QUESTIONS FOR TUT-8 DOUBLE INTEGRATION

TYPE-1 : EVALUATE

1. $\int_0^1 \int_0^y xye^{-x^2} dx\, dy$ **2.** $\int_0^\infty \int_0^\infty e^{-x^2(1+y^2)} x\, dx\, dy$ 3. $\int_0^1 \int_{x^2}^x xy(x+y) dy\, dx$

4. $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2-x^2-y^2}\, dx\, dy$ 5. $\int_0^{\pi/2} \int_0^{1-\sin\theta} r^2 \cos\theta\, dr d\theta$

6. $\int_0^{\pi/4} \int_0^{\sqrt{\cos 2\theta}} \frac{r}{(1+r^2)^2} dr\, d\theta$

TYPE-2: Evaluate Over given region

1. $\iint_R \frac{1}{x^4+y^2} dx\, dy$ where R is the region $x \geq 1, y \geq x^2$

2. $\iint \sqrt{xy(1-x-y)}\, dxdy$ over the area bounded by $x = 0,\ y = 0$ and $x + y = 1$

3. $\iint_R x(x-y) dx\, dy$ where R is the triangle with vertices $(0,0)$, $(1,2)$, $(0,4)$

4. $\iint (x^2+y^2) dxdy$ over the area of the triangle whose vertices are $(0,1)$, $(1,1)$, $(1,2)$

5. Evaluate $\iint_R (x+y)\, dxdy$ where R is the region bounded by $x = 0, x = 2, y = x, y = x+2$

6. Evaluate $\iint r \cos\theta \sin\theta\, d\theta dr$ over the upper half of the circle $r = 2a\cos\theta$

7. Evaluate $\iint r\, sin\theta\, dr\, d\theta$ over one loop of the lemniscate $r^2 = a^2 \cos\theta$

TYPE 3: CHANGE OF ORDER OF INTEGRATIONS

1. $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{cos^{-1}x}{\sqrt{1-x^2}\sqrt{1-x^2-y^2}} dx\, dy$ **2.** $\int_0^2 \int_{\sqrt{2x}}^2 \frac{y^2}{\sqrt{y^4-4x^2}} dy\, dx$

3. $\int_0^{1/2} \int_0^{\sqrt{1-4y^2}} \frac{1+x^2}{\sqrt{1-x^2}\sqrt{1-x^2-y^2}} dx\, dy$ **4.** $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dy\, dx$

5. $\int_0^a \int_{x^2/a}^{2a-x} xy\, dy\, dx$ **6.** $\int_0^3 \int_{y^2/9}^{\sqrt{10-y^2}} dxdy$

TYPE 4: TRANSFORMATION FROM CARTESIAN TO POLAR COORDINATES

1. $\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)}\, dy\, dx$ **2.** $\int_0^{a/\sqrt{2}} \int_x^{\sqrt{a^2-x^2}} \frac{xdydx}{\sqrt{(x^2+y^2)}}$

3. $\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2\sqrt{x^2+y^2}\, dydx$ **4.** $\int_0^{4a} \int_{y^2/4a}^{y} dxdy$

5. $\iint y^2 dx\, dy$ over the area outside $x^2+y^2-ax = 0$ and inside $x^2+y^2-2ax = 0$

6. $\iint_R \frac{1}{\sqrt{xy}} dxdy$ where R is the region of integration bounded by $x^2+y^2-x = 0$ and $y \geq 0$

7. Evalaute $\iint_R (3x+4y^2)\, dxdy$ where R is the region in the upper half of the area bounded by the circle $x^2+y^2 = 1, x^2+y^2 = 4$

8. Evaluate $\iint_R x^3 y\, dxdy$ over the positive quadrant of the ellipse $\frac{x^2}{4}+\frac{y^2}{16} = 1$