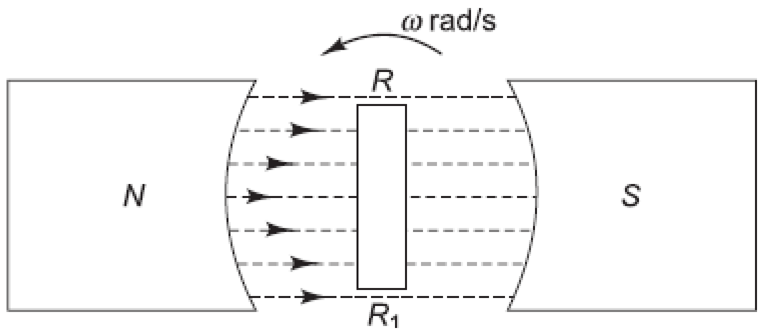
Generation of polyphase voltages

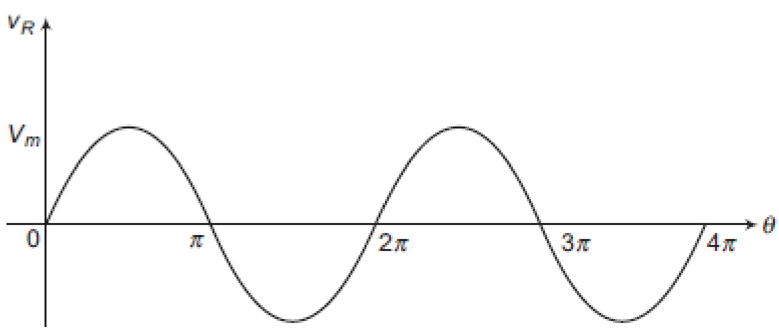
(i) Generation of single-phase voltage



When the winding is rotated in an anticlockwise direction with constant angular velocity ω rad/s in a uniform magnetic field, a voltage is induced in the winding. The equation of the induced voltage in the winding is given by

$$v_R = V_m \sin \theta$$

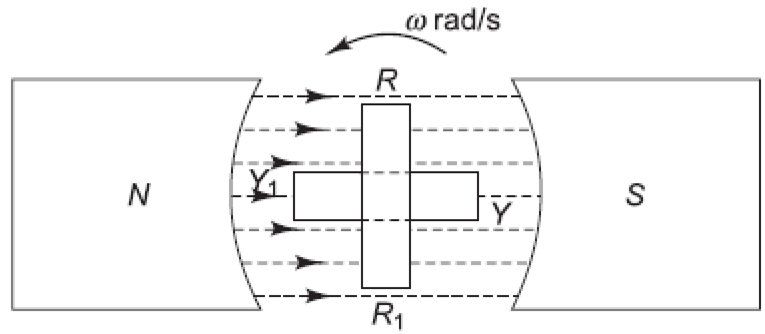
The voltage waveform is shown in Fig.



Phasor diagram

Reference: Basic electrical Engineering, By Ravish Singh, Mc Graw Hill Publication

(ii) Generation of two-phase voltages

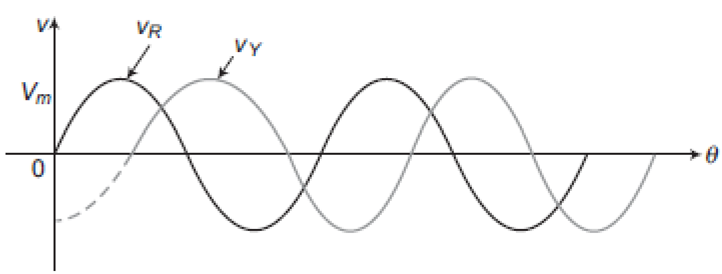


A two-phase system utilizes two identical windings that are displaced by 90 electrical degrees apart from each other. When these two windings are rotated in an anticlockwise direction with constant angular velocity in a uniform magnetic field, the voltages are induced in each winding which have the same magnitude and frequency but are displaced 90 electrical degrees from one another.

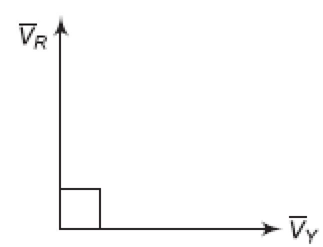
The instantaneous values of induced voltages in two windings RR_1 and YY_1 are given by

$$v_R = V_m \sin\theta$$

$$v_v = V_m\,(\theta - 90°)$$

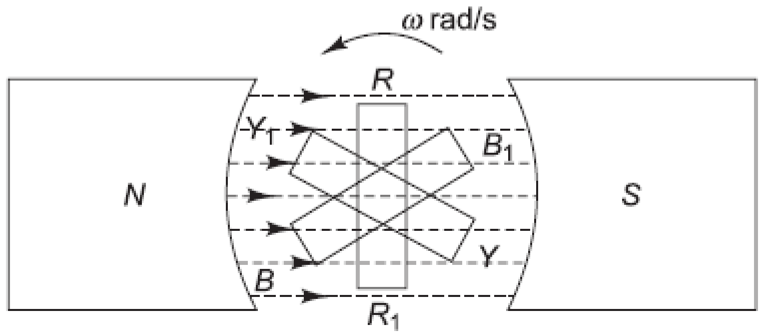


Voltage waveforms



Phasor diagram

(iii) Generation of three-phase voltages



A three-phase system utilizes three separate but identical windings that are displaced by 120 electrical degrees from each other. When these three windings are rotated in an anticlockwise direction with constant angular velocity in a uniform magnetic field, the voltages are induced in each winding which have the same magnitude and frequency but are displaced 120 electrical degrees from one another.

