Lab Report: Experiment 6

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Experiment: Design, implementation, and analysis of a bandpass filter constructed by cascading Sallen-Key second-order low-pass and high-pass filters. Analysis of Bode plots of HPF, LPF and BPF.

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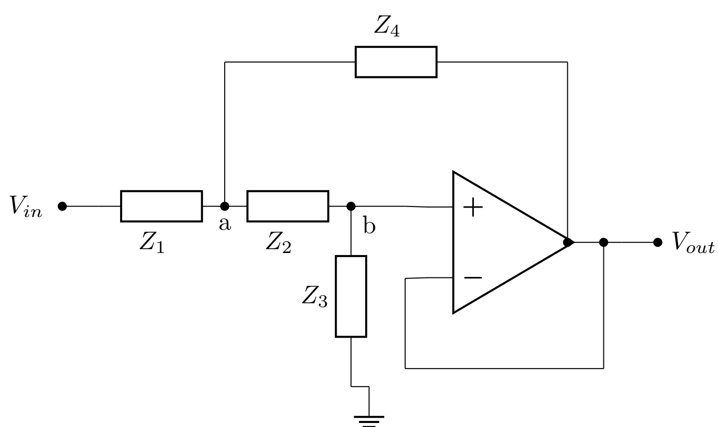
1 Introduction

This experiment focuses on implementing a bandpass filter using the Sallen-Key topology, a versatile active filter design that employs operational amplifiers.

The bandpass filter is constructed by cascading a high-pass filter (HPF) and a low-pass filter (LPF). The HPF attenuates frequencies below its cutoff frequency (ω_{c1}), while the LPF attenuates frequencies above its cutoff frequency (ω_{c2}). When properly designed with $\omega_{c2} > \omega_{c1}$, the combined response creates a bandpass characteristic.

The Sallen-Key topology offers a simple, low-component design with high input impedance and low output impedance, making it ideal for active filters with minimal component interaction and predictable performance.

Sallen Key Filter Circuit



Derivation of Transfer function for Sallen Key Filter:

For ideal Op Amp,

$$V_b = V_{out}$$

Applying KCL at a:

$$\frac{V_a - V_{in}}{Z_1} + \frac{V_a - V_b}{Z_2} + \frac{V_a - V_{out}}{Z_4} = 0 \quad \text{-(1)}$$

Applyting KCL at b:

$$\frac{V_b - V_a}{Z_2} + \frac{V_b - 0}{Z_3} = 0$$
$$V_b \left(\frac{1}{Z_2} + \frac{1}{Z_3} \right) = \frac{V_a}{Z_2}$$
$$V_a = V_b Z_2 \left(\frac{1}{Z_2} + \frac{1}{Z_3} \right)$$

Simplifying (1)

$$V_a \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_4} \right) - \frac{V_{in}}{Z_1} = V_{out} \left(\frac{1}{Z_2} + \frac{1}{Z_4} \right)$$
$$V_{out} Z_2 \left(\frac{1}{Z_2} + \frac{1}{Z_3} \right) \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_4} \right) - \frac{V_{in}}{Z_1} = V_{out} \left(\frac{1}{Z_2} + \frac{1}{Z_4} \right)$$

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{\frac{1}{Z_1}}{Z_2 \left(\frac{1}{Z_2} + \frac{1}{Z_3} \right) \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_4} \right) - \left(\frac{1}{Z_2} + \frac{1}{Z_4} \right)}$$
$$H(s) = \frac{\frac{1}{Z_1}}{\frac{(Z_2+Z_3)(Z_1Z_2+Z_1Z_4+Z_2Z_4)-(Z_2+Z_4)Z_1Z_3}{Z_1Z_2Z_3Z_4}}$$
$$= \frac{Z_2Z_3Z_4}{(Z_2 + Z_3)(Z_1Z_2 + Z_1Z_4 + Z_2Z_4) - Z_1Z_3(Z_2 + Z_4)}$$
$$= \frac{Z_3Z_4}{Z_1Z_2^2 + Z_1Z_2Z_4 + Z_2^2Z_4 + Z_1Z_3Z_4}$$
$$= \frac{Z_3Z_4}{Z_1Z_2 + Z_1Z_4 + Z_2Z_4 + Z_3Z_4}$$

