1

RANDOM EXPERIMENT: is an experiment whose outcome is not unique and is one of the possible outcomes

ex: Tossing coin.

P -> arrangement

C -> choosing

SAMPLE SPACE:- set of all possible outcomes of an experiment, $S = \{T, H\}$

EVENT: Each outcome of the random experiment

ex: getting H on tossing a coin

MUTUALLY EXCLUSIVE EVENTS:

Two events are said to be mutually exclusive if they can not occur at the same time & simultaneously

ex: getting heads & getting tails

EXHAUSTIVE EVENTS:

set of events are said to be exhaustive if one of them occurs when experiment is performed

ex: $S = \{1, 2, 3, 4, 5, 6\}$ for dice

INDEPENDENT EVENTS:

Two events are said to be independent events if happening of one event does not affect happening of other event

Sure event $P(E) = 1$

sunrise in east

IMPOSSIBLE EVENT

getting a 7 in a dice

EQUALLY LIKELY EEVENT

Every event has same chance of occuring

eg - {H, T}

1) What is a chance that a leap year directly selected has 53 sundays

366 → 52 weeks + 2 days

⇒ 2/7.

2) Among the digits (1, 2, 3, 4, 5) at first one number is chosen and a second selection is made among the remaining 4 digits. Assuming that all 20 possible outcomes have equal Probability. Find the prob that an odd digit will be selected

(i) the first time

(ii) second time → $\frac{4}{5} \times \frac{3}{4}$

(iii) both times → $\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$

(i) 4×3

(ii) 4×3

(iii) $\frac{4}{5} \times \frac{3}{4}$

3/5

3)

Func

i)

(ii

(i

(

3) 4 cards are drawn random from a pack of 52 cards Find the prob that they are

i) a king, a queen, a jack and an ace

(ii) 2 kings, 2 queens

(iii) 2 black, 2 red

(iv) 2 hearts and 2 diamonds

i) $\frac{4C_1 \times 4C_1 \times 4C_1 \times 4C_1}{52C_4}$

ii) $\frac{4C_2 \times 4C_2}{52C_4}$

(iii) $\frac{26C_2 \times 26C_2}{52C_4}$

IV) $\frac{13C_2 \times 13C_2}{52C_4}$

4) 25 books are placed at random in a shelf. Find the prob that a particular pair of books can be

i) always together

(ii) Never together

Sol. Total prob = 25!

(i) - - - - - $\underbrace{\_\ \_}_{2B}$ = $\frac{24! \times 2!}{25!} = \frac{2}{25}$

25books

(ii) $1 - \frac{24! \times 2!}{25!} = \frac{23}{25}$

5) A committee of 4 people is to be appointed from 3 officers of Prod dept, 4 officers of purchase dept, 2 officers of sales dept and one CA. Find the prob of forming the committee such that

Sol (i) 1 From each category

(ii) have atleast 1 from purchase dept

(iii) The CA must be in the committee

SOL

Total probability $40C_4$

(i) $\dfrac{3C_1 \times 4C_1 \times 2C_1 \times 1C_1}{10C_4}$

(ii) $1 - P(\text{nobody from purchase dept})$

$1 - \dfrac{6C_4}{10C_4}$

(iii) $\dfrac{1C_1 \times 9C_3}{10C_4}$

6) A committee of 7 members is to be formed from grp consisting of 8 men and 5 women. what is the Prob that the committee would comprise

(i) 2 women

(ii) atleast 2 women

SOL Tot prob $= 13C_7$

(i) $\dfrac{5C_2 \times 8C_5}{13C_7}$

(ii) $1-(0W\,7m + 1W\,6m)$

$$1-\left(\frac{8C_7}{13C_7} + \frac{5C_1 \times 8C_6}{13C_7}\right)$$

7) If the letters of the word "REGULATIONS" are arranged at random, what is the chance that there will be exactly 4 letters between R & E

R _ _ _ _ E

SOL

$$\frac{2! \times 9! \times 6!}{11!}$$

$$\frac{2! \times 9! \times 6}{11!}$$

R _ _ _ _ E

1 2 3 4 5 6 7 8 9

R E

$$\frac{2! \times 9! \times 6}{11!}$$

PROBABILITY AXIOMS

Let S be a sample space and A be any event of S

(i) Probability of A is defined, and is real and $P(A) \geq 0$

(ii) $P(S) = 1$

(iii) For any finite set of events $A_1, A_2 \ldots A_n$ which are mutually exclusive

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

1) If A, B, C are mutually exclusive & exhaustive find $P(A)$ such that $P(B) = \frac{3}{2} P(A)$, $P(C) = \frac{1}{2}(P(B))$

