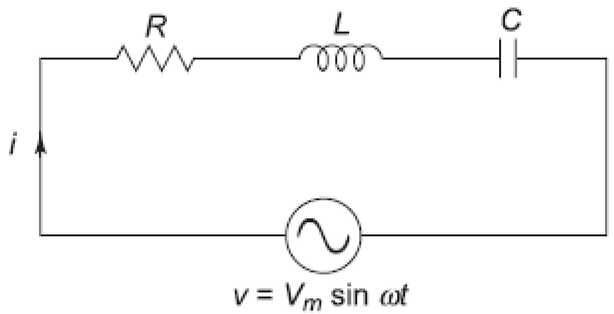
Series Resonance



Series circuit

A circuit containing reactance is said to be in resonance if the voltage across the circuit is in phase with the current through it. At resonance, the circuit thus behaves as a pure resistor and the net reactance is zero.

Consider the series *R-L-C* circuit as shown in Fig. The impedance of the circuit is

$$\bar{Z} = R + jX_L - jX_C$$

$$= R + j\omega L - j\frac{1}{\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Reference: Basic electrical Engineering, By Ravish Singh, Mc Graw Hill Publication

Continue…

At resonance, Z must be resistive. Therefore, the condition for resonance is

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$f = f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the resonant frequency of the circuit.

Power factor:

$$\text{Power factor} = \cos\phi = \frac{R}{Z}$$

$$\text{At resonance} \qquad Z = R$$

$$\text{Power factor} = \frac{R}{R} = 1$$

Current Since impedance is minimum, the current is maximum at resonance. Thus, the circuit accepts more current and as such, an *R-L-C* circuit under resonance is called an *acceptor circuit*.

$$I_0 = \frac{V}{Z} = \frac{V}{R}$$

Voltage At resonance,

$$\omega_0 L = \frac{1}{\omega_0 C}$$

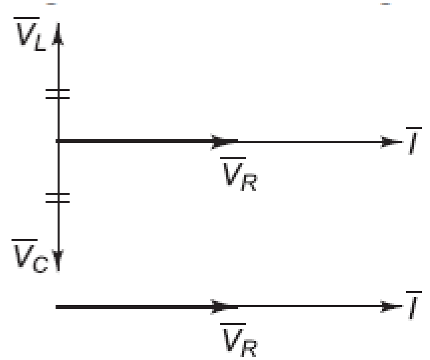
$$\omega_0 L\, I_0 = \frac{1}{\omega_0 C} I_0$$

$$V_{L_0} = V_{C_0}$$

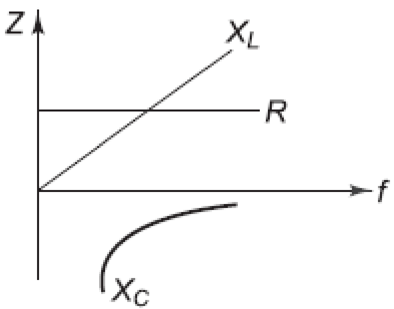
Thus, potential difference across inductor equal to potential difference across capacitor being equal and opposite cancel each other

since I_0 is maximum, V_{L_0} and V_{C_0} will also be maximum
Thus, voltage magnification takes place during resonance. Hence, it is also referred to as voltage magnification circuit.

Phasor Diagram



Behaviour of R, L and C with Change in Frequency



Resistance remains constant with the change in frequencies.

Inductive reactance X_L is directly proportional to frequency f.

It can be drawn as a straight line passing through the origin.

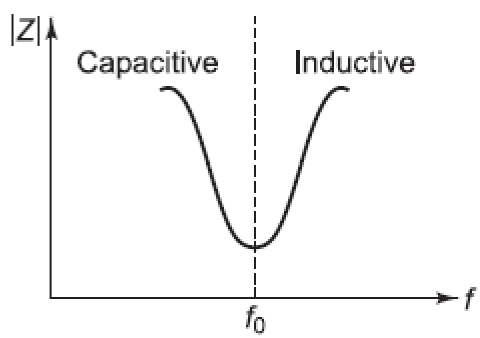
Capacitive reactance X_C is inversely proportional to the frequency f. It can be drawn as a rectangular hyperbola in the fourth quadrant.

Total impedance $\bar{Z} = R + j\,(X_L - X_C)$

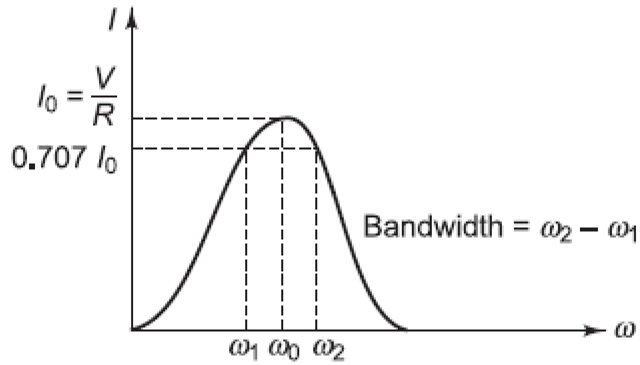
(a) When $f < f_0$, impedance is capacitive and decreases up to f_0. The power factor is leading in nature.

(b) At $f = f_0$, impedance is resistive. The power factor is unity.

(c) When $f > f_0$, impedance is inductive and goes on increasing beyond f_0. The power factor is lagging in nature.



