Sprog Presentation

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Problem Statement

$$\Pr(A) = 0.54 \tag{0.1}$$
$$\Pr(B) = 0.69 \tag{0.2}$$
$$\Pr(A \cap B) = 0.35 \tag{0.3}$$

Find $\Pr(A \cup B)$

Theoretical Solution

For 2 Boolean variables A and B, the axioms of Boolean Algebra are defined as:

$$A + A' = 1 \tag{0.4}$$

$$A + A = A \tag{0.5}$$

$$AB = BA \tag{0.6}$$

$$A + B = B + A \tag{0.7}$$

$$AA' = 0 \tag{0.8}$$

$$\Pr(1) = 1 \tag{0.9}$$

$$\Pr(A + B) = \Pr(A) + \Pr(B), \text{ if } \Pr(AB) = 0 \tag{0.10}$$

Using these axioms, we will try to prove that

$$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{0.11}$$

Theoretical Solution

We will start by representing A and B as:

$$A = AB + AB' \tag{0.12}$$

$$B = AB + A'B \tag{0.13}$$

$$\Pr(A) = \Pr(AB) + \Pr\left(AB'\right) \tag{0.14}$$

$$\Pr(B) = \Pr(AB) + \Pr\left(A'B\right) \tag{0.15}$$

Theoretical Solution

On adding (0.12) and (0.13),

$$A + B = AB + AB + AB' + A'B \tag{0.16}$$

$$A + B = AB + AB' + A'B \tag{0.17}$$

$$\Pr(A + B) = \Pr\left(AB + AB' + A'B\right) \tag{0.18}$$

$$\Pr(A + B) = \Pr(AB) + \Pr\left(AB'\right) + \Pr\left(A'B\right) \tag{0.19}$$

$$\Pr(A + B) = \Pr(AB) + \Pr(A) - \Pr(AB) + \Pr(B) - \Pr(AB) \tag{0.20}$$

$$\implies \Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{0.21}$$

Theoretical Solution

Using the given values of $\Pr(A), \Pr(B)$ and $\Pr(AB)$,

$$\Pr(A+B) = 0.54 + 0.69 - 0.35 \tag{0.22}$$

$$\Pr(A+B) = 0.88 \tag{0.23}$$

Therefore, the value of $\Pr(A+B)$ is 0.88.

Simulated Solution

Let X_1 be an indicator random variable of the event A.

X_1 is defined as:

$$X_1 = \begin{cases} 1, & A \\ 0, & A' \end{cases} \tag{0.24}$$

Let X_2 be the indicator random variable of the event B.

X_2 is defined as:

$$X_2 = \begin{cases} 1, & B \\ 0, & B' \end{cases} \tag{0.25}$$

Let X_3 be the indicator random variable of the event AB.

X_3 is defined as:

$$X_3 = \begin{cases} 1, & AB \\ 0, & (AB)' \end{cases} \tag{0.26}$$

Simulated Solution

The PMF of the random variable X_1 is:

$$p_{X_1}(n) = \begin{cases} p_1, & n = 1 \\ 1 - p_1, & n = 0 \end{cases} \tag{0.27}$$

The PMF of the random variable X_2 is:

$$p_{X_2}(n) = \begin{cases} p_2, & n = 1 \\ 1 - p_2, & n = 0 \end{cases} \tag{0.28}$$

The PMF of the random variable X_3 is:

$$p_{X_3}(n) = \begin{cases} p_3, & n = 1 \\ 1 - p_3, & n = 0 \end{cases} \tag{0.29}$$

where,

$$p_1 = 0.54 \tag{0.30}$$
$$p_2 = 0.69 \tag{0.31}$$
$$p_3 = 0.35 \tag{0.32}$$
$$\tag{0.33}$$

Let Y be the random variable which is defined as follows:

$$Y = X_1 + X_2 - X_3 \tag{0.34}$$

Simulated Solution

But we know that X_3 can never be 0 when X_1 and X_2 are 1 and it can never be 1 when either of the two are 0.
So, Y is another Indicator Random variable whose PMF is defined as:

$$p_Y(n) = \begin{cases} p, & n = 1 \\ 1 - p, & n = 0 \end{cases} \tag{0.35}$$

Simulated Solution

From (0.34),

$$
\begin{aligned}
E(Y) &= E(X_1 + X_2 - X_3) && (0.36)\\
E(Y) &= E(X_1) + E(X_2) - E(X_3) && (0.37)\\
1.(p) + 0.(1-p) &= 1.(p_1) + 0.(1-p_1) + 1.(p_2) && (0.38)\\
&\quad + 0.(1-p_2) - 1.(p_3) - 0.(1-p_3) && (0.39)\\
p &= p_1 + p_2 - p_3 && (0.40)
\end{aligned}
$$

Through our definition, we know that,

$$
\begin{aligned}
\Pr(A) &= p_1 && (0.41)\\
\Pr(B) &= p_2 && (0.42)\\
\Pr(AB) &= p_3 && (0.43)
\end{aligned}
$$

Simulated Solution

Therefore, by comparison of the axiom

$$\Pr(A+B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{0.44}$$

and the equation (0.40),

$$p = \Pr(A+B) \tag{0.45}$$

$$\Pr(A+B) = 0.54 + 0.69 - 0.35 \tag{0.46}$$

$$\implies \Pr(A+B) = 0.88 \tag{0.47}$$

Plot

Below is the plot for the simulation of the probabilities, where the grey stems represent the theretical probabilities and the coloured stems represent the simulated ones.
Through observation in the last stem, we have proved through the code that

$$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{0.48}$$

Plot

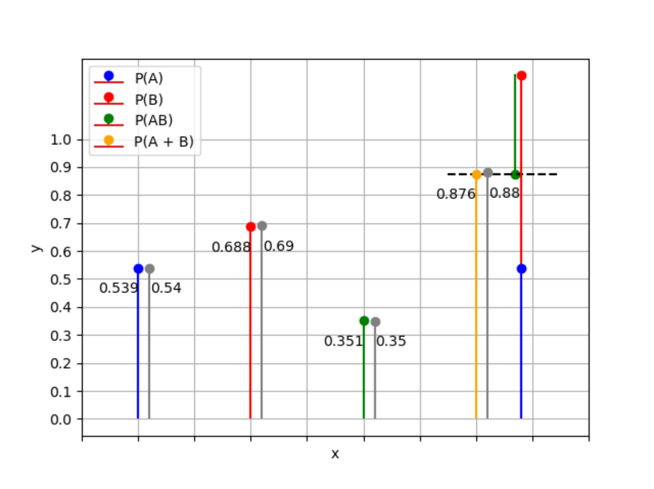

Figure 1: Plot of the probabilities