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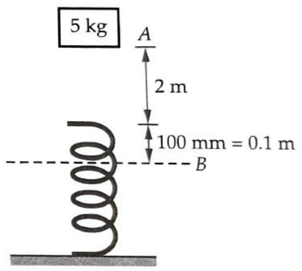
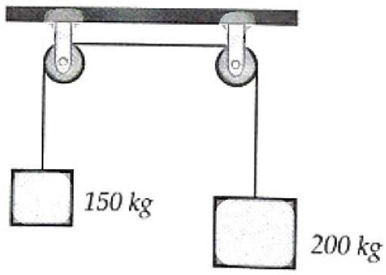
|---|---|---|
| **Maximum Marks: 50** | **Semester: July 2023 – December 2023**<br>**Examination: End-Semester Examination** | **Duration: 2 Hrs.** |

| | | |
|---|---|---|
| **Programme code:** 01<br>**Programme:** B. Tech. | **Class:** FY | **Semester:** I (SVU 2023) |
| **Name of the College:**<br>K. J. Somaiya College of Engineering | **Name of the department:** All | |
| **Course Code:** 216U06C104 | **Name of the Course:** Engineering Mechanics | |
| **Instructions: 1) Draw neat diagrams 2) All questions are compulsory 3) Assume suitable data wherever necessary** | | |

## Solution and Marking scheme

| Que. No. | Question Statement | Max. Marks |
|---|---|---|
| Q.1 | Attempt any **two** | |
| i) | A 5 Kg mass drops from 2 meters upon a spring whose modulus is 10 N/mm. What will be the speed of this mass when the spring is deformed by 100 mm?<br><br>Solution:<br>**<u>Diagram: 01 mark</u>**<br>**<u>Calculation: 04 marks</u>**<br><br>**Solution :** Consider the lowest position $B$ as shown in Fig. as reference for writing the gravitational potential energy. Let $v$ be the speed at $B$.<br>By conservation of energy principle,<br>$E_A = E_B$<br>$(K.E)_A + (G.P.E)_A + (S.P.E)_A = (K.E)_B + (G.P.E)_B + (S.P.E)_B$<br>$k = 10 \text{ N/mm} = 10\times 10^3 \text{ N/m}$<br>$\therefore 0 + 5\times 9.81\times(2+0.1)+0 = \frac{1}{2}\times 5\times v^2 + 0 + \frac{1}{2}\times 10\times 10^3\times 0.1^2$<br>$\therefore \boxed{v = 4.605 \text{ m/s}}$<br> | **05** |

ii) Two masses are connected by a string as shown in the figure below. Neglecting the inertia and friction of both pulleys calculate acceleration $a_1$ of 150 Kg block when the system is released from rest. If a force of (200 x 9.81) N is applied in place of a 200 Kg block what would be the acceleration of 150 Kg.? **05**



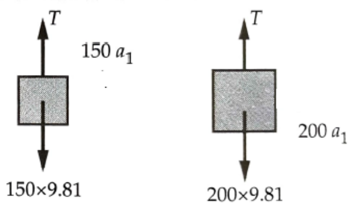

Solution:

**Diagram: 01 mark**

**Computation of $a_1$: 02 marks**

**Computation of $a_1$ when a force of (200 x 9.81) N is applied in place of a 200 Kg block:    02 marks**

**Solution :** Let acceleration of 200 kg block be $a_1 \downarrow$. Then acceleration of 100 kg block will be $a_1 \uparrow$. Let $T$ = Tension in string. The free body diagrams of the two blocks are shown in Fig.




