Module 3: Matrices, Rank, System of eq^n

Practice Problems

Q. 1 Express following Matrices as sum of Symmetric & skew Symmetric Matrices.

1) $\begin{bmatrix} 2 & -4 & 9 \\ 14 & 7 & 13 \\ 3 & 5 & 11 \end{bmatrix}$ 2) $\begin{bmatrix} 3+i & 2i & 5 \\ -7 & 1-i & -3i \\ 5-i & 2+3i & 2 \end{bmatrix}$ 3) $\begin{bmatrix} i & -i & 3 \\ 2 & 1+i & 7 \\ 3+2i & 2i & 1-i \end{bmatrix}$

Q. 2 Express following Matrices as Sum of Hermitian & Skew Hermitian Matrices & check your Result

1) $\begin{bmatrix} 2-i & 3+i & 3i \\ 2 & 5 & 4-i \\ -5 & 2-i & 3+i \end{bmatrix}$ 2) $\begin{bmatrix} 2 & 3-i & i \\ 0 & 1-i & 1+2i \\ 1 & 3i & 2 \end{bmatrix}$ 3) $\begin{bmatrix} 3+i & 6i & 4-i \\ -1+2i & -i & -3-2i \\ -1-i & 1+2i & 4+i \end{bmatrix}$

Q. 3 Express A as P+iQ where P & Q are Hermitian

1) $\begin{bmatrix} 1+3i & 2 & 3i-2 \\ 2+5i & 2i & 1-2i \\ 3+2i & 0 & 5 \end{bmatrix}$ 2) $\begin{bmatrix} 2i & -3 & 1-i \\ 0 & 2+3i & 2-5i \\ 7i & 1-i & 1+i \end{bmatrix}$ 3) $\begin{bmatrix} 1+i & 0 & 1-i \\ 3+i & 1+2i & 2+i \\ 1-i & 3i & 4i \end{bmatrix}$

Q. 4 Express following Hermitian Matrix as P+iQ where P is real Symmetric and Q is real skew Symmetric.

1) $\begin{bmatrix} 5 & 2-i & -1+i \\ 2+i & 3 & -2i \\ -1-i & 2i & 2 \end{bmatrix}$ 2) $\begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 0 & i \\ 2+3i & -i & 5 \end{bmatrix}$ 3) $\begin{bmatrix} 1 & 2+3i & 5i \\ 2-3i & 2 & 1+i \\ -5i & 1-i & -3 \end{bmatrix}$

Q. 5 Express following Skew Hermitian Matrix as P+iQ where P is real skew Symmetric & Q is real Symmetric. check your result.

1) $\begin{bmatrix} i & 1-i & -2+3i \\ -1-i & 2i & 5i \\ 2+3i & 5i & -i \end{bmatrix}$ 2) $\begin{bmatrix} 2i & -5i & -1-i \\ -5i & -i & -3-2i \\ 1-i & 3-2i & 0 \end{bmatrix}$ 3) $\begin{bmatrix} i & -3i & -1+3i \\ -3i & 2i & 2-i \\ 1+3i & -2-i & 3i \end{bmatrix}$

Q. 6 [There are 5 result of following type where proof is expected.]
Prove that Every Hermitian matrix A can be written as P+iQ where P is real symmetric & Q is real skew Symmetric. also this expression is Unique.

Q.7 ~~Express~~ Prove that following matrices are orthogonal & Hence find A^{-1}

1) $\frac{1}{\sqrt{6}}\begin{bmatrix} 0 & \sqrt{3} & \sqrt{3} \\ 2 & 1 & -1 \\ \sqrt{2} & -\sqrt{2} & \sqrt{2} \end{bmatrix}$ 2) $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 3) $\frac{1}{11}\begin{bmatrix} 2 & 6 & -9 \\ 6 & 7 & 6 \\ 9 & -6 & -2 \end{bmatrix}$

Q.8 If A is orthogonal Then find. a, b, c where.

$3A = \begin{bmatrix} a & b & c \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$ [2, 2, 1]

Q.9 Find a, b, c If $\frac{1}{9}\begin{bmatrix} a & 1 & b \\ c & b & 7 \\ 1 & a & c \end{bmatrix}$ is orthogonal
[±8, ±4, ±4]

Q.10 Prove that $\frac{1}{3}\begin{bmatrix} 2+i & 2i \\ 2i & 2-i \end{bmatrix}$ is Unitary & Find A^{-1}

Q.11 If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ Then Show that ~~$(I+N)^{-1}$~~
$(I-N)(I+N)^{-1}$ is Unitary matrix

Q.12 Find rank by Reducing Matrices to row echelon form

1) $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$ (r = 2) 2) $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 4 & 7 \\ 4 & -1 & 7 \\ 2 & 1 & 5 \end{bmatrix}$ (r = 2) 3) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 8 & 5 & 14 & 17 \\ 1 & 5 & 5 & 7 \end{bmatrix}$ (r = 2)

4) $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 4 \\ 2 & 5 & 11 & 6 \end{bmatrix}$ (r = 3) 5) $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 2 & 5 & 9 & 7 \\ 1 & 4 & 3 & 0 \end{bmatrix}$

Q.13 Reduce following Matrices to Normal form & find their rank.

