Engineering Mechanics

Module 2.1 – **Kinematics of Particles and Rigid Bodies**

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Brief Contents of module 2.1

- Variable motion, motion curves (a-t, v-t, s-t)
  (acceleration curves restricted to linear acceleration only)
- Motion along plane curved path,
- Velocity & acceleration in terms of rectangular components,
- Tangential & normal component of acceleration

Introduction

Dynamics includes:

- ***Kinematics***: study of the geometry of motion. Relates displacement, velocity, acceleration, and time ***without reference*** to the cause of motion. (i.e. *regardless of forces*).
- *Kinema* means movement in Greek
- Mathematical description of motion
  - Position
  - Time Interval
  - Displacement
  - Velocity; absolute value: speed
  - Acceleration
  - Averages of the latter two quantities.
- ***Kinetics***: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion

Introduction (Cont..)

Particle kinetics includes:

- ***Rectilinear motion***: position, velocity, and acceleration of a particle as it moves along a straight line.
- ***Curvilinear motion***: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.
- Please Recall
  1. Newton’s three laws of motion
  2. Position, Displacement, velocity, acceleration
  3. Horizontal motion
  4. Vertical motion

Introduction (Cont..)

- Rectilinear Motion: Position, Velocity & Acceleration

  - ***Rectilinear motion:*** particle moving along a straight line

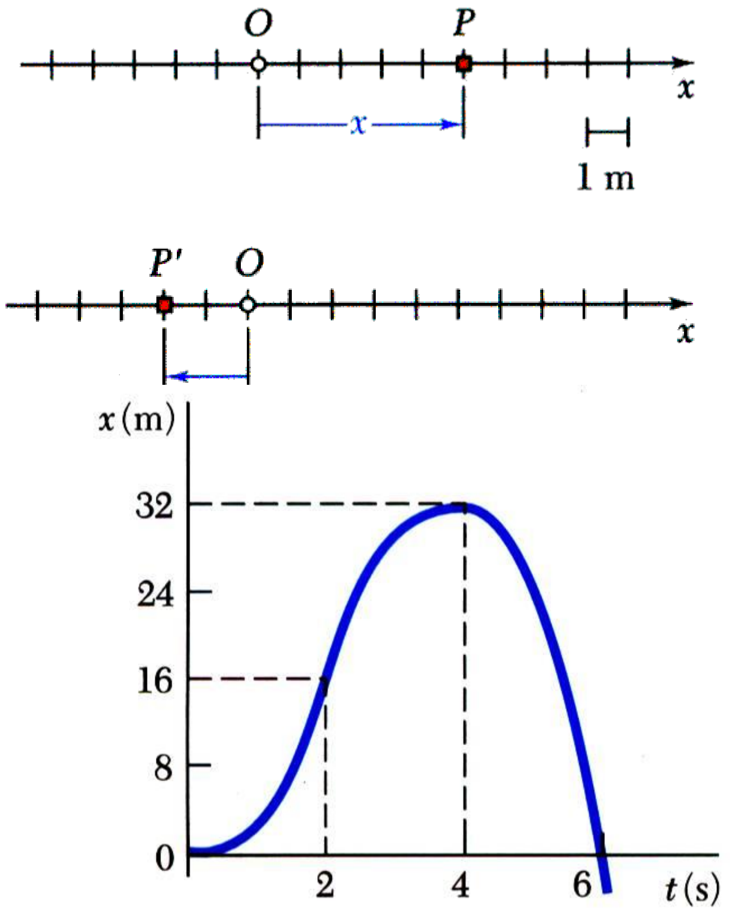
  - ***Position coordinate:*** defined by positive or negative distance from a fixed origin on the line.

- The ***motion*** of a particle is known if the position coordinate for particle is known for every value of time *t*.

- or in the form of a graph *x* vs. *t*.

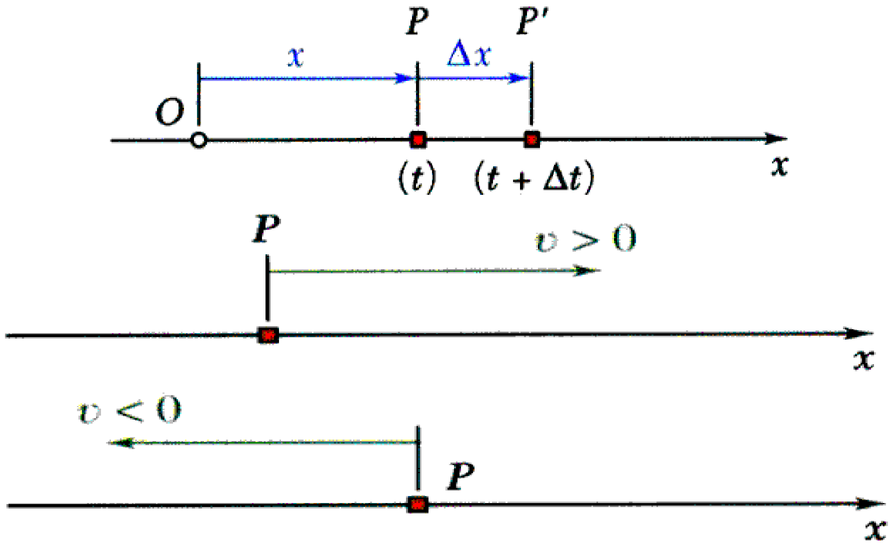
- May be expressed in the form of a function, e.g.,

$$x = 6t^2 - t^3$$



Introduction (Cont..)

- Rectilinear Motion: Position, Velocity & Acceleration



- Consider particle which occupies position *P* at time *t* and *P'* at *t* + *Δt*,

$$Average\ velocity = \frac{\Delta x}{\Delta t}$$

$$Instantaneous\ velocity = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

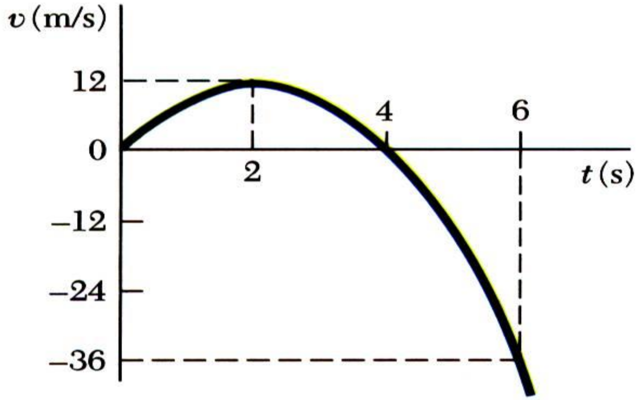
  - Instantaneous velocity may be positive or negative. Magnitude of velocity is referred to as *particle speed*.

  - From the definition of a derivative,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

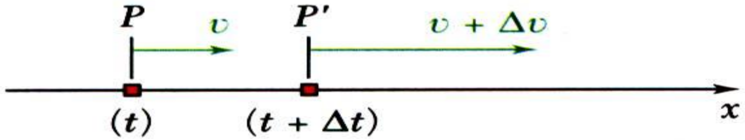
e.g., $x = 6t^2 - t^3$

$$v = \frac{dx}{dt} = 12t - 3t^2$$



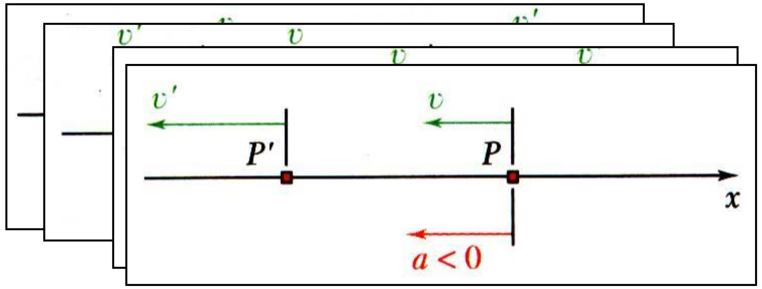
Introduction (Cont..)

- Rectilinear Motion: Position, Velocity & Acceleration

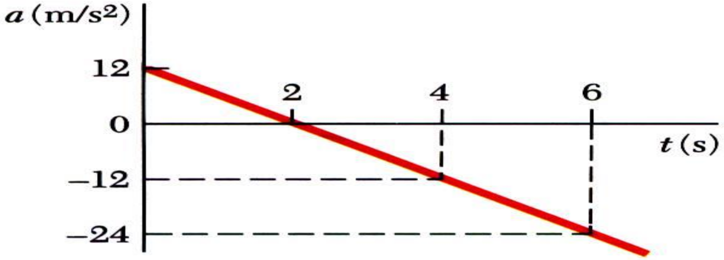


- Consider particle with velocity *v* at time *t* and *v’* at *t+Δt*,

$$Instantaneous\ acceleration = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$



- Instantaneous acceleration may be:
  - positive: increasing positive velocity
    or decreasing negative velocity
  - negative: decreasing positive velocity
    or increasing negative velocity.



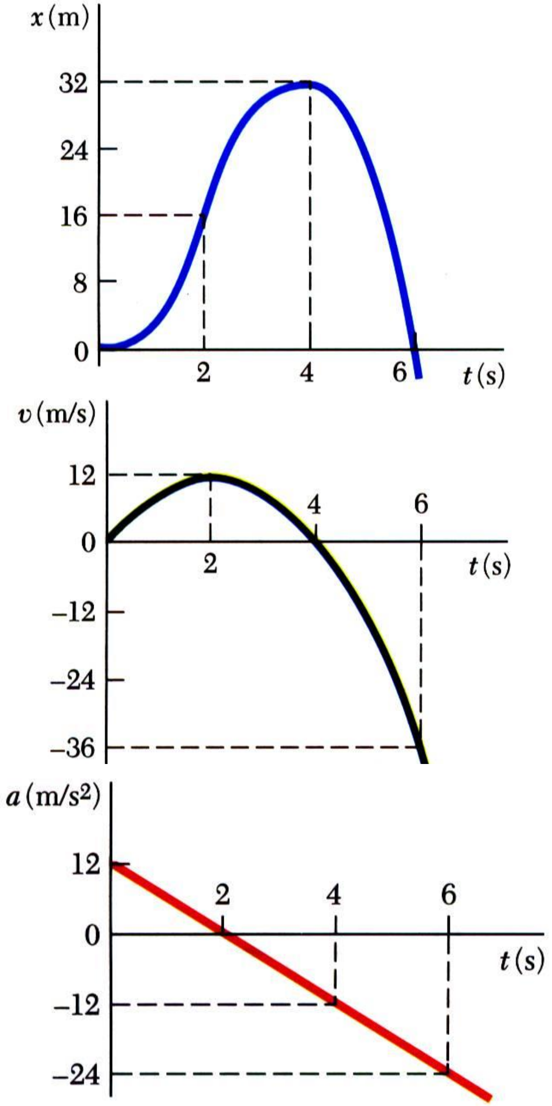
- From the definition of a derivative,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

e.g. $v = 12t - 3t^2$

$$a = \frac{dv}{dt} = 12 - 6t$$

Rectilinear Motion: Position, Velocity & Acceleration



- From our example,

$$x = 6t^2 - t^3$$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12 - 6t$$

- What are x, v, and a at t = 2 s ?

Ans: at t = 2 s, x = 16 m, $v = v_{max}$ = 12 m/s, a = 0

- Note that v_{max} occurs when a=0, and that the slope of the velocity curve is zero at this point.

- What are x, v, and a at t = 4 s ?

Ans: at t = 4 s, $x = x_{max}$ = 32 m, v = 0, a = -12 m/s^2

One minute break

- **What is true about the kinematics of a particle?**

a) The velocity of a particle is always positive
b) The velocity of a particle is equal to the slope of the position-time graph
c) If the position of a particle is zero, then the velocity must zero
d) If the velocity of a particle is zero, then its acceleration must be zero

Determination of the Motion of a Particle

- **Generally we have three classes of motion**
  - acceleration given as a function of *time*, $a = f(t)$
  - acceleration given as a function of *position*, $a = \mathrm{f}(x)$
  - acceleration given as a function of *velocity*, $a = \mathrm{f}(v)$

- **If the acceleration is given, we can determine velocity and position by two successive integrations.**

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} \qquad a = \frac{d^2x}{dt^2} \qquad a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

Rectilinear motion

For a particle in uniform rectilinear motion, the acceleration is zero and the velocity is constant.

$$\frac{dx}{dt} = v = \text{constant}$$

$$\int_{x_0}^{x} dx = v\int_{0}^{t} dt$$

$$x - x_0 = vt$$

$$x = x_0 + vt$$

If forces applied to a body are constant (and in a constant direction), then you have uniformly accelerated rectilinear motion.

Uniformly Accelerated Rectilinear Motion

For a particle in uniformly accelerated rectilinear motion, the acceleration of the particle is constant. You may recognize these constant acceleration equations from Physics courses.

$$\frac{dv}{dt} = a = \text{constant} \qquad \int_{v_0}^{v} dv = a\int_{0}^{t} dt \qquad v = v_0 + at$$

$$\frac{dx}{dt} = v_0 + at \qquad \int_{x_0}^{x} dx = \int_{0}^{t} \left(v_0 + at\right) dt \qquad x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v\frac{dv}{dx} = a = \text{constant} \qquad \int_{v_0}^{v} v\, dv = a\int_{x_0}^{x} dx \qquad v^2 = v_0^2 + 2a\left(x - x_0\right)$$

Rectilinear Motion

- Velocity as a Function of Time

Integrate

ac = dv/dt, assuming that initially v= v0 when t = 0.

$$\int_0^v dv = \int_0^t a_c dt$$

$$v = v_0 + a_c t$$

Constant acceleration

- Position as a Function of Time

Integrate

v = ds/dt = v0 + act, assuming that initially *s = s0 when t = 0*

$$\int_0^s ds = \int_0^t (v_0 + a_c t)dt$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

Constant acceleration

- Velocity as a Function of Position

Integrate

v dv = ac ds, assuming that initially v = v0 at s = s0

$$\int_{v_0}^{v} v dv = \int_{s_0}^{s} a_c ds$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Constant acceleration

Motion with variable acceleration:

- The governing equations are

$$V = dx/dt\ , \quad a = dv/dt, \quad a = V.dv/dx$$

- ~~Motion under gravity~~

1. ~~Motion in vertical direction is influenced by gravitational force~~
2. ~~Acceleration of particle remains constant and equal to g (gravitational force)~~
3. ~~Acceleration due to gravity is directed towards centre of earth~~
4. ~~It is taken as negative (-ve)~~
5. ~~It is a special case of uniformly accelerated motion hence equation of UAM are used with a = -g and s = y~~

Summary

Procedure:

1. Establish a coordinate system & specify an origin
2. Remember: x, v, a, t are related by:

$$\boxed{v=\frac{dx}{dt}} \qquad \boxed{a=\frac{dv}{dt}} \qquad \boxed{a=\frac{d^2x}{dt^2}} \qquad \boxed{a=\frac{dv}{dt}=\frac{dv}{dx}\frac{dx}{dt}=v\frac{dv}{dx}}$$

3. When integrating, either use limits (if known) or add a constant of integration

Problems

- The velocity of the particle is defined as $v = t^3 - 5t^2 + 3t + 4$ where v is in m/s and t is in seconds.

Assuming initial displacement of the particle to be 2 m, find (a) initial velocity, (b) initial acceleration, (c) time interval at which acceleration will be zero, (d) displacement in first 4 seconds, (e) displacement in 6^{th} second.

Problems

- motion of a particle is given by $x = t^4 - 3t^2 - t$ where x is in meter, t in seconds. Find position, velocity, acceleration at t = 3 seconds.

Steps are:

1. Differentiate the given displacement equation find velocity
2. Differentiate the velocity equation and find acceleration

Answer: (x = 51meter, v = 89 m/sec., a = 102m/sec. square)

Problems

Q the motion of particle is governed by $a = t^3 - 2t^2 + 7$. It moves in straight line at t=1 second, v=3.5 m/sec. and x = 9.30 m. Find displacement, velocity, acceleration when t = 2 seconds.

Steps are:

1. $a = t^3 - 2t^2 + 7 = dv/dt$ hence $dv = (t^3 - 2t^2 + 7)\,dt$
2. Integrate it find equation for v and value of C_1
3. Now $v = dx/dt$ hence $dx = (t^4/4 - 2t^3/3 + 7t - 3)dt$
4. Integrate it and find equation for x and value of C_2
5. Answer x =15.93m, v = 9.67m/sec., a = 7m/sec. square,
6. Displacement = (15.93-9.00) = 6.93m

- The acceleration of the particle starting from rest from initial position $x = 0$ is given by $(-6t + 180)\ m/s^2$ where t is in seconds. Determine the distance of the particle in the interval (a) 0 to 10 seconds, (b) 0 to 70 seconds, (c) maximum velocity attained by the particle.

- A sphere is fired downward into a medium with an initial speed of 27 m/s. Sphere experiences a deceleration $a = -6t$ m/s^2 where t is in seconds, determine the distance travelled before it comes to rest.

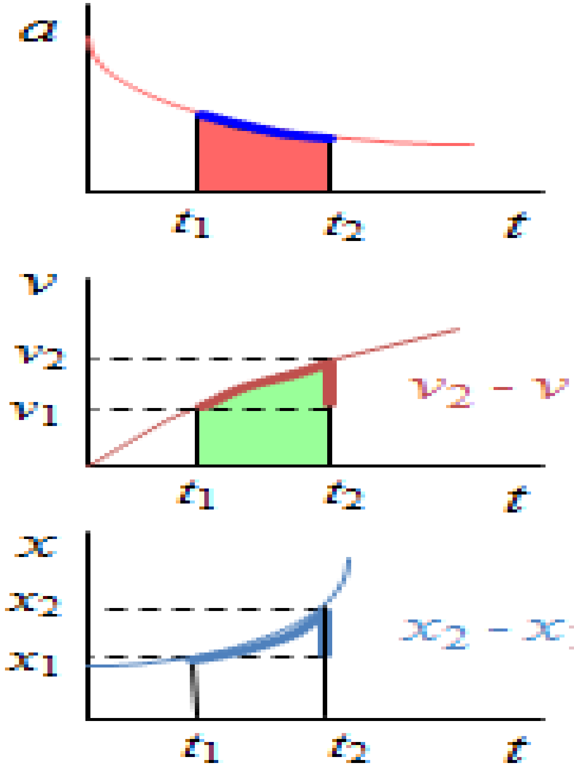
- When particle’s motion is erratic, it is best described graphically using a series of curves that can be generated experimentally from computer output.

