Engineering Mechanics Notes

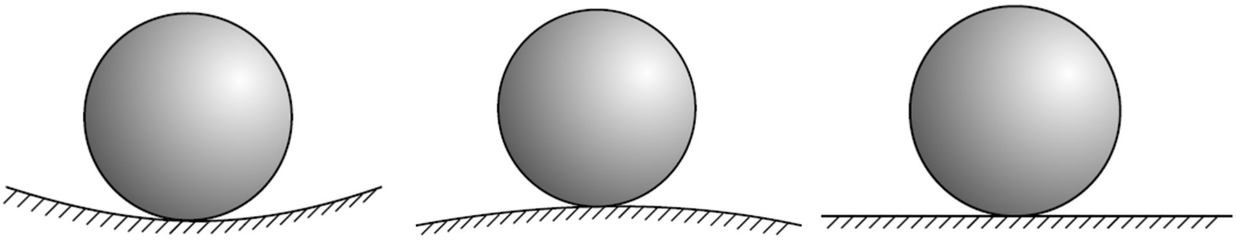
Module 4 – Equilibrium of Force System and Friction

Module Section 4.1 – Equilibrium of Force System

Class: FY BTech Faculty: Aniket S. Patil Date: 09/06/23

References: Engineering Mechanics, by M. D. Dayal & Engineering Mechanics – Statics and Dynamics, by N. H. Dubey.

Equilibrium: A body is said to be in equilibrium when it is in a state of rest or uniform motion. We can say there are three types of equilibria: Stable, Unstable and Neutral, as illustrated by following figures.



Conditions of Equilibrium (COE): From Newton's second law of motion, for a body to be in equilibrium the resultant of the system has to be zero. This implies that the sum of all forces should be zero, i.e., $\sum \bar{F} = 0$; and the sum of all moments should also be zero $\sum \bar{M} = 0$.

For a coplanar system of forces, the COE are: $\sum F_x = 0$, $\sum F_y = 0$ & $\sum M = 0$

COE for Various Force System:

1. Concurrent Force System: One of the following sets of equations can be used
    a. $\sum F_x = 0$ & $\sum F_y = 0$
    b. $\sum F_x = 0$ & $\sum M_A = 0$ (A should not lie on y-axis)
    c. $\sum F_y = 0$ & $\sum M_B = 0$ (B should not lie on x-axis)
2. Parallel Force System: One of the following sets of equations can be used
    a. $\sum F = 0$ & $\sum M = 0$
    b. $\sum M_A = 0$ & $\sum M_B = 0$ (line AB should not be parallel to the forces)
3. General Force System: One of the following sets of equations can be used
    a. $\sum F_x = 0$, $\sum F_y = 0$ & $\sum M = 0$
    b. $\sum F_x = 0$, $\sum M_A = 0$ & $\sum M_B = 0$
    (line AB should not be perpendicular to the x-axis)
    c. $\sum M_A = 0$, $\sum M_B = 0$ & $\sum M_C = 0$ (A, B, & C should not be collinear)

Free Body Diagram (FBD):

The Free Body Diagram (FBD) is a sketch of the body showing all active and reactive forces that acts on it after removing all supports with consideration of geometrical angles and distance given.

To investigate the equilibrium of a body, we remove the supports and replace them by the reactions which they exert on the body. The first step in equilibrium analysis is to identify all the forces that act on the body, which is represented by a free body diagram. Therefore, the free body diagram is the most important step in the solution of problems in mechanics.

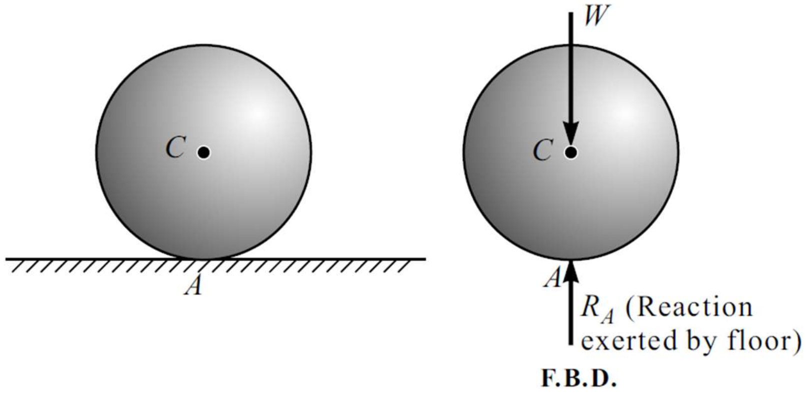
Importance of FBD:

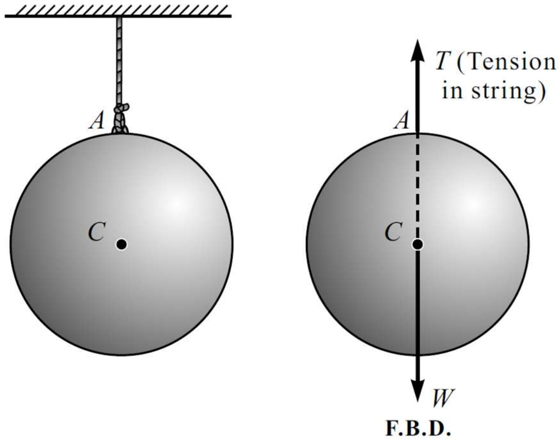
1. The sketch of FBD is the key step that translates a physical problem into a form that can be analysed mathematically.
2. The FBD is the sketch of a body, a portion of a body or two or more connected bodies completely isolated or free from all other bodies, showing the force exerted by all other bodies on the one being considered.
3. FBD represents all active (applied) forces and reactive (reactions) forces. Forces acting on the body that are not provided by the supports are called active force (weight of the body and applied forces). Reactive forces are those that are exerted on a body by the supports to which it is attached.
4. FBD helps in identifying known and unknown forces acting on a body.
5. FBD helps in identifying which type of force system is acting on the body so by applying appropriate condition of equilibrium, the required unknowns are calculated.

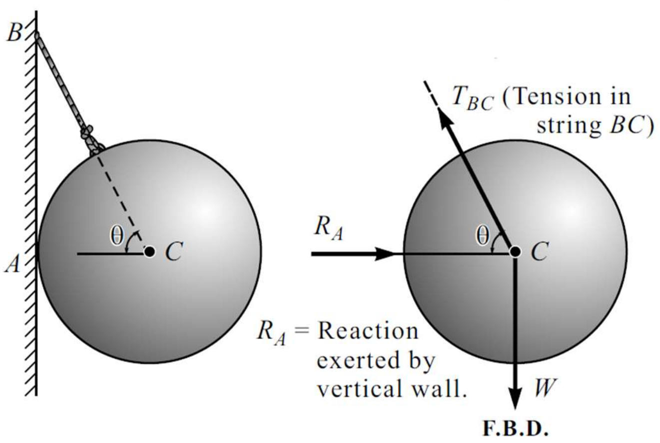
Procedure for Drawing an FBD:

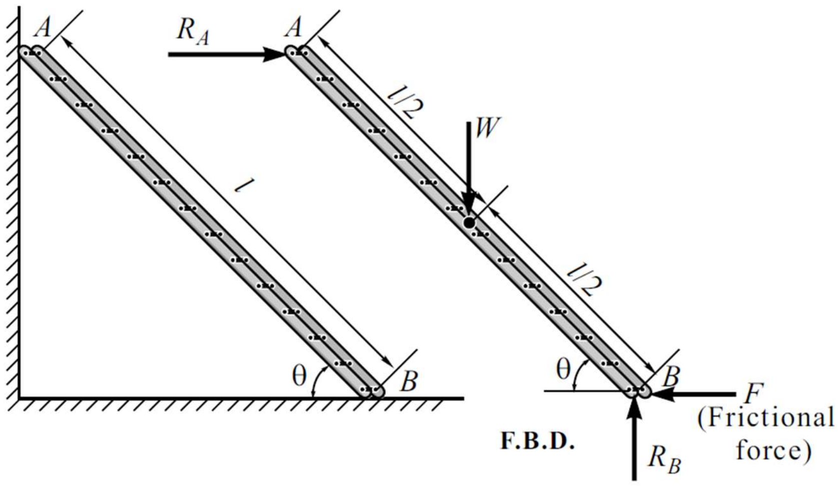
1. Draw a neat sketch of the body assuming that all supports are removed.
2. FBD may consist of an entire assembled structure or any combination or part of it.
3. Show all the relevant dimensions and angles on the sketch.
4. Show all the active forces on corresponding point of application and insert their magnitude and direction, if known.
5. Show all the reactive forces due to each support.
6. The FBD should be legible and neatly drawn, and of sufficient size, to show dimensions, since this may be needed in computation of moments of forces.
7. **If the sense of reaction is unknown, it should be assumed. The solution will determine the correct sense. A positive result indicates that the assumed sense is correct, whereas a negative result means the assumed sense is incorrect, so the correct sense is opposite to the assumed sense.**
8. Use principle of transmissibility wherever convenient.

