Engineering Mechanics Notes

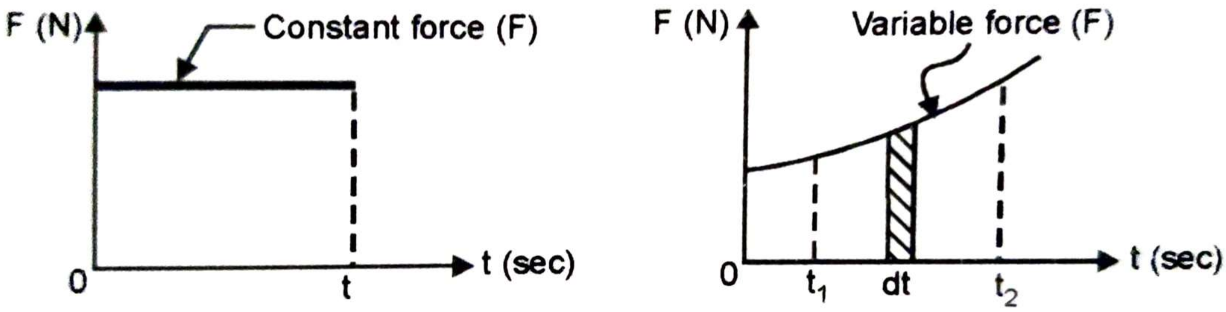
Module 5 – Kinetics of Particle

Module Section 5.3 – Kinetics – Impulse Momentum Equation

Class: FY BTech Faculty: Aniket S. Patil Date: 11/06/23

References: Engineering Mechanics, by M. D. Dayal & Engineering Mechanics – Statics and Dynamics, by N. H. Dubey.

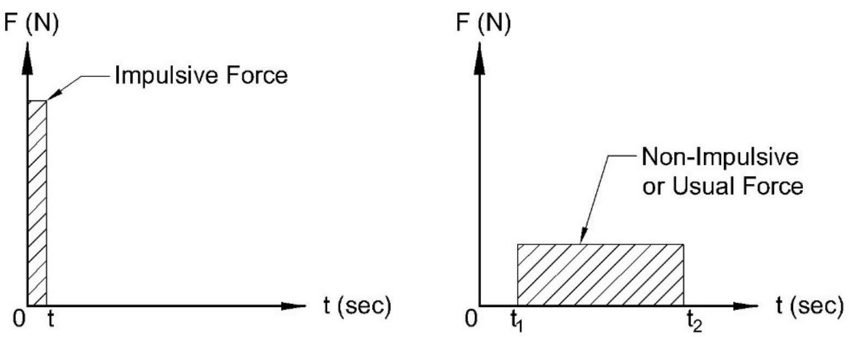
Impulse: For a particle acted upon by a force F for a duration of time t, the force is said to impart an impulse on the particle and the magnitude of this impulse is the product of the force and the duration for which it acts.



If the force F is constant during the time it acts, Impulse $= \mathrm{F} \times \mathrm{t}$

If the force F is variable from time t_1 to t_2, Impulse $= \int_{t_1}^{t_2} \mathrm{F}\, dt$

Impulsive Force: A large force when acts for a very small time and which causes a considerable change in a particle's momentum is called an impulsive force. E.g., bats hitting balls, two bodies colliding, hammering nails, guns firing, etc.



Impulsive forces are different from usual forces because the impulse generated is mainly due to the large force, and the time is less important; whereas usual forces also generate impulse, where the time, an equally important parameter, is large and equally contributes to the impulse generated.

Impulse Momentum Equation:

From Newton's Second Law, we have,

$$F = \frac{d(mv)}{dt} \Rightarrow F\,dt = d(mv)$$

$$\int_{t_1}^{t_2} F\,dt = \int_{v_1}^{v_2} d(mv) = mv_2 - mv_1$$

$$\therefore \text{Impulse}_{1-2} = \Delta\text{Momentum}$$

$$mv_1 + \text{Impulse}_{1-2} = mv_2$$

Or $\qquad$ Initial Momentum + Impulse Imparted = Final Momentum

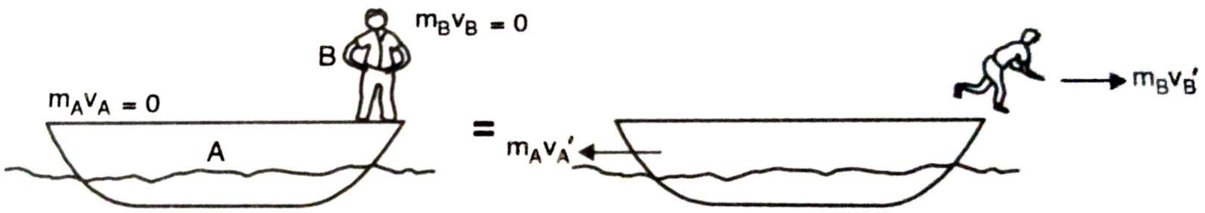
This gives rise to the Principle of Impulse Momentum which states that, "*for a particle or a system of particles acted upon by forces during a time interval, the total impulse acting on the system is equal to the difference between the final momentum and initial momentum during that period*".

Conservation of Momentum Equation:

In a system, if the resultant force is zero, the impulse momentum equation reduces to final momentum equal to initial momentum. Such situation arises in many cases because the force system consists of only action and reaction on the elements of the system. The resultant force is zero, only when entire system is considered, but not when the free body of each element of the system is considered.

