A detailed comparison of the performance of BJT and FET amplifiers is too involved to be included here, but the following general remarks may be made. R_n is generally smaller and I_{EQ} larger for BJTs compared to FETs. For input signal sources with low resistances, where the noise voltage $I_{na}R_s$ is small enough to be neglected, the BJT will produce lower noise because of its smaller value of R_n. Where, however, R_s is large such that the $I_{na}R_s$ voltage is significant, the FET will produce lower noise than the BJT because of its lower I_{EQ}. There will be an intermediate range for R_s where, in fact, the thermal noise generated by R_s itself dominates, and the type of transistor may have little bearing on the overall noise performance of the amplifier.

4.11 Signal-to-Noise Ratio

In a communications link it is the signal-to-noise ratio, rather than the absolute value of noise, that is important. Signal-to-noise is defined as a power ratio, and since at a given point in a circuit power it is proportional to the square of the voltage, then

$$\frac{S}{N} = \frac{P_s}{P_n} = \frac{V_s^2}{V_n^2} \qquad (4.11.1)$$

EXAMPLE 4.11.1

The equivalent noise resistance for an amplifier is 300 Ω, and the equivalent shot noise current is 5 μA. The amplifier is fed from a 150-Ω, 10-μV rms sinusoidal signal source. Calculate the individual noise voltages at the input and the input signal-to-noise ratio in decibels. The noise bandwidth is 10 MHz.

SOLUTION Assume room temperature so that $kT = 4 \times 10^{-21}$ J and $q_e = 1.6 \times 10^{-19}$ C. The shot noise current is $I_{na} = \sqrt{2q_e I_{EQ} B_n} = 4$ nA. The noise voltage developed by this across the source resistance is $I_{na}R_s = \mathbf{0.6}$ **μV.**

Note that the shot noise current *does not* develop a voltage across R_n. The noise voltage generated by R_n is $V_{na} = \sqrt{4R_n kT_o B_n} = 6.93$ μV. The thermal noise voltage from the source is

$$V_{ns} = \sqrt{4R_s kT_o B_n} = 4.9\ \mu\text{V}$$

The total noise voltage at the input to the amplifier is

$$V_n = \sqrt{4.9^2 + 6.93^2 + .6^2} = \mathbf{8.51\ \mu V}$$

The signal-to-noise ratio in decibels is

$$\frac{S}{N} = 20 \log \frac{V_s}{V_n} = \mathbf{1.4\ dB}$$

4.12 S/N Ratio of a Tandem Connection

In an analog telephone system it is usually necessary to insert amplifiers to make up for the loss in the telephone cables, the amplifiers being known as repeaters. As shown in Fig. 4.12.1, if the power loss of a line section is L, then the amplifier power gain G is chosen so that $LG = 1$. A long line will be divided into sections that are near enough identical, and each repeater adds its own noise, so the noise accumulates with the signal as it travels along the system.

Consider the situation where the input signal power to the first section of the line is P_s, and at this point the input noise may be assumed negligible. After traveling along the first section of line, the signal is attenuated by a factor L. At the output of the first repeater the signal power is again P_s since the gain G exactly compensates for the loss L. The noise at the output of the first repeater is shown as P_{n1} and consists of the noise added by the line section and amplifier, or what is termed the *first link* in the system.

As the signal progresses along the links, the power output at each repeater remains at P_s because $LG = 1$ for each link. However, the noise powers are additive, and the total noise at the output of the Mth link is $P_n = P_{n1} + P_{n2} + \cdots + P_{nM}$. If the links are identical such that each link contributes P_n, the total noise power becomes $P_{nM} = MP_n$. The output signal-to-noise ratio in this case is

$$\left(\frac{N}{S}\right)_o \text{dB} = 10 \log \frac{P_s}{MP_n}$$

$$= \left(\frac{S}{N}\right)_1 \text{dB} - (M)\,\text{dB} \qquad (4.12.1)$$

where (S/N) is the signal-to-noise ratio of any one link, and (M) dB is the number of links expressed as a power ratio in decibels (that is, in decilogs).