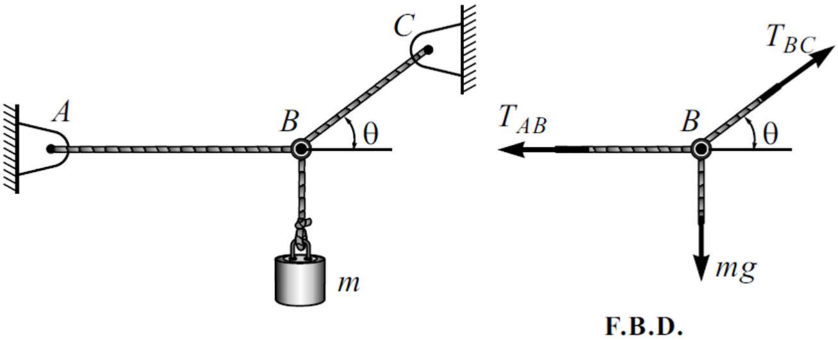
Examples:

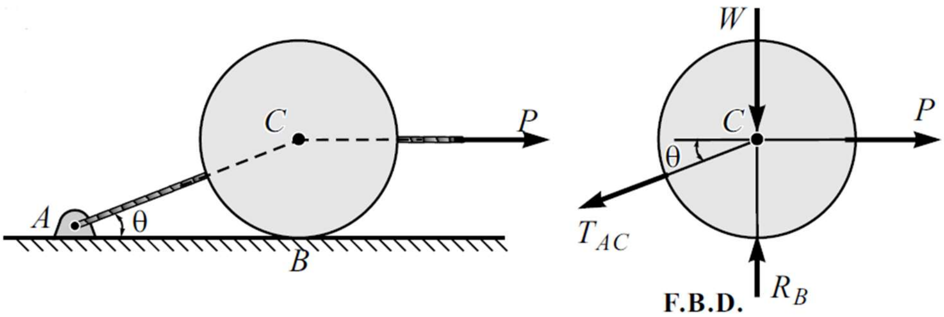

C
A
W
C
A
R_A (Reaction exerted by floor)
F.B.D.


A
C
T (Tension in string)
A
C
W
F.B.D.


B
θ
C
A
T_{BC} (Tension in string BC)
R_A
θ
C
R_A = Reaction exerted by vertical wall.
W
F.B.D.

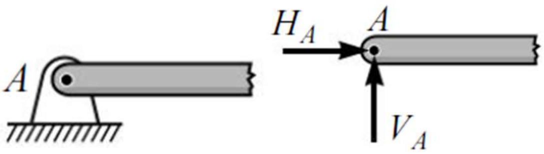

A
l
θ
B
R_A
A
l/2
W
l/2
θ
B
F (Frictional force)
F.B.D.
R_B

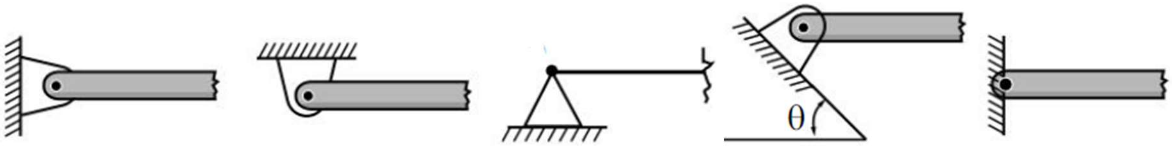




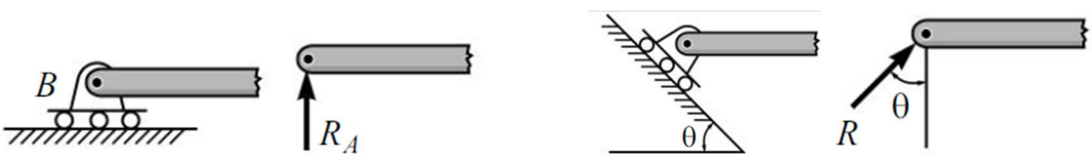
Types of Support:

1. Hinge (Pin) Support: The hinge support allows free rotation about the pin end but it does not allow linear displacement of that end. Since linear displacements are restricted in horizontal and vertical directions, the reactions offered at hinge support are H_A and V_A. E.g., doors on hinges, laptops, suitcases, etc.

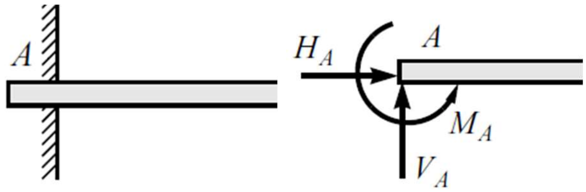




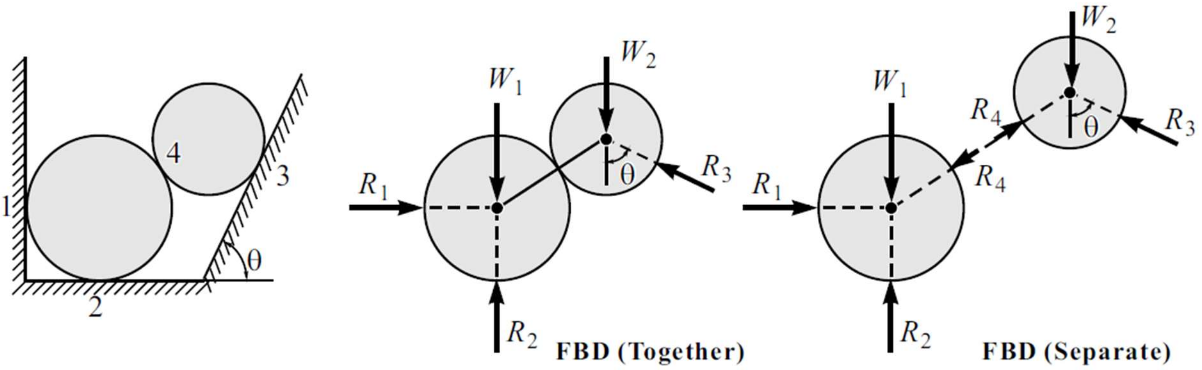
2. Roller Support: A roller support is equivalent to a frictionless surface. It can only exert a force that is perpendicular to the supporting surface. The roller support is free to roll along the surface on which it rests. Since the linear displacement in normal direction to surface of roller is restricted, it offers a reaction in normal direction to surface of roller (R_A). E.g., sliding doors, drawers, etc. Collar or slider free to move along smooth guides are also similar to roller support since they can support force normal to guide only.



3. <u>Fixed Support</u>: When the end of a beam is fixed then that support is said to be fixed support. Fixed support neither allows linear displacement nor rotation of a beam. Due to these restrictions, the reactions offered at fixed supports are horizontal component H_A, vertical component V_A and couple component M_A.



4. <u>Smooth Surface Contact</u>: When a body is in contact with a smooth (frictionless) surface at only one point, the reaction is a force normal to the surface, acting at the point of contact.

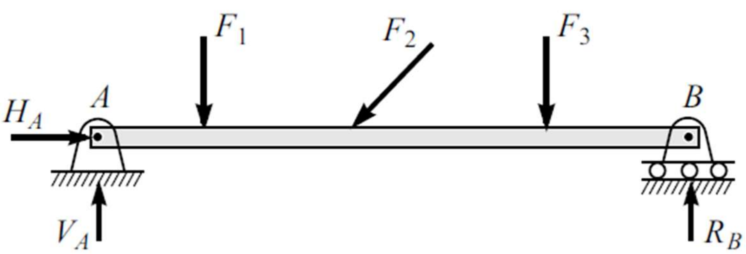


5. <u>Inextensible String, Cable, Belt Rope, Cord, Chain or Wire</u>: The force developed in rope is always a tension away from the body in the direction of rope. When one end of a rope is connected to a body, then the rope is not to be considered as a part of the system and it is replaced by tension in FBD.

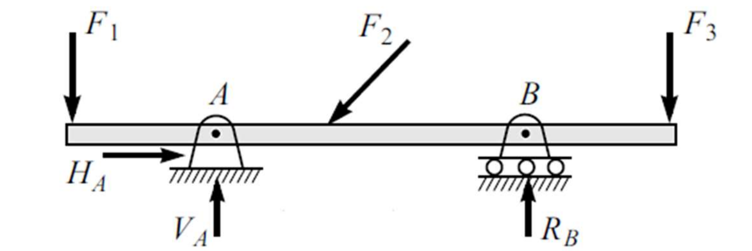
Types of Beams:

A horizontal member which takes transverse load (perpendicular to the length of the member) in addition to other loading is called beam. It is capable to take all types of loads, i.e., transverse load, tensile load, compressive load, twisting load, etc.

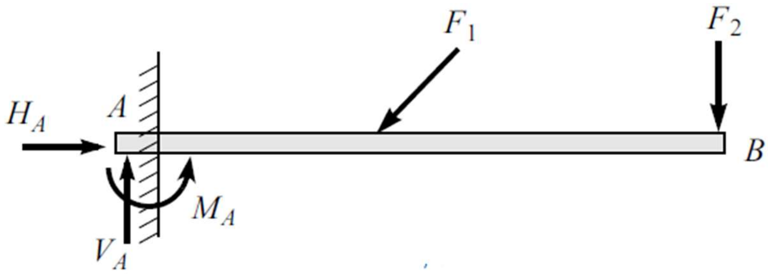
1. <u>Simply Supported Beam</u>: As the name indicates, it is the simplest of all beams which is supported by a hinge at one end and a roller at the other end.



