which is the same as that obtained in Example 2.1. A three-dimensional plot of the relative radiation intensity is also represented by Figure 2.2.

For an isotropic source, U will be independent of the angles θ and ϕ, as was the case for W_{rad}. Thus (2-13) can be written as

$$P_{rad} = \oiint_{\Omega} U_0 \, d\Omega = U_0 \oiint_{\Omega} d\Omega = 4\pi U_0 \tag{2-14}$$

or the radiation intensity of an isotropic source as

$$U_0 = \frac{P_{rad}}{4\pi} \tag{2-15}$$

2.5 DIRECTIVITY

In the 1983 version of the *IEEE Standard Definitions of Terms for Antennas,* there has been a substantive change in the definition of *directivity,* compared to the definition of the 1973 version. Basically the term *directivity* in the new 1983 version has been used to replace the term *directive gain* of the old 1973 version. In the new 1983 version the term *directive gain* has been deprecated. According to the authors of the new 1983 standards, "this change brings this standard in line with common usage among antenna engineers and with other international standards, notably those of the International Electrotechnical Commission (IEC)." Therefore *directivity of an antenna* defined as "the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. The average radiation intensity is equal to the total power radiated by the antenna divided by 4π. If the direction is not specified, the direction of maximum radiation intensity is implied." Stated more simply, the directivity of a nonisotropic source is equal to the ratio of its radiation intensity in a given direction over that of an isotropic source. In mathematical form, using (2-15), it can be written as

$$\boxed{D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}}} \tag{2-16}$$

If the direction is not specified, it implies the direction of maximum radiation intensity (maximum directivity) expressed as

$$\boxed{D_{max} = D_0 = \frac{U|_{max}}{U_0} = \frac{U_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}}} \tag{2-16a}$$

D = directivity (dimensionless)
D_0 = maximum directivity (dimensionless)
U = radiation intensity (W/unit solid angle)
U_{max} = maximum radiation intensity (W/unit solid angle)
U_0 = radiation intensity of isotropic source (W/unit solid angle)
P_{rad} = total radiated power (W)

which is related to the directivity of (2-21) by

$$\boxed{G(\theta, \phi) = e_{cd}\, D(\theta, \phi)} \tag{2-47}$$

In a similar manner, the maximum value of the gain is related to the maximum directivity by

$$\boxed{G_0 = G(\theta, \phi)|_{\max} = e_{cd}\, D(\theta, \phi)|_{\max} = e_{cd}\, D_0} \tag{2-47a}$$

As was done with the directivity, we can define the *partial gain of an antenna for a given polarization in a given direction* as "that part of the radiation intensity corresponding to a given polarization divided by the total radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically." With this definition for the partial directivity, then, in a given direction, "the total gain is the sum of the partial gains for any two orthogonal polarizations." For a spherical coordinate system, the total maximum gain G_0 for the orthogonal θ and ϕ components of an antenna can be written, in a similar form as was the maximum directivity in (2-17)–(2-17b), as

$$G_0 = G_\theta + G_\phi \tag{2-48}$$

while the partial gains G_θ and G_ϕ are expressed as

$$G_\theta = \frac{4\pi U_\theta}{P_{in}} \tag{2-48a}$$

$$G_\phi = \frac{4\pi U_\phi}{P_{in}} \tag{2-48b}$$

where

U_θ = radiation intensity in a given direction contained in θ field component
U_ϕ = radiation intensity in a given direction contained in ϕ field component
P_{in} = total input (accepted) power

For many practical antennas an approximate formula for the gain, corresponding to (2-27) or (2-27a) for the directivity, is

$$\boxed{G_0 \simeq \frac{30{,}000}{\Theta_{1d}\,\Theta_{2d}}} \tag{2-49}$$

In practice, whenever the term "gain" is used, it usually refers to the *maximum gain* as defined by (2-47a).