For 150 kg block, $\sum F_y = m\,a_y$:

$$T - 150 \times 9.81 = 150\,a_1$$

For 200 kg block, $\sum F_y = m\,a_y$ :

$$T - 200 \times 9.81 = -200\,a_1$$

From equations

$$\boxed{a_1 = 1.401 \ \mathrm{m/s^2} \uparrow}$$

$$T = 1681.8 \ \mathrm{N}$$

If a force of 200 × 9.81 is applied instead of 200 kg block, the tensile force is

$$T = 200 \times 9.81$$

$$\therefore \quad 200 \times 9.81 - 150 \times 9.81 = 150\,a_1$$

$$\therefore \quad \boxed{a_1 = 3.27 \ \mathrm{m/s^2} \uparrow}$$

When block is attached, tension is less than the weight due to inertia of the block. Hence if a force of 200 × 9.81 is applied instead of block B, we get more acceleration.

iii) Define the term coefficient of restitution. Discuss the different implications for each value of the coefficient of restitution. Write its formula. **05**

Solution:

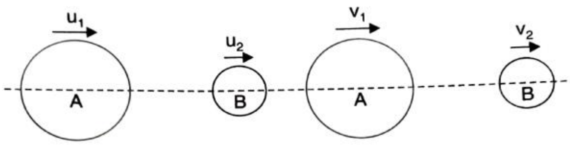
**Definition: 01 mark**

**Discussion for different implications: 03 marks**

**Formula: 01 mark**

It is defined as the ratio of velocity of separation (of the two moving bodies which collides with each other ) to their velocity of approach. It is also defined as the ratio of the relative velocities of colliding bodies after impact to their relative velocity before impact. It is denoted by symbol '$e$'. The relative velocities are measured along the line of impact, which is the common normal to the colliding surfaces.

According to Newton's Law of collision of elastic bodies, "*the velocity of separation, of the two moving bodies which collide with each other, bears a constant ratio to their velocity of approach*". And the constant of proportionality is known as co-efficient of restitution.




(*a*) Before Collision (*b*) After Collision

Let $u_1$ = Velocity of $A$ before collision along $x$-axis

$v_1$ = Velocity of $A$ after collision along $x$-axis

$u_2$ = Velocity of $B$ before collision along $x$-axis

$v_2$ = Velocity of $B$ after collision along $x$-axis.

The body $A$ will collide with body $B$ if velocity of $A$ is more than that of $B$. Hence velocity of approach (or relative velocity of colliding bodies before impact)

= Initial velocity of $A$ – Initial velocity of $B = (u_1 - u_2)$

After collision, the separation of the two bodies will take place if final velocity of $B$ is more than that of $A$.

Hence velocity of separation (or relative velocity of colliding bodies after impact)

= Final velocity of $B$ – Final velocity of $A = (v_2 - v_1)$

Now according to Newton's Law of collision of elastic bodies,

Velocity of separation $\propto$ Velocity of approach

or $(v_2 - v_1) \propto (u_1 - u_2)$

or $(v_2 - v_1) = e \times (u_1 - u_2)$

For most of bodies, the value of $e$ lies between 0 and 1. For perfectly elastic bodies $e = 1$ and for perfectly plastic bodies $e = 0$.

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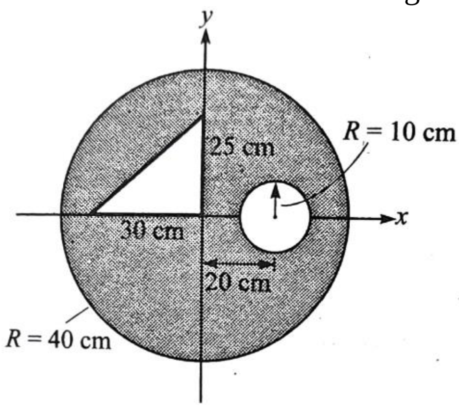
Q.2 Attempt any **one** **10**

i) Find the centroid of the shaded lamina as shown in figure below.




