Week 9 (Power Law)- Assignment

1. In $G(1000, 0.1)$ random network, each edge will be placed with the probability:

   A. 0.1
   B. 0.9
   C. 0.5
   D. 0.7

   Description : In $G(n, p)$ network, each edge is placed with the probability p, so in the given graph each edge will be placed with probability 0.1.

2. Which of the following correctly depicts the meaning of 'Height distribution of people in a city'?

   A. Plot different height values on the X axis and the fraction/percentage of people having those heights on the X axis.
   B. Plot different height values on the X axis and the fraction/percentage of people having heights greater than those heights on the X axis.
   C. Plot different height values on the X axis and the fraction/percentage of people having heights less than those heights on the X axis.
   D. None of the above

   Description : For plotting height distribution of people in a city, we plot different possible heights on the X axis and the fraction/percentage of people having those heights on the Y axis.

3. Which of the following distributions depict a normal/bell curve?

   A. Weights of people in a city
   B. Heights of people in a city
   C. Intelligence quotients of people in a city
   D. All of the above

   Description : All of the above have a normal distribution.

4. Given a random graph on 1000 nodes where each of the possible $\binom{1000}{2}$ edges is present with a probability of 0.1. Let N_1 represent the number of nodes having the least degree (i.e. 0), N_2 represent the number of nodes having the highest degree (i.e. 999) and N_3 represent the number of nodes having the median values of degrees (i.e. 499 or 500). Choose the correct statement.

   A. $N_1 > N_3$ and $N_2 > N_3$
   B. $N_1 < N_3$ and $N_2 < N_3$
   C. $N_1 > N_3$ and $N_2 < N_3$
   D. $N_1 < N_3$ and $N_2 > N_3$

   Description : The degree distribution of a random graph follows normal distribution. Here the number of very high degree and very low degree nodes is less as compared the number of nodes having medium degree. Hence the option B is correct.

5. Given the World Wide Web (WWW) network. Let N_1 represent the number of nodes having the least degree , N_2 represent the number of nodes having the highest degree and N_3 represent the number of nodes having the median values of degrees, in this network. Choose the correct statement.

   A. $N_1 > N_3$ and $N_2 > N_3$
   B. $N_1 < N_3$ and $N_2 < N_3$
   C. $N_1 > N_3$ and $N_2 < N_3$
   D. $N_1 < N_3$ and $N_2 > N_3$

   Description : The degree distribution of WWW graph shows a drop. Here, as the degree increases, the number of nodes having these degrees decrease. Hence the option C is correct.

6. Given set $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. We pick a value a_1 uniformly at random from this set E. Next, we pick another value a_2, again uniformly at random from this set E. Similarly we pick 8 more values, $a_3, a_4, ..., a_10$. Look at the sum $S = a_1 + a_2 + ... + a_{10}$. Which of the following sets define the range from which the sum S can have values from?

   A. $\{1, 2, ..., 9, 10\}$
   B. $\{10, 11, ..., 19, 20\}$
   C. $\{1, 2, ..., 9, 100\}$
   D. $\{10, 11, ..., 99, 100\}$

   Description : The minimum value of S will be 10 when all the selected elements $a_1, a_2, ..., a_{10}$ have a value 1 each. The maximum value of S will be 100 when all the selected elements $a_1, a_2, ..., a_{10}$ have a value 10 each. Hence the correct option is D.

7. Given set $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. We pick a value a_1 uniformly at random from this set E. Next, we pick another value a_2, again uniformly at random from this set E. Similarly we pick 8 more values, $a_3, a_4, ..., a_10$. Look at the sum $S = a_1 + a_2 + ... + a_{10}$. Let $p(i)$ be the probability that $S = i$, i.e., the probability that the sum of these randomly chosen 10 elements is i. Which of the following is true?

   A. $p(50) < p(100)$
   B. $p(100) < p(50)$
   C. $p(50) < p(10)$
   D. $p(100) < p(10)$

   Description : There is a less probability that all the chosen values are 1 and hence the sum is 10. Similarly, there is a very less probability that all the chosen elements are 10 and the sum S is 100. Medium value of S are more likely to be seen as compared to the extreme values (very less or very high). Hence, the option B is correct.