SOL

$$P(A) + P(B) + P(C) = 1$$

$$P(A) + \frac{3}{2}P(A) + \frac{1}{2} \cdot \frac{3}{2} P(A) = 1$$

$$P(A) = \frac{4}{13}$$

ALGEBRA OF SETS

$A \cup B = \{x \in S \mid x \in A \text{ or } x \in B\}$

$A \cap B = \{x \in S \mid x \in A \text{ and } x \in B\}$

$A - B$ or $A \setminus B = \{x \in S \mid x \in A \text{ and } x \notin B\}$

$A \Delta B$ x belongs to exactly one of A & B

$= (A \cup B) - (A \cap B)$

$\bar{A}$ or $A' = \{x \in S \mid x \in A'\}$

Let A, B, and C are 3 events

i) only A $A \cap \bar{B} \cap \bar{C}$

ii) Both A & B but not C : $A \cap B \cap \bar{C}$

iii) All A, B & C $A \cap B \cap C$

iv) At least one of A, B & C $A \cup B \cup C$

v) Atleast 2 occur $(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C) \cup (A \cap B \cap$

vi) one and no more occurs $(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B}$

vii) Two and no more occurs $(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B$

viii) None of them occurs $\bar{A} \cap \bar{B} \cap \bar{C}$
$= \overline{A \cup B \cup C}$

---

2) Let $P(A) = P_1$, $P(B) = P_2$, $P(C) = \overset{A \cap B}{P_3}$ Express the following in terms of P_1, P_2 and P_3

i) $P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - (P_1 + P_2 - P_3)$

ii $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - P_3$

iii) $P(\bar{A} \cap B) = P(B) - P(A \cap B) = P_2 - P_3$

iv) $P(\bar{A} \cup B) = P(A \cap \bar{B}) = P_1 - P_3$

v) $P(A \cap \bar{B}) = P(A) - P(A \cap B) = P_1 - P_3$

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3) if 2 dice are thrown what is probability sum is

i > 8 $\quad 5/18$ $\qquad$ ii neither 7 nor 11 $\quad 7/9$

i $P(S = 9) = 4/36$ $\qquad P(S > 8) = \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} =$

ii $P(S = 10) = 3/36$

iii) $P(S = 11) = 2/36$

iv) $P(S = 12) = 1/36$

ii) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - (P(A) + P(B))$

$= 1 - \left(\frac{6}{36} + \frac{2}{36}\right) = \frac{7}{9}$

ADDITIVE THEOREM OF PROBABILITY :

any events of a sample space

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

A and B are not mutually exclusive

2) An integer is chose from the digits 1, 2 . . 200. Find the probability that it is divisible by 6 or 8

SOL

Let A be the event that no. is divisible by 6

Let B be the event that no. is divisible by 8

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$n(A) = 33 \quad n(B) = 25 \quad n(A \cap B) = 8$

$P(A \cup B) = \frac{33 + 25 - 8}{200} = \frac{1}{4}$

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3) An investment consultant predicts that the odds against the price of a certain stock will go up during the next week are 2 : 1 and the odds in favour of the price remaining the same are 1 : 3. what is the probability that, the price of the stock will go down during the next week

SOL

odds in favour of A is the ratio of no. of favourable events to unfavourable events.

Let A be the event that price goes up $P(A) = 1/3$
Let B be the event that price remains same $P(B) = \frac{1}{4}$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cap B) = 1 - (P(A) + P(B))$$
$$= 1 - (1/3 + 1/4)$$
$$= 1 - \frac{7}{12} = 5/12$$

---

4) A card is drawn from a pack of 52 cards. Find the probability getting a king or a heart or a red card

<u>SOL</u> A be the probability of getting king
B be the probability of getting a heart
C be the probability of getting a red card

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$P(A) = 4/52$ $\quad P(A \cap B) = 1/52$ $\quad P(A \cap B \cap C) = \frac{1}{52}$

$P(B) = 13/52$ $\quad P(B \cap C) = 13/52$

$P(C) = 26/52$ $\quad P(A \cap C) = 2/52$

$$P(A \cup B \cup C) = \frac{4 + 13 + 26 - 1 - 13 - 2 + 1}{52}$$
$$= 7/13$$

5) ~~Three~~ Newspapers A, B, C are published in a city; it is estimated from a survey that 20% read A, 60% read B, 14% read C, 8% read both A and B, 5% read both B and C, 4% read both A and C, 2% read all the 3

Find what percentage read atleast 1 paper

SOL $P(A) = \frac{60}{100}$ $P(B) = \frac{14}{100}$ $P(C) = 8/100$

~~The probability~~

$P(A \cap B) = 8/100$. $P(B \cap C) = 5/100$. $P(A \cap C) = 4/100$

$P(A \cap B \cap C) = 2/100$

$$P(A \cup B \cup C) = \frac{60 + 14 + 8 - 8 - 5 - 4 + 2}{100}$$

---

6) The probability that a student passes a phy test 2/3, The probability that he passes both eng and phy is 14/45. The probability that he passes atleast ~~eng~~ one test is 4/5