EXAMPLE 4.12.1

Calculate the output signal-to-noise ratio in decibels for three identical links, given that the signal-to-noise ratio for any one link is 60 dB.

SOLUTION $(S/N)_o = 60 - 10 \log 3 = \mathbf{55.23\ dB}$

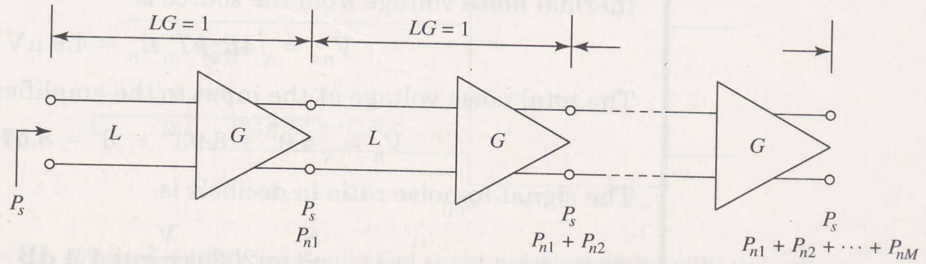

Figure 4.12.1 Tandem connection of repeaters.

If the S/N ratio of any one link is much worse than the others, that link will determine the overall S/N ratio. Suppose for example that the S/N ratio of the first link is much lower than the others; then the $(N/S)_1$ ratio will be much greater than the other noise-to-signal ratios. Hence

$$\begin{aligned}\left(\frac{N}{S}\right)_o &= \frac{P_{n1}}{P_s} + \frac{P_{n2}}{P_s} + \cdots \\ &= \left(\frac{N}{S}\right)_1 + \left(\frac{N}{S}\right)_2 + \cdots \\ &\cong \left(\frac{N}{S}\right)_1\end{aligned} \tag{4.12.2}$$

EXAMPLE 4.12.2

Calculate the output signal-to-noise ratio in decibels for three links, the first two of which have S/N ratios of 60 dB and the third an S/N of 40 dB.

SOLUTION The noise-to-power ratio of the first two links is −60 dB, or a power ratio of 10^{-6}, while that of the third link is −40 dB, or a power ratio of 10^{-4}. The overall noise-to-signal ratio is

$$\begin{aligned}\left(\frac{S}{N}\right)_o &= 10^{-6} + 10^{-6} + 10^{-4} \\ &\cong 10^{-4}\end{aligned}$$

Thus the output signal-to-noise is approximately **40 dB.**

This example shows that the S/N ratio is approximately equal to that of the worst link, and the old saying that "a chain is no stronger than its weakest link" applies here also!

4.13 Noise Factor

Consider a signal source at room temperature T_o = 290 K providing an input to an amplifier. As explained in Section 4.2, the available noise power from such a source would be $P_{ni} = kT_oB_n$. Let the available signal power from the source be denoted by P_{si}; then the available signal-to-noise ratio from the source is

$$\left(\frac{S}{N}\right)_{in} = \frac{P_{si}}{kT_oB_n} \tag{4.13.1}$$

With the source connected to an amplifier, this represents the available input signal-to-noise ratio and hence the use of the subscript *in*. If now the amplifier has an available power gain denoted by G, the available output signal power would be $P_{so} = GP_{si}$, and if the amplifier was entirely noiseless, the available output noise power would be $P_{no} = GkT_oB_n$, as shown in Fig. 4.13.1(a).

