International Astronomy and Astrophysics Competition

Pre-Final Round 2024

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Important: Read all the information on this page carefully!

General Information

- We recommend to print out this problem sheet. Use another paper to draft the solutions to the problems and write your final solution (with steps) on the provided space below the problems.
- You may use extra paper if necessary, however, the space under the problems is usually enough.
- Typing the solution on a computer is allowed but not recommended (no extra points).
- The six problems are separated into three categories: 2x basic problems (A; four points), 2x advanced problems (B; six points), 2x research problems (C; eight points). The research problems require you to read a short scientific article to answer the questions. There is a link to the PDF article.
- You receive points for the correct solution **and** for the performed steps. Example: You will not get all points for a correct value if the calculations are missing.
- Make sure to **clearly** mark your final solution values (e.g. underlining, red color, box).
- You can reach up to 36 points in total. You qualify for the final round if you reach at least 18 points (junior, under 18 years) or 24 points (youth, over 18 years).
- It is not allowed to work in groups on the problems. Help from teachers, friends, family, or the internet is prohibited. Cheating will result in disqualification! (Textbooks and calculators are allowed.)

Uploading Your Solution

- Please upload a file/pictures of (this sheet with) your written solutions: https://iaac.space/login
- Only upload **one single PDF file!** If you have multiple pictures, please compress them into one single file. Do not upload your pictures in a different format (e.g, no Word and Zip files).
- The deadline for uploading your solution is **Sunday 2. June 2024, 23:59 UTC+0**.
- The results of the pre-final round will be announced on Monday 10. June 2024.

Good luck!

Problem A.1: Rotation of the Earth (4 Points)

Rockets allow us to launch spacecraft and satellites into space, which are an essential part of our modern world. However, rocket launches need careful planning, and some places on Earth provide better launch conditions than others.

(a) Explain why most rocket launches take place close to the equator.
(b) Find an equation $v(\varphi)$ that calculates the rotational speed v of the Earth at latitude φ.
(c) Calculate the rotational speed at 5°S (near the equator) and 80°N (near the pole).

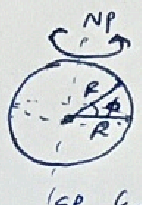
Note: The Earth's radius is 6371 km.

(a) Any rocket that needs to leave Earth's gravity needs a Minimum required Velocity (V-Escape) ≈ 11.2 Km/s. Due to rotation of Earth (West to East), any object on the surface has a tangential velocity due East. This velocity is dependent on latitude [$v(\phi)$ in part b] and is Maximum near Equator.

→ Thus a rocket launched near equator has a higher pre-launch velocity and thus requires less thrust velocity and power to reach orbit or exits Earth's influence than a rocket launched near poles.

→ This can also be thought of as a reduced effective acceleration due to Earth gravity at lower latitudes because of Centrifugal force from Earth's rotation. So $g_{eq} < g_{Pole}$, thus requiring less thrust for rocket launches near Equator.

(b)

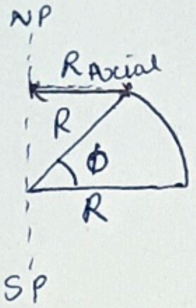


Earth

Since Earth has a rigid body rotation around an assumed fixed Axis. The tangential velocity $V_\perp$ is

$$V_\perp = R_{Axial}\,\omega \quad - \text{①}$$

where R_{axial} is distance of Surface from the axis and ω is the ~~axial~~ angular velocity of rotation.



From trignometric Relations:

$$R_{Axial} = R\cos\phi.$$

Hence the eqn ① becomes.

$$\boxed{v_{rot}(\phi) = (R\cos\phi)\,\omega_{rot}}$$

(c) We first calculate ω_{rot} for Earth.
Since Earth completes $360^\circ = 2\pi$ radians in 1 Sidereal day which is 23.93 hrs = 86164.1 sec.

$$\text{So} \quad \omega_{rot} = \frac{2\pi}{86164.1} = \frac{\text{Angle}}{\text{time period}} = 7.292\times10^{-5}\ \text{rad/sec}.$$