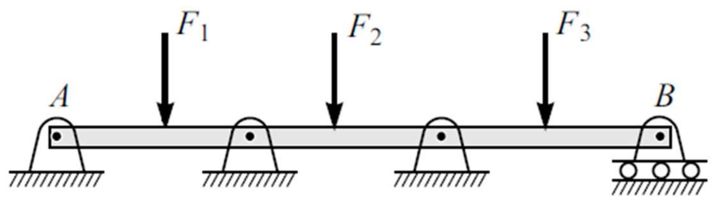
2. Overhang Beam: Here, one end or both the ends of simply supported beam is projected beyond the supports, which means that the portion of beam extends beyond the hinge and roller supports.



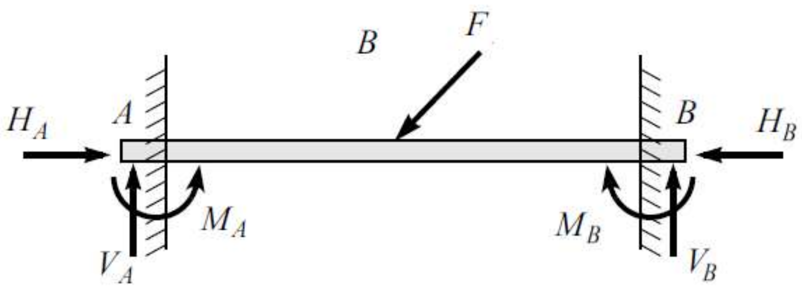
3. Cantilever Beam: A beam which is fixed at one end and free at the other end is called a cantilever beam. E.g., wall bracket, projected balconies, etc. One end of the beam is cast in concrete and is nailed, bolted, riveted, or welded.



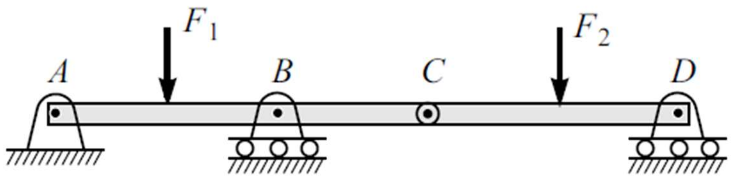
4. Continuous Beam: A beam which has more than two support is said to be a continuous beam. The extreme left and right supports are the end supports of the beam. Such beams are also called statically indeterminate beams because the reactions cannot be obtained by the equation of equilibrium.



5. Fixed Beam: A beam which is fixed at both the ends is called a fixed beam. E.g., supporting column in a building.

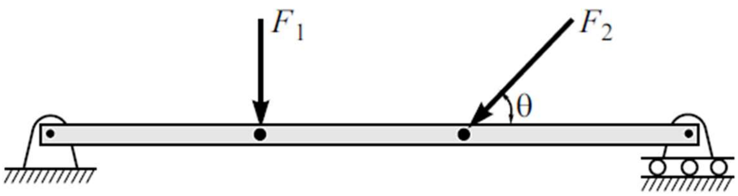


6. Beams Linked with Internal Hinges: Here two or more beams are connected to each other by pin joint and continuous beam is formed. Such joints are called internal hinges. Internal hinges allow us to draw FBD of beam at its joint, if required.



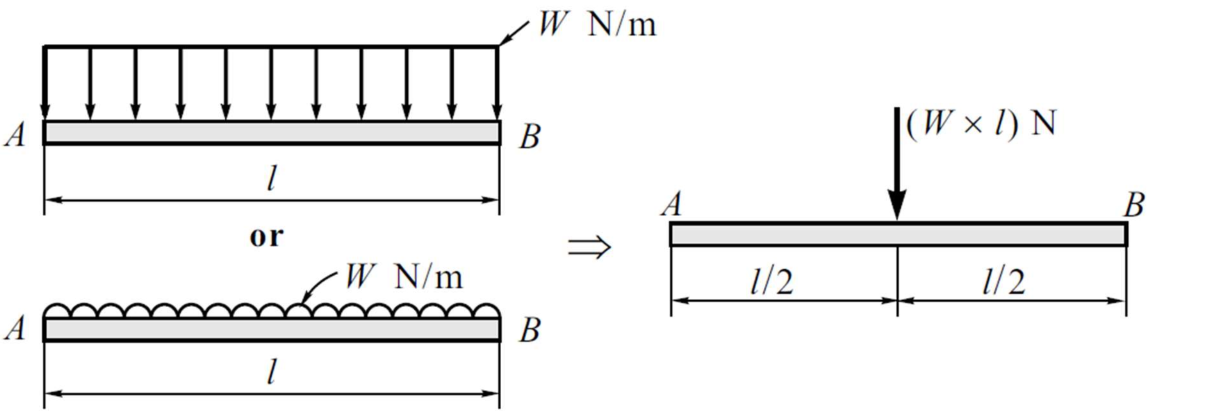
Types of Loads:

1. Point Load: If the whole intensity of load is assumed to be concentrated at a point, then it is known as point load.

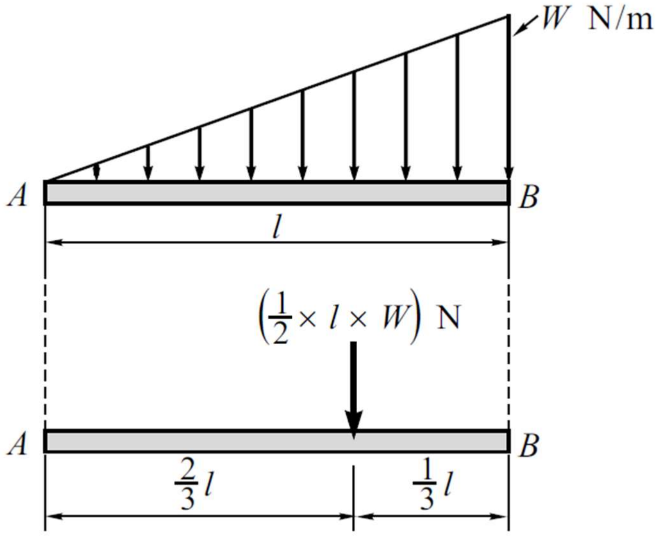


2. Distributed Load: When a load acts throughout the length of a beam or body in varying degree, it can be called as distributed load. This load may consist of the weight of materials supported directly or indirectly by the beam or it may be caused by wind or hydraulic pressure.
   A distributed load on a beam can be replaced by a concentrated point load. The magnitude of this equivalent point load is equal to the area under loading diagram and it acts through the centroid.

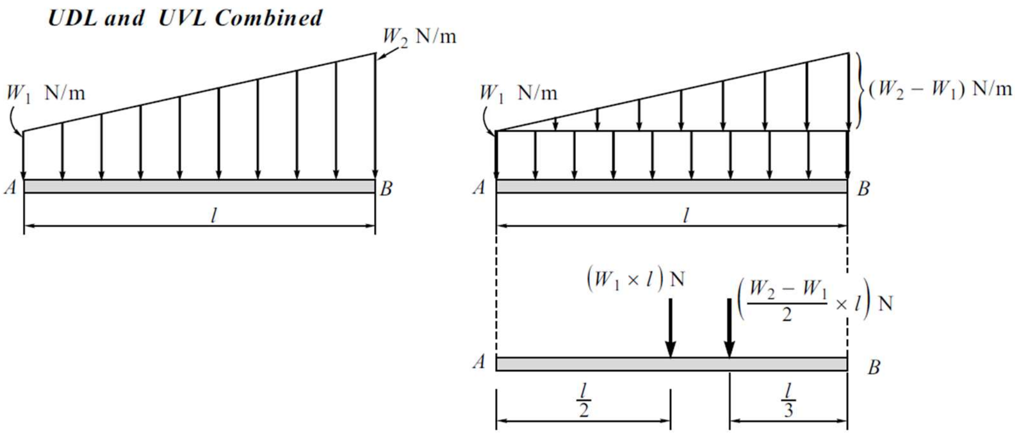
a) Uniformly Distributed Load (UDL): If the whole intensity of load is distributed uniformly along the length of loading, then it is called uniformly distributed load. E.g., weight of a slab of a building flooring.



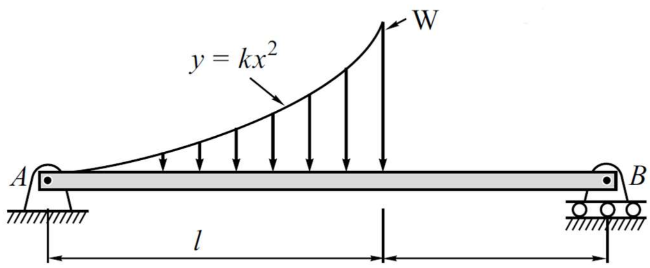
b) Uniformly Varying Load (UVL): If the whole intensity of load is distributed uniformly at varying rate along the length of loading, then, it is known as uniformly varying load. E.g., in a dam the hydraulic pressure varies linearly with the depth.



