UNIT - I LOGIC AND PROOFS

PREPOSITION → A declarative sentence which is True or False and are denoted by lower case alphabets p, q, r, s, t

Simple / atomic / primary Preposition

A preposition which cannot be split further

Compound / molecular / secondary preposition

The prepositions formed by simple prepositions and logical connectives

$(\neg, \wedge, \vee, \rightarrow, \longleftrightarrow)$

→ conditional, ↔ biconditional

Not preposition → doesn't tell T or F

1) **Negation** of variable p is denoted by $\neg p$ or $\bar{p}$ and if p is true then $\neg p$ is false & vice versa

p	$\neg p$
T	F
F	T

2) **conjunction** of two variables p & q denoted by $p \wedge q$ is true if both are true and False otherwise

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3) **Disjunction** of two variables is denoted by $p \vee q$ is false if both or ~~false~~ true otherwise

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

4) **conditional Statement** on two variables p & q denoted by $p \to q$ (p implies q) and is false if p is true but q is false and true otherwise

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$(\neg p \vee q)$

5) **Biconditional Statement** on 2 variables p & q denoted by $p \leftrightarrow q$ and is true iff both are of same truth values

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$(p \to q) \wedge (q \to p)$

$(\neg p \vee q) \wedge (\neg q \vee p)$

$(p \wedge q) \vee (\neg p \wedge \neg q)$.

Other ways of writing $P \to q$

1) if P then q
2) q whenever P
3) P is sufficient for q
4) q is necessary for P
5) P only if q

6) XOR of two variables P and q denoted by $P \oplus q$ is true if exactly one of them is true and False otherwise

P	q	$P \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

$(P \land \neg q) \lor (\neg P \land q)$ or $(P \lor q) \land \neg(P \land q)$

15/07

1) You can access the internet from the campus only if you're a CS major ~~and~~ or NOT a fresher

P → you can access internet
q : you are CS major
r : You are a fresher

$P \to (q \lor \neg r)$

2) I like cake but I don't like pizza

$P \to$ I like cake

q : I like Pizza

$P \wedge \neg q$

3) If it is not raining then the sun is not shining and there are clouds in the sky

P : It is raining

q : sun is shining

r : There are clouds in sky

$\neg P \to (\neg q \wedge r)$

4) John to get a Job it is sufficient to learn dM

$P \to$ John gets Job

$q \to$ To learn dM

~~PBO~~ $q \to P$

5) I want neither soup nor salad

P : I want soup

q : I want salad

$\neg P \wedge \neg q$

6) if moon is out and not snowing then Ram goes for a walk

$P \to$ moon is out

$q \to$ Snow

$r \to$ walk

$(P \wedge \neg q) \to r$

1) $p \to q$
2) $\neg p \to (\neg q \wedge r)$
3) $q \leftrightarrow \neg p$
4) $(p \vee \neg r) \to q$

p→ It is raining
q : There are clouds in sky
r : The sun is shining

1) if it is raining then there are clouds in sky
2) if it is not raining then there are no clouds and the sun shines
3) There are clouds in sky if only if it is not raining
4) if it is raining and the sun is not shining then there are clouds in sky

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The field is wet whenever it is raining

p : It is raining
q : The field is wet q

converse $q \to p$
if the field is wet then it is raining

inverse $\neg p \to \neg q$
if it is not raining then the field is not wet

contrapositive : $\neg q \to \neg p$
if the field is not wet then it is not raining

2) Today it is not sunday or there is DM class

P : It is sunday $\neg P$: It is not sunday

q : There is DM class

$\neg p \vee q \equiv p \to q$

converse: $q \to p$

If there is class it is not sunday

inverse : $\neg P \to \neg q$

If it is sunday then there is no dM class

contrapositive $\neg q \to \neg P$

if there is no dM class then it is sunday

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if P the q $\equiv p \to q$

P only if q $\equiv P \to q$

P if q $\equiv q \to P$

q only if P $\equiv q \to p$

Order of preference of operators

$\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

<u>**Construction of truth tables**</u>

1) $[P \wedge (P \rightarrow q)] \rightarrow q$

P	q	$P \rightarrow q$	$P \wedge (P \rightarrow q)$	$P \wedge (P \rightarrow q) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