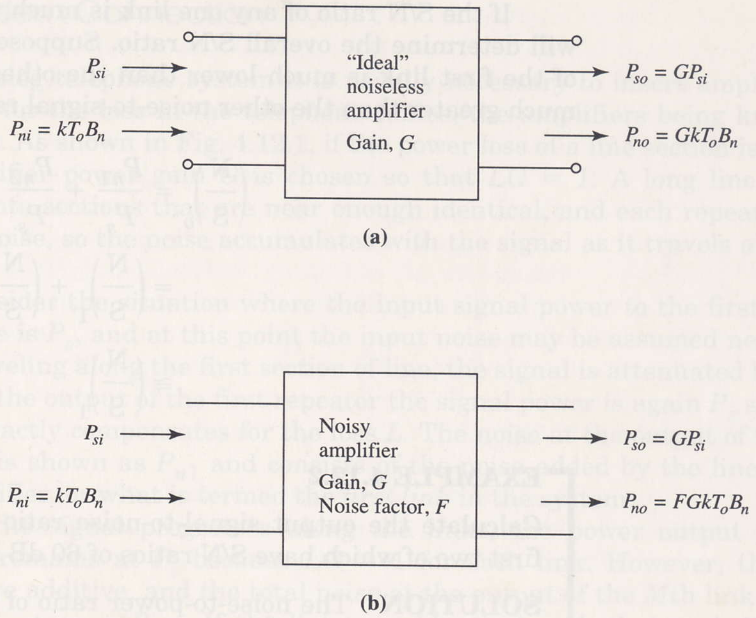

Figure 4.13.1 Noise factor F.

Hence the available output signal-to-noise ratio would be the same as that at the input since the factor G would cancel for both signal and noise.

However, it is known that all real amplifiers contribute noise, and the available output signal-to-noise ratio will be less than that at the input. The noise factor F is defined as

$$F = \frac{\text{available } S/N \text{ power ratio at the input}}{\text{available } S/N \text{ power ratio at the output}} \tag{4.13.2}$$

In terms of the symbols, this can be written as

$$\begin{aligned} F &= \frac{P_{si}}{kT_oB_n} \times \frac{P_{no}}{GP_{si}} \\ &= \frac{P_{no}}{GkT_oB_n} \end{aligned} \tag{4.13.3}$$

It follows from this that the available output noise power is given by

$$P_{no} = FGkT_oB_n \tag{4.13.4}$$

This is shown in Fig. 4.13.1(b). F can be interpreted as the factor by which the amplifier increases the output noise, for, if the amplifier were noiseless, the output noise would be GkT_oB_n.

A few comments are in order here regarding the definitions. Available power gain G is used because it can be defined unambiguously; that is, it does not depend on the load impedance. It may be thought that this definition requires the input to be matched for maximum power transfer, but this is not so. The available output power depends on the actual input power delivered to the amplifier and hence takes into account any input mismatch that may be present. It must also be noted that noise factor is defined for the source at room temperature $T = 290$ K.

Noise factor is a measured parameter and will usually be specified for a given amplifier or network (the definition given applies for any linear network). It is usually specified in decibels, when it is referred to as the *noise figure*. Thus

$$\text{noise figure} = (F)\ \text{dB} = 10 \log F \tag{4.13.5}$$

EXAMPLE 4.13.1

The noise figure of an amplifier is 7 dB. Calculate the output signal-to-noise ratio when the input signal-to-noise ratio is 35 dB.

SOLUTION From the definition of noise factor it follows that

$$\begin{aligned}(S/N)_o &= (S/N)_{in} - (F)\ \text{dB} \\ &= 35 - 7 \\ &= \mathbf{28\ dB}\end{aligned}$$

4.14 Amplifier Input Noise in Terms of F

Amplifier noise is generated in many components throughout the amplifier, but it proves convenient to imagine it to originate from some equivalent power source at the input of the amplifier. (This is somewhat similar to the equivalent input generator approach described in Section 4.10.) From Eq. (4.13.4), the total available input noise is

$$\begin{aligned}P_{ni} &= \frac{P_{no}}{G} \\ &= FkT_oB_n\end{aligned} \tag{4.14.1}$$

This is illustrated in Fig. 4.14.1.