1) $\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ (r = 3) 2) $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & 1 \\ 3 & 1 & 0 & 3 \end{bmatrix}$ (r = 2) 3) $\begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 4 & 3 & 7 & 10 & 17 \end{bmatrix}$ (r = 2)

4) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 4 & 3 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ (r = 3) 5) $\begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix}$ (r = 4) 6) $\begin{bmatrix} 3 & 1 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 1 \\ 2 & 4 & 6 & 8 \end{bmatrix}$ (r = 4)

Q.14 Find possible values of k for which rank of A is 1, 2, 3 where $A = \begin{bmatrix} k & 4 & 4 \\ 4 & k & 4 \\ 4 & 4 & k \end{bmatrix}$

Q.15 for Matrix A, Find value of k for which rank of A is 3, 2 or 1

$$A = \begin{bmatrix} 2 & 3k & 3k+4 \\ 1 & k+4 & 4k+2 \\ 1 & 2k+2 & 3k+4 \end{bmatrix}$$

Q.16 for real value of x, Find rank of $A = \begin{bmatrix} x-1 & x+1 & x \\ -1 & x & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(rank 3)

Q.17 Find rank of $A = [a_{ij}]_{3\times3}$ where $a_{ij} = i/j$. (rank 1)

$$\left[\text{Hint i.e. } A = \begin{bmatrix} 1/1 & 1/2 & 1/3 \\ 2/1 & 2/2 & 2/3 \\ 3/1 & 3/2 & 3/3 \end{bmatrix} \right]$$

Q.18 Find non-singular matrices P & Q such that PAQ is in normal form & find rank where matrices are.

1) $\begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$ 2) $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ 3) $\begin{bmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{bmatrix}$

($r = 3$)

Q.19 Find P & Q such that PAQ is normal. Hence find A^{-1}

1) $\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ 2) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

($r = 3$) ($r = 3$)

Q Test for consistancy the following equations and if possible solve them.

① $2x_1 + x_3 = 4$, $x_1 - 2x_2 + 2x_3 = 7$, $3x_1 + 2x_2 = 1$

(Ans: $x_1 = 2 - \frac{t}{2}$, $x_2 = \frac{3}{4}t - \frac{5}{2}$, $x_3 = t$) $\cancel{(x_1 = 1 - 3t}$

② $x_1 - x_2 + 2x_3 + x_4 = 2$, $3x_1 + 2x_2 + x_4 = 1$

$4x_1 + x_2 + 2x_3 + 2x_4 = 3$ $\left(x_1 = \frac{5 - 4t_1 - 5t_2}{5}, \ x_2 = \frac{-5 + 6t_1 + 2t_2}{5}\right.$

$\left. x_3 = t_1, \ x_4 = t_2 \right)$

③ $x_1 + 3x_2 - x_3 = 4$, $2x_1 + x_2 + x_3 = 7$, $2x_1 - 4x_2 + 4x_3 = 6$

$3x_1 + 4x_2 = 11$ $\left(x_1 = \frac{17 - 4t}{5}, \ x_2 = \frac{1 + 3t}{5}, \ x_3 = t\right)$

④ $2x_1 + x_2 - x_3 + 3x_4 = 11$, $x_1 - 2x_2 + x_3 + x_4 = 8$

$4x_1 + 7x_2 + 2x_3 - x_4 = 0$, $3x_1 + 5x_2 + 4x_3 + 4x_4 = 17$

$(x_1 = 2, \ x_2 = -1, \ x_3 = 1, \ x_4 = 3)$

⑤ $2x - y + z = 9$, $3x - y + z = 6$, $4x - y + 2z = 7$, $-x + y - z = 4$

(Inconsistant)

