2.1 TEMPERATURE AND DENSITY OF PLASMA

The temperature and the charged particle number density are the two important parameters of a plasma. Behaviour of a plasma can be distinguished by these two characteristic parameters. These parameters vary over wide ranges for naturally occurring and man-made plasmas. In conventional plasmas with low density and high temperature thermal de Broglie wavelength associated with plasma particles is small compared to average interparticle distance. The wave functions associated with neighbouring particles do not overlap and hence plasma can be assumed to behave classically. So plasma particles in thermal equilibrium may be assumed to follow classical Maxwellian distribution.

Concept of Temperature

A classical plasma in thermal equilibrium has particles of all velocities and the velocity distribution of these particles is known to be Maxwellian. In the simplest one-dimensional case the Maxwell's distribution is given by

$$f(u) = A \exp\left(-\frac{1}{2} m u^2 / k_B T\right)$$
 ...(2.1-1)

where f(u)du is the number of particles per unit volume with velocity between uand u+du, $\frac{1}{2}mu^2$ is the kinetic energy and $k_B = 1.38 \times 10^{-23}$ J·K⁻¹ is the Boltzmann constant. The density n, number of particles per unit volume, is given by

$$n = \int_{-\infty}^{\infty} f(u) du \qquad ...(2.1-2)$$

This on integration yields

$$A = n \left(\frac{m}{2\pi k_B T}\right)^{1/2} ...(2.1-3)$$

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The width of the distribution is characterized by the constant T which we call the temperature. In thermal equilibrium average kinetic energy per particle is

$$E_{av} = \frac{\int_{-\infty}^{\infty} \frac{1}{2} m u^2 f(u) du}{\int_{-\infty}^{+\infty} f(u) du} ...(2.1-4)$$

Using (2.1-1) it can be shown by simple integration that

$$E_{av} = \frac{1}{2}k_B T ...(2.1-5)$$

Generalizing the above results to three-dimensions one can get

$$A = n \left(\frac{m}{2\pi k_B T}\right)^{3/2} \text{ and } E_{av} = \frac{3}{2} k_B T$$
 ...(2.1-6)

Thus the temperature T may be considered as a measure of the mean kinetic energy of the random thermal motion of particles. It is customary in plasma physics to give temperature in units of energy (i.e., in electron-volt units). The energy corresponding to k_BT is used for this purpose. Thus the absolute temperature of 1 eV plasma is given by

$$k_B T = 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

 $T = \frac{1.6 \times 10^{-19}}{1.38 \times 10^{-23}} = 11600 \text{ K}$

or,

Thus the conversion formula is

$$1 \text{ eV} = 11600 \text{ K}$$
 ...(2.1-7)

Here it is interesting to point out that each component of a plasma can have temperature of its own which may be different from each other. This is primarily because of the fact that the collision rate among electrons or among ions themselves is larger than the rate of collisions between an electron and an ion. As a result electrons and ions can have separate Maxwellian distribution with different temperatures T_e and T_i . Plasma may not last long for the two temperatures to equalize. In presence of magnetic field even a single species, say electrons, can have two different temperatures. This is so because the force acting on electrons along the magnetic field is different from that acting normal to the magnetic field. As a result the components of velocity parallel and perpendicular to the magnetic field may then belong to different Maxwellian distributions with temperatures T_{\parallel} and T_{\perp} .

Degree of Ionization

A plasma may contain electrons, ions as well as neutral particles. Plasma behaviour is determined by the density of charge particles relative to neutral particles. This

relative density of charged particles in a plasma is described by a parameter called the *degree of ionization*.

The fractional ionization to be expected in a gas in thermal equilibrium is given by the

Saha equation:

$$\frac{n_i}{n_n} \approx 2.4 \times 10^{21} \cdot \frac{T^{3/2}}{n_i} \exp\left(-U_i/k_B T\right)$$
 (2.1-8)

where n_i and n_n are the densities (number per m³) of ion and neutral respectively, T is the temperature in kelvin (K) and U_i is the ionization energy of the gas. The fractional ionization (n_i/n_n) as given by the Eq. (2.1-8) represents a balance between the rate of ionization and the rate of recombination. The rate of ionization increases with temperature T. An ionized atom may recombine with an electron to become a neutral again. Clearly the recombination rate will increase and hence the ion density will decrease with increase in the density of electrons, which we can take as equal to n_i . This explains the appearance of the factor n_i^{-1} on the right-hand side of Eq. (2.1-8). The degree of ionization thus depends not only on the temperature but also on the density of the gas.

For ordinary air at room temperature $k_BT \ll U_i$ and the fractional ionization as given by the Eq. (2.1-8) is extremely low. The fractional ionization becomes significant only when k_BT becomes comparable to U_i . Then n/n_n rises abruptly and the gas is in plasma state. This is why plasma exists in astronomical bodies with very high temperatures. The interstellar plasma medium owes its origin to the low value of n_i and hence low recombination rate.