The source contributes an available power kT_oB_n and hence the amplifier must contribute an amount P_{na}, where

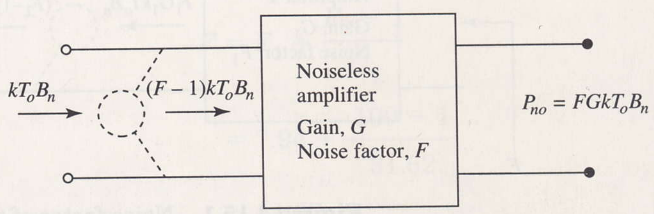

Figure 4.14.1 Equivalent input noise power source for an amplifier.

$$P_{na} = FkT_oB_n - kT_oB_n$$
$$= (F - 1)kT_oB_n \tag{4.14.2}$$

EXAMPLE 4.14.1

An amplifier has a noise figure of 13 dB. Calculate the equivalent amplifier input noise for a bandwidth of 1 MHz.

SOLUTION 13 dB is a power ratio of approximately 20 : 1. Hence

$$P_{na} = (20 - 1)4 \times 10^{-21}10^6 = \mathbf{1.44\ pW}$$

It will be noted in the example that the noise figure must be converted to a power ratio F to be used in the calculation.

4.15 Noise Factor of Amplifiers in Cascade

Consider first two amplifiers in cascade as shown in Fig. 4.15.1. The problem is to determine the overall noise factor F in terms of the individual noise factors and available power gains.

The available noise power at the output of amplifier 1 is $P_{no1} = F_1G_1kT_oB_n$ and this is available to amplifier 2. Amplifier 2 has noise $(F_2 - 1)kT_oB_n$ of its own at its input, and hence the total available noise power at the input of amplifier 2 is

$$P_{ni2} = F_1G_1kT_oB_n + (F_2 - 1)kT_oB_n \tag{4.15.1}$$

Now since the noise of amplifier 2 is represented by its equivalent input source, the amplifier itself can be regarded as being noiseless and of available power gain G_2, so the available noise output of amplifier 2 is

$$P_{no2} = G_2P_{ni2}$$
$$= G_2(F_1G_1kT_oB_n + (F_2 - 1)kT_oB_n) \tag{4.15.2}$$

The overall available power gain of the two amplifiers in cascade is $G = G_1G_2$, and let the overall noise factor be F; then the output noise power can also be expressed as [see Eq. 4.13.4]

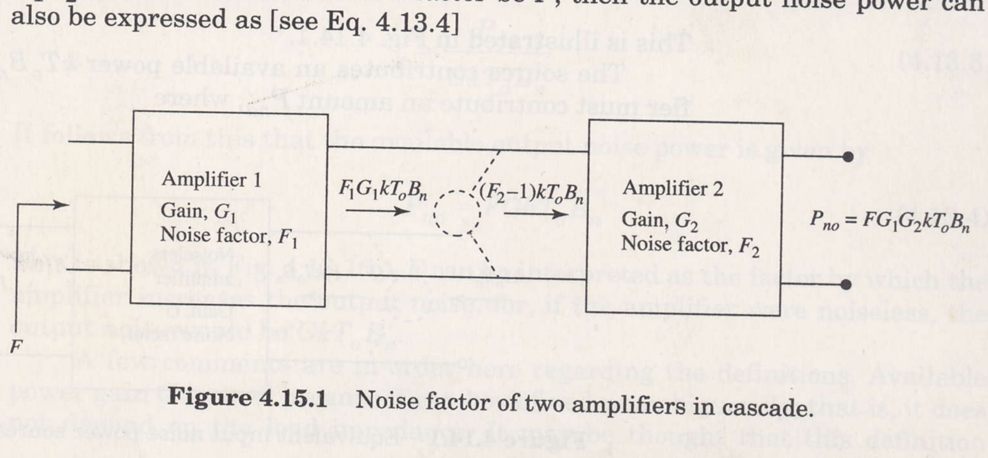

Figure 4.15.1 Noise factor of two amplifiers in cascade.

$$P_{no} = FGkT_oB_n \tag{4.15.3}$$

Equating the two expressions for output noise and simplifying yields

$$F = F_1 + \frac{F_2 - 1}{G_1} \tag{4.15.4}$$

This equation shows the importance of having a high-gain, low-noise amplifier as the first stage of a cascaded system. By making G_1 large, the noise contribution of the second stage can be made negligible, and F_1 must also be small so that the noise contribution of the first amplifier is low.