Solution:

**Computation of X: 05 marks**

**Computation of Y: 05 marks**

The shaded lamina consists of a 40-cm circle minus a triangle and a small 10-cm circle as shown

Area of the large circle, $A_1 = \pi \times 40^2 = 5026.5\ \text{cm}^2$

Area of the triangle, $A_2 = -\frac{1}{2} \times 30 \times 25 = -375\ \text{cm}^2$

Area of the small circle, $A_3 = -\pi \times 10^2 = -314.16\ \text{cm}^2$

The centroids of $A_1$, $A_2$ and $A_3$ from the $y$-axis are

$$\bar{x}_1 = 0,\ \bar{x}_2 = -\frac{1}{3} \times 30 = -10\ \text{cm},\ \bar{x}_3 = 20\ \text{cm}$$

The centroid of the shaded area from the $y$-axis,

$$\bar{X} = \frac{A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3}{A_1 + A_2 + A_3} = \frac{5026.5 \times 0 + (-375) \times (-10) + (-314.16 \times 20)}{5026.5 - 375 - 314.16} = -\mathbf{0.584\ cm}$$

The centroids of $A_1$, $A_2$ and $A_3$ from the $x$-axis,

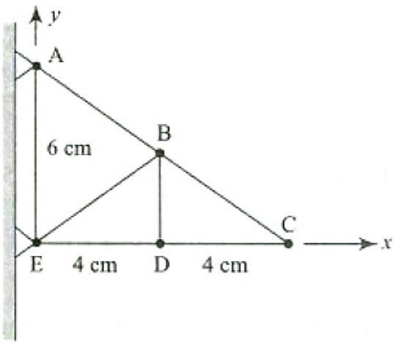
$$\bar{y}_1 = 0,\ \bar{y}_2 = \frac{1}{3} \times 25 = 8.33\ \text{cm},\quad \bar{y}_3 = 0$$

The centroid of the shaded area from the $x$-axis,

$$\bar{Y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3}{A_1 + A_2 + A_3} = \frac{5026.5 \times 0 + (-375 \times 8.33) + (-314.16 \times 0)}{5026.5 - 375 - 314.16} = -\mathbf{0.72\ cm}$$

C(−0.584, −0.72) lies in the third quadrant.

ii) Locate the centroid of the plane truss shown in the figure below. Assume all the bars have the same weight per unit length.
Assume the length of AB = BC = BE = 5 cm.




**Solution:**

<u>**Computation of X: 05 marks**</u>

<u>**Computation of Y: 05 marks**</u>

***Solution*** From geometry,

$$AC = \sqrt{6^2 + 8^2} = 10\ \text{cm}$$

So,

$$AB = BC = 5\ \text{cm} = BE$$

$$BD = 3\ \text{cm}$$

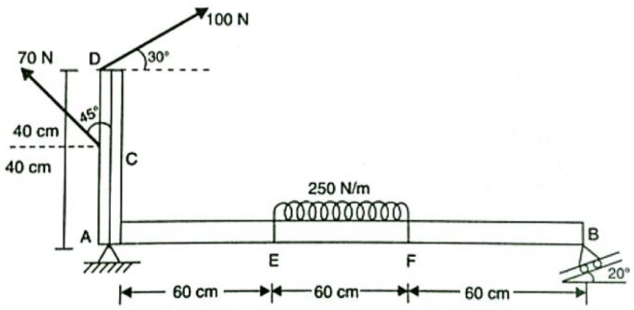
This problem can be easily solved using a tabular method.

| Bar | $L_i$ (cm) | $x_i$ (cm) | $y_i$ (cm) | $L_i x_i$ (cm$^2$) | $L_i y_i$ (cm$^2$) |
|---|---|---|---|---|---|
| AB | 5 | 2 | 4.5 | 10 | 22.5 |
| BC | 5 | 6 | 1.5 | 30 | 7.5 |
| CD | 4 | 6 | 0 | 24 | 0 |
| DE | 4 | 2 | 0 | 8 | 0 |
| BD | 3 | 4 | 1.5 | 12 | 4.5 |
| AE | 6 | 0 | 3 | 0 | 18 |
| BE | 5 | 2 | 1.5 | 10 | 7.5 |
| $\Sigma =$ | 32 | | | 94 | 60 |