① now at $\phi = 5^\circ S$ (near equator) for $R_{Earth} = 6371\,Km$

$$V_{rot}(5^\circ S) = \frac{6371\,(\cos 5^\circ)\times 2\pi}{86164.1} = 0.4629\ Km/sec$$

② at $\phi = 80^\circ N$ (near poles)

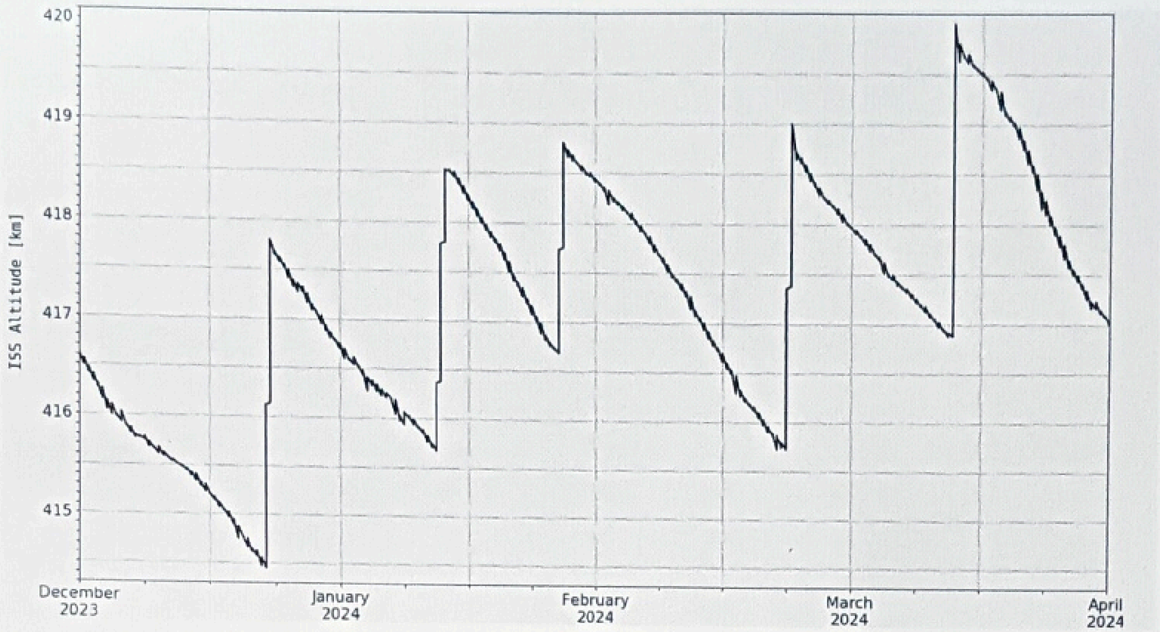
$$V_{rot}(80^\circ N) = \frac{6371\,(\cos 80^\circ)\times 2\pi}{86164.1} = 0.0807\ Km/sec$$

$$\boxed{v(5^\circ S) = 0.4629\ Km/sec \ ; \ v(80^\circ N) = 0.0807\ Km/sec}$$

→ the obtained values clearly show a higher tangential velocity near equator than near the poles.
This corroborates my answer in part (a).

Problem A.2: Altitude of the ISS – Part 1 (4 Points)

The International Space Station (ISS) does not orbit the Earth at a perfectly constant altitude. Instead, the ISS changes its altitude over time due to collisions with atmospheric particles (downwards) and boosters that are used to adjust the orbit (upwards). The diagram below displays the ISS's altitude above the ground for the last months (December 2023 to April 2024).[1]



(a) How much did the ISS descend and ascend between December 2023 and April 2024?

(b) Determine the average *rate of descent* of the ISS.

Space is often considered to start at an altitude of 100 km above ground (the *edge of space*).

(c) How long would it take for the ISS to naturally descend to the *edge of space*?

(a) Between Dec'23 and April'24, the ISS moved from 416.6 Km to 417.0 Km altitude above Earth Surface. But there were 6 Descents and 5 Ascension maneuvers, with Minimum altitude = 414.5 Km and Maximum altitude = 420.0 Km.