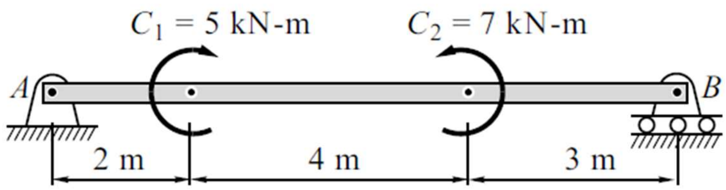
c) Trapezoidal Load (UDL + UVL): If the whole intensity of load is distributed uniformly at varying rate along the length of loading from some lower intensity at on end to a higher intensity at the other end, then, it is known as trapezoidal load.



d) Varying Load: The varying load is given by some relation.



3. Couple: A couple load acting on a body tends to cause rotation of the body. Its location on the body is of no significance because couples are free vectors.

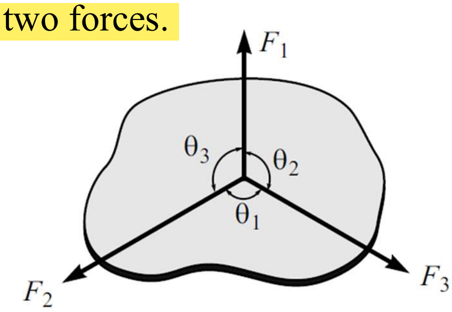


Equilibrium of Two Force System: If a body is in equilibrium and is acted by upon by only two forces, then these forces must be equal in magnitude, opposite in direction and collinear.

Equilibrium of Three Force System: If a body is in equilibrium and is acted by upon by three coplanar forces, then these forces must be form either a concurrent system or parallel system.

Lami's Theorem: If three concurrent coplanar forces acting on a body having same nature (i.e., pulling or pushing) are in equilibrium, then each force is proportional to the sine of angle included between the other two forces.

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

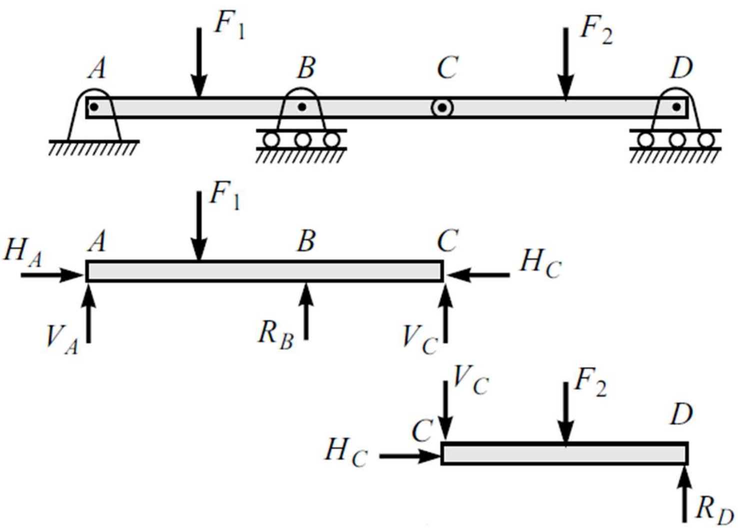


It is applicable to three non-parallel coplanar concurrent forces only. Nature of three forces must be same (i.e., pulling or pushing). If any force is in the opposite sense, then simply placing a negative sign with it, Lami's theorem can be applied.

Equilibrium of Connected Bodies:

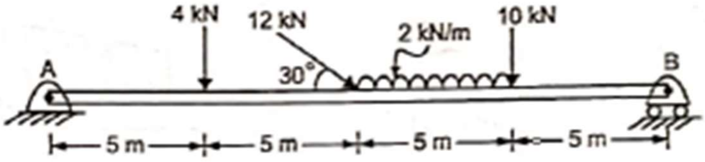
When two or more rigid bodies are connected to each other, they form a system of connected bodies. COE can be applied to the entire system or individual bodies can be isolated from internal connections and COE can be applied to them too. This is useful when the number of unknowns is more than 3.

At the internal connection (hinge, roller, smooth surface, etc.), the reactions of first body on the second are assumed in some direction and the reactions of second body on the first are assumed in the opposite sense, with equal magnitude. The reactions will depend upon the type of connection. (NOTE: Internal hinge is not in syllabus.)

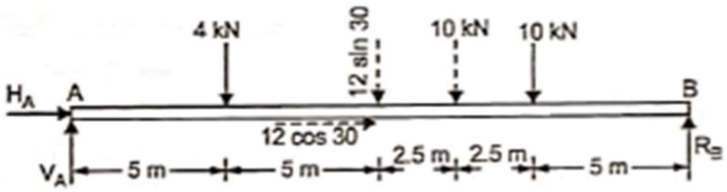


Numericals:

Ex. 3.1 A beam AB is hinged at end A and roller supported at end B. It is acted upon by loads as shown. Find the support reactions.



Solution:



F B D - Beam AB

Figure shows the FBD of the beam AB. Hinge at A gives reaction R_A having components H_A and V_A. Roller at B gives a vertical reaction R_B.

The u.d.l has been converted into a point load of 2 kN/m × 5 m = 10 kN acting at the center of u.d.l. The 12 kN inclined load has been resolved into components.

Applying Conditions of Equilibrium (COE) to the beam AB

$\Sigma M_A = 0$ + ve

$-(4 \times 5) - (12 \sin 30 \times 10) - (10 \times 12.5) - (10 \times 15) + (R_B \times 20) = 0$

$R_B = 17.75$ kN

$R_B = 17.75$ kN ↑ .... **Ans.**

$\Sigma F_X = 0$ → + ve

$H_A + 12 \cos 30 = 0$

$H_A = -10.39$ kN

$H_A = 10.39$ kN ←

$\Sigma F_Y = 0$ ↑ + ve

$V_A - 4 - 12 \sin 30 - 10 - 10 + 17.75 = 0$

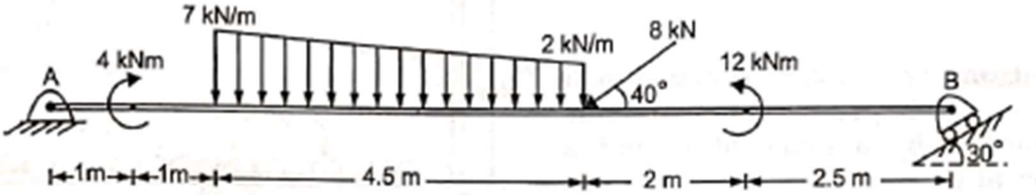
$V_A = 12.25$ kN ↑

Adding vectorially the components H_A and V_A, the reaction

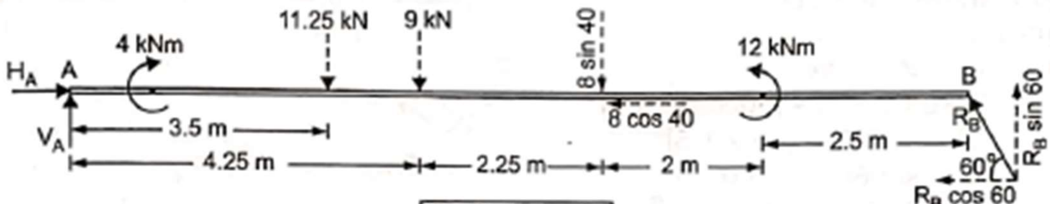
$R_A = 16.06$ kN $\theta = 49.69°$ .... **Ans.**

Note : Hinge reaction answers may also be written as $H_A = 10.39$ kN ←, $V_A = 12.25$ kN ↑

Ex. 3.2 The beam AB is loaded by forces and couples as shown. Find the reaction force offered by the supports to keep the system in equilibrium. ***(VJTI Apr 17)***



Solution:



F B D - Beam AB

Figure shows the FBD of the beam AB. The hinge at A offers reaction R_A having components H_A and V_A.

The roller at B offers reaction R_B normal to the surface on which the roller is supported.

The trapezoidal load has been replaced by two equivalent point loads.

The inclined 8 kN force has been resolved into components.

The beam is loaded with two couples viz. 4 kNm clockwise and 12 kNm anti-clockwise couples.

Applying COE to the beam AB.

$\Sigma M_A = 0$ ↺ + ve

$-4 - (11.25 \times 3.5) - (9 \times 4.25) - (8 \sin 40 \times 6.5) + 12 + (R_B \sin 60 \times 11) = 0$

$R_B = 10.82$ kN

$\therefore$ $R_B = 10.82$ kN $\theta = 60°$ .... **Ans.**

$\Sigma F_X = 0$ → + ve

$H_A - 8 \cos 40 - 10.82 \cos 60 = 0$

$H_A = 11.54$ kN

$H_A = 11.54$ kN → .... **Ans.**

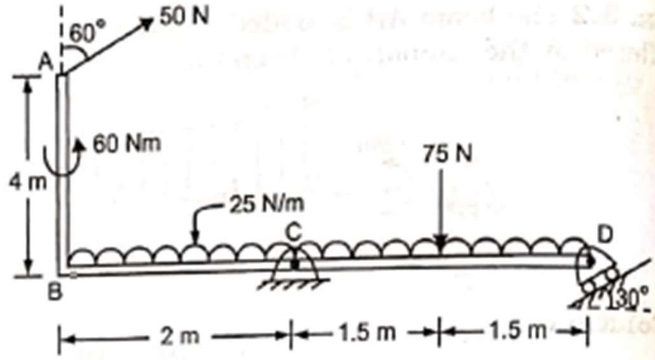
$\Sigma F_Y = 0$ ↑ + ve

$V_A - 11.25 - 9 - 8 \sin 40 + 10.82 \sin 60 = 0$

$V_A = 16.02$ kN

$V_A = 16.02$ kN ↑ .... **Ans.**

Ex. 3.3 A L shaped beam is loaded as shown. Find support reactions.