SOL let A be the prob of passing in phy $P(A) = 2/3$

B be the prob of passing eng is $P(B)$

$P(A \cup B) = 4/5$ $P(A \cap B) = 14/45$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$4/5 - 2/3 + \frac{14}{45} = P(B)$$

$$P(B) = 1$$

NDITIONAL PROBABILITY

Let A and B be any two events of sample space. probability that event A occurs given that B has already occurred.

$$\boxed{P(A/B) = \frac{P(A\cap B)}{P(B)}, \quad P(B/A) = \frac{P(A\cap B)}{P(A)}}$$

If A and B are independent $P(A/B) = P(A)$

$P(B/A) = P(B)$

Multiplicative theorem of probability:

Let A and B be two events

If A and B are dependent $P(A\cap B) = P(A/B)\cdot P(B)$

$P(A\cap B) = P(B/A)\cdot P(A)$

If A and B are independent $P(A\cap B) = P(A)\cdot P(B)$

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1) A problem in statistics to given 3 students A, B, C whose chances of solving it are 1/2, 3/4 and 1/4 respectively. What is the probability that the problem will be solved if all of them try independently

SOL

$P(A) = 1/2$; $P(B) = 3/4$, $P(C)$ $1/4$

$P(A\cup B\cup C) = 1 - P(\overline{A\cup B\cup C})$

$= 1 - P(\bar{A}\cap\bar{B}\cap\bar{C})$

$= 1 - P(\bar{A})\cdot P(\bar{B})\cdot P(\bar{C})$

$= 1 - (1 - 1/2)(1 - 3/4)(1 - \frac{1}{4}) = \frac{29}{32}$

2) One shot is fixed E_1, E_2 and E_3 denotes the event that the target is hit by the 1^{st}, 2^{nd} and 3^{rd} guns respectively. if $P(E_1) = 0.5$, $P(E_2) = 0.6$ and $P(E_3) = 0.8$. and E_1, E_2, E_3 are independent events.

i) Find the probability that exactly 1 hit is registered

ii) atleast 2 hit is registered

SOL

(i) $P(E_1 \cap \bar{E_2} \cap \bar{E_3}) + P(\bar{E_1} \cap \bar{E_2} \cap \bar{E_3}) + P(\bar{E_1} \cap E_2 \cap \bar{E_3})$

$= 0.26$

ii) $P(E_1 \cap E_2 \cap \bar{E_3}) + P(\bar{E_1} \cap \bar{E_2} \cap E_3) + P(E_1 \cap \bar{E_2} \cap E_3)$

$+ P(E_1 \cap E_2 \cap E_3) = 0.70$

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3) The odds that person x speaks the truth are 3:2 and the odds that the person y speaks the truth is 5:3. In what percentange of cases are they likely to contradict each other on an identical point

SOL

$P(A) = 3/5$

$P(B) = 5/8$

$\Rightarrow P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$P(A)\,P(\bar{B}) + P(\bar{A}) \cdot P(B)$

$= \left(\frac{3}{5}\left(1 - 5/8\right) + \left(1 - 3/5\right)\left(5/8\right) = \frac{19}{40}$

7/3 Groups of children contain respectively 3 girls, 1B, 2G and 2B, 1G and 3B.
One child is selected at random from each group. What is the chance of the 3 selected consists of 1G and 2B

SOL 3G 1B 2G 2B 1G 3B

GROUP	I	II	III	
	G	B	B	$= \left(\frac{3}{4} \times \frac{2}{4} \times \frac{3}{4}\right)$
	B	G	B	$= \left(\frac{1}{4} \times \frac{2}{4} \times \frac{3}{4}\right)$
	B	B	G	$= \left(\frac{1}{4} \times \frac{2}{4} \times \frac{1}{4}\right)$

$= \frac{13}{32}$

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5) A Box contains 6 Red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the prob that among the 4 balls drawn there's atleast 1 ball of each colour (without replacement)

SOL -

6 R, 4 W, 5 B

$\frac{720}{1365}$

$\frac{6C_1 \times 4C_1 \times 5C_2}{15C_4} + \frac{6C_1 \times 4C_2 \times 5C_1}{15C_4} + 6C_2 \times 4C_1 \times 5C_1$

