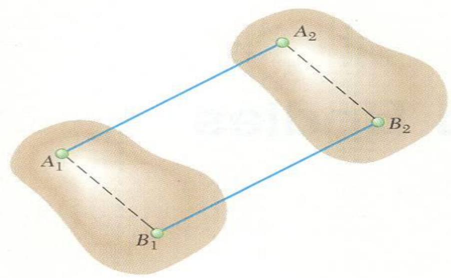
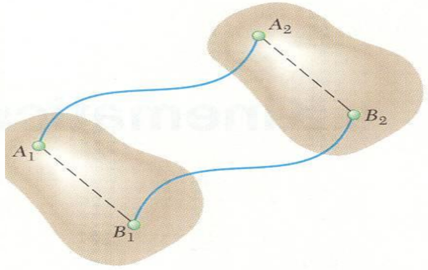
K J SOMAIYA COLLEGE OF ENGINEERING, MUMBAI-77

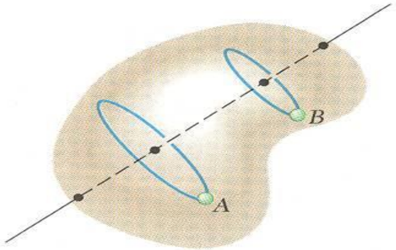
(CONSTITUENT COLLEGE OF SOMAIYA VIDYAVIHAR UNIVERSITY)

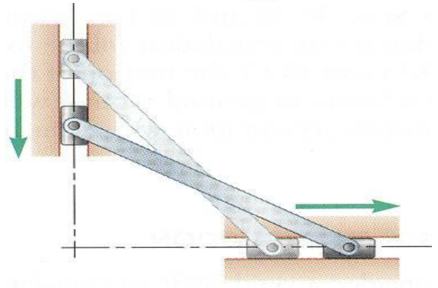
Presented by:
Prof. Abhishek Bhadauria

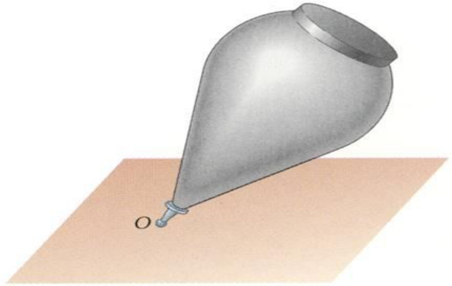
Introduction









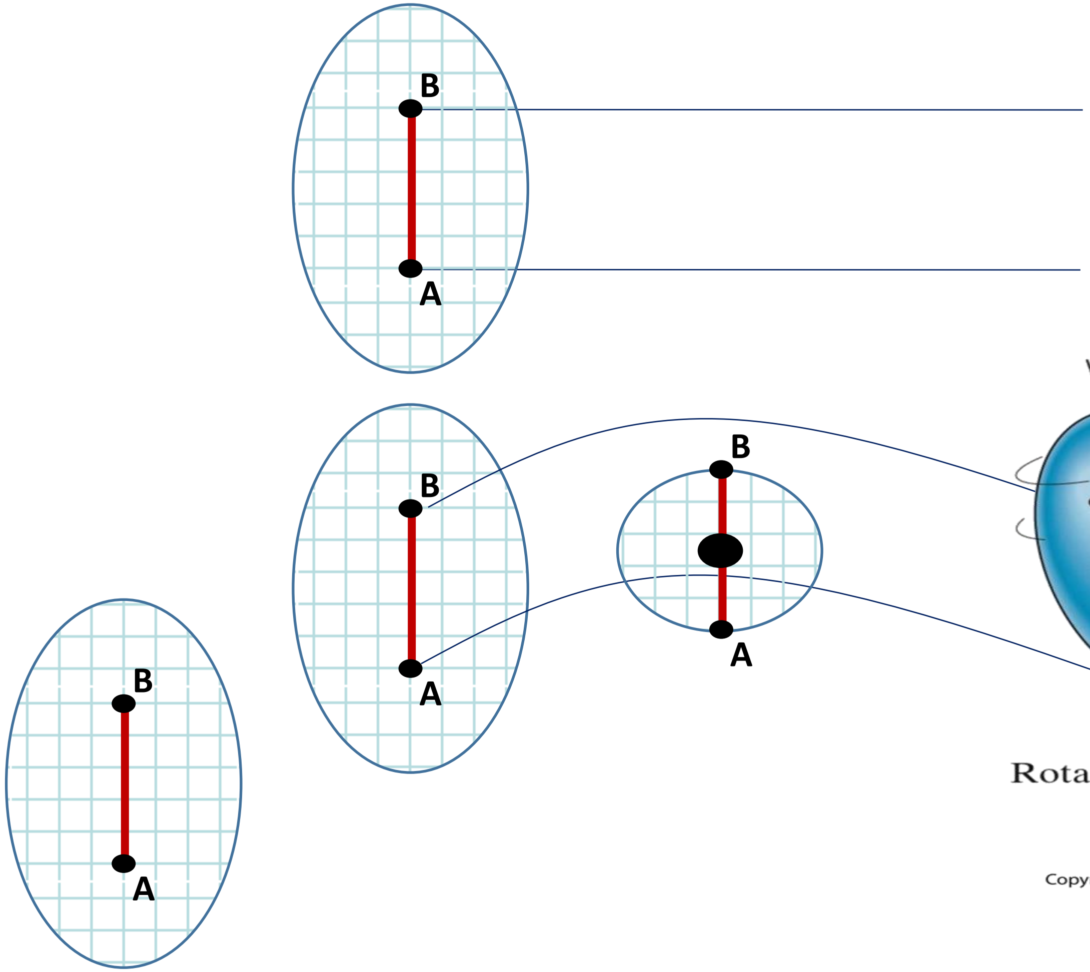


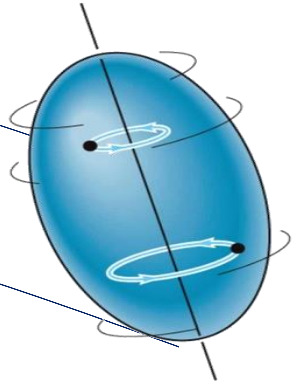
- Kinematics of rigid bodies: relations between time and the positions, velocities, and accelerations of the particles forming a rigid body.

- Classification of rigid body motions:
  - translation:
    - rectilinear translation
    - curvilinear translation
  - rotation about a fixed axis
  - general plane motion
  - motion about a fixed point
  - general motion

Types of rigid body motion

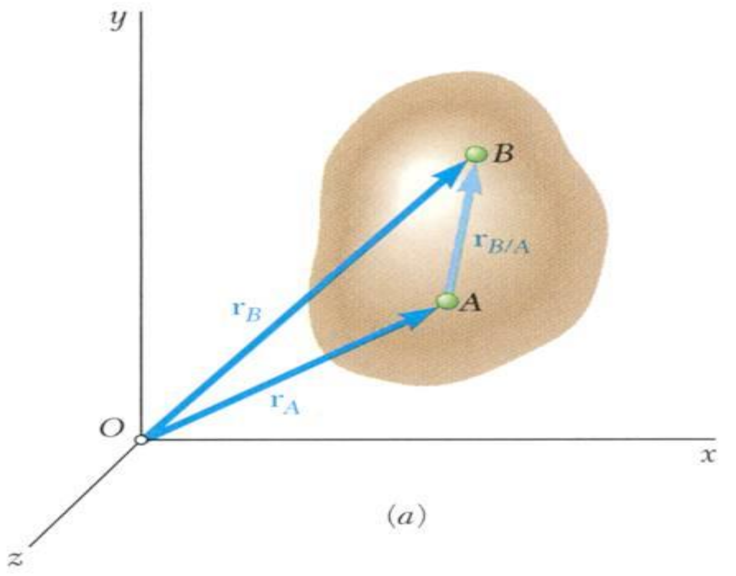
- Kinematically speaking…
  - Translation
    - –Orientation of AB constant
  - Rotation
    - –All particles rotate about fixed axis
  - General Plane Motion (both)
    - –Combination of both types of motion





Rotation about a fixed axis
(c)
fig16_01c.jpg

Translation



- Consider rigid body in translation:
  - direction of any straight line inside the body is constant,
  - all particles forming the body move in parallel lines.

- For any two particles in the body,

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

- Differentiating with respect to time,

$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A} = \dot{\vec{r}}_A$$

$$\vec{v}_B = \vec{v}_A$$

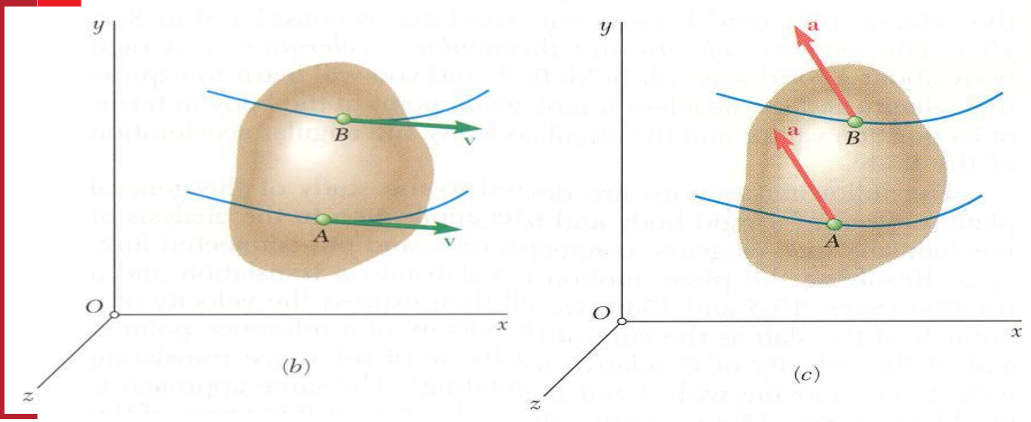
All particles have the same velocity.

- Differentiating with respect to time again,

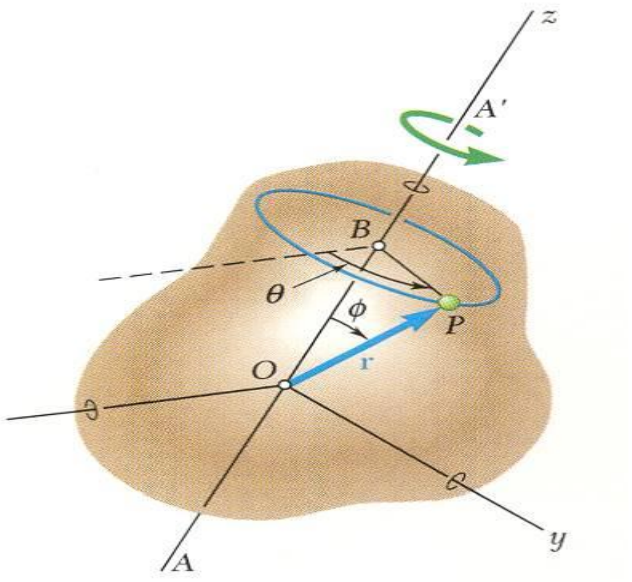
$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A} = \ddot{\vec{r}}_A$$

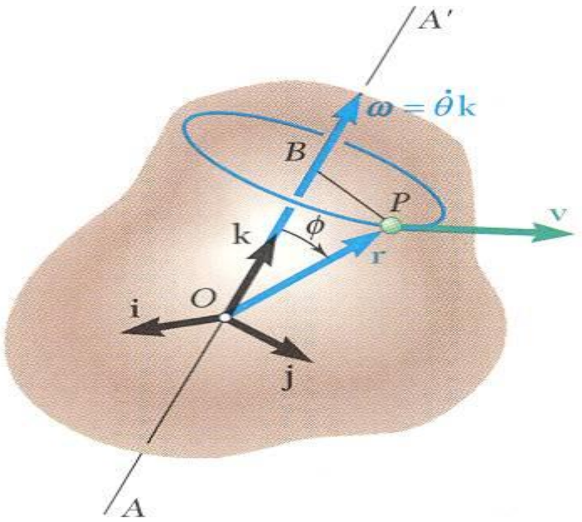
$$\vec{a}_B = \vec{a}_A$$

All particles have the same acceleration.



Rotation About a Fixed Axis. Velocity





- Consider rotation of rigid body about a fixed axis *AA'*

- Velocity vector $\vec{v} = d\vec{r}/dt$ of the particle *P* is tangent to the path with magnitude $v = ds/dt$

$$\Delta s = (BP)\Delta\theta = (r\sin\phi)\Delta\theta$$

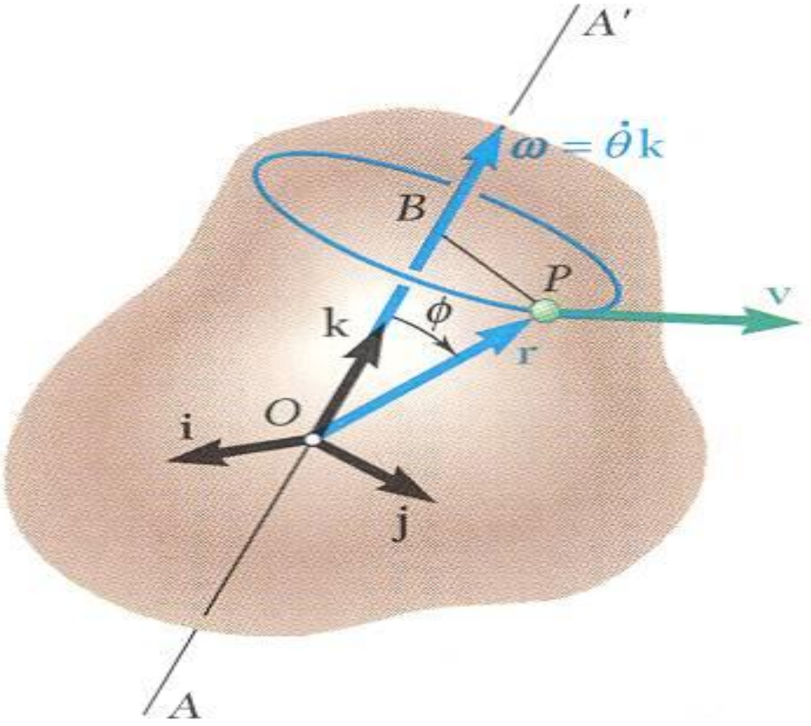
$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0}(r\sin\phi)\frac{\Delta\theta}{\Delta t} = r\dot{\theta}\sin\phi$$

- The same result is obtained from

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = angular\ velocity$$

Rotation About a Fixed Axis. Acceleration



- Differentiating to determine the acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$
$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$
$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \vec{v}$$

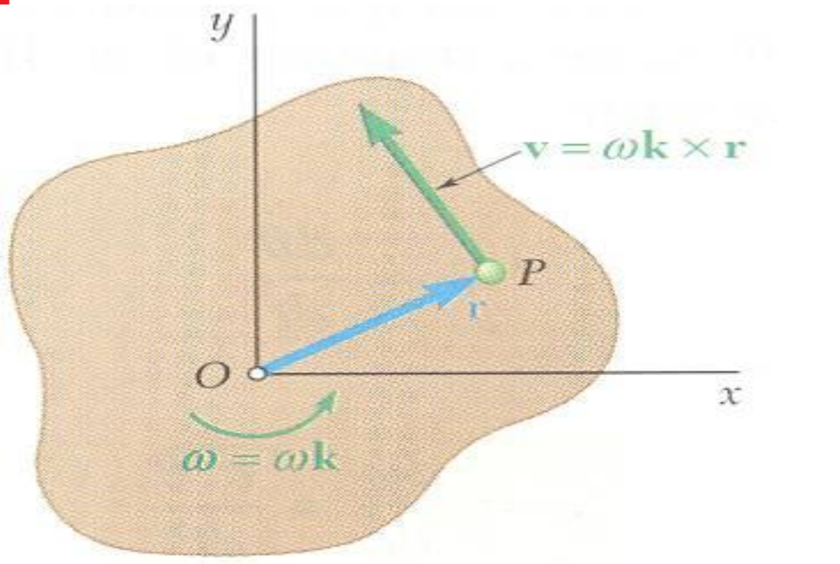
- $\frac{d\vec{\omega}}{dt} = \vec{\alpha} = angular\ acceleration$

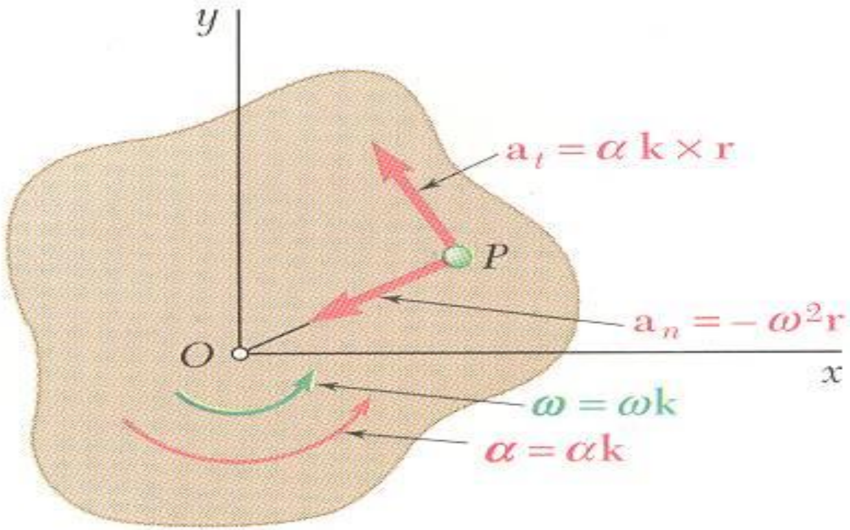
$$= \alpha\vec{k} = \dot{\omega}\vec{k} = \ddot{\theta}\vec{k}$$

- Acceleration of *P* is combination of two vectors,

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r}$$
$$\vec{\alpha} \times \vec{r} = \text{tangential acceleration component}$$
$$\vec{\omega} \times \vec{\omega} \times \vec{r} = \text{radial acceleration component}$$

Rotation About a Fixed Axis. Representative Slab





- Consider the motion of a representative slab in a plane perpendicular to the axis of rotation.