If a man jumps off a boat, the action of the man is equal and opposite to the reaction of the boat. Hence, the resultant is zero in the system. Similar equation holds good when we consider the system of a gun and shell.

$$\text{Initial momentum} = \text{Final momentum}$$



$$mv_1 + \text{Impulse}_{1-2} = mv_2$$

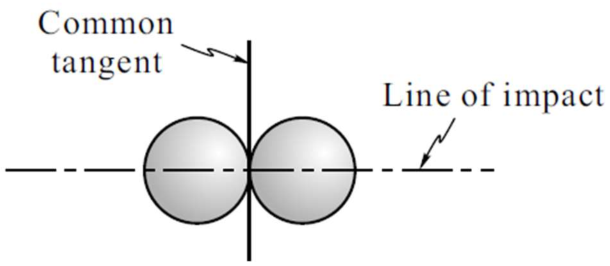
$$m_Av_A + m_Bv_B = m_Av_A' + m_Bv_B' \; (\because \text{Impulse}_{1-2} = 0)$$

$$m_Av_A' = -m_Bv_B' \; (\because m_Av_A = 0, m_Bv_B = 0)$$

"*For dynamic situations involving a system of particles, if the net impulse is zero, the momentum of the system is conserved.*"

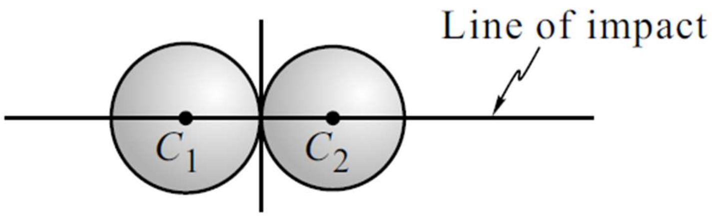
Impact: A collision of two bodies, which occurs for a very small interval of time and during which the two bodies exert relatively very large forces on each other, is called an impact.

Line of Impact: The common normal to the surfaces of two bodies in contact during the impact is called line of impact, and is perpendicular to the common tangent.

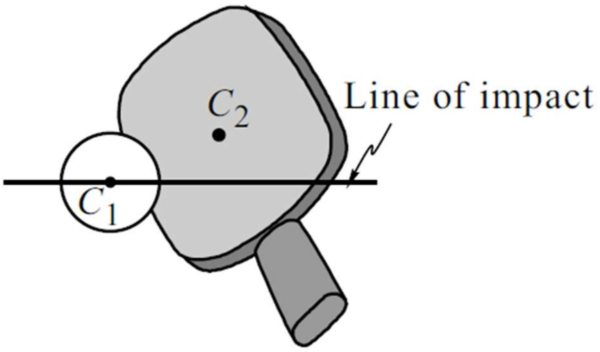


Types of Impact:

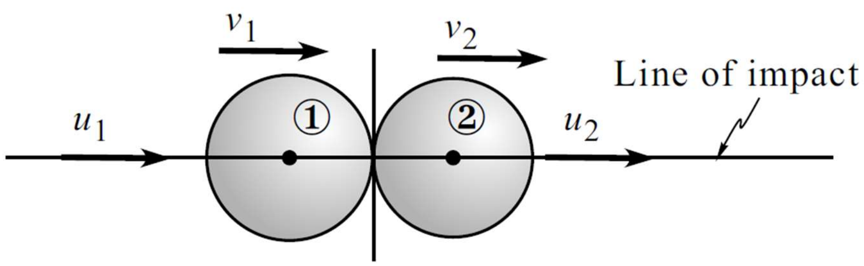
Central Impact: When the mass centres C_1 and C_2 of the colliding bodies lie on the line of impact, it is called central impact.



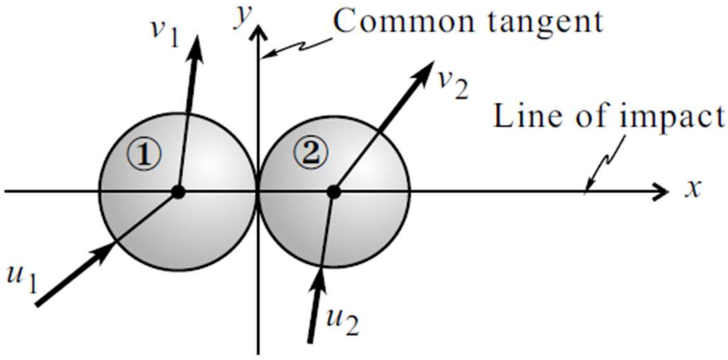
Non-Central Impact: When the mass centres C_1 and C_2 of the colliding bodies do not lie on the line of impact, it is called non-central or eccentric impact.



Direct Central Impact: When the direction of motion of the mass centres of two colliding bodies is along the line of impact then we say it is direct central impact. Here, the velocities of two bodies collision are collinear with the line of impact.

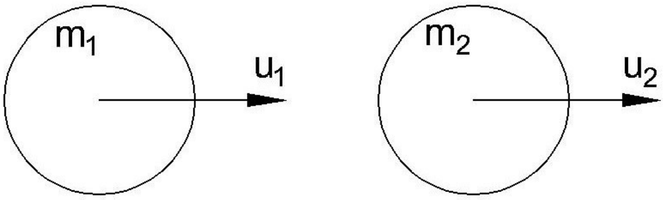


Oblique Central Impact: When the direction of motion of the mass centres of one or two colliding bodies is not along the line of impact (i.e., at the same angle with the line of impact) then we say it is oblique central impact. Here the velocities of two bodies collision are not collinear with the line of impact.

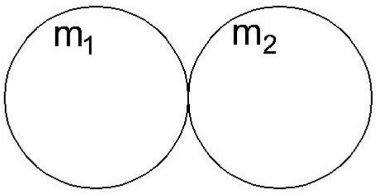


Direct Central Impact:

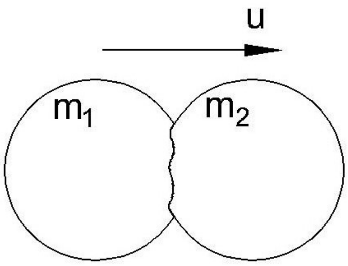
1. Two particles with masses m_1 & m_2 are traveling at velocities u_1 & u_2. If u_1 is greater than u_2, impact will occur.



2. When impact takes place, the period of impact is made of period of deformation and the period of restitution (regaining the original shape).

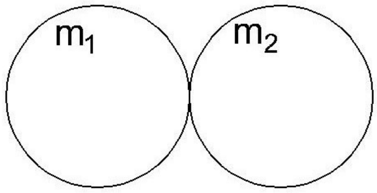


3. During the period of deformation, the particles exert large impulsive forces on each other. The deformation of both particles continues till maximum deformation, when both particles are momentarily united and are moving together with common velocity u.

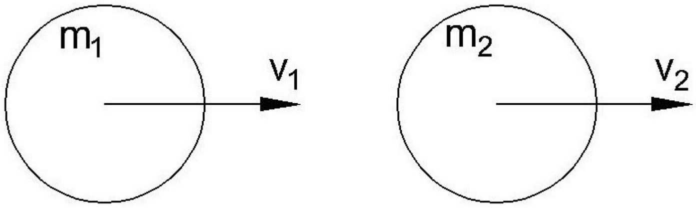


4. Now the period of restitution begins, during which the particles restore their shape. The particles may restore their shape completely, partially, or not at all, depending upon their properties. During this period also some impulsive

force is exerted by the particles on each other. At the end of this period the particles separate from each other.