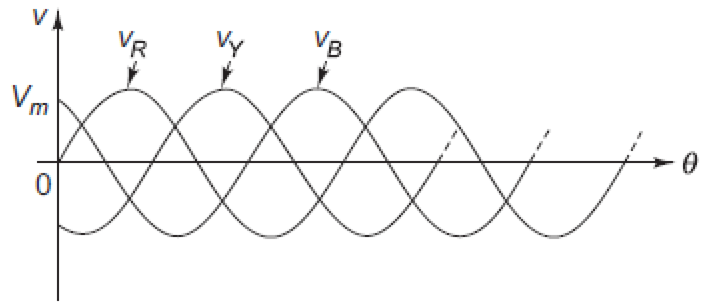
The instantaneous values of induced voltages in three windings RR_1, YY_1 and BB_1 are given by

$$v_R = V_m \sin \theta$$

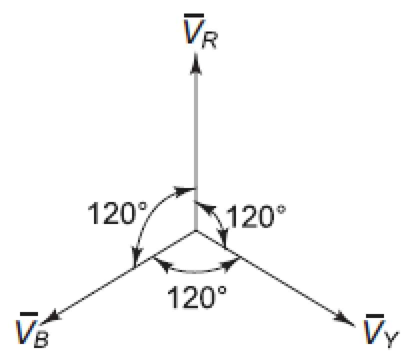
$$v_Y = V_m \sin (\theta - 120°)$$

$$v_B = V_m \sin (\theta - 240°)$$

The induced voltage in winding YY_1 lags behind that in winding RR_1 by 120° and the induced voltage in winding BB_1 lags behind that in winding RR_1 by 240°. The waveforms of these three voltages are shown in Fig.



Voltage waveforms



Phasor diagram

Advantages of Three Phase system

1. In a single-phase system, the instantaneous power is fluctuating in nature. However, in a three-phase system, it is constant at all times.
2. The output of a three-phase system is greater than that of a single-phase system.
3. Transmission and distribution of a three-phase system is cheaper than that of a single-phase system.
4. Three-phase motors are more efficient and have higher power factors than single-phase motors of the same frequency.
5. Three-phase motors are self-starting whereas single-phase motors are not self-starting.

DEFINITIONS

Phase Sequence The sequence in which the voltages in the three phases reach the maximum positive value is called the *phase sequence* or *phase order*. From the phasor diagram of a three-phase system, it is clear that the voltage in the coil R attains maximum positive value first, next in the coil Y and then in the coil B. Hence, the phase sequence is R-Y-B.

Phase Voltage The voltage induced in each winding is called the *phase voltage*.

Phase Current The current flowing through each winding is called the *phase current*.

Line Voltage The voltage available between any pair of terminals or lines is called the *line voltage*.

Line Current The current flowing through each line is called the *line current*.

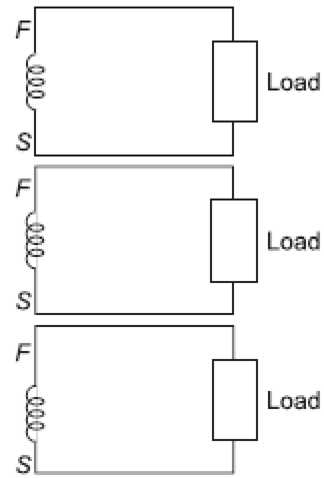
Symmetrical or Balanced System A three-phase system is said to be balanced if the

(a) voltages in the three phases are equal in magnitude and differ in phase from one another by 120°, and

(b) currents in the three phases are equal in magnitude and differ in phase from one another by 120°.

Balanced Load The load is said to be balanced if loads connected across the three phases are identical, i.e., all the loads have the same magnitude and power factor.

INTERCONNECTION OF THREE PHASES



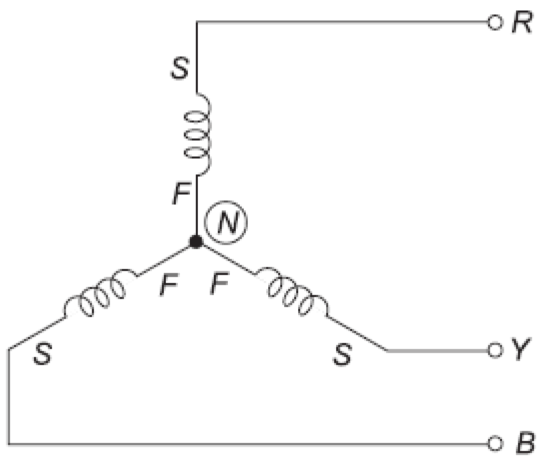
Non-interlinked three-phase system

In a three-phase system, there are three windings. Each winding has two terminals, viz., 'start' and 'finish'. If a separate load is connected across each winding as shown in Fig. six conductors are required to transmit and distribute power. This will make the system complicated and expensive.

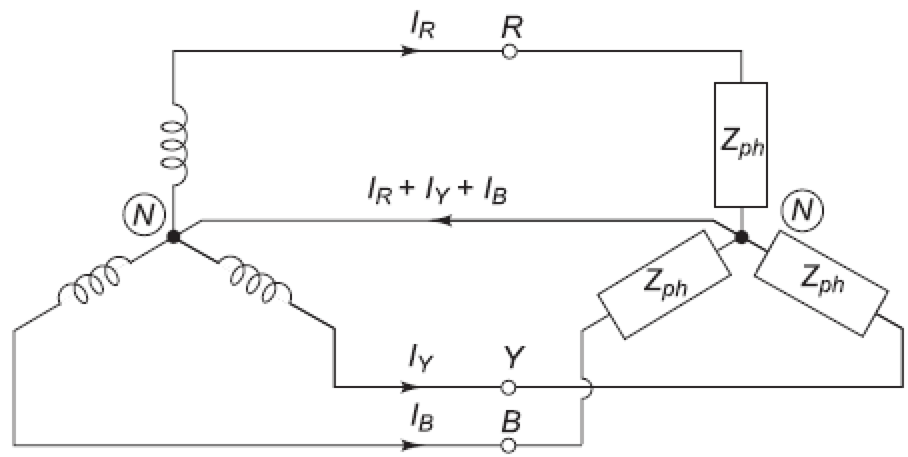
In order to reduce the number of conductors, the three windings are connected in the following two ways:

1. Star, or Wye, connection
2. Delta, or Mesh, connection

STAR OR WYE CONNECTION



Three-phase star connection



Three-phase, four-wire system

In this method, similar terminals (start or finish) of the three windings are joined together The common point is called *star* or *neutral point*.

