Qubits and Quantum Logic Gates

spin ↑
spin ↓

base qubits = states 0 and 1

gr state
excited state

32 bit → 1 word = 0 1 0 1 0 1 - - - - 0 1

left polarization
right - - -

$0 \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ or $|0\rangle$

Dirac notation

state of magnetisation

$1 \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ or $|1\rangle$

ket vector

$|0\rangle \longrightarrow |0\rangle^{\dagger}$ dagger $\rightarrow$ complex conjugate + transpose

$$|0\rangle^{\dagger} = |0\rangle^{*T} = \langle 0|$$

$$\langle 0| = [1 \quad 0]$$

$$|i\rangle = \begin{bmatrix} i \\ 0 \end{bmatrix}$$

$|i\rangle^{\dagger}$ $\quad i \rightarrow -i$

$\downarrow$

$\langle i| \quad [-i \quad 0]$

multiplication of qubits

(i) scalar product $\langle 0|0\rangle = \underset{1\times2}{\begin{bmatrix}1 & 0\end{bmatrix}}\underset{2\times1}{\begin{bmatrix}1\\0\end{bmatrix}} = 1$

number / scalar

HW 1 $\langle 0|1\rangle = ?$

(ii) vector product : $|0\rangle\langle 0| = \underset{2\times1}{\begin{bmatrix}1\\0\end{bmatrix}}\underset{1\times2}{\begin{bmatrix}1 & 0\end{bmatrix}} = \underset{2\times2}{\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}}$

matrix multiplications are non- commutative
(order dependent)

HW 2 $|0\rangle\langle 1| = ?$

(iii) tensor product $|0\rangle|0\rangle = \begin{array}{cc} p & q \\ \underset{2\times1}{\begin{bmatrix}1\\0\end{bmatrix}} & \underset{2\times1}{\begin{bmatrix}1\\0\end{bmatrix}}\end{array} = \underset{4\times1}{\begin{array}{c|c} \begin{bmatrix}1\\0\\0\\0\end{bmatrix} & \begin{array}{l} -\ p_1q_1 \\ -\ p_1q_2 \\ -\ p_2q_1 \\ -\ p_2q_2 \end{array}\end{array}}$

↑ rank of matrix

$|00\rangle$

HW 3 $|0\rangle|1\rangle = ?$ $|0\rangle \oplus |0\rangle$

$$|0\rangle \oplus |0\rangle = |0\rangle|0\rangle = |00\rangle$$

└ 2-qubit state

3-qubit state $|000\rangle$

n-qubit state $|0\rangle^{\oplus n}$

} linear algebra
matrices

Probability of ~~an~~ computation

superposed $|\psi\rangle$ is said to be in superposed state if $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

α, β : arbitrary consts.

what is the prob that $|\psi\rangle$ can be found in state $|0\rangle$ after operation?

$$P = |\langle 0|\psi\rangle|^2 = |\langle 0|(\alpha|0\rangle + \beta|1\rangle)|^2$$

$$= |\langle 0|\alpha|0\rangle + \langle 0|\beta|1\rangle|^2$$

$$= |\alpha\langle 0|0\rangle + \beta\langle 0|1\rangle|^2$$

$\langle 0|0\rangle = 1$

$\langle 0|1\rangle = 0$

$$\Rightarrow P = |\alpha\cdot 1 + \beta\cdot 0|^2$$

$$= |\alpha|^2$$

HW. $|\psi\rangle$ is profound to be $|1\rangle$?

superposed

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \to |+\rangle \qquad \alpha = \beta = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \to |-\rangle$$

Quantum logic gates

Classical logic gates

NOT gate

NOT(0) = 1

NOT(1) = 0

Quantum NOT gate

Pauli-x gate σ_x

$$\sigma_x \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

HW

can u express σ_x in $|\ \rangle$ notation

$|+\rangle$

$|-\rangle$

$$\sigma_x |0\rangle = \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

HW Check $\sigma_x |1\rangle = ?$

↳ $\sigma_x |+\rangle = ?$

Pauli-Z gate $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

HW. $\sigma_z |+\rangle = |-\rangle$

Hadamard gate

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Imp. $\rightarrow$ produces a superposed state

