4 Impedance Matching and Two-Port Network Analysis

The design of impedance matching networks is an important part of microwave engineering. Some of the more widely used techniques are described in the first two sections of this chapter. The remaining sections illustrate the use of matrix methods and flow graphs in analyzing two-port networks. Attenuation, transducer loss, and insertion loss and phase are defined and expressed in terms of the matrix elements.

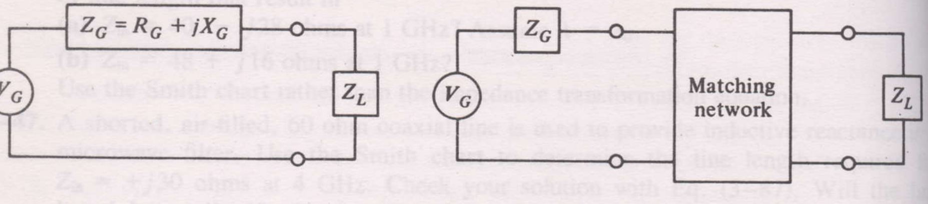
4-1 SOME IMPEDANCE MATCHING TECHNIQUES

Beatty (Ref. 4-14) has defined two types of impedance matching. Namely,

Conjugate Matching: The matching of a load impedance to a generator for maximum transfer of power.

Z_0 Matching: The matching of a load impedance to a transmission line to eliminate wave reflections at the load.

Conjugate matching is illustrated in Fig. 4-1. From ac circuit theory, maxi-



(a) (b)

Figure 4-1 Matching a load impedance Z_L to a generator having an internal impedance Z_G. **(Conjugate matching)**

mum power is delivered to a load when Z_L is set equal to the complex conjugate of the generator impedance. That is, $Z_L = Z_G^* = R_G - jX_G$. For this condition, the power absorbed by the load is exactly the available power of the generator (P_A). Denoting the rms value of V_G by V_G,

$$P_A = \frac{V_G^2}{4R_G} \tag{4–1}$$

In situations where the load impedance is not adjustable, a matching network may be placed between the generator and the fixed load. This is shown in part b of Fig. 4-1. The matching network is designed so that with Z_L connected at the output, its input impedance satisfies the conjugate match condition. With $Z_{in} = R_G - jX_G$, the power into the network equals P_A. If the matching network is dissipationless, all of the available generator power is delivered to the load Z_L. For a matching network with dissipation, the load power is less than P_A. When Z_G is real (that is, $Z_G = R_G$), the conjugate match condition reduces to $Z_{in} = R_G$.

The term Z_0 *matching* is used to denote matching a load impedance to the characteristic impedance of a transmission line (that is, $Z_L = Z_0$). In this case $\Gamma_L = 0$ and hence the SWR along the line is unity. If $Z_L \neq Z_0$, a matching network may be used to eliminate the standing waves on the line. This arrangement is shown in Fig. 4-2, where in most cases the network is essentially dissipationless. With Z_L connected as shown, Z_0 matching requires that the input impedance of the network equal Z_0 or $Z_{in}/Z_0 = 1 + j0$. For a Smith chart normalized to Z_0, this means that $\bar{Z}_{in} \equiv Z_{in}/Z_0$ must be at the center of the chart.

For a well-designed source, $Z_G = Z_0$, the characteristic impedance of its output line. With Z_0 real, matching the load to the line ($Z_L = Z_0$) results in a conjugate match between the generator and the load (namely, $Z_G = Z_0 = Z_L$). If $Z_G \neq Z_0$, a matching network can be placed at the generator end to match the generator to the line. This and the load matching network are shown in Fig. 4–3. For properly designed networks, $Z_L' = Z_0$ and $Z_G' = Z_0$, where Z_L' is the impedance to the right of plane B and Z_G' is the Thevenin impedance to the left of plane A. When the matching network is dissipationless, the power available from the Thevenin equivalent must be the same as the available generator power. Hence the rms voltages are related by

$$V_G' = V_G\sqrt{\frac{Z_0}{R_G}} \tag{4–2}$$

If the generator matching network is not built into the source, it should be placed as close as possible to its output terminals. Similarly, it is good engineering practice to

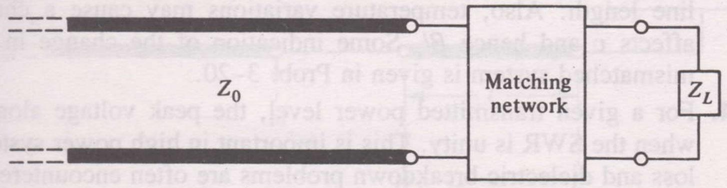


Figure 4–2 Matching a load impedance Z_L to a transmission line of characteristic impedance Z_0. (**Z_0 matching**)