The argument is easily extended for additional amplifiers to give

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \cdots \tag{4.15.5}$$

This is known as *Friis's formula.*

There are two particular situations where a low-noise, front-end amplifier is employed to reduce noise. One of these is in satellite receiving systems, and this is discussed more fully in Chapter 19. The other is in radio receivers used to pick up weak signals, such as short-wave receivers. In most receivers, a stage known as the *mixer stage* is employed to change the frequency of the incoming signal, and it is known that mixer stages have notoriously high noise factors. By inserting an RF amplifier ahead of the mixer, the effect of the mixer noise can be reduced to negligible levels. This is illustrated in the following example.

EXAMPLE 4.15.1

A mixer stage has a noise figure of 20 dB, and this is preceded by an amplifier that has a noise figure of 9 dB and an available power gain of 15 dB. Calculate the overall noise figure referred to the input.

SOLUTION It is first necessary to convert all decibel values to the equivalent power ratios:

$F_2 = 20 \text{ dB} = 100 : 1$ power ratio

$F_1 = 9 \text{ dB} = 7.94 : 1$ power ratio

$G_1 = 15 \text{ dB} = 31.62 : 1$ power ratio

$$F = F_1 + \frac{F_2 - 1}{G_1}$$

$$= 7.94 + \frac{100 - 1}{31.62}$$

$$= 11.07$$

This is the overall noise factor. The overall noise figure is

$$(F)\,\mathrm{dB} = 10 \log 11.07$$
$$= \mathbf{10.44\ dB}$$

4.16 Noise Factor and Equivalent Input Noise Generators

The noise factor is a function of source resistance as well as amplifier input noise. Referring once again to Fig. 4.10.2, the total mean-square input noise voltage is V_n^2, while the noise from the source alone is V_{ns}^2. In terms of these quantities, the noise factor is

$$F = \frac{V_n^2}{V_{ns}^2} \tag{4.16.1}$$

Substituting from Eqs. (4.10.1) through (4.10.3) and simplifying gives

$$F = 1 + \frac{R_n}{R_s} + \frac{q_e I_{EQ} \cdot R_s}{2kT_o}$$
$$= 1 + \frac{R_n}{R_s} + \frac{I_{EQ} R_s}{2V_T} \tag{4.16.2}$$

Here, $V_T = kT_o/q_e = 26$ mV is a constant.

The second term is inversely proportional to R_s, and the third term is proportional to R_s, which means that there must be an optimum value for R_s that minimizes F. This can be found by differentiating Eq. (4.16.2) and equating to zero. After simplifying, this results in

$$R_{s\,\mathrm{opt}} = \frac{V_{na}}{I_{na}}$$
$$= \sqrt{\frac{2R_n V_T}{I_{EQ}}} \tag{4.16.3}$$

Thus, knowing the input generator parameters allows the optimum value of source resistance to be determined. An input transformer coupling circuit may be necessary in order to transform the actual source resistance to the optimum value.

4.17 Noise Factor of a Lossy Network

When a signal source is matched through a lossy network, such as a connecting cable, the available signal power at the output of the network is reduced by the insertion loss of the network. The output noise remains unchanged at kT_oB_n

(assuming source and network to be at room temperature), since available noise power is independent of source resistance. In effect, the network attenuates the source noise, but at the same time adds noise of its own. The S/N ratio is therefore reduced by the amount that the output power is attenuated.

