Lab Report: Experiment 2

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Experiment:
Studying the transient
and steady state response
of an RC Circuit
with Square Wave Input

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1 Introduction

In this experiment, we investigate the frequency response of the RC low-pass filter for single-stage, two-stage (cascade), and three-stage (cascade) filters. The objective is to derive the transfer function for each case and plot the Bode magnitude and phase responses. The components used in the experiment have the following values:

- Resistance, $R = 100\,\Omega$
- Capacitance, $C = 0.1\,\mu F$

2 Transfer Function and Bode Plot for Single-Stage RC Filter

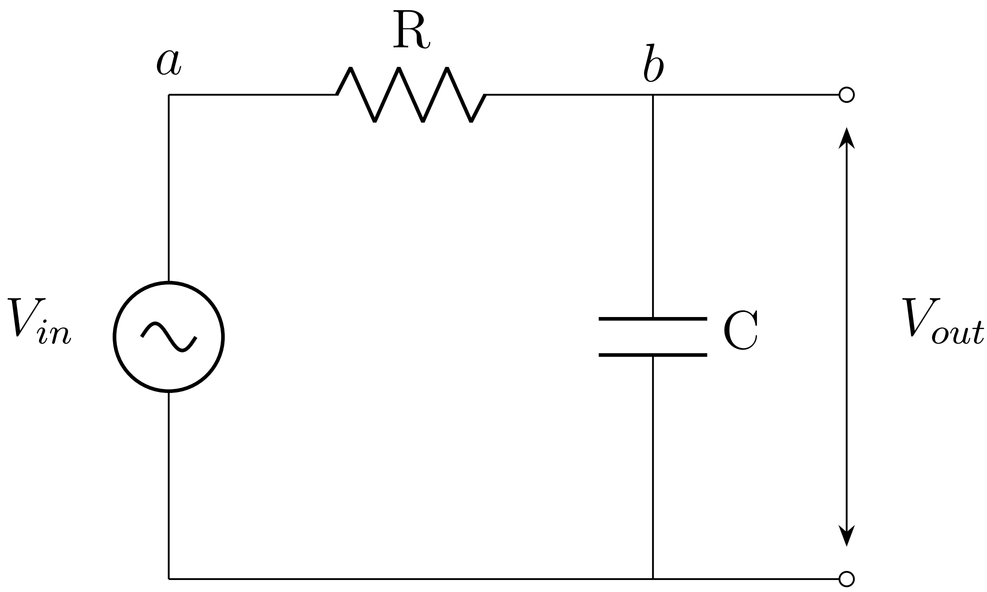
The transfer function of a single-stage RC low-pass filter is given by:

$$H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{1}{1 + sRC}$$

Where:

- R is the resistance,
- C is the capacitance,
- $s = j\omega$ is the complex frequency.
- $X_c = \frac{1}{j\omega C}$ is the impedance due to capacitor

2.1 One-Stage RC Low-Pass Filter



Applying KVL at node b

$$\frac{\vec{v}_{out} - \vec{v}_{in}}{R} + \frac{\vec{v}_{out}}{X_c} = 0$$

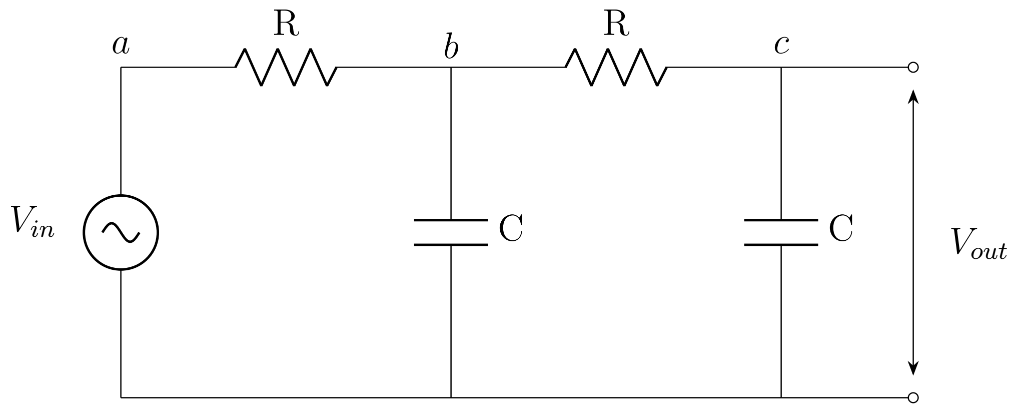
We get,

$$\vec{v}_{out}\left(\frac{R + X_c}{X_c}\right) = \vec{v}_{in}$$
$$H(jw) = \frac{1}{1 + j\omega RC}$$