- Velocity of any point *P* of the slab,

$$\vec{v} = \vec{\omega} \times \vec{r} = \omega \vec{k} \times \vec{r}$$
$$v = r\omega$$

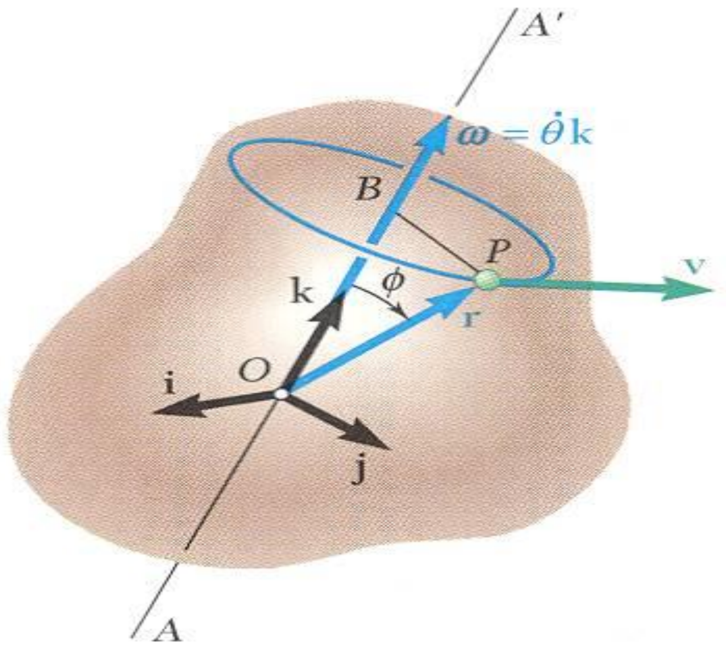
- Acceleration of any point *P* of the slab,

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r}$$
$$= \alpha \vec{k} \times \vec{r} - \omega^2 \vec{r}$$

- Resolving the acceleration into tangential and normal components,

$$\vec{a}_t = \alpha \vec{k} \times \vec{r} \qquad a_t = r\alpha$$
$$\vec{a}_n = -\omega^2 \vec{r} \qquad a_n = r\omega^2$$

Equations Defining the Rotation of a Rigid Body About a Fixed Axis



- Motion of a rigid body rotating around a fixed axis is often specified by the type of angular acceleration.

- Recall $$\omega = \frac{d\theta}{dt} \quad \text{or} \quad dt = \frac{d\theta}{\omega}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega \frac{d\omega}{d\theta}$$

- *Uniform Rotation*, $\alpha = 0$:

$$\theta = \theta_0 + \omega t$$

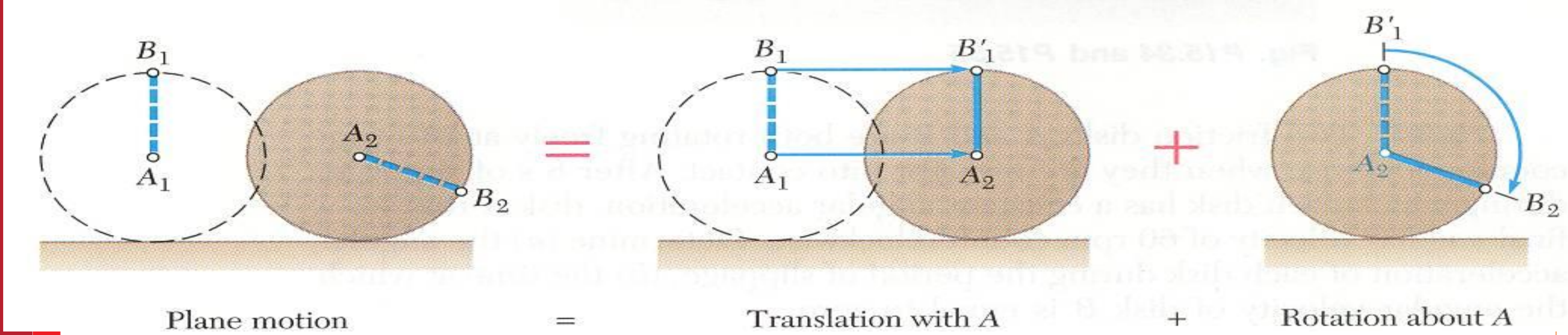
- *Uniformly Accelerated Rotation*, α = constant:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

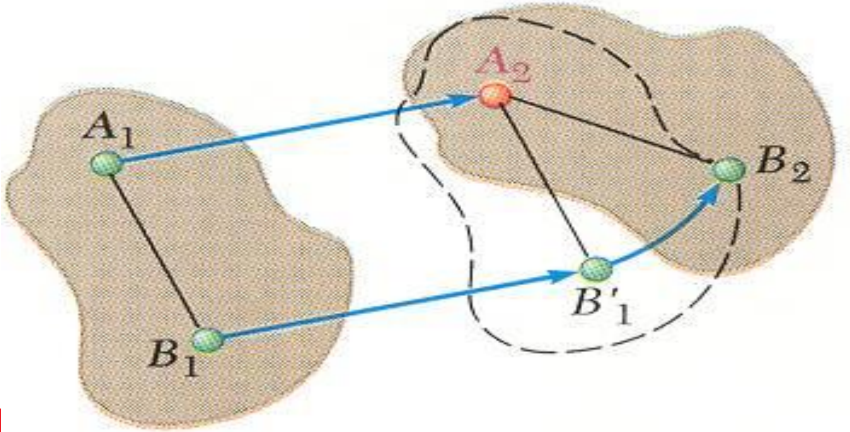
General Plane Motion



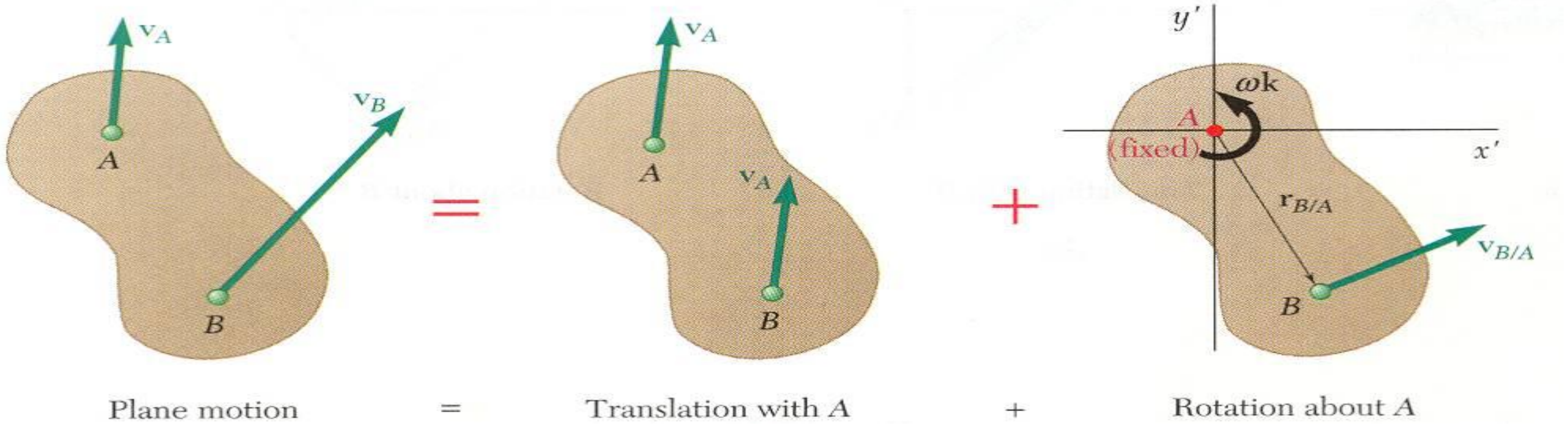
- *General plane motion* is neither a translation nor a rotation.

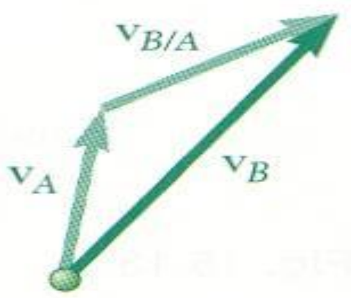
- General plane motion can be considered as the *sum* of a translation and rotation.

- Displacement of particles A and B to A_2 and B_2 can be divided into two parts:
  - translation to A_2 and B_1'
  - rotation of B_1' about A_2 to B_2



Absolute and Relative Velocity in Plane Motion





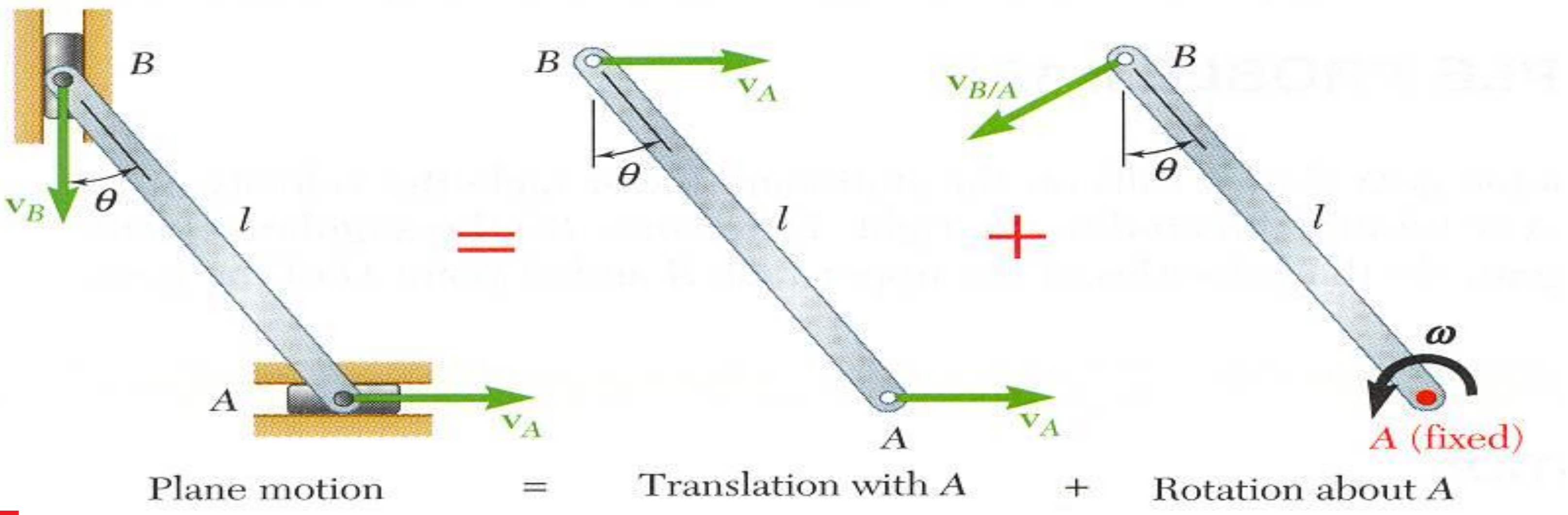
- Any plane motion can be replaced by a translation of an arbitrary reference point *A* and a simultaneous rotation about *A*.

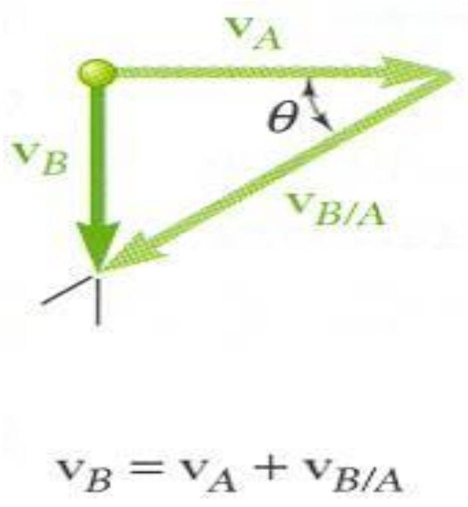
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \omega\vec{k} \times \vec{r}_{B/A} \qquad v_{B/A} = r\omega$$

$$\vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A}$$

Absolute and Relative Velocity in Plane





- Assuming that the velocity v_A of end *A* is known, wish to determine the velocity v_B of end *B* and the angular velocity ω in terms of v_A, *l*, and θ.
- The direction of v_B and $v_{B/A}$ are known. Complete the velocity diagram.