This system is called a three-phase, four-wire system. If three identical loads are connected to each phase, the current flowing through the neutral wire is the sum of the three currents I_R, I_Y and I_B. Since the impedances are identical, the three currents are equal in magnitude but differ in phase from one another by 120°.

$$i_R = I_m \sin \theta$$

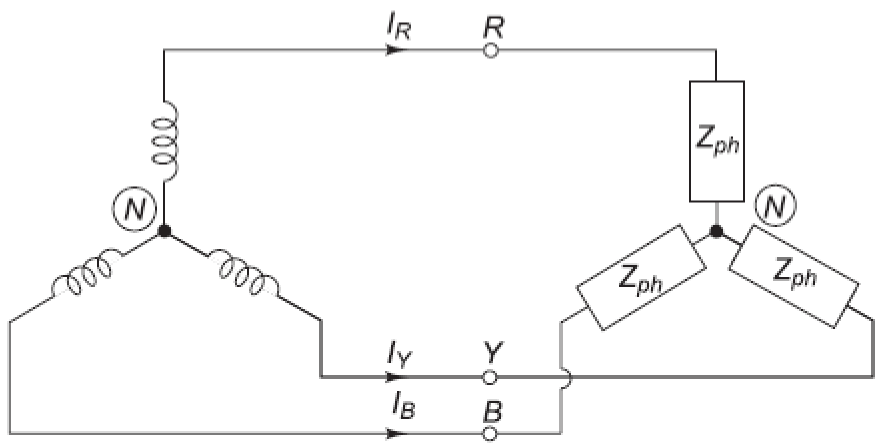
$$i_Y = I_m \sin (\theta - 120°)$$

$$i_B = I_m \sin (\theta - 240°)$$

Continue……

$$i_R + i_Y + i_B = I_m \sin\theta + I_m \sin(\theta - 120°) + I_m \sin(\theta - 240°) = 0$$

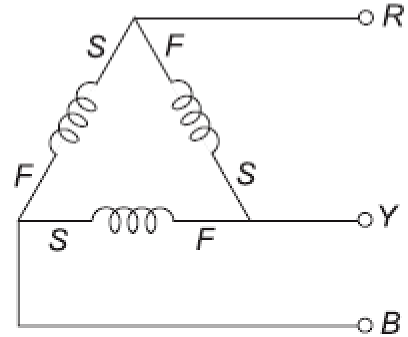
Therefore, the neutral wire can be removed without any way affecting the voltages or currents in the circuit as shown in Fig. 5.13. This constitutes a three-phase, three-wire system. If the load is not balanced, the neutral wire carries some current.



Three-phase, three-wire system

Reference: Basic electrical Engineering, By Ravish Singh, Mc Graw Hill Publication

DELTA OR MESH CONNECTION



Three-phase delta connection

In this method, dissimilar terminals of the three windings are joined together,

the 'finish' terminal of one winding is connected to the 'start' terminal of the other winding, and so on,

For a balanced system, the sum of the three phase voltages round the closed mesh is zero. The three emfs are equal in magnitude but differ in phase from one another by 120°.

$$v_R = V_m \sin \theta$$

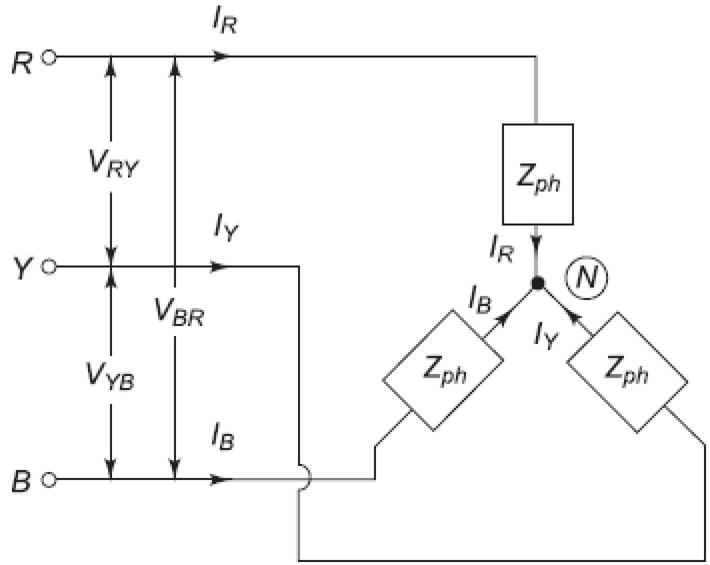
$$v_Y = V_m \sin (\theta - 120°)$$

$$v_B = V_m \sin (\theta - 240°)$$

$$v_R + v_Y + v_B = V_m \sin \theta + V_m \sin (\theta - 120°) + V_m \sin (\theta - 240°) = 0$$

VOLTAGE, CURRENT AND POWER RELATIONS IN A BALANCED STAR-CONNECTED LOAD

Relation between Line Voltage and Phase Voltage



Star connection

Since the system is balanced, the three-phase voltages V_{RN}, V_{YN} and V_{BN} are equal in magnitude and differ in phase from one another by 120°.