Hence, $x_c = \dfrac{\Sigma L_i x_i}{\Sigma L_i} = 2.9375$ cm

and $y_c = \dfrac{\Sigma L_i y_i}{\Sigma L_i} = 1.875$ cm

| Q.3 | Attempt any **one** | **10** |
|---|---|---|
| i) | Find the reactions at the supports of an L bent shown in figure below. | |




Solution:

**FBD: 02 marks**

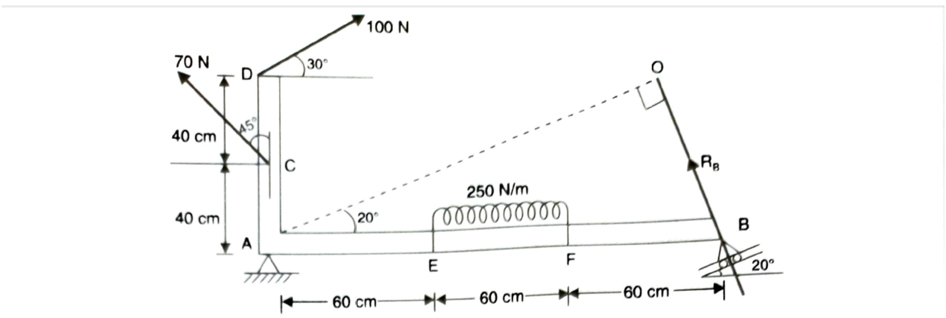
**Reaction at B: 02 marks**

**Reaction at A in x direction ($R_{AX}$) :02 marks**

**Reaction at A in y direction ($R_{AY}$) :02 marks**

**Resultant reaction at A ($R_A$) : 01 mark**

**Inclination of resultant resultant reaction at A (θ) :01 mark**



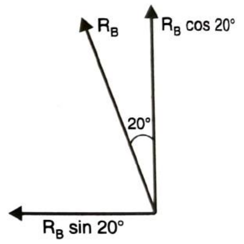

The load on $EF$ will be acting at the middle point of $EF$ *i.e.*, at a distance of $0.6/2 = 0.3$ m from $E$ or at a distance of $0.6 + 0.3 = 0.9$ m from $A$. The point $B$ is placed on roller at an angle of 20° with the horizontal. Hence reaction at $B$ will be normal to the surface of the roller.

The perpendicular distance from $A$ on the line of action of $R_B = AO = AB \cos 20° = 180 \times \cos 20°$ cm $= 1.8 \cos 20°$ m as shown in Fig. For equilibrium of the beam, the moments of all forces about any point should be zero. Taking moments of all forces about point $A$, we get

$$[\text{Horizontal component at } D] \times AD - [\text{Horizontal component at } C] \times AC + \text{Load on } EF \times 90 - R_B \times AO = 0$$

or $$(100 \cos 30) \times 80 - (70 \times \sin 45) \times 40 + 150 \times 90 - R_B \times 180 \cos 20 = 0$$

(**Note.** The vertical components at $D$ and $C$, pass through the point $A$. Hence moments of these vertical components about $A$ are zero).




or $6928 - 1979.6 + 13500 - 169.14\, R_B = 0$

or $18448.4 = 169.14\, R_B$

$$\therefore \quad R_B = \frac{18448.4}{169.14} = 109.07$$

Let $R_A$ = Reaction at the point $A$

The reaction at $A$ can be resolved in two components *i.e.*, $R_{AX}$ and $R_{AY}$.