8. Given set $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. We pick a value a_1 uniformly at random from this set E. Next, we pick another value a_2, again uniformly at random from this set E. Similarly we pick 8 more values, $a_3, a_4, ..., a_10$. Look at the sum $S = a_1 + a_2 + ... + a_{10}$. Let $p(i)$ be the probability that $S = i$, i.e., the probability that the sum of these randomly chosen 10 elements is i. We plot i on the X-axis and $p(i)$ on the Y axis. Choose the correct statement from the following.

   A. The plot has very high values in the beginning but then drops.
   B. The plot is a constant curve.
   C. The plot is a bell shaped curve.
   D. The plot is linear.

   Description : The probability distribution of the sum of random variables depicts a bell shaped curve.

9. Which of the following represents the correct equation for power law?

   A. $y = \frac{1}{x^2}$
   B. $y = \frac{1}{x^3}$
   C. $y = \frac{1}{x^4}$
   D. All of the above

   Description : Power law in general follows the equation $y = \frac{1}{k^\alpha}$

10. Which of the following distributions follow a power law?

    A. Telephone conversation duration
    B. Number of song downloaded from a website
    C. Number of incoming links for nodes in the web graph
    D. All of the above

    Description : All of these distributions follow power law.

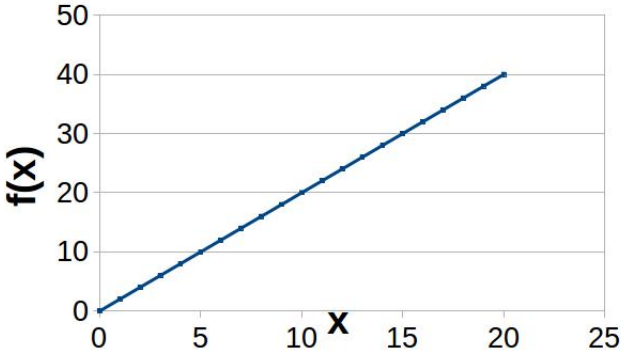
11. Power law degree distribution in real world networks follow the characteristic equation $y = \frac{1}{k^{\alpha}}$. What is the value of α here?

    A. $1 < \alpha < 2$
    B. $2 < \alpha < 3$
    C. $3 < \alpha < 4$
    D. None of the above

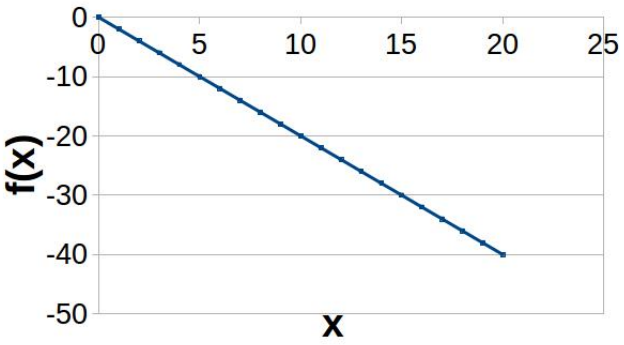
    Description : In real world power law degree distributions, the value of α has been observed to be between 2 and 3.

12. Consider the equation $f(k) = \frac{1}{k^2}$. Let $x = \log k$ and $f(x) = \log f(k)$. How does the plot of $f(x)$ look like?

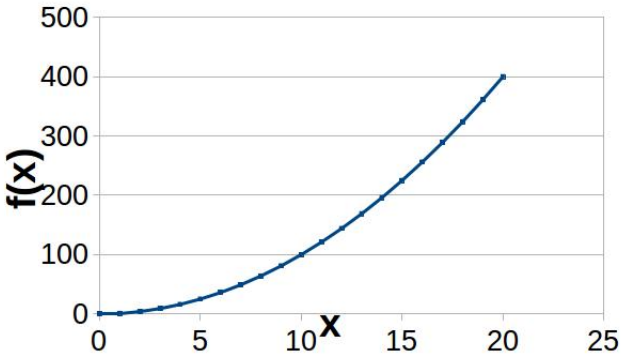
A.

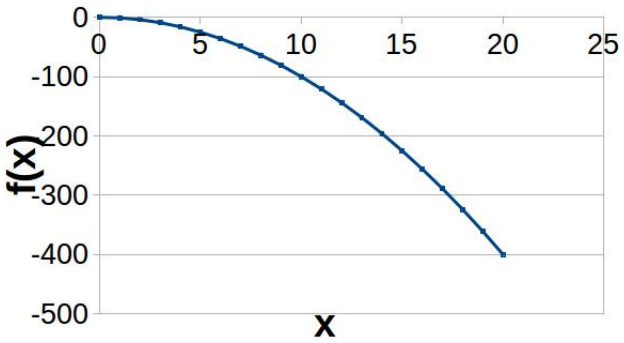


B.