Theory

Bandpass Filter Principle

A bandpass filter allows signals within a specific frequency band to pass while attenuating signals outside this band. The transfer function of an ideal bandpass filter can be expressed as:

$$H(j\omega) = \begin{cases} 1, & \text{if } \omega_1 < \omega < \omega_2 \\ 0, & \text{otherwise} \end{cases}$$

Where $\omega_1 = 2\pi f_{c1}$ and $\omega_2 = 2\pi f_{c2}$ represent the lower and upper cutoff frequencies, respectively.

Sallen-Key Filter Topology

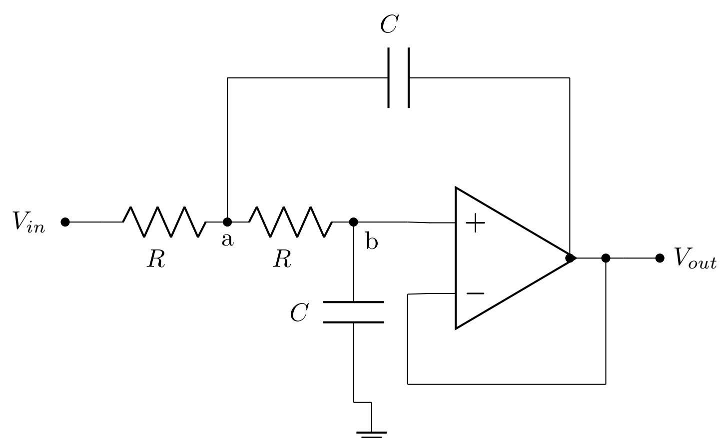
The Sallen-Key topology is a versatile active filter configuration that uses an operational amplifier with RC components to create second-order filter responses. The general transfer function of a second-order Sallen-Key filter is:

$$H(s) = \frac{A}{s^2 + (\omega_c/Q)s + \omega_c^2}$$

Where:

- ω_c is the cutoff frequency
- Q is the quality factor
- A is the gain at passband

Sallen-Key Low-Pass Filter



$$Z_1 = Z_2 = R_1$$

$$Z_3 = Z_4 = \frac{1}{SC_1}$$

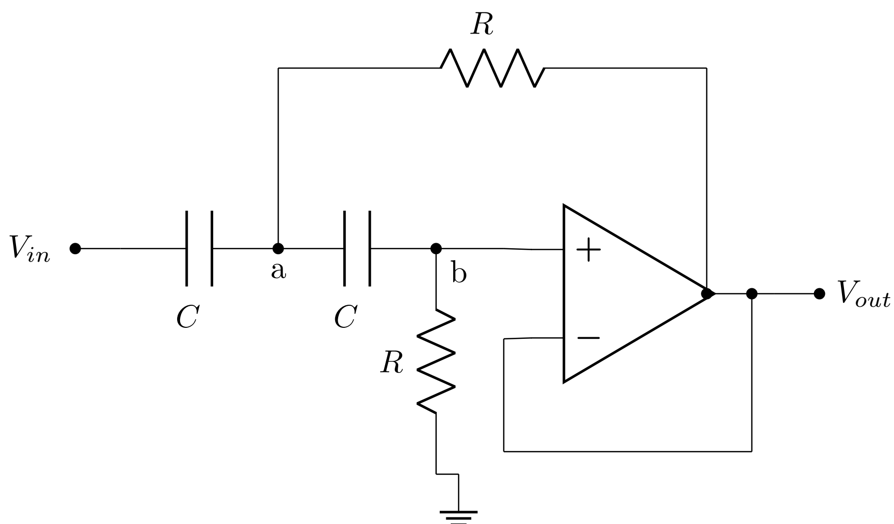
$$H(s) = \frac{\left(\frac{1}{SC_1}\right)^2}{R_1^2 + \frac{2R_1}{SC_1} + (\frac{1}{SC_1})^2}$$