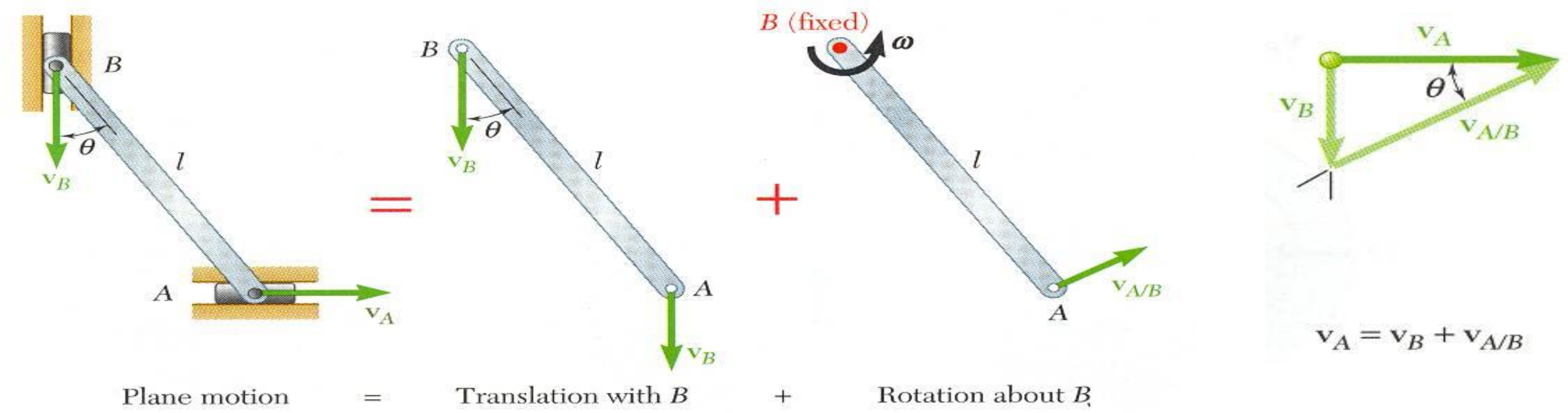
$$\frac{v_B}{v_A} = \tan\theta$$

$$v_B = v_A \tan\theta$$

$$\frac{v_A}{v_{B/A}} = \frac{v_A}{l\omega} = \cos\theta$$

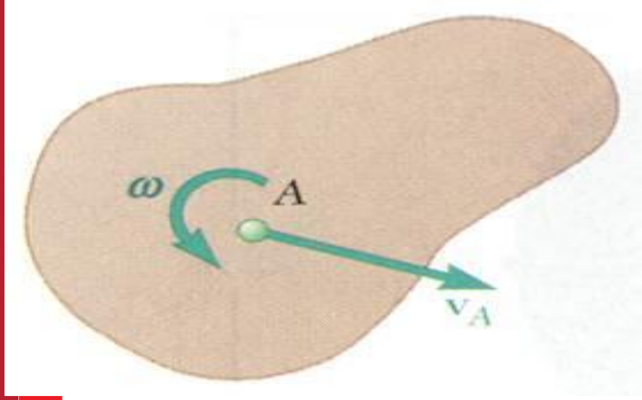
$$\omega = \frac{v_A}{l\cos\theta}$$

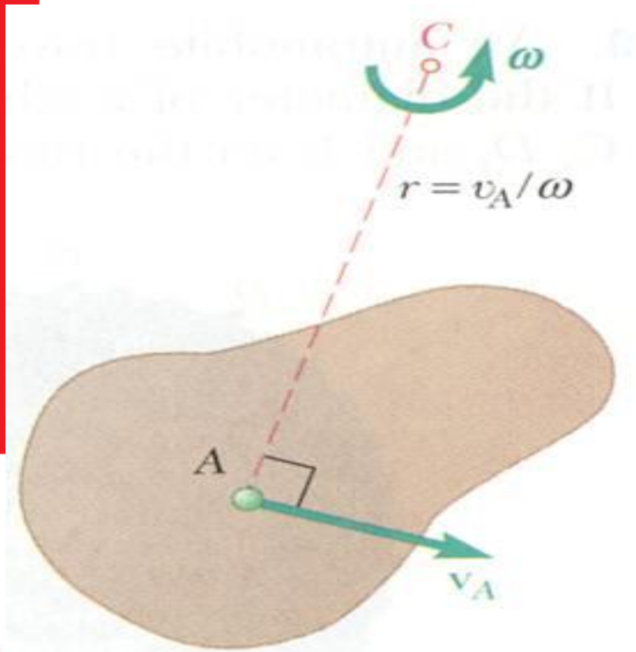
Absolute and Relative Velocity in Plane



- Selecting point *B* as the reference point and solving for the velocity v_A of end *A* and the angular velocity ω leads to an equivalent velocity triangle.
- $v_{A/B}$ has the same magnitude but opposite sense of $v_{B/A}$. The sense of the relative velocity is dependent on the choice of reference point.
- Angular velocity ω of the rod in its rotation about *B* is the same as its rotation about *A*. Angular velocity is not dependent on the choice of reference point.

Instantaneous Center of Rotation in Plane Motion





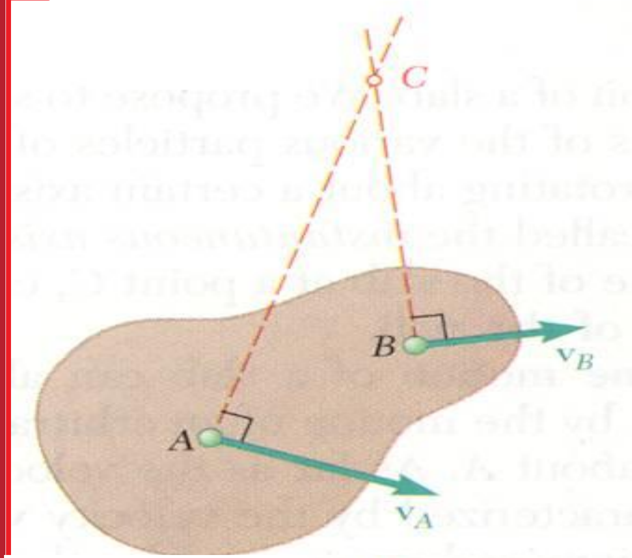
- Plane motion of all particles in a slab can always be replaced by the translation of an arbitrary point *A* and a rotation about *A* with an angular velocity that is independent of the choice of *A*.

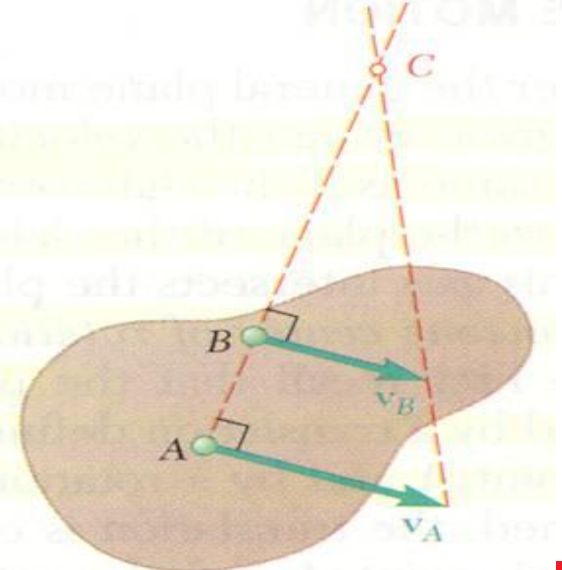
- The same translational and rotational velocities at *A* are obtained by allowing the slab to rotate with the same angular velocity about the point *C* on a perpendicular to the velocity at *A*.

- The velocity of all other particles in the slab are the same as originally defined since the angular velocity and translational velocity at *A* are equivalent.

- As far as the velocities are concerned, the slab seems to rotate about the *instantaneous center of rotation C*.

Instantaneous Center of Rotation in Plane Motion





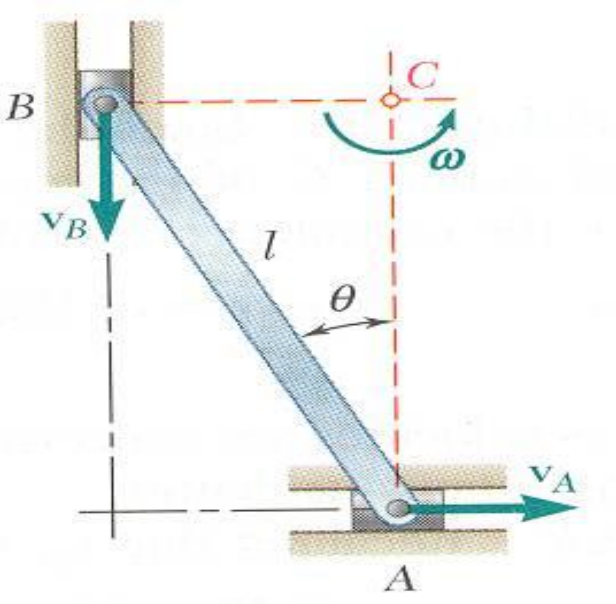
- If the velocity at two points *A* and *B* are known, the instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through *A* and *B* .

- If the velocity vectors are parallel, the instantaneous center of rotation is at infinity and the angular velocity is zero.

- If the velocity vectors at *A* and *B* are perpendicular to the line *AB*, the instantaneous center of rotation lies at the intersection of the line *AB* with the line joining the extremities of the velocity vectors at *A* and *B*.

- If the velocity magnitudes are equal, the instantaneous center of rotation is at infinity and the angular velocity is zero.

Instantaneous Center of Rotation in Plane Motion

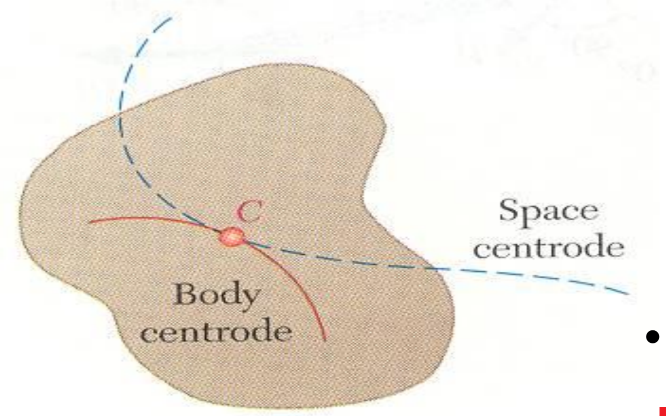


- The instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through A and B .

$$\omega = \frac{v_A}{AC} = \frac{v_A}{l\cos\theta}$$

$$v_B = (BC)\omega = (l\sin\theta)\frac{v_A}{l\cos\theta} = v_A\tan\theta$$

- The velocities of all particles on the rod are as if they were rotated about C.
- The particle at the center of rotation has zero velocity.
- The particle coinciding with the center of rotation changes with time and the acceleration of the particle at the instantaneous center of rotation is not zero.

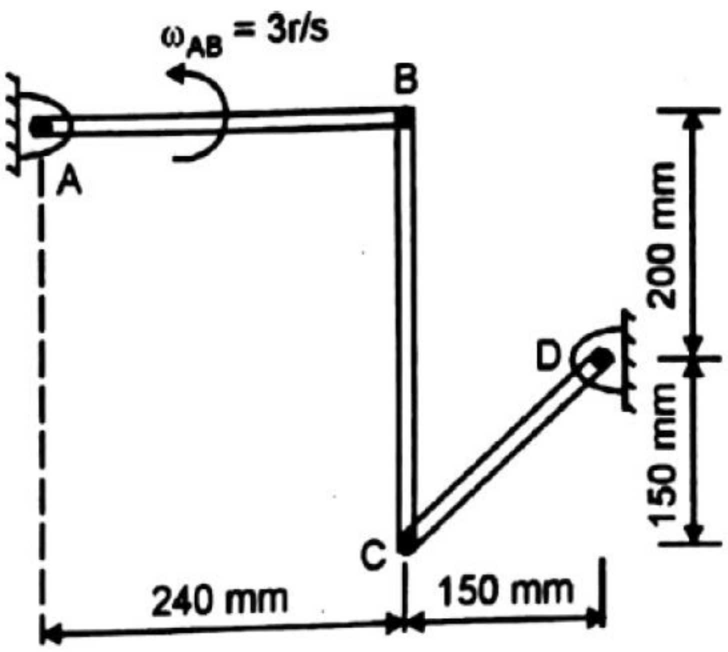


The acceleration of the particles in the slab cannot be determined as if the slab were simply rotating about C.

- The trace of the locus of the center of rotation on the body is the body centrode and in space is the space centrode.

Problems

Problem



In the position shown, bar AB has constant angular velocity of 3 rad/s anticlockwise, determine the angular velocity of bar CD.

Solution: The system consists of three bodies in motion. Rods AB and CD perform rotation motion, while rod BC performs General Plane Motion.

Rod AB rotates about A

Using $v = r\omega$

$$v_B = r_{BA} \times \omega_{AB}$$

$$= 0.24 \times 3 = 0.72 \text{ m/s} \uparrow$$

Direction velocity of end B i.e. Dov_B is $\perp$ to radial length AB.

Similarly, direction of velocity of end C, i.e. Dov_C is $\perp$ to radial length CD. Since rod CD rotates about D.

General Plane Motion of rod BC

Locating the instantaneous centre of rotation I of rod BC by drawing $\perp^{lers}$ to Dov_B and Dov_C and getting the point of intersection, which is I as shown in figure.

From geometry of Δ BCI

$$r_{BI} = 350 \text{ mm} = 0.35 \text{ m} \quad \text{and} \quad r_{CI} = 495 \text{ mm} = 0.495 \text{ m}$$

The General Plane rod BC perform rotation motion @ I at this instant

Using $v = r\omega$

$$v_B = r_{BI} \times \omega_{BC}$$

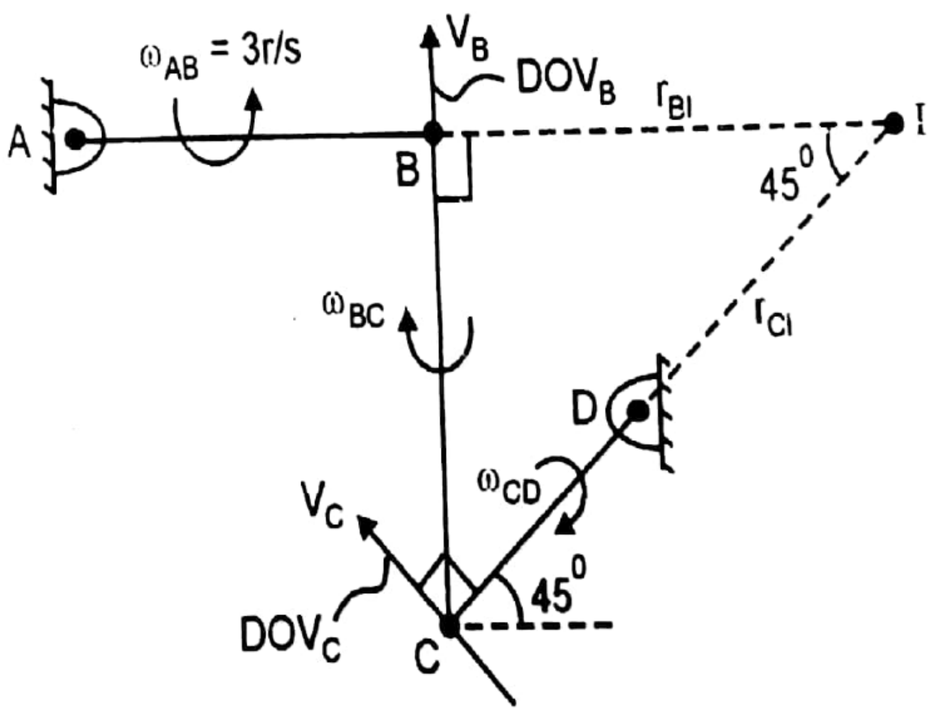
$$0.72 = 0.35 \times \omega_{BC}$$

$$\therefore \quad \omega_{BC} = 2.057 \text{ rad/sec}$$

also $v_C = r_{CI} \times \omega_{BC}$

$$= 0.495 \times 2.057$$

$$= 1.018 \text{ m/s}$$



The rod CD is rotating @ D

Using $v = r\omega$

$$v_C = r_{CD} \times \omega_{CD}$$

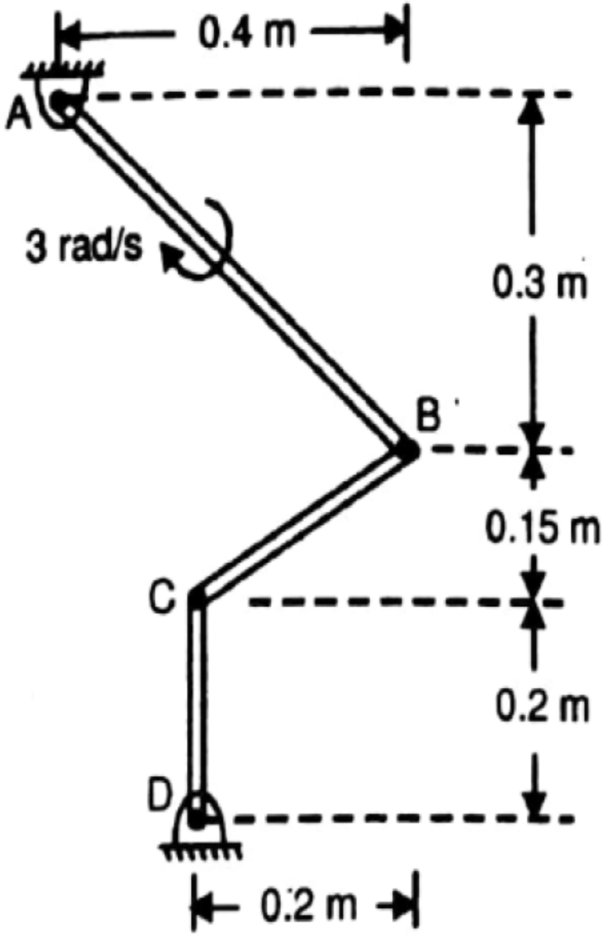
$$1.018 = 0.212 \times \omega_{CD}$$

$$\therefore \quad \omega_{CD} = 4.8 \text{ rad/sec}$$

.... **Ans.**

Problem

Figure shows a mechanism in motion. Rod AB has a constant angular velocity of 3 rad/s clockwise. Find angular velocity of rod BC and rod CD.