Solution: The system consists of a single L shaped beam externally supported by a hinge at C and a roller at D.

Figure shows the FBD

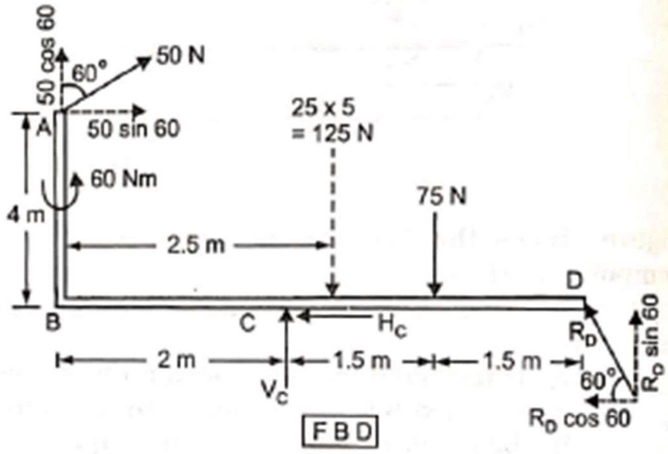
Applying COE

$\Sigma M_C = 0$ ↺ +ve

$+ 60 - (50\sin 60 \times 4) - (50\cos 60 \times 2)$
$- (125 \times 0.5) - (75 \times 1.5)$
$+ (R_D \sin 60 \times 3) = 0$

$\therefore \quad R_D = 130.17$ N, $\theta = 60°$

.... **Ans.**



$\Sigma F_X = 0 \quad \rightarrow +$ve

$50 \sin 60 - H_C - 130.17 \cos 60 = 0$

$\therefore \quad H_C = -21.78$ N

or $\quad H_C = 21.78$ N $\rightarrow$

(– ve value indicates assumption about the sense of unknown force is incorrect) .... **Ans.**

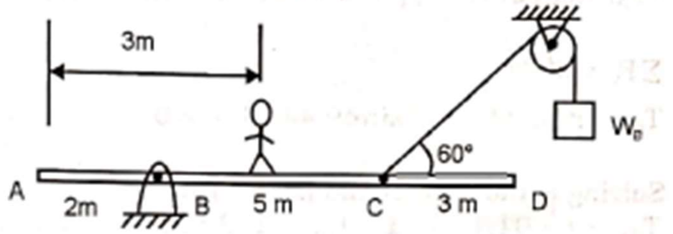
$\Sigma F_Y = 0 \quad \uparrow +$ve

$50 \cos 60 + V_C - 125 - 75 + 130.17 \sin 60 = 0$

$\therefore \quad V_C = 62.27$ N

or $\quad V_C = 62.27$ N $\uparrow$ (+ ve value indicates assumption about the sense of unknown force is correct) .... **Ans.**

Ex. 3.5 A man of 800 N weight stands on a 10 m long uniform beam ABCD of self weight 2000 N. The beam is supported by hinge at B and a rope whose one end is attached at C and the other end carries a counterweight W_B. Find the value of W_B needed to keep the beam in a horizontal position as shown and also the hinge reactions.



Solution: The beam is supported by a hinge at B giving reactions H_B and V_B as shown and a rope. Since the rope passes over a smooth pulley the tension T in it is equal to the counterweight W_B. The weight 2000 N of the beam acts through its centre of gravity G i.e. its mid point.

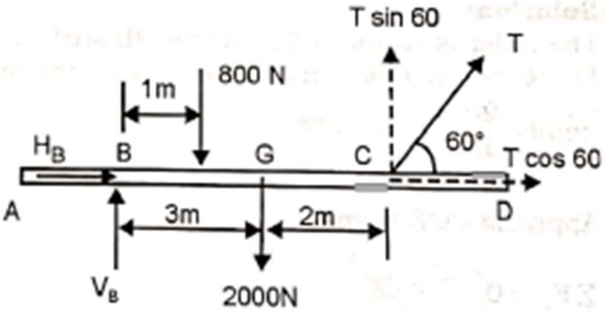
Applying C O E

$\Sigma M_B = 0$ ↺ +ve

$-(800 \times 1) - (2000 \times 3) + (T\sin 60 \times 5) = 0$

$\therefore \quad T = 1570.4$ N

$\therefore \quad W_B = T = 1570.4$ N .... **Ans.**



F B D - Beam AD

$\Sigma F_x = 0 \quad \rightarrow$ +ve

$H_B + T\cos 60 = 0$

$\therefore \quad H_B = -T\cos 60$

$= -1570.4 \cos 60$

$= -785.19$

or $H_B = 785.19$ N ← .... **Ans.**

$\Sigma F_y = 0 \quad \uparrow$ +ve

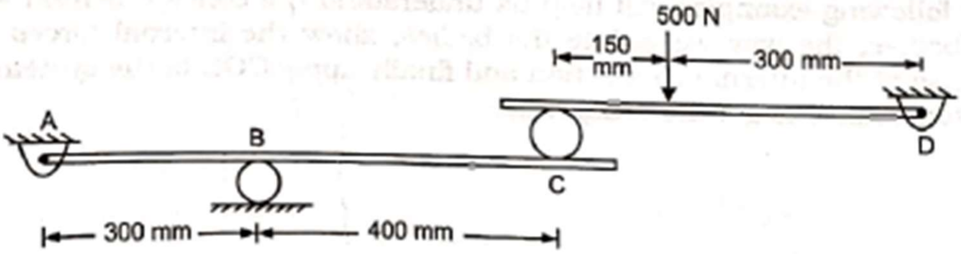
$V_B - 800 - 2000 + T\sin 60 = 0$

$\therefore \quad V_B - 2800 + 1570.4 \sin 60 = 0$

$\therefore \quad V_B = 1450$ N

or $V_B = 1450$ N ↑ .... **Ans.**

Ex. 3.10 For a lever system shown, find the support reactions.

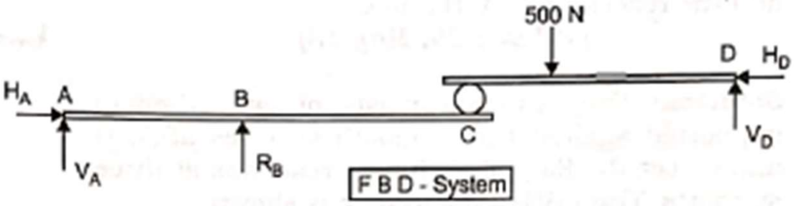


Solution: The system contains two bodies AC and CD. The external supports are;

1) Hinge at A giving reaction R_A. Let H_A and V_A be the components of R_A.
2) Roller at B giving reaction R_B.
3) Hinge at D giving reaction R_D. Let H_D and V_D be the components of R_D

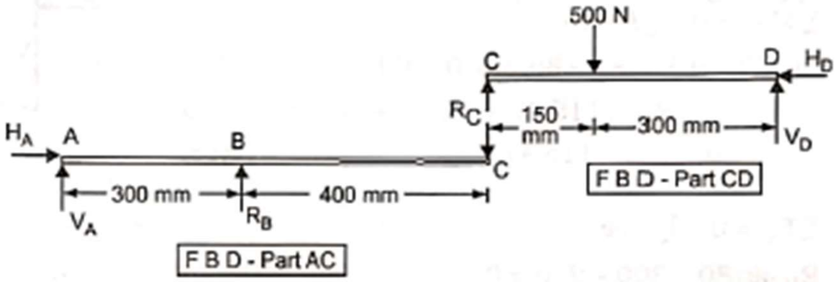
The bodies are internally connected by a roller at C.

Figure shows the FBD of the system of two connected bodies. There are in all five unknowns viz., H_A, V_A, H_D, V_D, and R_B and we have three COE for the system.



We are therefore not in a position to find the unknowns.

Let us therefore isolate the two bodies and apply COE to each of them. Refer figure. Note that the internal force R_C occurs in pair, of same magnitude, collinear and opposite in sense.



Applying COE to body CD.

$M_D = 0$ ↺ + ve

$+ (500 \times 300) - (R_C \times 450) = 0$

$R_C = 333.33$ N $\quad \therefore \quad R_C = 333.33$ N ↑ on body CD.

