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EE24BTECH11003 - Akshara Sarma Chennubhatla

1) The distinct eigenvalues of the matrix $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ are
   a) 0 and 1
   b) 1 and -1
   c) 1 and 2
   d) 0 and 2
2) The minimal polynomial of the matrix $\begin{pmatrix} 3 & 3 & 0 \\ 3 & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ is
   a) $x(x-1)(x-6)$
   b) $x(x-3)$
   c) $(x-3)(x-6)$
   d) $x(x-6)$
3) Which of the following is the imaginary part of a possible value of $\ln\left(\sqrt{i}\right)$?
   a) π
   b) $\frac{\pi}{2}$
   c) $\frac{\pi}{4}$
   d) $\frac{\pi}{8}$
4) Let $f\colon \mathbb{C} \to \mathbb{C}$ be analytic except for a simple pole at $z = 0$ and let $g\colon \mathbb{C} \to \mathbb{C}$ be analytic. Then, the value of $\frac{Res_{z=0}\{f(z)g(z)\}}{Res_{z=0}\{f(z)\}}$ is
   a) $g(0)$
   b) $g'(0)$
   c) $lim_{z\to 0} zf(z)$
   d) $lim_{z\to 0} zf(z)g(z)$
5) Let $I = \oint_C \left(2x^2 + y^2\right) dx + e^y dy$, where C is the boundary (oriented anticlockwise) of the region in the first quadrant bounded by $y = 0, x^2 + y^2 = 1$ and $x = 0$. The value of I is
   a) -1
   b) $-\frac{2}{3}$
   c) $\frac{2}{3}$
   d) 1
6) The series $\sum_1^\infty x^{ln(m)}$, $x > 0$, is convergent on the interval
   a) $\left(0, \frac{1}{e}\right)$
   b) $\left(\frac{1}{e}, e\right)$
   c) $(0, e)$
   d) $(1, e)$
7) While solving the equation $x^2 - 3x + 1 = 0$ using the Newton-Raphson method with the initial guess of a root as 1, the value of the root after one iteration is
   a) 1.5
   b) 1
   c) 0.5
   d) 0

8) Consider the system of equations $\begin{pmatrix} 5 & 2 & 1 \\ -2 & 5 & 2 \\ -1 & 2 & 8 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 13 \\ -22 \\ 14 \end{pmatrix}$. With the initial guess of the solution $\left[x_1^{(0)}, x_2^{(0)}, x_3^{(0)}\right]^T = [1, 1, 1]^T$, the approximate value of the solution $\left[x_1^{(0)}, x_2^{(0)}, x_3^{(0)}\right]^T$ after one iteration by the Gauss-Seidel method is
   a) $[2, -4.4, 1.625]^T$
   b) $[2, -4, -3]^T$
   c) $[2, 4.4, 1.625]^T$
   d) $[2, -4, 3]^T$

9) Let y be the solution of the initial value problem
$$\frac{dy}{dx} = \left(y^2 + x\right); y(0) = 1$$
   Using Taylor series method of order 2 with the step size $h = 0.1$, the approximate value of $y(0.1)$ is
   a) 1.315
   b) 1.415
   c) 1.115
   d) 1.215

10) The partial differential equation
$$x^2 \frac{\partial^2 z}{\partial x^2} - \left(y^2 - 1\right) x \frac{\partial^2 z}{\partial x \partial y} + y(y-1)^2 \frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$$
   is hyperbolic in a region in the XY- plane if
   a) $x \neq 0$ and $y = 1$
   b) $x = 0$ and $y \neq 1$
   c) $x \neq 0$ and $y \neq 1$
   d) $x = 0$ and $y = 1$

11) Which of the following functions is a probability density function of a random variable X?
   a) $f(x) = \begin{cases} x(2-x) & 0 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$
   b) $f(x) = \begin{cases} x(1-x) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$
   c) $f(x) = \begin{cases} 2xe^{-x^2} & -1 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$
   d) $f(x) = \begin{cases} 2xe^{-x^2} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$

12) Let X_1, X_2, X_3 and X_4 be independent standard normal random variables. The distribution of
$$W = \frac{1}{2}\left\{(X_1 - X_2)^2 + (X_3 - X_4)^2\right\}$$
   is
   a) $N(0, 1)$
   b) $N(0, 2)$
   c) χ_2^2
   d) χ_4^2

13) For $n \geq 1$, let $\{X_n\}$ be a sequence of independent random variables with
$$P(X_n = n) = P(X_n = -n) = \frac{1}{2n^2}, P(X_n = 0) = 1 - \frac{1}{n^2}.$$

Then, which of the following statements is **TRUE** for the sequence $\{X_n\}$?

a) Weak Law of Large Numbers holds but Strong Law of Large Numbers does not hold
b) Weak Law of Large Numbers does not hold but Strong Law of Large Numbers holds
c) Both Weak Law of Large Numbers and Strong Law of Large Numbers hold
d) Both Weak Law of Large Numbers and Strong Law of Large Numbers do not hold