Denoting the power insertion loss ratio as L, the output S/N ratio will be $1/L$ times the input S/N ratio, and, from the definition of noise factor given by Eq. (4.13.2),

$$F = \frac{\text{available } S/N \text{ power ratio at the input}}{\text{available } S/N \text{ power ratio at the output}} = L \tag{4.17.1}$$

In Section 1.3 the insertion loss IL was defined in terms of currents. In terms of power, the power insertion loss is $L = (\mathrm{IL})^2$. Alternatively, specifying the insertion loss in decibels, which apply equally to current and power ratios, also specifies the noise figure in decibels.

EXAMPLE 4.17.1

Calculate the noise factor of an attenuator pad that has an insertion loss of 6 dB.

SOLUTION The insertion loss is 6 dB, and therefore the noise figure is 6 dB. This is equivalent to a noise factor of 4.

The available power gain of a lossy network is $1/L$, and therefore when a lossy network, such as a connecting cable, is placed ahead of an amplifier, Friis's formula gives for the overall noise factor

$$\begin{aligned} F &= F_{nw} + \frac{F_a - 1}{G_{nw}} \\ &= L + (F_a - 1) \cdot L \end{aligned} \tag{4.17.2}$$

The subscript nw refers to the lossy network and a to the amplifier. It will be seen therefore that the loss L adversely affects the overall noise factor in two ways: by its direct contribution and by increasing the effect of the amplifier noise.

Alternatively, if the amplifier is placed ahead of the network, the overall noise factor is

$$\begin{aligned} F &= F_a + \frac{F_{nw} - 1}{G_a} \\ &= F_a + \frac{L - 1}{G_a} \end{aligned} \tag{4.17.3}$$

In this case, provided the amplifier has high gain, the overall noise factor of the system is essentially that of the amplifier alone. This situation is met with in satellite receiving systems (see Problem 4.42 and Chapter 19).

4.18 Noise Temperature

The concept of noise temperature is based on the available noise power equation given in Section 4.2, which is repeated here for convenience:

$$P_n = kT_aB_n \tag{4.18.1}$$

Here, the subscript a has been included to indicate that the noise temperature is associated only with the available noise power. In general, T_a will not be the same as the physical temperature of the noise source. As an example, an antenna pointed at deep space will pick up a small amount of cosmic noise. The equivalent noise temperature of the antenna that represents this noise power may be a few tens of kelvins, well below the physical ambient temperature of the antenna. If the antenna is pointed directly at the sun, the received noise power increases enormously, and the corresponding equivalent noise temperature is well above the ambient temperature.

When the concept is applied to an amplifier, it relates to the equivalent noise of the amplifier referred to the input. If the amplifier noise referred to the input is denoted by P_{na}, the equivalent noise temperature of the amplifier referred to the input is

$$T_e = \frac{P_{na}}{kB_n} \tag{4.18.2}$$

In Section 4.14, it was shown that the equivalent input power for an amplifier is given in terms of its noise factor by $P_{na} = (F - 1)kT_oB_n$. Substituting this in Eq. (4.18.2) gives for the equivalent input noise temperature of the amplifier

$$T_e = (F - 1)T_o \tag{4.18.3}$$

This shows the proportionality between T_e and F, and knowing one automatically entails knowing the other. In practice, it will be found that noise temperature is the better measure for low-noise devices, such as the low-noise amplifiers used in satellite receiving systems, while noise factor is a better measure for the main receiving system.

Friis's formula can be expressed in terms of equivalent noise temperatures. Denoting by T_e the overall noise of the cascaded system referred to the input, and by T_{e1}, T_{e2}, and so on, the noise temperatures of the individual stages, then Friis's formula is easily rearranged to give

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \cdots \tag{4.18.4}$$

EXAMPLE 4.18.1

A receiver has a noise figure of 12 dB, and it is fed by a low-noise amplifier that has a gain of 50 dB and a noise temperature of 90 K. Calculate the noise temperature of the receiver and the overall noise temperature of the receiving system.