6) From a city population, find the probability that

(i) A male or a smoker is $7/10 \rightarrow P(A \cup B)$

(ii) male smoker is $2/5 \rightarrow P(A \cap B)$

(iii) a male, if a smoker is already selected is $2/3$ $P(A|B)$

a) a non-smoker

b) a male

c) a smoker, if a male is first selected

SOL

a) $P(\bar{B}) = 1 - P(B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(B) = \frac{P(A \cap B)}{P(A|B)}$$

$$= \frac{2/5}{2/3} = \frac{3}{5}$$

$$P(\bar{B}) = 1 - \frac{3}{5} = \frac{2}{5}$$

b) $P(A) = ?$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) = P(A \cup B) - P(B) + P(A \cap B)$$

$$= \frac{7}{10} - \frac{3}{5} + \frac{2}{5}$$

$$= \frac{7 - 6 + 4}{10} = \frac{5}{10} = 1/2$$

c) $$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{2/5}{1/2} = \frac{4}{5}$$

7) Two computers A, B are to be marketed. A salesman who is assigned the job of finding customers for them has 60% and 40% respectively of succeeding in case of computer A and B. The 2 computers can be sold independently given that he was able to sell atleast 1 computer. What is the prob that computer A has been sold.

SOL

A : computer A is sold

B : computer B is sold

$$P(A/A\cup B) = \frac{P(A\cap(A\cup B))}{P(A\cup B)} = \frac{P(A)}{1-P(\bar{A})\cdot P(\bar{B})}$$

$$= \frac{\frac{60}{100}}{1-\left(\frac{40}{100}\times\frac{60}{100}\right)} = \frac{60}{76} = \frac{15}{19}$$

---

8) An urn contains 4 tickets numbered 1, 2, 3 and 4 and another contains 6 tickets number 2, 4, 6, 7, 8 and 9. If one of the 2 urns is chosen at random and a ticket is drawn random from the chosen urn, Find the probabilities that the ticket drawn bears the number

(i) 2 or 4

ii) 3

(iii) 1 or 9

let P(A) be selecting from urn 1

let P(B) be selecting from urn 2

Probability of selecting urn 1 is $1/2$
urn 2 is $1/2$

(i) 2 or 4

$$\text{urn } 1 = \frac{1}{2} \times \frac{2}{4} = \frac{1}{4}$$

$$= \frac{1}{4} + \frac{1}{6} = \frac{10}{24}$$

$$\text{urn } 2 = \frac{1}{2} \times \frac{2}{6} = \frac{1}{6}$$

(ii) 3

$$\text{urn } 1 = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

$$= \frac{1}{8}$$

$$\text{urn } 2 = 0$$

(iii) 1 or 9

$$\text{urn } 1 = \frac{1}{2} \times \frac{1}{\cancel{4}} = \frac{1}{\cancel{8}}$$

$$\text{urn } 2 = 1/2 \times 1/6 = \frac{1}{12}$$

$$= \frac{2}{\cancel{12}} = \frac{1}{\cancel{6}} = \frac{5}{24}$$

---

9) A Bag contains 10 Gold and 8 silver coins & successive drawing of 4 coins are made such that

(i) coins are replaced before the 2nd trial

(ii) " " not " " "

Find the probability that first drawing will give 4 gold and 2nd draw consists 4 silv

SOL

A : drawing 4 Gold coins in first draw

B " silver "

(i) $P(A \cap B) = P(A) \cdot P(B) = \frac{^{10}C_4}{^{18}C_4} \times \frac{^{8}C_4}{^{18}C_4}$

ii) $P(A \cap B) = P(A) \cdot P(B/A) = \frac{^{10}C_4}{^{18}C_4} \times \frac{^{8}C_4}{^{14}C_4}$

BAYE'S THEOREM:

Let $E_1, E_2 \dots E_n$, $P(E_i) \neq 0$ be n events of a sample space and for any arbitrary event E, such that $P(E) > 0$

$$P(E_i/E) = \frac{P(E_i \cap E)}{P(E)} = \frac{P(E_i)\,P(E/E_i)}{\sum^{n} P(E_i)\,P(E/E_i)}$$

---

1) in a certain town there are equal number of male and female residence. It's known that 5% females and 20% males are unemployed. If any unemployed is picked up at random what's the probability the person is

(i) a male

(ii) a Female

SOL

E_1 : Person is male

E_2 : Person is female

E : Person is employed

(i) $$P(E_1/E) = \frac{P(E_1)\,P(E/E_1)}{P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{20}{100}}{\frac{1}{2} \times \frac{20}{100} + \frac{1}{2} \times \frac{5}{100}} = 0.8 = \frac{4}{5}$$

ii) $P(E_2/E) = \dfrac{P(E_2)\ P(E/E_2)}{P(E_1)P(E/E_1) + P(E_2)P(E/E_2)} = \dfrac{\frac{1}{2} \times \frac{5}{100}}{\frac{1}{2} \times \frac{20}{100} + \frac{1}{2} \times \frac{5}{100}} = 1/5$