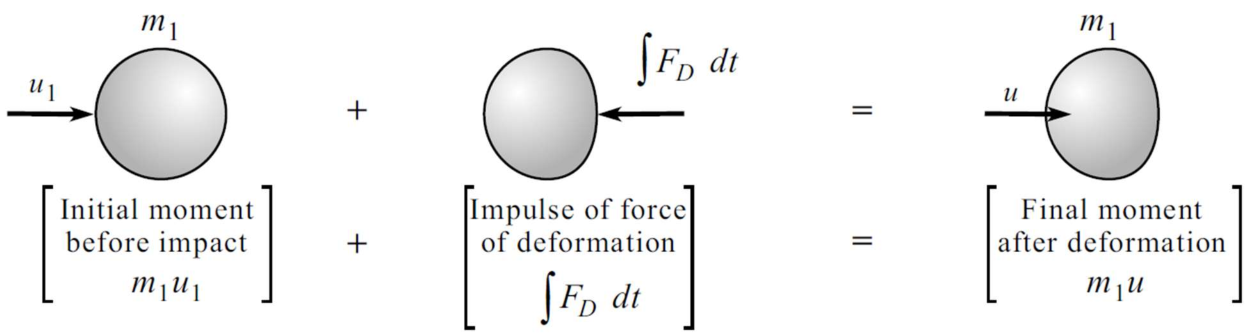


5. The particles now move independently with new velocities v_1 and v_2.



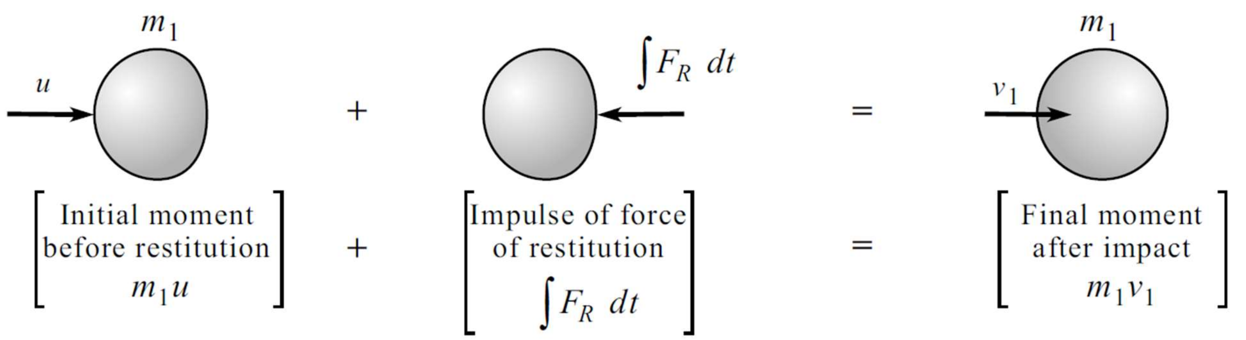
Coefficient of Restitution: *It is the ratio of the impulse exerted between the colliding particles during the period of restitution to the impulse exerted during the period of deformation*. This indicates the fraction of the shape that is regained by the particles, that was deformed during the collision.

During period of deformation, from beginning of impact till maximum deformation,



$$m_1u_1 - \int F_D\, dt = m_1u$$

During period of restitution, from maximum deformation till end of impact,



$$m_1u - \int F_R\, dt = m_1v_1$$

Hence, the deformation impulse and restitution impulse can be written as,

$$\int F_D\, dt = m_1u_1 - m_1u \quad \& \quad \int F_R\, dt = m_1u - m_1v_1$$

Therefore, coefficient of restitution, e, is given by,

$$\frac{\int F_R\, dt}{\int F_D\, dt} = \frac{m_1u - m_1v_1}{m_1u_1 - m_1u} = \frac{u - v_1}{u_1 - u} = e$$

Similarly, for particle 2, using the same impulse momentum equations,

$$\frac{v_2 - u}{u - u_2} = e$$

Using the addendo property from mathematics,

$$e = \frac{u - v_1 + v_2 - u}{u_1 - u + u - u_2} = \frac{v_2 - v_1}{u_1 - u_2}$$

Hence, another definition of coefficient of restitution for direct central impact is "*the ratio of velocity of separation to the velocity of approach*".

Equations to solve Direct Central Impact problems:

1. Total momentum of system is conserved along the line of impact.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

2. Coefficient of restitution equation,

$$v_2 - v_1 = e(u_1 - u_2)$$

Special Case of Direct Central Impact**:** When a ball hits a wall or ground, which can be considered as having infinite mass as compared to the ball.

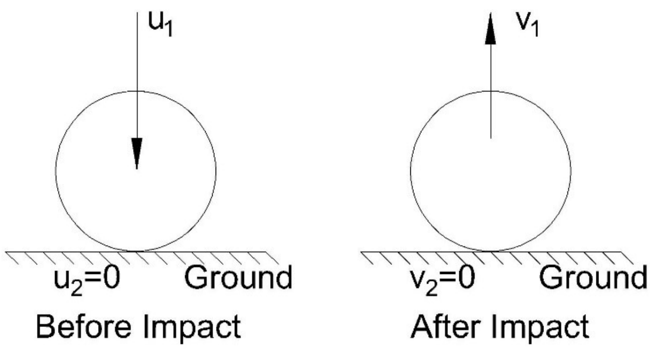
$v_2 - v_1 = e(u_1 - u_2)$

$0 - v_1 = e(u_1 - 0)$

$v_1 = -eu_1$

Only considering the magnitude,

$v_1 = eu_1$



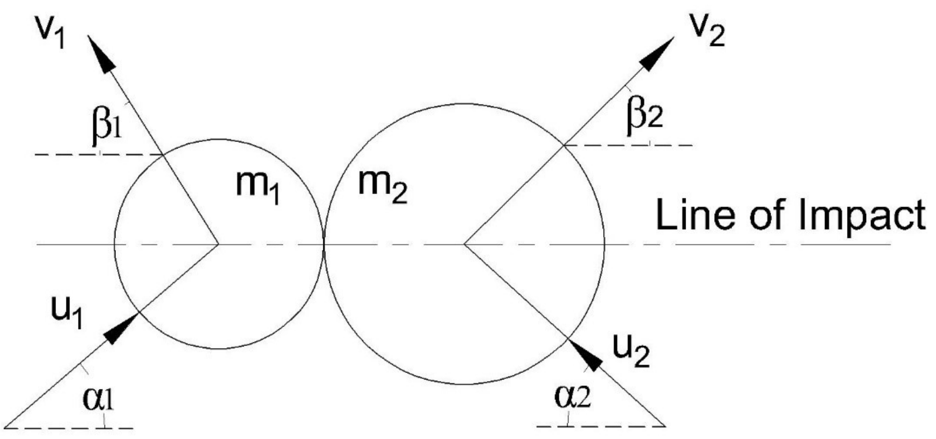
Let a ball be dropped from a height h on the ground and it reaches height h_n after n bounces, and the coefficient of restitution between ball and ground is e.