⑥ $6x + y + z = -4$, $2x - 3y - z = 0$, $-x - 7y - 2z = 7$

$(x = -1, \ y = -2, \ z = 4)$

⑦ $2x_1 + x_2 - x_3 + 3x_4 = 8$, $x_1 + x_2 + x_3 - x_4 = -2$

$3x_1 + 2x_2 - x_3 = 6$, $4x_1 + 3x_2 + 2x_4 = -8$

$(x_1 = 2, \ x_2 = -1, \ x_3 = -2, \ x_4 = 1)$

⑧ $2x - 3y + 7z = 5$, $3x + y - 3z = 13$, $2x + 19y - 47z = 32$

(Inconsistant)

⑨ $2x_1 - 3x_2 + 5x_3 = 1$, $3x_1 + x_2 - x_3 = 2$

$x_1 + 4x_2 - 6x_3 = 1$,

$x_1 = \frac{7}{11} - \frac{2t}{11}$ $x_2 = \frac{1}{11} + \frac{17t}{11}$ $x_3 = t$

10) Discuss for all values of K the system of equations.

$2x + 3Ky + (3K+4)z = 0$, $x + (K+4)y + (4K+2)z = 0$

$x + 2(K+1)y + (3K+4)z = 0$ $(K = \pm 2)$

11) Investigate for what value of λ and μ the equations $2x+3y+5z=9$, $7x+3y-2z=8$, $2x+3y+\lambda z=\mu$ have (i) No solution (ii) a unique solution (iii) an infinite number of solutions. Find solution for (iii) case:

12) For what value λ the equations $x+2y+z=3$, $x+y+z=\lambda$. $3x+y+3z=\lambda^2$ have a solution and solve them completely in each case?

13) For what value of λ the equations

$x+y+4z=1$, $2x+2y+3z=5$, $\lambda x+3y+6z=4$

will have ~~no~~ ~~have~~ no unique sol^n? will the equation have any solution for this value of λ?

14) For what value λ the equations

$\lambda x+2y-2z=1=0$, $4x+2\lambda y-z=2$, $6x+6y+\lambda z=3$

have (i) a unique solution (ii) infinity of solutions? Find the ~~set~~ solutions in the second case.

15) Find the values of K for which the following system of equations has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

$Kx+y+z=1$, $x+Ky+z=1$, $x+y+Kz=1$

16) Find the values of λ for which the system of eq^n

$x+y+4z=1$, $x+2y-2z=1$, $\lambda x+y+z=1$

will have (i) unique solution (ii) no solution?

Q2 Solve the following equations

(1) $x_1 + x_2 + x_3 + x_4 = 0$, $2x_1 + x_2 - x_4 = 0$

$x_1 + 3x_2 + 2x_3 + 4x_4 = 0$ $(t, -t, -t, t)$

(2) $7x_1 + x_2 - 2x_3 = 0$ $x_1 + 5x_2 - 4x_3 = 0$

$3x_1 - 2x_2 + x_3 = 0$ $2x_1 - 7x_2 + 5x_3 = 0$

(3)

$\left(\frac{3t}{17}, \frac{13}{17}t, t\right)$

(3) $3x_1 + 4x_2 - x_3 - 9x_4 = 0$, $2x_1 + 3x_2 + 2x_3 - 3x_4 = 0$

$2x_1 + x_2 - 14x_3 - 12x_4 = 0$, $x_1 + 3x_2 + 13x_3 + 3x_4 = 0$

$(11t, -8t, t, 0)$

(4) $4x - y + \cancel{3z}\ 2z + t = 0$ $2x + 3y - z - 2t = 0$

$7y - 4z - 5t = 0$ $2x - 11y + 7z + 8t = 0$

$\left(\frac{-5t_1 - t_2}{7}, \frac{4t_1 + 5t_2}{7}, t_1, t_2\right)$

(5) $x_1 + 2x_2 + 3x_3 = 0$ $2x_1 + 3x_2 + x_3 = 0$ $4x_1 + 5x_2 + 4x_3 = 0$

$x_1 + 2x_2 - 2x_3 = 0$ $(x_1 = x_2 = x_3 = 0)$

(6) $3x + y - 5z = 0$ $5x + 3y - 6z = 0$ $x + y - 2z = 0$

$x - 5y + z = 0$ $(x = y = z = 0)$

(7) Find the value of λ for which the following equation have non-zero solutions, obtain the solution for real values of λ.

$x + 2y + 3z = \lambda x$, $3x + y + 2z = \lambda y$, $2x + 3y + z = \lambda z$

$(\lambda = 6, x = y = z = t)$

(8) Show that the system of equations

$ax + by + cz = 0$, $bx + cy + az = 0$, $cx + ay + bz = 0$

has a non-trivial solution if $a + b + c = 0$ or if $a = b = c$

Find the non-trivial solution when the condition is satisfied.