2) $(P \vee q) \wedge (\neg P \wedge \neg q)$

P	q	$P \vee q$	$\neg P \wedge \neg q$	$(P \vee q) \wedge (\neg P \wedge \neg q)$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

3) $(P \wedge q) \vee (P \wedge r)$

P	q	r	$(P \wedge q)$	$(P \wedge r)$	$(P \wedge q) \vee (P \wedge r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

Tautology

A compound preposition is called tautology if its truth value is true

Contradiction.

Truth value is False

Contigency :

neither tautology nor contradiction

Prepositional Equivalence

Two prepositions are said to be equivalent if their truth values are same for every assignment of variables

Equivalence law

1) Idempodent Law

$$P \wedge P \equiv P$$

$$P \vee P \equiv P$$

2) commutative law

$$P \wedge q \equiv q \wedge P$$

$$P \vee q \equiv q \vee P$$

3) Associative Law

$$P \wedge (q \wedge r) \equiv (P \wedge q) \wedge r$$

$$P \vee (q \vee r) \equiv (P \vee q) \vee r$$

4) Distribut

5) De mor

6) Ident

7) Con

8 Dom

9) Do

10) A

Laws

1) P

2) -

3)

4

4) Distributive law

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

5) De morgan's law

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

6) Identity law : $p \wedge T \equiv p$
$p \vee F \equiv p$

7) complement law : $p \wedge \neg p = F$
$p \vee \neg p \equiv T$

8 Dominant law : $p \vee T \equiv T$
$p \wedge F \equiv F$

9) Double negation law : $\neg\neg p \equiv p$

10) Absorption law $p \wedge (p \vee q) \equiv p$
$p \vee (p \wedge q) \equiv p$

Laws of conditional Statements

1) $p \rightarrow q \equiv \neg p \vee q$

2) $\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$
$\equiv p \wedge \neg q$

3) $p \rightarrow q \equiv \neg q \rightarrow \neg p$

4) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
$(\neg p \vee q) \wedge (\neg q \vee p)$

1) Prove that $P \to (q \to P)$ is a tautology

$P \to (q \to P)$

$\equiv P \to (\neg q \vee P)$

$\equiv \neg P \vee (\neg q \vee P)$

~~$\neg P \vee \neg q$~~

$\equiv \neg P \vee (P \vee \neg q)$ commutative

$\equiv (\neg P \vee P) \vee \neg q$ associative

$\equiv T \vee \neg q$ complement

$\equiv T$ Dominant

2) $(\neg P \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (P \wedge r) \equiv r$

$\equiv (\neg P \wedge (\neg q \wedge r)) \vee ((q \vee P) \wedge r)$ (Distributive)

$\equiv ((\neg P \wedge \neg q) \wedge r) \vee (P \vee q) \wedge r)$ associative

$\equiv (\neg (P \vee q) \vee (P \vee q) \wedge r)$ demorgans

$\equiv (T \wedge r)$ complement

$\equiv r$ (Identity law)

3) $[(P \vee q) \wedge \neg(\neg P \wedge (\neg q \wedge \neg r))] \vee [(\neg P \wedge \neg q) \vee (\neg P \wedge \neg r)] \equiv T$

$[(P \vee q) \wedge \neg(\neg P \wedge \neg(q \wedge r))] \vee [\neg(P \vee q) \vee \neg(P \vee r)]$

$\equiv [(P \vee q) \wedge (P \vee (q \wedge r))] \vee \neg[(P \vee q) \wedge (P \vee r)]$

$\equiv [(P \vee q) \wedge [(P \vee q) \wedge (P \vee r)]] \vee \neg(P \vee q) \wedge (P \vee r)]$