We finally get,

$$|H(jw)| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$
$$\phi = \tan^{-1}(\omega RC)$$

2.2 Two-Stage RC Low-Pass Filter (Cascade of Two Filters)



Applying KVL at node c

$$\frac{\vec{v}_{out}}{X_c} + \frac{\vec{v}_{out} - \vec{v}_b}{R} = 0$$
$$\vec{v}_b = \vec{v}_{out}\left(\frac{R + X_c}{X_c}\right)$$

Applying KVL at node b

$$\frac{\vec{v}_b - \vec{v}_{out}}{R} + \frac{\vec{v}_b}{X_c} + \frac{\vec{v}_b - \vec{v}_{in}}{R} = 0$$
$$\vec{v}_{out}\left(\frac{R}{X_c}\right) + \vec{v}_{out}\left(\frac{R^2 + RX_c}{X_c^2}\right) + \vec{v}_{out}\left(\frac{R + X_c}{X_c}\right) = \vec{v}_{in}$$
$$\vec{v}_{out}\left(\frac{R^2 + X_c^2 + 3RX_c}{X_c^2}\right) = \vec{v}_{in}$$

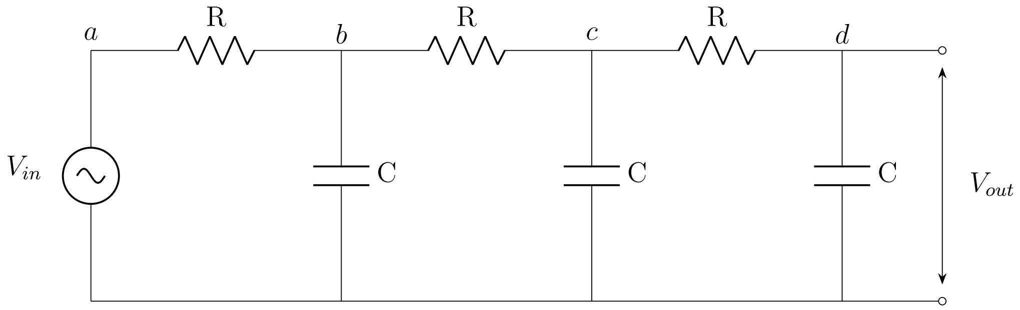
The transfer function becomes:

$$H(j\omega) = \frac{1}{(1 - \omega^2R^2C^2) + j(3\omega RC)}$$

We finally get,

$$|H(jw)| = \frac{1}{\sqrt{(1 - \omega^2R^2C^2)^2 + (3\omega RC)^2}}$$
$$\phi = \tan^{-1}\left(\frac{3\omega RC}{1 - \omega^2R^2C^2}\right)$$

2.3 Three-Stage RC Low-Pass Filter (Cascade of Three Filters)



Applying KVL on node d,

$$\frac{\vec{v}_{out}-\vec{v}_c}{R}+\frac{\vec{v}_d}{X_c}=0$$
$$\vec{v}_{out}\left(\frac{X_c+R}{X_c}\right)=\vec{v}_c$$

Applying KCL on node c we get,

$$\frac{\vec{v}_c-\vec{v}_d}{R}+\frac{\vec{v}_c-\vec{v}_b}{R}+\frac{\vec{v}_c}{X_c}=0$$
$$\vec{v}_c=\vec{v}_{out}\left(\frac{(R+Z_c)^2+RX_c}{X_c^2}\right)$$

Applying KVL on node b,

$$\frac{\vec{v}_b-\vec{v}_{in}}{R}+\frac{\vec{v}_b-\vec{v}_c}{R}+\frac{\vec{v}_b}{X_c}=0$$

On simplifying we get,

$$\vec{v}_{out}\left(\frac{RX_c(R+X_c)+(R+X_c)^3}{X_c^3}+\frac{R}{X_c}+\frac{R(X_c+R)}{X_c^2}\right)=\vec{v}_{in}$$
$$\vec{v}_{out}\left(\frac{2RX_c(R+X_c)+(R+X_c)^3+RX_c^2}{X_c^3}\right)=\vec{v}_{in}$$