$$e = \frac{v_1}{u_1} = \frac{\sqrt{2gh_1}}{\sqrt{2gh}} = \left(\frac{h_1}{h}\right)^{\frac{1}{2}} \Rightarrow \text{For n bounces, } e = \left(\frac{h_n}{h}\right)^{\frac{1}{2n}}$$

Types of Impact Based on Coefficient of Restitution:

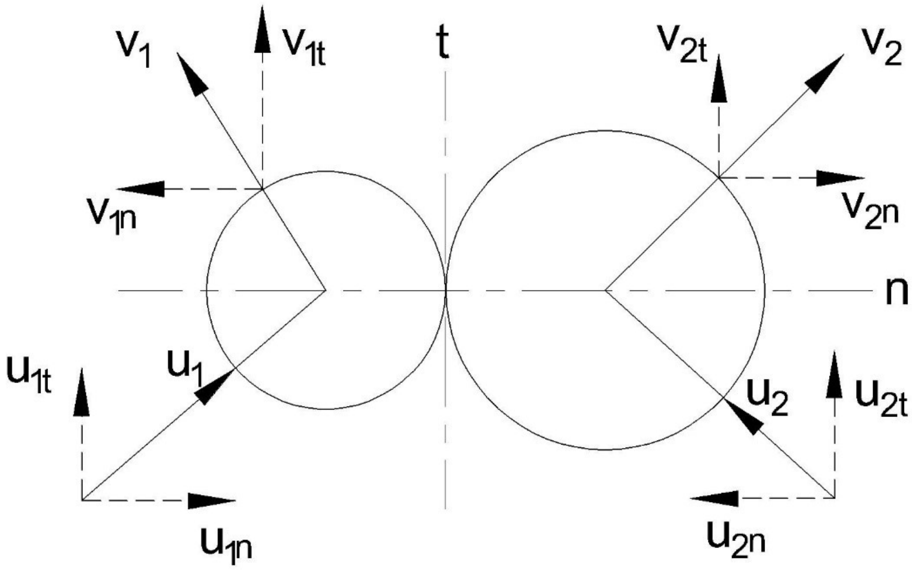
1. Perfectly Elastic Impact ($e = 1$)
   a. Momentum is conserved along the line of impact
   $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
   b. KE is conserved. No loss of KE.
   $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
2. Perfectly Plastic Impact ($e = 0$)
   a. After impact both the bodies collide and move together, and Momentum is conserved.
   $$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
   b. There is loss of KE during impact. Thus, KE is not conserved.
   $$KE_{loss} = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$
3. Semi-elastic Impact ($0 < e < 1$)
   a. Momentum is conserved along the line of impact
   $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
   b. Coefficient of restitution value is to be used.
   $$v_2 - v_1 = e(u_1 - u_2)$$
   c. There is loss of KE during impact.
   $$KE_{loss} = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

Oblique Central Impact:



In oblique central impact, the impulsive force acts along the line of impact (common normal). Thus, the velocity changes occur only along the line of impact and no change in velocity takes place in a direction perpendicular to the line of impact (common tangent).

Here, not only the magnitudes of velocities after impact are unknown, but also the new directions of travel are unknown.



$$u_{1n} = u_1 \cos\alpha_1 \ ; \ u_{1t} = u_1 \sin\alpha_1$$

$$u_{2n} = u_2 \cos\alpha_2 \ ; \ u_{2t} = u_2 \sin\alpha_2$$

$$v_{1n} = v_1 \cos\beta_1 \ ; \ v_{1t} = v_1 \sin\beta_1$$

$$v_{2n} = v_2 \cos\beta_2 \ ; \ v_{2t} = v_2 \sin\beta_2$$

<u>Equations to solve Oblique Central Impact problems</u>:

1. The component of the total momentum of the two bodies along the line of impact is conserved.

$$m_1u_{1n} + m_2u_{2n} = m_1v_{1n} + m_2v_{2n}$$

2. Coefficient of restitution relation along the line of impact is,

$$e = \frac{v_{2n} - v_{1n}}{u_{1n} - u_{2n}}$$

3. Component of the momentum along the common tangent is conserved, which means the component of velocities along the tangent remains unchanged.

$$u_{1t} = v_{1t} \ \& \ u_{2t} = v_{2t}$$

4. Magnitudes of velocities after impact are given by,

$$v_1 = \sqrt{(v_{1n})^2 + (v_{1t})^2} \ \& \ v_2 = \sqrt{(v_{2n})^2 + (v_{2t})^2}$$

5. Directions of velocities after impact are given by,

$$\beta_1 = \tan^{-1}\frac{v_{1t}}{v_{1n}} \ \& \ \beta_2 = \tan^{-1}\frac{v_{2t}}{v_{2n}}$$

Problem 33

A cannon gun is nested by three springs each of 250 kN/cm stiffness as shown in Fig. 15.33(a). The gun fires a 500 kg shell with a muzzle velocity of 1000 m/s. Calculate the total recoil and the maximum force developed in each spring if the gun has a mass of 80,000 kg.