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

H.W. $H|1\rangle = ?$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \frac{1}{\sqrt{2}}\left\{\underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{|0\rangle} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{|1\rangle}\right\}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$$

clan

AND

$0\,0 \rightarrow 0$

$0\,1 \rightarrow 0$

$1\,0 \rightarrow 0$

$1\,1 \rightarrow 1$

$?x,\ ?x \rightarrow \boxed{AND} \rightarrow 0$

$? \rightarrow \boxed{H} \rightarrow 0$

$|+\rangle$

$|-\rangle$

$|0\rangle$

$|1\rangle$

$\rightarrow \boxed{H} \rightarrow |0\rangle$

$|+\rangle$ $\qquad |1\rangle$

$|-\rangle$

check

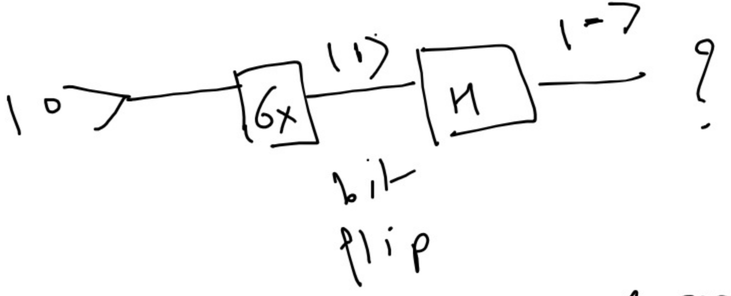
$H|+\rangle = |0\rangle$

H-gate is reversible

Quantum Circuits

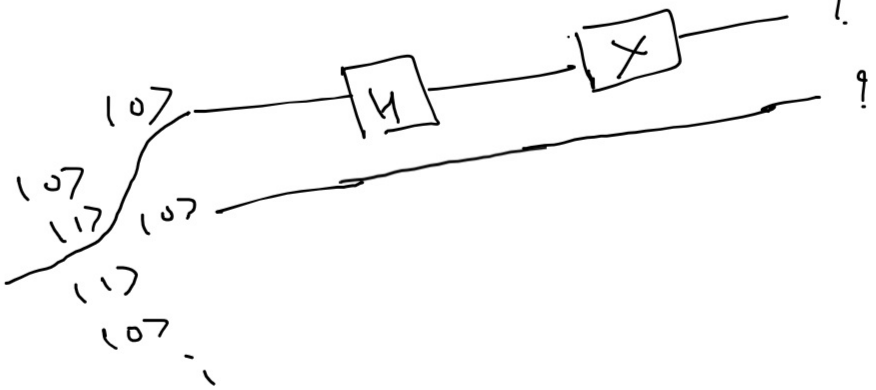
a combination of several logic gates

(i)



Check $|1\rangle$ then o/p ?

(ii)



Controlled NOT gate (2 bit gate)

XOR gate

A	B	O/P
0	0	0
0	1	1
1	0	1
1	1	0

$|00\rangle$

control bit

data bit

$CNOT|00\rangle \longrightarrow$

if control bit is 1 → flip databit

"　　"　　　　0 → do nothing

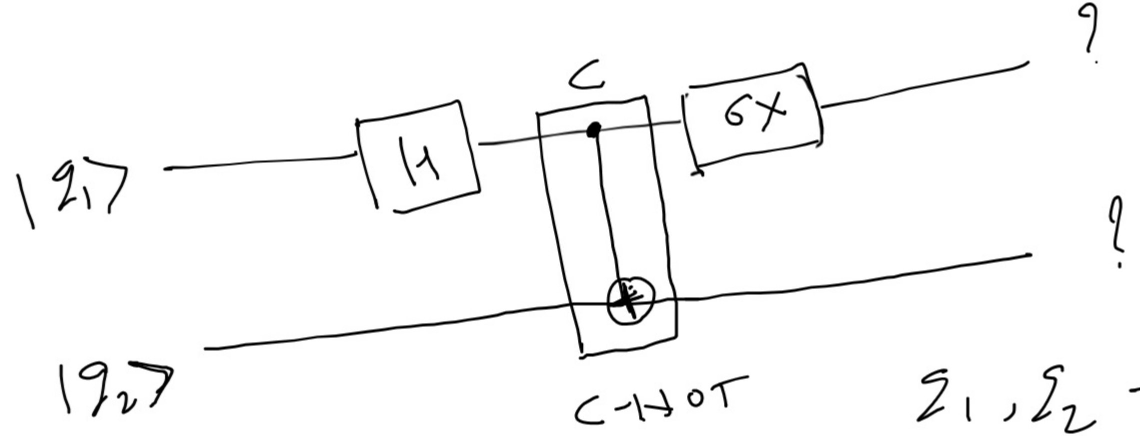
$CNOT|10\rangle \longrightarrow |11\rangle$

$CNOT|11\rangle \rightarrow |10\rangle$

$CNOT|01\rangle \rightarrow |01\rangle$

HW:

find output of following quantum ckt:



$q_1, q_2 \rightarrow$ 00

01

10

11