Practice Problems

Type – I (Non-homogeneous)

1. Test for consistency the following set of equations and obtain the solution if consistent.

(i) $$\begin{aligned} 3x+3y+2z&=1\\ x+2y&=4\\ 10y+3z&=-2\\ 2x-3y-z&=5 \end{aligned}$$

(ii) $$\begin{aligned} 2x-y-z&=2\\ x+2y+z&=2\\ 4x-7y-5z&=2. \end{aligned}$$

(iii) $$\begin{aligned} 2x_1+2x_2&=-11\\ 6x_1+20x_2-6x_3&=-3\\ 6x_2-18x_3&=-1 \end{aligned}$$

(iv) $$\begin{aligned} x-2y+3t&=0\\ 2x+y+z+t&=-4\\ 4x-3y+z+7t&=8 \end{aligned}$$

(v) $$\begin{aligned} x_1+x_2+x_3&=4\\ 2x_1+5x_2-2x_3&=3\\ x_1+7x_2-7x_3&=5. \end{aligned}$$

(vi) $$\begin{aligned} 5x_1-3x_2-7x_3+x_4&=10\\ -x_1+2x_2+6x_3-3x_4&=-3\\ x_1+x_2+4x_3-5x_4&=0. \end{aligned}$$

(vii) $$\begin{aligned} 2x_1-x_2+x_3&=4\\ 3x_1-x_2+x_3&=6\\ 4x_1-x_2+2x_3&=7\\ -x_1+x_2-x_3&=9 \end{aligned}$$

(viii) $$\begin{aligned} x+2y&=1\\ -3x+2y&=-2\\ -x+6y&=0 \end{aligned}$$

(ix) $$\begin{aligned} 2x-y+3z&=9\\ x+y+z&=6\\ x-y+z&=2 \end{aligned}$$

(x) $$\begin{aligned} x+y+4z&=6\\ 3x+2y-2z&=9\\ 5x+y+2z&=13 \end{aligned}$$

(xi) $$\begin{aligned} x_1+x_2+x_3&=4\\ 2x_1+5x_2-2x_3&=3 \end{aligned}$$

2. Show that the system $$\begin{aligned} 2x_1-3x_2+7x_3&=5\\ 3x_1+x_2-3x_3&=13\\ 2x_1+19x_2-47x_3&=32 \end{aligned}$$ is inconsistent.

3. Investigate for what values of a and b the simultaneous equations $$\begin{aligned} 2x-y+3z&=2\\ x+y+2z&=2\\ 5x-y+az&=b \end{aligned}$$ will have **(i)** no solution: **(ii)** a unique solution: **(iii)** an infinite number of solutions.

4. Investigate for what values of λ and μ the simultaneous equations $$\begin{aligned} x+y+z&=6\\ x+2y+3z&=10\\ x+2y+\lambda z&=\mu \end{aligned}$$ will have **(i)** no solution: **(ii)** a unique solution: **(iii)** an infinite number of solutions.

5. Find the values of λ for which the system of equations $$\begin{aligned} x+y+4z&=1\\ x+2y-2z&=1\\ \lambda x+y+z&=1 \end{aligned}$$ will have **(i)** a unique solutions **(ii)** no solution

6. Find values of λ for which the set of equations $$\begin{aligned} x_1+2x_2+x_3&=3\\ x_1+x_2+x_3&=\lambda\\ 3x_1+x_2+3x_3&=\lambda^2 \end{aligned}$$ are consistent and solve equations for those values.

7. For what value of λ the equations $x+y+z=1, x+2y+4z=\lambda, x+4y+10z=\lambda^2$ have a solution and solve them completely in each case.

8. Show that the system of equation $\begin{aligned} -2x+y+z&=a \\ x-2y+z&=b \\ x+y-2z&=c \end{aligned}$ have no solution unless $a+b+c=0$, in which case they have infinitely many solutions. Find these solutions when $a=1, b=1, c=-2$.