[1]A high-resolution version of the diagram is available online: https://iaac.space/A2-AltitudeISS.png

(b) Calculating average rate of Descent.

S. No.	Min. Alt (Km)	Max. Alt (km)	Duration (days)	Rate of Descent (km/day)
1.	414.5	416.6	21.5	0.0977
2.	415.7	417.8	20	0.1050
3.	416.7	418.5	14	0.1286
4.	415.7	418.8	26.5	0.1170
5.	416.9	419.0	19	0.1105
6.	417.0	420.0	18	0.1667*

* 6th descent is not used to calculate mean

→ Now we calculate the average rate of descent for ISS from all given descents (except 6th which has an anomalously high rate).

$$\boxed{\text{Avg. Rate of Descent for ISS} = 0.112 \text{ Km/day}}$$

(c) Given the Edge of space at altitude = 100 Km.

The time to descend naturally from current altitude of 417 Km to 100 Km would be.

$$\Rightarrow \text{Time} = \frac{\text{Initial Alt.} - \text{Final Alt.}}{\text{Rate of Descent}} = \frac{417.0 - 100.0}{0.112}$$

$$\boxed{\text{Time of Descent} = 2830.36 \text{ days} = 7.754 \text{ yrs.}}$$

You can view the Jupyter Notebook (Python Code) I used to Calculate Rate of Descent and time of descent here: https://github.com/AkshankTyagi/IAAC/blob/main/ISS_Descent.ipynb

Problem B.1: Altitude of the ISS – Part 2 (6 Points)

As mentioned in Problem A.2, the ISS loses altitude due to the collision with atmospheric particles. This causes the ISS to experience a drag force F_d according to the drag equation

$$F_d = \frac{1}{2} \cdot \rho \cdot C_d \cdot A \cdot v^2$$

with the atmospheric density ρ, the dimensionless drag coefficient C_d, the ISS's cross-sectional area A, and the ISS's speed relative to the particles v.

Use the rate of descent from Problem A.2 to estimate

(a) the vacuum density ρ at the current position of the ISS and

(b) the total amount of matter (in kg) which collides with the ISS every day.

Note: The following constants may be helpful: the gravitational constant: 6.67×10^{-11} m^{3}kg^{-1}s^{-2}, the mass of Earth: 5.97×10^{24} kg, the drag coefficient of the ISS: 1.3, the cross-sectional area of the ISS: 4800 m^2, the total mass of the ISS: 450 tons.

(a) We know that gravitational force provides the required centripetal force to keep ISS in a stable Earth orbit. Thus a stable orbit at altitude = 417.0 km follows the condition:

$$\rightarrow F_{cent} = \frac{mv^2}{r} = F_g = \frac{G M_e m}{r^2} \quad — \text{①}$$

here m is mass of ISS, M_e is mass of Earth, v is velocity of ISS in orbit.

and $r = R_{earth} + \text{altitude} = 6371 + 417 = 6788$ Km

$$\text{So } v = \sqrt{\frac{G M_e}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6788 \times 10^3}}$$

$$\boxed{v \approx 7{,}659 \text{ m/s} = 7.66 \text{ Km/s}}$$

→ now using eqn ①

$$mv^2 = \frac{G M_e m}{r}$$

taking derivatives on both sides w.r.t time.

$$\Rightarrow 2mv\dot{v} = -\frac{GM_e m}{r^2}\dot{r}$$

now $F_d = -m\dot{v} = +\frac{GM_e m}{2vr^2}\dot{r} = \frac{+1}{2}\rho C_d A v^2$ (given)
(Drag Force)

$$\Rightarrow \rho = +\frac{GM_e m}{v^3 r^2} \times \frac{\dot{r}}{C_d A}$$

here the constants are given v and r were calculated earlier and $\dot{r}$ (rate of descent) calculated in A.2.b.

$$\rho = \frac{\underbrace{6.67\times10^{-11}m^3}_{G}\underbrace{(5.97\times10^{24})}_{M_e}\underbrace{(4.5\times10^5)}_{m}\times\underbrace{(0.112\ Km/sec)}_{\dot{r}}}{\underbrace{(7659\ m/s)^3}_{v}\underbrace{(6788\ Km)^2}_{r}\underbrace{1.3}_{C_d}\underbrace{(4800\ m^2)}_{A}}$$

$$\boxed{\rho = 1.554\times10^{-7}\ Kg/m^3}$$

(b) to calculate amount of matter that collides with ISS every day.

