Shiv Nadar University Chennai

End Semester Examinations, 2023-2024 Odd

Question Paper

Name of the Program: Common to B.Tech. AI & DS, B.Tech. CSE (IoT) and B.Tech. CSE (Cyber Security)	Semester: I
Course Code & Name: MA1001 LINEAR ALGEBRA	
Regulation 2021	
Time: 3 Hours	Maximum: 100 Marks

Q.No		Questions	Marks	CO#	KL#
1	a	Check whether $W = \{(a_1, a_2): 2a_1 + 3a_2 = 0;\ a_1, a_2 \in R\}$ is a subspace of a vector space R.	2	CO1	KL2
	b	Verify if $\alpha = \{(1,1,2), (1,2,5), (5,3,4)\}$ is a basis for the vector space $V = R^3$.	2	CO1	KL2
2	a	Verify if $T: R^2 \to R^2$ defined by $T(x, y) = (xy, x)$ is a linear transformation.	2	CO2	KL2
	b	Let $T: R^3 \to R^2$ be a linear transformation defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$. Find the rank of T.	2	CO2	KL3
3	a	Let $T: R^2 \to R^2$ be defined by $T(1,1) = (0,1)$ and $T(-1,1) = (2,3)$. Find the matrix representation of T with respect to the standard basis.	2	CO3	KL3
	b	Let $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$. If $v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigenvector of A then find its corresponding eigenvalue.	2	CO3	KL3
4	a	In an inner product space $V(F)$, if x and y are orthogonal vectors then prove that $\|x + y\|^2 = \|x\|^2 + \|y\|^2$.	2	CO4	KL2
	b	The inner product on a vector space V of polynomials of degree less than or equal to 2 together with the zero polynomial is defined by $\langle f, g\rangle = \int_0^1 f(t)g(t)dt$. Let $f(t) = t + 2$ and $g(t) = t^2 - 2t - 3$. Find $\langle f, g\rangle$.	2	CO4	KL3
5	a	Find the singular values of the matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.	2	CO5	KL3
	b	What do you mean by least squares approximation?	2	CO5	KL1
6	a	Prove that the set $M_2(R)$ of all 2×2 matrices over R is a vector space under matrix addition and scalar multiplication defined by $\alpha \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{bmatrix}$.	10	CO1	KL3
7	a	Let $T: R^3 \to R^3$ be a linear transformation defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$. Find dim $(N(T))$, dim $(R(T))$ and then verify dimension theorem (rank-nullity theorem).	10	CO2	KL3
8	a	Find the eigen values and eigen vectors of the linear transformation $T: R^3 \to R^3$ defined by $T(x, y, z) = (2x - y, -x + 3y + z, -z)$.	10	CO3	KL3

	b	Let T be the linear operator on $P_2(R)$ defined by $T(f(x)) = f(x) + (x+1)f'(x)$. Let β be the standard basis of $P_2(R)$. Find matrix A of the transformation. Hence find the eigenvalues and eigenvectors of the matrix A.	10	CO3	KL3
9	a	Use Frobenius inner product to compute $\|A\|$, $\|B\|$, $\langle A, B\rangle$ and verify Cauchy-Schwarz inequality given $A = \begin{bmatrix} 1 & 2+i \\ 3 & i \end{bmatrix}$, $B = \begin{bmatrix} 1+i & 0 \\ i & -1 \end{bmatrix}$.	10	CO4	KL3
	b	Find an orthonormal basis of the inner product space R^3 with the standard inner product, given the basis $\{(1,0,1), (0,1,1), (1,3,3)\}$ using Gram-Schmidt's process. Also, find the Fourier coefficients of the vector $(1,1,2)$ relative to the orthonormal basis.	10	CO4	KL3
10	a	Compute a QR decomposition of $A = \begin{bmatrix} 3 & -6 \\ 4 & -8 \\ 0 & 1 \end{bmatrix}$.	10	CO5	KL3
	b	Find a singular value decomposition of $A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$.	10	CO5	KL3

KL – Bloom's Taxonomy Levels
(KL1: Remembering, KL2: Understanding, KL3: Applying, KL4: Analyzing, KL5: Evaluating, KL6: Creating)
CO – Course Outcomes