$\equiv [(P \vee q) \wedge (P \vee r)] \vee \neg[(P \vee q) \wedge (P \vee r)]$

$\equiv T$

4) $\neg(P$

NOTE

Two

P q

taut

S.T

(or)

4) $\neg(P \leftrightarrow q) \equiv P \leftrightarrow \neg q$

$\neg[(P \to q) \wedge (q \to P)]$

$\neg[(\neg P \vee q) \wedge (\neg q \vee P)]$

$[\neg(\neg P \vee q) \vee \neg(\neg q \vee P)$

$[(P \wedge \neg q) \vee (q \wedge \neg P)]$

$= [(P \vee q) \wedge (P \vee \neg P) \wedge (\neg q \vee q) \wedge (\neg q \neg P)]$

$= [(P \vee q) \wedge (\neg q \vee \neg P)]$

$= (\neg P \to q) \wedge (q \to \neg P)$

$= (\neg q \to P) \wedge (P \to \neg q)$ contrapositive

$\equiv P \leftrightarrow (\neg q)$

<u>**NOTE**</u>

Two statement

P & q are equivalent $P \equiv Q$ if $P \leftrightarrow q$ is a tautology

S.T $(P \to q) \equiv (\neg q \to \neg P)$

(or) S.T $(P \to q) \leftrightarrow (\neg q \to \neg P)$ is a tautology

$(\neg(P \to q) \vee (\neg q \to \neg P)) \wedge (\neg(\neg q \to \neg P) \vee (P \to q))$

$(\neg(\neg P \vee q) \vee (q \vee \neg P)) \wedge (\neg(q \vee \neg P) \vee (\neg P \vee q))$

Demorgans)

$((P \wedge \neg q) \vee (q \vee \neg P)) \wedge ((\neg q \wedge P) \vee (\neg P \vee q))$

$(P \wedge \neg q) \vee (q \vee \neg P)$ (idempodent)

$(P \vee (q \vee \neg P) \wedge (\neg q \vee (q \vee \neg P))$ Distributive

$(P \vee \neg P) \vee q) \wedge (\neg q \vee q) \vee \neg P)$ Associative

$(T \vee q) \wedge (T \vee \neg P)$ complement

Duality: Dual of a statement / compound Preposition S is denoted by S^* and is obtained replacing T by F, $\vee$ by $\wedge$ and Vice Versa

Ex: $(p \vee \neg q)^* \equiv p \wedge \neg q$

$\neg p \vee (T \wedge q) \equiv \neg p \wedge (F \vee q)$

<u>Theorem</u>

Two statements S_1 & S_2 are equivalent iff $S_1^* \equiv S_2^*$ i.e their duals are equivalent

1) S.T $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$ by showing that their duals are equivalent

dual $\neg((\neg p \vee q) \wedge (\neg p \vee \neg q)) \wedge (p \vee q) \equiv p$

<u>Sol</u>

$\neg((p \wedge \neg q) \vee \neg(p \wedge q)) \wedge (p \vee q)$ (de morgan's)

$(p \vee (q \wedge \neg q)) \wedge (p \vee q)$ (Distributive)

$(p \vee F) \wedge (p \vee q)$ complement

$p \wedge (p \vee q)$ identity

$\equiv p$ absorbtion

2) $(P \wedge (P \leftrightarrow Q)) \rightarrow Q \equiv T$

$S = \neg(P \wedge (P \leftrightarrow Q)) \vee Q \equiv T$

$= \neg(P \wedge ((\neg P \vee Q) \wedge (\neg Q \vee P))) \vee Q$

$S^* = \neg(P \vee ((\neg P \wedge Q) \vee (\neg Q \wedge P))) \wedge Q$

$\neg(P \vee ((\neg P \vee \neg Q) \wedge \overset{T}{(\neg P \vee P)} \wedge \overset{T}{(Q \vee \neg Q)} \wedge (Q \wedge P)) \wedge Q$