Solution: The system consists of three bodies in motion. Rods AB and CD perform rotation motion and rod BC performs General Plane Motion.

Rotation Motion of rod AB

Rod AB rotates about A

$\therefore$ velocity of end B $= v_B = r_{BA} \times \omega_{AB}$

$= 0.5 \times 3$

$\therefore \quad v_B = 1.5 \text{ m/s}$

also the direction of velocity of v_B is $\perp$ to radial length AB.

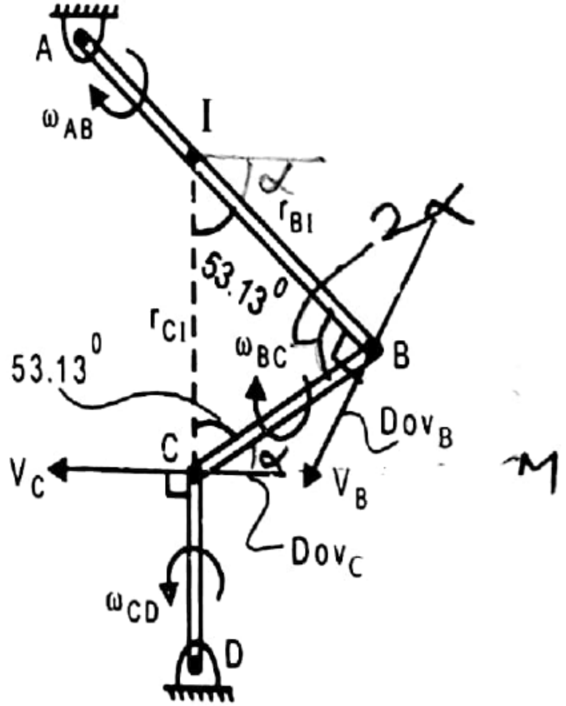
$\tan^{-1}\left(\frac{0.3}{0.4}\right) = 36.86°$

<u>General Plane Motion of rod BC</u>

Let us use Instantaneous Centre Method. Working as per the procedure discussed earlier.

1) Velocity of end B, v_B =1.5 m/s and Dov_B is $\perp$ to radial length AB.
Also direction of velocity of end C i.e. Dov_C is $\perp$ to radial length CD since C is also common to rod CD, which rotates about D.

2) Drawing the perpendiculars to the directions of velocity, Dov_B and Dov_C. Let the point of intersection be I.



3) From geometry the radial lengths r_{BI} and r_{CI} need to be worked out.

Solving Δ BCI

$$L(BC) = 0.25 \text{ m}$$

Since Δ BCI is isosceles, $r_{BI} = L(BC) = 0.25$ m

also $r_{CI} = 0.3$ m

$\frac{BC}{\sin 53.13} = \frac{CI}{\sin 36.87}$

$\therefore CI = 0.25 \times \frac{\sin 73.74}{\sin 53.13}$

$CI = 0.3 m$

$\because \angle BCm = \alpha \therefore \angle CBI = 2\alpha = 73.74$

$\therefore \angle BCI = \angle CIB = 53.13°$

$\alpha = \tan^{-1}\left(\frac{0.15}{0.2}\right)$

$\alpha = 36.869°$

$\therefore \angle BCI = 90 - 36.87$

$= 53.13°$

$\because r_{BI} = L(BC) = 0.25 m$

4) Since I is the instantaneous centre of rotation of the G.P body BC, we have

$$v_B = r_{BI} \times \omega_{BC}$$
$$1.5 = 0.25 \times \omega_{BC}$$
$$\therefore \quad \omega_{BC} = 6 \text{ rad/s} \circlearrowright \qquad \text{.........}\textbf{Ans.}$$

also
$$v_C = r_{CI} \times \omega_{BC}$$
$$= 0.3 \times 6$$
$$\therefore \quad = 1.8 \text{ m/s} \leftarrow$$

Rotation Motion of rod CD

Rod CD rotates about D

Knowing C is a common end to both rods BC and rod CD, we use $v_C = 1.8$ m/s from above

$$v_C = r_{CD} \times \omega_{CD}$$
$$1.8 = 0.2 \times \omega_{CD}$$
$$\therefore \quad \omega_{CD} = 9 \text{ rad/s} \circlearrowleft \qquad \text{.........}\textbf{Ans.}$$

Example **8** : The bar BC shown in *figure Ex.8(a)* has an angular velocity of 5 r/s clockwise when it is in the position given. Determine the angular velocity of the bar AB and also linear velocity of the point P on the bar BC.

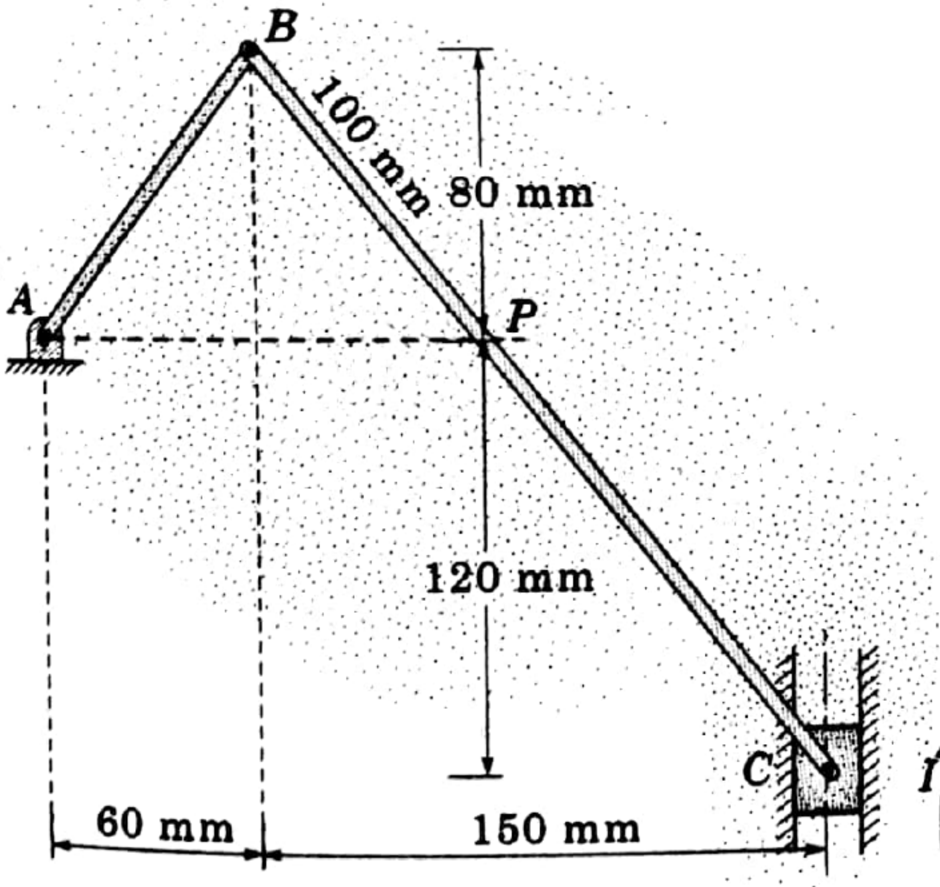


Fig. Ex.8(a)

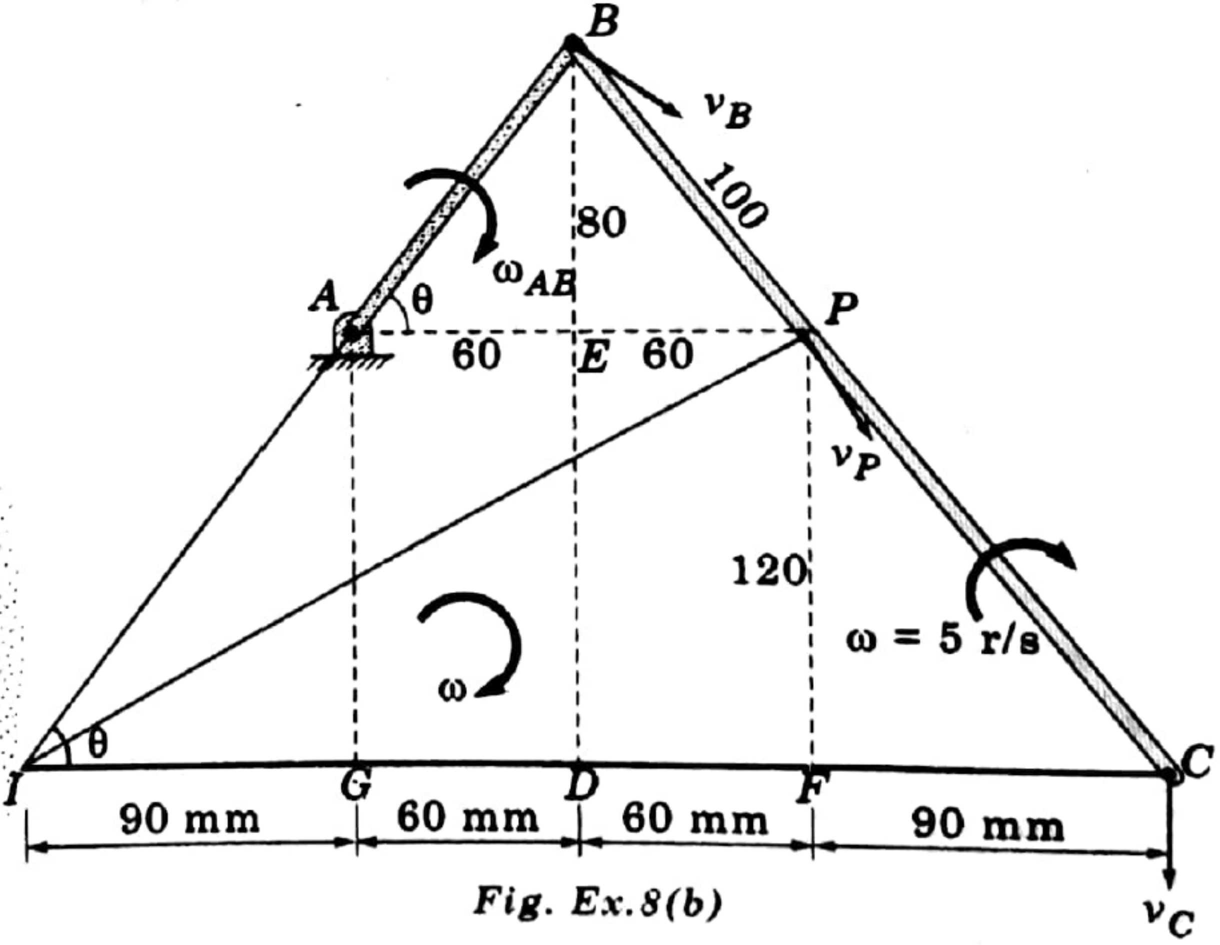


Fig. Ex.8(b)

Solution : Rod AB rotates about point A with angular velocity ω_{AB}. Velocity of point B is perpendicular to AB. Slider at C moves vertically up or down and hence v_C is in the vertical direction. ICR is located by drawing lines perpendicular to v_B [along line AB] and v_C as shown in *figure Ex.8(b)*. Now we have from *figure Ex.8(b)*.

$$v_B = AB \times \omega_{AB} = IB \times \omega$$

$$v_B = \sqrt{(60)^2 + (80)^2} \times \omega_{AB} = IB \times 5$$

$$[\because \omega_{BC} = \omega = 5 \text{ r/s given}]$$

$$\therefore 100\, \omega_{AB} = 5\, IB \qquad \text{..... (I)}$$

To find IB, we use geometry

From triangle *ABE*

$$\tan\theta = \frac{80}{60}$$

$$\therefore\ \theta = 53.130^\circ$$

From triangle *AIG*

$$\tan\theta = \frac{AG}{IG} \quad \therefore\ \tan 53.130^\circ = \frac{120}{IG}$$

$$\therefore\ IG = 90 \text{ mm}$$

From triangle *IBD*

$$\tan\theta = \tan 53.130^\circ = \frac{BD}{ID} = \frac{200}{ID} \qquad \therefore\ ID = 150 \text{ mm}$$

$$IB = \sqrt{(ID)^2 + (DB)^2} = \sqrt{(150)^2 + (200)^2} = 250 \text{ mm}$$

Put this value of *IB* in equation (I) to get

$$\therefore\ 100\,\omega_{AB} = 5\,IB = 5 \times 250$$

$\therefore$ Angular velocity of bar *AB*, $\omega_{AB} = 12.5$ r/s (↻) ... *Ans.*

To find velocity of point *P*

Join point *I* and *P*

$$\text{Now } v_P = IP \times \omega = IP \times 5 \quad \text{... (II)}$$

From triangle *IFB*

$$IP = \sqrt{(IF)^2 + (FP)^2} = \sqrt{(210)^2 + (120)^2} = 241.8677 \text{ mm}$$

Put this value in equation (II) to get

Velocity of *P*, $v_P = 241.8677 \times 5 = 1209.34$ mm/sec ... *Ans.*

Example 9 : For the link and slider mechanism shown in *figure Ex.9(a)* locate the instantaneous centre of rotation of link AB. Find also the angular velocity of link OA. Take velocity of slider at B = 2500 mm/sec.