For equilibrium, $\Sigma F_x = 0$

or $R_{AX} + 100 \cos 30° - 70 \sin 45° - R_B \sin 20 = 0$

or
$$R_{AX} = R_B \sin 20° + 70 \sin 45° - 100 \cos 30°$$
$$= 109.07 \times 0.342 + 70 \times 0.707 - 100 \times 0.866$$
$$= 37.3 + 49.49 - 86.6 = 0.19 \text{ N}$$

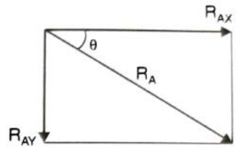
For equilibrium, $\Sigma F_y = 0$

or $R_{AX} + 100 \sin 30° + 70 \cos 45° + R_B \cos 20° = 150$

or
$$R_{AY} = 150 - 100 \sin 30° - 70 \cos 45° - R_B \cos 20°$$
$$= 150 - 50 - 49.49 - 109.07 \times 0.9396 = -51.98 \text{ N}$$

(–ve sign means, $R_{AY}$ will be acting vertically downward

Refer to Fig.




$$\therefore \quad R_A = \sqrt{R_{AX}^2 + R_{AY}^2}$$

or
$$R_A = \sqrt{0.19^2 + (-51.98)^2}$$
$$= \sqrt{0.0361 + 2701.92}$$
$$= \mathbf{51.9803\ kN.} \quad \textbf{Ans.}$$

The angle made by $R_A$ with $x$-axis is given by

$$\tan\theta = \frac{R_{AY}}{R_{AX}} = \frac{51.98}{0.19} = 273.57$$

$$\therefore \quad \theta = \tan^{-1} 273.57 = \mathbf{89.79°}. \quad \textbf{Ans.}$$

ii) A uniform ladder of length 15 meters rests against a vertical wall making an angle of $60^0$ with horizontal. The coefficient of friction between the wall and the ladder is 0.30 and between the ground and ladder is 0.25. A man weighing 500 N ascends the ladder. How long will he be able to go before the ladder slips? Find the weight that is necessary to be put at the bottom of the ladder to be sufficient to permit the man to go to the top. Assume the weight of the ladder to be 850 N.

Solution:

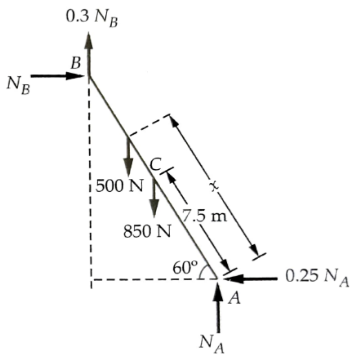
**<u>FBD:02 marks</u>**

**<u>Computation of distance X:03 marks</u>**

**<u>FBD:02 marks</u>**

**<u>Computation of W:03 marks</u>**

**Solution :** The F.B.D. of ladder is shown in Fig.



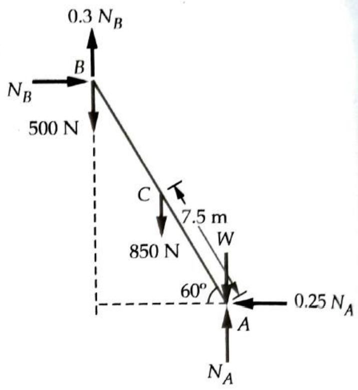

$$\sum F_x = 0$$
$$N_B - 0.25 N_A = 0$$
$$\therefore \quad N_A = 4 N_B$$
$$\sum F_y = 0$$
$$N_A + 0.3 N_B - 850 - 500 = 0$$
$$4 N_B + 0.3 N_B = 1350$$
$$\therefore \quad N_B = 313.95 \text{ N}$$
$$\sum M_A = 0$$

$$(850)(7.5 \cos 60) + (500)(x \cos 60) - (N_B)(15 \sin 60) - (0.3 N_B)(15 \cos 60) = 0$$

Substituting for $N_B$, we get

$$\boxed{x = 6.39 \text{ m}}$$

The F.B.D. of ladder when weight $W$ is put at the bottom is shown in Fig.