$\neg(P \vee ((\neg P \vee \neg Q) \wedge (P \wedge Q)) \wedge Q$

$\neg(P \vee (\overset{F}{(\neg P \wedge P)} \vee (\neg P \wedge Q) \vee (\neg Q \wedge P) \vee \overset{F}{(\neg Q \wedge Q)}) \wedge Q$

$\neg(P \vee ((\neg P \wedge Q) \vee (\neg Q \wedge P))) \wedge Q$

$\neg P \wedge ((\neg Q \vee P) \wedge (\neg P \vee Q)) \wedge Q$

$((\neg P \wedge \neg Q \vee \overset{T}{P}) \wedge (\neg P \wedge \neg P \vee Q)) \wedge Q$

$((T \wedge \neg Q) \wedge (\neg P \vee Q)) \wedge Q$

$((\neg Q \wedge \neg P) \vee (\neg Q \wedge Q)) \wedge Q$

$= ((\neg Q \wedge \neg P) \vee F) \wedge Q$

$(\neg P \wedge \neg Q) \wedge Q$

$= (\neg P \wedge \neg Q \wedge Q)$

$= (\neg P \wedge F)$

$\equiv F$

3) Show that $\neg(p \leftrightarrow q) \equiv (p \vee q) \wedge \neg(p \wedge q)$ using duality

$S_1 : \neg(p \leftrightarrow q)$

$\equiv \neg((\neg p \vee q) \wedge (\neg q \vee p))$

$S_1^* : \neg((\neg p \wedge q) \vee (\neg q \wedge p))$ $\quad$ $S_2^* : (p \wedge q) \vee \neg(p \vee q)$

LHS

$(p \vee \neg q) \wedge (q \vee \neg p)$ (demorgans)

$= (p \wedge q) \vee (p \wedge \neg p) \vee (\neg q \wedge q) \vee (\neg q \wedge \neg p)$ (distributive)

$= (p \wedge q) \vee (\neg p \wedge \neg q)$ (negation)

$\equiv (p \wedge q) \vee \neg(p \vee q)$

$P \equiv Q$ if

1) P is derived from Q or vice versa
2) $P \leftrightarrow Q$ is a tautology
3) LHS = RHS
4) $P^* \equiv Q^*$

4) Show that $p \vee q \vee r$ and $(p \vee q) \wedge (r \rightarrow p)$ are not logically equivalent

$p : F$, $q : F$: $r : F$

$S_1 : p \vee q \vee \neg r = F \vee F \vee \neg F \equiv T$

$S_2 : (p \vee q) \wedge (r \rightarrow p)$, $(F \vee F) \wedge (F \rightarrow F) \equiv F \wedge T$

$\equiv F$

Tautological implication

A statement P implies another statement Q if $P \to Q$ is a tautology

1) Show that $((P \to Q) \land (R \to Q)) \to ((P \lor R) \to Q)$ implies (or)

$((P \to Q) \land (R \to Q)) \to ((P \lor R) \to Q)$

$\equiv \neg((P \to Q) \land (R \to Q)) \lor ((P \lor R) \to Q)$

$\equiv \neg((\neg P \lor Q) \land (\neg R \lor Q)) \lor (\neg(P \lor R) \lor Q)$

$\equiv ((P \land \neg Q) \lor (R \land \neg Q)) \lor ((\neg P \land \neg R) \land \neg Q)$

$\equiv ((P \lor R) \land \neg Q)) \lor ((\neg P \land \neg R) \land \neg Q)$

$\equiv ((P \lor R) \lor \neg(P \lor R)) \land \neg Q)$

$\equiv T \land \neg Q$

$\equiv T$

2) $[(P \lor \neg P) \to Q)] \to ((P \lor \neg P) \to R)] \to (Q \to R)$

SOL

$([(P \lor \neg P) \to Q)] \to ((P \lor \neg P) \to R)]) \to (Q \to R) \equiv T$

$\equiv ([(T \to Q)] \to (T \to R)]) \to (\neg Q \lor R) \equiv$

$\equiv ((Q \to R)) \to (\neg Q \lor R)$

$\equiv (\neg Q \lor R) \to (\neg Q \lor R)$

$\equiv \neg(\neg Q \lor R) \lor (\neg Q \lor R)$

$\equiv T$

Satisfiability

A compound statement is said to be satisfiable if there exists an assignment of truth values such that the statement results into true.