- A graph can be established describing the relationship with any two of the variables, *a*, *v*, *s*, *t*

- using the kinematics equations $a = dv/dt$, $v = ds/dt$, $a\,ds = v\,dv$

Motion Diagrams

Sometimes it is convenient to use a *graphical solution* for problems involving rectilinear motion of a particle. The graphical solution most commonly involves $x - t$, $v - t$, and $a - t$ curves.



$$v_2 - v_1 = \int_{t_1}^{t_2} a \, dt$$

$$x_2 - x_1 = \int_{t_1}^{t_2} v \, dt$$

At any given time t,

v = slope of $x - t$ curve
a = slope of $v - t$ curve

while over any given time interval t_1 to t_2,

$v_2 - v_1$ = area under $a - t$ curve
$x_2 - x_1$ = area under $v - t$ curve

Displacement – Time diagram

Given the *s-t Graph, construct the v-t Graph*

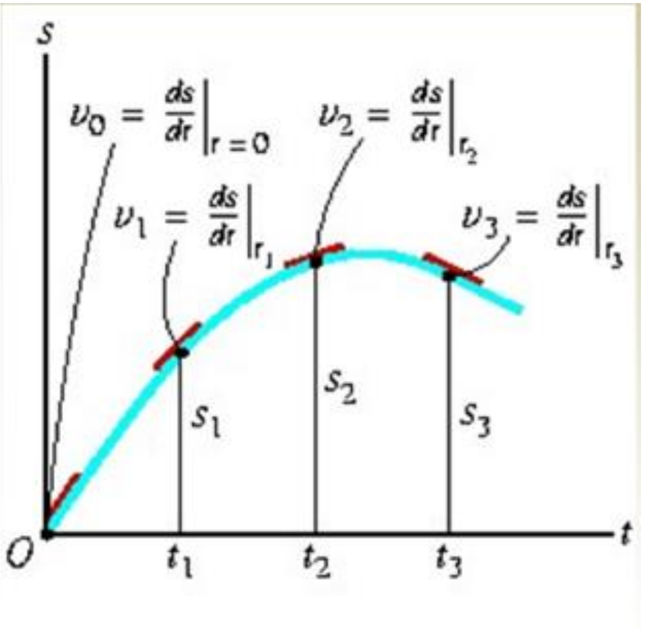
•The *s-t graph can be plotted if the position of the* particle can be *determined experimentally during a* period of time *t.*

•To determine the particle's velocity as a function of time, the *v-t Graph, use v = ds/dt*

•Velocity as any instant is determined by measuring the slope of the *s-t graph*

When displacement of particle is maximum or minimum velocity of particle is zero.

$$\frac{ds}{dt} = v$$



(a)

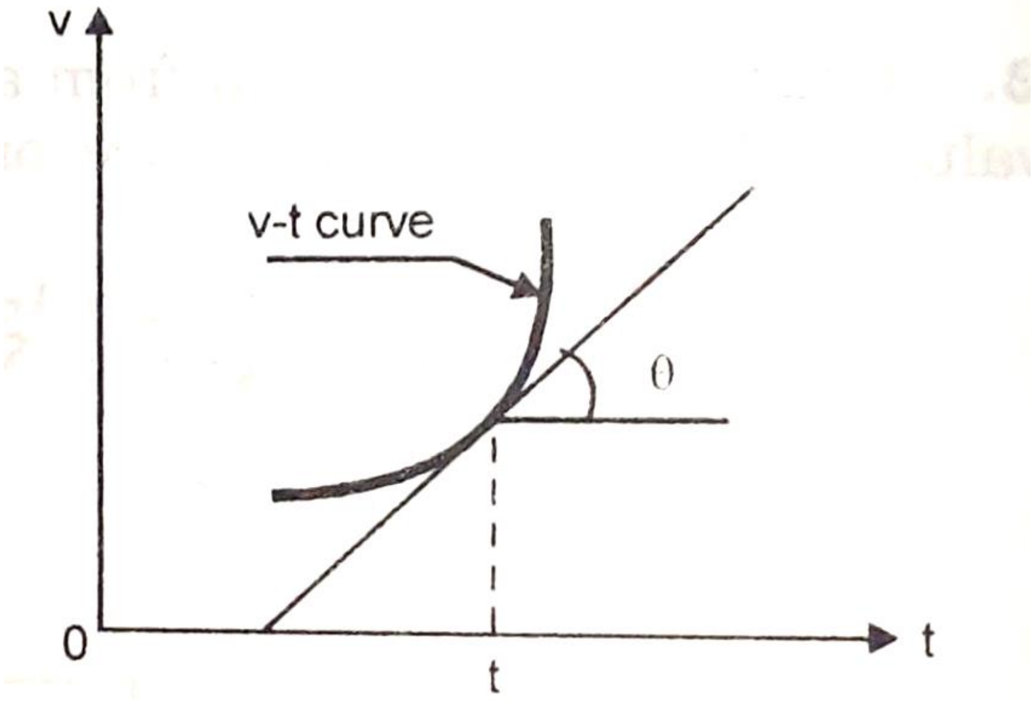
Slope of *s-t graph=velocity*

Velocity Time Diagram

- This is drawn with velocity on y axis and time on x axis.
- As $a = dv/dt$, slope of v-t curve gives acceleration of particle at that instant.
- Now $v = dx/dt$

so $dx = vdt$

When velocity of particle is maximum or minimum acceleration of particle is zero.

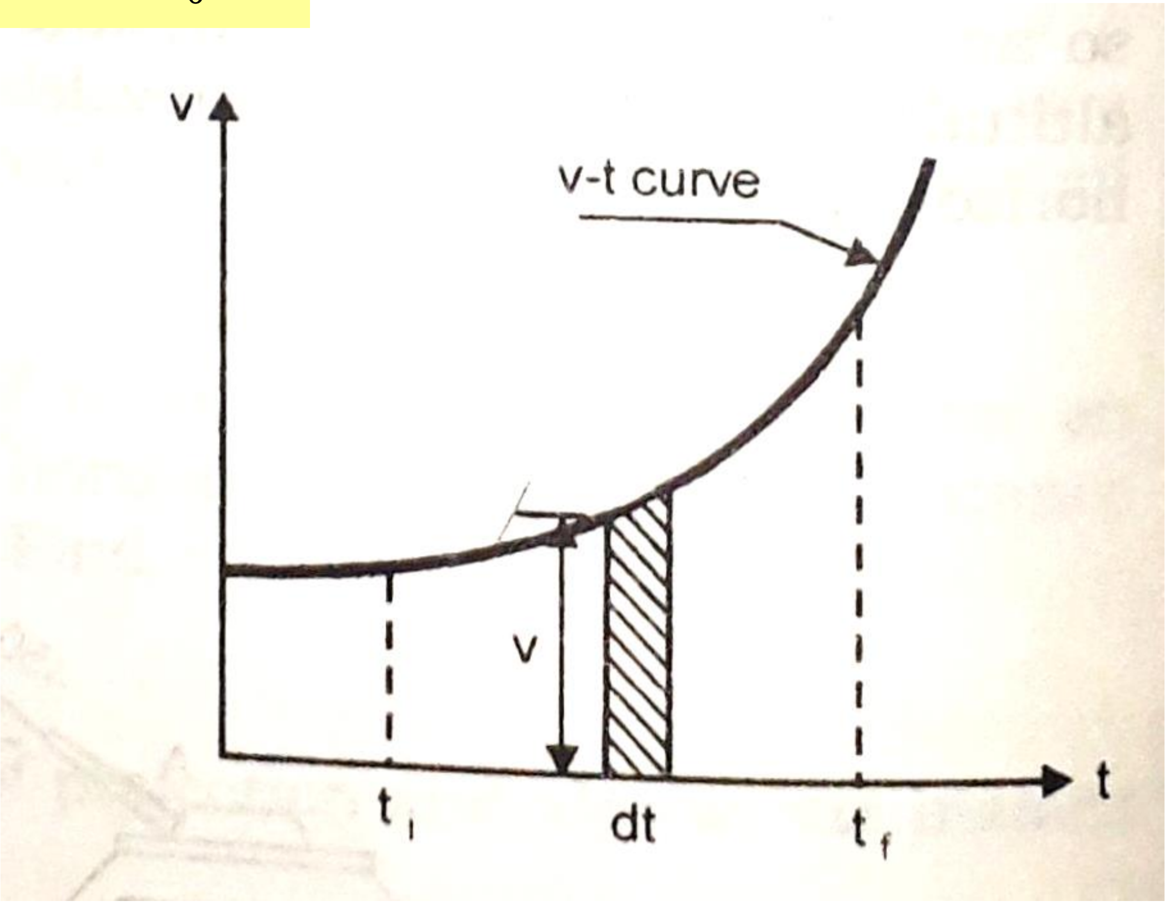


Velocity Time Diagram

$$v = \frac{dx}{dt} \quad \Rightarrow dx = vdt \quad \Rightarrow \int_{x_0}^{x} dx = \int_{0}^{t} vdt \quad \Rightarrow x - x_0 = \int_{0}^{t} vdt$$

Or

$x-x_0$ = area under v-t curve

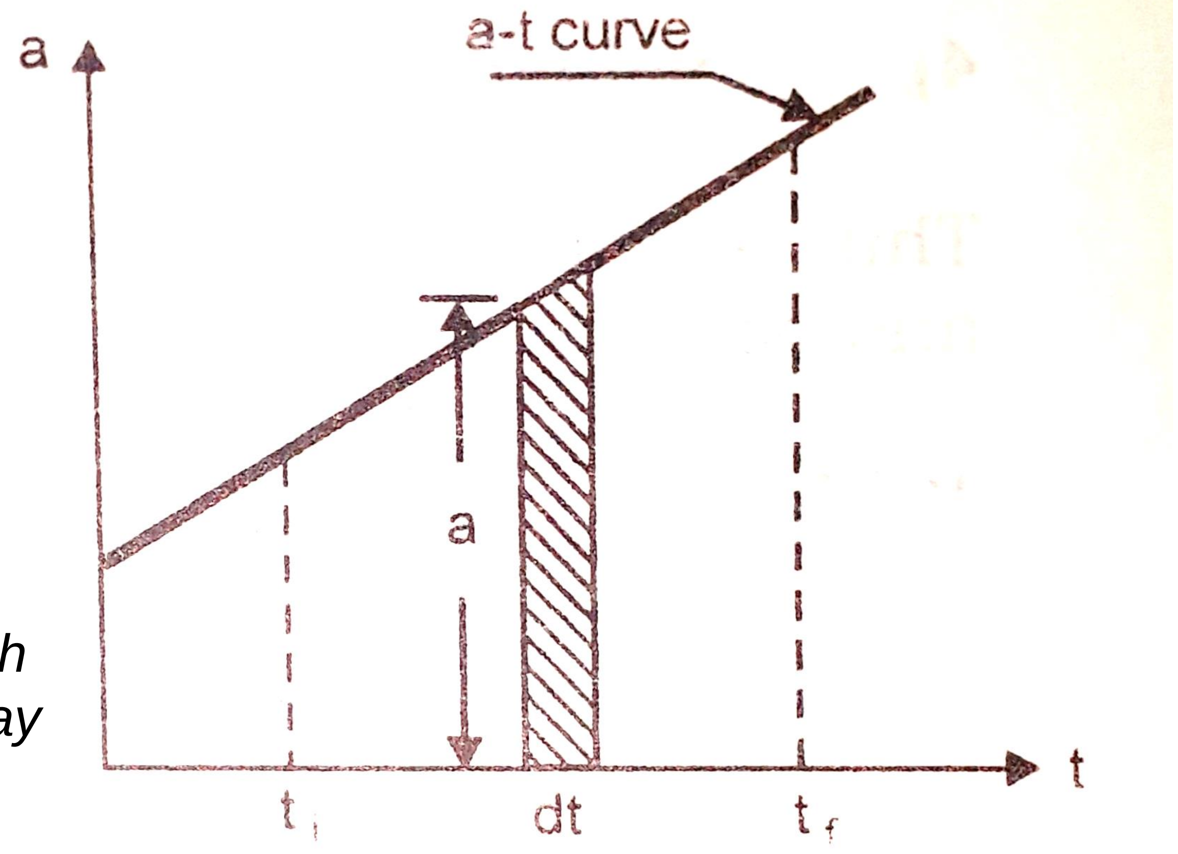


Acceleration Time Diagram

- Given the *a-t Graph, construct the v-t Graph*
- • When the *a-t graph is known, the v-t graph may* be constructed using *a = dv/dt*

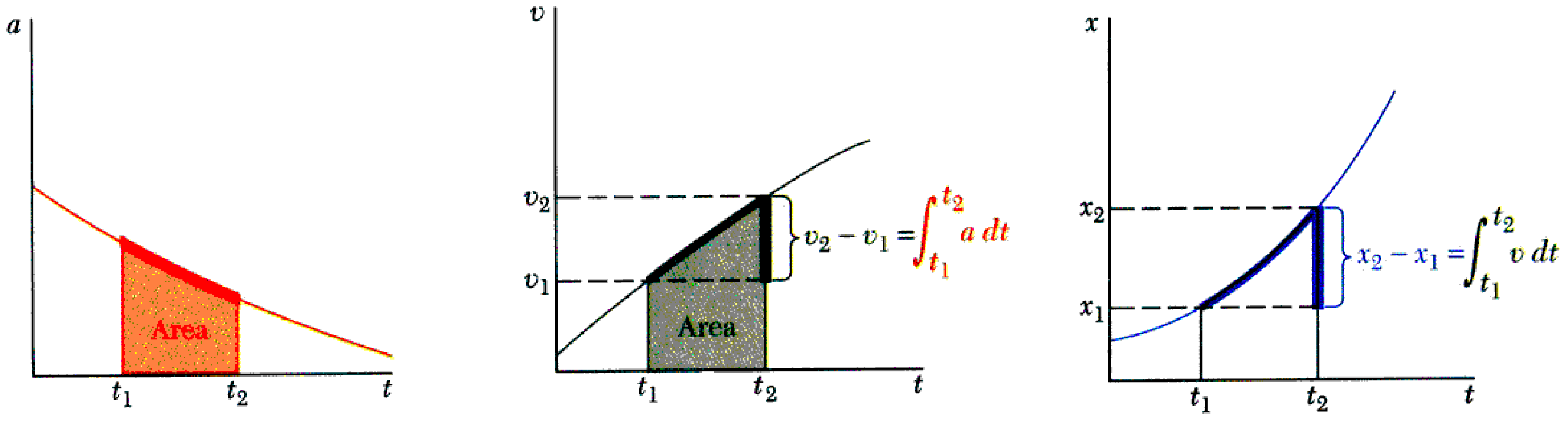
$$\Delta v = \int a\, dt$$

Change in velocity = Area under *a-t* graph



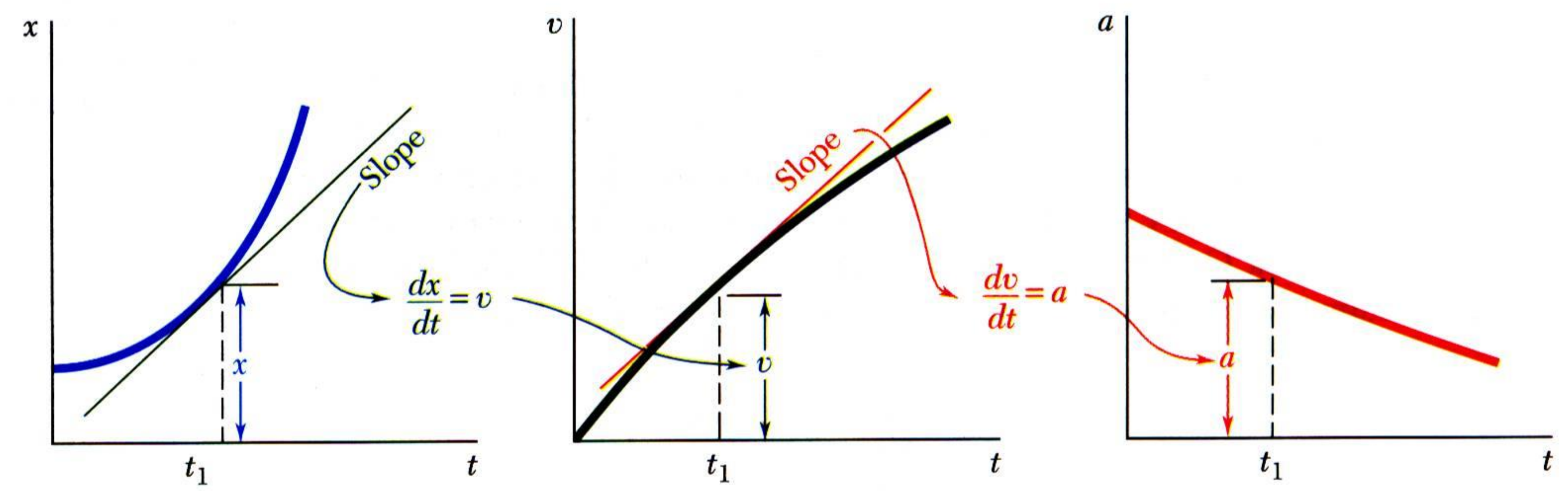
• Knowing particle's initial velocity *v0, and add to this small increments* of area (*Δv*)
• Successive points *v1 = v0 + Δv, for* the *v-t graph*
• Each eqn. for each segment of the *a-t graph may be integrated to yield* eqns. for corresponding segments of the *v-t graph*

Graphical Solution of Rectilinear-Motion Problems



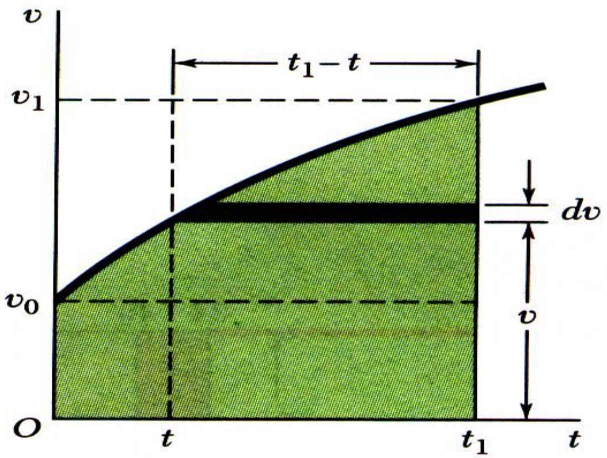
- Given the *a*-*t* curve, the change in velocity between t_1 and t_2 is equal to the area under the *a*-*t* curve between t_1 and t_2.
- Given the *v*-*t* curve, the change in position between t_1 and t_2 is equal to the area under the *v*-*t* curve between t_1 and t_2.

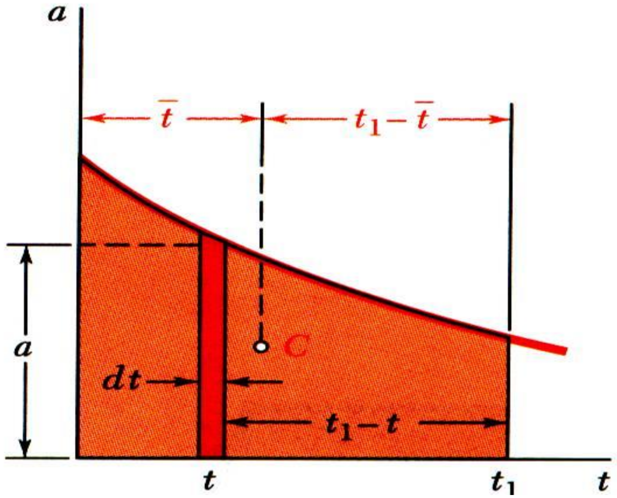
Graphical Solution of Rectilinear-Motion Problems



- Given the *x*-*t* curve, the *v*-*t* curve is equal to the slope of *x*-*t* curve
- Given the *v*-*t* curve, the *a*-*t* curve is equal to the slope *v*-*t* curve

