MATRIX THEORY: RANK OF MATRIX
-
NUMERICAL METHODS FOR SYSTEM OF LINEAR EQUATIONS

FY BTECH SEM-I

MODULE-2

SUB-MODULE 2.5

Knowledge Alone Liberates
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2	**Matrix Theory: Rank of Matrix**		8	CO 2
	2.1	Types of matrices: Hermitian, Skew-Hermitian, Unitary and Orthogonal matrix		
	2.2	Rank of a matrix using row echelon forms, reduction to normal form, and PAQ form		
	2.3	System of homogeneous and non-homogeneous equations, their consistency and solutions		
	2.4	Linearly dependent and independent vectors		
	2.5	Solution of system of linear algebraic equations by **(a)** Gauss Seidal method **(b)** Jacobi iteration method		
		#Self-learning topics: Symmetric, Skew-symmetric matrices and properties, Properties of adjoint and inverse of a matrix		

ITERATIVE METHODS

- **Definition:** Numerical method in which we start with some random (Initial) solution of system of equations and use previous iteration values in rearranged equations to find next values are called **Iterative methods**.
- **Example: 1) Gauss Jacobi's Method 2) Gauss Seidel Method**
- **Convergence condition for these two methods:**
- A sufficient condition for method to converge is that the coefficient matrix A of order n should be **strictly or irreducibly diagonally dominant.** i.e. $a_{ii} > \sum_{j \neq i} |a_{ij}| \;, for\ every\ 1 < i < n$
- **Note:** If the initial value to start the iterations is not provided in the problem then we can assume it to be $x = 0, y = 0\ and\ z = 0$

GAUSS JACOBI'S METHOD

- Solve the following equations by Gauss-Jacobi's Method
- $20x + y - 2z = 17$
- $3x + 20y - z = -18$
- $2x - 3y + 20z = 25$
- **Solution:** Rewrite given equations as,
- $x = \frac{1}{20}(17 - y + 2z)$
- $y = \frac{1}{20}(-18 - 3x + z)$
- $z = \frac{1}{20}(25 - 2x + 3y)$
- **(i) First iteration:**
- start with $x_0 = 0, y_0 = 0$ and $z_0 = 0$
- $x_1 = \frac{17}{20} = 0.85$, $y_1 = \frac{-18}{20} = -0.9$, $z_1 = \frac{25}{20} = 1.25$
- **(ii) Second iteration:**
- Use $x_1 = 0.85, y_1 = -0.9$ and $z_1 = 1.25$
- $x_2 = \frac{1}{20}\big(17 - (-0.9) + 2(1.25)\big)$
- $= 1.02$
- $y_2 = \frac{1}{20}\big(-18 - 3(0.85) + (1.25)\big)$
- $= -0.965$
- $z_2 = \frac{1}{20}\big(25 - 2(0.85) + 3(-0.9)\big)$
- $= 1.03$

Example 1 (contd…)

- (iii) **Third iteration:**
- Use $x_2 = 1.02,\ y_2 = -0.965\ and\ z_2 = 1.03$
- $x_3 = \frac{1}{20}\big(17 - (-0.965) + 2(1.03)\big)$
- $= 1.00125$
- $y_3 = \frac{1}{20}\big(-18 - 3(1.02) + (1.03)\big)$
- $= -1.0015$
- $z_3 = \frac{1}{20}\big(25 - 2(1.02) + 3(-0.965)\big)$
- $= 1.00325$
- (iv) **Fourth iteration:**
- Use $x_3 = 1.00125,\ y_3 = -1.0015\ and\ z_3 = 1.00325$
- $x_4 = \frac{1}{20}\big(17 - (-1.0015) + 2(1.00325)\big)$
- $= 1.0004$
- $y_4 = \frac{1}{20}\big(-18 - 3(1.00125) + (1.00325)\big)$
- $= -1.000025$
- $z_4 = \frac{1}{20}\big(25 - 2(1.00125) + 3(-1.0015)\big)$
- $= 0.99965$
- Hence the final answer (correct up to 4 decimal places) after fourth iteration is
- $x = 1.0004,\ y = -1.0000\ and\ z = 0.9997$