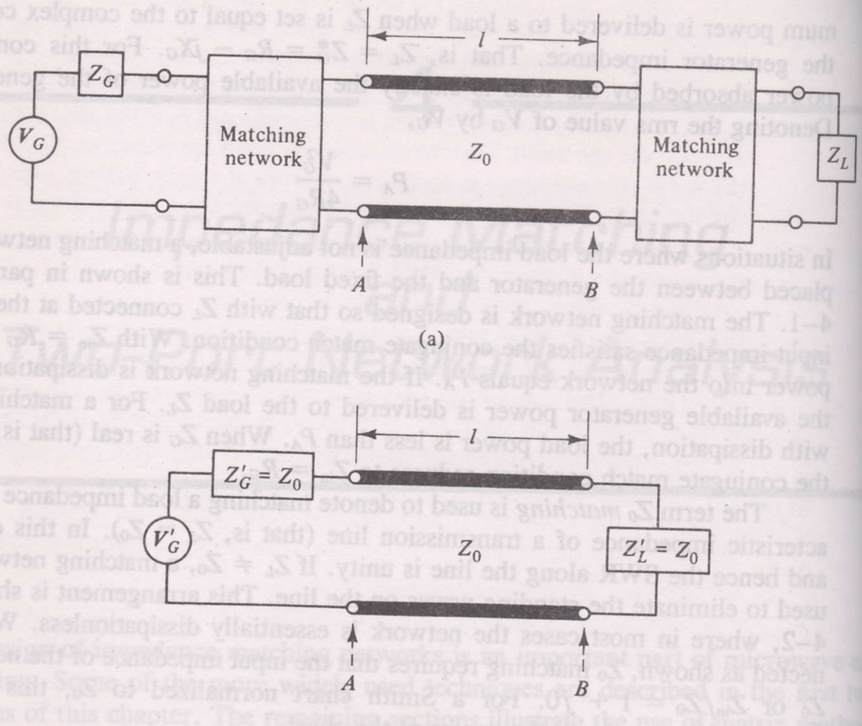


Figure 4–3 A matched transmission-line system and its equivalent circuit.

place the load matching network as close as possible to the load. This arrangement, which is shown in Fig. 4–3, has several advantages. These include

1. For dissipationless matching networks, maximum power is delivered from the generator to the load. If the transmission line is lossless, $P_L = P_A$ otherwise $P_L = P_A e^{-2\alpha l}$, where α is the attenuation constant of the line.
2. For a line with finite attenuation, power transmission is most efficient with no standing waves on the line. With SWR = 1.00, $P_L = P_{\text{in}} e^{-2\alpha l}$. For SWR $\neq$ 1.00, the load power is less. The transmission efficiency is derived in Sec. 1.37 of Ref. 4–3.
3. The load power is independent of βl, an important advantage, particularly for low-loss lines. Since $\beta l = \omega l / v$, its value is a function of frequency as well as line length. Also, temperature variations may cause a change in ϵ_R which affects v and hence βl. Some indication of the change in P_L with βl for a mismatched system is given in Prob. 3–20.
4. For a given transmitted power level, the peak voltage along the line is less when the SWR is unity. This is important in high power systems where corona loss and dielectric breakdown problems are often encountered.
5. If neither the load nor the generator is matched to the line, the transmission line may cause the output phase versus frequency characteristic to be nonlinear.

This results in modulation distortion, which reduces the information content of the signal.

6. If the SWR along the line is not unity, the impedance seen by the generator is a function of βl. For long lines and certain type sources, the generator may shift frequency intermittently or even oscillate at two frequencies simultaneously.
7. Precise measurement systems require that both the source and the load (usually a detector) be well matched to minimize errors. This is illustrated in Sec. 4–4 (Ex. 4–10).

For the reasons summarized here, many systems and components are designed to have low SWR (typically 1.25 or less) over their useful frequency range.

Recognizing the importance of impedance matching, let us consider various methods of achieving it at high frequencies. Although the following examples involve matching a load to a transmission line, the same techniques may be used to match a generator to a line or a load directly to a generator with Z_G real.

4–1a Reactive Matching Networks

One of the most widely used matching networks consists of a reactive element placed either in series or shunt with a small length of low-loss line. The design and analysis for both cases will now be explained with the aid of the Smith chart.

Series reactive matching. Figure 4–4 illustrates the series reactance technique of matching a load Z_L to a transmission line. Specific values of Z_L and Z_0 are given in the figure. The subscripts L, A, and *in* are used to denote the impedances at the load, plane A, and the input, respectively. Note that the characteristic impedance of the line section (Z_{01}) has been chosen to equal the characteristic impedance of the input line (Z_0). This choice makes the Smith chart solution easier and usually results in a simplified configuration. With $Z_{01} = Z_0 = 50$ ohms, $\bar{Z}_L \equiv Z_L/50 = 0.50 + j0.60$. The requirement for a match condition is $\bar{Z}_{in} \equiv Z_{in}/50 = 1 + j0$. To achieve this, two steps are required. First, the line length must be chosen so that the real part of $\bar{Z}_A$ equals unity. Next a reactance X must be inserted in a series with Z_A so

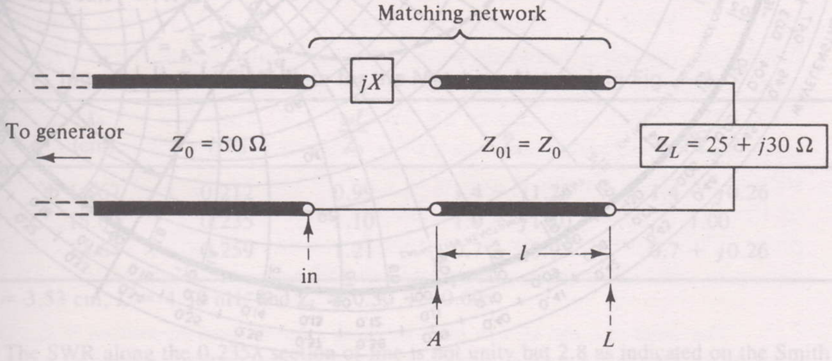


Figure 4–4 A series reactance matching network.