-> Distance ISS travels perday $= v \times 1$ day.

$= 7659 \times 86400$

$D = 6.617\times10^8\ m$

-> Total Mass of colliding particles $= \rho \cdot A \cdot D$

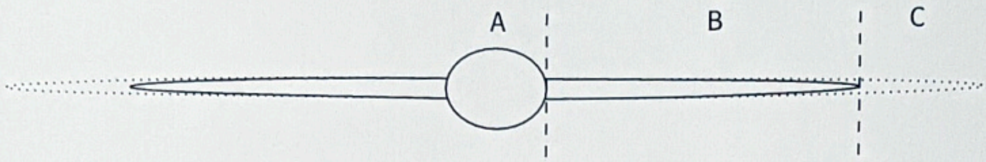
$= 1.554\times10^{-7}(4800)(6.617\times10^8)$.

$\boxed{M_{collision} = 49.36\times10^5\ Kg} \approx 493.6$ tons

Problem B.2: Stars in the Milky Way (6 Points)

In Problem B of the Qualification Round, you estimated the number of stars in the Milky Way by assuming a constant density of stars throughout the galaxy. However, the density of stars is not constant and varies significantly across different regions.

(a) Name the three regions A, B, C marked in the horizontal Milky Way drawing below.



Scientists have developed a basic model for the Milky Way to describe the density distribution of stars $\rho(r)$ at distance r from the center by evaluating the three regions A, B, C:

$$\rho(r) = \Psi \cdot \left[\exp\left(\Omega_A - \frac{r}{R_A}\right) + \exp\left(\Omega_B - \frac{r}{R_B}\right) + \exp\left(\Omega_C - \frac{r}{R_C}\right)\right]$$

The model parameters have the values below:

$\Psi = 10^{-4}$ *stars/(light-year)*^{3}
$R_A = 20$ *light-years*
$R_B = 12 \cdot 10^3$ *light-years*
$R_C = 5 \cdot 10^4$ *light-years*
$\Omega_A = 21, \; \Omega_B = -3, \; \Omega_C = -8$

(b) Create a *double logarithmic* plot of the density distribution $\rho(r)$ with respect to r.

(c) Using this model, calculate the number of stars in the Milky Way ($r \leq$ 130,000 light-years).

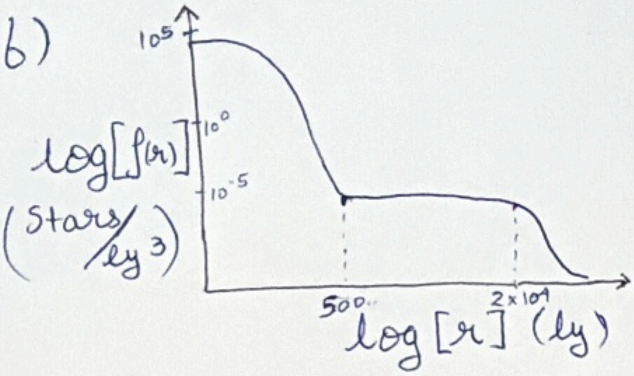
Note: Assume that the Milky Way has a constant thickness of 1,000 light-years.

(a) For the given diagram of Milky Way:

A → Galactic Bulge.
B → Thin disk with spiral arms.
C → either Monoceros Ring or thick Disk of the galaxy.

(Here C cannot be the galactic halo since its thickness is similar to B, the disk and is not spherical).