Other Graphical Methods





- *Moment-area method* to determine particle position at time *t* directly from the *a*-*t* curve:

$$x_1 - x_0 = \text{area under } v - t \text{ curve}$$

$$= v_0 t_1 + \int_{v_0}^{v_1} (t_1 - t)\,dv$$

using $dv = a\,dt$,

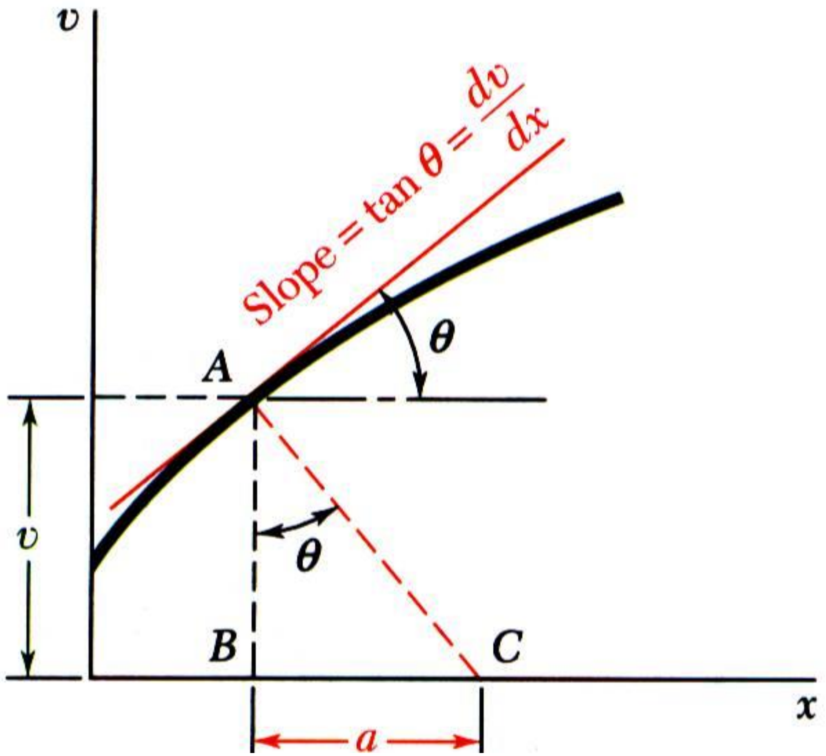
$$x_1 - x_0 = v_0 t_1 + \int_{v_0}^{v_1} (t_1 - t)\,a\,dt$$

$$\int_{v_0}^{v_1} (t_1 - t)\,a\,dt = \text{first moment of area under } a\text{-}t \text{ curve with respect to } t = t_1 \text{ line.}$$

$$x_1 = x_0 + v_0 t_1 + (\text{area under } a\text{-}t \text{ curve})(t_1 - \bar{t})$$

$$\bar{t} = \text{abscissa of centroid } C$$

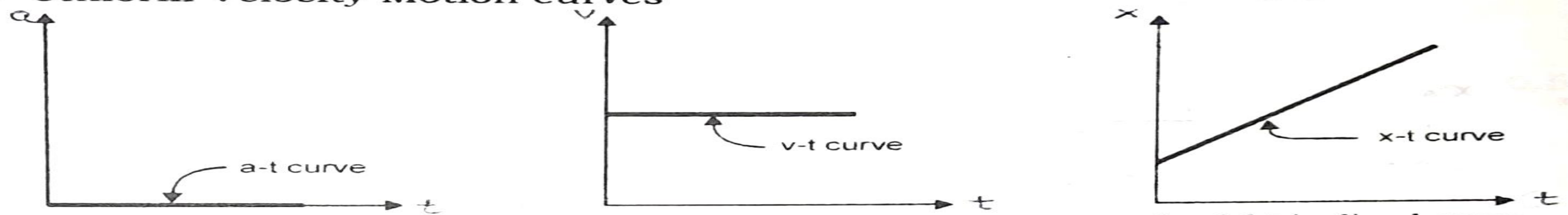
Other Graphical Methods



- Method to determine particle acceleration from *v*-*x* curve:

$$a = v\frac{dv}{dx}$$
$$= AB\tan\theta$$
$$= BC = \textit{subnormal} \text{ to } v\text{-}x \text{ curve}$$

a) Uniform Velocity Motion curves

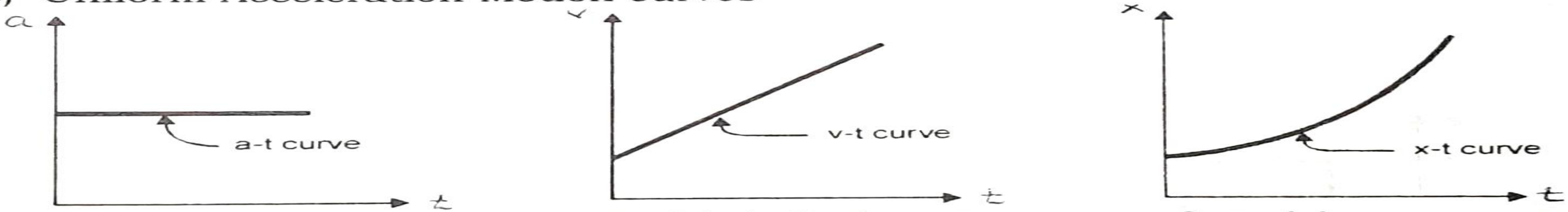


Straight horizontal curve on the acceleration axis

Straight horizontal curve parallel to velocity axis

Straight inclined curve

b) Uniform Acceleration Motion curves

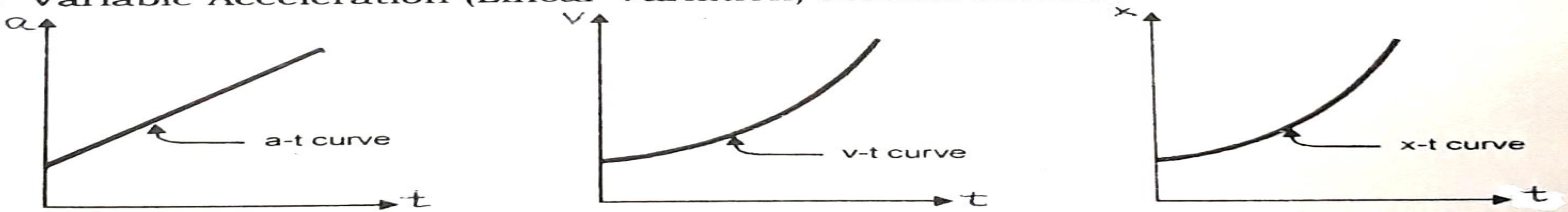


Straight horizontal curve parallel to acceleration axis

Straight inclined curve

Second degree curve

c) Variable Acceleration (Linear Variation) Motion curves

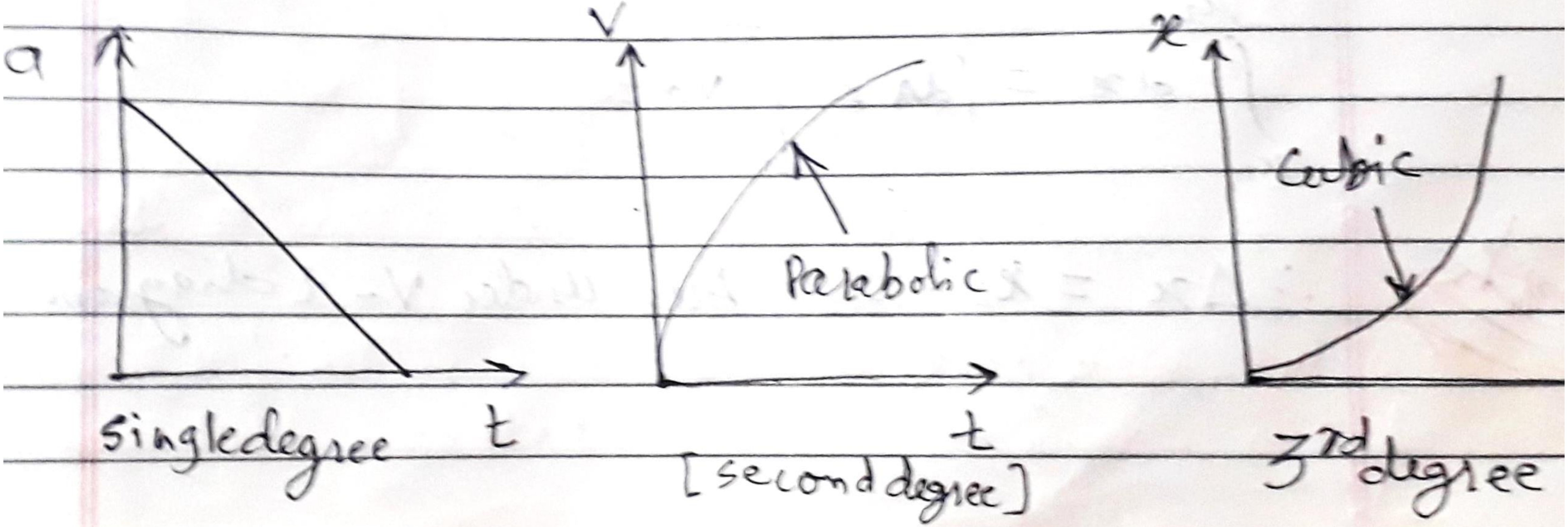


Straight inclined curve

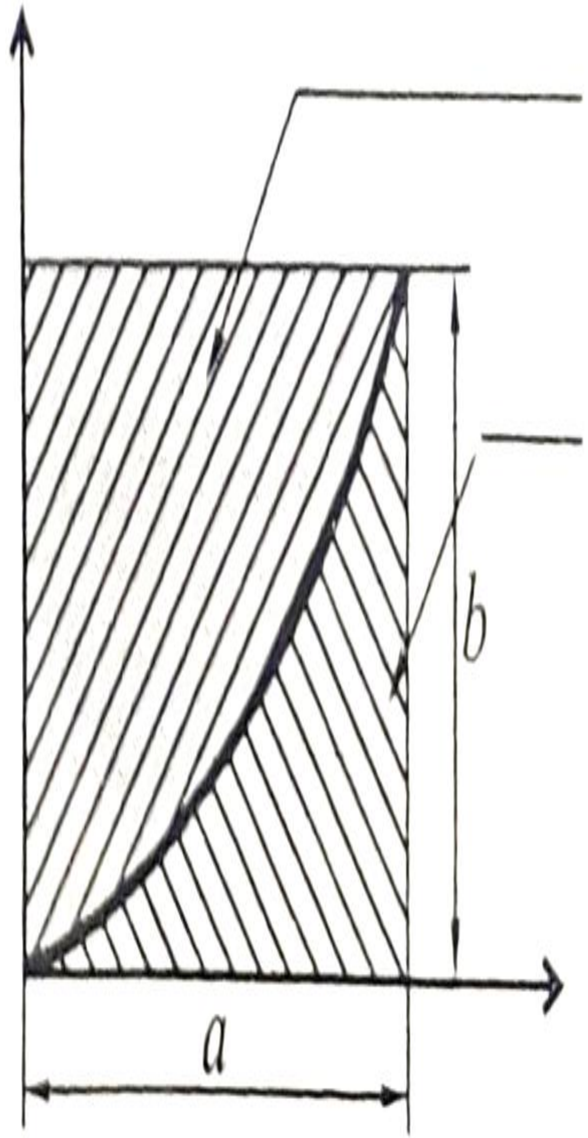
Second degree curve

Third degree curve

Acceleration decreases.



Area bounded by curve



$$A_1 = \frac{nab}{n+1}$$

$$A_2 = \frac{ab}{n+1}$$

$$A = A_1 + A_2 = ab$$

n = Curve of degree of polynomial

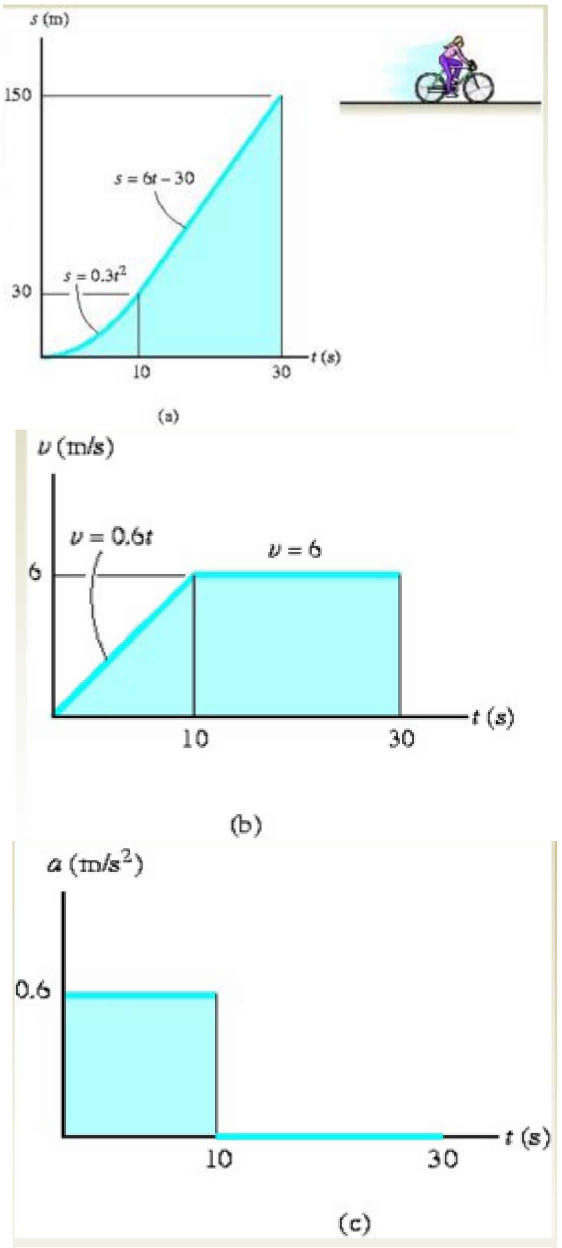
Important points to remember

- If a-t curve is horizontal line (zero degree) then v-t curve is inclined line (single degree) and x-t curve is parabolic curve (second degree)
- Slope of motion curve increases from a-t curve towards v-t curve.

Problems

- Q1 A bicycle moves along a straight road such that it position is

described by the graph as shown. Construct the *v-t and a-t graphs for 0 ≤ t ≤ 30s.*



v-t Graph. *The v-t graph can be determined by* differentiating the eqns. defining the *s-t graph*

$$0 \le t \le 10s; \quad s = 0.3t^2 \quad v = \frac{ds}{dt} = 0.6t$$

$$10s \le t \le 30s; \quad s = 6t - 30 \quad v = \frac{ds}{dt} = 6$$

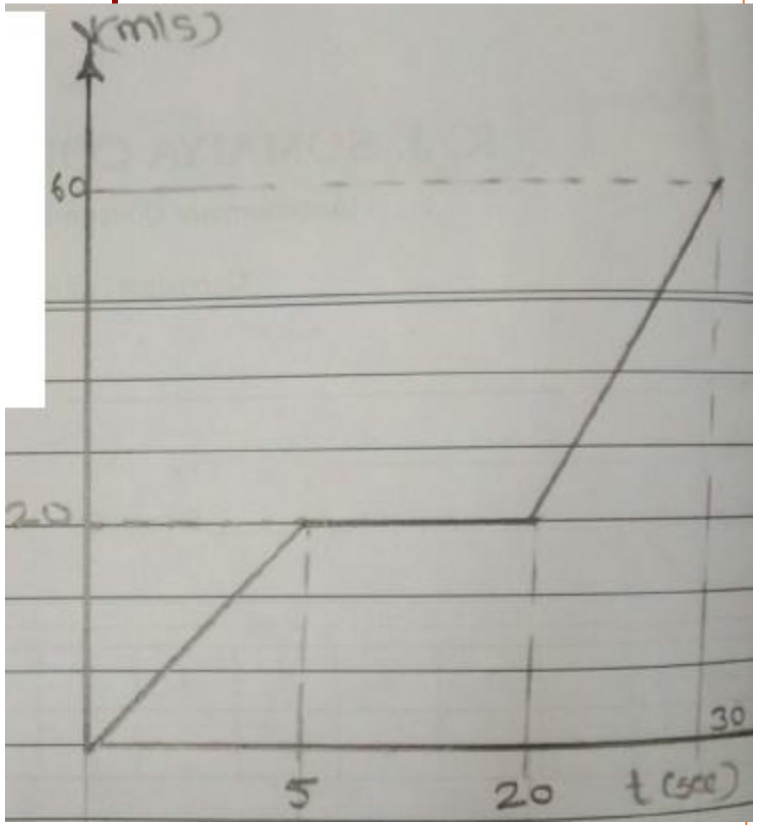
$$v = \frac{\Delta s}{\Delta t} = \frac{150 - 30}{30 - 10} = 6m/s$$

a-t Graph. *The a-t graph can be determined by* differentiating the eqns. defining the lines of the *v-t graph.*

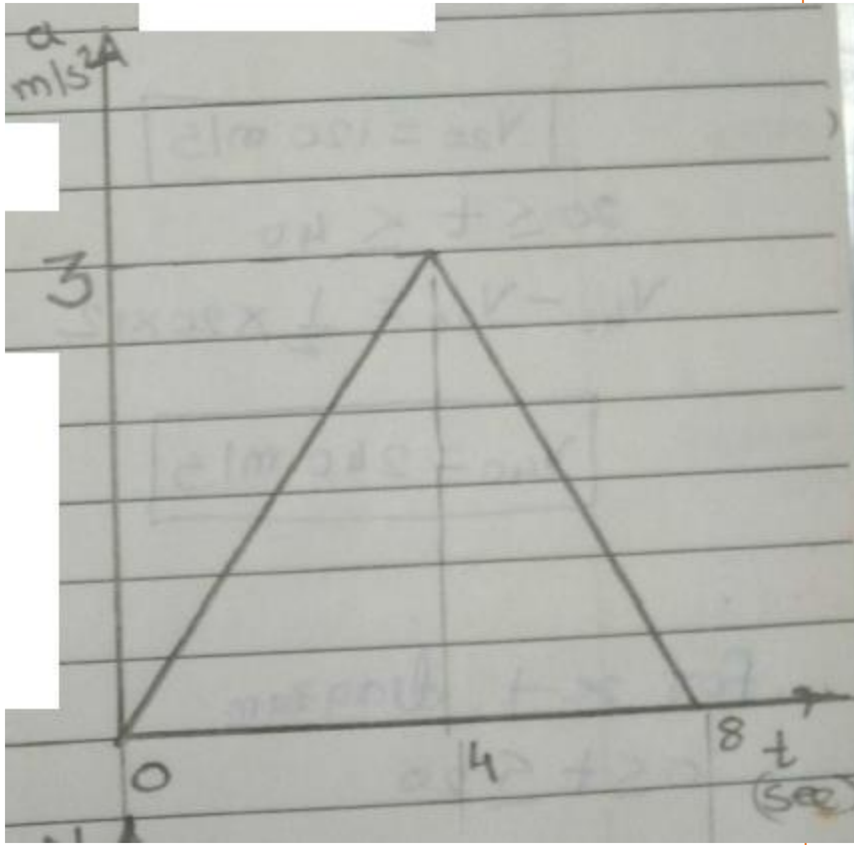
$$0 \le t \le 10s; \quad v = 0.6t \quad a = \frac{dv}{dt} = 0.6$$

$$10 < t \le 30s; \quad v = 6 \quad a = \frac{dv}{dt} = 0$$

The motion of a jet plane while travelling along a runway is defined by v-t curve. Construct x-t and v-t graphs for the motion. The plane starts from the rest.



The a-t diagram for a car is shown in the figure. Draw v-t and x-t diagrams. Find the maximum speed attained and maximum distance covered. The car starts from rest from the origin in a straight line.



Q 2 A test car starts from rest and travels along a straight track such that it accelerates at a constant rate for 10 s and then decelerates at a constant rate. Draw the v-t and s-t graphs and determine the time t' needed to stop the car. How far has the car traveled?

