8.3.7

EE24BTECH11003 - Akshara Sarma Chennubhatla

Question: Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.
Solution:
The 2 curves meet at the points $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 12 \end{pmatrix}$. So, the area between the curves is given by,

$$\int \left(\frac{3}{2}x + 6\right) - \left(\frac{3}{4}x^2\right) \tag{1}$$

Theoretical Solution:

$$\int \left(\frac{3}{2}x + 6\right) - \left(\frac{3}{4}x^2\right) \tag{2}$$

$$= \left[\frac{3}{2}\left(\frac{x^2}{2}\right) + 6x\right]_{-2}^{4} - \left[\frac{3}{4}\left(\frac{x^3}{3}\right)\right]_{-2}^{4} \tag{3}$$

$$= (24 + 12 - 16) - (2 + 3 - 12) \tag{4}$$

$$= 27 sq.units \tag{5}$$

Simulated Solution:

First, we divide the interval $b - a$ into n intervals of equal sizes, each of size $\frac{b-a}{n}$. We shall call each of the x values at the boundaries as $x_1, x_2, x_3, ...x_{n+1}$, where $x_1 = a, x_{n+1} = b$ and the step size as h. Individual areas are in the shape of a trapezoid. So, we sum the values of the areas of the individual trapezoids to get the value of the definite integral between a and b. For the n_{th} trapezoid, the area is given by,

$$A_n = \frac{1}{2}(h)(y(x_n) + y(x_{n+1})) \tag{6}$$

Also,

$$A_{n+1} = A_n + \frac{1}{2}h(y_n + y_{n+1}) \tag{7}$$

Using this equation we can get the total area under the curve by taking the sum of A_1 to A_n.

$$A = \frac{1}{2}h(y(x_1) + y(x_0)) + \frac{1}{2}h(y(x_2) + y(x_1)) + \cdots + \frac{1}{2}h(y(x_n) + y(x_{n-1})) \tag{8}$$

$$= h\left[\frac{1}{2}(y(x_0) + y(x_n)) + y(x_1) + \cdots + y(x_{n-1})\right] \tag{9}$$

By the first principle of derivatives,

$$y(x+h) = y(x) + hy'(x) \tag{10}$$

In this case, to calculate the area enclosed between the line and the parabola, we subtract the y coordinate of the parabola from the y coordinate of the line and then apply the trapezoidal rule on that function.
For the parabola,

$$\frac{dy}{dx} = \frac{3x}{2} \tag{11}$$

For the line,

$$\frac{dy}{dx} = \frac{3}{2} \tag{12}$$

The general area element in this case is given by,

$$A_n = \frac{1}{2}h\left(y(x_n) + \left(y(x_n) + hy'(x_n)\right)\right)(\text{for line}) - \frac{1}{2}h\left(y(x_n) + \left(y(x_n) + hy'(x_n)\right)\right)(\text{for parabola}) \tag{13}$$

$$A_n = \frac{1}{2}h\left(3x_n + 12 + h\frac{3}{2} - \frac{3}{2}x_n^2 - h\frac{3x_n}{2}\right) \tag{14}$$

$$\tag{15}$$

The general difference equation is given by,

$$A_{n+1} = A_n + \frac{1}{2}h\left(3x_n + 12 + h\frac{3}{2} - \frac{3}{2}x_n^2 - h\frac{3x_n}{2}\right) \tag{16}$$

$$x_{n+1} = x_n + h \tag{17}$$

By iterating through the required value of n, we get the area enclosed between the line and the parabola.
Theoretical area = 27 sq.units
Calculated area through trapezoidal rule = 26.9815636 sq.units

Below is the plot for line and the parabola

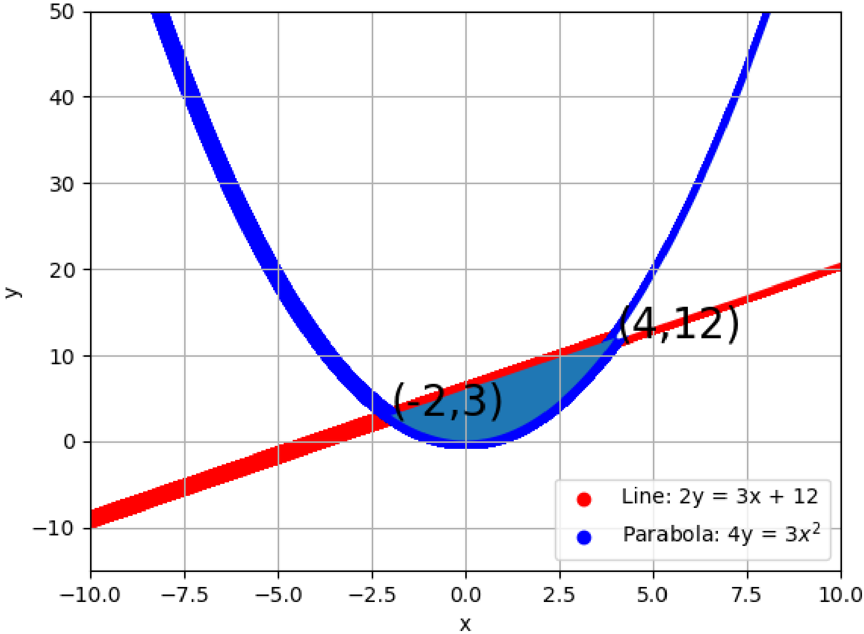

Fig. 1: Plot of the line $2y = 3x + 12$ and the parabola $= 4y = 3x^2$