Type – II (homogeneous, linear dependence)

9. Find (trivial or non-trivial) solutions of the following linear equations.

(i) $\begin{aligned} x_1 - x_2 + 2x_3 &= 0 \\ x_1 + 2x_2 + x_3 &= 0 \\ 2x_1 + x_2 + 3x_3 &= 0 \end{aligned}$

(ii) $\begin{aligned} x_1 + 2x_2 + 3x_3 + x_4 &= 0 \\ x_1 + x_2 - x_3 - x_4 &= 0 \\ 3x_1 - x_2 + 2x_3 + 3x_4 &= 0 \end{aligned}$

(iii) $\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ x_1 - 2x_2 - x_3 &= 0 \\ 2x_1 - 4x_2 - 5x_3 &= 0 \end{aligned}$

(iv) $\begin{aligned} 2x_1 + 3x_2 - x_3 + x_4 &= 0 \\ 3x_1 + 2x_2 - 2x_3 + 2x_4 &= 0 \\ 5x_1 - 4x_3 + 4x_4 &= 0 \end{aligned}$

10. Find the solution of the system given by $\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ x_1 - 2x_2 - x_3 &= 0 \\ 2x_1 - 4x_2 - 5x_3 &= 0 \end{aligned}$

Also find the relation between column vectors of coefficient matrix.

11. Solve the following system of linear equation $\begin{aligned} x_1 - 2x_2 - x_3 &= 0 \\ -2x_1 + 4x_2 + 2x_3 &= 0 \\ -3x_1 - x_2 + 7x_3 &= 0 \\ 4x_1 + 3x_2 + 6x_3 &= 0 \end{aligned}$

12. Find k if the system $\begin{aligned} 2x - 3y + 4z &= 0 \\ 3x + 4y + 6z &= 0 \\ 4x + 5y + kz &= 0 \end{aligned}$ has non trivial solution

13. If the following system has non – trivial solutions, prove that $a+b+c=0$ or $a=b=c$, Where $ax+by+cz=0, bx+cy+az=0, cx+ay+bz=0$. Find the non – trivial solution when the condition is satisfied.

14. Show that the rows of the matrix $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 3 & 1 & 2 & 1 \\ 4 & 6 & 2 & -4 \\ -6 & 0 & -3 & -4 \end{bmatrix}$ are linearly dependent and find the relationship between them.

15. Are the following vectors linearly dependent? If so find the relation between them.

(i) $X_1 = [1\ 1\ 1\ 3], X_2 = [1\ 2\ 3\ 4], X_3 = [2\ 3\ 4\ 9]$

(ii) $X_1 = [2\ 3\ 4\ -2], X_2 = [-1\ -2\ -2\ 1], X_3 = [1\ 1\ 2\ -1]$

(iii) $X_1 = [1\ 2\ 1\ ], X_2 = [2\ 1\ 4], X_3 = [4\ 5\ 6], X_4 = [1\ 8\ -3]$

(iv) $X_1 = [1\ -1\ 1\ ], X_2 = [2\ 1\ 1], X_3 = [3\ 0\ 2]$

(v) $X_1 = [1\ 2\ 3\ ], X_2 = [2\ -2\ 6]$

(vi) $X_1 = [3\ 1\ -4\ ], X_2 = [2\ \ 2\ -3], X_3 = [0\ -4\ \ 1]$

(vii) $X_1 = [1\ 1\ 1\ 3], X_2 = [1\ 2\ 3\ 4], X_3 = [2\ 3\ 4\ 7]$

(viii) $X_1 = [1\ 1\ -1\ 1], X_2 = [1\ -1\ 2\ -1], X_3 = [3\ 1\ 0\ 1]$

(ix) $X_1 = [1\ -1\ 2\ 0], X_2 = [2\ 1\ 1\ 1], X_3 = [3\ -1\ 2\ -1], X_4 = [3\ 0\ 3\ 1]$