$$H(s) = \frac{1}{(1 + SR_1C_1)^2}$$

The cutoff frequency for the Sallen-Key LPF is given by:

$$\omega_{c_1} = \frac{1}{R_1C_1}$$

Sallen-Key High-Pass Filter

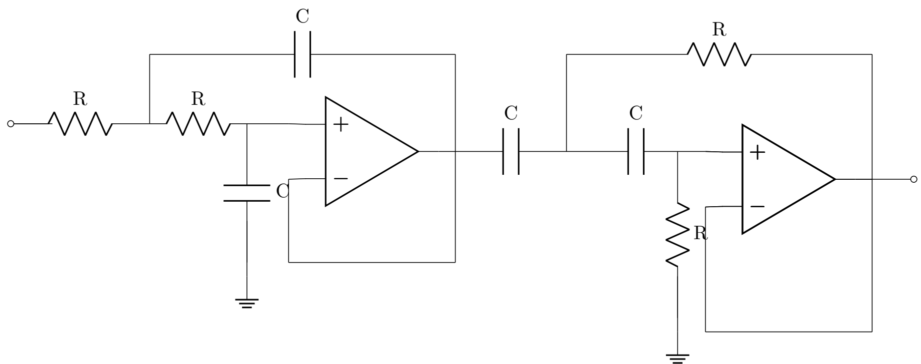


$$Z_1 = Z_2 = \frac{1}{SC_2}$$
$$Z_3 = Z_4 = R_2$$
$$H(s) = \frac{R_2^2}{R_2^2 + \frac{2R_2}{SC_2} + (\frac{1}{SC_2})^2}$$
$$H(s) = \frac{(SR_2C_2)^2}{(1 + SR_2C_2)^2}$$

The cutoff frequency for the Sallen-Key LPF is given by:

$$\omega_{c_2} = \frac{1}{R_2C_2}$$

Cascaded Bandpass Filter



When an HPF with cutoff frequency ω_{c1} is cascaded with an LPF with cutoff frequency ω_{c2} (where $\omega_{c2} > \omega_{c1}$), the resulting system functions as a bandpass filter with:

- Lower cutoff frequency: ω_{c1}
- Upper cutoff frequency: ω_{c2}
- Bandwidth: $BW = \omega_{c2} - \omega_{c1}$
- Center frequency: $\omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$

The overall transfer function becomes the product of the individual transfer functions:

$$H_{BPF}(s) = H_{HPF}(s) \cdot H_{LPF}(s)$$

$$H_{BPF}(s) = \frac{(SR_1C_1)^2}{(1 + SR_1C_1)^2(1 + SR_2C_2)^2}$$

Procedure

Components

For this experiment, we select the following components:

- Operational Amplifier: LM358 (or equivalent)
- Resistors: $R_1 = R_2 = 1k\Omega$ (for LPF), $R_3 = R_4 = 15k\Omega$ (for HPF)
- Capacitors: $C_1 = C_2 = 1nF$ (for LPF), $C_3 = C_4 = 1nF$ (for HPF)

The theoretical cutoff frequencies are:

- HPF: $\omega_{c1} = \frac{1}{(\sqrt{R_1R_2C_1C_2})} = \frac{10^6}{15}$
- LPF: $\omega_{c2} = \frac{1}{(\sqrt{R_3R_4C_3C_4})} = 10^6$