Let $V_{RN} = V_{YN} = V_{BN} = V_{ph}$ where V_{ph} indicates the rms value of phase voltage.

$$\overline{V}_{RN} = V_{ph} \angle 0°$$

$$\overline{V}_{YN} = V_{ph} \angle -120°$$

$$\overline{V}_{BN} = V_{ph} \angle -240°$$

Let $V_{RY} = V_{YB} = V_{BR} = V_L$ where V_L indicates the rms value of line voltage.

Continue…

Applying Kirchhoff's voltage law,

$$\overline{V}_{RY} = \overline{V}_{RN} + \overline{V}_{NY}$$

$$= \overline{V}_{RN} - \overline{V}_{YN}$$

$$= V_{ph} \angle 0° - V_{ph} \angle -120°$$

$$= (V_{ph} + j0) - (-0.5\ V_{ph} - j0.866\ V_{ph})$$

$$= 1.5\ V_{ph} + j0.866\ V_{ph}$$

$$= \sqrt{3}\ V_{ph} \angle 30°$$

Similarly,

$$\overline{V}_{YB} = \overline{V}_{YN} + \overline{V}_{NB} = \sqrt{3}\, V_{ph} \angle 30°$$

$$\overline{V}_{BR} = \overline{V}_{BN} + \overline{V}_{NR} = \sqrt{3}\, V_{ph} \angle 30°$$

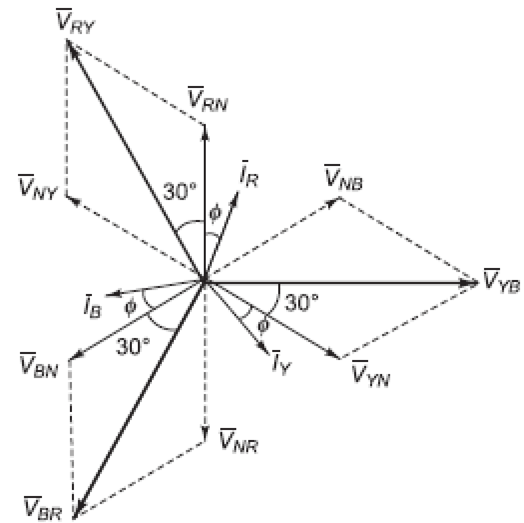
Thus, in a star-connected, three-phase system, $V_L = \sqrt{3}\, V_{ph}$ and line voltages lead respective phase voltages by 30°.

Relation between line current and phase current

$$I_L = I_{ph}$$

Phasor diagram (Lagging Power factor)

1. First draw V_{RN} as reference voltage.
2. Since three phase voltages are equal in magnitude and differ in phase from one another by 120°, draw $\overline{V}_{YN}$ and $\overline{V}_{BN}$ lagging 120° behind w.r.t. each other.
3. Draw $\overline{V}_{NY}$ equal and opposite to $\overline{V}_{YN}$.
4. Add $\overline{V}_{RN}$ and $\overline{V}_{NY}$ using the parallelogram law of vector addition such that
$$\overline{V}_{RY} = \overline{V}_{RN} + \overline{V}_{NY}$$
5. Draw $\overline{V}_{NB}$ equal and opposite to $\overline{V}_{BN}$.
6. Add $\overline{V}_{YN}$ and $\overline{V}_{NB}$ using the parallelogram law of vector addition such that
$$\overline{V}_{YB} = \overline{V}_{YN} + \overline{V}_{NB}$$
7. Draw $\overline{V}_{NR}$ equal and opposite to $\overline{V}_{RN}$.
8. Add $\overline{V}_{BN}$ and $\overline{V}_{NR}$ using the parallelogram law of vector addition such that
$$\overline{V}_{BR} = \overline{V}_{BN} + \overline{V}_{NR}$$
9. Assuming inductive load, draw three phase currents $\overline{I}_R$, $\overline{I}_Y$ and $\overline{I}_B$ lagging behind its respective phase voltages by an angle ϕ. The phase currents are equal in magnitude and differ in phase from one another by 120°.
10. Line currents are same as the phase currents in star connected load. Hence, separate line currents are not drawn.
11. Since $\overline{V}_{NY}$ is antiphase with $\overline{V}_{YN}$, angle between $\overline{V}_{RN}$ and $\overline{V}_{NY}$ is 60°. The line voltage $\overline{V}_{RY}$ leads phase voltage $\overline{V}_{RN}$ by 30°. Similarly, line voltage $\overline{V}_{YB}$ leads phase voltage $\overline{V}_{YN}$ by 30° and line voltage $\overline{V}_{BR}$ leads phase voltage $\overline{V}_{BN}$ by 30°.



Power :

The total power in a three-phase system is the sum of powers in the three phases. For a balanced load, the power consumed in each load phase is the same.

Total active power $P = 3 \times \text{power in each phase} = 3\, V_{ph}\, I_{ph} \cos \phi$

In a star-connected, three-phase system,

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L$$

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos \phi = \sqrt{3}\, V_L\, I_L \cos \phi$$

where ϕ is the phase difference between phase voltage and corresponding phase current.