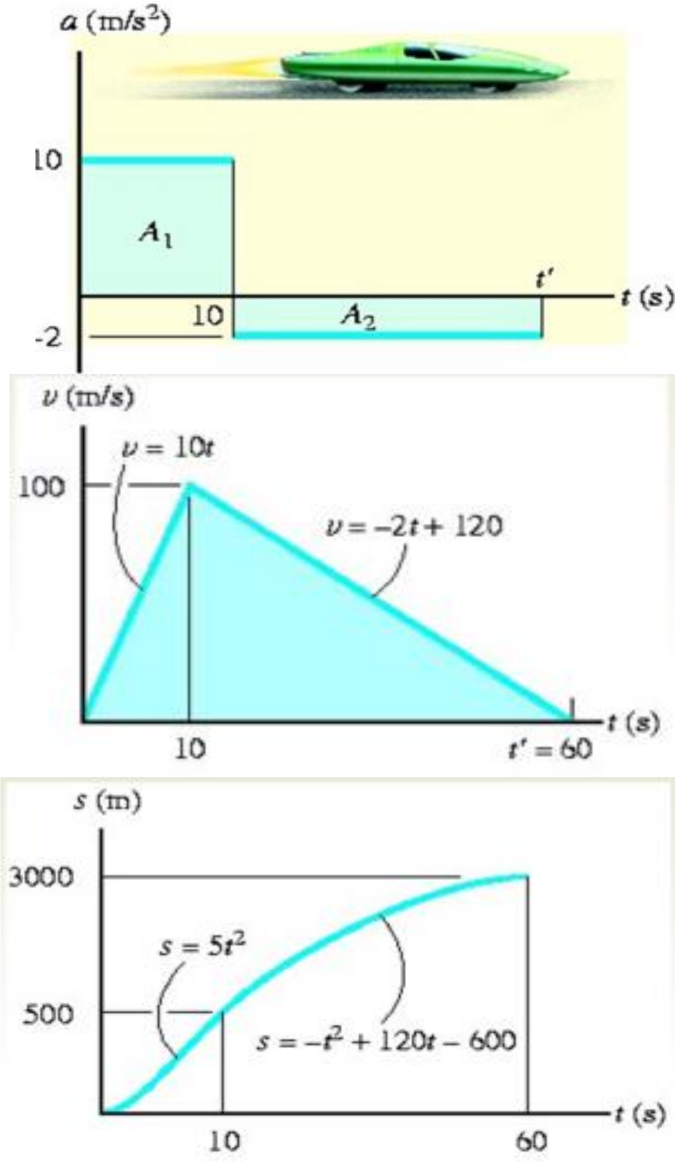
v-t Graph. The v-t graph can be determined by integrating the straight-line segments of the *a-t* graph. Using *initial condition v = 0 when t = 0,*

$$0 \le t \le 10s \quad a = 10; \quad \int_0^v dv = \int_0^t 10\,dt, \quad v = 10t$$

When *t = 10s, v = 100m/s, using this as initial* condition for the next time period, we have

$$10s \le t \le t'; \quad a = -2; \quad \int_{100}^v dv = \int_{10}^t -2\,dt, \quad v = -2t + 120$$

When *t = t' we require v = 0. This yield t' = 60 s*



(c)

s-t Graph. Integrating the eqns. of the v-t graph yields the corresponding eqns. of the *s-t graph*. Using the *initial conditions s = 0 when t = 0,*

$$0 \le t \le 10s; \quad v = 10t; \quad \int_0^s ds = \int_0^t 10t\,dt, \quad s = 5t^2$$

When t = 10s, s = 500m. Using this initial condition,

$$10s \le t \le 60s; \quad v = -2t + 120; \quad \int_{500}^s ds = \int_{10}^t (-2t + 120)dt$$

$$s = -t^2 + 120t - 600$$

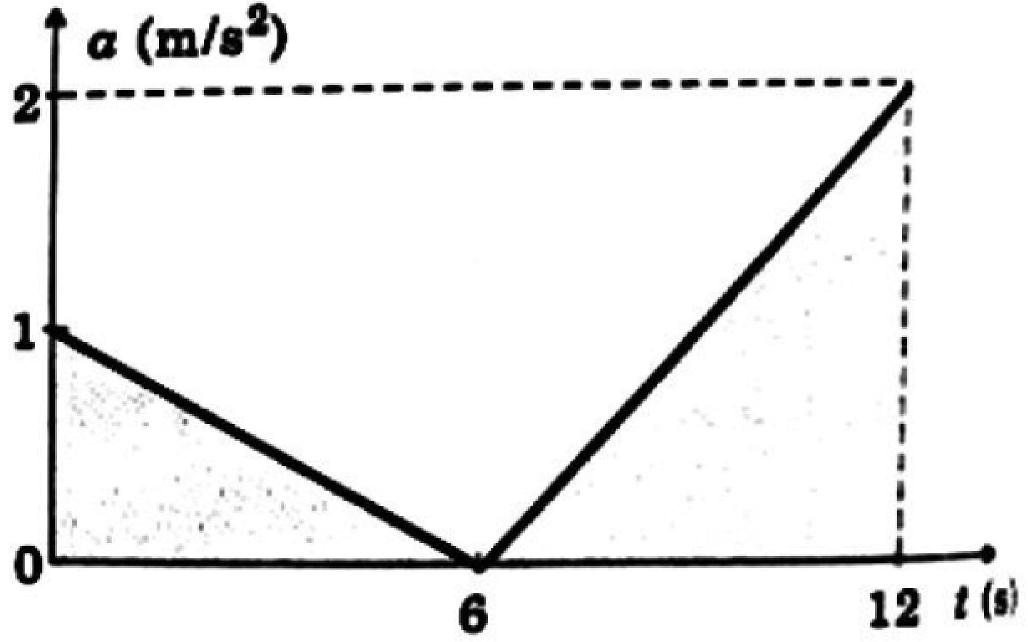
When t’ = 60s, the position is s = 3000m

Problem

Example 49 : The acceleration - time diagram for the linear motion is shown in *figure Ex.49(a)*. Construct velocity - time diagram and displacement - time diagram for the motion assuming that the motion starts with initial velocity of 5 m/s from the starting point.

Solution : Initial condition : At $t = 0$, $x_0 = 0$, $v_0 = 5$ m/s

Area under a-t diagram = Change in velocity (Δv)

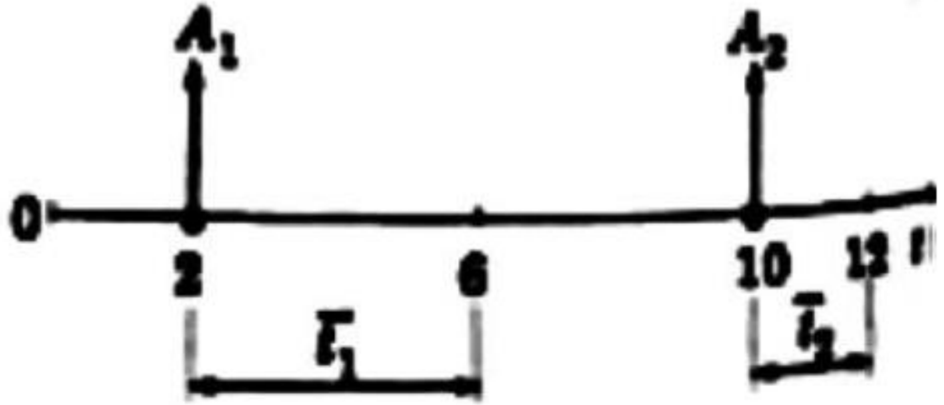


For $0 \le t \le 6$ s, Area $A_1 = \frac{1}{2} \times 6 \times 1 = 3 = v_6 - v_0 = v_6 - 5$ $\quad \therefore v_6 = 8$ m/s

For $6 \le t \le 12$ s, Area $A_2 = \frac{1}{2} \times 6 \times 2 = 6 = v_{12} - v_6 = v_{12} - 8$ $\quad \therefore v_{12} = 14$ m/s

To find position of the particle for *x-t* diagram (using *a-t* diagram), we use

$$x_t = x_0 + v_0 t + A\,\bar{t}$$



For $0 \le t \le 6$ s, $\quad x_6 = x_0 + v_0 t_1 + A_1 \bar{t}_1$

$$\therefore x_6 = 0 + 5 \times 6 + 3 \times 4$$

$$= 42 \text{ m}$$

For $0 \le t \le 12$ s, $\quad x_{12} = x_6 + v_6 t_2 + A_2 \bar{t}_2$

$$\therefore x_{12} = 42 + 8 \times 6 + 6 \times 2$$

$$= 102 \text{ m}$$

$$\bar{t}_1 = \frac{2}{3} \times 6 = 4$$

$$\bar{t}_2 = \frac{1}{3} \times 6 = 2$$

Fig. Ex. 19(b)

Plot *v-t* and *x-t* diagram as shown in *figure Ex.49(c)* and *(d)*.

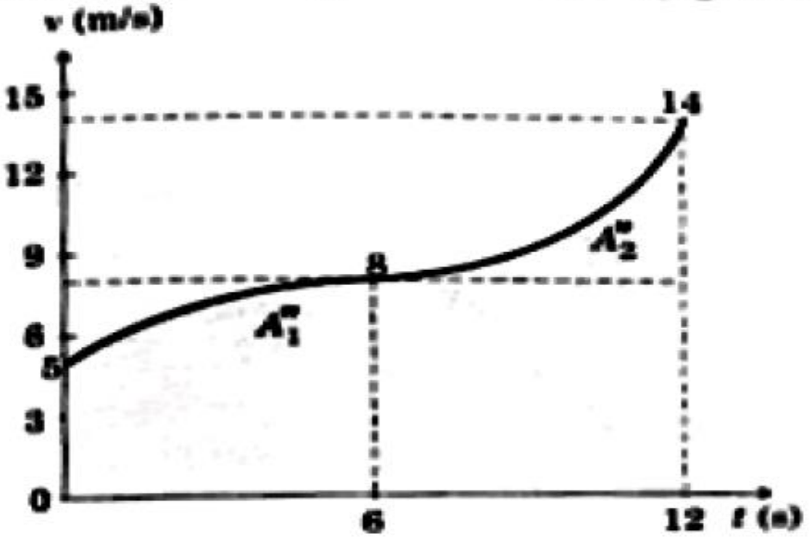


Fig. Ex.49(c) : v-t diagram

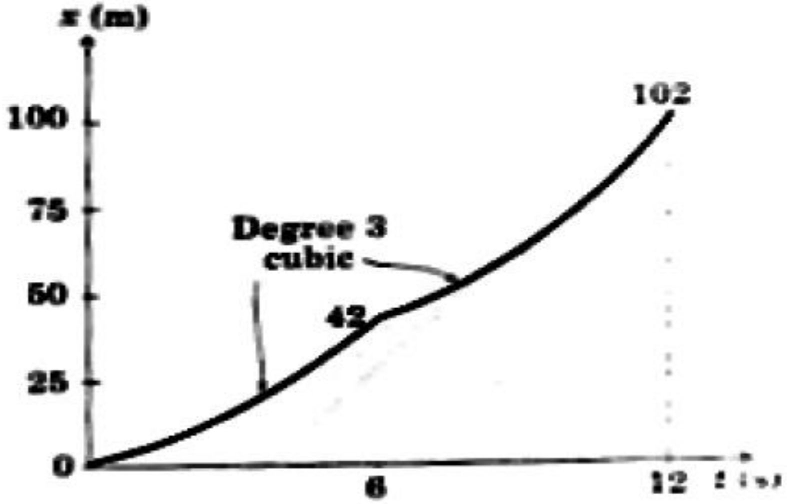


Fig. Ex.49(d) : x-t diagram

Alternative solution for *x-t* diagram. (Using *v-t* diagram)

Area under *v-t* diagram = Change in position (Δx) [Refer *figure Ex.49(c)*]

For $0 \le t \le 6$ s, Area $A_1 = A_1' + A_1'' = 5 \times 6 + \frac{nab}{n+1}$. Here $n = 2$, $a = 6$, $b = 3$

$$\therefore\ A_1 = 30 + \frac{2 \times 6 \times 3}{2+1} = 42 = x_6 - x_0 = x_6 - 0 \qquad \therefore\ x_6 = 42 \text{ m}$$

For $6 \le t \le 12$ s, Area $A_2 = A_2' + A_2'' = 6 \times 8 + \frac{ab}{n+1}$. Here $n = 2$, $a = 6$, $b = 6$

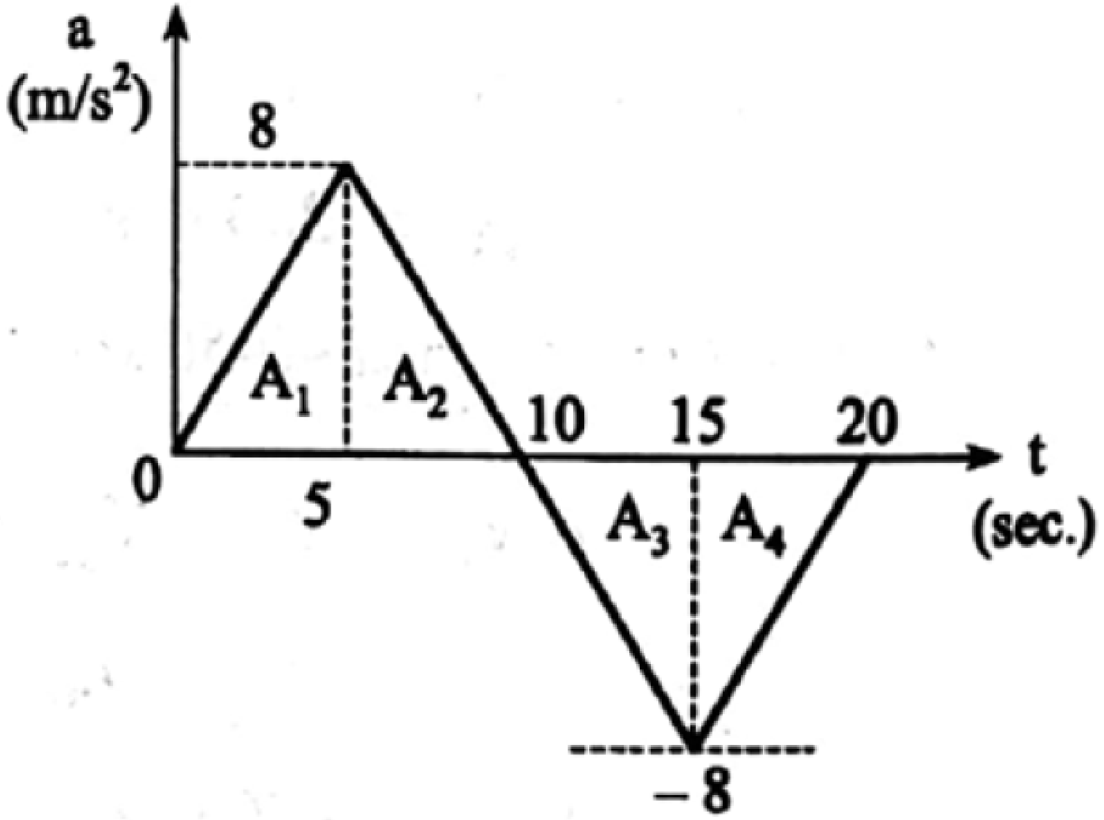
$$\therefore\ A_2 = 48 + \frac{6 \times 6}{2+1} = 60 = x_{12} - x_6 = x_{12} - 42 \qquad \therefore\ x_{12} = 102 \text{ m}$$

Now, plot *x-t* diagram as shown in *figure Ex.49(d)*.

Problem

For the acceleration time diagram for the linear motion is shown in figure. Construct velocity time diagram and displacement time diagram for the motion. Assume that the motion starts from rest.

Solve the problem by motion curve (graphical) method. Also show type (nature) of each curve on all the diagrams.



Solution

(i) Velocity-Time diagram

Change in velocity = Area under a-t diagram

(a) At $t = 5$ sec

$$v_5 - v_0 = \frac{1}{2} \times 5 \times 8 \quad (\because v_0 = 0)$$

$$v_5 = 20 \text{ m/s}$$

(b) At $t = 10$ sec

$$v_{10} - v_5 = \frac{1}{2} \times 5 \times 8$$

$$v_{10} = 20 + 20 = 40 \text{ m/s}$$

(c) At $t = 15$ sec

$$v_{15} - v_{10} = \frac{1}{2} \times 5 \times (-8)$$

$$v_{15} = 40 - 20 = 20 \text{ m/s}$$

(d) At $t = 20$ sec

$$v_{20} - v_{15} = \frac{1}{2} \times 5 \times (-8)$$

$$v_{15} = 20 - 20$$

$$v_{15} = 0 \text{ m/s}$$

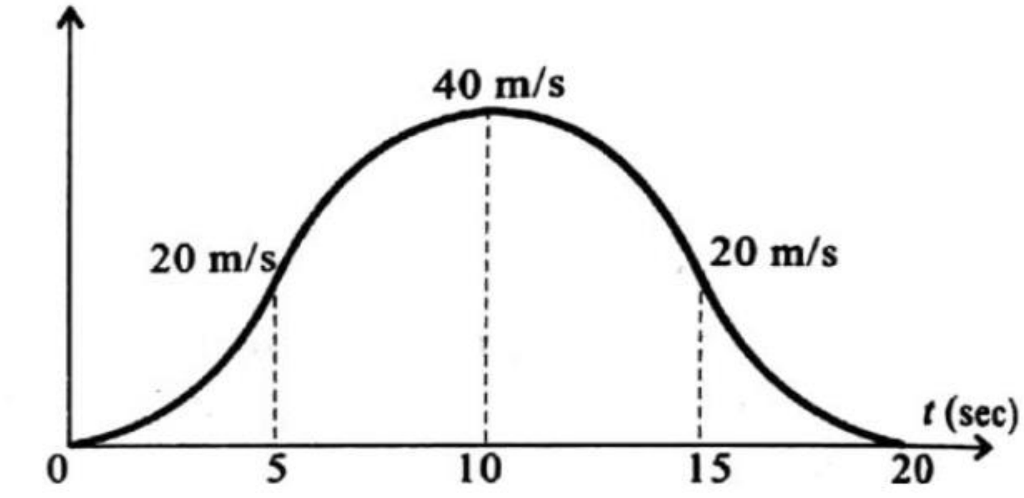


Fig. 11.53(b)

(ii) Displacement-Time diagram

Method I : Finding displacement

Change in displacement = Area under v-t diagram

(i) At $t = 5$ sec

$$s_5 - s_0 = \frac{1}{3} \times 5 \times 20$$

$$s_5 = 33.33 \text{ m}$$

(ii) At $t = 10$ sec

$$s_{10} - s_5 = 5 \times 20 + \frac{2}{3} \times 5 \times 20$$

$$s_{10} = 33.33 + 100 + 66.67$$

$$s_{10} = 200 \text{ m}$$

(iii) At $t = 15$ sec

$$s_{15} - s_{10} = 5 \times 20 + \frac{2}{3} \times 5 \times 20$$

$$s_{15} = 200 + 100 + 66.67$$

$$s_{15} = 366.67 \text{ m}$$

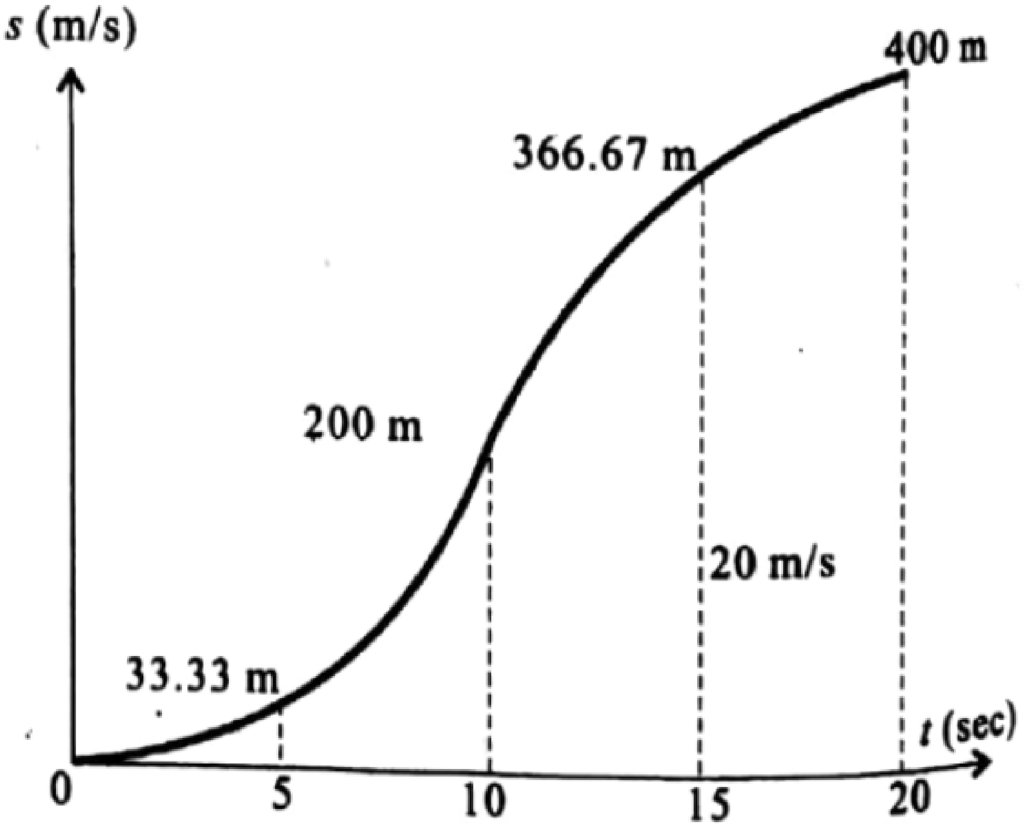


Fig. 11.53(c)