The transfer function becomes:

$$H(j\omega)=\frac{1}{(1-5\omega^2R^2C^2)+j(6\omega RC-\omega^3R^3C^3)}$$

We finally get,

$$|H(jw)|=\frac{1}{\sqrt{(1-5\omega^2R^2C^2)^2+(6R\omega C-\omega^3R^3C^3)^2}}$$
$$\phi=\tan^{-1}\left(\frac{6\omega RC-\omega^3R^3c^3}{1-5\omega^2R^2C^2}\right)$$

3 Bode Plot Analysis

The Bode plot is typically plotted in two parts: magnitude and phase

Magnitude:
20 $log_{10}(|H(jw)|)$ vs $log_{10}(w)$ is plotted

Phase:
ϕ vs $log_{10}(w)$ is plotted

The frequency response will be plotted for single, two-stage, and three-stage filters.

4 Experimental Data

One thing to note is that when the value of capacitance was verified using a multimeter, capacitance was found to be 0.88 times the labelled value. So capacitance for theoretical calculations was taken to be $0.088\mu F$.
The following table summarizes the experimental data collected. It includes the frequency (f), the magnitude of the transfer function $H(j\omega)$, and the change in the time (Δt)
One stage RC low pass filter:

Frequency (Hz)	$\|H(j\omega)\|$	Δt
50	0.96	0
100	0.96	0
200	0.96	0
400	0.96	0
800	0.96	0
1600	0.96	3
3200	0.94	4
6400	0.88	8
12800	0.77	6.4
25600	0.55	5.6
51200	0.34	3.6

Two stage RC low pass filter:

Frequency (Hz)	$\lvert H(j\omega)\rvert$	Δt
50	0.96	0
100	0.96	0
200	0.96	0
400	0.96	5
800	0.96	10
1600	0.96	24
3200	0.916	18
6400	0.77	16
12800	0.55	14
25600	0.277	10
51200	0.13	6

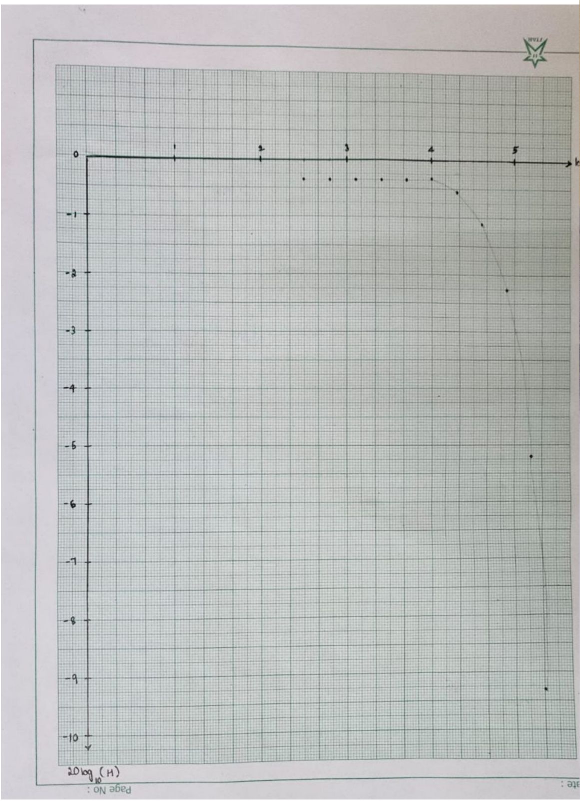
Three stage RC low pass filter:

Frequency (Hz)	$\lvert H(j\omega)\rvert$	Δt
50	0.9615	-19.8
100	0.96	-9.85
200	0.96	-5
400	0.96	-2.47
800	0.96	-1.22
1600	0.92	-0.57
3200	0.833	-0.282
6400	0.651	-0.13
12800	0.409	-0.053
25600	0.203	-0.025
51200	0.03	-0.0105

