Parallel Resonance

Consider a parallel circuit consisting of a coil and a capacitor as shown in Fig. 4.99. The impedances of two branches are

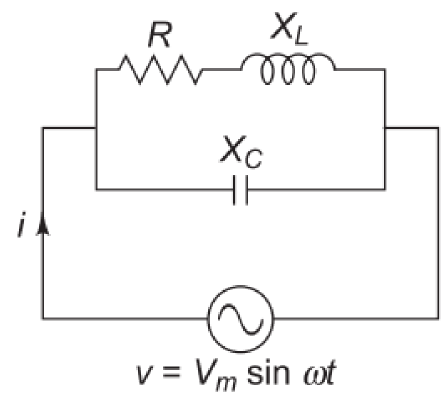

Fig. 4.99 *Parallel circuit*

$$\bar{Z}_1 = R + jX_L$$

$$\bar{Z}_2 = -jX_C$$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{-jX_C} = \frac{j}{X_C}$$

Admittance of the circuit $\bar{Y} = \bar{Y}_1 + \bar{Y}_2$

$$= \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} - j\left(\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C}\right)$$

At resonance, the circuit is purely resistive. Therefore, the condition for resonance is

$$\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C} = 0$$

$$\frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C}$$

$$X_L X_C = R^2 + X_L^2$$

$$\omega_0 L \frac{1}{\omega_0 C} = R^2 + \omega_0^2 L^2$$

$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_0^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

where f_0 is called the resonant frequency of the circuit.

If R is very small as compared to L then

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the resonant frequency of the circuit.

Dynamic Impedance of a Parallel Circuit At resonance, the circuit is purely resistive. The real part of admittance is $\frac{R}{R^2 + X_L^2}$. Hence, the dynamic impedance at resonance is given by

$$Z_D = \frac{R^2 + X_L^2}{R}$$

At resonance,

$$R^2 + X_L^2 = X_L X_C = \frac{L}{C}$$

$$Z_D = \frac{L}{CR}$$

Current Since impedance is maximum at resonance, the current is minimum at resonance.

$$I_0 = \frac{V}{Z_D} = \frac{V}{\frac{L}{CR}} = \frac{VCR}{L}$$

CR

Phasor Diagram At resonance, power factor of the circuit is unity and the total current drawn by the circuit is in phase with the voltage. This will happen only when the current I_C is equal to the reactive component of the current in the inductive branch, i.e., $I_C = I_L \sin\phi$

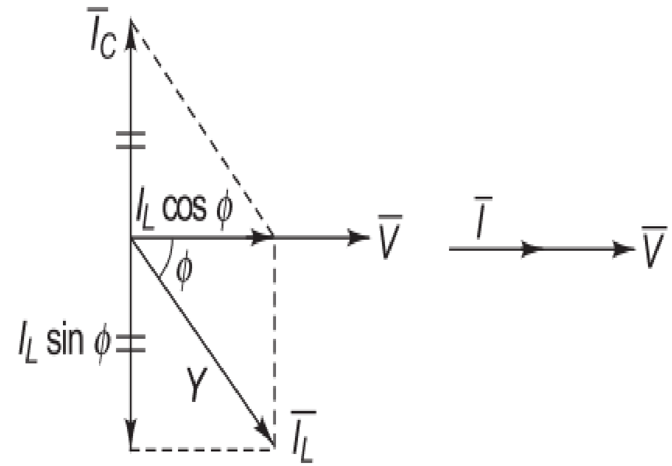

Fig. 4.100 *Phasor diagram*

Hence, at resonance

$$I_C = I_L \sin\phi$$

and
$$I = I_L \cos\phi$$

Behaviour of Conductance G, Inductive Susceptance B_L and Capacitive Susceptance with Change in Frequency Conductance remains constant with the change in frequencies.

Inductive susceptance B_L is

$$B_L = \frac{1}{jX_L} = -j\frac{1}{X_L} = -j\frac{1}{2\pi f L}$$

It is inversely proportional to the frequency. Thus, it decreases with the increase in the frequency. Hence, it can be drawn as a rectangular hyperbola in the fourth quadrant.