---

2) There are 3 candidates for the position of principal Mr. A, Mr. B, Mr. C whose chances of getting the appointment are in the proportion 4 : 2 : 3 respectively. The prob that if MR. A is selected introduce co-ed in the clg is 0.3. The Prob of Mr. B and C doing the same are respectively are 0.5 and 0.8. What is the Prob i) co-ed is introduced
ii) if there is a co-ed in the clg, what is the probability that C is the principal

SOL

E_1 : Mr. A is principal
E_2 : B is principal
E_3 : C is principal
E : coeducation is introduced

$P(E_1) = \frac{4}{9}$ $P(E_2) = \frac{2}{9}$ $P(E_3) = \frac{3}{9}$

$P(E/E_1) = 0.3$ $P(E/E_2)$ 0.5 $P(E/E_3) = 0.8$

(i) $P(E) = P(E_1)P(E/E_1) + P(E_2)P(E/E_2) + P(E/E_3)P(E_3)$

$= \frac{4}{9} \times 0.3 + \frac{2}{9} \times 0.5 + \frac{3}{9} \times 0.8$

$= \dfrac{1.2 + 1.0 + 2.4}{9} = \dfrac{4.6}{9} = \dfrac{23}{45}$

(ii) $= P(E_3/E) = \dfrac{P(E_3).P(E/E_3)}{P(E)} = \dfrac{\frac{3}{9}.0.8}{\frac{23}{45}} = \dfrac{12}{23}$

3) The chances that Doctor A will diagnose a disease x correcty is 60%. The chances that the patient will die with his treatment after crd diagonosis is 40% and the chance of death after wrong diagonosis is 10%. The Patient of A who had diease x died. What is the chance that his disease was diagonect correctly

SOL

E - Patient dies

E_1 -> ~~died coz of~~ correct diagonis

E_2 -> ~~died coz of~~ wrong diagonis

$P(E_1) = \cancel{\frac{40}{100}}$ $\quad P(E_1) = \frac{60}{100}$ $\quad P(E_2) = \frac{40}{100}$

$\cancel{P(E_2) = \frac{70}{100}}$ $\quad P(E/E_1) = \frac{40}{100}$ $\quad P(E/E_2) = \frac{70}{100}$

~~A Fact~~

$$P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)}$$

$$= \frac{\frac{60}{100} \cdot \frac{40}{100}}{\frac{60}{100} \cdot \frac{40}{100} + \frac{40}{100} \cdot \frac{70}{100}} = \frac{2400}{2400 + 2800}$$

$$= \frac{24}{52}$$

4) A Factory produces certain types of outputs with 3 machines

The respective production figures are

Machine I : 3000 units

II : 2500

III : 4500

Past experience shows that 1% of the output produced by I is defective. The corresponding fraction of defectives of others are 1.2% an 2% respectively.. An item is draw at random from the production is found to be defective. What is the prob that it comes from

(i) Machine I

(ii) II

(iii) III

SOL

$P(E_1) = \frac{3000}{10000}$ $P(E_2) = \frac{2500}{10000}$ $P(E_3) = \frac{4500}{10000}$

$P(E/E_1) = 0.01$ $P(E/E_2) = \frac{1.2}{100}$ $P(E/E_3) = \frac{2}{1000}$

(i) $P(E_1/E) = \frac{P(E_1)\,P(E/E_1)}{P(E_1)\,P(E/E_1) + P(E_2)\,P(E_2/E) + P(E_3)\,P(E/E_3)}$

$$= \frac{\frac{3000}{10000} \times \frac{1}{100}}{\frac{3000}{10000} \times \frac{1}{100} + \frac{2500}{10000} \times \frac{1.2}{100} + \frac{4500}{10000} \times \frac{2}{1000}}$$

$$= \frac{3000}{15000} = \frac{1}{5}$$

5) There are 2 Bags A and B. A contains n white and 2 Black Balls. B contains 2 white n black balls. One of the 2 bags is chosen at random and 2 Balls are drawn from it without replacement. If Both the balls drawn are white and the prob that Bag A chosen was $\frac{6}{7}$. Find the value of n