1) $p \vee q$ is satisfiable

   $p : T \quad q : T$

Tautology and contigency is satisfiable

contradiction is never satisfiable

1) verify if $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is satisfiable

   $p : T \quad q : F \quad r : F$

   $T \wedge T : T$

Normal Forms : Standard form of expressing statements

Elementary Product : Product of variable and its negation

ex : $p, q, \neg p, \neg q \Rightarrow p \wedge q ; \neg p \wedge \neg q, \neg p \wedge q ; p \wedge \neg q$

elementary Sum : Sum of variables and its negation

ex - $p, q, \neg p, \neg q \Rightarrow p \vee q, \neg p \vee \neg q, \neg p \vee q ; p \vee \neg q$

CNF : conjunctive Normal Form : A statement which is equivalent to given statement and is in the form of product of elementary sums

ex: $(P) \wedge (P \vee q) \wedge (\neg P \vee q) \wedge \neg q$

DNF: A statement which is equivalent to given statement and is the form of sum of elementary products

$P \vee (P \wedge q) \vee (\neg P \wedge q) \vee \neg P$

Minterms:

elementary products in which each variable or its negation is present exactly once 2 variable

$P \wedge q$, $\neg P \wedge q$, $\neg P \wedge \neg q$, $P \wedge \neg q$

Maxterms:

elementary sums in which each variable or its negation is present exactly once 3 variables

$P \vee q \vee r$, $P \vee \neg q \vee r$, $\neg P \vee q \vee \neg r$

PDNF: Principle DNF: sum of minterms

ex: $(P \wedge q) \vee (P \wedge \neg q) \vee (\neg P \wedge q)$

PCNF: Principle CNF product of Maxterms

1) Find PDNF of $(\neg P \vee \neg q) \rightarrow (\neg P \wedge r)$

<u>SOL</u>

$\neg(\neg P \vee \neg q) \vee (\neg P \wedge r)$ (∵ implication)

$\equiv (P \wedge q) \vee (\neg P \wedge r)$ (∵ demorgans)

$\equiv (P \wedge q \wedge T) \vee (\neg P \wedge r \wedge T)$ (∵ identity)

$\equiv (P \wedge q \wedge (r \vee \neg r)) \vee (\neg P \wedge (q \vee \neg q) \wedge r)$ (complement)

$\equiv (P \wedge q \wedge r) \vee (P \wedge q \wedge \neg r) \vee (\neg P \wedge q \wedge r) \vee (\neg P \wedge \neg q \wedge r)$ (∵ Distribution)

2) <u>PCNF</u>

$(P \wedge q) \vee (\neg P \wedge r)$

$\equiv (P \vee \neg P) \wedge (P \vee r) \wedge (q \vee \neg P) \wedge (q \vee r)$ (distributive)

$\rightarrow$ CNF

$\equiv (P \vee r) \wedge (q \vee \neg P) \wedge (q \vee r)$ - (complement)

$(P \vee F \vee r) \wedge (q \vee \neg P \vee F) \wedge (q \vee r \vee F)$ (Identity)

$\equiv (P \vee (q \wedge \neg q) \vee r) \wedge (q \vee \neg P \vee (\neg r \wedge r) \wedge ((P \wedge \neg P) \vee q \vee r$

$\equiv (P \vee q \vee r) \wedge (P \vee \neg q \vee r) \wedge (\neg P \vee q \vee r) \wedge (\neg P \vee q \vee \neg r)$