Solution : Rod OA rotates about point O with angular velocity ω_{OA}. Velocity of point A is perpendicular to OA. Slider at B moves along the incline with velocity v_B. Drawing lines perpendicular to v_A (along line OA) and v_B to intersect at ICR I. Now we have from *figure Ex.9(b)*.

$$v_B = 2500 \text{ mm/sec} = IB \times \omega \quad \text{..... (I)}$$

$$v_A = OA \times \omega_{OA} = IA \times \omega$$

$$= 200\, \omega_{OA} = IA \times \omega \quad \text{..... (II)}$$

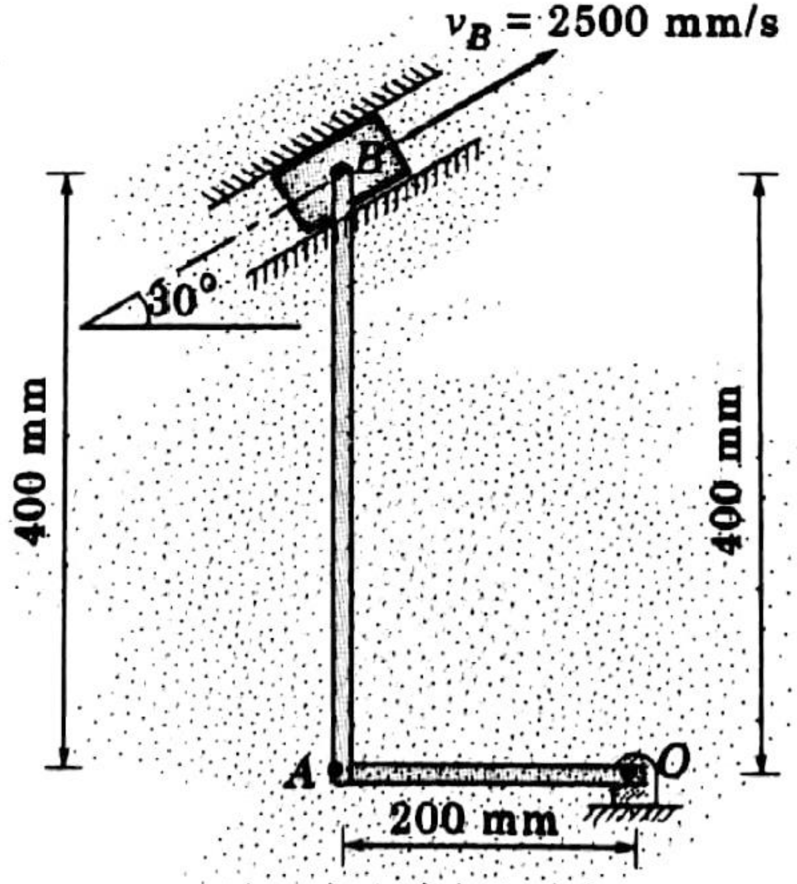


Fig. Ex.9(a)

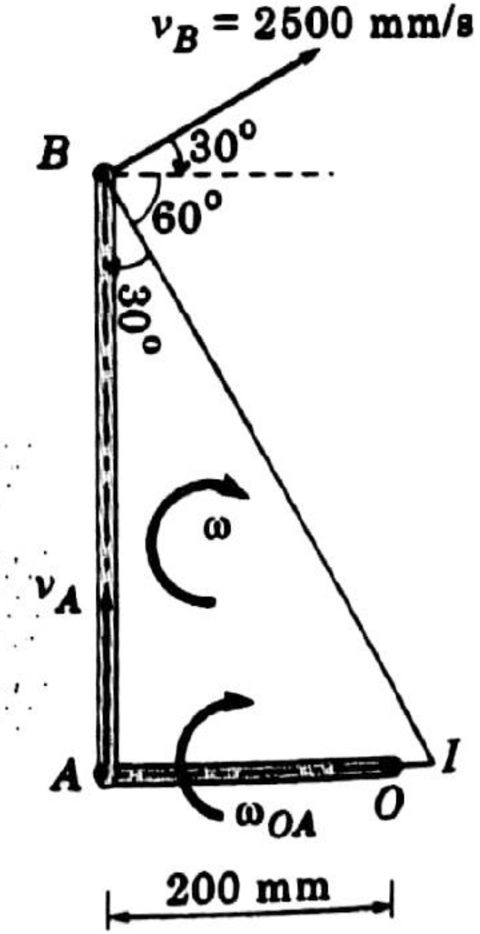


Fig. Ex.9(b)

o find lengths IA and IB, we use geometry

rom triangle IAB, of *figure Ex.9(b)*

$$\cos 30^\circ = \frac{AB}{IB} = \frac{400}{IB} \qquad \therefore\ IB = 461.88 \text{ mm}$$

$$\tan 30^\circ = \frac{IA}{AB} = \frac{IA}{400} \qquad \therefore\ IA = 230.94 \text{ mm}$$

ubstituting these values in equations (I) and (II), we get

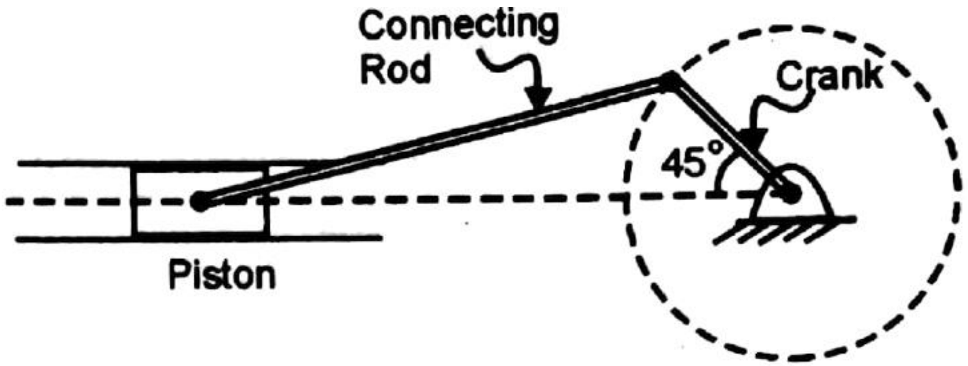
From equation (I), $2500 = 461.88 \times \omega$

Angular velocity of rod AB, $\omega = 5.4127$ r/s

From equation (II), $200\ \omega_{OA} = IA \times \omega = 230.94 \times 5.4127$

$\therefore$ Angular velocity of link OA, $\omega_{OA} = 6.25$ r/s (↻) ... *Ans.*

Ex. 13.4 In a crank and connected rod mechanism, the length of crank and connecting rod are 300 mm and 1200 mm respectively. The crank is rotating at 180 rpm anticlockwise. Find the velocity of piston, when the crank is at an angle of 45° with the horizontal.



Solution: The system consists of three bodies. The crank OA performs rotation motion about fixed axis at O, the connecting rod AB performs General Plane Motion (GPM), while the piston B performs translation motion.

Let us fist analyze rotation motion of crank OA.

Using $v = r\,\omega$

$v_A = r_{AO} \times \omega_{OA}$

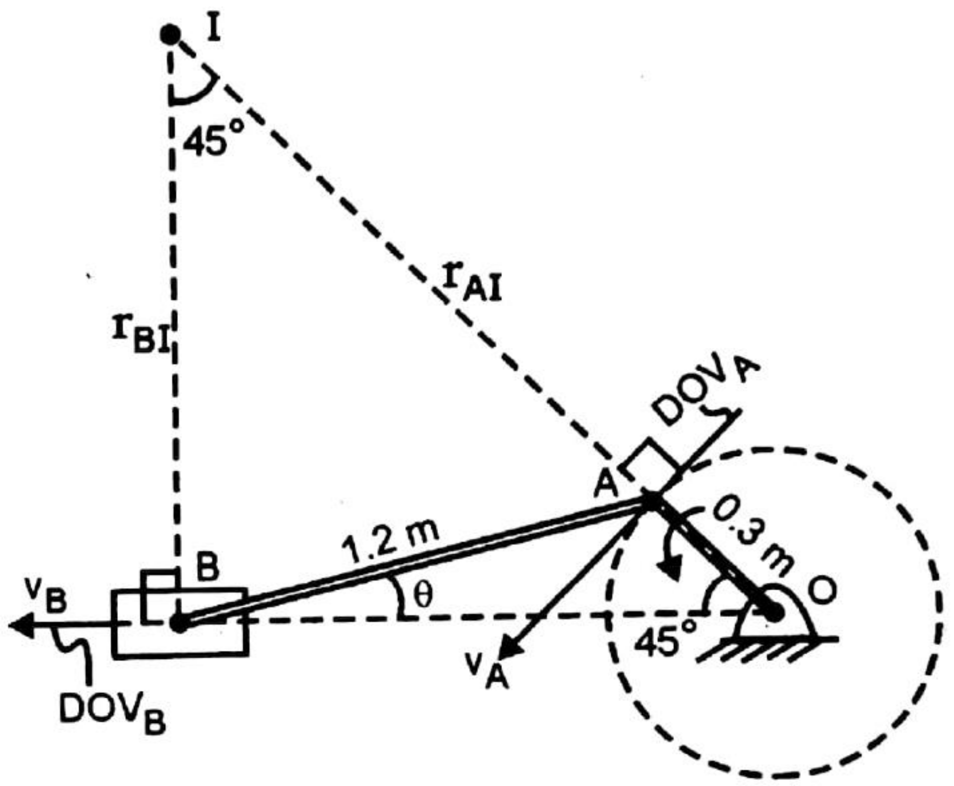
$v_A = 0.3 \times 18.849$

$= 5.655 \ \text{m/s}$ ↙

$\omega_{OA} = 180 \text{ rpm}$ ↺

$= 180 \times \dfrac{2\pi}{60}$

$= 18.849 \text{ r/s}$ ↺



General Plane Motions of rod AB

Direction of velocity of end A i.e. DOV_A is $\perp^{er}$ to radius OA.

Also direction of velocity of piston B i.e DOV_B is horizontal since it translates horizontally.

To locate the instantaneous centre of rotation I of rod AB, draw $\perp^{er}$ to DOV_A and DOV_B and get the point of intersection I as shown in figure.

The GP body AB is rotating about I at this instant.

Using $v = r\,\omega$

$$v_A = r_{AI} \times \omega_{AB}$$

$$5.655 = 1.6703 \times \omega_{AB}$$

$$\therefore \quad \omega_{AB} = 3.392 \ r/s \ \circlearrowleft$$

also $v_B = r_{BI} \times \omega_{AB}$

$$= 1.39 \times 3.392$$

$$= 4.714 \ m/s \leftarrow$$

$\therefore$ Velocity of piston

$$= 4.714 \ m/s \leftarrow$$

.... **Ans.**

From Δ ABO,

Using sine rule

$$\frac{0.3}{\sin\theta} = \frac{1.2}{\sin 45}$$

$$\therefore \quad \theta = 10.18^\circ$$

In Δ ABI

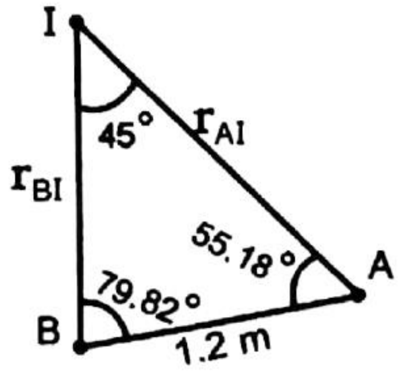
$$\angle ABI = 90 - 10.18$$

$$= 79.82^\circ$$

also

$$\angle BAI = 180 - 79.82 - 45$$

$$= 55.18^\circ$$



Using sine rule for Δ ABI

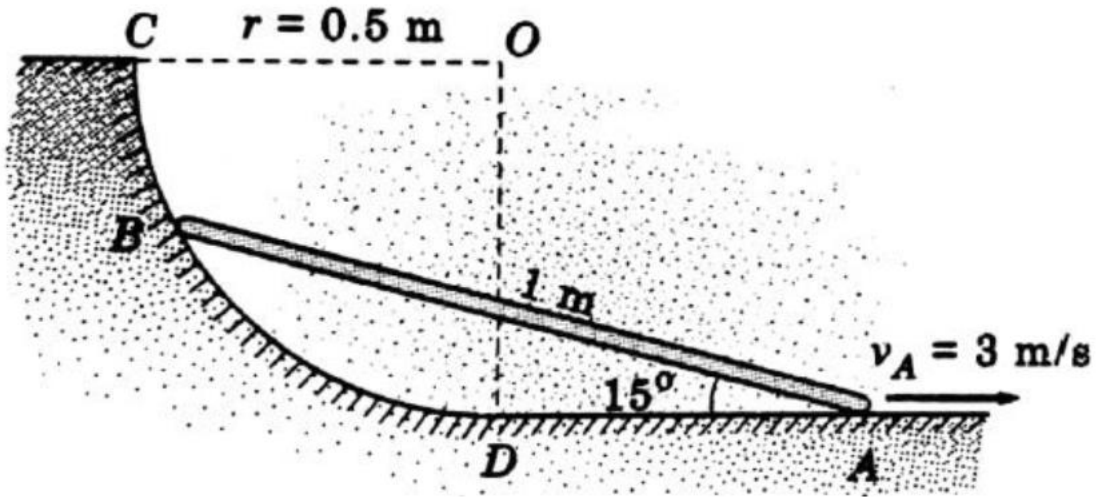
$$\frac{1.2}{\sin 45} = \frac{r_{AI}}{\sin 79.82} = \frac{r_{BI}}{\sin 55.18}$$

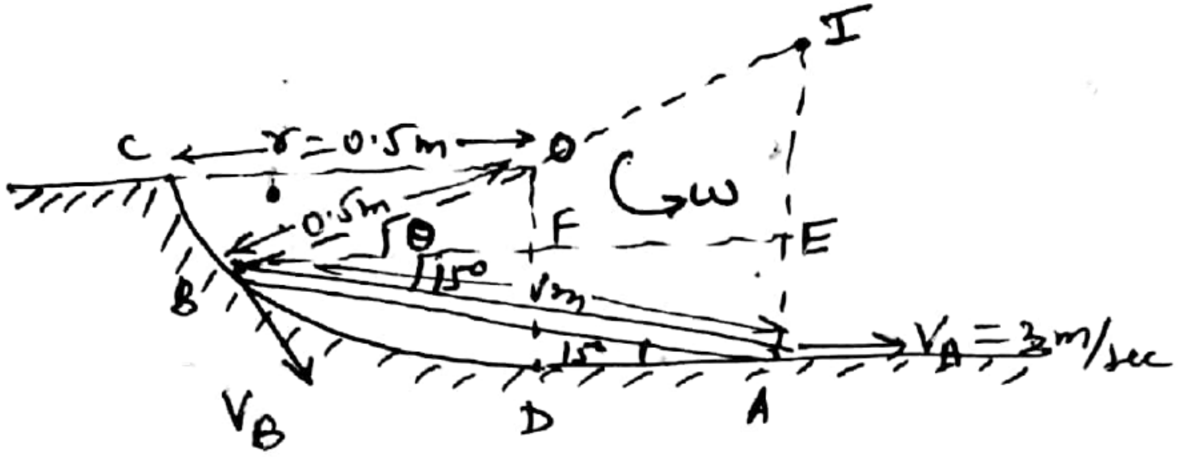
$$\therefore \quad r_{AI} = 1.6703 \ m$$

and $\quad r_{BI} = 1.39 \ m$

Problem

Bar AB is 1 m long. End A of the bar moves with a velocity of 3 m/s on the horizontal plane. End B travels along circular path CD of radius 0.5 m. Find the velocity of B for the given position.





when end A moves with vel. $V_A = 3$ m/sec to right on horizontal plane, end B travels along circular curve with velocity V_B which will be tangential to curve at B.