Example 2

- Solve the following equations by Gauss-Jacobi's Method (Take three iterations)
- $2x + 20y - 3z = 19$
- $3x - 6y + 25z = 22$
- $15x + 2y + z = 18$
- **Solution:** First checking the condition of strictly diagonally dominant, we rearrange the system as,
- $15x + 2y + z = 18$
- $2x + 20y - 3z = 19$
- $3x - 6y + 25z = 22$
- Rewrite given equations as,
- $x = \frac{1}{15}(18 - 2y - z)$
- $y = \frac{1}{20}(19 - 2x + 3z)$
- $z = \frac{1}{25}(22 - 3x + 6y)$
- (i) **First iteration:**
- start with $x_0 = 0,\ y_0 = 0\ and\ z_0 = 0$
- $x_1 = \frac{18}{15} = 1.2\ ,\ y_1 = \frac{19}{20} = 0.95\ ,\ z_1 = \frac{22}{25} = 0.88$
- (ii) **Second iteration:**
- Use $x_1 = 1.2,\ y_1 = 0.95\ and\ z_1 = 0.88$
- $x_2 = \frac{1}{15}\big(18 - 2(0.95) - (0.88)\big)$
- $= 1.0147$
- $y_2 = \frac{1}{20}\big(19 - 2(1.2) + 3(0.88)\big)$
- $= 0.962$
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Example 2 (contd…)

- $z_2 = \frac{1}{25}\left(22 - 3(1.2) + 6(0.95)\right)$
- $= 0.964$
- (iii) **Third iteration:**
- Use $x_2 = 1.0147, y_2 = 0.962\ and\ z_2 = 0.964$
- $x_3 = \frac{1}{15}\left(18 - 2(0.962) - (0.964)\right)$
- $= 1.0075$
- $y_3 = \frac{1}{20}\left(19 - 2(1.0147) + 3(0.964)\right)$
- $= 0.9931$
- $z_3 = \frac{1}{25}\left(22 - 3(1.0147) + 6(0.962)\right)$
- $= 0.9891$

- Hence the final answer (correct up to 4 decimal places) after third iteration is
- $x = 1.0075,\ y = 0.9931\ and\ z = 0.9891$

GAUSS SEIDEL METHOD:

- In this method, we will use **latest two values** instead of previous iteration values to calculate next value. All other conditions and calculation is same as Gauss Jacobi's Method
- Use Gauss-Seidel method to solve the following equations (Take three iterations)
- $3x - 0.1y - 0.2z = 7.85$
- $0.1x + 7y - 0.3z = -19.3$
- $0.3x - 0.2y + 10z = 71.4$
- **Solution:** Rewrite given equations as,
- $x = \frac{1}{3}(7.85 + 0.1y + 0.2z)$ ……(1)
- $y = \frac{1}{7}(-19.3 - 0.1x + 0.3z)$….. (2)
- $z = \frac{1}{10}(71.4 - 0.3x + 0.2y)$ …….(3)

- (i) **First iteration:** start with $y = 0 \ and \ z = 0$
- $x = \frac{7.85}{3} = 2.6167$,
- We use this value to find y,
- i.e. we put $x = 2.6167 \ and \ z = 0$
- $y = \frac{1}{7}\left(-19.3 - 0.1(2.6167) + 0.3(0)\right)$
- $= -2.7945$,
- We use latest two value to find z, i.e.
- we put $x = 2.6167 \ and \ y = -2.7945$
- $z = \frac{1}{10}\left(71.4 - 0.3(2.6167) + 0.2(-2.7945)\right)$
- $= 7.0056$

Example 1 (contd…)

- (ii) **Second iteration:** We use latest two value to find x, we put $y = -2.7945 \; and \; z = 7.0056$
- $x = \frac{1}{3}(7.85 + 0.1(-2.7945) + 0.2(7.0056))$
- $= 2.9906$
- We use latest two value to find y, we put $x = 2.9906 \; and \; z = 7.0056$
- $y = \frac{1}{7}\big(-19.3 - 0.1(2.9906) + 0.3(7.0056)\big)$
- $= -2.4996$
- We use latest two value to find z, i.e. we put $x = 2.9906 \; and \; y = -2.4996$
- $z = \frac{1}{10}\big(71.4 - 0.3(2.9906) + 0.2(-2.4996)\big)$
- $= 7.0003$
- (iii) **Third iteration:** We use latest two value to find x, we put $y = -2.4996 \; and \; z = 7.0003$
- $x = \frac{1}{3}(7.85 + 0.1(-2.4996) + 0.2(7.0003))$
- $= 3.0000$
- We use latest two value to find y, we put $x = 3 \; and \; z = 7.0003$
- $y = \frac{1}{7}\big(-19.3 - 0.1(3) + 0.3(7.0003)\big)$
- $= -2.500$
- We use latest two value to find z, i.e. we put $x = 3 \; and \; y = -2.5$
- $z = \frac{1}{10}\big(71.4 - 0.3(3) + 0.2(-2.5)\big)$
- $= 7.000$
- Hence the final answer after third iteration is
- $x = 3, \; y = -2.5 \; and \; z = 7$