High-Pass Filter Implementation

1. Assemble the Sallen-Key HPF circuit on the breadboard according to the circuit diagram.

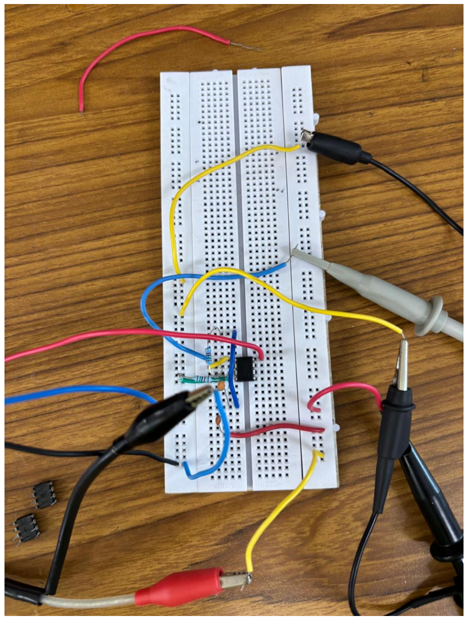


Figure 1: High Pass Filter circuit

2. Connect the function generator to provide a sine wave input.
3. Connect the oscilloscope to measure both input and output signals.
4. Vary the input frequency by atleast doubling each time.
5. For each frequency, measure the output voltage and calculate the gain (V_{out}/V_{in}).
6. Record the measurements in a table and plot the bode plot.

4.3 Low-Pass Filter Implementation

1. Assemble the Sallen-Key LPF circuit on the breadboard as shown.

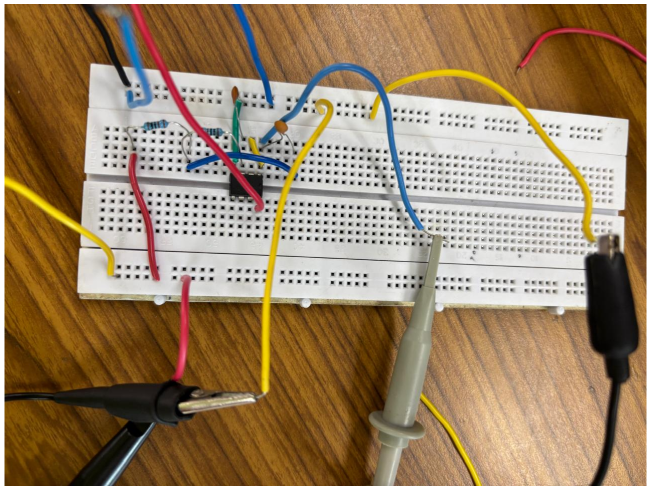


Figure 2: High Pass Filter circuit

2. Repeat the measurement procedure as in the HPF implementation.
3. Record the measurements and plot the bode plot.

4.4 Bandpass Filter Implementation

1. Connect the output of the HPF to the input of the LPF to form the bandpass filter.

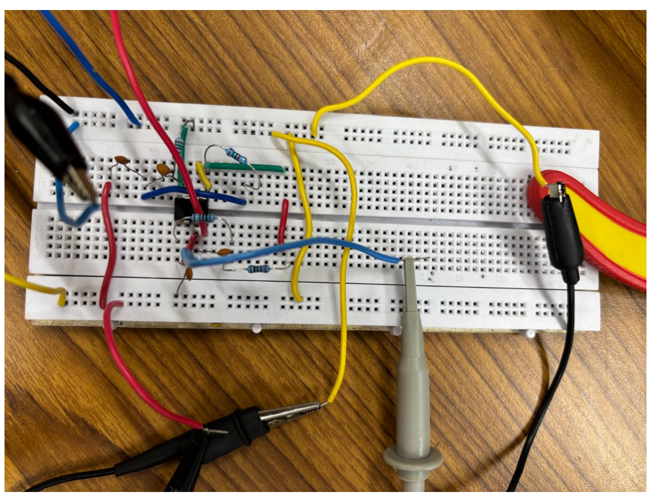


Figure 3: High Pass Filter circuit

2. Repeat the measurement procedure across the frequency range.
3. Record the measurements and plot the bode plot.

5 Results and Observations

Low Pass, HighPass,

$\log\omega$	$\mathbf{20}\log\mathbf{H}(\mathbf{j}\omega)$
2.50	0.00
2.80	0.00
3.10	0.00
3.50	0.00
3.97	0.00
4.50	0.00
5.10	0.00
5.40	0.00
5.70	-6.38
6.10	-13.98
6.50	-27.96