$\wedge ((P \vee q \vee r) \wedge (\neg P \vee q \vee r)$

3) Find PDNF of $(\neg p \vee \neg q) \rightarrow (p \leftrightarrow \neg q)$

SOL

$(\neg p \vee \neg q) \rightarrow ((\neg p \vee q) \wedge (\neg q \vee p))$

$\equiv \neg(\neg p \vee \neg q) \vee ((\neg p \vee q) \wedge (\neg q \vee p))$

$\equiv (p \wedge q) \vee ((\neg p \wedge \neg q) \vee (\neg p \wedge p) \vee (q \wedge \neg q) \vee (p \wedge q))$

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q) \vee (p \wedge q)$

4) $(\neg p \rightarrow r) \wedge (q \leftrightarrow p) \rightarrow$ PCNF

$(p \vee r) \wedge (\neg q \vee p) \wedge (\neg p \vee q)$

$\equiv (p \vee r) \wedge (\neg q \vee p) \wedge (\neg p \vee q)$

$\equiv (p \vee F \vee r) \wedge (\neg q \vee p \vee F) \wedge (\neg p \vee q \vee F)$

$\equiv (p \vee (q \wedge \neg q) \vee r) \wedge (p \vee \neg q \vee (r \wedge \neg r)) \wedge (\neg p \vee q \vee (r \wedge \neg r))$

$\equiv (p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee \neg q \vee \neg r)$
$\wedge (\neg p \vee q \vee r) \wedge (\neg p \vee q \neg r)$

5) $(q \vee (p \wedge r)) \wedge \neg((p \vee r) \wedge q)) \rightarrow$ PCNF

$(q \vee p) \wedge (q \vee r) \wedge ((\neg p \wedge \neg r) \vee \neg q)$

$\equiv (p \vee q) \wedge (q \vee r) \wedge (\neg p \vee \neg q) \wedge (\neg r \vee \neg q)$

$\equiv (p \vee q \vee (r \wedge \neg r)) \wedge ((p \wedge \neg p) \vee q \vee r) \wedge (\neg p \vee \neg q \vee (r \wedge \neg r))$
$\wedge ((p \wedge \neg p) \vee \neg q \vee \neg r)$

$\equiv (p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (p \vee q \vee r) \wedge (\neg p \vee q \vee r)$
$\wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee \neg r) \wedge$
$(\neg p \vee \neg q \vee \neg r)$

6) Find PDNF & PCNF of $P \to (\neg p \wedge (q \to p))$

$\equiv p \to (\neg p \wedge (\neg q \vee p))$

$\equiv \neg p \vee (\neg p \wedge (\neg q \vee p))$

$\neg p \vee \neg p$

$\equiv \neg p \vee ((\neg p \wedge \neg q) \vee (\neg p \wedge p))$

$\equiv \neg p \vee ((\neg p \wedge \neg q) \vee F)$

$\equiv (\neg p \vee \neg p) \wedge (\neg p \vee \neg q)$

$\neg p \wedge (\neg p \vee \neg q)$

$\equiv (\neg p \wedge \neg p) \vee (\neg p \wedge \neg q)$

$\equiv \neg p \wedge \neg q$

$\equiv \neg p \vee \neg p$

$\equiv \neg p$

PCNF	PDNF
$\neg p \vee F$	$\neg p \wedge T$
$\neg p \vee (q \wedge \neg q)$	$\neg p \wedge (q \vee \neg q)$
$(\neg p \vee q) \wedge (\neg p \vee \neg q)$	$\equiv (\neg p \wedge q) \vee (\neg p \vee \neg q)$

Two methods to find Normal forms

1) using equivalence laws

2) using truth table

P	q	q
T	T	
T	F	
F	T	
F	F	

Minterms

$(\neg p \wedge q)$

Maxterm

$(\neg p \vee \neg q$

For P

1) Select

2) Disj

select

For

1) S

2) C

corre

p	q	$q \to p$	$\neg p$	$\neg p \wedge (q \to p)$	$p \to (\neg p \wedge (q \to p))$
T	T	T	F	F	F
T	F	T	F	F	F
F	T	F	T	F	T
F	F	T	T	T	T