C.





D.

Description : $f(x) = x^{-2}$. $\log f(x) = -2 \log x$

13. How does the power law degree distribution come by in real world networks?:

    A. By preferential attachment
    B. By random linking
    C. By uniform edge connection
    D. No hypothesis is found.

    Description : Barabasi and Albert proposed a hypothesis that the real world networks are evolved using the preferential attachment and this gives birth to power law degree distribution.

14. In the preferential attachment, a new coming node will prefer to make the connection with the node having _____:

    A. fewer friends
    B. More friends
    C. Average number of friends
    D. None of the above

    Description : In preferential attachment model, a new coming node prefers to make the connection with the node having the higher degree. So, this model gives birth to power law degree distribution.

15. Given a network being generated by 'rich get richer' phenomenon. Figure 1 shows the snapshot of the network at time t. A new node u enters the network at time $t + 1$ and makes an edge with one of the existing nodes. What is the probability that u will make an edge with C?

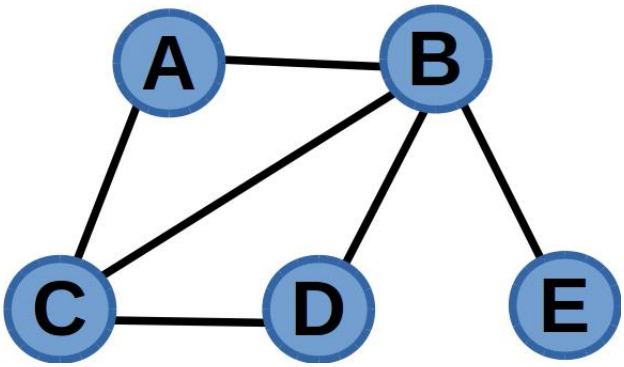


Figure 1: Network at time t

    A. 1/2
    B. 1/5
    C. 1/3
    D. 1/4

Description : The sum of degrees of all the nodes in the network is 12. The degree of the node C is 1. Hence, $P(link\ formation = 3/12 = 1/4)$

16. Given a network being generated by 'rich get richer' phenomenon. Figure 2 shows the snapshot of the network at time t. A new node u enters the network at time $t+1$ and makes an edge with one of the existing nodes. The probability of u making an edge with an existing node w is defined as $p(w)$. Which of the following equations is correct?

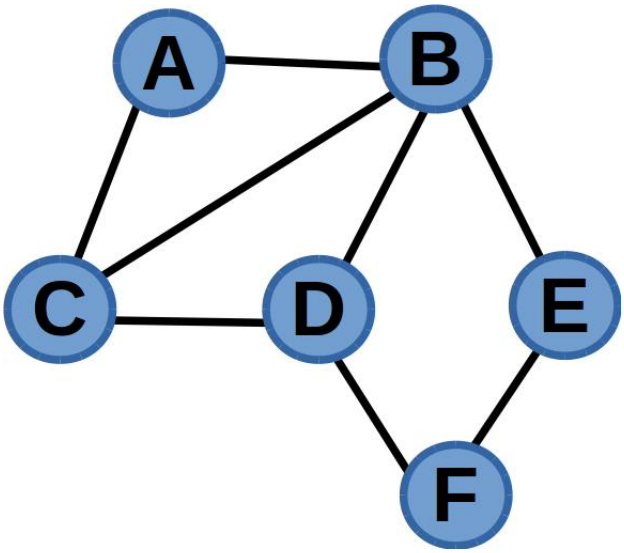


Figure 2: Network at time t

A. $p(A) < p(C) < p(B)$
B. $p(E) < p(D) < p(B)$
C. $p(F) < p(C) < p(B)$
D. All of the above

Description : $p(A) = 2/16, p(B) = 4/16, p(C) = 3/16, p(D) = 3/16, p(E) = 2/16, p(F) = 2/16$

17. Given a network being generated by 'rich get richer' phenomenon. Figure 2 shows the snapshot of the network at time t. A new node u enters the network at time $t+1$ and makes an edge with one of the existing nodes. Which of the node it makes a link to?

A. A
B. B
C. C
D. Can't say

Description : We can only comment about the probabilities with which u will get connected to different nodes but can't comment deterministically which node it will connect to.