(b)

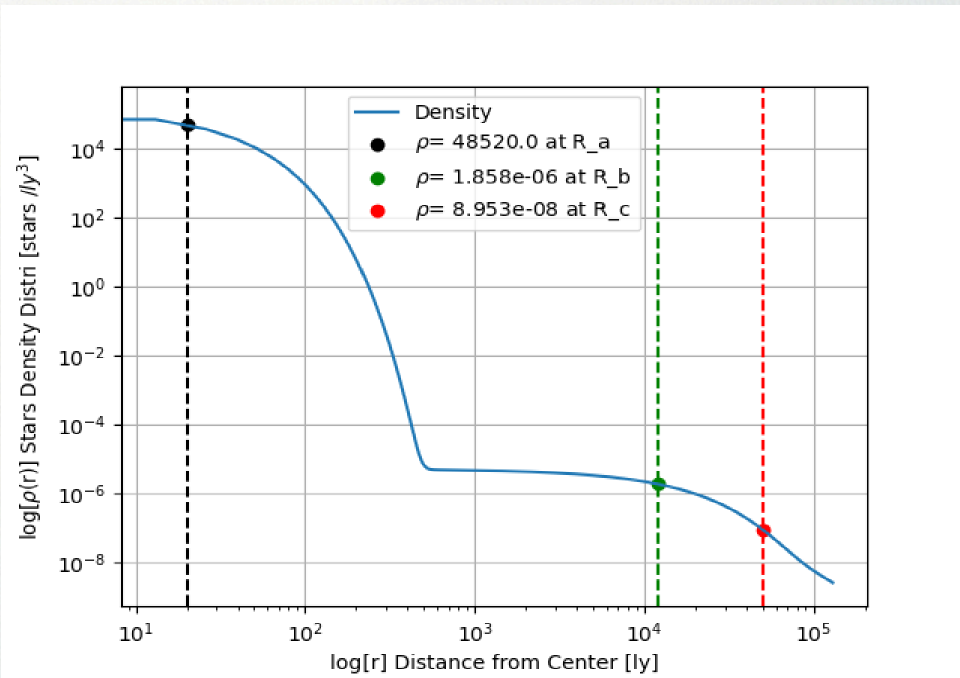


$$\rightarrow f(r) = \Psi\left[\exp\left(\Omega_A - \frac{r}{R_A}\right) + \exp\left(\Omega_B - \frac{r}{R_B}\right) + \exp\left(\Omega_C - \frac{r}{R_C}\right)\right]$$

(c) calculating no. of stars in the Milky way by integrating: $\int_0^{1.3\times10^5} f(r)\times \pi r^2 \times t \, dr$.

$\boxed{t = 1000 \text{ ly}}$ & ($r \le 1.3\times10^5$ ly)

$\boxed{\text{Number of stars} = 6.696 \times 10^{12}}$



You can Find the Jupyter Notebook (python Code) that I used to create the Above plot and Calculate Number of Stars here:
https://github.com/AkshankTyagi/IAAC/blob/main/Milky_Way.ipynb

Problem C.1 : Einstein Ring around Galaxy (8 Points)

This problem requires you to read the following recently published scientific article:

A massive compact quiescent galaxy at z = 2 with a complete Einstein ring in JWST imaging.

van Dokkum, P., Brammer, G., Wang, B. et al. Nat Astron 8, 119-125 (2024).

Link: https://www.nature.com/articles/s41550-023-02103-9.pdf

Answer the following questions related to this article:

(a) What kind of object is the article about, and how was it discovered?

→ The article is about a compact, early-type, red and dead galaxy, which is gravitationally lensing a background galaxy to form an Einstein ring around itself. The lens galaxy dubbed JWST-ER1g, was discovered in NIRCam images from JWST in the Cosmic Evolution Survey (COSMOS)-Web data.

(b) Describe all elements visible in Figure 1a, including the names JWST-ER1g and JWST-ER1r.

→ The image of the object has 3 main components: ~~the~~ compact lens galaxy (JWST-ER1g) with $Z_{phot} = 1.94$ at center and a complete Einstein ring (JWST-ER1r) a blue ring $Z = 2.89$ & 2 mirrored ~~red~~ red arches on the ring. There is also a small nearby galaxy visible outside the Einstein Ring.

(c) Find the galaxy's age, mass, and radius and compare them to our Milky Way.