Similarly, total reactive power $Q = 3\, V_{ph}\, I_{ph} \sin \phi$

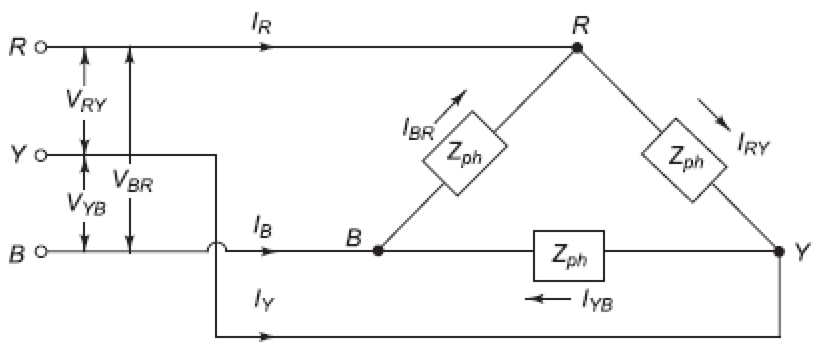
$$= \sqrt{3}\ V_L\, I_L \sin \phi$$

Total apparent power $S = 3 V_{ph}\, I_{ph} = \sqrt{3}\ V_L\, I_L$

VOLTAGE CURRENT and POWER Relationship in Balanced Delta connected Load

Relation between Line voltage and Phase voltage

$$V_L = V_{ph}$$



Delta connection

Relation between Line current and Phase current

Since the system is balanced, the three-phase currents I_{RY}, I_{YB} and I_{BR} are equal in magnitude but differ in phase from one another by 120°.

Let $I_{RY} = I_{YB} = I_{BR} = I_{ph}$ where I_{ph} indicates rms value of the phase current.

$$\bar{I}_{RY} = I_{ph} \angle 0°$$

$$\bar{I}_{YB} = I_{ph} \angle -120°$$

$$\bar{I}_{BR} = I_{ph} \angle -240°$$

Let $I_R = I_Y = I_B = I_L$ where I_L indicates rms value of the line current.

Applying Kirchhoff's current law,

$$\bar{I}_R + \bar{I}_{BR} = \bar{I}_{RY}$$

$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR} = I_{ph} \angle 0° - I_{ph} \angle -240°$$

$$= (I_{ph} + j0) - (-0.5\, I_{ph} + j0.866\, I_{ph})$$

$$= 1.5\, I_{ph} - j0.866\, I_{ph}$$

$$= \sqrt{3}\; I_{ph} \angle -30°$$

Similarly,
$$\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} = \sqrt{3}\; I_{ph} \angle -30°$$

$$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} = \sqrt{3}\; I_{ph} \angle -30°$$

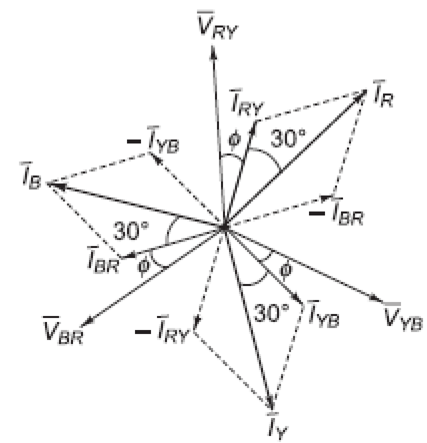
Thus, in a delta-connected, three-phase system, $I_L = \sqrt{3}\; I_{ph}$ and line currents are 30° behind the respective phase currents.

1. First draw $\overline{V}_{RY}$ as reference voltage.

2. Since three phase voltages are equal in magnitude and differ in phase from one another by 120°, draw $\overline{V}_{YB}$ and $\overline{V}_{BR}$ lagging 120° behind w.r.t. each other.

3. Line voltages are same as the phase voltages for balanced delta connected load. Hence, separate line voltages are not drawn.

4. Assuming inductive load, draw three phase currents $\overline{I}_{RY}$, $\overline{I}_{YB}$ and $\overline{I}_{BR}$ lagging behind respective phase voltages by an angle ϕ. The phase currents are equal in magnitude and differ in phase from one another by 120°.



5. Draw $-\overline{I}_{BR}$ equal and opposite to $\overline{I}_{BR}$.

6. Add $\overline{I}_{RY}$ and $-\overline{I}_{BR}$ using the parallelogram law of vector addition such that
$$\overline{I}_R = \overline{I}_{RY} - \overline{I}_{BR}$$

7. Draw $-\overline{I}_{RY}$ equal and opposite to $\overline{I}_{RY}$.

8. Add $\overline{I}_{YB}$ and $-\overline{I}_{RY}$ using the parallelogram law of vector addition such that
$$\overline{I}_Y = \overline{I}_{YB} - \overline{I}_{RY}$$

9. Draw $-\overline{I}_{YB}$ equal and opposite to $\overline{I}_{YB}$.

10. Add $\overline{I}_{BR}$ and $-\overline{I}_{YB}$ using the parallelogram laws of vector addition such that
$$\overline{I}_B = \overline{I}_{BR} - \overline{I}_{YB}$$

11. Since $-\overline{I}_{BR}$ is antiphase with $\overline{I}_{BR}$, angle between $\overline{I}_{RY}$ and $-\overline{I}_{BR}$ is 60°. The line current $\overline{I}_R$ lags behind phase current $\overline{I}_{RY}$ by 30°. Similarly, the line current $\overline{I}_Y$ lags behind phase current $\overline{I}_{YB}$ by 30° and the line current $\overline{I}_B$ lags behind phase current $\overline{I}_{BR}$ by 30°.

Power

$$P = 3\, V_{ph}\, I_{ph} \cos \phi$$

In a delta-connected, three-phase system,

$$V_{ph} = V_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos \phi = \sqrt{3}\; V_L\, I_L \cos \phi$$

Total reactive power $Q = 3\; V_{ph}\, I_{ph} \sin \phi = \sqrt{3}\; V_L\, I_L \sin \phi$

Total apparent power $S = 3\; V_{ph}\, I_{ph} = \sqrt{3}\; V_L\, I_L$

BALANCED Y/Δ AND Δ/Y CONVERSIONS

Any balanced star-connected system can be converted into the equivalent delta-connected system and vice versa.