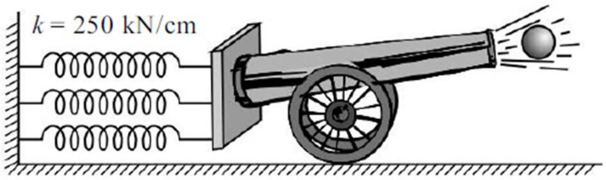


Fig. 15.33(a)

Solution

(i) By law of conservation of momentum, we have

Initial momentum = Final momentum

$$0 = 500 \times 1000 + 80000 \times v_{gun}$$

$$v_{gun} = -6.25 \text{ m/s}$$

$$\therefore \; v_{gun} = 6.25 \text{ m/s } (\leftarrow)$$

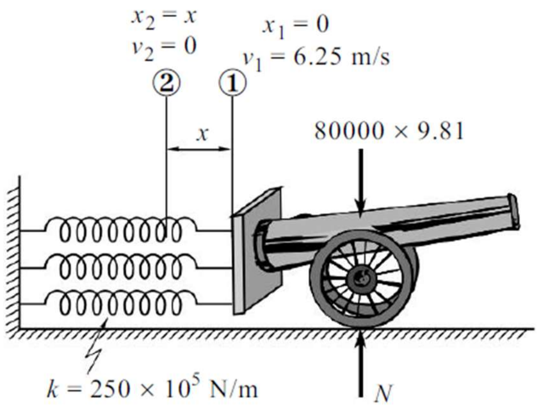


Fig. 15.33(b)

(iii) By work - energy principle, we have

Work done = Change in kinetic energy

$$3\left[\frac{1}{2} \times 250 \times 10^5 \,(0^2 - x^2)\right] = 0 - \frac{1}{2} \times 80000 \times 6.25^2$$

$$x = 0.204 \text{ m (maximum compression of spring)}$$

Spring force $F = kx$

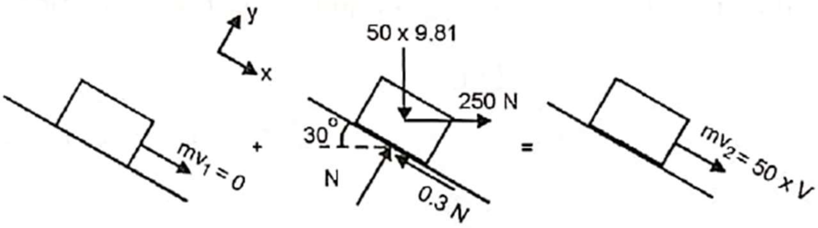
$$\therefore \; F = 250 \times 10^5 \times 0.204$$

$$\therefore \; F = 51.025 \times 10^5 \text{ N}$$

W

Ex. 12.3 A block of mass of 50 kg is placed on a plane inclined at 30° with the horizontal. A horizontal force of 250 N acts on the block tending to move the block down the plane. Determine its velocity 4 sec after starting from rest. Take $\mu_k = 0.3$.

Solution: We shall apply the Impulse Momentum Equation to the block for the first 4 sec of its motion.



Applying Impulse Momentum Equation in the y direction

$$mv_1 + \text{Impulse}_{1\text{-}2} = mv_2$$

Forces in the y direction are the weight component, the component of 250 N and the normal reaction.

$$\therefore \quad 0 + [-50\times 9.81 \cos 30 + 250 \sin 30 + N] \times 4 = 0$$

or $\quad N = 300$ Newton

Applying Impulse Momentum Equation in the x direction

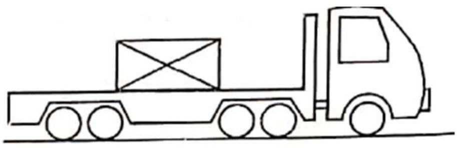
$$mv_1 + \text{Impulse}_{1\text{-}2} = mv_2$$

Forces in the x direction are the weight component, the component of 250 N and the frictional force.

$$\therefore \quad 0 + [50\times 9.81 \sin 30 + 250 \cos 30 - 0.3 \times 300] \times 4 = 50 \text{ v}$$

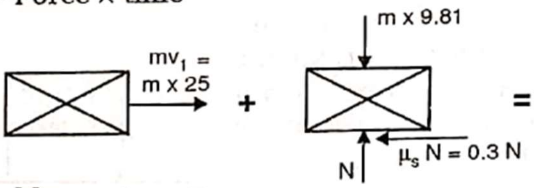
or $\quad v = 29.74$ m/s .... **Ans.**

Ex. 12.2 A truck traveling at a constant speed of 90 kmph on a straight highway carries a package on its flat bed trailor. μ_s = 0.3 and μ_k = 0.2 between the package and the flat bed. If the truck suddenly wants to come to a halt determine the minimum time in which it can do so without the package slipping on the flat bed.



Solution: As the truck driver applies the brakes, the package kept on it tends to slip forward. However the static frictional force prevents the package from slipping. Since the truck has to come to a halt in a minimum possible time implies that the static frictional force reaches its maximum value i.e. $\mu_s N$.

Let us analyse the kinetics of only the package. We shall draw three figures of the package. The L.H.S. and R.H.S. figures represent the initial and final momentum, while the central figure represents the FBD and is use to calculate the impulse,
since Impulse = Force × time



Applying Impulse Momentum Equation in the x direction $\rightarrow$ + ve

$$mv_1 + \text{Impulse}_{1\text{-}2} = mv_2$$

$$m \times 25 + [-0.3 \times (m \times 9.81) \times t] = 0$$

or $t = 8.495$ sec ……… **Ans.**

Force in the x direction is 0.3 N

Impulse = Force × time
= (0.3 N) × t
= 0.3 × (m × 9.81) × t

Problem 36

A boy of 60 kg mass and a girl of 50 kg mass dive off the end of a boat of mass 160 kg with a horizontal velocity of 2 m/s relative to the boat as shown in Fig. 15.36. Considering the boat to be initially at rest, find its velocity just after **(i)** both the boy and girls dive off simultaneously, and **(ii)** the boy dives first followed by the girl.

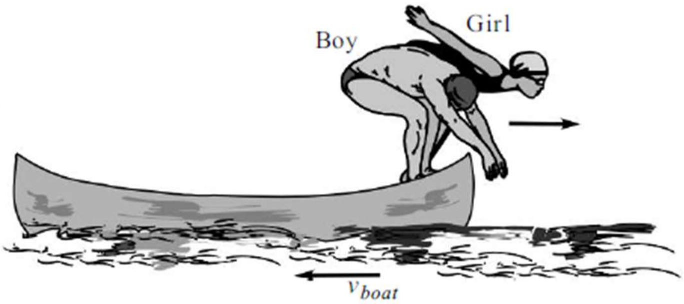


Fig. 15.36

Solution

(i) Both boy and girl dive off simultaneously

When boy and girl will jump together towards right the boat will move in opposite direction, i.e., towards left.