$$\sum M_A = 0$$

$$(850)(7.5 \cos 60) + (500)(15 \cos 60) -$$
$$(N_B)(15 \sin 60) - (0.3 N_B)(15 \cos 60) = 0$$

$$\therefore \quad N_B = 455.205 \text{ N}$$

$$\sum F_x = 0$$

$$N_B - 0.25 N_A = 0$$

$$\therefore \quad N_A = 1820.82 \text{ N}$$

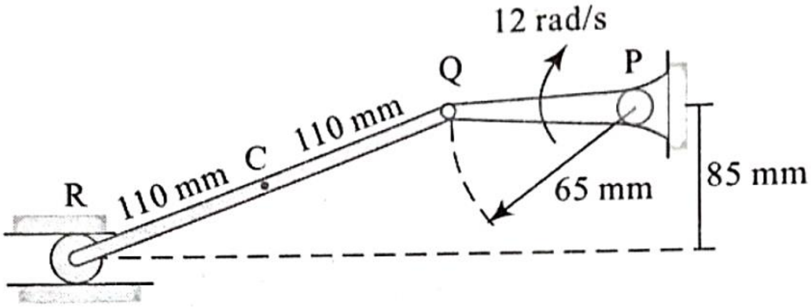
$$\sum F_y = 0$$

$$N_A + 0.3 N_B - 500 - 850 - W = 0$$

$$\therefore \quad \boxed{W = 607.38 \text{ N}}$$

| | | |
|---|---|---|
| Q.4 | Attempt the following | **10** |

For the instant represented in the figure below, when the crank PQ passes the horizontal position determine the velocity of the center C of the link QR by the method of instantaneous center.



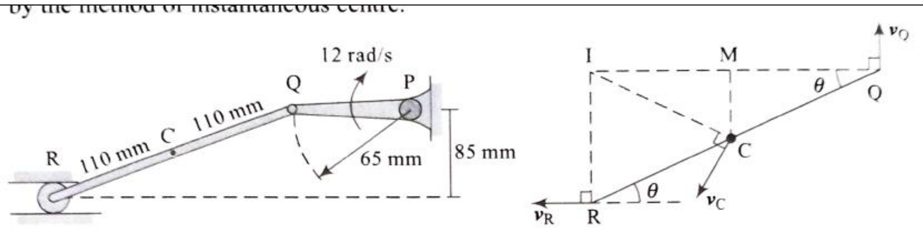

Solution:

**<u>Diagram</u>: 04 marks**

**<u>Calculations</u>: $\omega_{RQ}$ 03 marks**

**$v_c$ 03 marks**

by the method of instantaneous centre.




(a) (b)

***Solution*** Draw the velocity diagram considering instantaneous centre of rotation I [Fig.

From geometry, $\theta = \sin^{-1}\left(\frac{85}{220}\right) = 22.728°$

Now, $v_Q = PQ \times \omega_{PQ} = 0.065 \times 12 = 0.78$ m/s

Again, $v_Q = \omega_{RQ} \times IQ$

or $\omega_{RQ} = \frac{v_Q}{RQ\cos\theta} = \frac{0.78}{0.22\cos 22.728°} = 3.844$ rad/s