For a balanced star-connected load,

Line voltage $= V_L$

Line current $= I_L$

Impedance/phase $= Z_Y$

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L$$

$$Z_Y = \frac{V_{ph}}{I_{ph}} = \frac{V_L}{\sqrt{3} I_L}$$

For an equivalent delta-connected system, the line voltages and currents must have the same values as in the star-connected system, i.e.,

Line voltage $= V_L$

Line current $= I_L$

Impedance/phase $= Z_\Delta$

$$V_{ph} = V_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$Z_\Delta = \frac{V_{ph}}{I_{ph}} = \frac{V_L}{\frac{I_L}{\sqrt{3}}} = \sqrt{3}\frac{V_L}{I_L} = 3Z_Y$$

$$Z_Y = \frac{1}{3} Z_\Delta$$

Thus, when three equal phase impedances are connected in delta, the equivalent star impedance is one third of the delta impedance.

RELATION BETWEEN POWER IN DELTA AND STAR SYSTEM

Let a balanced load be connected in star having impedance per phase as Z_{ph}.

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{\sqrt{3}Z_{ph}}$$

$$I_L = I_{ph} = \frac{V_L}{\sqrt{3}Z_{ph}}$$

$$P_Y = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$= \sqrt{3} \times V_L \times \frac{V_L}{\sqrt{3}Z_{ph}} \times \cos\phi$$

$$= \frac{V_L^2}{Z_{ph}} \cos\phi$$

For a delta-connected load,

$$V_{ph} = V_L$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{Z_{ph}}$$

$$I_L = \sqrt{3}\, I_{ph} = \sqrt{3}\frac{V_L}{Z_{ph}}$$

$$P_\Delta = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$= \sqrt{3} \times V_L \times \sqrt{3}\, \frac{V_L}{Z_{ph}} \times \cos\phi$$

$$= 3\, \frac{V_L^2}{Z_{ph}} \cos\phi$$

$$= 3P_Y$$

$$P_Y = \frac{1}{3} P_\Delta$$

Thus, power consumed by a balanced star-connected load is one-third of that in the case of a delta-connected load.

Example on three phase system

Three similar resistors are connected in star across 400 V, three-phase lines. The line current is 5 A. Calculate the value of each resistor. To what value should the line voltage be changed to obtain the same line current with the resistors connected in delta?

$$V_L = 400 \text{ V}$$
$$I_L = 5 \text{ A}$$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$I_{ph} = I_L = 5 \text{ A}$$

$$Z_{ph} = R_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{230.94}{5} = 46.19 \ \Omega$$

For a delta-connected load,

$$I_L = 5 \text{ A}$$
$$R_{ph} = 46.19 \ \Omega$$
$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{5}{\sqrt{3}} \text{A}$$

$$V_{ph} = I_{ph} R_{ph} = \frac{5}{\sqrt{3}} \times 46.19 = 133.33 \text{ V}$$
$$V_L = 133.33 \text{ V}$$

Voltage needed is one-third of the star value.

Three identical coils connected in delta to a 440 V, three-phase supply take a total power of 50 kW and a line current of 90 A. Find the (i) phase current, (ii) power factor, and (iii) apparent power taken by the coils.

$$V_L = 440 \text{ V}$$
$$P = 50 \text{ kW}$$
$$I_L = 90 \text{ A}$$

For a delta-connected load,

(i) Phase current

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{90}{\sqrt{3}} = 51.96 \text{ A}$$

(ii) Power factor

$$P = \sqrt{3}\, V_L I_L \cos\phi$$
$$50 \times 10^3 = \sqrt{3} \times 440 \times 90 \times \cos\phi$$
$$\text{pf} = \cos\phi = 0.73 \text{ (lagging)}$$

(iii) Apparent power

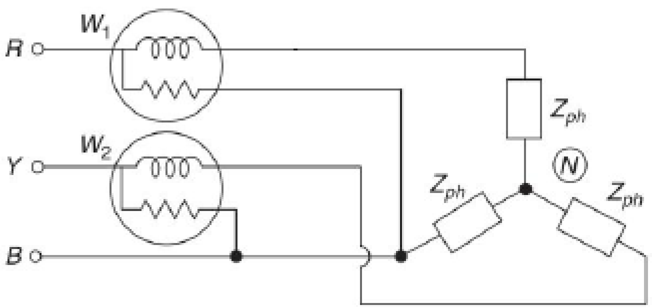
$$S = \sqrt{3}\, V_L I_L = \sqrt{3} \times 440 \times 90 = 68.59 \text{ kVA}$$

Measurement of Three Phase Power

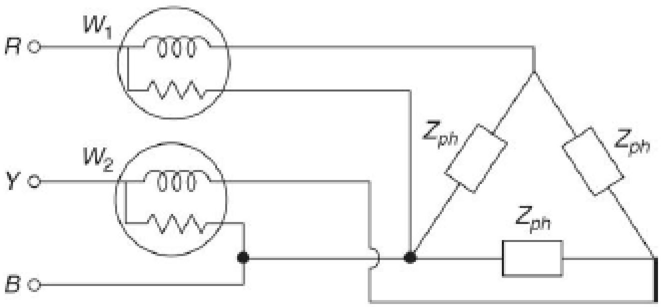
There are three methods to measure three-phase power:

1. Three-wattmeter method
2. Two-wattmeter method
3. One-wattmeter method

Two-Wattmeter Method



(a)