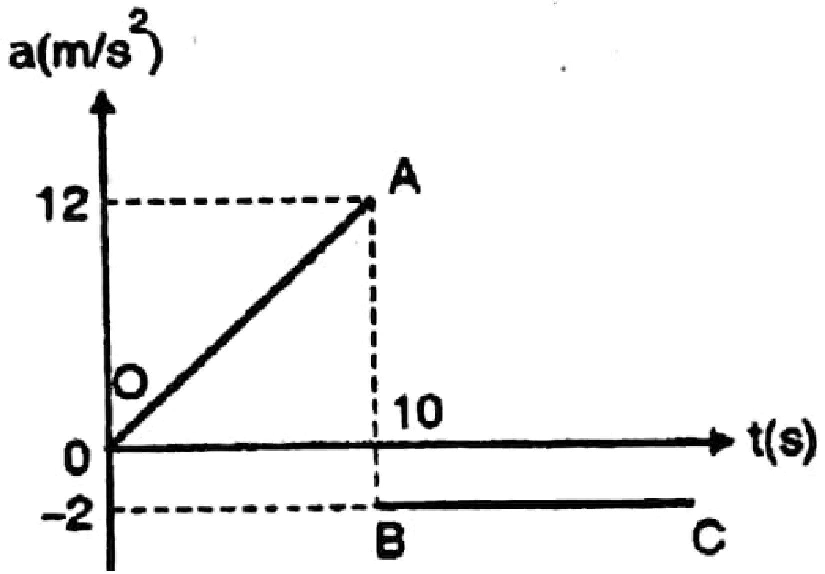
(d) At $t = 20$ sec

$$s_{20} - s_{15} = \frac{1}{2} \times 5 \times 20$$

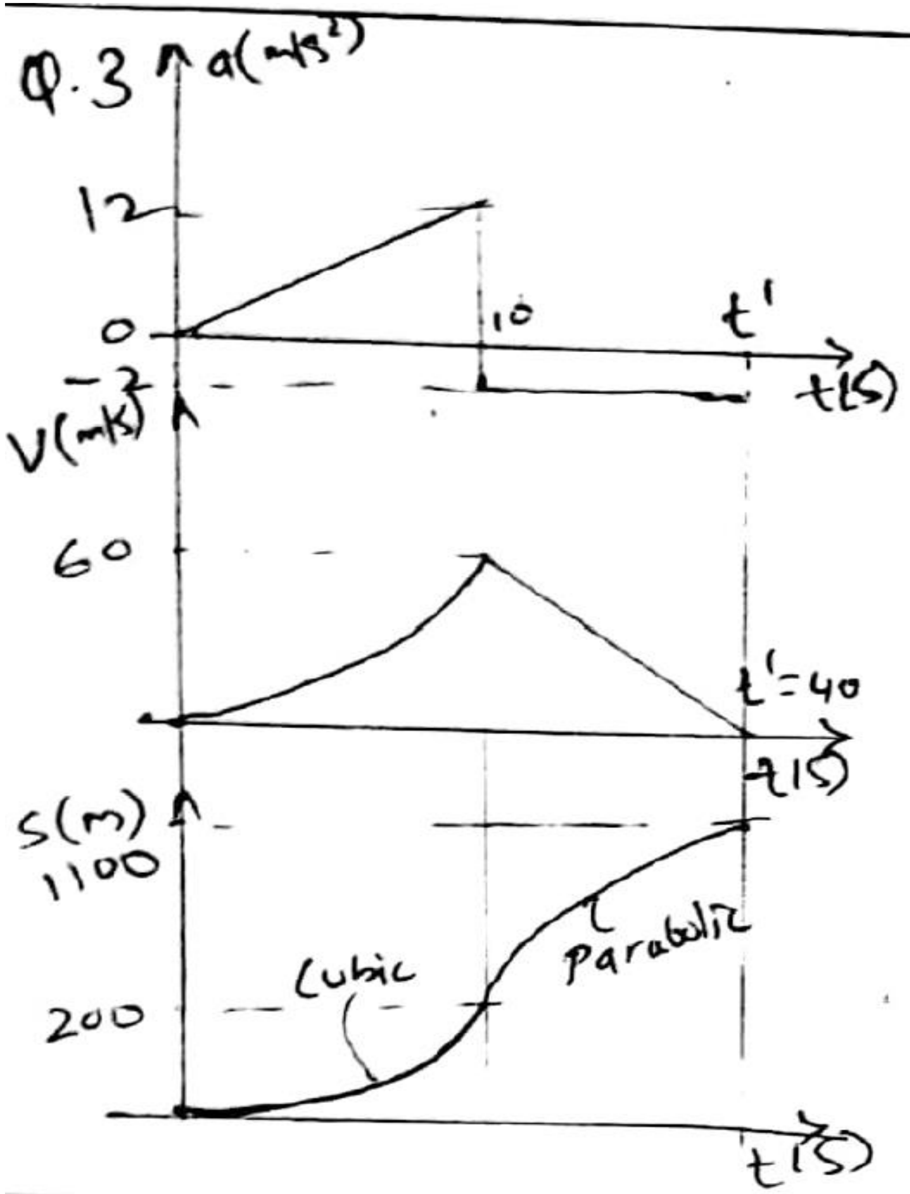
$$s_{20} = 366.67 + 33.33 = 400 \text{ m}$$

Problem

A particle having rectilinear motion has its a-t graph as shown in figure. After 10 seconds the particle has a uniform retardation of $2 m/s^2$ till it stops. Draw v-t and x-t graphs for the entire motion. At t=0, x=0, v=0.



Q.3



$V_{10} - V_0$ = area under a-t curve

$V_{10} = 60$ m/s

$V_{t'} - V_{10} = 2(t' - 10)$

$t' = 40$ sec.

$x_{10} - x_0$ = Area under v-t c

$x_{10} = 200$ m

$x_{40} - x_{10} = \frac{1}{2} \times 30 \times 60$

$x_{40} = 1100$ m

V-t curve

S-t curve

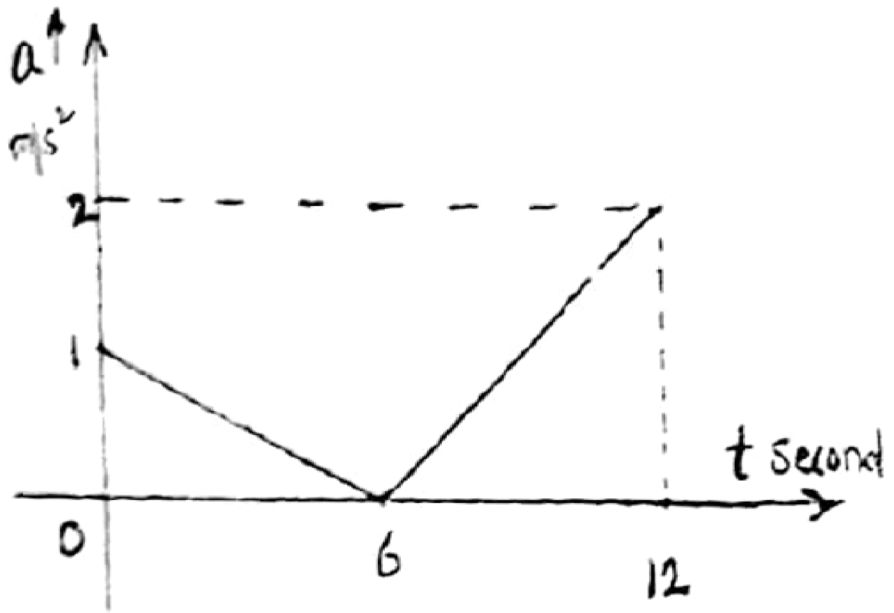
Problem

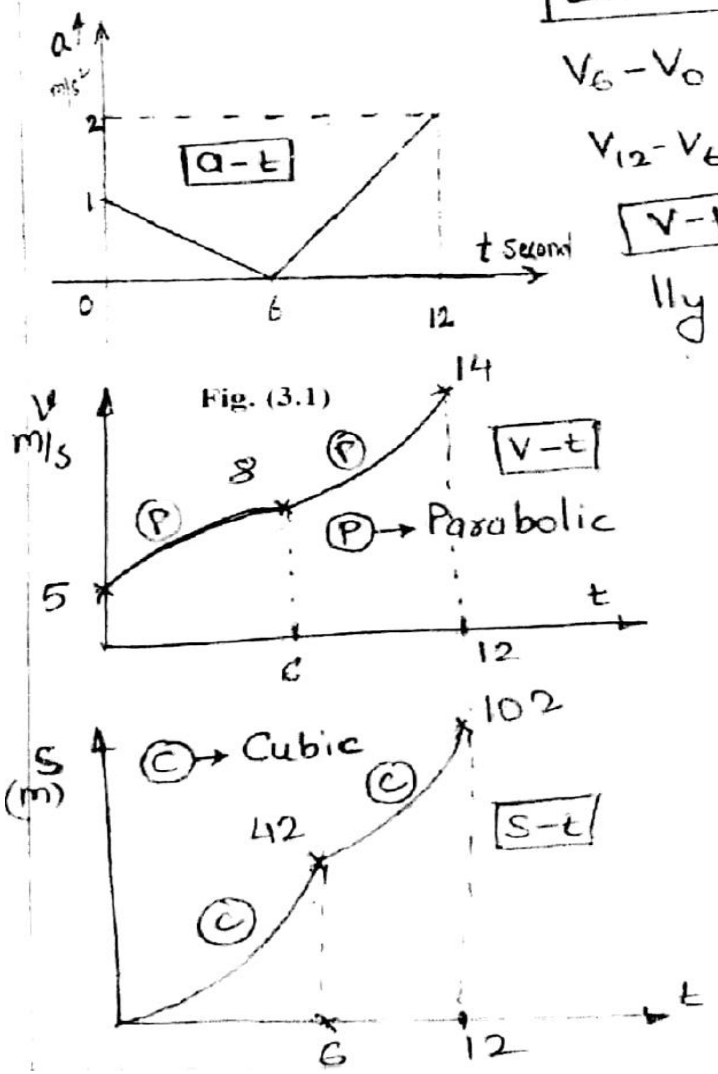
The acceleration-time diagram for the linear motion in **Fig. (3.1)**.

(i) Construct velocity – time assuming that the motion starts with initial velocity of **5 m/s** from starting point.

(ii) What is the maximum velocity,

(iii) Construct displacement – time diagram.





$$\boxed{\Delta V = \text{Area}(a-t)}$$

$$V_6 - V_0 = \frac{1}{2} \times 6 \times 1 \Rightarrow \boxed{V_6 = 8\ m/s}$$

$$V_{12} - V_6 = \frac{1}{2} \times 6 \times 2 \Rightarrow \boxed{V_{12} = 14\ m/s}$$

V-t plot

||ly, $\boxed{\Delta S = \text{Area}(v-t)}$

$$S_6 - S_0 = \frac{2 \times 6 \times 3}{2+1} + 5 \times 6$$

$$\boxed{S_6 = 42\ m}$$

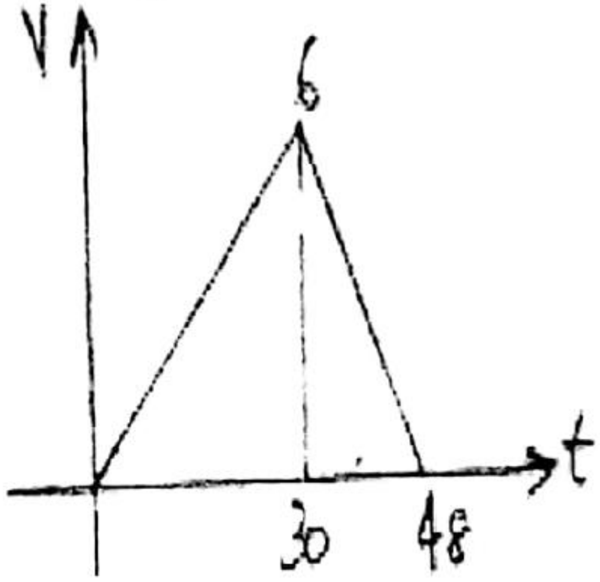
$$S_{12} - S_6 = 8 \times 6 + \frac{6 \times 6}{3}$$

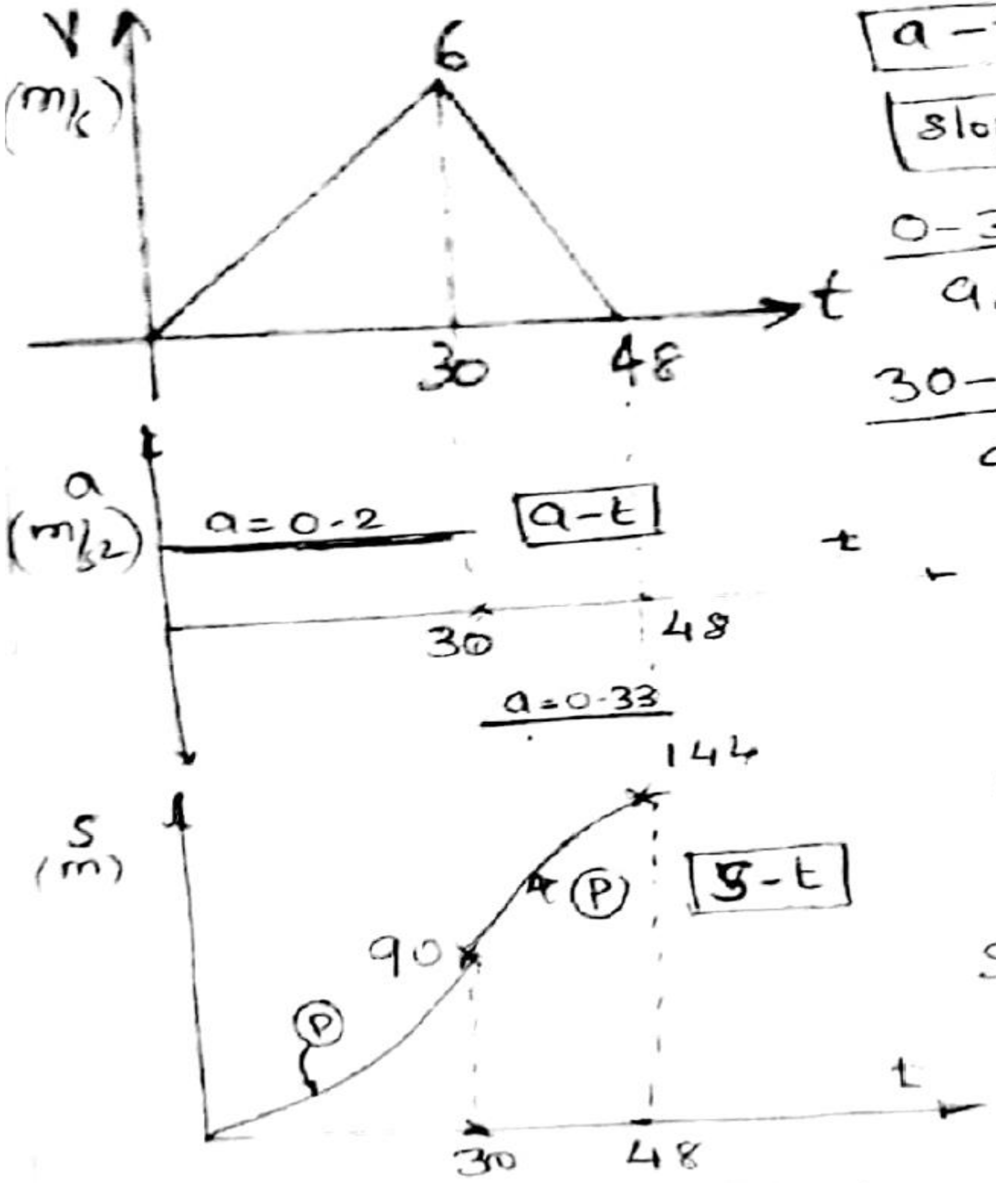
$$\boxed{S_{12} = 102\ m}$$

S-t plot

Problem

A car travels along a straight road with the speed shown by V -t graph in **Fig (3.2)**. The velocity of car at **t = 30** sec is **6** m/s. Find the total distance the car travels until it stops when t = **48** sec. Also plot s-t & a-t graphs.





$\boxed{\text{a-t curve}}$:

$\boxed{\text{slope of v-t = accel}^n}$

0-30 sec.

$$a_{0-30} = \frac{6}{30} = 0.2 \text{ m/s}^2$$

30-48 sec:

$$a_{48-30} = \frac{-6}{18} = -0.333$$

$\boxed{\text{a-t plot}}$

$$\Delta S = A(v-t)$$

$$S_{30} - S_0 = \frac{1}{2} \times 30 \times 6$$

$$S_{30} = 90\text{ m}$$

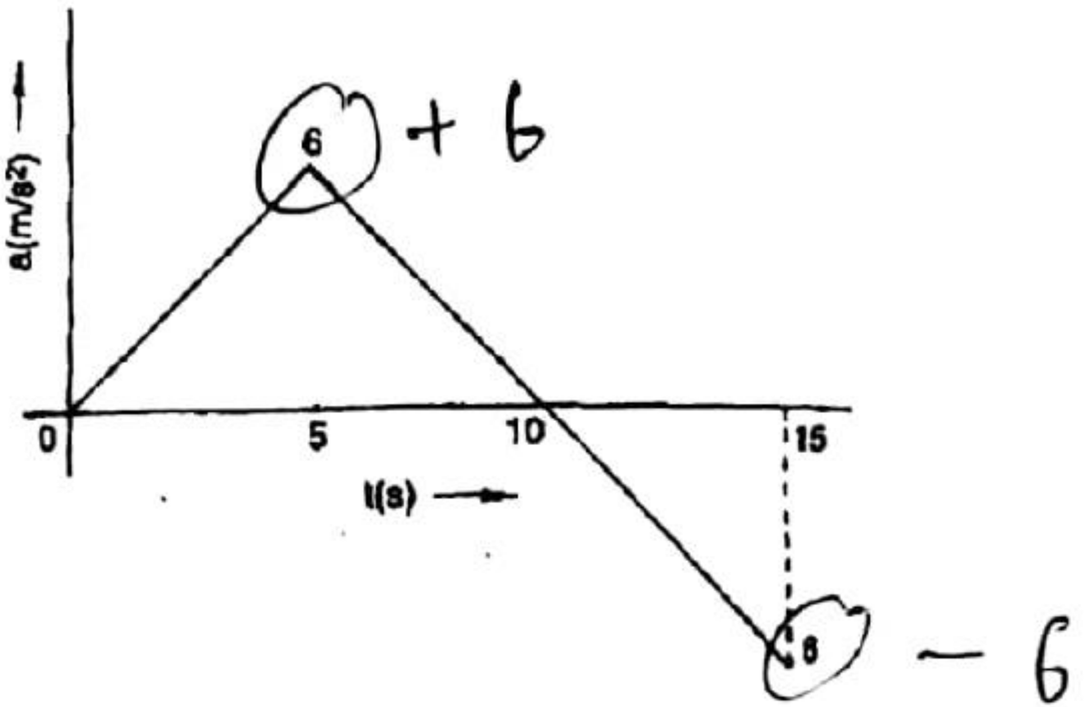
$$S_{48} - S_{30} = 54$$

$$S_{48} = 144\text{ m}$$

S-t plot

Problem

The a-t diagram of a car during continuous period of 15 seconds is shown in figure. The car starts from rest and from the origin. Draw v-t and s-t diagram for the motion of the car. Find velocity and position of the car at t = 5 sec, 10 sec, and 15 sec.



Sol^n - 4: For v-t diagram:

Change in velocity = Area under a-t diagram

i) At $t = 5$ sec,

$$V_5 - V_0 = \frac{1}{2} \times 5 \times 6$$

$$\therefore V_5 - 0 = 15 \qquad \therefore V_5 = 15 \text{ m/s}$$

ii) At $t = 10$ sec,

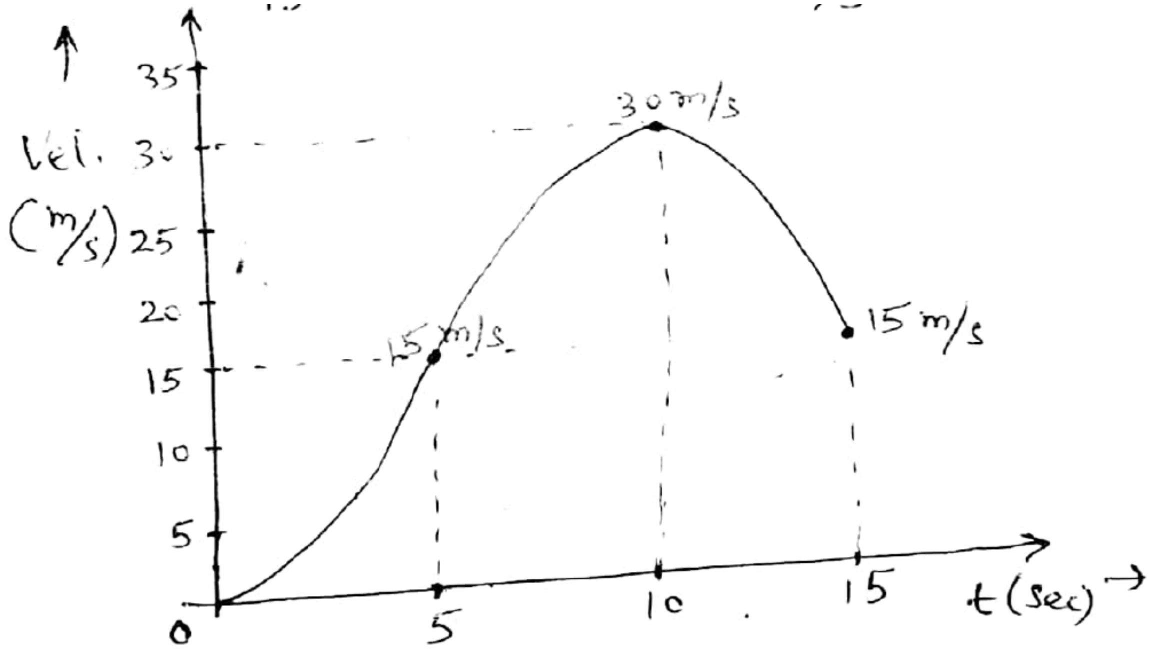
$$V_{10} - V_5 = \frac{1}{2} \times 5 \times 6$$

$$\therefore V_{10} - 15 = 15 \qquad \therefore V_{10} = 30 \text{ m/s}$$

iii) At $t = 15$ sec,

$$V_{15} - V_{10} = \frac{1}{2} \times 5 \times (-6)$$

$$\therefore V_{15} = -15 + 30 = 15 \text{ m/s} \qquad \therefore V_{15} = 15 \text{ m/s}$$



For x-t diagram :

Change in displacement = Area under ~~x~~-t diagram

i) At $t = 5$ Sec.