$\Sigma F_Y = 0$ ↑ + ve

$333.33 - 500 + V_D = 0$

$V_D = 166.67$ N $\quad \therefore \quad V_D = 166.67$ N ↑ .... **Ans.**

$\Sigma F_X = 0 \rightarrow$ + ve

$H_D = 0$ .... **Ans.**

Applying COE to body AC

using $R_C = 333.33$ N ↓ on body AC

$\Sigma M_A = 0$ ↺ + ve

$-(333.33 \times 700) + (R_B \times 300) = 0$

$R_B = 777.7$ N $\quad \therefore \quad R_B = 777.7$ N ↑ .... **Ans.**

$\Sigma F_Y = 0$ ↑ +ve

$V_A + 777.7 - 333.3 = 0$

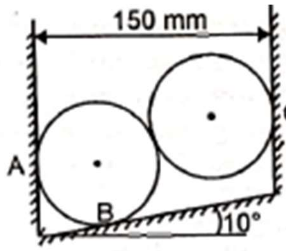
$V_A = -444.4 \quad \therefore \quad V_A = 444.4$ N ↓ .... **Ans.**

$\Sigma F_X = 0 \rightarrow$ + ve

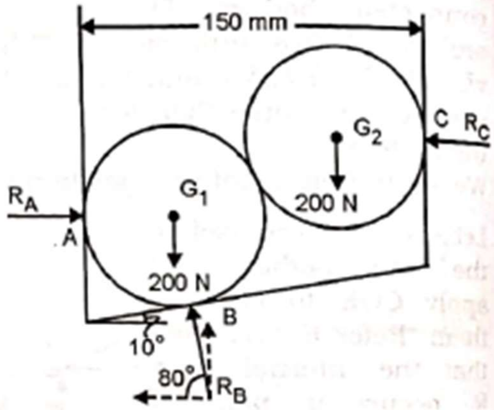
$H_A = 0$ .... **Ans.**

Ex. 3.11 Two cylinders each of diameter 100 mm and each weighing 200 N are placed as shown in figure. Assuming that all the contact surfaces are smooth find the reactions at A, B and C.

(MU Dec 09, May 13)



Solution: The system consists of two cylinders supported against three smooth surfaces at A, B and C. Let R_A, R_B and R_C be the reactions at three supports. The FBD of the system is shown.



FBD - System

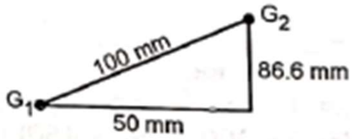
Applying COE to the system

$\Sigma M_{G1} = 0$ ↺ +ve

$$-(200 \times 50) + (R_C \times 86.6) = 0$$

$$\therefore \quad R_C = 115.47\text{ N}$$

or $R_C = 115.47\text{ N} \leftarrow$ .... **Ans.**



$\Sigma F_y = 0 \quad \uparrow +\text{ve}$

$$R_B \sin 80 - 200 - 200 = 0$$

$$\therefore \quad R_B = 406.17\text{ N}$$

or $R_B = 406.17\text{ N}$, $\theta = 80°$ .... **Ans.**

$\Sigma F_x = 0$

$$R_A - R_B \cos 80 - R_C = 0$$

$$R_A - 406.17 \cos 80 - 115.47 = 0$$

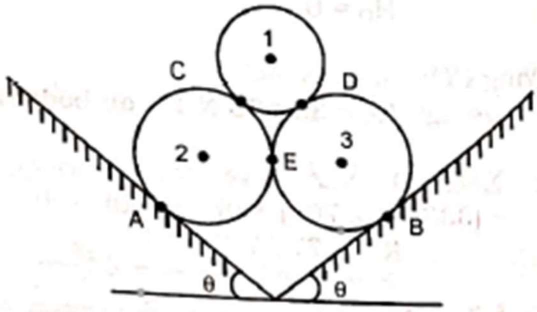
$$\therefore \quad R_A = 186\text{ N}$$

or $R_A = 186\text{ N} \rightarrow$ .... **Ans.**

Ex. 3.12 Three smooth spheres rest against two inclined smooth planes as shown. Determine

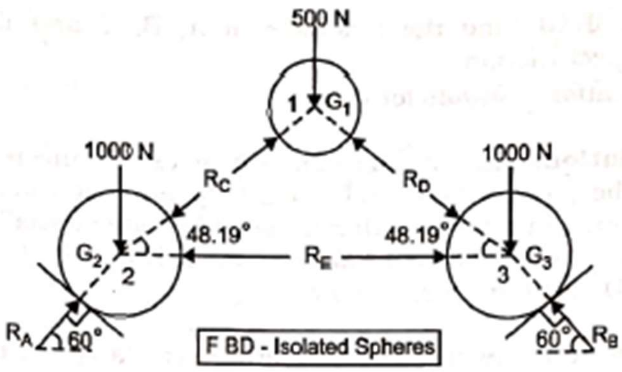
a) The reaction force at contact points when $\theta = 30°$

b) The minimum angle θ for which the spheres remain in equilibrium.
Take for sphere 1 weight = 500 N and radius = 0.2 m
for spheres 2 and 3 weight = 1000 N and radius = 0.4 m



Solution: a) Given: $\theta = 30°$

Let us isolate the bodies as shown in figure. Since the external supports at A and B and internal supports at C, D and E are smooth surfaces, these offer a reaction force normal to the smooth surface.



F BD - Isolated Spheres

Applying COE to sphere 1

$\Sigma F_X = 0 \rightarrow +ve$

$R_C \cos 48.19 - R_D \cos 48.19 = 0$

$\therefore \quad R_C = R_D$ .... (1)

$\Sigma F_Y = 0 \uparrow +ve$

$-500 + R_C \sin 48.19 + R_D \sin 48.19 = 0$ .... (2)

Solving equations (1) and (2) we get, $R_C = R_D = 335.4$ N

Applying COE to sphere 2

$\Sigma F_Y = 0 \uparrow +ve$

$R_A \sin 60 - R_C \sin 48.19 - 1000 = 0$ .... (3)

Substituting $R_C = 335.4$ N, we get, $R_A = 1443.3$ N .... **Ans.**

$\Sigma F_X = 0 \rightarrow +ve$

$R_A \cos 60 - R_C \cos 48.19 - R_E = 0$ .... (4)

Substituting $R_A = 1443.3$ N, and $R_C = 335.4$ N, we get $R_E = 498.1$ N .... **Ans.**

By symmetry of loading and symmetry of supports we can say,

$R_B = R_A = 1443.3$ N .... **Ans.**

b) To find minimum angle θ

As the angle θ is slowly reduced, a stage will be reached when the pyramid of spheres will collapse.

At the minimum angle θ when the system is about to collapse, the reaction at E becomes zero. i.e. $R_E = 0$

By analysis of sphere (1) as done earlier, the reactions $R_C = R_D = 335.4$ N remain the same.

Let us now apply COE and analyse sphere (2) to find the minimum angle θ

$\Sigma F_X = 0 \rightarrow +ve$

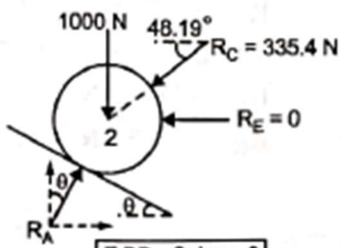
$R_A \sin\theta - 335.4 \cos 48.19 = 0$

$\therefore \quad R_A \sin\theta = 223.6$ .... (5)

$\Sigma F_Y = 0 \uparrow +ve$

$R_A \cos\theta - 335.4 \sin 48.19 - 1000 = 0$

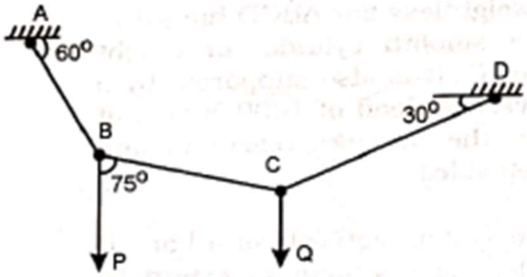
$\therefore \quad R_A \cos\theta = 1250$ .... (6)



F BD - Sphere 2

Solving equations (5) and (6) we get, $\theta = 10.14°$ ... **Ans.**

Ex. 3.15 A string ABCD carries two loads P and Q. If P = 50 kN, find force Q and tensions in different portions of the string.

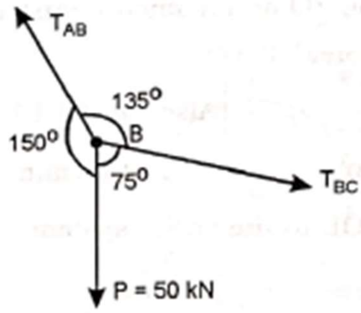


Solution: Isolating joint B of the string. Let T_{AB} and T_{BC} be the tensions in the string portions AB and BC respectively.
Using Lami's equation

$$\frac{T_{AB}}{\sin 75} = \frac{T_{BC}}{\sin 150} = \frac{50}{\sin 135}$$

$\therefore$ $T_{AB} = 68.3$ kN .... **Ans.**

$T_{BC} = 35.35$ kN ....**Ans.**



F B D - Joint B

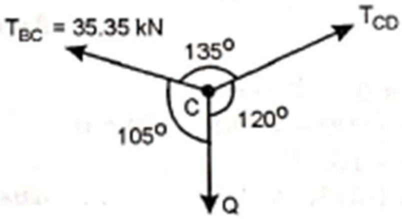
Now isolating joint C.
Let T_{CD} be the tension in portion CD.

Using Lami's equation

$$\frac{35.35}{\sin 120} = \frac{T_{CD}}{\sin 105} = \frac{Q}{\sin 135}$$

$\therefore$ $T_{CD} = 39.43$ kN .... **Ans.**

Q = 28.86 kN .... **Ans.**



F B D - Joint C

We have solved the problem using Lami's equation. As an Exercise solve the problem applying COE.