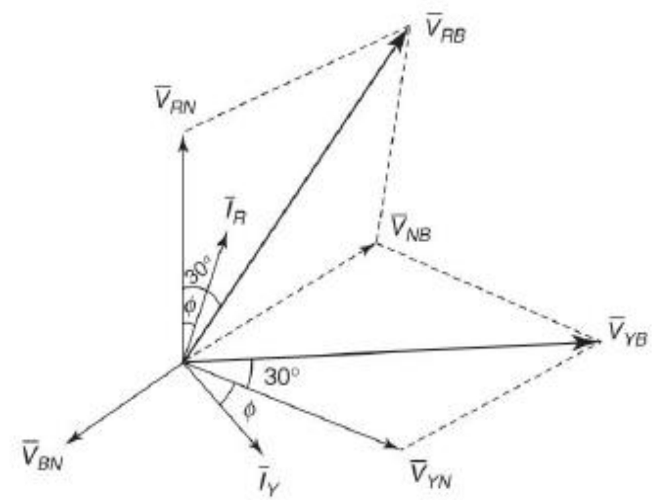
(b)

` Total power $P = W_1 + W_2$

Measurement of Three Phase Power using Two Wattmeter Method (Star connected load)

Let V_{RN}, V_{YN} and V_{BN} be the three phase voltages and I_R, I_Y and I_B be the phase currents.

The phase currents lag behind their respective phase voltages by an angle ϕ.



Phasor diagram of a balanced star connected inductive load

Current through current coil of $W_1 = I_R$

Voltage across voltage coil of $W_1 = V_{RB} = V_{RN} + V_{NB} = V_{RN} - V_{BN}$

From the phasor diagram, it is clear that the phase angle between V_{RB} and I_R is $(30° - \phi)$.

$$W_1 = V_{RB} I_R \cos(30° - \phi)$$

Current through current coil of $W_2 = I_Y$

Voltage across voltage coil of $W_2 = V_{YB} = V_{YN} + V_{NB} = V_{YN} - V_{BN}$

From the phasor diagram, it is clear that phase angle between V_{YB} and I_Y is $(30° + \phi)$.

$$W_2 = V_{YB}\ I_Y \cos(30° + \phi)$$

But, $$I_R = I_Y = I_L$$

Continue…..

$$V_{RB} = V_{YB} = V_L$$

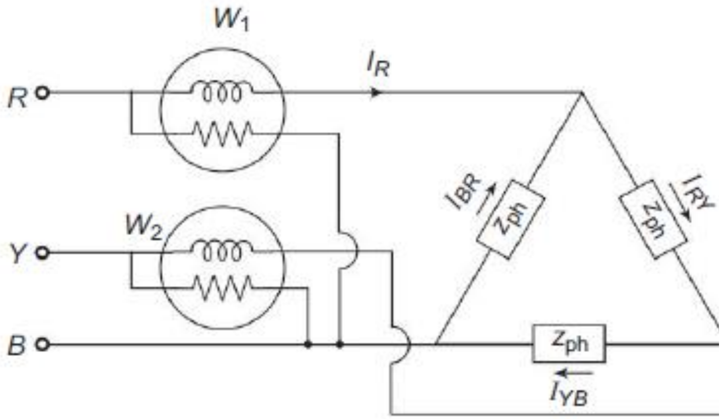
$$W_1 = V_L I_L \cos(30° - \phi)$$

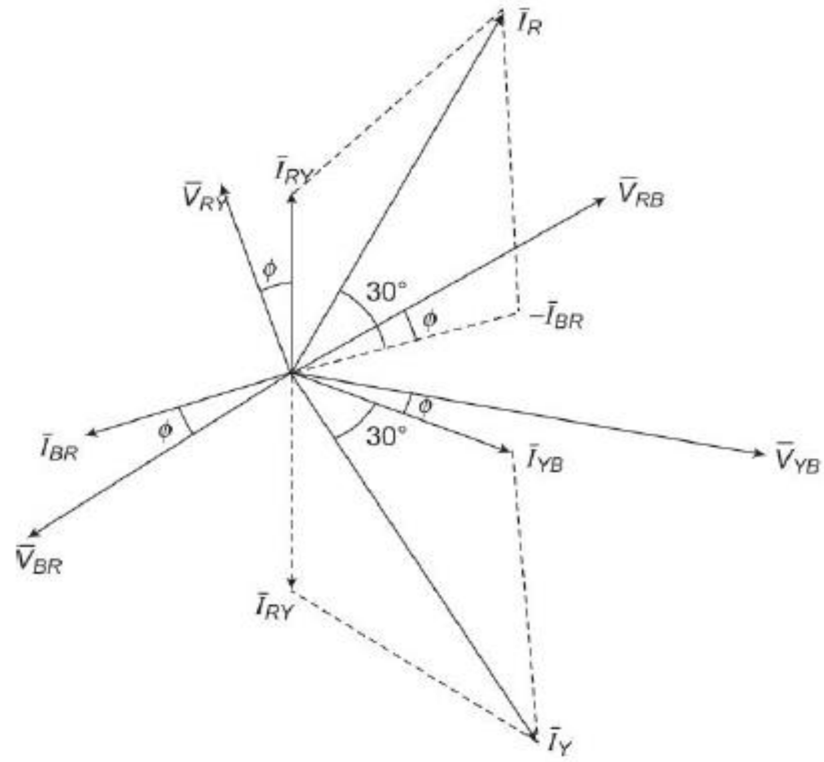
$$W_2 = V_L I_L \cos(30° + \phi)$$

$$W_1 + W_2 = V_L I_L [\cos(30° + \phi) + \cos(30° - \phi)]$$
$$= V_L I_L (2\cos 30° \cos\phi) = \sqrt{3} V_L I_L \cos\phi$$

Thus, the sum of two wattmeter readings gives three-phase power.