$S_5 - \cancel{S_0}^{\,0} = \frac{1}{3} \times 5 \times 15$

$\therefore$ ~~$S_5 = 25\,m$~~ $S_5 = 25\,m$

ii) At $t = 10$ sec.

$$S_{10} - S_5 = (5 \times 15) + \left(\frac{2}{\cancel{3}} \times 5 \times \cancel{15}^{\,5}\right)$$

$$\therefore S_{10} - 25 = 75 + 50$$

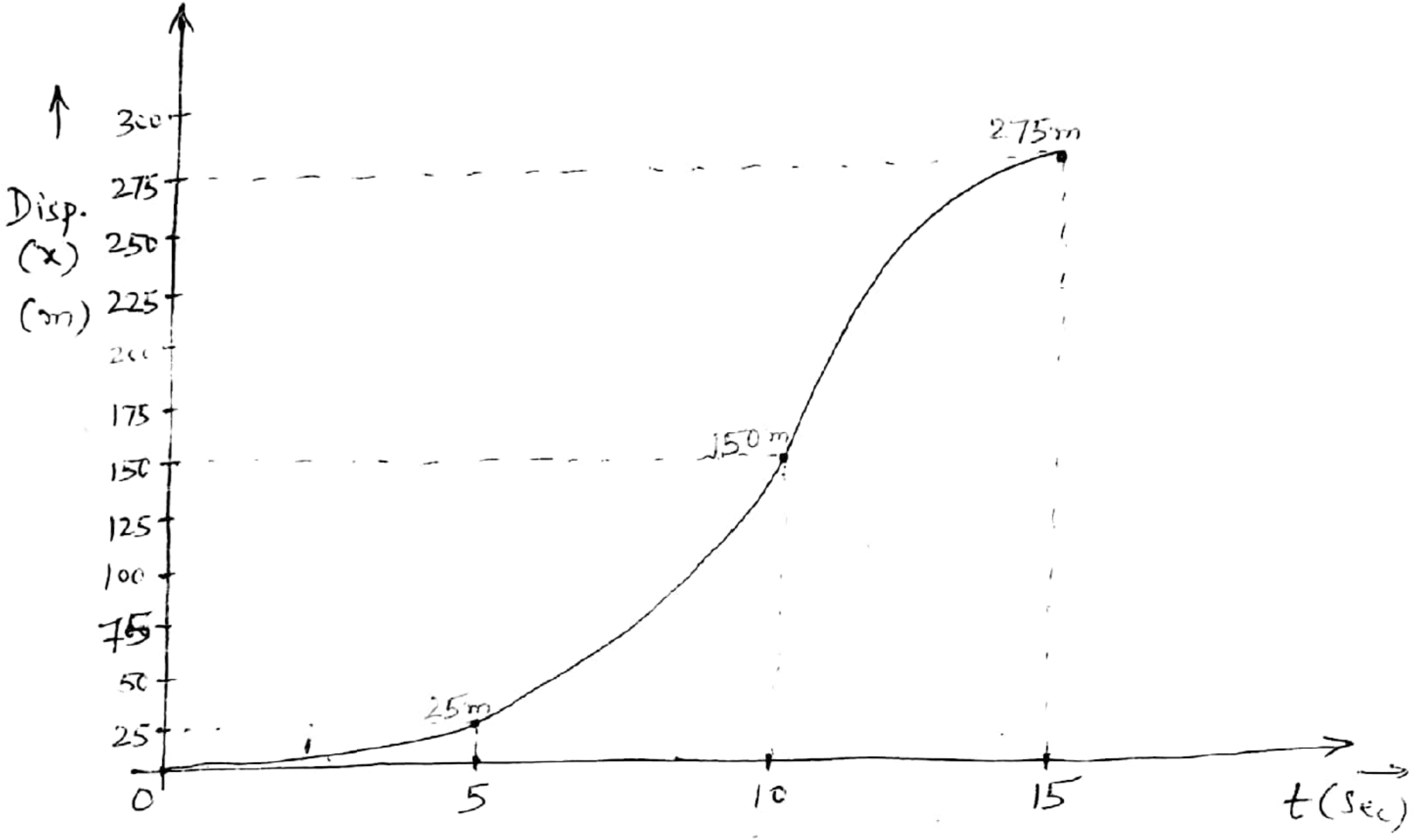
$$\therefore S_{10} = 150 \text{ m}$$

iii) At $t = 15$ sec,

$$S_{15} - S_{10} = (5 \times 15) + \left(\frac{2}{\cancel{3}} \times 5 \times \cancel{15}^{\,5}\right)$$

$$S_{15} - 150 = 75 + 50$$

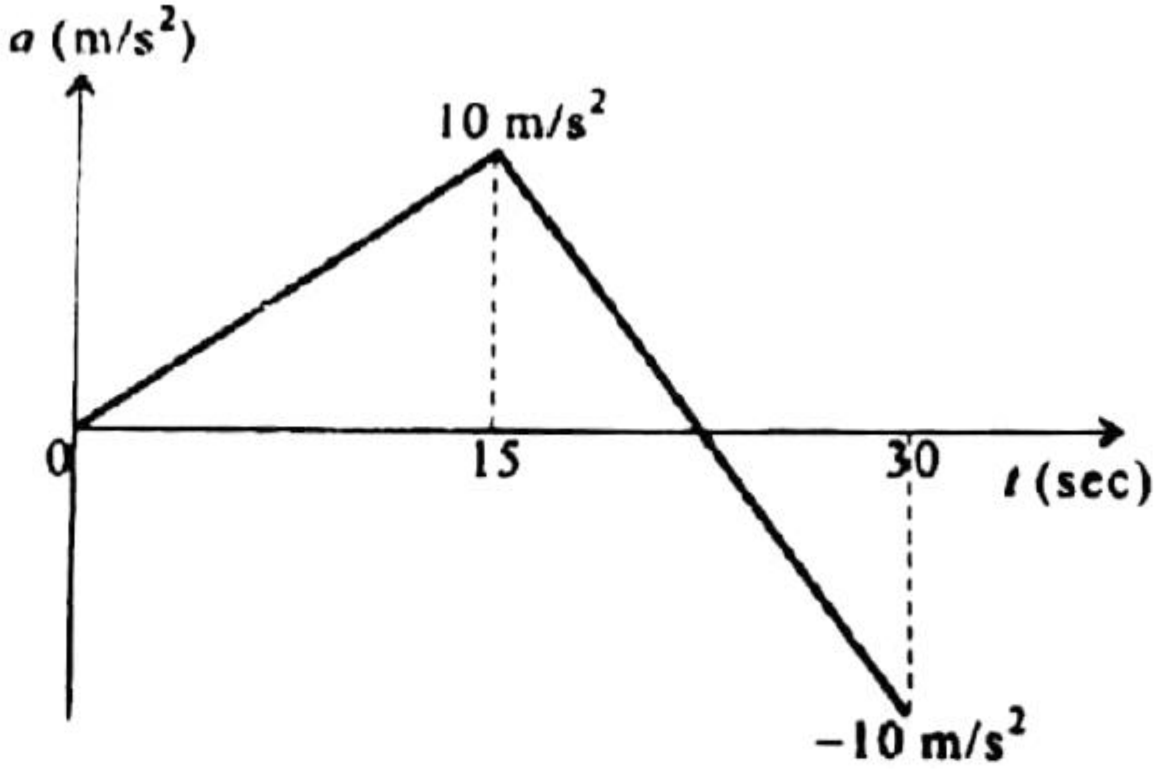
$$\therefore S_{15} = 275 \text{ m}$$

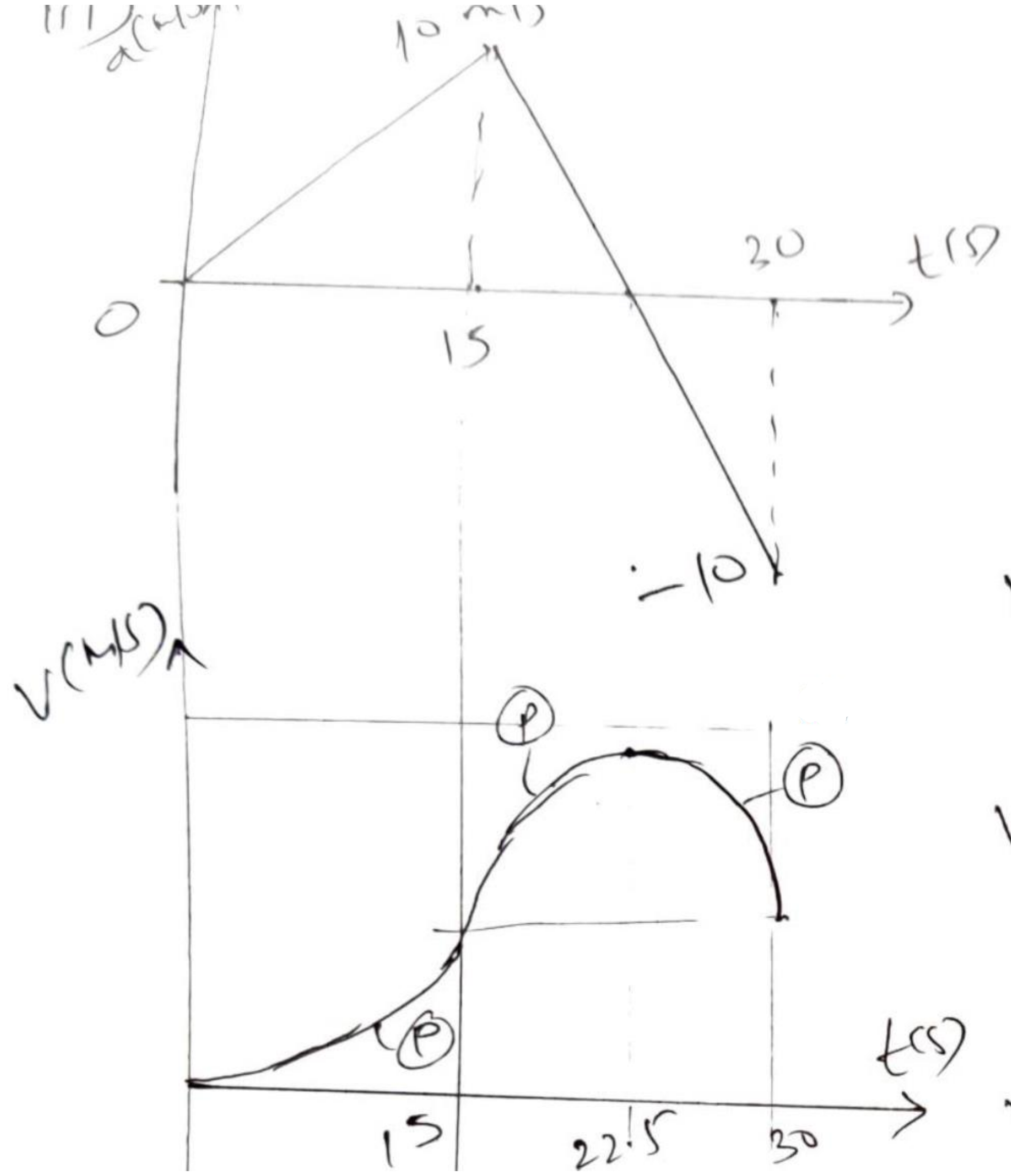


Disp. (x) (m)
300
275
250
225
200
175
150
125
100
75
50
25
0
5
10
15
t (sec)
275 m
150 m
25 m

Problem

A particle is moving along a straight path with an acceleration shown in Fig. in a-t graph. Draw v-t and x-t graph.





$$V_{15} - V_0 = \frac{1}{2} \times 15 \times 10$$

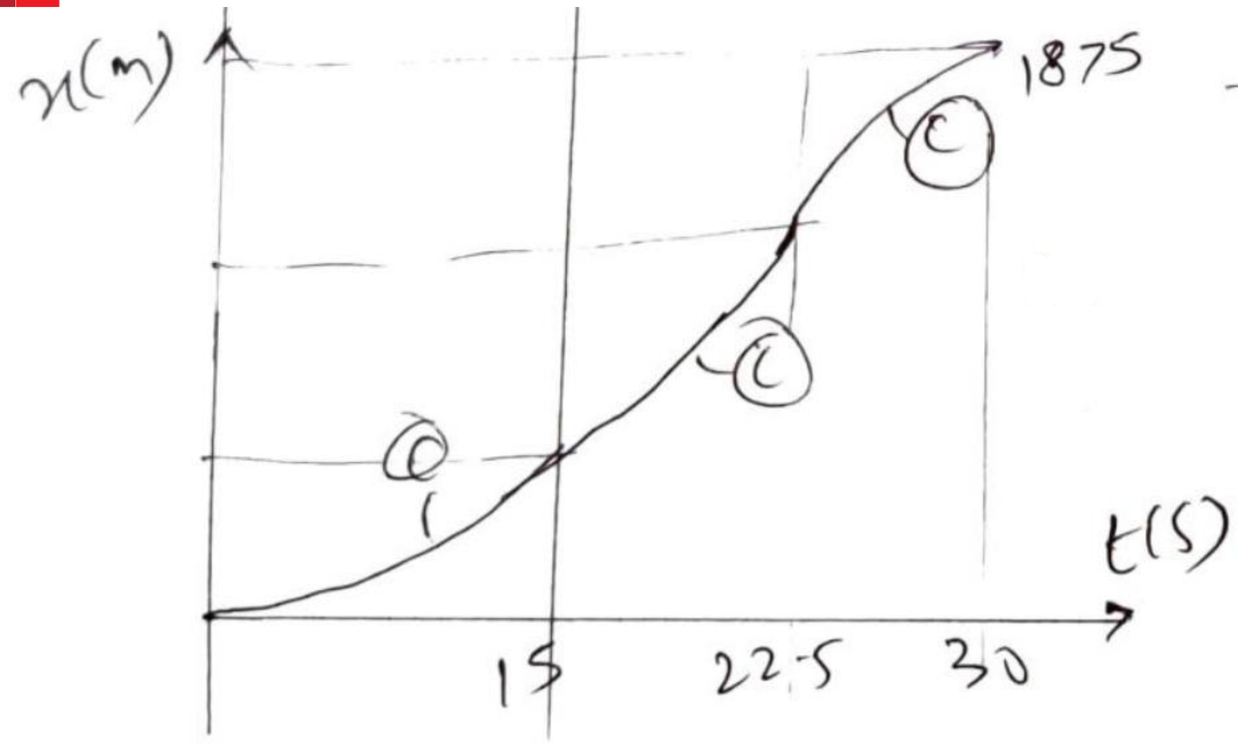
$$V_{15} = 75 \text{ m/s} \quad \because V_0 = 0$$

$$V_{22.5} - V_{15} = \frac{1}{2} \times 7.5 \times 10$$

$$= 75 + 37.5$$

$$V_{22.5} = 112.5 \text{ m/s}$$

$$V_{30} - V_{22.5} = \frac{1}{2} \times 7.5(-10)$$



$V_{30} = 75$ m/s

$S_{15} - S_0 = \frac{1}{3} \times 15 \times 75$

$S_{15} = 375\ m$

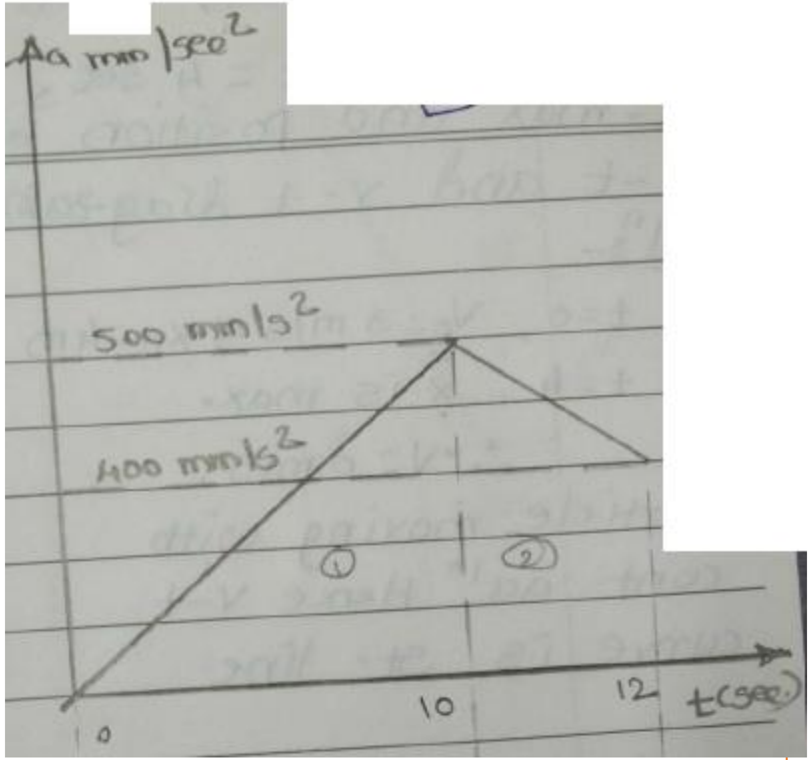
$S_{22.5} - S_{15} = (7.5 \times 75) + \frac{2}{3}(7.5 \times 37.5)$

$= 1125\ m$

$S_{30} - S_{22.5} = (7.5 \times 75) + (\frac{2}{3} \times 7.5 \times 37.5)$

$= 1875\ m$

The motion of a particle from rest is given by a-t diagram as shown. Sketch v-t diagram and hence calculate velocity.



A particle moves in a straight line with a velocity-time diagram shown in figure. If s = – 25 m at t = 0, draw displacement-time and acceleration-time diagrams for 0 to 24 seconds.