SOL

A -> n white 2 Black

B -> 2 white n Black

$P(E)$ = white ball

E_1 = from Bag A

E_2 = from Bag B

$\frac{1}{2}$

$$P(E_2/E) = \frac{P(E_1)\,P(E/E_1)}{P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2)}$$

$$P(E|A) = \frac{n(n-1)}{(n+2)(n+1)}$$

$$P(E|B) = \frac{2 \times 1}{(n+2)(n+1)} = 2$$

$$\frac{n(n-1)}{2 + n(n-1)}$$

$$\frac{n(n-1)}{(n(n-1)+(2)} = \frac{6}{7}$$

-12

$-4 \quad 3$

$$\frac{n^2-n}{n^2-n+2} = \frac{6}{7}$$

$$7n^2 - 7n - 6n^2 + 6n - 12 = 0$$

$$n^2 - n - 12 = 0$$

$$\boxed{n = 4}$$

CIA - I = 5 × 4M = 20M

INEQUALITIES

1) if $B \subseteq A$, $P(B) \leq P(A)$

2) Boolean's inequality

i) $P(\bigcap_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$

ii) $P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$

EXAMPLE

i) $P(A) = 40\%$ $P(B) = 20\%$

$P(A \cup B) \leq 60\%$

(ii) $P(A) = 3/4$ $P(B) = 5/8$

$P(A \cap B) \geq 3/4 + 5/8 - 1 = 3/8$

RANDOM VARIABLE

let S be a sample . R.V is a Function from $S \to R$, in which each outcome is associated with a real value

ex for tossing two coins let x be the no. of heads

$S = \{HH, HT, TH, TT\}$

DISCRETE R.V → assumes atmost countable infinite no. of real values

CONTINOUS R.V → assumes infinite uncountable no. of real values
ex. length, weight, time etc

PROBABILITY MASS FUNCTION

let $x_1, x_2 \dots x_n$ be the values of a R.V . A func which describes prob at each value of the r.v is called pmf and is denoted by $P_X(x) = P_X(x = x_i)$
$P_i = 1, 2$

PROPERTIES

(i) $\sum_{i=1}^{n} P(X = x_i) = 1$

(ii) $P(x = x_i) = P_i \geq 0$ for $1, 2 \dots n$

1) $P(x = x_i)$ $P \geq 0$ for $i = 1, 2, ..., n$. assume that 2 dice are thrown

$$X = \begin{cases} i & \text{if } (i = j) \\ \max(i, j) & i \neq j \end{cases}$$

Pmf

X	1	2	3	4	5	6
$P(x_i)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

$$P(X = x) = \frac{2x-1}{36}, \quad i = 1, 2, 3, 4, 5, 6$$

2)

x	1	2	3	4	5	6	7
$P(X = x)$	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

for given pmf find

(i) K

$$K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$$

$$10K^2 + 9K - 1 = 0$$

-10, 10, -1

$$K = \frac{1, -10}{10}$$

$$K = \frac{1}{10}$$

(ii) $P(X \geq 6) = 1 - P(X < 6)$

$$7K^2 + K$$

$$= \left(7\left(\frac{1}{100}\right) + \frac{1}{10}\right) = 8 = 81/100$$

(iii) ~~$P(0 < x < 5)$~~ $P(x \geq 6)$

$$= 2K^2 + 7K^2 + K$$

$$9K^2 + K = \cancel{10}$$

$$= \frac{9}{100} + \frac{1}{10} = \frac{19}{100}$$

iv) $P(0 < x < 5)$

$K + 2K + 2K + 3K$

$= 8K = 8\left(\frac{1}{10}\right) = \frac{8}{10}$

v) a such that $P(x \leq a) > \frac{1}{2}$

$a = 4$ [sub K and check

$$3)\ P(x) = \begin{cases} x/15 & x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

(i) $P(X = 1 \text{ or } 2)$

$= P(X=1) + P(X=2)$

$= \frac{1}{15} + \frac{2}{15} = \frac{1}{5}$

ii) $P\left(\frac{1}{2} < X < \frac{5}{2} \middle| X > 1\right)$

$$\frac{P(0.5 < x < 2.5 \cap x > 1)}{P(X > 1)}$$

$$= \frac{\frac{2}{15}}{\frac{14}{15}} = \frac{1}{7}$$

---

4) The p.m.f of a RV is given as

X	1	2	3	4
P(x)	0.4	0.3	0.2	0.1

find $P(1/2 < X < 7/2 \,/\, X > 1)$

<u>Sol</u>

$$= \frac{P(X=2 \text{ or } P=3)}{P(X=2 \text{ or } X=3 \text{ or } x=4)} = \frac{0.3+0.2}{0.3+0.2+0.1} = \frac{5}{6}$$

<u>DISTRIBUTION FUNCTION OR CUMULATIVE DISTRIBUTION FUNCTION (c.d.f)</u>

(increasing fun)

is defined as $F_X(x) = P(X \leq x)$

$$= \sum_{x_i \leq x} P(x_i)$$

<u>PROPERTIES OF cmf</u>

1) $0 \leq F_X(x) \leq 1$

2) $P(a < X \leq b) = F_X(b) - F_X(a)$

3) $\lim_{x \to -\infty} F(x) = 0$

$\lim_{x \to \infty} F(x) = 1$

1)