Capacitive susceptance B_C is

$$B_C = \frac{1}{-jX_C} = j\frac{1}{X_C} = j2\pi fC$$

It is directly proportional to the frequency. It can be drawn as a straight line passing through the origin.

(a) When $f < f_0$, inductive susceptance predominates. Hence, the current lags behind the voltage and the power factor is lagging in nature.

(b) When $f = f_0$, net susceptance is zero. Hence, the admittance is minimum and impedance is maximum. At f_0, the current is in phase with the voltage and the power factor is unity.

(c) When $f > f_0$, capacitive susceptance predominates. Hence, the current leads the voltage and power factor is leading in nature.

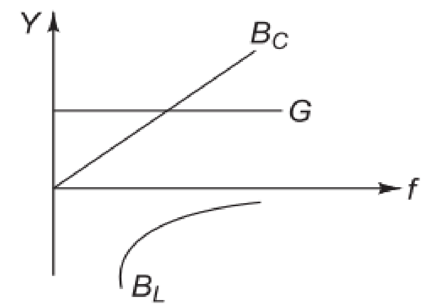

Fig. 4.101 *Behaviour of G, B_L and B_C with change in frequency*

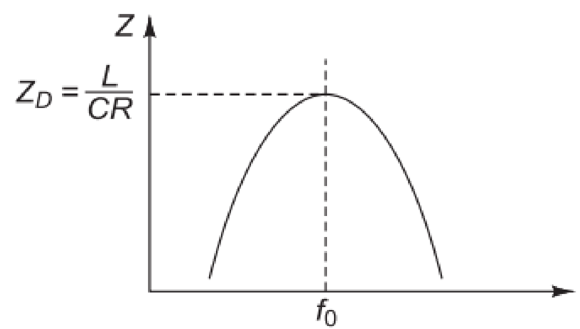

Fig. 4.102 *Impedance*

Bandwidth The bandwidth of a parallel resonant circuit is defined in the same way as that for a series resonant circuit.

Quality Factor It is a measure of current magnification in a parallel resonant circuit.

$$Q_0 = \frac{\text{Current through inductor or capacitor}}{\text{Current at resonance}} = \frac{I_{C_0}}{I_0}$$

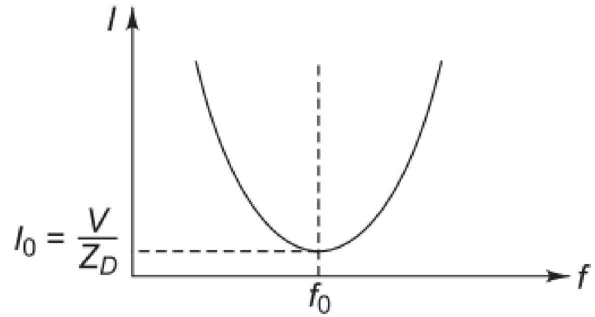

Fig. 4.103 *Resonance curve*

Substituting values of I_{C_0} and I_0,

$$Q_0 = \frac{\dfrac{V}{X_{C_0}}}{\dfrac{VCR}{L}} = \frac{\dfrac{1}{X_{C_0}}}{\dfrac{CR}{L}} = \frac{\omega_0 C}{\dfrac{CR}{L}} = \frac{\omega_0 L}{R}$$

Neglecting the resistance R, the resonant frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_0 = \frac{\left(\dfrac{1}{\sqrt{LC}}\right)L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Parameter	*Series Circuit*	*Parallel Circuit*
Current at resonance	$I = \frac{V}{R}$ and is maximum	$I = \frac{VCR}{L}$ and is minimum
Impedance at resonance	$Z = R$ and is minimum	$Z = \frac{L}{CR}$ and is maximum
Power factor at resonance	Unity	Unity
Resonant frequency	$f_0 = \frac{1}{2\pi\sqrt{LC}}$	$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
Q-factor	$Q = \frac{2\pi f_0 L}{R}$	$Q = \frac{2\pi f_0 L}{R}$
It magnifies	Voltage across L and C	Current through L and C