Lab Report: Experiment 3

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Experiment:
Studying the transient
and steady state response
of an RC Circuit
with Square Wave Input

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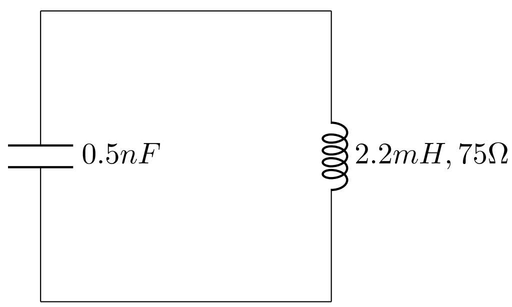
1 Introduction

In this experiment, we study and analyze the transient response of an LC circuit, determine the natural frequency (ω_n), and calculate the damping ratio (ξ) using theoretical and experimental methods.
An LC circuit consists of an inductor (L) and a capacitor (C) connected in parallel. When a charged capacitor is connected to an inductor, energy oscillates between the capacitor's electric field and the inductor's magnetic field. The system follows the second-order differential equation.

- Inductance, $2.2mH$
- Capacitance, $450pF$

2 Theory



Writing KVL equation,

$$L\frac{di}{dt} + iR + \frac{1}{C}\int i\,dt = 0$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

$$\frac{d^2q}{dt^2} + \left(\frac{R}{L}\right)\frac{dq}{dt} + \frac{q}{LC} = 0$$

Let

$$\omega_n = \frac{1}{\sqrt{LC}},$$
$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}.$$

Substituting these into the equation:

$$\frac{d^2q}{dt^2} + 2\xi\omega_n\frac{dq}{dt} + (\omega_n^2)q = 0$$

Here, $\xi < 1$, so the system is underdamped.
The solution for $q(t) = e^{st}$ will have complex conjugate roots:

$$s = -\xi\omega_n \pm j\omega_d,$$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$.
Thus, the current $i(t)$ can be written as:

$$q = Ce^{-\xi\omega_n t}e^{j\omega_d t} + C^* e^{-\xi\omega_n t}e^{-j\omega_d t}$$

Expanding this:

$$q = Ae^{-\xi\omega_n t}\cos(\omega_d t + \phi).$$

ϕ, A are constants that can be determined using inital conditions. We know $i\Big|_{t=0} = 0$, and $q\Big|_{t=0} = CV_{DC}$

Substituting $q(0) = CV_{DC}$,

$$CV_{DC} = A\cos(\phi). \tag{1}$$

The current $i(t)$ is given by,

$$i(t) = \frac{dq}{dt},$$
$$i(t) = -Ae^{-\xi\omega_n t}(\xi\omega_n\cos(\omega_d t + \phi) + \omega_d\sin(\omega_d t + \phi)).$$

At $t = 0$,

$$i(0) = -A(\xi\omega_n\cos(\phi) + \omega_d\sin(\phi)).$$

Substituting $\frac{dq}{dt}\Big|_{t=0} = 0$,

$$0 = -A(\xi\omega_n\cos(\phi) + \omega_d\sin(\phi)).$$

Simplifying,

$$0 = \xi\omega_n\cos(\phi) + \omega_d\sin(\phi). \tag{2}$$

From $(1), (2)$ we get,

$$\phi = \tan^{-1}\left(-\frac{\xi\omega_n}{\omega_d}\right)$$
$$A = \frac{CV_{DC}}{\cos(\phi)}$$

We finally get,

$$q(t) = \left(\frac{CV_{DC}}{\sqrt{1-\xi^2}}\right) e^{-\xi\omega_n t}\cos\left(\omega_d t + \tan^{-1}\left(-\frac{\xi\omega_n}{w_d}\right)\right).$$

Where, $\xi = \frac{R}{2}\sqrt{\frac{C}{L}}, \omega_n = \frac{1}{\sqrt{LC}}, \omega_d = \omega_n\sqrt{1-\xi^2}$

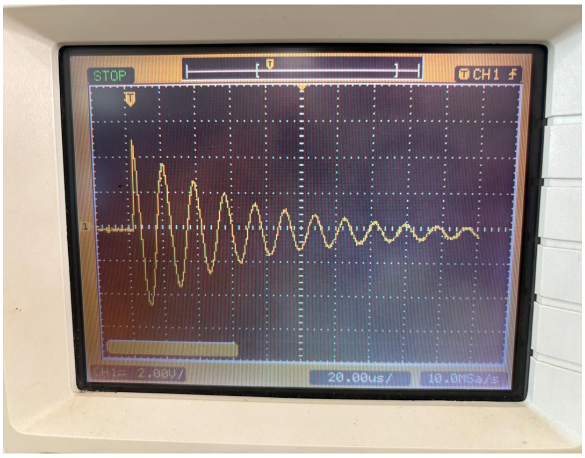
3 Procedure

- First, fully charge the capacitor using a DC voltage source (here, we have used $5V$).
- Then carefully disconnect the capacitor from charging and connect it in parallel with an inductor (preferably one of large inductance).
- Measure the voltage across either the inductor or capacitor (we have measured across inductor for convenience) using an oscilloscope.
- Using the oscilloscope, set it in the *single* mode and adjust the trigger level accordingly so that the voltage decay across the inductor (or capacitor) can be observed and captured properly.
- Be careful not to charge the capacitor when it is connected in parallel to the inductor, as the inductor might blow up.

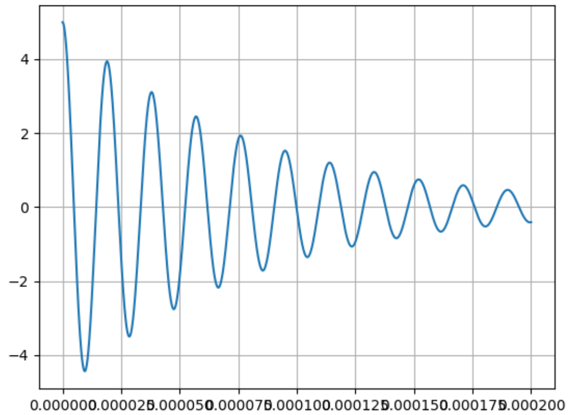
4 Experimental Data

Resistance 25Ω,
Capacitance $460pF$
Inductance, $2.2mH$
Python code used for verifcation can be found below,
`https://github.com/ArjunPavanje/EE1200/blob/main/Experiment_4/codes/`

(a) Experimental plot



(b) Theoretical plot



5 Conclusion

In this experiment, we derived the equation for the damped oscillation of a series RLC circuit and verified it experimentally.