Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

    A. 0.3875
    B. 0.2676
    C. 0.5
    D. 0.6987

Ans: Given,

Mean = 45 min
Standard deviation = 8 min
Here the service starts after 10 min of drop off and completes in 1 hour.
60-10 =50, so the work can be completed in 50 minutes.
Probability that the service manager cannot meet his commitments =
(1-pnorm(50,45,8)) = 0.267

Answer is option B

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
    A. More employees at the processing center are older than 44 than between 38 and 44.
    B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

A. **False.**

Empolyees older than 44 years of age, p(x>44)
1-stats.norm.cdf(44,38,6) = 0.1586
Empolyees between 38 to 44 years of age, p(38<x<44)
Stats.norm.cdf(44,38,6) - stats.norm.cdf(38,38,6) = 0.3413

B. True

Employess under 30 years of age
Stats.norm.cdf(30,38,6)
P(x<30) = 0.0912
No. of employees attending training program from 400 numbers is
N*P(x<30)= 36.4844

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans: Both the distributions are normally distributed but the parameters is varies.

Distribution of $2X_1$: In this distribution, when we are multiplying with 2 the mean is multiplied with 2 but the variance is multiplied with the square of the constant that is 4.
$2X_1 \sim N(2\mu, 4\sigma^2)$
Distribution of $X_1 + X_2$: In this distribution we are adding the individual variables so the mean is doubled and same with the variance.
$X_1 + X_2 \sim N(2\mu, 2\sigma^2)$
Parameters of $2X_1$: μ, σ^2 These are the initial values
Mean = 2μ
Variance = $4\sigma^2$
Parameters of $X_1 + X_2$: μ, σ^2 These are the initial values.
Mean = 2μ
Variance = $2\sigma^2$

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

    A. 90.5, 105.9
    B. 80.2, 119.8
    C. 22, 78
    D. 48.5, 151.5
    E. 90.1, 109.9

Ans:

The probability of the random variable taking value between a & b = 0.99
So, The probability of getting value outside a & b = 1 - 0.99 = 0.01
The probability towards left of a = -0.01/2 = 0.05
The probability towards right of b = 0.01/2 =0.05
$Z = (X-\mu)/\sigma$
$X = z^* \sigma + \mu$
For a probability of 0.05, Z value is -2.57
Therefore, -(-2.57)*20+100 = 151.4
(-2.57)*20+100 = 48.6

Answer is option D (48.5,151.5)

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

ANS: Mean profits from two different division of company = Mean 1 +Mean 2

Mean = (5+7)*45 =540

Variance of profits from two different division of a company is

SD = (np.sqrt(9+16))*45 =225

A) Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Range is Rs (99.00810347848784, 980.9918965215122) in Millions

B) Specify the 5^{th} percentile of profit (in Rupees) for the company

Ans: To find 5th percentile , we use the formula $X = Z*\sigma + \mu$

from the z table

X = 540+(-1.645)*225 = 170

C) Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

Probability of Division 1 making loss P(X<0)

stats.norm.cdf(0,5,3) —> 0.047

Probability of Division 2 making loss P(X<0)

stats.norm.cdf(0,7,4) —> 0.040