From $\Delta$CIM,

$IM = \frac{1}{2} IQ = \frac{1}{2} RQ \cos 22.728° = 0.1015$ m

and $CM = \frac{1}{2} IR = \frac{0.085}{2} = 0.0425$ m

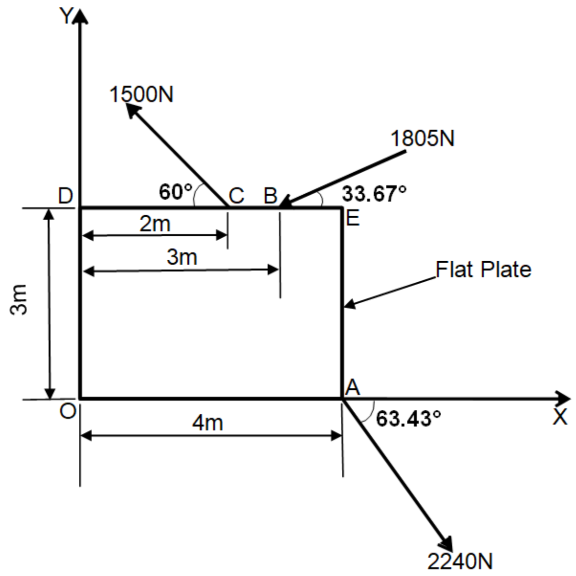
So, $IC = (IM^2 + CM^2)^{1/2} = 0.11$ m

Here, $\omega_C = \omega_{RQ} = 3.844$ rad/s

Hence, $v_C = \omega_C \times IC = 3.844 \times 0.11 = 0.4228$ m/s

Q.5 Attempt the following **10**

The figure shows the coplanar system of forces acting on a flat plate. Determine the resultant and x as well as y-intercept of the resultant.




Solution:

**FBD: 03 marks**

**Computation of Resultant with direction: 03 marks**

**x as well as y-intercept of the resultant with inclination:04 marks**

**Sol.** Given :

Force at $A = 2240$ N

Angle with $x$-axis = 63.43°

Force at $B = 1805$ N

Angle with $x$-axis = 33.67°

Force at $C = 1500$ N

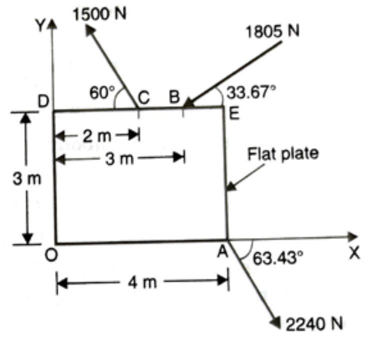
Angle with $x$-axis = 60°

Lengths $OA = 4$ m,

$DB = 3$ m,

$DC = 2$ m

and $OD = 3$ m.




Each force is resolved into $X$ and $Y$ components as shown in Fig.

(*i*) Force at $A = 2240$ N.

Its $X$-component $= 2240 \times \cos 63.43° = 1001.9$ N

Its $Y$-component $= 2240 \times \sin 63.43° = 2003.4$ N

(*ii*) Force at $B = 1805$ N.

Its $X$-component $= 1805 \times \cos 33.67° = 1502.2$ N

Its $Y$-component $= 1805 \times \sin 33.67° = 1000.7$ N

(*iii*) Force at $C = 1500$ N.

