

2011-MA-1-13

EE24BTECH11003 - Akshara Sarma Chennubhatla

- 1) The distinct eigen values of the matrix $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ are
 - a) 0 and 1
 - b) 1 and -1
 - c) 1 and 2
 - d) 0 and 2
- 2) The minimal polynomial of the matrix $\begin{pmatrix} 3 & 3 & 0 \\ 3 & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ is
 - a) $x(x-1)(x-6)$
 - b) $x(x-3)$
 - c) $(x-3)(x-6)$
 - d) $x(x-6)$
- 3) Which of the following is the imaginary part of a possible value of $\ln(\sqrt{i})$?
 - a) π
 - b) $\frac{\pi}{2}$
 - c) $\frac{\pi}{4}$
 - d) $\frac{\pi}{8}$
- 4) Let $f: \mathbb{C} \rightarrow \mathbb{C}$ be analytic except for a simple pole at $z = 0$ and let $g: \mathbb{C} \rightarrow \mathbb{C}$ be analytic. Then, the value of $\frac{\text{Res}_{z=0}\{f(z)g(z)\}}{\text{Res}_{z=0}\{f(z)\}}$ is
 - a) $g(0)$
 - b) $g'(0)$
 - c) $\lim_{z \rightarrow 0} z f(z)$
 - d) $\lim_{z \rightarrow 0} z f(z) g(z)$
- 5) Let $I = \oint_C (2x^2 + y^2) dx + e^y dy$, where C is the boundary (oriented anticlockwise) of the region in the first quadrant bounded by $y = 0$, $x^2 + y^2 = 1$ and $x = 0$. The value of I is
 - a) -1
 - b) $-\frac{2}{3}$
 - c) $\frac{2}{3}$
 - d) 1
- 6) The series $\sum_1^\infty x^n n(m)$, $x > 0$, is convergent on the interval
 - a) $(0, \frac{1}{e})$
 - b) $(\frac{1}{e}, e)$
 - c) $(0, e)$
 - d) $(1, e)$
- 7) While solving the equation $x^2 - 3x + 1 = 0$ using the Newton-Raphson method with the initial guess of a root as 1, the value of the root after one iteration is
 - a) 1.5
 - b) 1
 - c) 0.5
 - d) 0

- 8) Consider the system of equations $\begin{pmatrix} 5 & 2 & 1 \\ -2 & 5 & 2 \\ -1 & 2 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 13 \\ -22 \\ 14 \end{pmatrix}$. With the initial guess of the solution

$[x_1^{(0)}, x_2^{(0)}, x_3^{(0)}]^T = [1, 1, 1]^T$, the approximate value of the solution $[x_1^{(0)}, x_2^{(0)}, x_3^{(0)}]^T$ after one iteration by the Gauss-Seidel method is

- a) $[2, -4.4, 1.625]^T$
- b) $[2, -4, -3]^T$
- c) $[2, 4.4, 1.625]^T$
- d) $[2, -4, 3]^T$

- 9) Let y be the solution of the initial value problem

$$\frac{dy}{dx} = (y^2 + x); y(0) = 1$$

Using Taylor series method of order 2 with the step size $h = 0.1$, the approximate value of $y(0.1)$ is

- a) 1.315
- b) 1.415
- c) 1.115
- d) 1.215

- 10) The partial differential equation

$$x^2 \frac{\partial^2 z}{\partial x^2} - (y^2 - 1) x \frac{\partial^2 z}{\partial x \partial y} + y(y - 1)^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$$

is hyperbolic in a region in the XY - plane if

- a) $x \neq 0$ and $y = 1$
- b) $x = 0$ and $y \neq 1$
- c) $x \neq 0$ and $y \neq 1$
- d) $x = 0$ and $y = 1$

- 11) Which of the following functions is a probability density function of a random variable X ?

- a) $f(x) = \begin{cases} x(2-x) & 0 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$
- b) $f(x) = \begin{cases} x(1-x) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$
- c) $f(x) = \begin{cases} 2xe^{-x^2} & -1 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$
- d) $f(x) = \begin{cases} 2xe^{-x^2} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$

- 12) Let X_1, X_2, X_3 and X_4 be independent standard normal random variables. The distribution of

$$W = \frac{1}{2} \{(X_1 - X_2)^2 + (X_3 - X_4)^2\}$$

is

- a) $N(0, 1)$
- b) $N(0, 2)$
- c) χ_2^2
- d) χ_4^2

- 13) For $n \geq 1$, let $\{X_n\}$ be a sequence of independent random variables with

$$P(X_n = n) = P(X_n = -n) = \frac{1}{2n^2}, P(X_n = 0) = 1 - \frac{1}{n^2}.$$

Then, which of the following statements is **TRUE** for the sequence $\{X_n\}$?

- a) Weak Law of Large Numbers holds but Strong Law of Large Numbers does not hold
- b) Weak Law of Large Numbers does not hold but Strong Law of Large Numbers holds
- c) Both Weak Law of Large Numbers and Strong Law of Large Numbers hold
- d) Both Weak Law of Large Numbers and Strong Law of Large Numbers do not hold