X	-2	-1	0	1	2	3
$P(X = x)$	0.1	K	0.2	$2K$	0.3	$3K$

(i) Find K

$0.1 + K + 0.2 + 2K + 0.3 + 3K = 1$

$6K + 0.6 = 1$

$6K = 0.4$

$K = \frac{0.4}{6} = \frac{0.2}{3}$

(ii) $P(X < 2)$

$= 1 - (P(X \geq 3))$

$= 1 - (3K + 0.2) = 1 - (0.2 + 0.3) = 0.5$

(iii) $P(-2 < X < 2)$

$= P(X = -1) + P(X = 0) + P(X = 1)$

$= K + 0.2 + 2K$

$= 0.2 + 3K$

$= 0.2 + 0.2 = 0.4$

iv) Find cdf

X	-2	-1	0	1	2	3
$P(X = x)$	0.1	$\frac{0.2}{3}$	0.2	$\frac{0.4}{3}$	0.3	0.2

cdf

X	-2	-1	0	1	2	3
$P(X = x)$	0.1	$\frac{0.5}{3}$	$\frac{1.1}{3}$	$\frac{1.5}{3}$	$\frac{2.4}{3}$	$\frac{3.0}{3}$

CONTINOUS R.V

Probability Density Function or Pdf of a continous R.V denoted by $f_x(x)$, where $P(a \le x \le b) = \int_a^b f(x)\,dx$

PROPERTIES

(i) $f_x(x) \ge 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(iii) $P(x = c) = 0$

---

1) let $f(x) = 6x(1-x)$, $0 \le x \le 1$ be the pdf of R.V X

i) check that $f(x)$ is a p.d.f

ii) find b such that $P(x<b) = P(x>b)$

SOL

(i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty} \Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_0^1 f(x)\,dx) + \int_1^{\infty} f(x)\,dx$$

$$= \int_0^1 6x - 6x^2 = \left[3x^2 - \frac{2x^3}{3}\right]_0^1$$

$$= 3 - \frac{2}{3} = 1$$

(ii) $\int_{-\infty}^{b} f(x)\,dx = \int_0^b f(x)\,dx = \int_b^1 f(x)\,dx$

$$= \int_0^b 6x - 6x^2 = \int_b^1 6x - 6x^2$$

$$\cancel{6x}\ [3x^2 - 2x^3]_0^b = [3x^2 - 2x^3]_b^1$$

$$= 3b^2 - 2b^3 = 3\cancel{x^2} - 2 - 3b^2 + 2b^3$$

$$= 3b^2 - 2b^3 + 3b^2 - 2b^3 \cancel{=} 1 = 0$$

$$= 6b^2 - 4b^3 - 1 = 0$$

---

2) Let p.d.f of R.V x be

$$f(x) = \begin{cases} 4x^3 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

find $P(x \le 2/3 \mid x > 1/3)$

<u>SOL</u>

$$= \frac{P(1/3 < x < 2/3)}{P(x > 2/3)}$$

$$= \frac{\int_{1/3}^{2/3} 4x^3 dx}{\int_{1/3}^{1} 4x^3 dx} = \frac{3}{16}$$

---

3) $$b(x) = \begin{cases} Ke^{-3x} & , x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find K and $P(0.5 \le x \le 1)$

<u>SOL</u>

$$\int_{-\infty}^{0} f(x) + \int_{0}^{\infty} f(x) = 1$$

$$K\int_0^\infty e^{-3x} = 1$$

$$K\left[\frac{e^{-3x}}{-3}\right]_0^\infty = 1$$

$$K\left[e^{-\infty} - e^0\right] = \frac{K}{-3}[0-1] = 1 \qquad \boxed{K = 3}$$

$$P(0.5 \le x \le 1) = \int_{0.5}^{1} 3e^{-3x}\,dx = \left[\frac{3e^{-3x}}{-3}\right]_{0.5}^{1}$$

$$= e^{-1.5} - e^{-3}$$

4) $f(x) = 3x^2,\ 0 \le x \le 1$

find a, b such that

i) $P(x \le a) = P(x > a)\ (\tfrac{1}{2})$

ii $P(X > b) = 0.05$

Sol

$$\int_{-\infty}^{a} f(x) = \int_{a}^{1} f(x)$$

$$= \int_{0}^{a} 3x^2 = \int_{a}^{1} 3x^2$$

$$= [x^3]_0^a = [x^3]_a^1$$

$$= a^3 = 1 - a^3$$

$$2a^3 = 1$$

$$a = (\tfrac{1}{2})^{1/3}$$

b) $\int_{b}^{1} 3x^2 = [x^3]_b^1$

$$1 - b^3 = 0.05$$

$$b^3 = 0.95$$

$$b = (0.95)^{1/3}$$

cumulative distribution function for continous R.V X is defined as $F_X(x) = P(X \le x)$