ICR is pt. of intersection of lines drawn $\perp^{ar}$ to vel. V_A & V_B as shown in fig.

$$\therefore\ \omega = \frac{V_A}{IA} = \frac{V_B}{IB} \Rightarrow \omega = \frac{3}{IA} = \frac{V_B}{IB} \quad \text{—— (1)}$$

$\therefore$ BE = 1 cos 15° = 0.966 m & AE = 1 sin 15° = 0.259 m

But FD = AE = 0.259 m $\therefore$ OF = OD − FD = 0.5 − 0.259

$\therefore$ OF = 0.241 m

From ΔOBF,

$$\sin\theta = \frac{OF}{OB} = \frac{0.241}{0.5} \quad \therefore \boxed{\theta = 28.816^\circ}$$

From ΔBIE,

$$\tan\theta = \frac{IE}{BE} \quad \therefore \tan 28.816 = \frac{IE}{0.966} \quad \therefore IE = 0.5314\ m$$

$$\sin\theta = \frac{IE}{IB} \Rightarrow \sin 28.816 = \frac{0.5314}{IB} \quad \therefore IB = 1.103\ m$$

But $AI = AE + IE = 0.259 + 0.5314 = 0.7904\ m$

Putting values of IE & IB in eq (1), we get

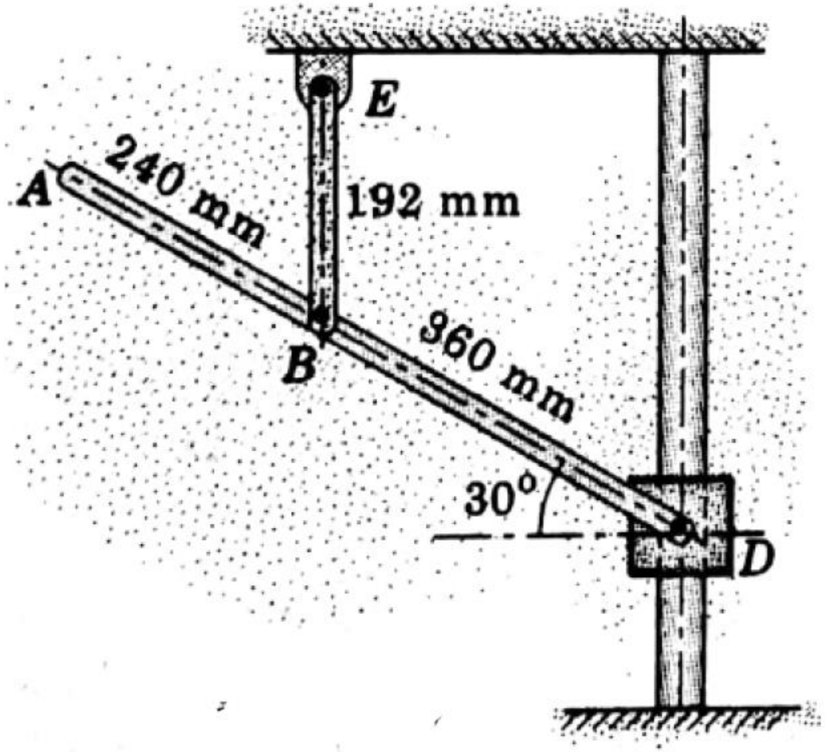
$$\omega = \frac{3}{0.7904} = \frac{V_B}{1.103}$$

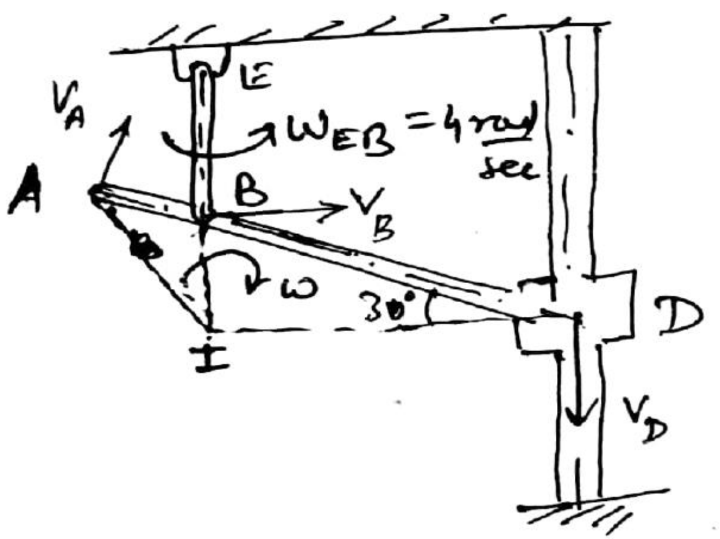
$$\therefore \omega = 3.796\ \frac{rad}{sec} \quad \& \quad V_B = 4.187\ \frac{m}{sec} \quad 61.184^\circ$$

Problem

Rod *EB* in the mechanism shown in *figure Ex.7(a)* has angular velocity of 4 r/s at the instant under observation in counter clockwise direction.

Calculate **(i)** angular velocity of rod *AD*, **(ii)** velocity of collar *D*, **(iii)** velocity of point *A*.





Rod EB rotates about E with angular vel. $W_{EB} = 4$ rad/sec. in CCW. Hence $V_B = EB \times W_{EB}$

$$\therefore V_B = 192 \times 4 = 768 \frac{mm}{sec} (\rightarrow)$$

Collar D reciprocates vertically with velocity V_D.

Locate I.

$$V_B = 768 = IB \times \omega_{AD}$$

$$\& \quad V_D = ID \times \omega_{AD} \quad \text{—— (1)}$$

ΔBID, $\sin 30° = \frac{IB}{DB} = \frac{IB}{360}$ $\quad \therefore IB = 180$ mm

$\cos 30° = \frac{ID}{DB} = \frac{ID}{360}$ $\quad \therefore ID = 311.769$ mm

$\therefore$ Putting value in (1) eq, we get

$$768 = IB \times W_{AD} = 180 \times \omega_{AD} \quad \therefore W_{AD} = 4.267 \frac{rad}{sec} \quad (2)$$

$$V_D = ID \times \omega_{AD} = 311.769 \times 4.267 = 1330.215 \ \frac{mm}{sec} \ (\downarrow)$$

To find $V_A = ?$

$$V_A = IA \times \omega_{AD}$$

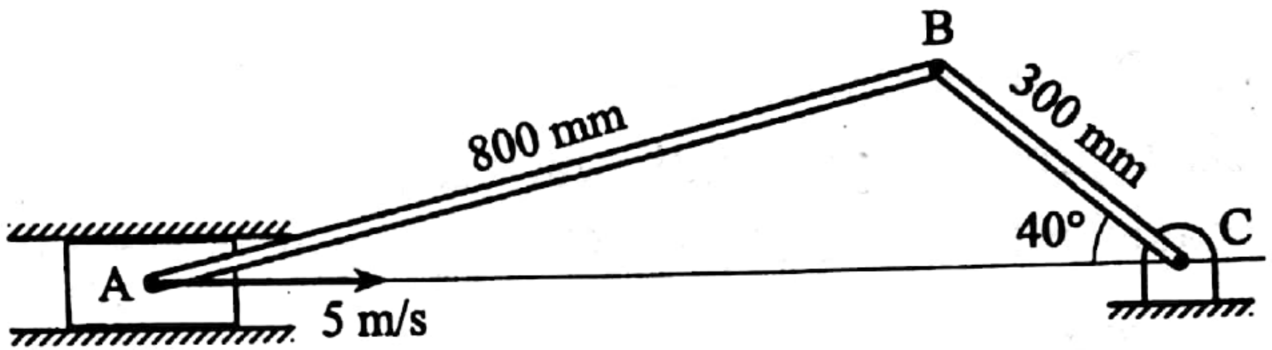
$$V_A = \sqrt{(AD)^2 + (ID)^2 - \{2(AD)(ID)\cos 30^\circ\}} \times 4.267$$

$$V_A = \left\{\sqrt{(600)^2 + (311.769)^2 - 2(600)(311.769)\cdot\cos 30^\circ}\right\} \times 4.267$$

$$\therefore V_A = 1557.31 \ \frac{mm}{sec} \ (\perp^{ar} \text{ to } IA)$$

Problem

In the crank and connecting rod mechanism shown in the figure, the crank BC is 300 mm long and the connecting rod is 800 mm long. If the piston 'A' moves at 5 m/sec. to the right when crank BC makes an angle of 40° with AC, find the angular velocities of the crank and the connecting rod.



Solution :

(i) As piston A moves towards right, show velocity V_A of end A is towards right. The rod BC is hinged at C *i.e.*, it rotates about the C. Hence, show the velocity V_B of end B perpendicular to rod BC as shown in Fig. 9.13 (a).

(ii) Erect perpendicular on velocity V_A and V_B at point A and B respectively as shown in Fig. 9.13 (a). The point of intersection of these perpendiculars is the ICR I of rod AB. The motion performed by the links in the system is as follows.

V_B is perpendicular to BC.

Rod BC performs pure rotational motion. Centre of rotation is point 'C'.

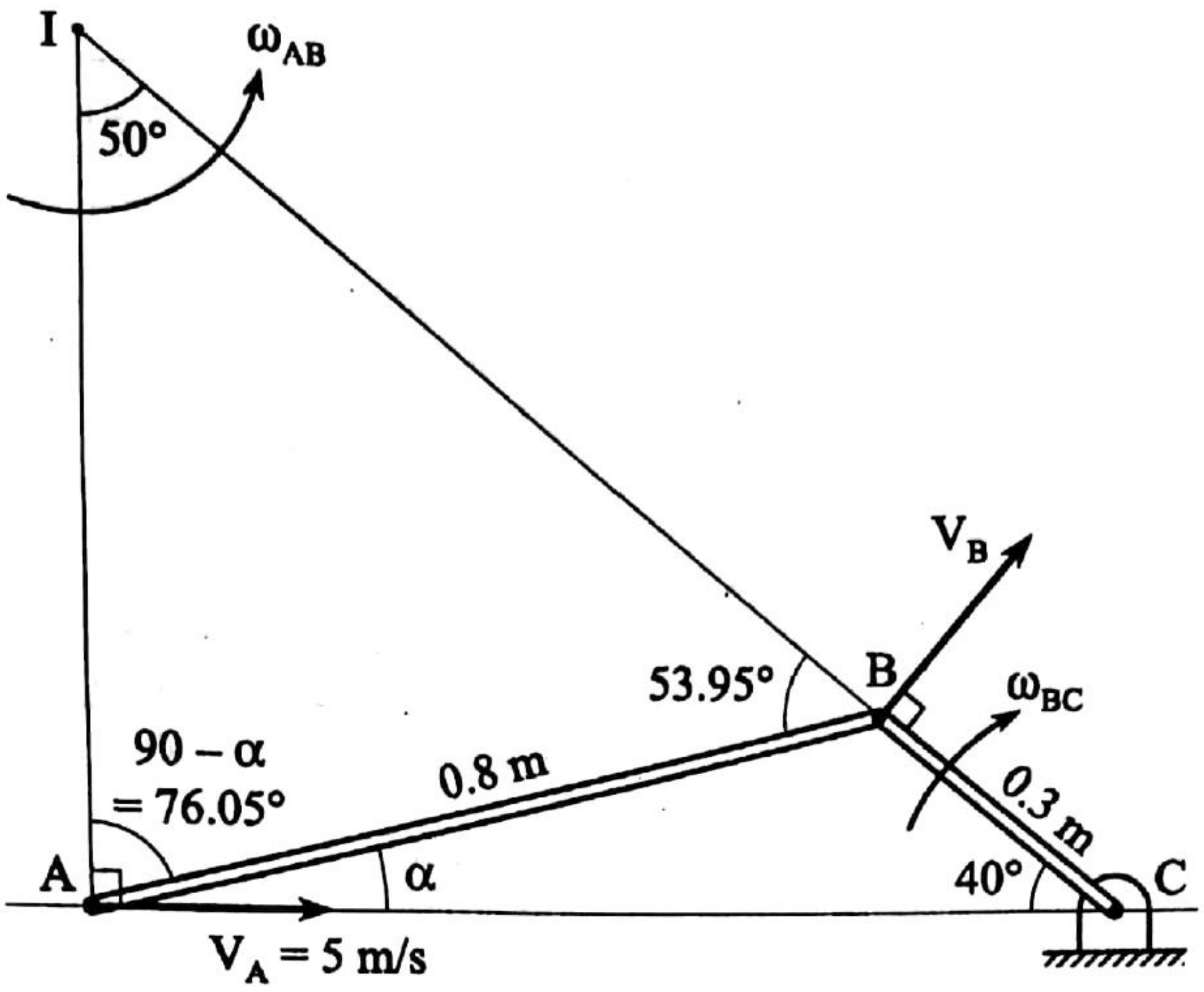
Rod AB performs General plane motion, position of ICR is 'I'.

In Δ ABC, sine rule,

$$\frac{0.3}{\sin\alpha} = \frac{0.8}{\sin 40} \qquad \therefore \qquad \alpha = 13.95^\circ$$

In Δ IAB,

$$\angle ABI = 180 - (50 + 76.05) = 53.95^\circ$$



In Δ IAB, sine rule,

$$\frac{IA}{\sin 53.95^\circ} = \frac{IB}{\sin 76.05^\circ} = \frac{AB}{\sin 50^\circ} \quad \text{.... [AB = 0.8 m]}$$

$$\therefore \quad IA = 0.844 \text{ m};$$

$$IB = 1.013 \text{ m}$$

For Rod AB :

$$V_A = IA \times \omega_{AB} \qquad \therefore \quad 5 = 0.844 \times \omega_{AB}$$

$$\therefore \quad \omega_{AB} = \mathbf{5.924 \ r/s} \ (\circlearrowleft) \qquad \text{.................... Ans.}$$

$$V_B = IB \times \omega_{AB}$$

$$= 1.013 \times 5.924 = 6 \text{ m/s}$$

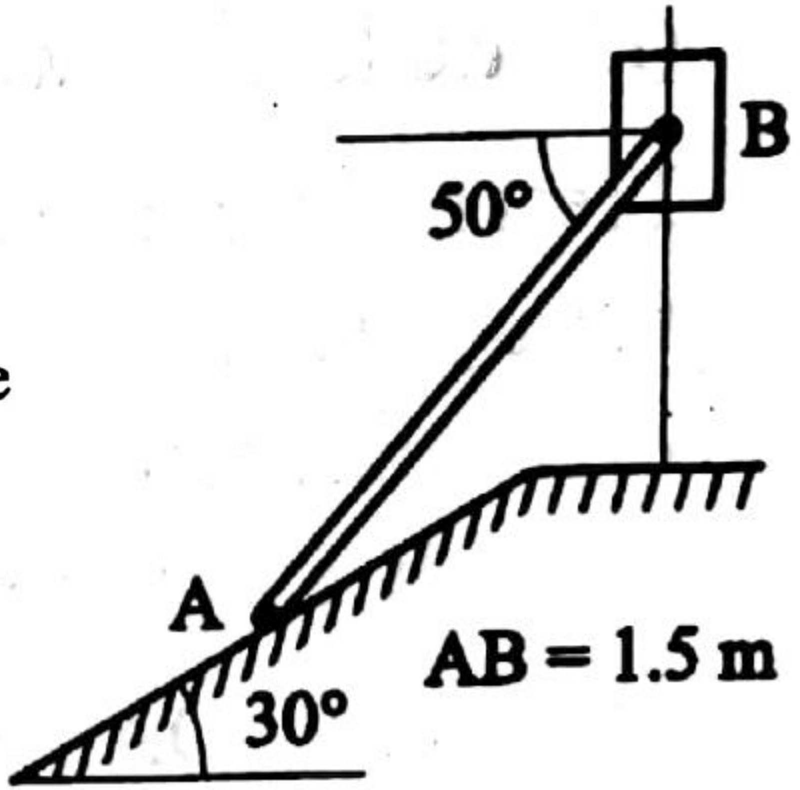
For Rod BC :

$$V_B = BC \times \omega_{BC} \qquad \therefore \quad 6 = 0.3 \times \omega_{AB}$$

$$\therefore \quad \omega_{BC} = \mathbf{20 \ r/s} \ (\circlearrowright) \qquad \text{.................... Ans.}$$

Problem

Collar B moves up with constant velocity $V_B = 2$ m/s. Rod AB is pinned at B. Find out angular velocity of AB and velocity of A.



Solution :

Erect perpendiculars on velocity V_A and V_B and locate ICR I of rod AB as shown in Fig. 9.19 (a).