Here, velocity of boy and girl is 2 m/s relative to the boat.

$\therefore\ v_{boy/boat} = v_{boy} - v_{boat}$

$2 = v_{boy} - (-v_{boat})$

$v_{boy} = 2 - v_{boat}$

and $v_{girl/boat} = v_{girl} - v_{boat}$

$2 = v_{girl} - (-v_{boat})$

$v_{girl} = 2 - v_{boat}$

By conservation of momentum principle to the system of boy, girl and boat.

Initial momentum = Final momentum

$0 = (\text{mass} \times \text{velocity})_{boy} + (\text{mass} \times \text{velocity})_{girl} + (\text{mass} \times \text{velocity})_{boat}$

$0 = 60(2 - v_{boat}) + 50(2 - v_{boat}) + 160(-v_{boat})$

$0 = 120 - 60v_{boat} + 100 - 50v_{boat} - 160v_{boat}$

$-220 = -270v_{boat}$

$v_{boat} = 1.227 \text{ m/s } (\leftarrow)$

(ii) The boy dives first followed by the girl

Here, boy is jumping first and girl is still on boat.

By conservation of momentum principle,

Initial momentum = Final momentum

$0 = (\text{Mass} \times \text{Velocity})_{boy} + (\text{Mass} \times \text{Velocity})_{boat}$

$0 = 60(2 - v_{boat}) + 160(-v_{boat})$

$0 = 120 - 60v_{boat} - 160v_{boat}$

$-220v_{boat} = -120$

$v_{boat} = 0.5455 \text{ m/s } (\leftarrow)$

Later the girl jumps from the boat when the boat is moving back with a velocity of 0.5455 m/s.

By conservation of momentum principle.

Initial momentum = Final momentum

$$(\text{mass} \times \text{velocity})_{boat} = (\text{mass} \times \text{velocity})_{girl} + (\text{mass} \times \text{velocity})_{boat}$$

$$160(-0.5455) = 50(2 - v_{boat}) + 160(-v_{boat})$$

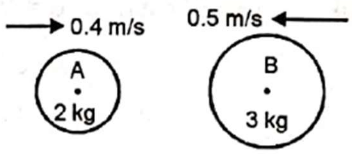
$$-87.28 = 100 - 50v_{boat} - 160v_{boat}$$

$$-210v_{boat} = -187.28$$

$$v_{boat} = 0.8918 \text{ m/s } (\leftarrow)$$

Ex. 12.8 A 2 kg ball moving with 0.4 m/s towards right, collides head on with another ball of mass 3 kg, moving with 0.5 m/s towards left. Determine the velocities of the balls after impact and the corresponding percentage loss of kinetic energy, when

i) the impact is perfectly elastic
ii) the impact is perfectly plastic
iii) the impact is such that e = 0.7



Solution: This is a case of Direct Central Impact.

i) Impact is perfectly elastic i.e. e = 1

Using Conservation of Momentum Equation $\rightarrow +ve$

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$2 \times 0.4 + 3 \times (-0.5) = 2 v_A' + 3 v_B'$$

$$-0.7 = 2 v_A' + 3 v_B' \quad \text{........... (1)}$$

Using Coefficient of Restitution Equation $\rightarrow +ve$

$$v_B' - v_A' = e[v_A - v_B]$$

$$v_B' - v_A' = 1[0.4 - (-0.5)]$$

$$v_B' = 0.9 + v_A' \quad \text{........... (2)}$$

Solving equations (1) and (2), we get

$$v_A' = -0.68 \text{ m/s} = 0.68 \text{ m/s} \leftarrow \quad \text{.......} \textbf{ Ans.}$$

$$v_B' = 0.22 \text{ m/s} = 0.22 \text{ m/s} \rightarrow \quad \text{.......} \textbf{ Ans.}$$

Since impact is perfectly elastic, implies that the energy is conserved i.e. there will be no loss of kinetic energy.

ii) Impact is perfectly plastic i.e. e = 0

In this case, the particles move together with a common velocity after impact,
i.e $v_A' = v_B' = v'$

Using Conservation of Momentum Equation

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \rightarrow +ve$$

$$2 \times 0.4 + 3 \times (-0.5) = 2v' + 3v'$$

$$\therefore \quad v' = -0.14 \text{ m/s}$$

i.e. $v_A' = v_B' = 0.14 \text{ m/s} \leftarrow$ .... **Ans.**

Kinetic energy of the system before impact

$$= \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2$$

$$= ½ \times 2 \times (0.4)^2 + ½ \times 3 \times (0.5)^2 = 0.535 \text{ J}$$

Kinetic energy of the system after impact

$$= ½ \times 2 \times (0.14)^2 + ½ \times 3 \times (0.14)^2 = 0.049 \text{ J}$$

∴ Percentage loss of kinetic energy $= \dfrac{0.535 - 0.049}{0.535} \times 100 = 90.84$ .... **Ans.**

iii) Impact when $e = 0.7$

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \rightarrow +ve$$

$$2 \times 0.4 + 3 \times (-0.5) = 2 v_A' + 3 v_B'$$

$$-0.7 = 2 v_A' + 3 v_B' \quad \text{........... (1)}$$

Using Coefficient of Restitution Equation

$$v_B' - v_A' = e[v_A - v_B]$$

$$v_B' - v_A' = 0.7[0.4 - (-0.5)]$$

$$v_B' = 0.63 + v_A' \quad \text{........... (2)}$$

Solving equations (1) and (2)

$v_A' = -0.518$ m/s $= 0.518$ m/s ← .... **Ans.**

$v_B' = 0.112$ m/s $= 0.112$ m/s → .... **Ans.**

Kinetic energy of the system after impact

$$= ½ \times 2 \times (0.518)^2 + ½ \times 3 \times (0.112)^2 = 0.287 \text{ J}$$

∴ Percentage loss of kinetic energy $= \dfrac{0.535 - 0.287}{0.535} \times 100 = 46.33$ .... **Ans.**

Problem 4

Two balls having 20 kg and 30 kg masses are moving towards each other with velocities of 10 m/s and 5 m/s respectively as shown in Fig.15.4. If after impact the ball having 30 kg mass is moving with 6 m/s velocity to the right then determine the coefficient of restitution between the two balls.