→ A Prospector fit gives JWST-ER1g an approx. age ≈ 1.9 Gyr, Total Stellar Mass $\approx 1.3 \times 10^{11} M_\odot$ and effective Radius, $r_e \approx 1.9$ Kpc. Whereas the Milky way has age ≈ 13.6 Gyr, $M_{star} = 9 \times 10^9 M_\odot$, $R_{200} = 26.8$ Kpc. Here the observed galaxy is more massive but smaller in size than Milky Way.

(d) According to the study, what is and what is not likely to cause the lensing mass?

→ The total Lensing mass $6.5 \times 10^{11} M_\odot$ is composed of $M_{stars} \approx 1.1 \times 10^{11} M_\odot$ and $M_{DM} \approx 2.6 \times 10^{11} M_\odot$. The unaccounted $2.8 \times 10^{11} M_\odot$ could either be extra Dark Matter from a higher DM halo mass ($\sim 10^{14} M_\odot$) or a deviated density profile for DM due to baryonic processes. More likely than these, the missing mass can be present as low mass stars explained by a bottom-heavy IMF.

(e) Explain what the IMF is by using Figure 3a.

→ IMF (Initial Mass Function) is an empirically obtained function that describes distribution of masses for a population of stars during formation. Fig. 3a plots the probability distri. function for Number of stars that form at a particular Mass. → Saltpeter IMF: $\xi\, dm = \xi_0\, m^{-2.35}\, dm$.

(f) Why is the ring in Figure 1a a gravitational lens and not a ring galaxy?

→ The observed photometric redshifts from best-fit for JWST-ER1g and JWST-ER1ring are very different, $Z \approx 1.94$ & $Z \approx 2.9$ respectively. Moreover the symmetries of the ring are uncanny features of an Einstein Ring. The double mirror images of the red knots stretched into arcs and multiple images of the blue knots are result of gravitational lensing.

Problem C.2 : Volcanic Activity on Io (8 Points)

This problem requires you to read the following recently published scientific article:

Io's polar volcanic thermal emission indicative of magma ocean and shallow tidal heating.

Davies, A.G., Perry, J.E., Williams, D.A. et al. Nat Astron 8, 94-100 (2024).

Link: https://www.nature.com/articles/s41550-023-02123-5.pdf

Answer the following questions related to this article:

(a) What is different about the presented observations compared to previous studies?

→ This study uses new observations of Io's polar caps by Juno's JIRAM at 4.8um, at high resolutions (≤20Km/pixel). The data showed that Io's polar volcanoes are less energetic than those at lower latitudes contradicting previous studies. They identified 266 Hotspots and showed that the north pole had considerably more volcanic activity than south pole on Io.

(b) Explain how the north pole, south pole and the lower latitudes differ in volcanic activity.

→ The study found that the polar regions collectively have only a slightly lower hotspot density per unit area than lower latitudes. While by 4.8 um Spectral radiance, South had least (7 kW/um Km²), half of North polar caps (15 kW/umKm²) and quarter of those at lower lats (26 kW/um Km²). This suggests lithospheric dichotomies.

(c) What is Loki Patera and Tvashtar Paterae?

→ Loki Patera is a powerful thermal source 180Km wide located at 310°W, 12°N on Io. It has a very high magnitude of Thermal emissions.

→ Tvashtar Paterae is a vigorous volcanic site near the North pole of Io, which has multiple hotspots now resolvable through JIRAM images.

(d) How many hot spots did the researchers find in the north and south polar caps?

→ The researchers found a total of 266 hotspots on Io's surface. and by defining polar regions at latitudes ≥ 60°, they identified 20 hotspots in the North polar caps and 12 in the south polar regions.

(e) According to the study's findings, which mechanism likely causes Io's volcanic activity?

→ The observed volcanic pattern is consistent with models of a global magma ocean or tidal heating in shallow astenosphere. The finding disfavours tidal heating in deep Mantle and suggests structural dichotomies that inhibit volcanic advection towards Io's poles (mainly South pole)

(f) Describe how the scientists identified hot spots from the observation pixel data.

→ Hotspots were obtained from Surface images after processing them to normalize the spectral radiance at 4.8um. Then in nightside, the authors used a threshold brightness 50% > background. In day side, the threshold could be as low as 20% > background, with additional criterias applied to Identify hotspots.