$$= \int_{-\infty}^{x} f(x)\,dx$$

PROPERTIES

① $0 \le F_X(x) \le 1$

② $\lim_{x \to -\infty} F(x) = 0$, $\lim_{x \to \infty} F(x) = 1$

③ $P(a \le x \le b) = F_X(b) - F_X(a)$

1) Find c.d.f of $f(x) = \begin{cases} 0 & x < 0 \\ 1/8 & 0 \le x < 2 \\ x/8 & 2 \le x < 4 \\ 0 & x \ge 4 \end{cases}$

SOL

$$F_X x = \begin{cases} \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{x} 0\,dx = 0 \\ \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} 1/8\,dx = \frac{x}{8} \\ \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} 1/8\,dx + \int_{2}^{x} x/8 = \frac{x^2}{16} \\ \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{2} 1/8\,dx + \int_{2}^{4} x/8\,dx + \int_{4}^{x} 0\,dx = 1 \end{cases}$$

2) find c.d.f of $f(x) = \begin{cases} Kx & 0 \le x < 1 \\ K & 1 \le x < 2 \\ -Kx + 3K & 2 \le x < 3 \\ 0 & \text{elsewhere} \end{cases}$

i) $x < 0$

$$\int_{-\infty}^{x} f(x)\,dx) = \int_{-\infty}^{0} 0\,dx = 0$$

ii) $0 \le x < 1$

$$\int_{-\infty}^{0} f(x) + \int_{0}^{x} f(x) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{x} Kx\,dx$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx + \int_{3}^{\infty} f(x\,dx)$$

$$0 + K\int_{0}^{1} x + K\int_{1}^{2} dx + \int_{2}^{3} Kx + 3K + 0 = 1$$

$$= K\left[\frac{x^2}{2}\right]_0^1 + K[x]_1^2 + K\left[-\frac{x^2}{2} + 3x\right]_2^3 = 1$$

$$= \frac{K}{2} + 2K - K + K\left[\frac{-9}{2} + 9 + \frac{4}{2} - 6\right] = 1$$

$$= \frac{K}{2} + 2K - K + K\left[\frac{-5}{2} + 3\right] = 1$$

$$\frac{K}{2} + 2K + \frac{K}{2} - K = 1$$

$$2K = 1$$

$$K = 1/2$$

answer = 1/2

$$F_X(x) = \begin{cases} \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{0} 0\,dx = 0 \\ \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{x} \frac{1}{2}x\,dx = \frac{x^2}{4} \\ x \int_{-\infty} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} \frac{1}{2}x\,dx + \int_{1}^{x} K\,dx = \frac{2x-1}{4} \\ x \int_{-\infty} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} \frac{1}{2}x\,dx + \int_{1}^{2} \frac{1}{2}\,dx + \int \frac{x}{2} + \frac{3}{2} \\ \frac{1}{2} + \frac{1}{2} + = -\frac{x^2}{4} + \frac{3x}{2} - \frac{5}{4} \\ = 1 \end{cases}$$

3) $f(x) = \begin{cases} \frac{100+x}{10000} & -100 \le x < 0 \\ \frac{100-x}{10000} & 0 \le x < 100 \\ 0 & \text{elsewhere} \end{cases}$

find c.d.f & $P(|x|<40)$ = $\frac{16}{25}$

SOL

$$F(x) = \begin{cases} 0 & x < -100 \quad \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty} 0\,dx = 0 \\ \frac{1}{10000}\left(100x + \frac{x^2}{2} + \frac{10^4}{2}\right) \\ \cancel{\frac{1}{10000}(\ )} & -100 \le x < 0 \quad \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{-100} f(x)\,dx + \int_{-100}^{x} f(x)\,dx \quad \frac{100+x}{10000} \\ \frac{1}{10000}\left(100x - \frac{x^2}{2} + \frac{10^4}{2}\right) & 0 \le x < 100 \quad = \int_{x} 0\,dx + \int_{-100}^{0} \frac{100+x}{10000} + \int_{0}^{x} \frac{100-x}{10000}\,dx \\ & x > 100 \quad \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{-100} 0\,dx + \int_{-100}^{0} \frac{100+x}{10000}\,dx \\ 1 & + \int_{0}^{100} \frac{100-x}{10000}\,dx + \int_{100}^{x} 0\,dx \end{cases}$$

$F'(x) = f(x)$

1) find $P(x)$ from the following c.d.f

X	0	1	2	3
$F(x)$	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	$\frac{8}{8} = 1$

$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$