Module 3

INTRODUCTORY QUANTUM MECHANICS

(As per Revised Curriculum SVU R-2023)

Physical constants:

1. Avogadro's number $N_0 = 6.023 \times 10^{23}$/mol
2. Elementary charge $q = 1.6 \times 10^{-19}$ C
3. Planck's constant $h = 6.63 \times 10^{-34}$ J-s
4. Speed of light in vacuum $c = 3 \times 10^{8}$ m/s
5. Electron mass/rest mass $= 9.1 \times 10^{-31}$ kg

Classwork:

1. Calculate de' Broglie wavelengths of an electron orbiting in hydrogen atom at a speed of 10^6 m/s.
2. Calculate de' Broglie wavelength of a cricket ball of mass 0.2 kg thrown at a speed of 100 m/s. Compare your results of this and earlier example and comment on the same.
3. What is de' Broglie wavelength of a neutron having energy 1 MeV. Neutron mass = 1.67×10^{-27} kg?
4. By how much potential difference a proton has to be accelerated in order to have the same de' Broglie wavelength as above? Proton charge $q = 1.6 \times 10^{-19}$ C, mass = 1.67×10^{-27} kg.
5. Find kinetic energy of an electron whose de' Broglie wavelength is the same as 1 keV X-ray photon.
6. An electron and a proton have the same kinetic energies. Compare their de' Broglie wavelengths. Given $m_p = 1800\ m_e$ and $q_p = q_e = q$
7. An electron and a muon (μ) are accelerated through the same potential difference. How do their de' Broglie wavelengths compare? Given $m_\mu = 207\ m_e$ and $q_\mu = q_e = q$
8. Calculate uncertainty in the determination of momentum of an electron confined to a quantum well of size 1 nm. What is the percentage uncertainty in the momentum if its average speed is 10^6 m/s?
9. Determine percentage uncertainty in the measurement of momentum of a marble of mass 10 gm confined to a box of dimensions 50 cm if it is moving with a speed of 20 cm/s. What can you say about the measurement?
10. Find minimum energy possessed by an electron in an atom.
11. The lifetime of an excited state of nucleus is usually 1 ps. Estimate uncertainty in energy of a γ-ray emitted by a nucleus.
12. What is the uncertainty in the determination of position of a particle if its momentum is measured to be 2×10^{-24} kg-m/s with an uncertainty of 0.05%
13. Calculate the width of a spectral line if the transition giving rise to this spectral line has occurred during 0.01 μs seconds.
14. Calculate the percentage uncertainty in the measurement of momentum of a neutron having energy 20 MeV confined to a region of width equal to 3 nuclei.
15. The wave function of a particle is given by $\varphi(x) = \sqrt{\frac{\pi}{2}}\, x;\ \ 0 \leq x \leq 1$. Find the probability that the particle can be found between x = 0.45 to x = 0.55.
16. Find the probability that a particle confined to an infinite square well of size "a" can be found within 0.3a to 0.7a in its ground state. Its wave function is given by:

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$\varphi(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}; \; -\frac{a}{2} < x < +\frac{a}{2}.$

17. Determine normalization constant for a particle whose wave function is given by $\varphi(x) = Ae^{-2x}$ in the interval x = 0 to 1.
18. Calculate the energy, momentum and de' Broglie wavelength of an electron trapped in a one-dimensional quantum well of size 10 Å in its ground state.
19. Calculate the difference in energies of the first two allowed states for an electron confined to a strip of 1 nm.
20. Perform a similar calculation for a marble of mass 10 gm confined to a box of size 1 cm. What can you say by comparing both the results?

Homework:

1. Calculate de' Broglie wavelength of a proton accelerated through 100 kV potential difference.
2. Through what potential difference must a deuteron be accelerated in order to have de' Broglie wavelength of 15 femto metres? Given $m_d = 2m_p$ and $q_d = q$.
3. Recently, wave-particle duality is demonstrated with C-60 fullerene molecules (Bucky balls). The average speed of Bucky balls was around 220 m/s. Molecular weight of C-60 is 720 gm/mol. Determine its de' Broglie wavelength. Can the claim that de' Broglie hypothesis was verified by this demonstration is acceptable?
4. The uncertainty in the measurement of velocity of an electron belonging to the first Bohr orbit in hydrogen atom is 500 m/s. Calculate uncertainty in the measurement of its angular position.
5. The position of a proton is determined within an accuracy of 1 Å. Determine uncertainty in the measurement of its position 1 ns later.
6. How many revolutions does an electron in n = 2 state in hydrogen atom make before dropping to n = 1 state? The average lifetime of an excited state is about 10^{-8} sec.
7. The 2023 Physics Nobel Prize is awarded for a critical work involving attoseconds measurements. Determine the spectral bandwidth and uncertainty in energy corresponding to a 5 attosecond laser pulse.
8. Consider an infinite square well with wall boundaries x = 0 to L. What is the probability of finding a quantum particle in its first excited state somewhere between x = 0 to L/4?
9. Solid-state blue lasers are made using lower band gap $In_xGa_{1-x}N$ layers sandwiched between higher band gap GaN layers. A typical commercial blue laser diode emits light at 445 nm wavelength. The thin $In_xGa_{1-x}N$ layers act as quantum well. Treat it to be one-dimensional infinite potential well and determine width of this $In_xGa_{1-x}N$ layer. (Assume transition between lowest allowed energy levels).
10. Consider a one dimensional quantum well of certain width oscillates with an amplitude of 0.1 nm. Calculate the percentage change in the ground state energy of a particle trapped in this quantum well.

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