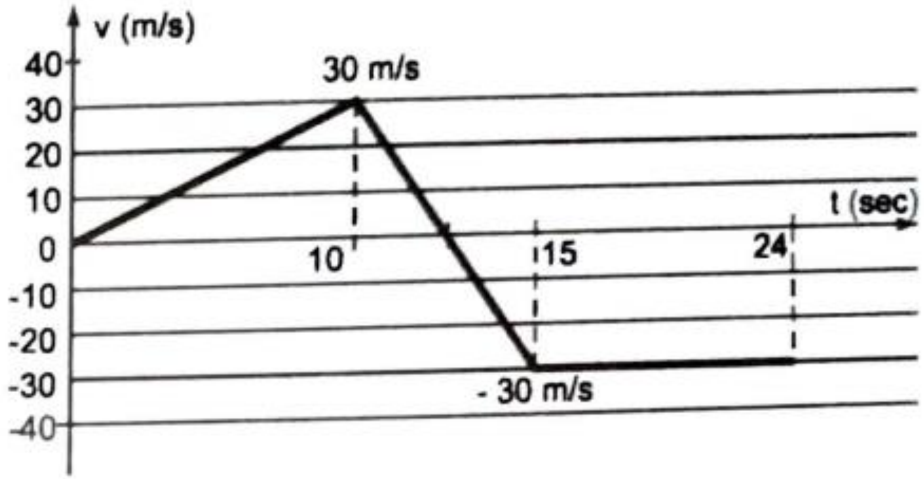
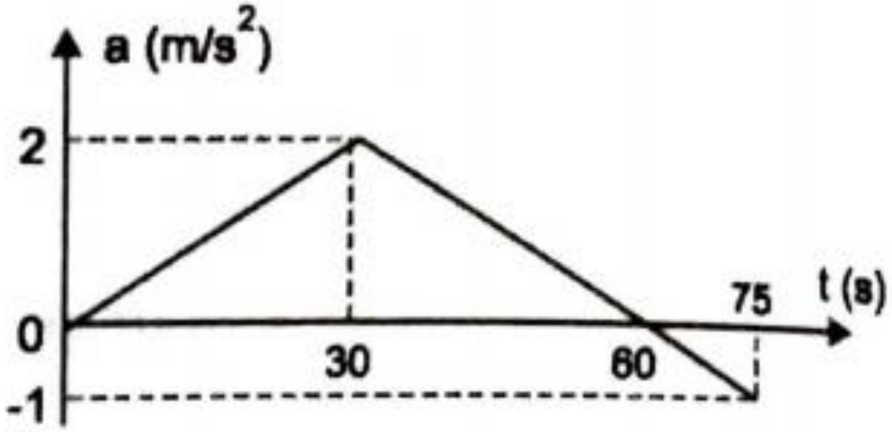


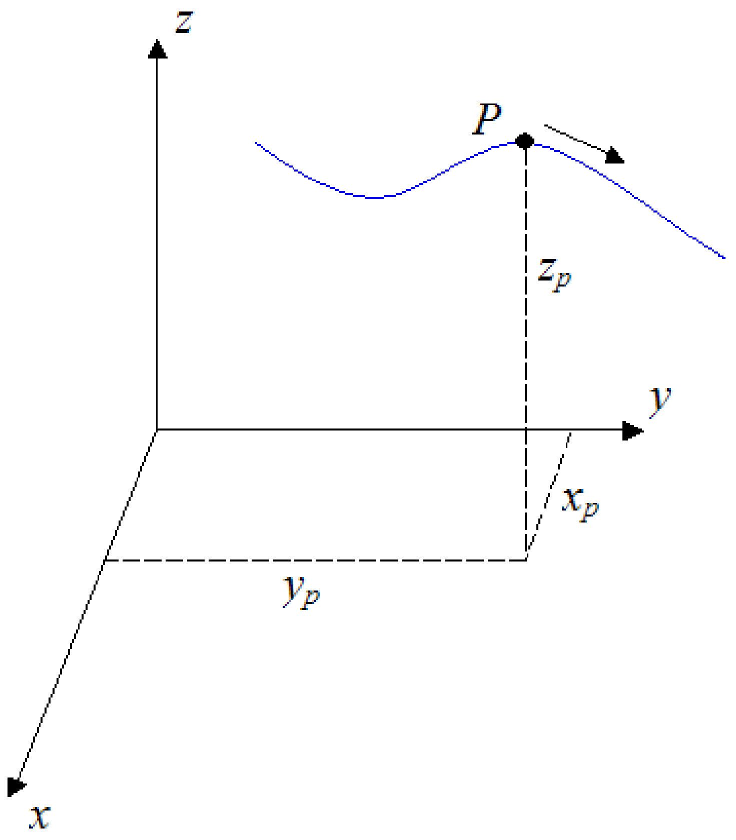
Figure shows (a – t) diagram for particle moving along a straight path for a time interval 0 – 75 sec. Plot (v – t) and (x – t) diagrams and hence find the maximum speed attained by the particle. The particle started from rest from origin.



Curvilinear motion

$$x_p = x_p(t)$$
$$y_p = y_p(t)$$
$$z_p = z_p(t)$$

- Curvilinear motion is defined as motion that occurs when a particle travels along a curved path.
- The curved path can be in two dimensions (in a plane), or in three dimensions.
- To find the velocity and acceleration of a particle experiencing curvilinear motion one only needs to know the position of the particle as a function of time.
- The velocity and acceleration of the particle *P* is given by



$$v_x = \frac{dx_p}{dt} \qquad a_x = \frac{d^2 x_p}{dt^2} \qquad v_p = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$v_y = \frac{dy_p}{dt} \qquad a_y = \frac{d^2 y_p}{dt^2}$$

$$a_p = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$v_z = \frac{dz_p}{dt} \qquad a_z = \frac{d^2 z_p}{dt^2}$$

Curvilinear Motion: Position, Velocity & Acceleration

The softball and the car both undergo curvilinear motion.

- A particle moving along a curve other than a straight line is in *curvilinear motion*.

Analysis of curvilinear motion

- Rectangular Co-ordinate system
- Normal and Tangential co-ordinate system

Rectangular Coordinates (*x*-*y*)

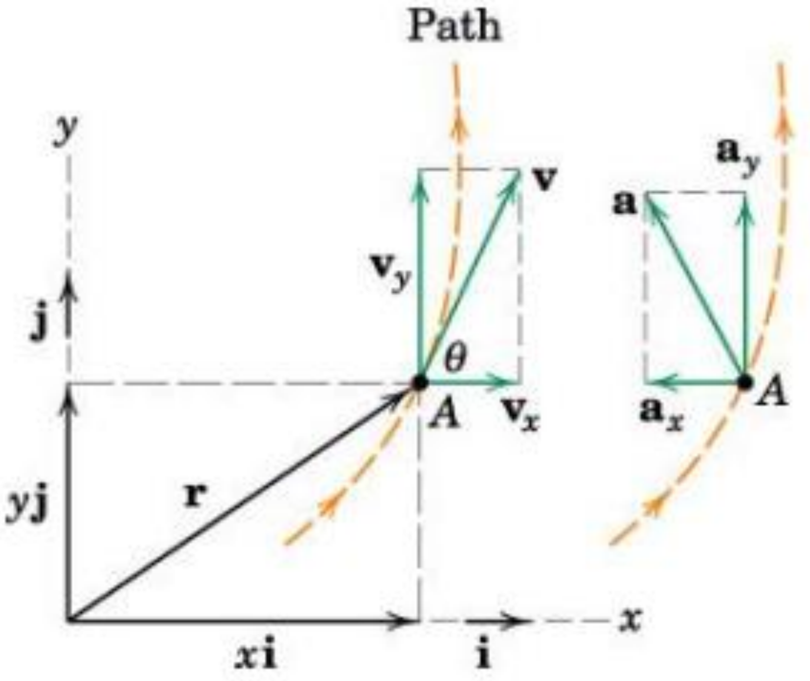
If all motion components are directly expressible in terms of horizontal and vertical coordinates

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

$v_x = \dot{x}$, $v_y = \dot{y}$ and $a_x = \dot{v}_x = \ddot{x}$, $a_y = \dot{v}_y = \ddot{y}$



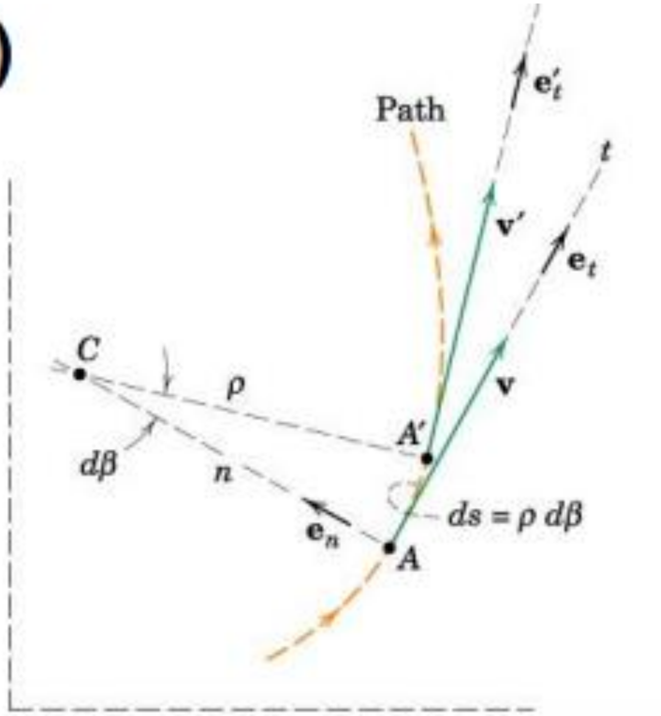
$$v^2 = v_x^{\,2} + v_y^{\,2} \qquad v = \sqrt{v_x^{\,2} + v_y^{\,2}} \qquad \tan\theta = \frac{v_y}{v_x}$$

$$a^2 = a_x^{\,2} + a_y^{\,2} \qquad a = \sqrt{a_x^{\,2} + a_y^{\,2}}$$

Time derivatives of the unit vectors are zero because their magnitude and direction remains constant.

Also, $dy/dx = \tan\theta = v_y/v_x$

Normal and Tangential Coordinates (*n*-*t*)



Determination of $\dot{\mathbf{e}}_t$:

→ change in $\mathbf{e}_t$ during motion from *A* to *A'*

→ The unit vector changes to $\mathbf{e}'_t$

The vector difference $d\mathbf{e}_t$ is shown in the bottom figure.

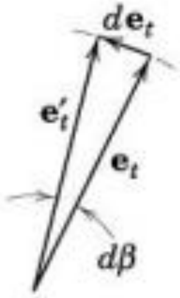
- In the limit $d\mathbf{e}_t$ has magnitude equal to length of the arc $|\mathbf{e}_t|\ d\beta = d\beta$
- Direction of $d\mathbf{e}_t$ is given by $\mathbf{e}_n$

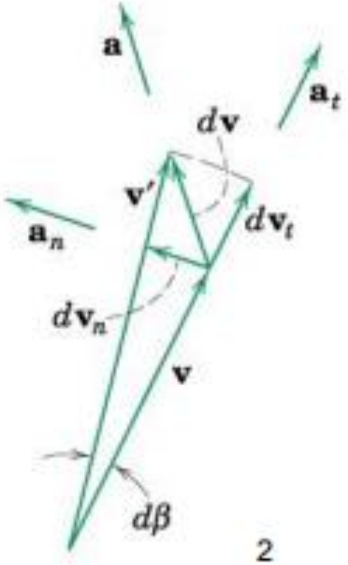
→ We can write: $d\mathbf{e}_t = \mathbf{e}_n\, d\beta$ → $\dfrac{d\mathbf{e}_t}{d\beta} = \mathbf{e}_n$

Dividing by *dt*: $d\mathbf{e}_t/dt = \mathbf{e}_n\,(d\beta/dt)\,\mathbf{e}_n$ → $\boxed{\dot{\mathbf{e}}_t = \dot{\beta}\mathbf{e}_n}$

Substituting this and $v = \rho\, d\beta/dt = v = \rho\dot{\beta}$ in equation for acceleration:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d(v\mathbf{e}_t)}{dt} = v\dot{\mathbf{e}}_t + \dot{v}\mathbf{e}_t \quad \rightarrow \quad \boxed{\mathbf{a} = \frac{v^2}{\rho}\mathbf{e}_n + \dot{v}\mathbf{e}_t}$$





Here:

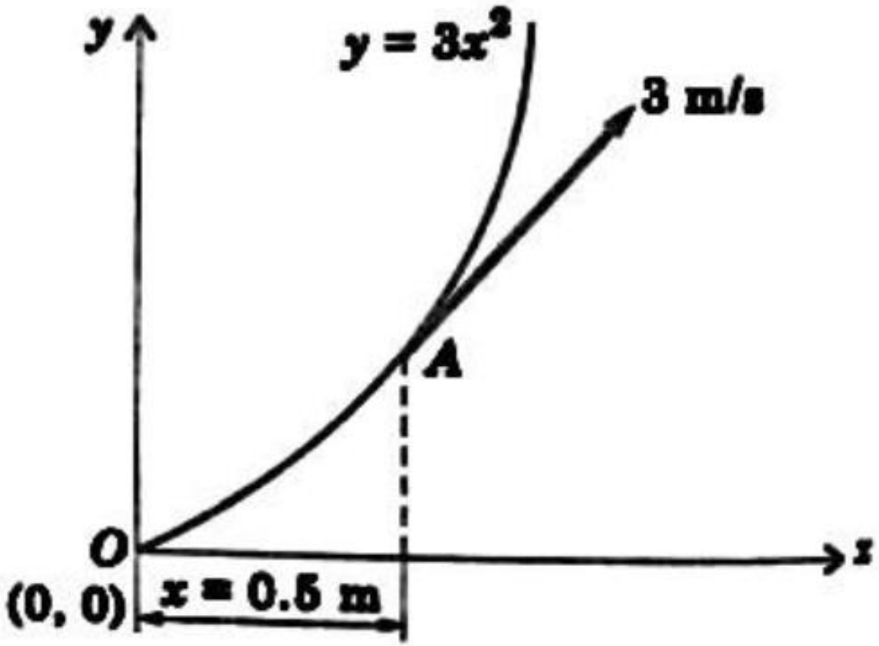
$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta}$$

$$a_t = \dot{v} = \ddot{s}$$

$$a = \sqrt{a_n^2 + a_t^2}$$

Problem

A particle moves with a constant speed of 3 m/s along the path as shown in figure below. What is the resultant acceleration at a position on the path where x=0.5 m Also represent the acceleration in the vector form



Solution : Given data :

Speed of the particle, $v = 3$ m/s [Constant]

Equation of the curve, $y = 3x^2$

To find : Acceleration a in vector form = ?

We have, acceleration, $a = \sqrt{a_t^2 + a_N^2}$ ...(i)

Body is moving with constant speed. ...(ii)

Hence $a_t = 0$

Normal acceleration at A, $a_N = \frac{v^2}{\rho} = \frac{3^2}{\rho} = \frac{9}{\rho}$ ...(iii)

To find : Radius of curvature, ρ = ? at $t = 2$ s

Given : $y = 3x^2$

$$\therefore \frac{dy}{dx} = 6x. \text{ At point } A, \left(\frac{dy}{dx}\right)_{x=0.5} = 6 \times 0.5 = 3$$

$$\frac{d^2y}{dx^2} = 6 \text{ [Constant]}$$

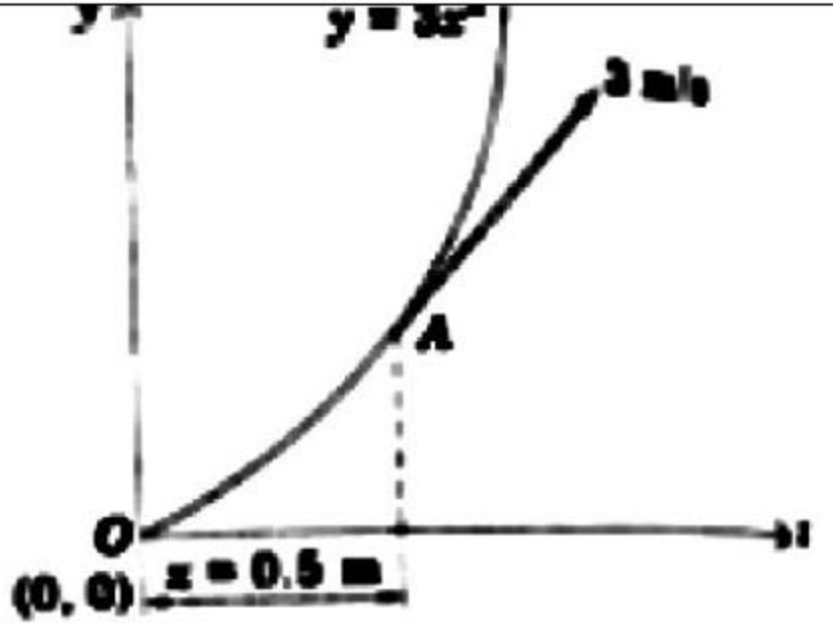


Fig. Ex. 6.1(a)

We have

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \qquad \therefore \rho_{at\ x=0.5} = \frac{\left[1 + (3)^2\right]^{3/2}}{6}$$

$\therefore \rho_{at\ x=0.5} = 5.27$ m. Put this value in equation (iii) to get

$$a_N = \frac{9}{5.27} = 1.708 \text{ m/s}^2$$

Using equation (i) we get

$$\text{Acceleration, } a = \sqrt{0 + (1.708)^2} = 1.708 \text{ m/s}^2 \quad \text{...Ans.}$$

To represent acceleration in vector form

As shown in *figure Ex.61(b)*, tangential acceleration is tangent to the curve at point A and normal acceleration is towards the centre of curvature and is perpendicular to the direction of tangential acceleration.

From the figure

$$\tan\alpha = \left(\frac{dy}{dx}\right)_{x=0.5} = 3 \qquad \therefore \alpha = 71.565°$$

Fig. Ex.61(b)

We have $\boldsymbol{a} = a_x \, \boldsymbol{i} + a_y \, \boldsymbol{j}$

$$\boldsymbol{a} = (-a_N \sin \alpha) \, \boldsymbol{i} + (a_N \cos \alpha) \, \boldsymbol{j}$$

$$\boldsymbol{a} = (-1.708 \sin 71.565^\circ) \, \boldsymbol{i} + (1.708 \cos 71.565^\circ) \, \boldsymbol{j}$$

$$\boldsymbol{a} = -1.62 \, \boldsymbol{i} + 0.54 \, \boldsymbol{j} \quad \ldots \textit{Ans.}$$

Problem

A particle moves along a hyperbolic path $\frac{x^2}{16} - y^2 = 28$

If the x-component of velocity is $v_x = 4$ m/s and remains constant, determine the magnitudes of particles velocity and acceleration when it is at point (32,6) m.

Solution

Given : $v_x = 4$ m/sec is constant $\therefore \ a_x = 0$ or $\frac{d^2x}{dt^2} = 0$

$$\frac{x^2}{16} - y^2 = 28$$

Differentiating with respect to t, we get

$$\frac{2x}{16}\frac{dx}{dt} - 2y \cdot \frac{dy}{dt} = 0$$

$$\frac{x}{8}\frac{dx}{dt} - 2y \cdot \frac{dy}{dt} = 0 \qquad \text{..... (I)}$$

$$\frac{32}{8} v_x - 2 \times 6 \cdot v_y = 0$$

$$4 \times 4 - 12\, v_y = 0$$

$$v_y = 1.33 \text{ m/sec} \quad \textit{Ans.}$$

$$\therefore\ v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{4^2 + 1.33^2}$$

$$\therefore\ v = 4.22 \text{ m/sec} \quad \textit{Ans.}$$

We have Eq. (I),

$$\frac{x}{8}\,\frac{dx}{dt} - 2y \cdot \frac{dy}{dt} = 0$$

Again differentiating with respect to t, we get

$$\frac{1}{8} \cdot \left[x \cdot \frac{d^2x}{dt^2} + \frac{dx}{dt} \cdot \frac{dx}{dt}\right] - 2\left[y \cdot \frac{d^2y}{dt^2} + \frac{dy}{dt} \cdot \frac{dy}{dt}\right] = 0$$

$$\frac{1}{8} \cdot \left[32 \cdot \frac{d^2x}{dt^2} + (4)^2\right] - 2\left[6 \cdot \frac{d^2y}{dt^2} + (1.33)^2\right] = 0$$

$$\left[\begin{array}{l} \because\ P(32, 6)\ \text{m} \\ \therefore\ x = 32,\ y = 6 \\ \quad \dfrac{dy}{dx} = v_x = 4\ \text{m/s} \\ \quad \dfrac{dy}{dt} = v_y = 1.33\ \text{m/sec} \\ \quad \dfrac{d^2x}{dt^2} = 0 \end{array}\right]$$

$$\therefore \frac{1}{8} \cdot [32 \times 0 + 16] - 2\,[6 \cdot a_y + (1.33)^2] = 0$$

$$\therefore a_y = -0.129\ \text{m/s}^2$$

$$\therefore a = \sqrt{a_x^{\,2} + a_y^{\,2}} = \sqrt{0 + (-0.129)^2}$$

$$a = 0.129\ \text{m/sec}^2$$ ***Ans.***

Problem

A particle moving in straight line with an acceleration $a = \sqrt{v}$. its displacement and velocity at time $t = 2$ s are 128/3 m and 16 m/s respectively. Find the displacement, velocity and acceleration at $t = 3$ s.