Problem 7

A roller of weight W = 1000 N rests on a smooth inclined plane. It is kept from rolling down the plane by string AC as shown in Fig. 3.7(a). Find the tension in the string and reaction at the point of contact D.

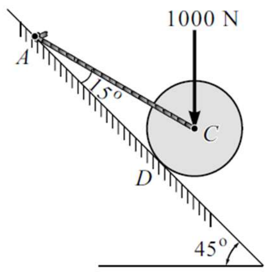


Fig. 3.7(a)

Solution

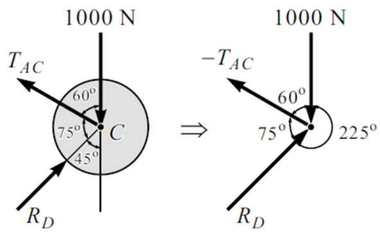
(i) Draw the F.B.D. of the roller.

(ii) By Lami's theorem,

$$\frac{1000}{\sin 75^\circ} = \frac{R_D}{\sin 60^\circ} = \frac{-T_{AC}}{\sin 225^\circ}$$

$\therefore R_D = 896.58$ N $(\angle 45^\circ)$

$\therefore T_{AC} = 732$ N



Problem 8

A cylinder of 50 kg mass is resting on a smooth surface which are inclined at 30° and 60° to horizontal as shown in Fig. 3.8(a). Determine the reaction at contact A and B.

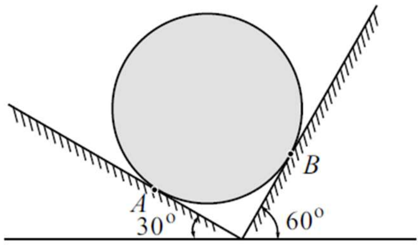


Fig. 3.8(a)

Solution

(i) Consider the F.B.D. of the cylinder.

(ii) By Lami's theorem, we have

$$\frac{50 \times 9.81}{\sin 90^\circ} = \frac{R_A}{\sin 120^\circ} = \frac{R_B}{\sin 150^\circ}$$

$$R_A = 424.79 \text{ N}$$

$$R_B = 245.25 \text{ N}$$

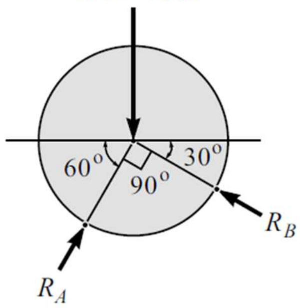


Fig. 3.8(b) : F.B.D. of Cylinder

Problem 15

Two spheres A and B are resting in a smooth through as shown in Fig. 3.15(a). Draw the free body diagrams of A and B showing all the forces acting on them, both in magnitude and direction. Radii of spheres A and B are 250 mm and 200 mm, respectively.

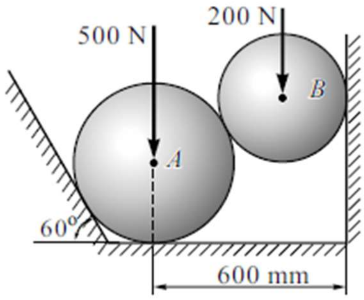


Fig. 3.15(a)

Solution

(i) From Fig. 3.15(b). $AB = 450$ mm and $AC = 400$ mm

$$\cos\theta = \frac{AC}{AB} = \frac{400}{450} \qquad \therefore \ \theta = 27.27^\circ$$

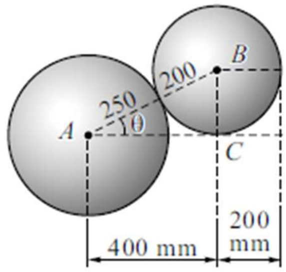


Fig. 3.15(b)

(ii) Consider the F.B.D. of sphere B [Fig. 3.15(c)]

By Lami's theorem,

$$\frac{200}{\sin 152.73^\circ} = \frac{R_1}{\sin 117.27^\circ} = \frac{R_2}{\sin 90^\circ}$$

$\therefore \ R_1 = 388$ N (←) and

$R_2 = 436.51$ N (∠27.27°)

(iii) Consider the F.B.D. of sphere A [Fig. 3.15(d)]

$$\Sigma F_x = 0$$

$$R_4 \cos 30^\circ - 436.51 \cos 27.27^\circ = 0$$

$R_4 = 448$ N (∠30°)

$$\Sigma F_y = 0$$

$$-500 + R_3 - 436.51 \sin 27.27^\circ + 448 \sin 30^\circ = 0$$

$R_3 = 476$ N (↑)

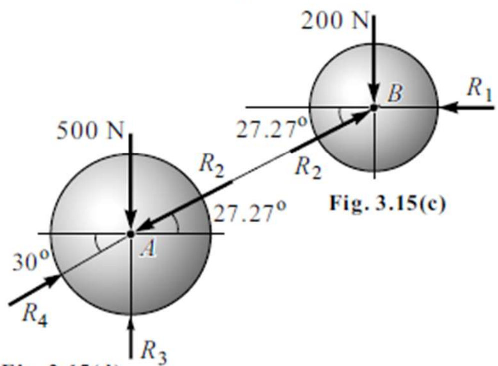


Fig. 3.15(c)

Fig. 3.15(d)

Problem 21

Three cylinders are piled up in a rectangular channel as shown in Fig. 3.21(a). Determine the reactions at point 6 between the cylinder A and the vertical wall of the channel.

(Cylinder A : radius = 4 cm, m = 15 kg,
Cylinder B : radius = 6 cm, m = 40 kg,
Cylinder C : radius = 5 cm, m = 20 kg).

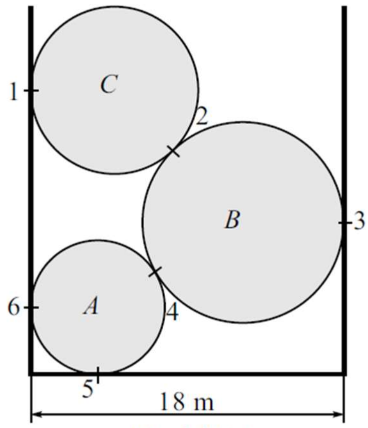


Fig. 3.21(a)

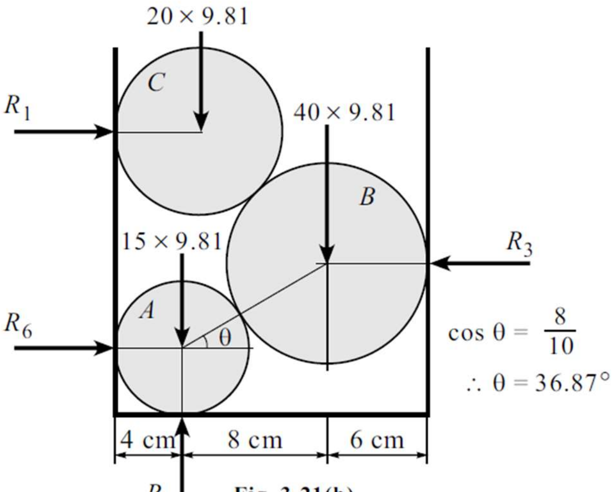


Fig. 3.21(b)

Solution

(i) Consider F.B.D. of entire system as shown in Fig. 3.21(b).

$$\Sigma F_y = 0$$

$$R_5 - 20 \times 9.81 - 40 \times 9.81 - 15 \times 9.81 = 0$$

$$R_5 = 735.75 \text{ N}$$

(ii) Consider the F.B.D. of cylinder A [Refer to Fig. 3.21(c)].

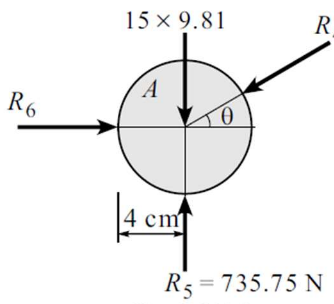


Fig. 3.21(c)

$$\Sigma F_y = 0$$

$$735.75 - 15 \times 9.81 - R_4 \sin 36.87° = 0$$

$$R_4 = 981 \text{ N}$$

$$\Sigma F_x = 0$$

$$R_6 - R_4 \cos 36.87° = 0$$

$$R_6 = 784.8 \text{ N } (\rightarrow)$$

Problem 35

Find the support reactions at A and B for the beam loaded as shown in Fig. 3.35(a).

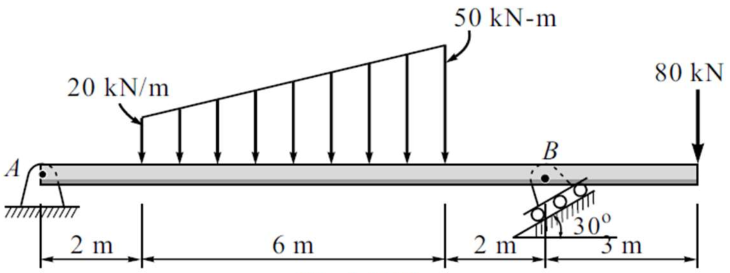


Fig. 3.35(a)

Solution

(i) Consider the F.B.D. of beam AB [Fig. 3.35(b)].