Solution

(i) By law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$20 \times 10 + 30 \times (-5) = 20v_1 + 30 \times 6$$

$$v_1 = -6.5 \text{ m/s}$$

$$\therefore v_1 = 6.5 \text{ m/s} (\leftarrow)$$

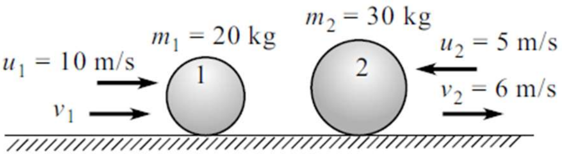


Fig. 15.4

(ii) For coefficient of restitution, we have the relation

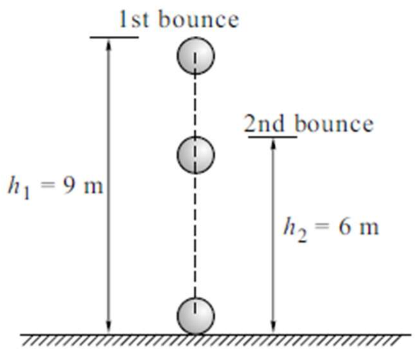
$$e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right]$$

$$e = -\left[\frac{6-(-6.5)}{-5-10}\right]$$

$$e = 0.8333$$

Problem 7

A glass ball is dropped onto a smooth horizontal floor from which it bounces to a height of 9 m as shown in Fig.15.7(a). On the second bounce, it attains a height of 6 m. What is the coefficient of restitution between the glass and the floor? Also determine the height from where the glass ball was dropped.



$u_2 = v_2 = 0$ (velocity of horizontal floor)

Fig. 15.7(a)

Solution

(i) $u_1 = \sqrt{2gh_1}$ $(\downarrow)$ (velocity before impact)

$v_1 = \sqrt{2gh_2}$ $(\uparrow)$ (velocity after impact)

(ii) Coefficient of restitution gives

$$e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right] = -\left[\frac{0 - v_1}{0 - u_1}\right] = -\frac{v_1}{u_1}$$

$$e = -\frac{\sqrt{2gh_2}}{-\sqrt{2gh_1}} = \frac{\sqrt{h_2}}{\sqrt{h_1}} = \sqrt{\frac{6}{9}}$$

$$e = 0.816$$

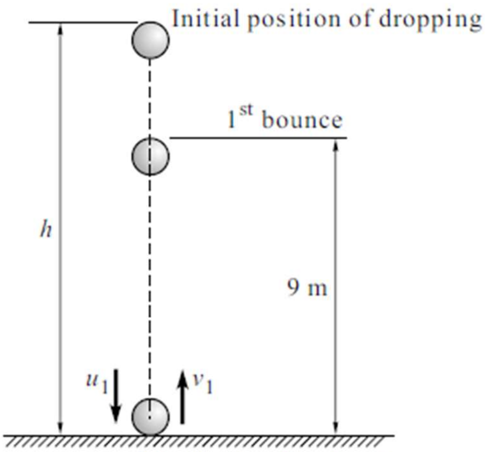
(iii) Coefficient of restitution gives

$$e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right]$$

$$0.816 = -\left[\frac{0 - \sqrt{2 \times 9.81 \times 9}}{0 - \left(-\sqrt{2 \times 9.81 \times h}\right)}\right]$$

$$0.816 = \sqrt{\frac{9}{h}}$$

$$\therefore h = 13.52 \text{ m}$$



$u_2 = v_2 = 0$ (velocity of surface)

Fig. 15.7(b)

Ex. 12.10 A ball drops from the ceiling of a room. After rebounding twice from the floor it reaches a height equal to half that of ceiling, find the coefficient of restitution.

(MU Dec 08)

Solution: Let e be the coefficient of restitution between the ball and the floor.

Using standard relation $e = \left(\frac{h'}{h}\right)^{\frac{1}{2n}}$

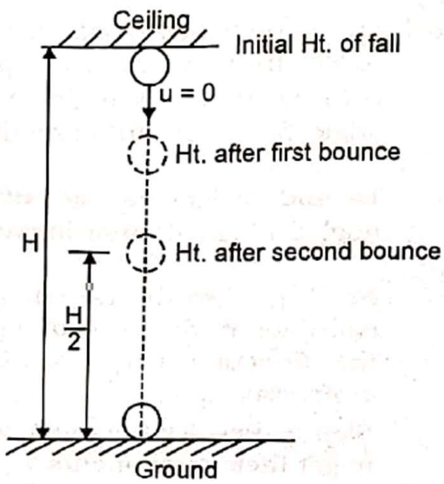
Where n = no. of bounces

h = Initial height of fall

h′ = height of rebound after nth bounce.

Let the ball fall from initial height of H and after 2 bounces it reaches a height of $\frac{H}{2}$ as given.

$$\therefore \quad e = \left(\frac{\frac{H}{2}}{H}\right)^{\frac{1}{2\times2}} \quad \text{or} \quad e = \left(\frac{1}{2}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{1}{2}} = 0.841 \text{ ... } \textbf{Ans.}$$



Problem 12

Two smooth spheres ① and ② having masses of 2 kg and 4 kg respectively collide with initial velocities as shown in Fig. 15.12(a). If the coefficient of restitution for the spheres is $e = 0.8$, determine the velocities of each sphere after collision.

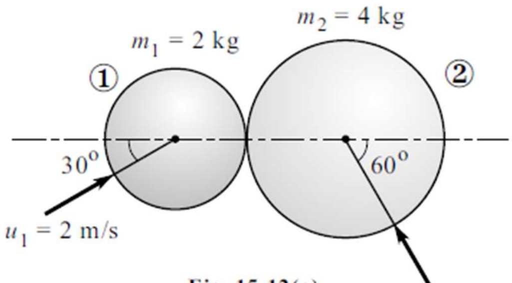


Fig. 15.12(a)