$a = \sqrt{v} \qquad 2v^{1/2} = t + c$

At $t = 2s \qquad v = 16\ m/s \qquad c = 6$

$$2v^{1/2} = t + 6 \qquad \text{—— (i)}$$

$$\frac{2}{3} v^{3/2} = x + c_1 \qquad c_1 = 0$$

$$2/3\, v^{3/2} = x. \qquad \text{—— (ii)}$$

At $t = 3\ s \qquad v = 20.25\ m/s$

$x = 60.75\ m$

$a = 4.5\ m/s^2$

Problem

A particle moving in x-y plane with y components of velocity $v_y = 6t$ m/s where t is in seconds. The x components of acceleration of particle is $a_x = 3t$ m/s^2 where t is in seconds. When t=0, x = 3m and y = 0, and $v_x = 0$. Find the equation of path of particle. Determine the magnitude of velocity of particle at the instant when y = 10m.

$$V_y = 6t = dy/dt \qquad y = \frac{6t^2}{2} + C_1$$

$$y = 3t^2 \qquad \because C_1 = 0 \qquad \text{—(i)}$$

$$a_x = \frac{d^2x}{dt^2} = 3t \quad \therefore V_x = \frac{3t^2}{2} + C_2$$

$$V_x = 3t^2/2 \qquad \because C_2 = 0$$

Integrating

$$x = \frac{3t^3}{6} + c_3 \quad \text{at } t=0 \;\; x=3$$

$$\therefore c_3 = 3$$

$$x = 0.5t^3 + 3 \qquad \text{(ii)}$$

eqn. of path – $y^3 = 108(x-3)^2$

putting $y = 10$ $\quad t = 1.82$ s.

$V_x = 4.99$ m/s $\qquad V_y = 10.95$ m/s

$v = 12.036$ m/s.

Problem

The acceleration of a particle is given by a = k/x, when x = 250 mm v is 4 m/sec. When x = 500 mm v is 3 m/sec. Determine the velocity of the particle when x = 750 mm. Find the position of the particle when it comes to the rest.

Solⁿ :- $a = K/x$ $\therefore v \cdot \frac{dv}{dx} = \frac{K}{x}$

$$v \cdot dv = \frac{K}{x} \cdot dx$$

Taking integration on both sides, we get

$$K \int \frac{1}{x} \cdot dx = \int v \cdot dv$$

$$K \cdot \log_e x = \frac{v^2}{2} + C_1$$

Given: when $x = 0.25\,m$, $v = 4\ m/s$

& $x = 0.5\,m$, $v = 3\ m/s$

$\therefore \quad K \log_e(0.25) = \frac{(4)^2}{2} + C_1$ — ① eq

$K \log_e(0.5) = \frac{(3)^2}{2} + C_1$ — ② eq.

① eq − ② eq. we get

$K \log_e(0.25) - K \log_e(0.5) = (8 + C_1) - (4.5 + C_1)$

$\therefore \quad K[\log_e(0.25) - \log_e(0.5)] = 3.5$

$$\therefore \quad K = \frac{3.5}{\log_e(0.25) - \log_e(0.5)} = \frac{3.5}{-0.6931} = -5.05$$

Putting value of K in eq^n (1), we get

$$-5.05 \log_e (0.25) = 8 + C_1$$

$$\therefore C_1 = -1$$

i) Vel. of particle when $x = 0.75$ m : ~~we get~~

$$K \log_e x = \frac{V^2}{2} + C_1$$

$$-5.05 \log_e (0.75) = \frac{V^2}{2} - 1$$

$$\therefore \boxed{V = 2.215 \text{ m/s}}$$

ii) Position of particle when it comes to rest :

$x = ?$ $\quad V = 0$

$$K \log_e x = \cancel{\frac{V^2}{2}} + C_1$$

0

$$-5.05 \log_e(x) = -1$$

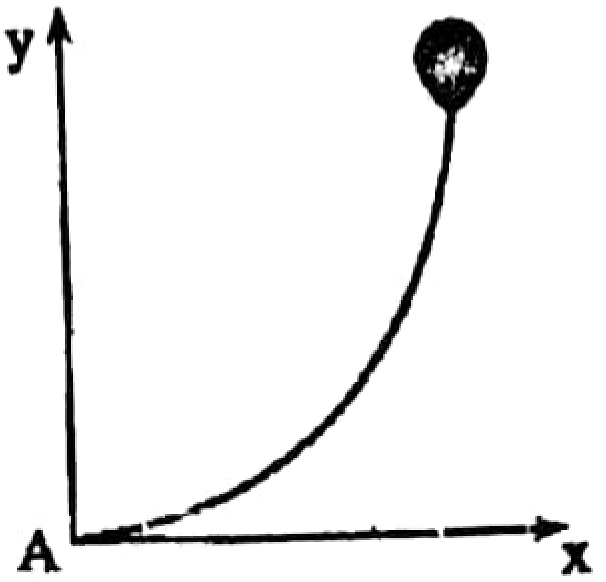
$$\therefore \boxed{x = 1.219 \text{ m}}$$

Problem

At any instant the horizontal position of the weather balloon in figure is described by $x = 9t$ m where t is in second. If the equation of path is $y = x^2/30$. Determine at t=2 sec.,

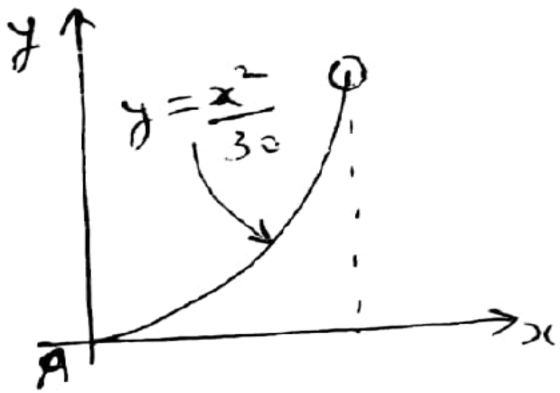
i) The distance of the balloon from the station A

ii) The magnitude and direction of the acceleration.



Sol^n: At $t = 2$ sec, $x = 9t = 9 \times 2 = 18$ m

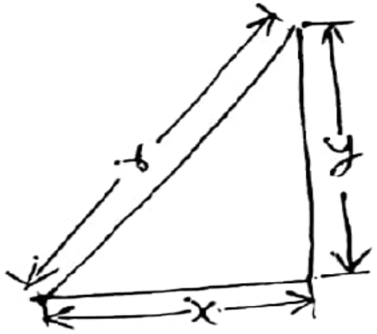
$$y = \frac{x^2}{30} = \frac{(18)^2}{30} = 10.8 \text{ m}$$



$$r = \sqrt{x^2 + y^2} = \sqrt{(18)^2 + (10.8)^2}$$

$$r = 21 \text{ m}$$

$$x = 9t \quad \therefore \quad \frac{dx}{dx} = V_x = 9$$



$$\frac{dV_x}{dt} = a_x = 0$$

$$y = \frac{x^2}{30} = \frac{(9t)^2}{30} = \frac{81t^2}{30}$$

$$\frac{dy}{dt} = V_y = \frac{81}{30} \times 2t \quad \& \quad \frac{dV_y}{dt} = a_y = \frac{81}{30} \times 2$$

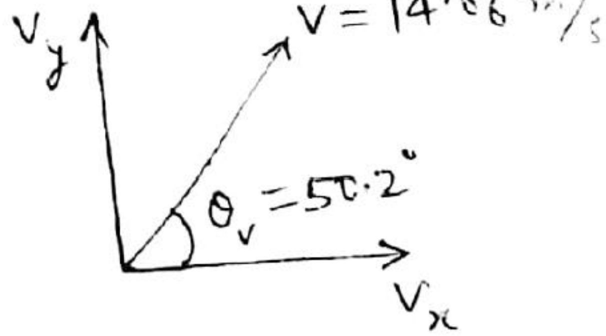
∴ At 2 sec,

$V_x = 9 \ m/s \ (\rightarrow)$

$V_y = \frac{81}{30} \times 2 \times 2 = 10.8 \ m/s \ (\uparrow)$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(9)^2 + (10.8)^2} = 14.06 \ m/s$

$\theta_v = \tan^{-1}\left(\frac{V_y}{V_x}\right) = 50.2^\circ$



At $t = 2$ sec,

$a_x = 0$

$a_y = \frac{81}{30} \times 2 = 5.4 \ m/s^2 \ (\uparrow)$

Problem

- A particle moves along the path $\bar{r} = (8t^2)i + (t^3 + 5)j$ meters. Where t is in seconds. Determine magnitudes of particles velocity and acceleration when t = 3 seconds. Also determine the equation y = f(x) of the path.

Ex. A particle moves in a circular path of 4 m radius. Calculate 4 sec later the particle's total acceleration and distance traveled if,

a) speed is constant at 2 m/s

b) speed is 2 m/s at the instant and is increasing at a rate of 0.7 m/s^2.

Solution: a) Speed is constant at 2 m/s i.e $a_t = 0$

also $$a_n = \frac{v^2}{\rho} = \frac{2^2}{4} = 1 \ m/s^2$$

Using $$a = \sqrt{a_n^2 + a_t^2}$$

$\therefore \quad a = 1 \ m/s^2$ ..... **Ans.**

This is a case of curvilinear motion with uniform speed.
To calculate distance traveled, using

$$v = \frac{s}{t}$$

$$2 = \frac{s}{4} \qquad \therefore s = 8 \ m$$ ......**Ans.**

b) Speed is increasing at $0.7\ m/s^2$ i.e $a_t = 0.7\ m/s^2$

This is a case of curvilinear motion with uniform tangential acceleration

$$u = 2\ m/s$$
$$v = ?$$
$$s = ?$$
$$a_t = 0.7 m/s^2$$
$$t = 4\ sec$$

using
$$v = u + a_t \times t$$
$$v = 2 + 0.7 \times 4$$
$$v = 4.8\ m/s$$

using
$$s = ut + \tfrac{1}{2} a_t \times t^2$$
$$s = 2 \times 4 + \tfrac{1}{2} \times 0.7 \times (4)^2$$
$$s = 13.6\ m$$
..... **Ans.**

Now
$$a_n = \frac{v^2}{\rho} = \frac{(4.8)^2}{4} = 5.76\ m/s^2$$

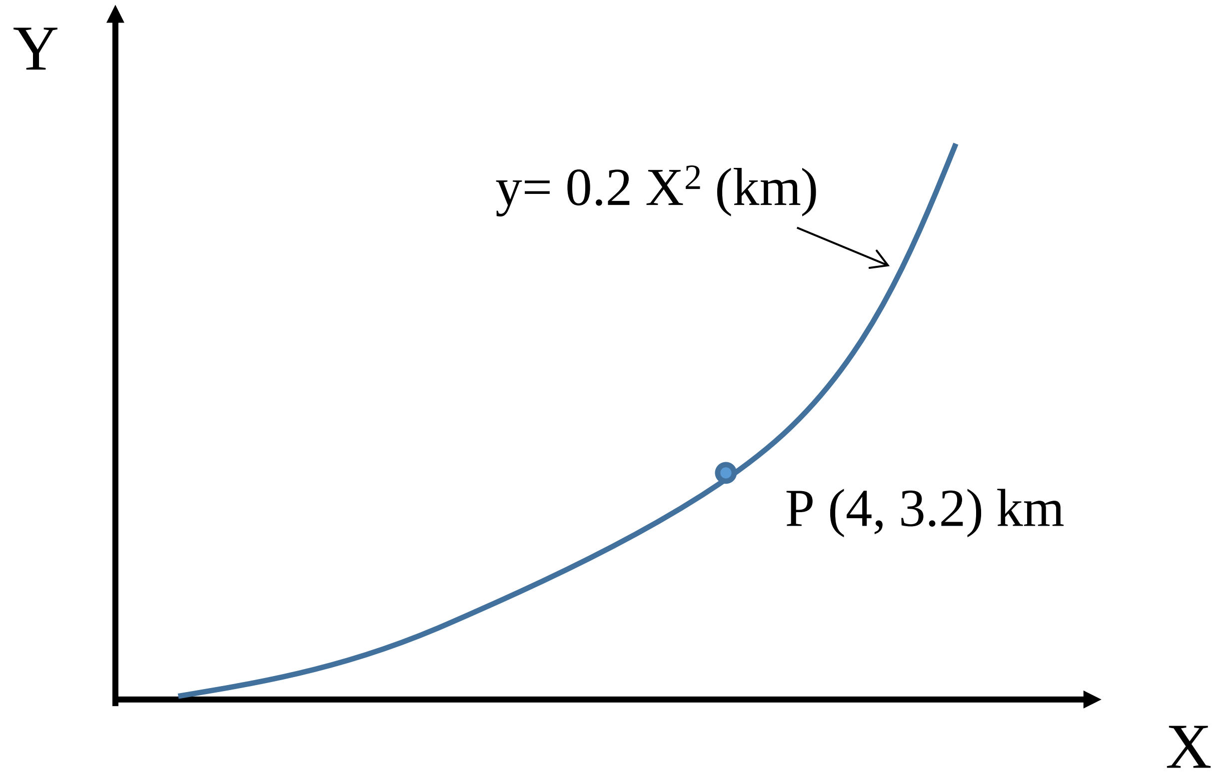
$\therefore$ total acceleration $= a = \sqrt{(5.76)^2 + (0.7)^2}$

$$a = 5.802\ m/s^2$$
..... **Ans.**

Problem

An airplane travels on a curved path. At point ‘P’ it has a speed of 360kmph which is increasing at the rate of 0.5 m/s^2. Determine at ‘P’,

- Magnitude of total acceleration
- Angle made by the acceleration vector with the positive X-axis.



Solution: Given equation of path as $y = 0.2\,x^2$

$$\frac{dy}{dx} = 0.4x \qquad \left(\frac{dy}{dx}\right)_{x=4km} = 1.6$$

$$\frac{d^2y}{dx^2} = 0.4 \qquad \left(\frac{d^2y}{dx^2}\right)_{x=4km} = 0.4$$

using $$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+(1.6)^2\right]^{3/2}}{0.4} = 16.792\text{ km} \quad \therefore \quad \rho = 16792\text{ m}$$

Now $$a_n = \frac{v^2}{\rho} = \frac{(100)^2}{16792} = 0.595\text{ m/s}^2$$

also $a_t = 0.5\text{ m/s}^2$ ....given

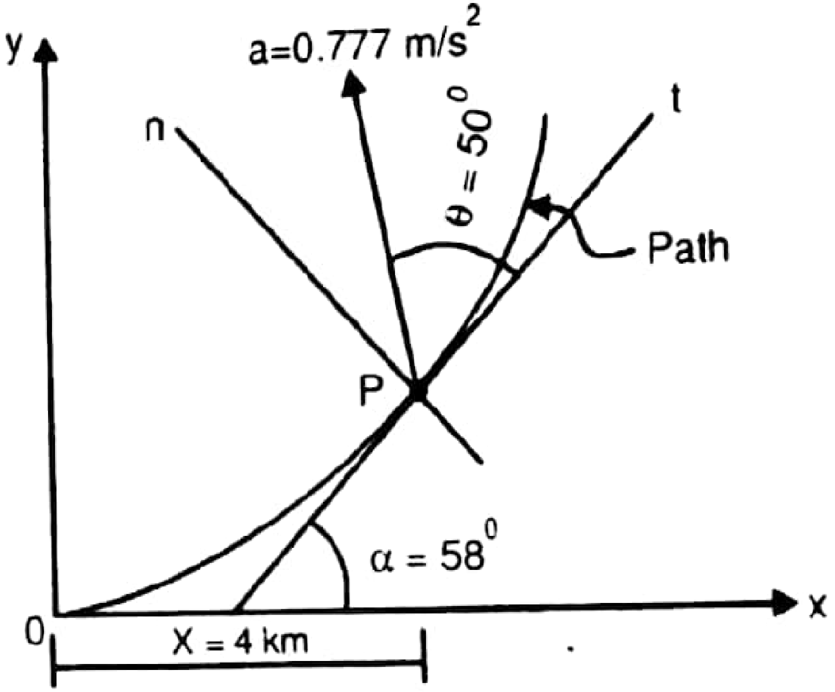
$\therefore$ total acceleration $a = \sqrt{a_n{}^2 + a_t{}^2}$ $\quad \therefore \quad a = 0.777\text{ m/s}^2$ ....**Ans.**

Let θ be the angle made by the acceleration vector with the tangent at x = 4 km.

$$\tan\theta = \frac{a_n}{a_t} = \frac{0.595}{0.5} \qquad \therefore \quad \theta = 50^\circ$$

Let α be the angle made by the tangent with the x axis then

$$\tan\alpha = \frac{dy}{dx} \qquad \therefore \quad \tan\alpha = 1.6 \quad \therefore \quad \alpha = 58^\circ$$



The total angle made by the acceleration vector with the positive x axis

$= 50 + 58 = 108^\circ$ .... **Ans.**

Problem

A particle moves along a hyperbolic path $\frac{x^2}{16} - y^2 = 28$. If the x component of velocity V_x is 4 m/s, and remains constant, determine the magnitude of its velocity and acceleration when it is at point (32m, 6m).

Problem

The position of the charged particle moving in a horizontal plane is measured electronically. This information is fed into a computer, which employees a curve fitting techniques to generate analytical expression for its position given by $\bar{r} = (t^3)i + (t^4)j$, where $\bar{r}$ is in meters and t is in seconds. For t = 1 seconds, determine,

- The acceleration of the particle in rectangular components
- Its normal and tangential acceleration,
- The radius of curvature of the path

Problem

A particle moving in x-y plane and its position is defined by,

$\bar{r} = \left(\frac{3}{2}t^2\right)i + \left(\frac{2}{3}t^3\right)j$. Find radius of curvature when t = 2 seconds.

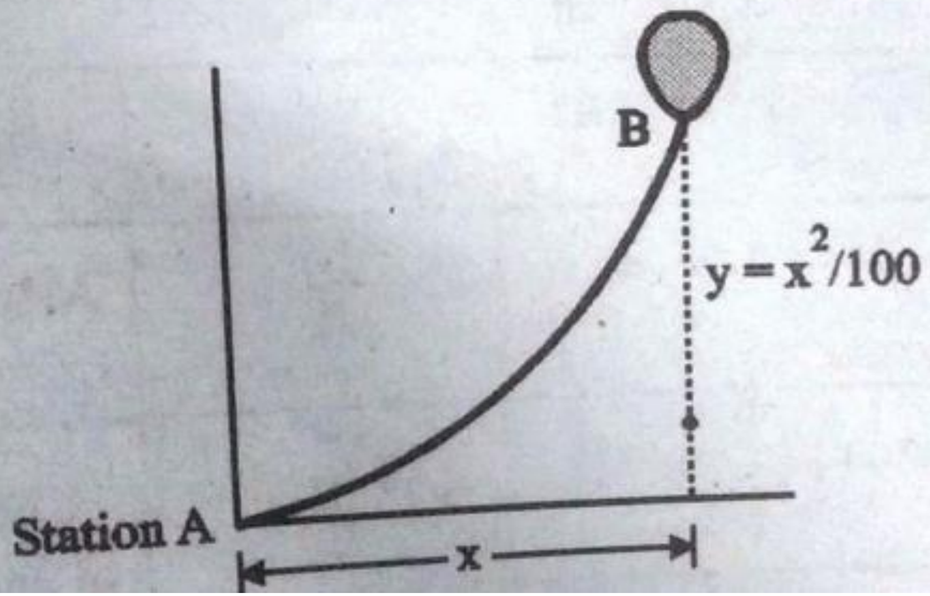
Problem

At any instant the horizontal position of a balloon is defined by $x = 30t$. If path equation is $y = \frac{x^2}{100}$ determine,

a) The distance of the balloon from the station at A when $t = 2$ sec

b) Magnitude and direction of velocity when $t = 2$sec.

c) The magnitude and direction of acceleration when $t = 2$ sec.



Problem

- A car is moving along a curve of radius 300m at a speed 90 kmph. The brakes are suddenly applied, causing speed to decrease at a constant rate of 1.3 m/s^2. Determine the total acceleration,

  a. immediately after brakes have been applied.

  b. after 5 sec.

Problems

- The motion of a particle is defined by $x=4t^2$ and $y=2t^3$ in meters. Determine normal and tangential component acceleration at t=2 sec.
- The velocity of a particle is defined by $v_x = 100 - t^{3/2}$ and $v_y = 100 + 10t - 2t^2$. Determine radius of curvature (1) at the top of its path (2) At t = 12 sec.
- A train enters a curve of radius 800 meters with a speed of 72 kmph. Determine magnitude of total acceleration at the instant the brakes are applied so that train stops by covering a distance of 500 meters along the curve. Also determine the time required by train to come to rest.