Its $X$-component $= 1500 \times \cos 60° = 750$ N

Its $Y$-component $= 1500 \times \sin 60° = 1299$ N

The net force along $X$-axis,

$$R_x = \Sigma F_x = 1001.9 - 1502.2 - 750 = -1250.3 \text{ N}$$

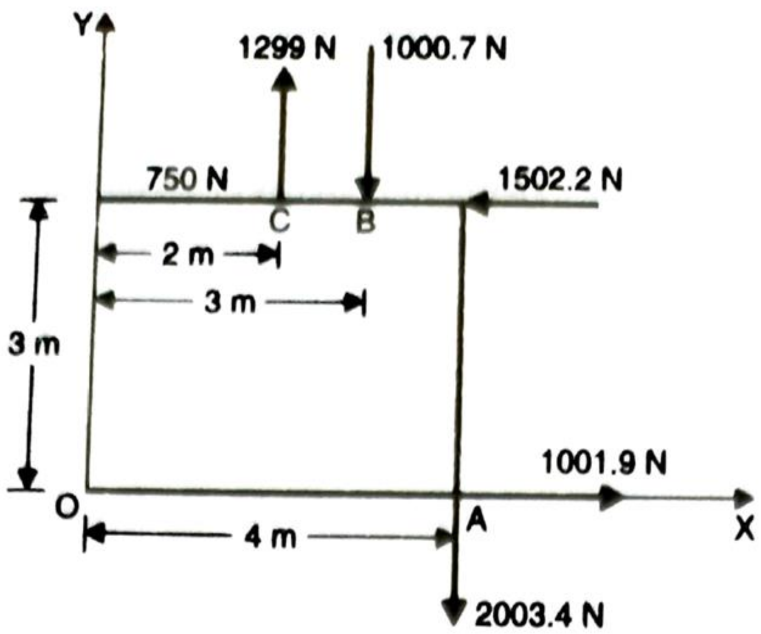
The net force along $Y$-axis,

$$R_y = \Sigma F_y = -2003.4 - 1000.7 + 1299 = -1705.1 \text{ N}$$

(*i*) *The resultant force is given by,*

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-1250.3)^2 + (-1705.1)^2}$$

$$= \sqrt{1563250 + 2907366} = \mathbf{2114.4\ N.} \quad \textbf{Ans.}$$



The angle made by the resultant with $x$-axis is given by

$$\tan\theta = \frac{R_y}{R_x} = \frac{-1705.1}{-1250.3} = 1.363$$

$\therefore$ $$\theta = \tan^{-1} 1.363 = 53.70$$

The net moment* about point $O$,

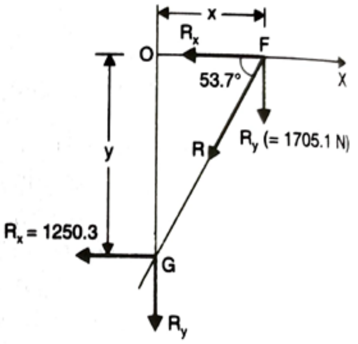
$$M_0 = 2003.4 \times 4 + 1000.7 \times 3 - 1299 \times 2 - 1502.2 \times 3 - 750 \times 3$$
$$= 8012.16 + 3002.1 - 2598 - 4506.6 - 2250$$
$$= 11014.26 - 9354.6 = 1659.55 \text{ Nm}$$

As the net moment about $O$ is clockwise, hence the resultant must act towards right of origin $O$, making an angle = 53.7° with $x$-axis as shown in Fig.

The components $R_x$ and $R_y$ are also negative. Hence this condition is also satisfied.

(*ii*) *Intercepts of resultant on x-axis and y-axis*

[Refer to Fig.




Let $x$ = Intercept of resultant along $x$-axis.

$y$ = Intercept of resultant along $y$-axis.

The moment of a force about a point is equal to the sum of the moments of the components of the force about the same point. Resolving the resultant ($R$) into its component $R_x$ and $R_y$ at $F$.

Moment of $R$ about $O$ = Sum of moments of $R_x$ and $R_y$ at $O$

But moment of $R$ about $O$ = 1659.66 $\quad (M_0 = 1659.66)$

$\therefore$ $$1659.66 = R_x \times O + R_y \times x$$

(as $R_x$ at $F$ passes through $O$ hence it has no moment)

$\therefore$ $$1659.66 = 1705.1 \times x \qquad (\because \; R_y = 1705.1)$$

$\therefore$ $$x = \frac{1659.66}{1705.10} = \textbf{0.97 m right of O.} \quad \textbf{Ans.}$$

To find $y$-intercept, resolve the resultant $R$ at $G$ into its component $R_x$ and $R_y$.

$\therefore$ Moment of $R$ about $O$ = Sum of moments of $R_x$ and $R_y$ at $O$

or $$1659.66 = R_x \times y + R_y \times O.$$

(At $G$, $R_y$ passes through $O$ and hence has no moment)

$\therefore$ $$1659.66 = 1250.3 \times y$$

$\therefore$ $$y = \frac{1659.66}{1250.30} = \textbf{1.32 m below O.}$$