(ii) $\Sigma M_A = 0$

$R_B \sin 60^\circ \times 10 - 120 \times 5 - 90 \times 6 - 80 \times 13 = 0$

$R_B = 251.73$ kN (60°)

(iii) $\Sigma F_x = 0$

$H_A - 251.73 \cos 60^\circ = 0$

$H_A = 125.87$ kN $(\rightarrow)$

(iv) $\Sigma F_y = 0$

$V_A - 120 - 90 + 251.73 \sin 60^\circ - 80 = 0$

$V_A = 72$ kN $(\uparrow)$

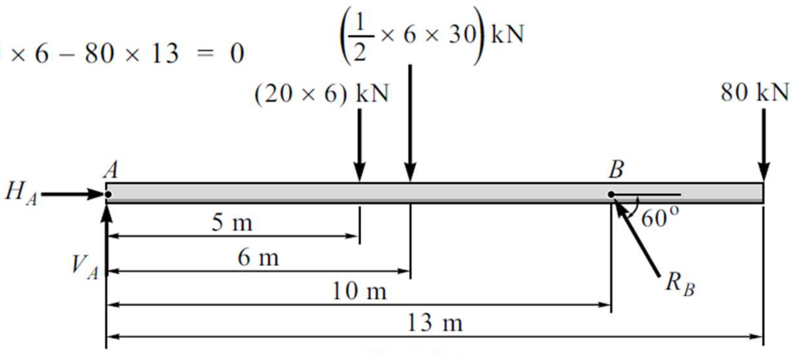


Fig. 3.35(b)

Problem 36

Find analytically the support reaction at B and the load P, for the beam shown in Fig. 3.36(a), if the reaction of support A is zero.

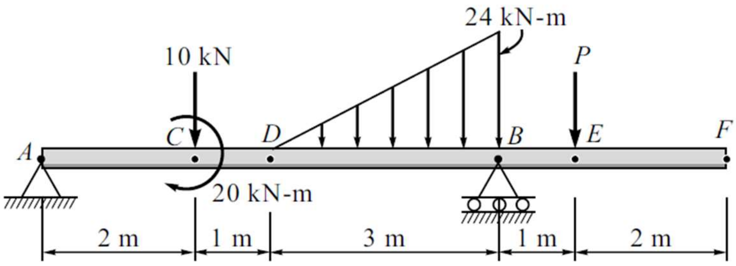


Fig. 3.36(a)

Solution

(i) Consider the F.B.D. of beam AF.

(ii) $\Sigma F_y = 0$

$V_A + R_B - 10 - 36 - P = 0$ ($V_A = 0$ given)

$R_B - P = 46$ ...(I)

(iii) $\Sigma M_A = 0$

$R_B \times 6 - 10 \times 2 - 20 - 36 \times 5 - P \times 7 = 0$

$6 R_B - 7 P = 220$ ...(II)

(iv) Solving Eqs. (I) and (II)

$R_B = 102$ kN $(\uparrow)$

(v) From Eq. (I)

$P = 102 - 46$

$P = 56$ kN $(\downarrow)$

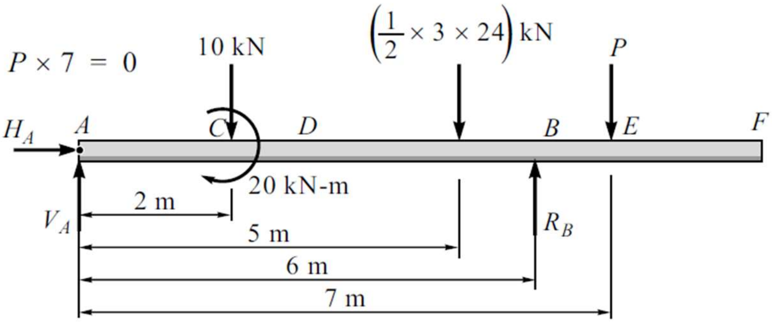


Fig. 3.36(b) : F.B.D of Beam AF

Problem 37

Find the support reactions at A and F for the given Fig. 3.37(a).

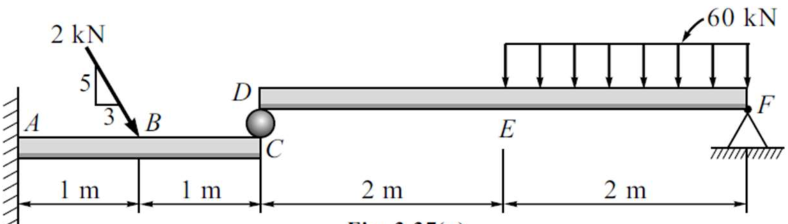


Fig. 3.37(a)

Solution

(i) Consider the F.B.D. of beam DF [Fig. 3.37(b)].

$\Sigma M_F = 0$

$120 \times 1 - R_D \times 4 = 0 \quad \therefore \; R_D = 30 \text{ kN}$

$\Sigma F_x = 0 \qquad \therefore \; H_F = 0$

$\Sigma F_y = 0$

$R_D + V_F - 120 = 0$

$V_F = 90 \text{ kN } (\uparrow)$

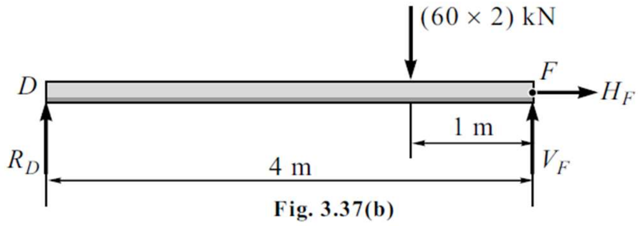


Fig. 3.37(b)

(ii) Consider the F.B.D. of beam AC [Fig. 3.37(c)].

$\Sigma M_A = 0$

$M_A - 2 \sin 59.04^\circ \times 1 - 30 \times 2 = 0$

$M_A = 61.72 \text{ kN-m } (\circlearrowleft)$

$\Sigma F_x = 0$

$2 \cos 59.04^\circ - H_A = 0$

$H_A = 1.03 \text{ kN } (\leftarrow)$

$\Sigma F_y = 0$

$V_A - 2 \sin 59.04^\circ - 30 = 0$

$V_A = 31.72 \text{ kN } (\uparrow)$

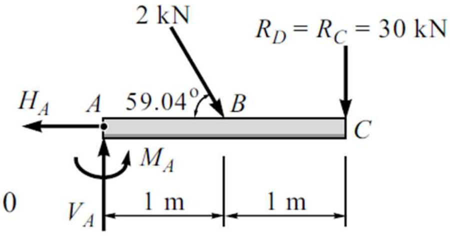


Fig. 3.37(c)

Problem 38

Two beams AB and CD are arranged as shown in Fig. 3.38(a). Find the support reactions at D.

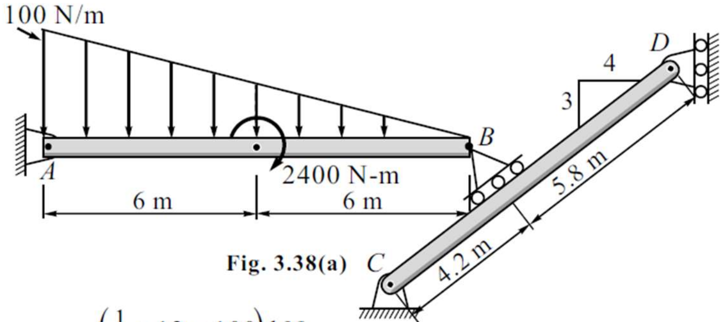


Fig. 3.38(a)

Solution

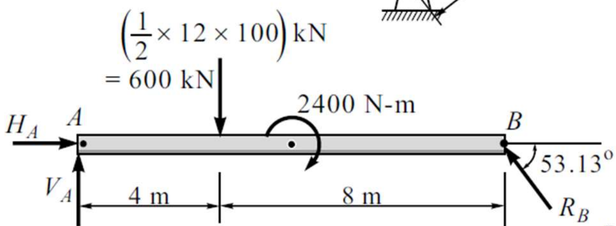


Fig. 3.38(b)

(i) Consider the F.B.D. of beam AB.

$\Sigma M_A = 0$

$R_B \sin 53.13^\circ \times 12 - 600 \times 4 - 2400 = 0$

$R_B = 500 \text{ N}$

(ii) Consider the F.B.D. of beam CD.

$\Sigma M_C = 0$

$R_D \sin 36.87^\circ \times 10 - 500 \times 4.2 = 0$

$R_D = 350 \text{ N } (\leftarrow)$

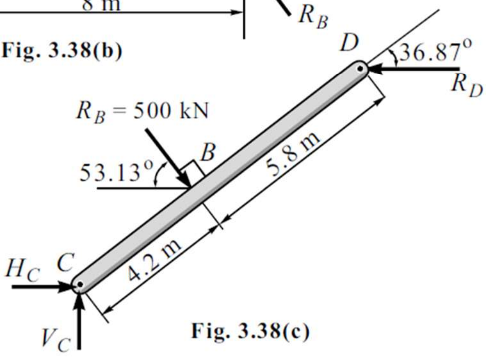


Fig. 3.38(c)