Table 1: Low pass filter

$\log\omega$	$\mathbf{20}\log\mathbf{H}$
3.28	-58.06
3.50	-54.62
3.80	-42.92
4.10	-31.93
4.50	-18.06
4.80	-9.28
5.10	-4.53
5.40	-3.25
5.50	-2.87
5.70	-1.58

Table 2: High pass filter

BandPass,

Images of oscilloscope while capturing response may be found in the respective folders in the below link, `https://github.com/ArjunPavanje/EE1200/tree/main/Experiment_6/figs`

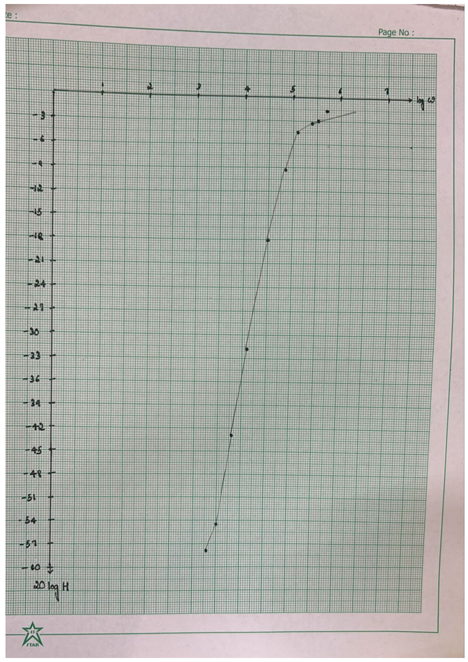


Figure 4: Low Pass Filter Graph

$\log \omega$	$\mathbf{20} \log \mathbf{H}$
4.50	-20.67
4.80	-9.90
4.97	-5.85
5.28	-3.44
5.50	-3.44
5.80	-5.69
6.10	-15.88
6.20	-20.98

Table 3: Band pass filter

7 Conclusion

This experiment demonstrates how Sallen-Key filters can be constructed, and how a high pass and a lowpass filter together can be used to create a bandpass filter.

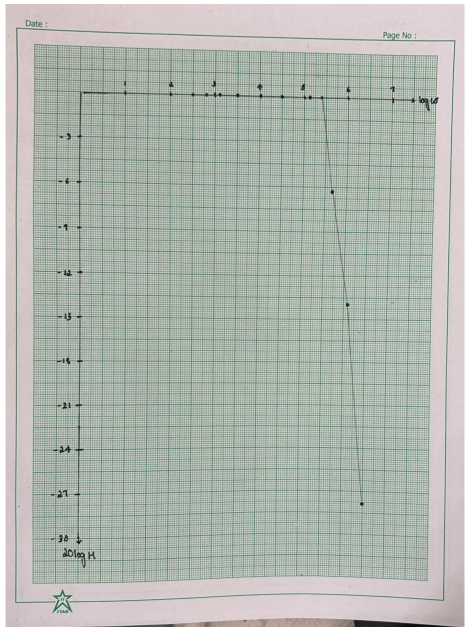


Figure 5: High Pass Filter Graph

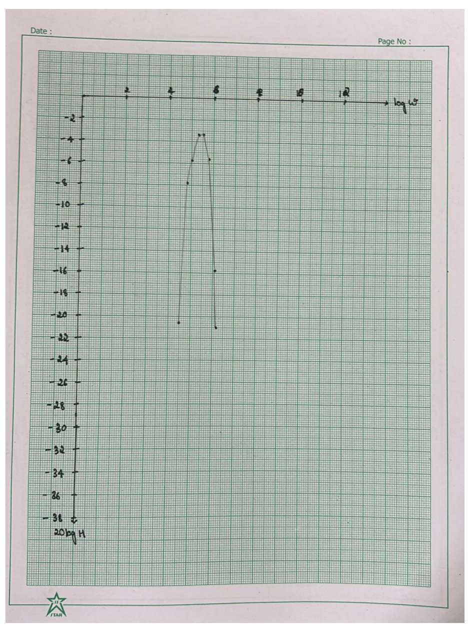


Figure 6: Band Pass Filter Graph