PDNF of statement is equal PCNF of negation statement

Minterms

$(\neg p \wedge q) \vee (\neg p \wedge \neg q)$

Maxterms

$(\neg p \vee \neg q) \wedge (\neg p \vee q)$

For PDNF

1) Select the rows with truth value T
2) Disjunction of minterms corresponding to selected rows

For PCNF

1) select the rows with truth value F
2) conjunction of maxterms (in negation form) corresponding to selected rows

Let S be the statement PDNF of ¬S consists missing minterms in its PDNF

→ $S \equiv \neg(\neg S)$

$S \equiv$ PCNF of S

ex → PDNF of $(\neg p \wedge q) \vee (\neg p \wedge \neg q)$

missing minterms $(p \wedge q) \vee (p \wedge \neg q)$

PCNF $\equiv \neg((p \wedge q) \vee (p \wedge \neg q)) \vee (\neg p \vee \neg q) \wedge (\neg p \vee q))$

PDNF to PCNF

PCNF ≡ ¬(disjunction of missing minterms in PDNF)

PCNF to PDNF

PDNF ≡ ¬(conjunction of missing maxterms in PCNF)

1) Find PCNF hence find PDNF of following

$(p \rightarrow (q \wedge r)) \wedge (\neg p \rightarrow (\neg q \wedge \neg r))$

i) using laws

$(\neg p \vee (q \wedge r)) \wedge (p \vee (\neg q \wedge \neg r))$

$(\neg p \vee q) \wedge (\neg p \vee r) \wedge (p \vee \neg q) \wedge (p \vee \neg r)$

$(\neg p \vee q \vee (r \wedge \neg r)) \wedge (\neg p \vee (q \wedge \neg q) \vee r) \wedge (p \vee \neg q \vee (r \wedge \neg r$

$\wedge (p \vee (q \vee \neg q) \vee \neg r))$

$\equiv (\neg p \vee q$

$(\neg p \vee \neg q$

$\wedge (p \vee \neg q$

2) Find

truth t

SOL

P	q
T	T
T	T
T	F
T	F
F	T
F	T
F	F
F	F

PDNF

$(p \wedge$

PCNF

$(\neg p$

$\equiv (\neg p \vee q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge$
$(\neg p \vee \neg q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee q \vee \neg r)$
$\wedge (\cancel{p \vee \neg q \vee \neg r})$

2) Find PDNF & PCNF of $(p \wedge q) \vee (\neg p \wedge r)$ using truth table

<u>SOL</u>

p	q	r	$\neg p$	$p \wedge q$	$\neg p \wedge r$	$(p \wedge q) \vee (\neg p \wedge r)$
T	T	T	F	T	F	T
T	T	F	F	T	F	T
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	T	T
F	T	F	T	F	F	F
F	F	T	T	F	T	T
F	F	F	T	F	F	F

PDNF

$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$

PCNF

$(\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee r)$

2) Find PDNF and hence find PCNF of $(\neg p \to r) \wedge (p \vee \neg q)$ without using truthtable

<u>SOL</u>

$(p \vee r) \wedge (p \vee \neg q)$

PDNF

$\equiv \cancel{(p \vee r \vee (q \wedge \neg q)) \wedge (p \vee \neg q \vee (r \wedge \neg r))}$

$\equiv \cancel{(p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee \neg q \vee \neg r)} \wedge$

$\equiv p \vee (r \wedge \neg q)$

$\equiv (p \wedge (q \vee \neg q)) \vee ((p \vee \neg p) \wedge r \wedge \neg q)$

$\equiv (p \wedge q) \vee (p \wedge \neg q) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$

$\equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r)$

$\vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$ $\quad \neg p \wedge q \wedge r$

B.

PDNF to PCNF

$\equiv \neg((\neg p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r))$

$\equiv ((p \vee q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee r)$

Rules

1) Set

2) Set as

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