In Δ IAB, by sine rule

$$\frac{1.5}{\sin 60} = \frac{IA}{\sin 50} = \frac{IB}{\sin 70}$$

$\therefore$ $IA = 1.326\ m$; $IB = 1.627\ m.$

$V_B = IB \times \omega_{AB}$

$\therefore$ $2 = 1.627 \times \omega_{AB}$

$\therefore$ $\omega_{AB} = \mathbf{1.229\ r/s}$ (↺)

$V_A = IA \times \omega_{AB} = 1.326 \times 1.229$

$\therefore$ $V_A = \mathbf{1.629\ m/s}$ ($\angle 30°$)

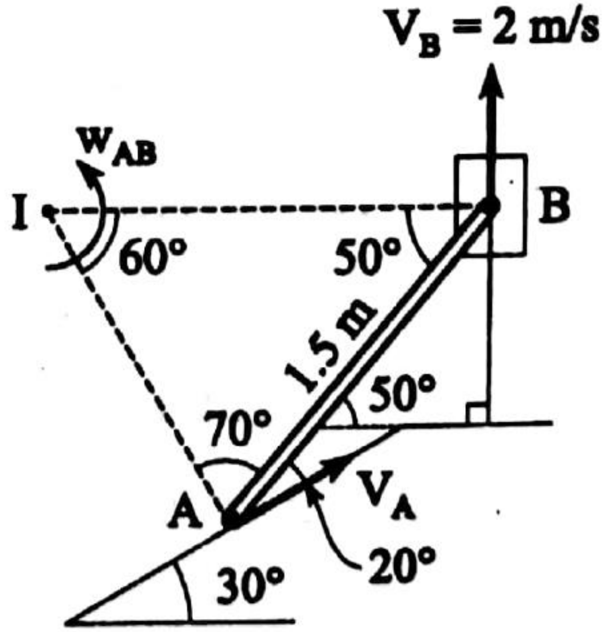


Fig. 9.19 (a)

Problem

For the link and slider mechanism shown in figure, locate the instantaneous centre of rotation of link AB. Also find the angular velocity of link OA. Take velocity of slider at B = 2500 mm/sec.

Solution :

Motion performed by the links is as follows :

(i) OA - pure rotation centre of rotation is point O.

(ii) Perform general plane motion, ICR is at I.

In Δ IAB,

$$\sin 60^\circ = \frac{AB}{IB}$$

$$\therefore \quad IB = \frac{AB}{\sin 60} = \frac{400}{\sin 60} = 461.88 \text{ mm}$$

$$\cos 60^\circ = \frac{IA}{IB}$$

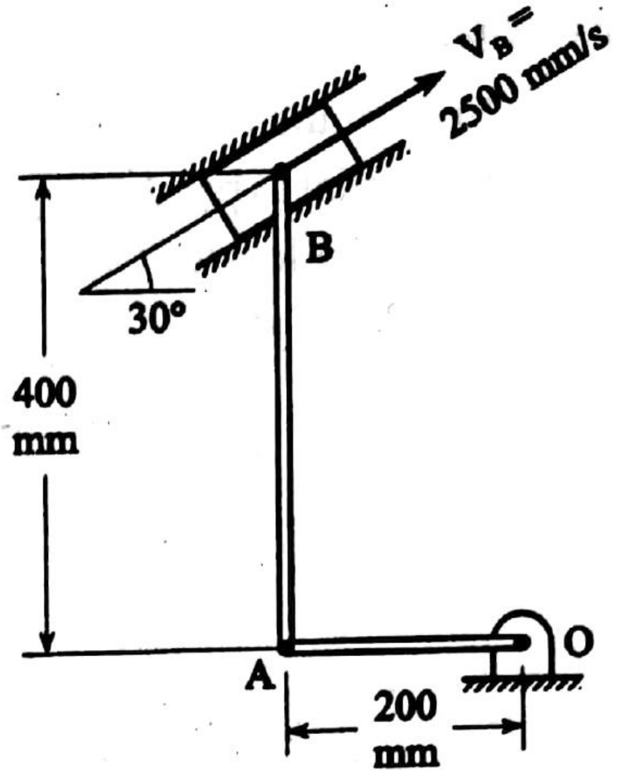


Fig. 9.20

$$\therefore \quad IA = IB \cos 60 = 461.88 \cos 60^\circ$$

$$\therefore \quad IA = 230.94 \text{ mm}$$

For Rod AB :

$$V_B = IB \times \omega_{AB}$$

$$\therefore \quad 2500 = 461.88 \times \omega_{AB}$$

$$\therefore \quad \omega_{AB} = 5.413 \text{ r/s} (\circlearrowright)$$

$$V_A = IA \times \omega_{AB} = 230.94 \times 5.413$$

$$\therefore \quad V_A = 1250.07 \text{ m/s}$$

For Rod OA :

$$V_A = OA \times \omega_{OA}$$

$$\therefore \quad 1250.07 = 200 \times \omega_{OA}$$

$$\therefore \quad \omega_{OA} = 6.25 \text{ r/s} (\circlearrowright)$$

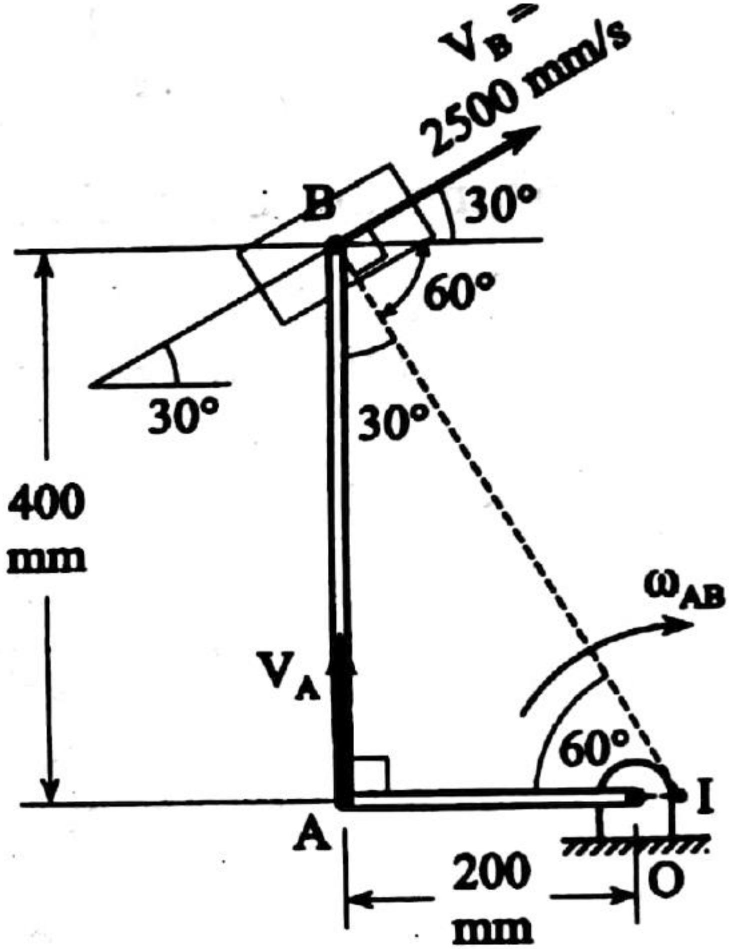


Fig. 9.20 (a)

Problem

A bar AB, 24 cm long, is hinged to a wall at A as shown in Fig. 13.23(a). Another bar CD 32 cm long is connected to it by a pin at B such that $CB = 12$ cm and $BD = 20$ cm. At the instant shown, ($AB \perp CD$) the angular velocities of the bars are $\omega_{AB} = 4$ rad/sec and $\omega_{CD} = 6$ rad/sec. Determine the linear velocities of C and D. (**Hint:** Bar CD is in plane motion.)

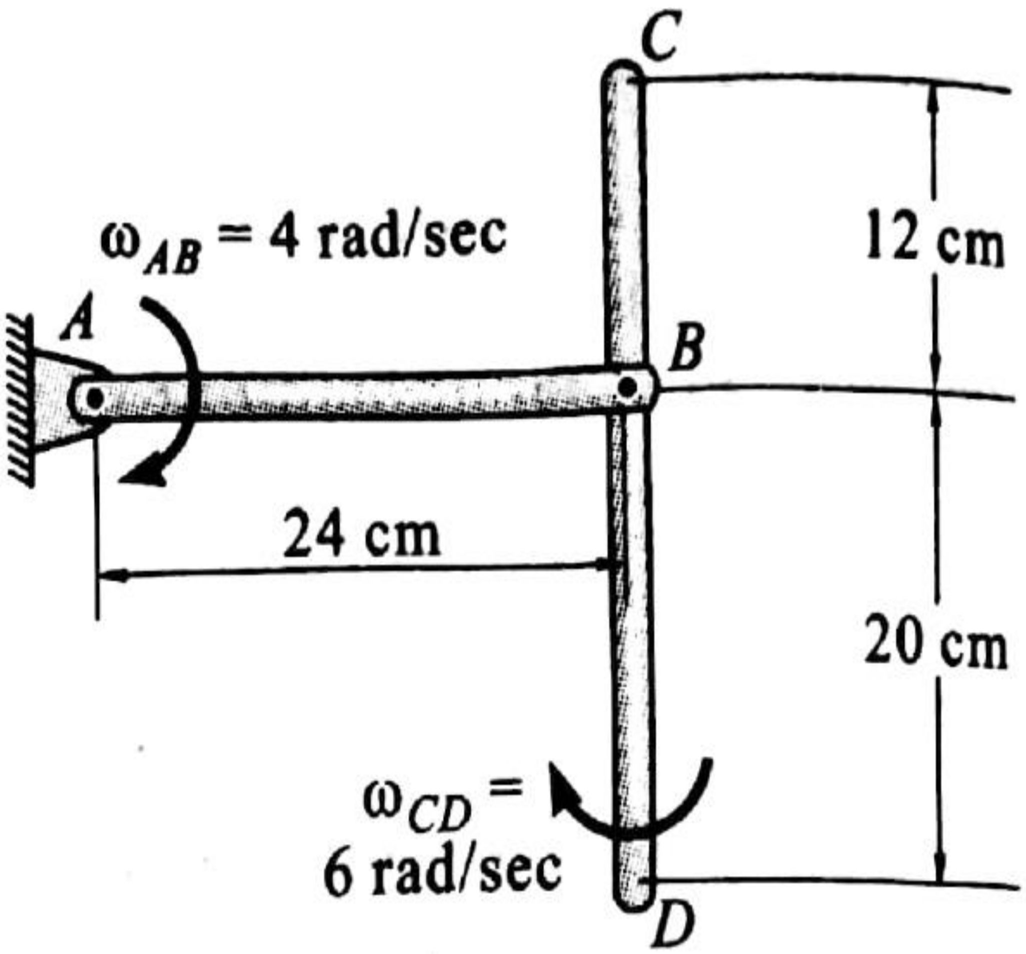


Fig. 13.23(a)

(i) Rod *AB* (Performs rotational motion about point *A*)

$$\therefore \ v_B = (AB)\,\omega_{AB} = (24)\,(4)$$

$$v_B = 96 \text{ cm/sec } (\downarrow)$$

(ii) Rod *CD* (Performs general plane motion)

Let us assume the point *I* to be ICR

$$v_B = (IB)\,\omega_{CD}$$

$$IB = \frac{96}{6} = 16 \text{ cm}$$

$$IC = \sqrt{16^2 + 12^2} = 20 \text{ cm}$$

$$v_C = (IC)\,(\omega_{CD}) = 20 \times 6$$

$$v_C = 120 \text{ cm/sec } (\theta) \quad \textit{Ans.}$$

$$v_D = (ID)\,(\omega_{CD}) = \left(\sqrt{16^2 + 20^2}\right)(6)$$

$$v_D = 153.67 \text{ cm/sec } (\alpha) \quad \textit{Ans.}$$

Fig. 13.23(a)

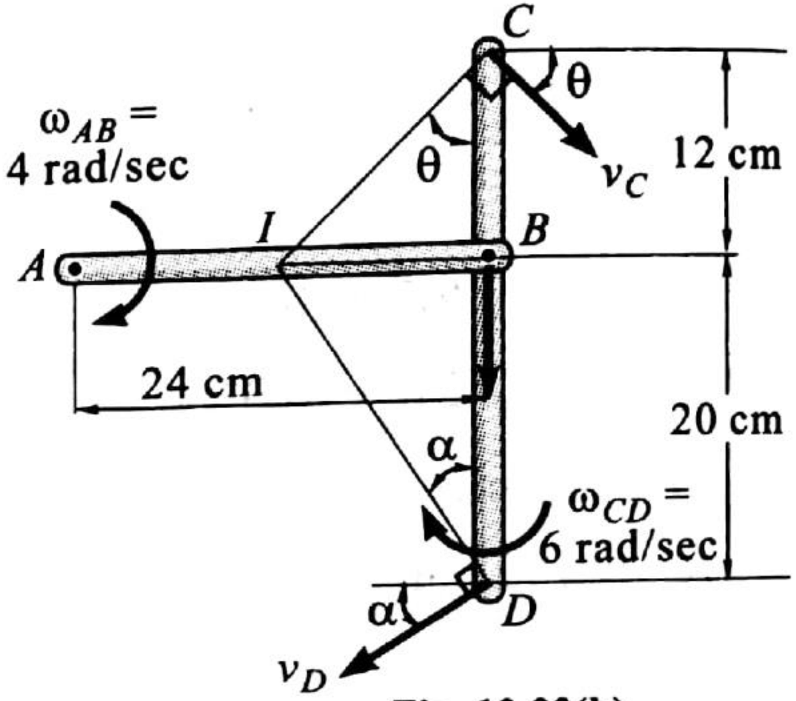


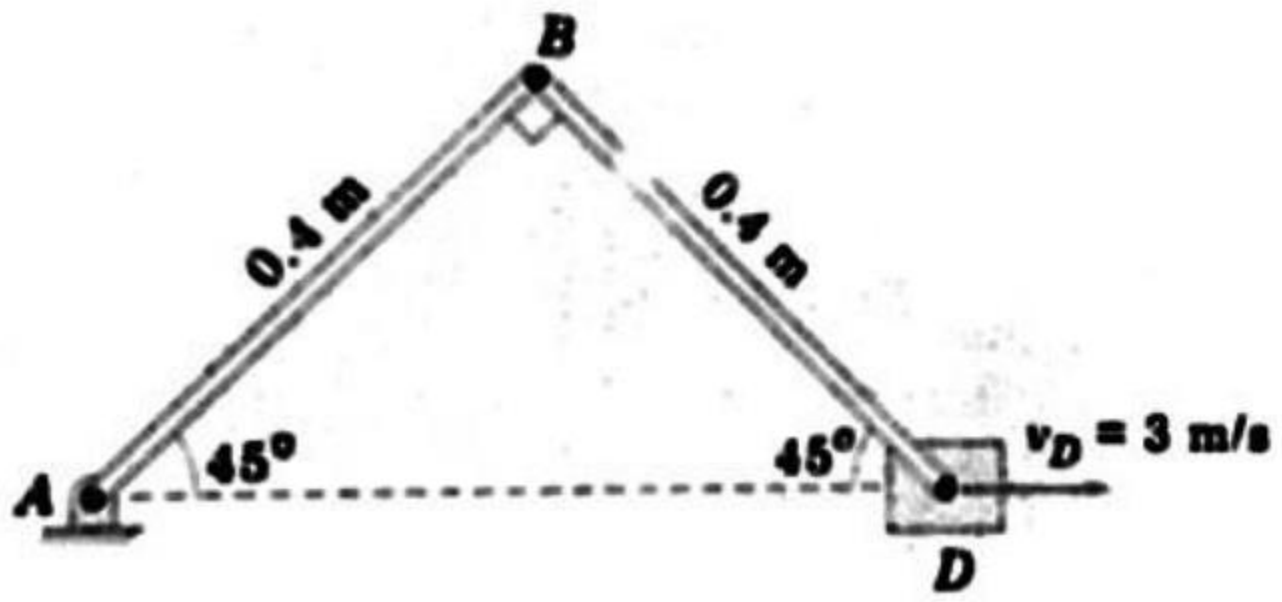
Fig. 13.23(b)

$$\tan\theta = \frac{16}{12} \quad \therefore \ \theta = 53.13^\circ$$

$$\tan\alpha = \frac{16}{12} \quad \therefore \ \alpha = 38.66^\circ$$

Problem

Block D shown in the figure given below, moves with a speed of 3 m/s. Determine the angular velocity of the links BD **(3 marks)** and AB **(3 marks)** and the velocity of point B **(4 marks)** at the instant known. Take length of AB = BD = 0.4 m.



