which of the following is correct?

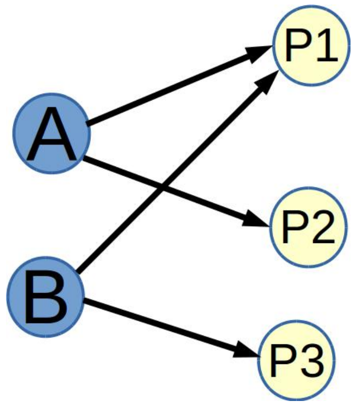

Figure 1: The Graph

A. $A = P1 \times P2, B = P1 \times P3, P1 = A + B, P2 = A, P3 = B$
B. $A = P1 + P2, B = P1 + P3, P1 = A \times B, P2 = A, P3 = B$
C. $A = P1 + P2, B = P1 + P3, P1 = A + B, P2 = A, P3 = B$
D. $A = P1 + P2, B = P1 + P3, P1 = A \times B, P2 = 0, P3 = 0$

Explanation: Using the concept of hubs and authorities, A receives points from $P1$ and $P2$; B gets points from $P1$ and $P3$; $P1$ gets pointed by both A and B; hence gets points from both; $P2$ is pointed only by A; $P3$ gets pointed only by B. All the points one node gets added up.

2. In the graph shown in Figure 2, assume that the current pagerank values of A, B and C are 0.2, 0.4 and 0.4 respectively. What will be their pagerank values after one iteration?

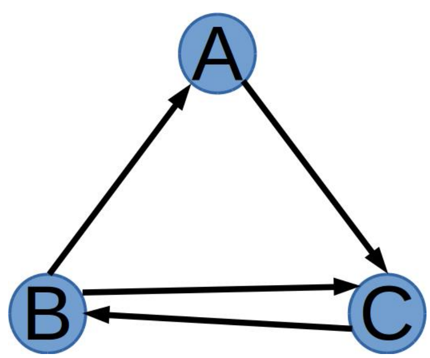

Figure 2: The Graph

A. $A : 0.4, B : 0.4, C : 0.4$
B. $A : 0.2, B : 0.4, C : 0.4$
C. $A : 0.4, B : 0.2, C : 0.4$
D. $A : 0.4, B : 0.4, C : 0.2$

Explanation: A gets half of the points from B, i.e. 0.2. B gets all the points of C, i.e. 0.4 points. C gets all the points of A and half of the points of B, i.e. $0.2+0.2=0.4$ points. Hence, the correct answer is **B**.

3. Given a vector $(3, 4)$ in the XY plane, what will this vector become after being pulled to the unit circle as shown in the Figure 3?

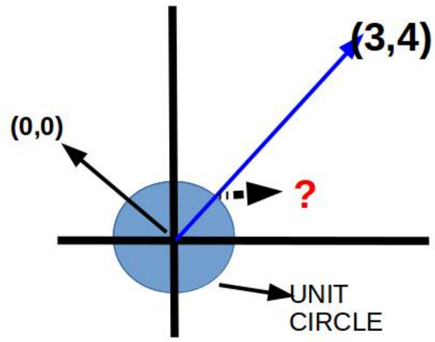

Figure 3: Figure

A. $4/5, 3/5$
B. $4/25, 3/25$
C. $3/5, 4/5$
D. $3/25, 4/25$

Explanation: As discussed in the lecture, the normalised vector is $(\frac{3}{\sqrt{3^2+4^2}}, \frac{4}{\sqrt{3^2+4^2}})$ = (3/5,4/5).

4. When we normalise a vector by pulling it on the unit circle

A. Its magnitude as well as the direction change.
B. Its magnitude remains the same but the direction changes.
C. Its magnitude might change but the direction remains the same.
D. Both the magnitude as well as the direction remains the same.

Explanation: When we normalize a vector by pulling it on the unit circle, its magnitude changes but the direction remains the same.

5. When we add two vectors in the XY plane, where one vector has a very high magnitude as compared to the other, then the resultant vector is closer towards (in terms of direction) to

A. the bigger vector
B. the smaller vector
C. origin
D. none of the above

Explanation: When we add two vectors in the XY plane, where one vector has a very high magnitude as compared to the other, then the resultant vector is closer towards (in terms of direction) to the bigger vector. This has been discussed in the lecture.

6. Given two linearly independent vector $v1$ and $v2$, which of the following is true?

A. Any other vector can be written as the linear combination of $v1$ and $v2$. i.e. $z = \alpha\ v1 + \beta\ v2$.
B. Any other vector can be written as sum of $v1$ and $v2$. i.e. $z = v1 + v2$.
C. Any other vector can be written as difference of $v1$ and $v2$. i.e. $z = |v1 - v2|$.
D. Any other vector can be written as multiplication of $v1$ and $v2$. i.e. $z = v1 \times v2$.

Explanation: In a 2D plane, any vector can be written as the linear combination of two linearly independent vectors. Hence the answer is **A**

7. When we repeatedly apply a matrix A to a vector v, k times; we get $A^k v$. For a very large value of k

   A. $A^k v$ converges in the direction of the eigen vector corresponding to the bigger eigen value of the matrix A.
   B. $A^k v$ converges in the direction of the eigen vector corresponding to the smaller eigen value of the matrix A.
   C. $A^k v$ converges towards the origin.
   D. None of the above.

   Explanation: $A^k v$ converges in the direction of bigger eigen vector of the matrix A. This has been shown in the lecture video.

8. Given the graph as shown in Figure 4. While calculating the pagerank using matrix multiplication method on this graph, how does the first matrix operation look like?

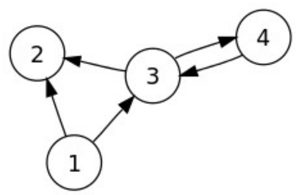

Figure 4: The Graph

A.
$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1/2 & 1 & 1/2 & 0 \\ 1/2 & 0 & 0 & 1 \\ 0 & 0 & 1/2 & 0 \end{pmatrix} \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix} \tag{1}$$

B.
$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 1/2 & 0 & 0 & 1 \\ 0 & 0 & 1/2 & 0 \end{pmatrix} \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix} \tag{2}$$

C.
$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix} \tag{3}$$

D.
$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix} \tag{4}$$

Explanation: In the matrix, element in the row i and column j represents the fraction of points node j gives to node i. Hence the answer is **A**. Another way of finding the answer is looking at the sum of elements in a column. It should always sum up to 1. This holds true only in the case of option A.

9. In a Markov matrix

   A. The sum of elements in every row is 1.
   B. The sum of elements in every column is 1.
   C. The sum of diagonal elements is 1.
   D. None of the above.

   Explanation: In a Markov matrix, elements in every column add up to 1.

10. Highest eigen value of a Markov matrix is

    A. -1
    B. 0
    C. 1
    D. None of the above.

    Explanation: It has been shown in the lecture videos that the highest eigen value of a Markov matrix is 1.