Solutions I Insem 27/08/2018

Digital Communications

Q.1 (a) RV - mapping from random expt space to a real line / real number

RP - mapping from random expt to space of signals. It is a probability space composed of sample space, ensemble of time functions and a probability measure OR it is a RV which is a function of time.

(b) 2 (c) $-\infty < t < \infty$ (d) $1/2$ (e) No. randomness comes from the uncertainty as to which waveform will occur in a given trial.

Q.2. $R_x(\tau) = \cos(\tau)$, $|\tau| \leq \frac{\pi}{2}$. Not a valid auto correlation function

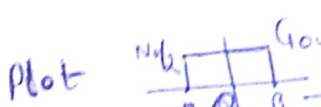
Reason: Discontinuities are not allowed in the $R_x(\tau)$ or the psd (which has to be always +ve) becomes -ve because of the convolution of FTs of $\cos(\tau)$, $|\tau| < \infty$ and rectangular pulse of height unity, $|\tau| \leq \pi/2$ ($R_x(\tau) = \cos(\tau)$, $|\tau| \leq \pi/2$ is obtained by multiplying $\cos(\tau)$, $|\tau| < \infty$ and pulse, $|\tau| \leq \pi/2$)

Q.3. (a) psd of output process can be found as $G_{output}(f) = |H(f)|^2 G_{input}(f)$

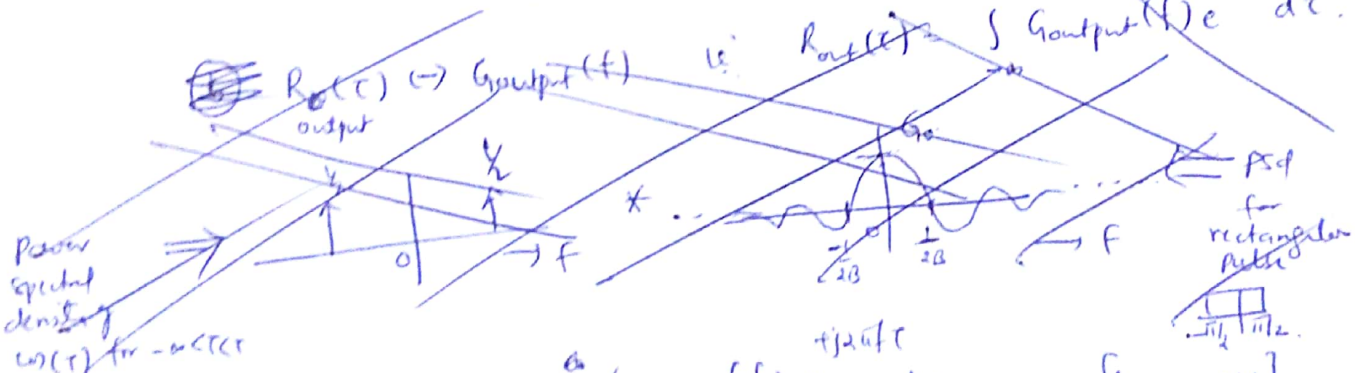
$= 1 \times N_0/2$,

$-B \leq f \leq B$.

Plot



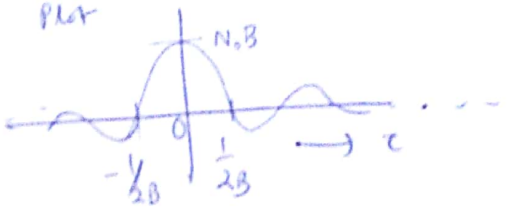
$R_{out}(\tau) = \int_{-\infty}^{\infty} G_{output}(f) e^{j2\pi f\tau} d\tau$.



(b) $R_{output}(\tau) = \int_{-\infty}^{\infty} G_{output}(f) e^{j2\pi f\tau} d\tau = FT^{-1}\left[G_{output}(f)\right]$

$= N_0 B \operatorname{sinc}(2B\tau)$

Plot



Q4: (a) infinite (b) 5.7 and 6.3 kHz (c) no cannot be reconstructed back from sampled signal because of aliasing.

Q5: (a) Sine wave (of single frequency) extends from $-\infty$ to ∞ in time $(-\infty < t < \infty)$, hence time Unlimited [So cannot be generated since we need to wait upto infinity time $(-\infty$ to $\infty)$]

Any of the two sentences can be written

(b) (Ideal filters are non causal) (1) So (output at at (2) time depends on all previous time inputs) (3) So (we need to wait for infinite (very very long) time to get the current output). → So not realizable

Any of these three sentences can be written

(d) Baseband transmission — no carrier is used, the source signal is transmitted after encoding

Bandpass transmission — The carrier gets modulated using the baseband signal.

(c) Periodic signals have infinite energy but finite power.

Dhirubhai Ambani Institute of Information and Communication Technology (DA-IICT)
First In Semester Examination
CT303 (Digital Communications)
Date of Examination: August 30, 2018
Duration: 1:30 Hours
Maximum Marks: ~~30~~ 25

Instructions:

1. Attempt all questions.
2. Use of scientific non programmable calculator is permitted.
3. Figures in brackets indicate full marks.
4. All the acronyms carry their usual meaning.

Q1: (a) What is the difference between a random variable and a random process? (b) Consider the experiment of tossing a fair coin. How many sample functions (also called realizations) are there for this experiment and (c) for what time interval they should exist? (d) Give the probability of occurrence of each sample function. (e) Are the sample functions random? If your answer is NO, from where do you think the randomness comes from in the random process? (6 marks)

Q2: Consider $R_X(\tau) = \cos(\tau)$, for $|\tau| \leq \frac{\pi}{2}$. Is this a valid autocorrelation function? Why? (Hint: One of the properties of autocorrelation function is: its Fourier transform represents power spectral density (PSD) of the random process) (2 marks)

Q3: White noise with PSD of $\frac{N_0}{2}$ is applied as input to an ideal low-pass filter of bandwidth B, where No represents a positive constant. Here, bandwidth B means the filter has the frequency response characteristic H(f), which is unity in the interval of –B to B. (a) Find the PSD of the output process and plot it. (b) Obtain the expression for autocorrelation function of the output process and plot it. (7 marks)

Q4: A band limited signal $x(t) = \cos 2\pi(400)t + \cos 2\pi(700)t$ is uniformly sampled at 1000 samples/sec using a rectangular pulse train c(t) having width of each pulse as $\frac{10^{-3}}{6}$ and height as 6000 (i.e., natural sampling is performed). (a) Write the total number of frequency components present in the sampled signal (b) Which frequency components are present in the sampled signal in the range of 5.5 kHz to 6.5 KHz (Not required to write the amplitude of each frequency component). (c) Can x(t) be reconstructed back from sampled signal by passing it through a low pass filter. Give proper reason. (6)

Q5: Give answer in **one sentence** for each of the following questions. (a) In practice you cannot generate a sinusoidal signal (a single frequency component). Why? (b) Why ideal filters are not realizable? (c) Periodic signals are power signals. Why? (d) What is the difference between baseband transmission and band pass transmission? (4 marks)