Solution : Rod AB rotates with angular velocity ω_{AB} about point A. Linear velocity of point B is perpendicular to AB. Point D has velocity $v_D = 3$ m/s to the right. ICR is located by drawing lines perpendicular to v_B (extension of AB) and v_D as shown in *figure Ex.6(b)*. Now we have from *figure Ex.6(b)*.

$$v_B = AB \times \omega_{AB} = IB \times \omega$$

$$\therefore \; v_B = 0.4\,\omega_{AB} = IB \times \omega \qquad \text{----- (I)}$$

$$v_D = ID \times \omega$$

$$3 = ID \times \omega \qquad \text{----- (II)}$$

To find IB and ID, we use geometry

From triangle ABD

$$AD = \sqrt{(0.4)^2 + (0.4)^2} = 0.5659 \text{ m}$$

From triangle AID

$$\tan 45° = \frac{ID}{AD} \qquad \therefore\; 1 = \frac{ID}{0.5659} \qquad \therefore\; ID = 0.5659 \text{ m}$$

Also,

$$\cos 45° = \frac{AD}{AI} = \frac{0.5659}{AI} \quad \therefore\; AI = 0.8 \text{ m} \quad \therefore\; IB = AI - 0.4$$

$$\therefore\; IB = 0.8 - 0.4 = 0.4 \text{ m}$$

Substituting these values in equation (I) and (II)

$$\text{From (II)}, \quad 3 = 0.5659 \times \omega$$

$$\therefore \quad \omega = 5.30 \text{ r/s}$$

$$\text{From (I)}, \quad 0.4\,\omega_{AB} = IB \times \omega = 0.4 \times 5.30 = v_B$$

$$\therefore \quad \omega_{AB} = 5.3 \text{ r/s} \quad \text{and } v_B = 2.12 \text{ m/s}$$

Angular velocity of link BD, $\omega = 5.3$ r/s ... *Ans.*

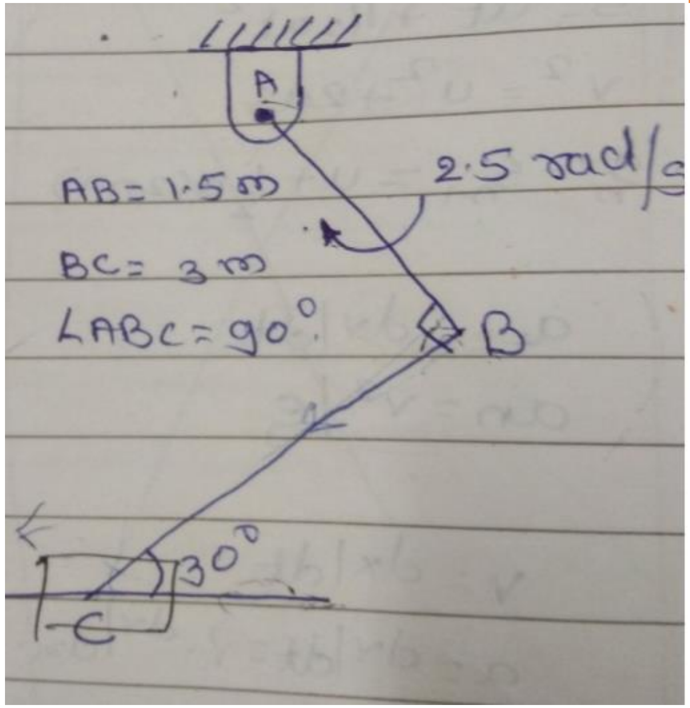
Angular velocity of link $AB = \omega_{AB} = 5.3$ r/s ... *Ans.*

Velocity of point $B = v_B = 2.12$ m/s ($45°$, v_B) ... *Ans.*

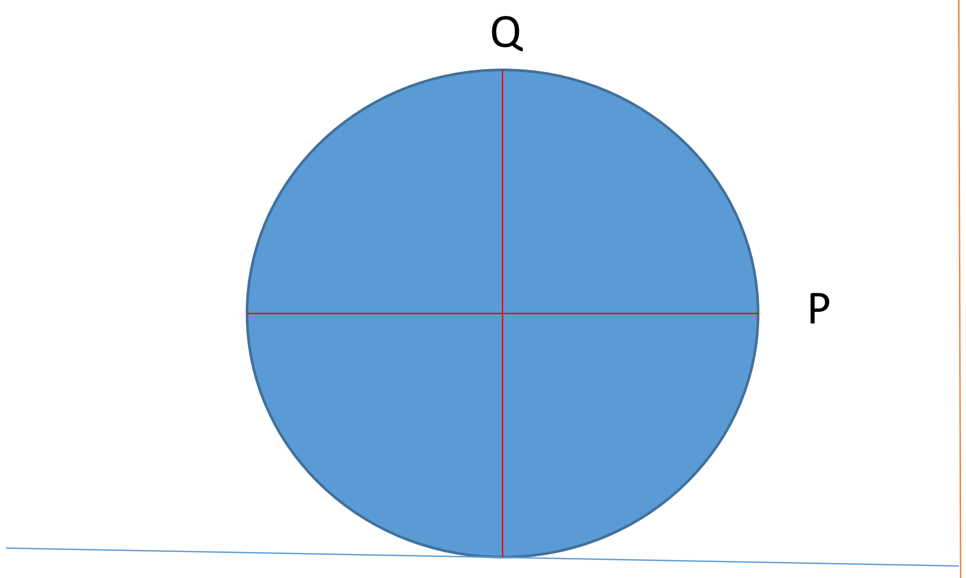
Problems

- A rod AB 26 m long leans against a vertical wall. The end A on the floor is drawn away from the wall at a rate of 24 m/s. When the end A of the rod is 10 m from the wall, determine the velocity of B sliding down vertically and the angular velocity of the rod.

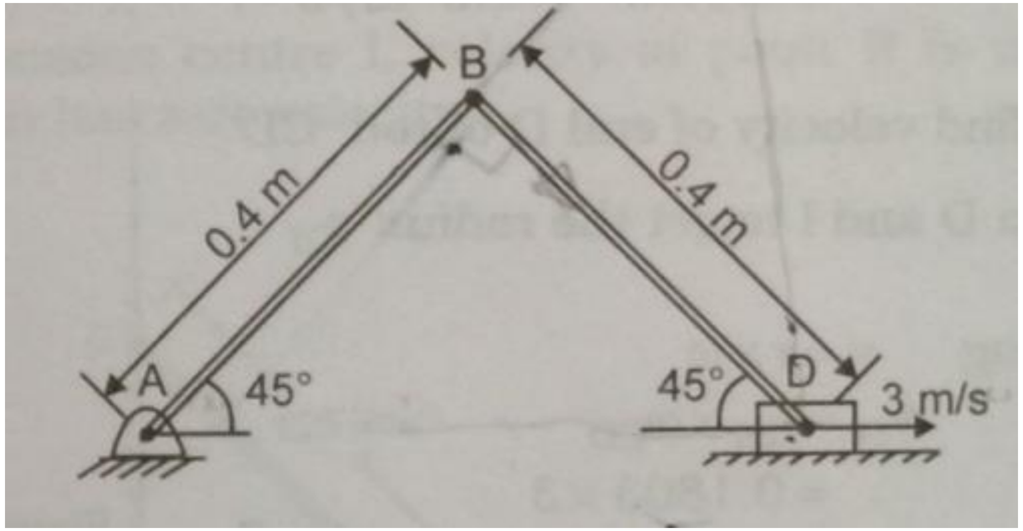
- At the instant shown in figure, the rod AB is rotating clockwise at 2.5 rad/sec. If the end C of the rod BC is free to move on horizontal surface, find the angular velocity of the point C.



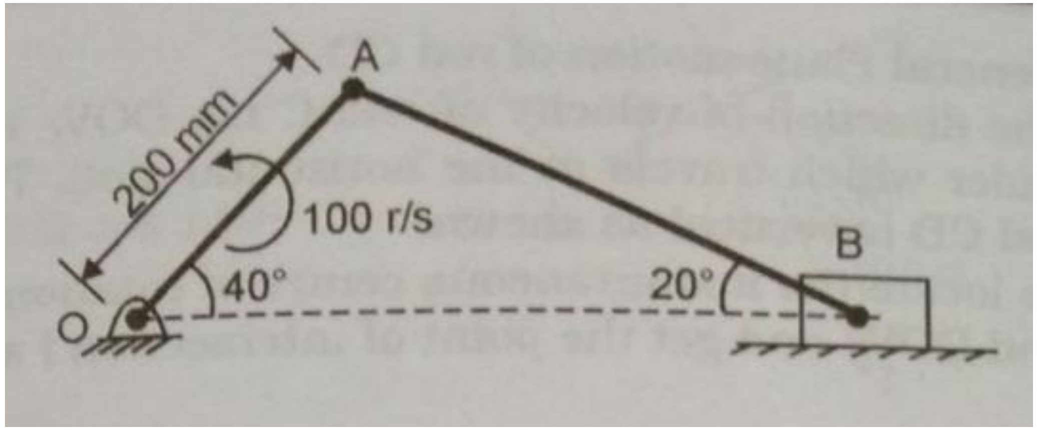
- A wheel of radius o.75 m rolls without slipping on a horizontal surface to right. Determine the velocities of the points P and Q shown in figure when the velocity of the wheel is 10 m/s towards right.



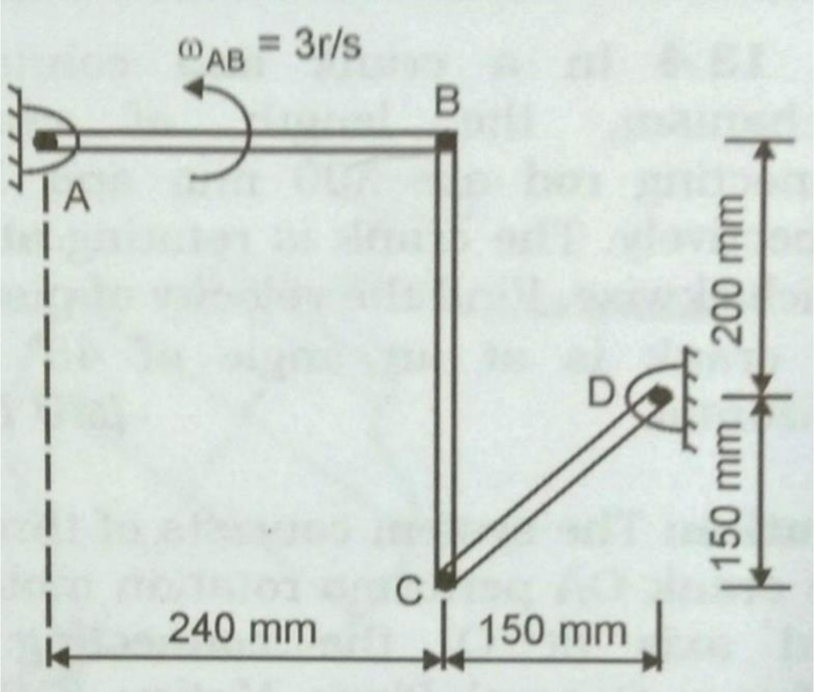
- Block D shown in figure moves with a speed of 3 m/s. Determine the angular velocities of link BD and AB and the velocity of point B at the instant shown.



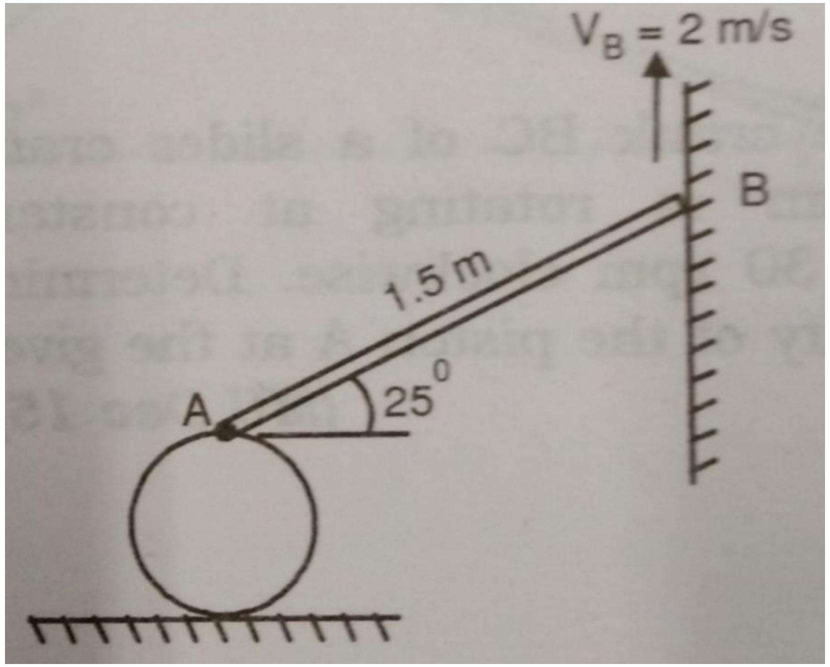
- A slider crank mechanism is shown in the figure. The crank OA rotates anticlockwise at 100 rad/sec. Find the angular velocity of the rod AB and the velocity of the slider B.



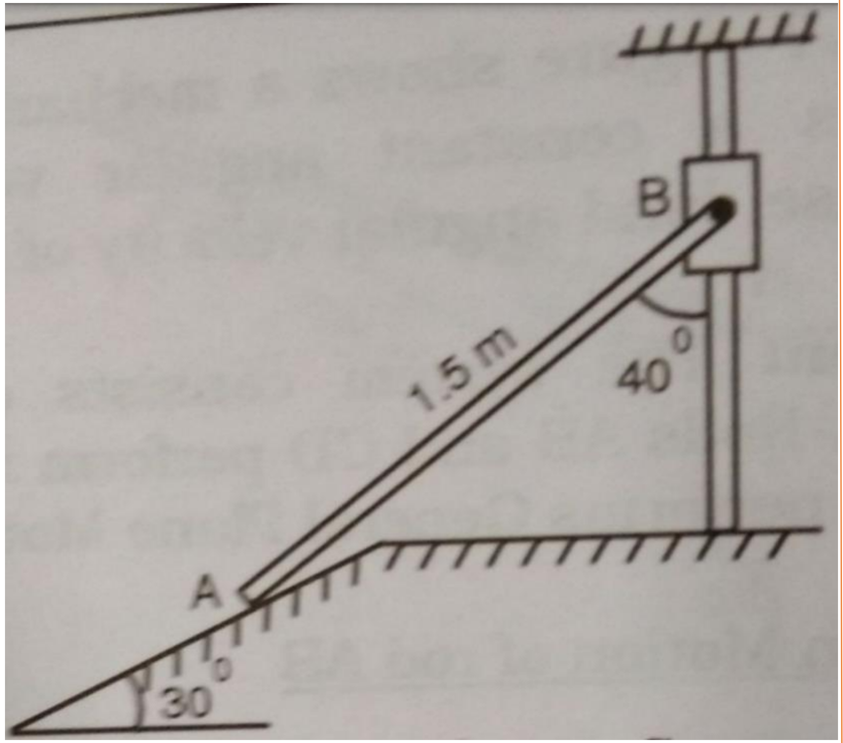
- In the position shown, bar AB has constant angular velocity of 3 rad/sec anticlockwise, determine the angular velocity of bar CD.



- One end of rod AB is pinned to the cylinder of diameter 0.5 m while the other end slides vertically up the wall with a uniform speed 2 m/s. For the instant, when the end A is vertically over the center of the cylinder, find the angular velocity of the cylinder, assuming it to roll without slip.



- Figure shows a collar B which moves up with constant velocity of 2 m/s. To the collar is pinned a rod AB, the end A of which slides freely against a 30° sloping ground. For this instant, determine the angular velocity of the rod and velocity of end A of the rod.



- Locate the Instantaneous center of rotation for the link ABC and determine the velocity of points B and C. Angular velocity of rod OA is 15 rad/sec counter clock wise. Length of OA is 200 mm, AB is 400 mm and BC is 150 mm.