Bandwidth



Bandwidth of a series resonance ckt is defined as the range of frequency over which Ckt current is equal to or greater than 70.7% of maximum current

BW= (f_2-f_1) Hz

OR = $(\omega_2- \omega_1)$ rad / sec

Expression for bandwidth

at any frequency ω,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \qquad \text{..........(1)}$$

At half-power points,

$$I = \frac{I_0}{\sqrt{2}}$$

But $\quad I_0 = \frac{V}{R}$

$$I = \frac{V}{\sqrt{2}R} \qquad \text{................(2)}$$

From equation 1 & 2

$$\frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2}R}$$

Continue……..

$$\frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{2}R}$$

Squaring both the sides,

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} \pm R = 0$$

$$\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC} = 0$$

$$\omega = \pm\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

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For low values of R, the term $\left(\dfrac{R^2}{4L^2}\right)$ can be neglected in comparison with the term $\dfrac{1}{LC}$.

Then ω is given by, $$\omega = \pm\frac{R}{2L} \pm \sqrt{\frac{1}{LC}} = \pm\frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$$

The resonant frequency for this circuit is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega = \pm\frac{R}{2L} + \omega_0$$ (considering only positive sign of ω_0)

$$\omega_1 = \omega_0 - \frac{R}{2L}$$

$$\omega_2 = \omega_0 + \frac{R}{2L}$$

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$$f_1 = f_0 - \frac{R}{4\pi L}$$

$$f_2 = f_0 + \frac{R}{4\pi L}$$

$$\text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L}$$

or $$\text{Bandwidth} = f_2 - f_1 = \frac{R}{2\pi L}$$

Quality Factor It is a measure of voltage magnification in the series resonant circuit. It is also a measure of selectivity or sharpness of the series resonant circuit.

$$Q_0 = \frac{\text{Voltage across inductor or capacitor}}{\text{Voltage at resonance}}$$

$$= \frac{V_{L_0}}{V} = \frac{V_{C_0}}{V}$$

Substituting values of V_{L_0} and V,

$$Q_0 = \frac{I_0 X_{L_0}}{I_0 R}$$

$$= \frac{X_{L_0}}{R}$$

$$= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

Substituting values of ω_0,

$$Q_0 = \frac{\left(\frac{1}{\sqrt{LC}}\right)L}{R}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

Example

A series RLC circuit has the following parameter values: R = 10 Ω, L = 0.014 H, C = 100 μF. Compute the resonant frequency, quality factor, bandwidth, lower cut-off frequency and upper cut-off frequency.

Solution $R = 10\ \Omega$

$L = 0.014\ \text{H}$

$C = 100\ \mu\text{F}$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.014 \times 100 \times 10^{-6}}} = 134.51\ \text{kHz}$$

(ii) Qualify factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.014}{100 \times 10^{-6}}} = 1.18$$

(iii) Bandwidth

$$BW = \frac{R}{2\pi L} = \frac{10}{2\pi \times 0.014} = 113.68\ \text{Hz}$$

(iv) Lower cut-off frequency (f_1)

$$f_1 = f_0 - \frac{BW}{2} = 134.51 - \frac{113.68}{2} = 77.67\ \text{Hz}$$

(v) Upper cut-off frequency (f_2)

$$f_2 = f_0 + \frac{BW}{2} = 134.51 + \frac{113.68}{2} = 191.35\ \text{Hz}$$

Example

An inductor having a resistance of 25 Ω and Q_0 of 10 at a resonant frequency of 10 kHz is fed from a 100 V supply. Calculate (i) value of series capacitance required to produce resonance with the coil, (ii) the inductance of the coil, (iii) Q_0 using L/C ratio, (iv) voltage across capacitor, and (v) voltage across the coil.

Solution

$$R = 25\ \Omega$$
$$Q_0 = 10$$
$$f_0 = 10\ \text{kHz}$$
$$V = 100$$

(i) Value of series capacitance

$$Q_0 = \frac{V_{C0}}{V}$$
$$10 = \frac{V_{C0}}{100}$$
$$V_{C0} = 1000\ \text{V}$$
$$I_0 = \frac{V}{R} = \frac{100}{25} = 4\ \text{A}$$
$$V_{C0} = I_0 X_{C0}$$
$$1000 = 4 X_{C0}$$
$$X_{C0} = 250\ \Omega$$

$$X_{C0} = \frac{1}{2\pi f_0 C}$$
$$250 = \frac{1}{2\pi \times 10 \times 10^3 \times C}$$
$$C = 6.37 \times 10^{-8}\ \text{F}$$

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(ii) Inductance of the coil

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$10 \times 10^3 = \frac{1}{2\pi\sqrt{L \times 6.37 \times 10^{-8}}}$$

$$L = 3.98 \text{ mH}$$

(iii) Quality factor

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{25}\sqrt{\frac{3.98 \times 10^{-3}}{6.37 \times 10^{-8}}} = 10$$

(iv) Voltage across capacitor

$$V_{L0} = V_{C0} = 1000 \text{ V}$$

(v) Voltage across coil

$$Z_{\text{coil}} = \sqrt{R^2 + X_L^2} = \sqrt{(25)^2 + (250)^2} = 251.25\ \Omega$$

$$V_{\text{coil}} = IZ_{\text{coil}} = 4 \times 251.25 = 1005 \text{ V}$$