Solution

(i) By law of conservation of momentum along line of impact, we have

$$m_1 u_{1x} + m_2 u_{2x} = m_1 v_{1x} + m_2 v_{2x}$$

$$2 \times 2 \cos 30^\circ + 4 \times (-4 \cos 60^\circ) = 2(-v_{1x}) + 2v_{2x}$$

$$-v_{1x} + 2v_{2x} = -2.268 \quad \text{... (I)}$$

Coefficient of restitution along the line of impact gives

$$e = -\left[\frac{v_{2x} - v_{1x}}{u_{2x} - u_{1x}}\right]$$

$$0.8 = -\left[\frac{v_{2x} - (-v_{1x})}{-4 \cos 60^\circ - 2 \cos 30^\circ}\right]$$

$$v_{2x} + v_{1x} = 2.986 \text{ m/s} \quad \text{... (II)}$$

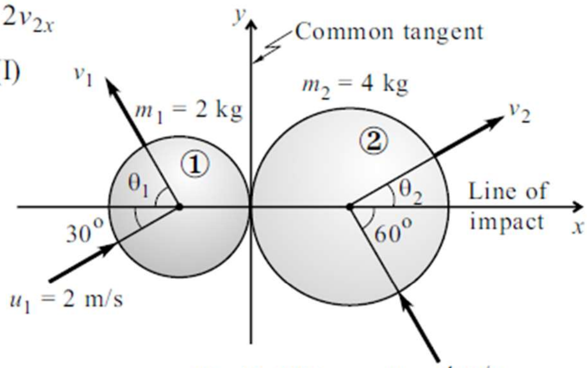


Fig. 15.12(b)

Solving Eqs. (I) and (II), we get

$$v_{1x} = 2.747 \text{ m/s } (\leftarrow) \quad \text{and} \quad v_{2x} = 0.239 \text{ m/s } (\rightarrow)$$

(ii) Component of velocity before and after impact along a common tangent is conserved.

$v_{1y} = 2 \sin 30^\circ$	$v_{2y} = 4 \sin 60^\circ$
$v_{1y} = 1$ m/s $(\uparrow)$	$v_{2y} = 3.464$ m/s $(\uparrow)$

For v_1, we have

$$\tan\theta_1 = \frac{v_{1y}}{v_{1x}} = \frac{1}{2.747}$$

$$\theta_1 = 20^\circ$$

$$v_1 = \sqrt{v_{1x}^2 + v_{1y}^2} = \sqrt{2.747^2 + 1^2}$$

$$v_1 = 2.923 \text{ m/s } (\theta_1)$$

Velocity of sphere ①

For v_2, we have

$$\tan\theta_2 = \frac{v_{2y}}{v_{2x}} = \frac{3.464}{0.239}$$

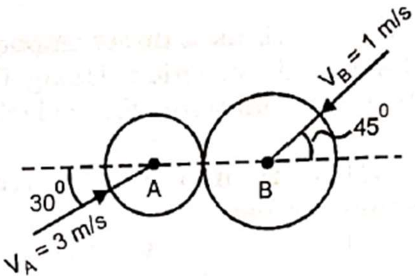
$$\theta_2 = 86.05^\circ$$

$$v_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{0.239^2 + 3.464^2}$$

$$v_2 = 3.472 \text{ m/s } (\theta_2)$$

Velocity of sphere ②

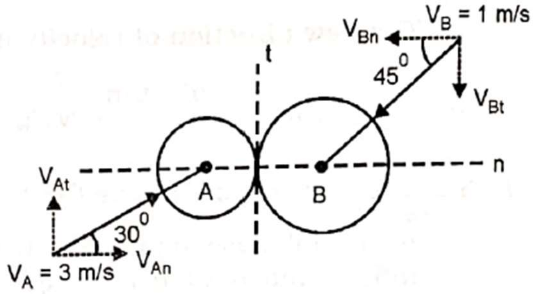
Ex. 12.12 Two smooth balls collide as shown. Find the velocities after impact.
Take $m_A = 1$ kg, $m_B = 2$ kg and $e = 0.75$



Solution: This is a case of Oblique Central Impact
Let the line of impact be the n direction and a perpendicular to it be the t direction.
Resolving the velocities along n and t direction.

$v_{An} = 2.6$ m/s $\rightarrow$, $\quad v_{At} = 1.5$ m/s $\uparrow$

$v_{Bn} = 0.707$ m/s $\leftarrow$, $\quad v_{Bt} = 0.707$ m/s $\downarrow$



Working in n direction

Using Conservation of Momentum Eqn. $\rightarrow$ + ve

$$m_A v_{An} + m_B v_{Bn} = m_A v_{A}'_{n} + m_B v_{B}'_{n}$$

$$1 \times 2.6 + 2 \times (-0.707) = 1 \times v_{A}'_{n} + 2 v_{B}'_{n}$$

$$1.186 = v_{A}'_{n} + 2 v_{B}'_{n} \quad \ldots\ldots\ldots\ldots (1)$$

Using Coefficient of Restitution Equation $\rightarrow$ + ve

$$v_{B}'_{n} - v_{A}'_{n} = e[v_{An} - v_{Bn}]$$

$$v_{B}'_{n} - v_{A}'_{n} = 0.75[2.6 - (-0.707)]$$

$$v_{B}'_{n} = v_{A}'_{n} + 2.48 \quad \ldots\ldots\ldots\ldots (2)$$

Solving equations (1) and (2), we get

$$v_{A'n} = -\ 1.26\ m/s = 1.26\ m/s \leftarrow$$
$$v_{B'n} = \ 1.22\ m/s = 1.22\ m/s \rightarrow$$

Working in t direction

Since velocities don't change in t direction

$$v_{A't} = v_{At} = 1.5\ m/s \uparrow$$
$$v_{B't} = v_{Bt} = 0.707\ m/s \downarrow$$

$\therefore$ Total velocity $\quad v_A' = \sqrt{(v_{A'n})^2 + (v_{A't})^2} = \sqrt{(1.26)^2 + (1.5)^2} = 1.96\ m/s$

at angle $\quad \alpha' = \tan^{-1}\left(\dfrac{v_{A't}}{v_{A'n}}\right) = \tan^{-1}\left(\dfrac{1.5}{1.26}\right) = 50^\circ$

$\therefore \quad v_A' = 1.96\ m/s,\ \alpha' = 50^\circ$ .... **Ans.**

Similarly total velocity $\quad v_B' = \sqrt{(v_{B'n})^2 + (v_{B't})^2} = \sqrt{(1.22)^2 + (0.707)^2} = 1.41\ m/s$

at angle $\quad \beta' = \tan^{-1}\left(\dfrac{v_{B't}}{v_{B'n}}\right) = \tan^{-1}\left(\dfrac{0.707}{1.22}\right) = 30.1^\circ$

$\therefore \quad v_B' = 1.41\ m/s,\ \beta' = 30.1^\circ$ .... **Ans.**