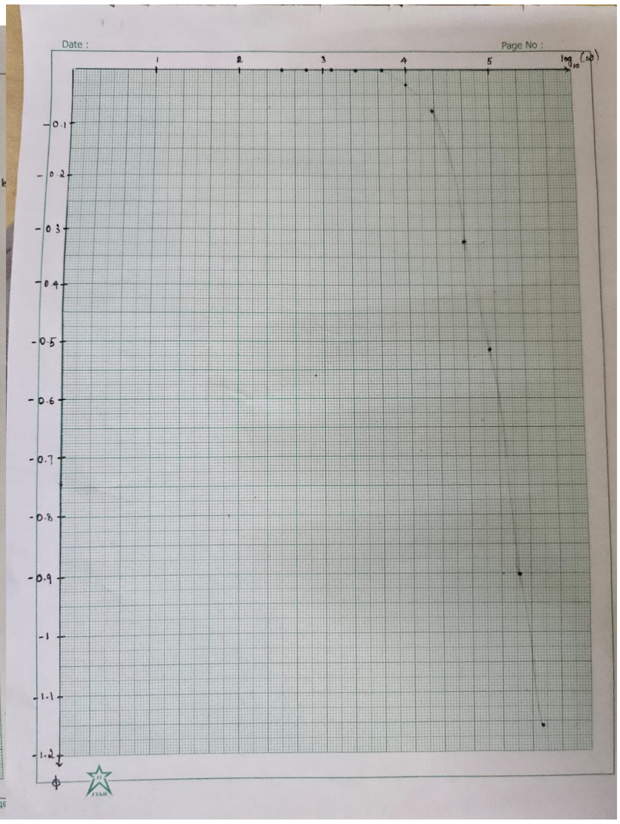
5 Plots

5.1 One Stage:

(a) Magnitude plot

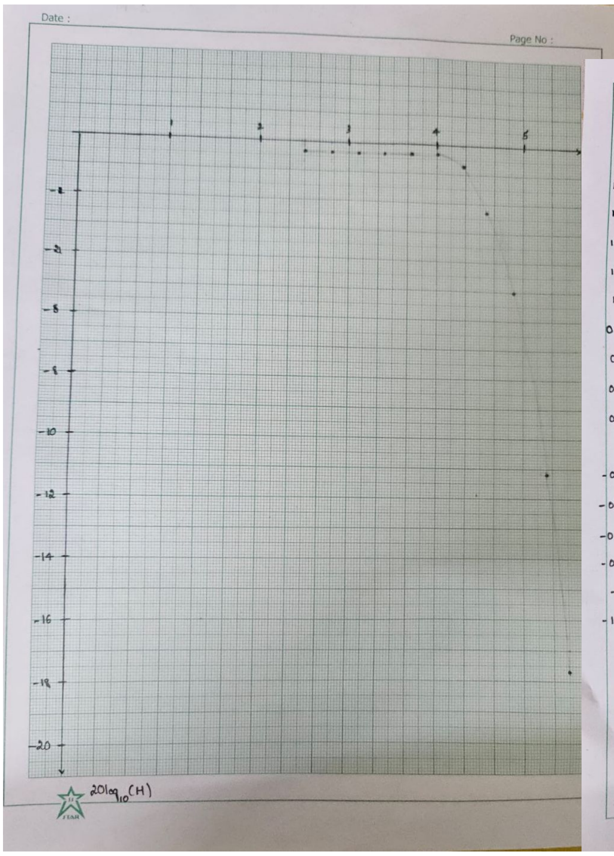


(b) Phase plot

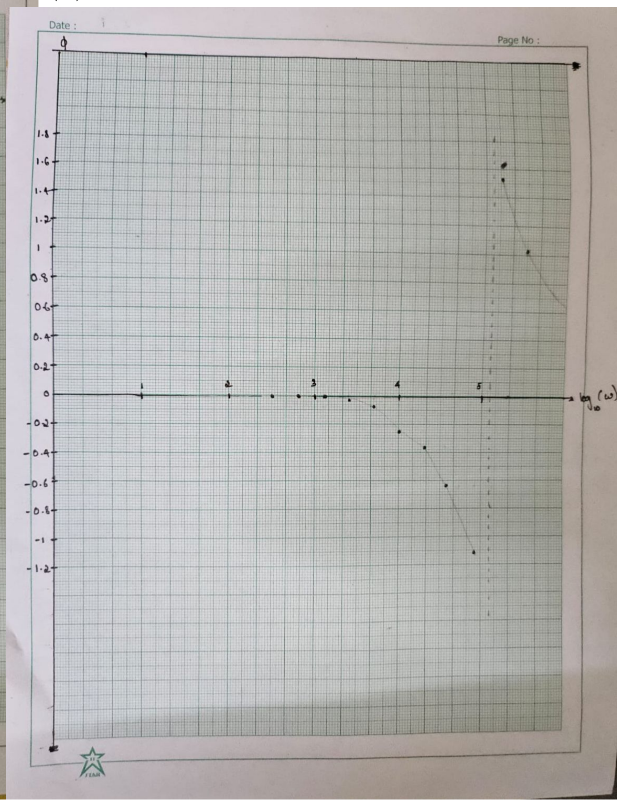


5.2 Two Stage:

(a) Magnitude plot

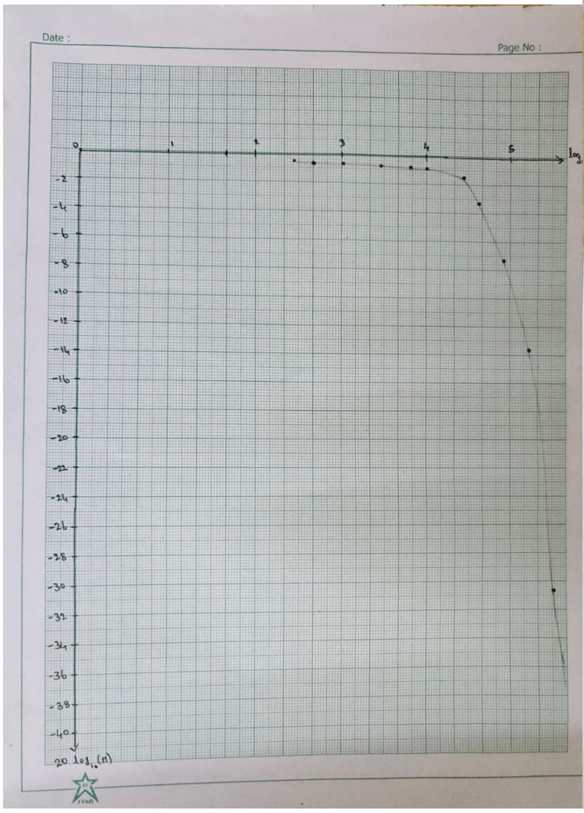


(b) Phase plot

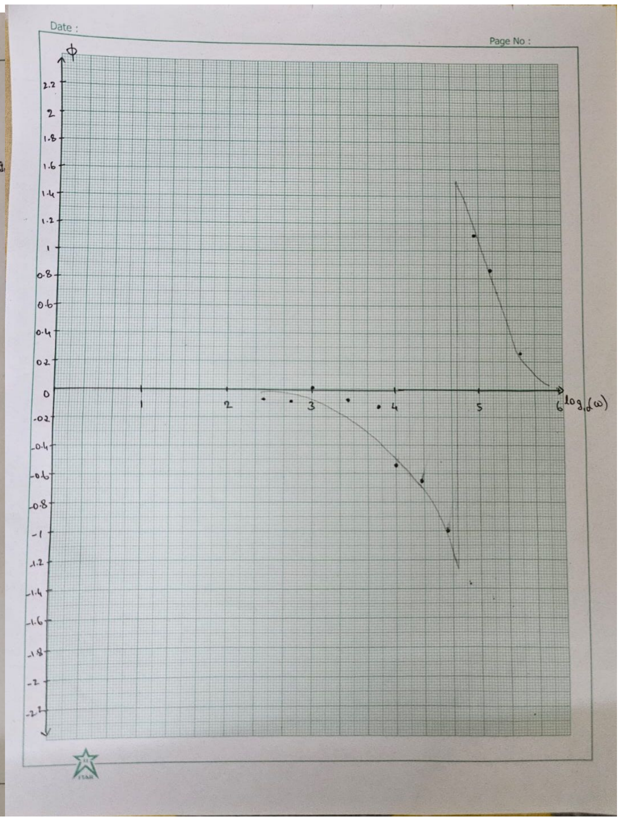


5.3 Three Stage:

(a) Magnitude plot



(b) Phase plot



If images are not clear,

`https://github.com/ArjunPavanje/EE1200/tree/main/Experiment_3/figs`

To view theoretical verifcation of obtained data,

`https://github.com/ArjunPavanje/EE1200/tree/main/Experiment_3/codes`

6 Conclusion

In this experiment, we derived the transfer functions for single-stage, two-stage, and three-stage RC low-pass filters, analyzed the frequency response, and generated Bode plots for each case. The cascade of stages leads to a steeper roll-off in the frequency response as the number of stages increases.