Measurement of Active power using Delta connected Load





Measurement of Active power using Delta connected Load

Current through current coil of $W_1 = \overline{I}_R = \overline{I}_{RY} - \overline{I}_{BR}$

Voltage across voltage coil of $W_1 = V_{RB}$

Current through current coil of $W_2 = I_Y = \overline{I}_{YB} - \overline{I}_{RY}$

Voltage across voltage coil of $W_2 = V_{YB}$

From phasor diagram, it is clear that the phase angle between I_R and V_{RB} is $(30° - \phi)$.

$$W_1 = I_R V_{RY} \cos(30° - \phi) = I_L V_L \cos(30° - \phi)$$

From phasor diagram, it is clear that the phase angle between I_Y and V_{YB} is $(30° + \phi)$

$$W_2 = I_Y V_{YB} \cos(30° + \phi) = I_L V_L \cos(30° + \phi)$$

$$W_1 + W_2 = V_L I_L [\cos(30° - \phi) + \cos(30° + \phi)]$$

$$= V_L I_L (2 \cos 30° \cos \phi)$$

$$= \sqrt{3} V_L I_L \cos \phi$$

Thus, sum of two wattmeter readings gives three-phase power.

Measurement of power factor by two wattmeter method

Lagging Power Factor

$$W_1 = V_L I_L \cos(30° - \phi)$$
$$W_2 = V_L I_L \cos(30° + \phi)$$
$$\therefore \quad W_1 > W_2$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$
$$W_1 - W_2 = V_L I_L [\cos(30° - \phi) - \cos(30° + \phi) = V_L I_L \sin\phi$$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi}$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$\phi = \tan^{-1}\left(\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$\text{pf} = \cos\phi = \cos\left\{\tan^{-1}\left(\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}\right)\right\}$$

Effect of power factor for Two wattmeter reading

pf	ϕ	W_1 Reading	W_2 Reading	Remark
0	90°	Positive	Negative	$W_1 = -W_2$
$0 < \text{pf} < 0.5$	$90° < \phi < 60°$	Positive	Negative	
0.5	60°	Positive	0	
$0.5 < \text{pf} < 1$	$60° < \phi < 0°$	Positive	Positive	
1	0°	Positive	Positive	$W_1 = W_2$

Reference: Basic electrical Engineering, By Ravish Singh, Mc Graw Hill Publication

Two wattmeters are used to measure power in a three-phase balanced load. Find the power factor if (i) two readings are equal and positive, (ii) two readings are equal and opposite, and (iii) one wattmeter reads zero. **[Dec 2013]**

(i) $W_1 = W_2$

(ii) $W_2 = 0$ $W_1 = -W_2$

(i) Power factor if two readings are equal and positive

$$W_1 = W_2$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,(0) = 0$$

$$\phi = 0^\circ$$

$$\text{Power factor} = \cos\phi = \cos(0^\circ) = 1$$

(ii) Power factor if two readings are equal and opposite

$$W_1 = -W_2$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \infty$$

$$\phi = 90^\circ$$

$$\text{Power factor} = \cos\phi = \cos(90^\circ) = 0$$

(iii) Power factor if one wattmeter reads zero

$$W_2 = 0$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\left(\frac{W_1}{W_1}\right) = \sqrt{3}$$

$$\phi = 60^\circ$$

$$\text{Power factor} = \cos\phi = \cos(60^\circ) = 0.5$$

Reference: Basic electrical Engineering, By Ravish Singh, Mc Graw Hill Publication

Two wattmeters are used to measure power in a 3ϕ balanced delta connected load using two wattmeter method. The readings of the 2 wattmeters are 500 W and 2500 W respectively. Calculate the total power consumed by the 3ϕ load and the power factor. [illegible]

$$W_1 = 500 \text{ W}$$
$$W_2 = 2500 \text{ W}$$

(i) Total power

$$P = W_1 + W_2 = 500 + 2500 = 3 \text{ kW}$$

(ii) Power factor

The power factor is leading in nature since $W_1 < W_2$.

$$\tan\phi = -\sqrt{3}\,\frac{(W_1 - W_2)}{W_1 + W_2}$$

$$\phi = \tan^{-1}\left[-\sqrt{3}\,\frac{(W_1 - W_2)}{W_1 + W_2}\right] = \tan^{-1}\left[-\sqrt{3}\left(\frac{-2000}{3000}\right)\right] = 49.12$$

$$\text{pf} = \cos\phi = 0.65 \text{ (leading)}$$

A three-phase, star-connected load draws a line curent of 20 A. The load kVA and kW are 20 and 11 respectively. Find the readings on each of the two wattmeters used to measure the three-phase power.

$$I_L = 20 \text{ A}$$
$$S = 20 \text{ kVA}$$
$$P = 11 \text{ kW}$$

$$S = \sqrt{3}\ V_L I_L$$
$$20 \times 10^3 = \sqrt{3}\ V_L I_L$$

$$V_L I_L = 11.55 \text{ kVA}$$

$$P = \sqrt{3}\ V_L I_L \cos\phi$$

$$11 \times 10^3 = 20 \times 10^3 \times \cos\phi$$

$$\cos\phi = 0.55$$

$$\phi = 56.63°$$

$$W_1 = V_L I_L \cos(30° - \phi) = 11.55 \times \cos(30° - 56.63°) = 10.32 \text{ kW}$$
$$W_2 = V_L I_L \cos(30° + \phi) = 11.55 \times \cos(30° + 56.63) = 0.68 \text{ kW}$$

Three similar coils, connected in star, take a total power of 18 kW at a power factor of 0.866 lagging from a three-phase, 400-volt, 50 Hz system. Calculate the resistance and inductance of each coil. **[May 2014]**

Reference: Basic electrical Engineering, By Ravish Singh, Mc Graw Hill Publication