Usually the gain is given in terms of decibels instead of the dimensionless quantity of (2-47a). The conversion formula is given by

$$G_0(\text{dB}) = 10 \log_{10}[e_t\, D_0 \text{ (dimensionless)}] \tag{2-50}$$

2.8 ANTENNA EFFICIENCY

The total antenna efficiency e_o is used to take into account losses at the input terminals and within the structure of the antenna. Such losses may be due, referring to Figure 2.17(b), to

1. reflections because of the mismatch between the transmission line and the antenna
2. I^2R losses (conduction and dielectric)

In general, the overall efficiency can be written as

$$\boxed{e_o = e_r e_c e_d} \tag{2-51}$$

where

e_o = total efficiency (dimensionless)

e_r = reflection (mismatch) efficiency = $(1 - |\Gamma|^2)$ (dimensionless)

e_c = conduction efficiency (dimensionless)

e_d = dielectric efficiency (dimensionless)

Γ = voltage reflection coefficient at the input terminals of the antenna [$\Gamma = (Z_{in} - Z_0)/(Z_{in} + Z_0)$ where Z_{in} = antenna input impedance, Z_0 = characteristic impedance of the transmission line]

Usually e_c and e_d are very difficult to compute, but they can be determined experimentally. Even by measurements they cannot be separated, and it is usually more convenient to write (2-51) as

$$e_o = e_r e_{cd} = e_{cd}(1 - |\Gamma|^2) \tag{2-52}$$

where $e_{cd} = e_c e_d$ = antenna radiation efficiency, which is used to relate the gain and directivity.

Example 2.8

A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is to be connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by

$$U = B_0 \sin^3 \theta$$

find the overall maximum gain of this antenna.

SOLUTION

Let us first compute the maximum directivity of the antenna. For this

$$U|_{\max} = U_{\max} = B_0$$

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi = 2\pi B_0 \int_0^{\pi} \sin^4 \theta \, d\theta = B_0\left(\frac{3\pi^2}{4}\right)$$

$$D_0 = 4\pi \frac{U_{\max}}{P_{\text{rad}}} = \frac{16}{3\pi} = 1.697$$

Since the antenna was stated to be lossless, then the radiation efficiency

$$e_{cd} = 1$$

duced if either one of them possesses a polarization with a finite axial ratio. In addition, these techniques are accurate if the tests can be performed in a free-space, a ground-reflection, or an extrapolation range. These requirements place a low-frequency limit of 50 MHz.

Below 50 MHz, the ground has a large effect on the radiation characteristics of the antenna, and it must be taken into account. It usually requires that the measurements are performed on full scale models and *in situ*. Techniques that can be used to measure the gain of large HF antennas have been devised [47]–[49].

16.5 DIRECTIVITY MEASUREMENTS

If the directivity of the antenna cannot be found using solely analytical techniques, it can be computed using measurements of its radiation pattern. One of the methods is based on the approximate expressions of (2-27) by Kraus or (2-30b) by Tai and Pereira, whereas the other relies on the numerical techniques that were developed in Section 2.6. The computations can be performed very efficiently and economically with modern computational facilities and numerical techniques.

The simplest, but least accurate method, requires that the following procedure is adopted:

1. Measure the two principal E- and H-plane patterns of the test antenna.
2. Determine the half-power beamwidths (in degrees) of the E- and H-plane patterns.
3. Compute the directivity using either (2-27) or (2-30b).

The method is usually employed to obtain rough estimates of directivity. It is more accurate when the pattern exhibits only one major lobe, and its minor lobes are negligible.

The other method requires that the directivity be computed using (2-35) where P_{rad} is evaluated numerically using (2-43). The $F(\theta_i, \phi_j)$ function represents the radiation intensity or radiation pattern, as defined by (2-42), and it will be obtained by measurements. $U_{\max}$ in (2-35) represents the maximum radiation intensity of $F(\theta, \phi)$ in all space, as obtained by the measurements.

The radiation pattern is measured by sampling the field over a sphere of radius r. The pattern is measured in two-dimensional plane cuts with ϕ_j constant $(0 \le \phi_j \le 2\pi)$ and θ variable $(0 \le \theta \le \pi)$, as shown in Figure 2.15, or with θ_i fixed $(0 \le \theta_i \le \pi)$ and ϕ variable $(0 \le \phi \le 2\pi)$. The first are referred to as elevation or great-circle cuts, whereas the second represent azimuthal or conical cuts. Either measuring method can be used. Equation (2-43) is written in a form that is most convenient for elevation or great-circle cuts. However, it can be rewritten to accommodate azimuthal or conical cuts.

The spacing between measuring points is largely controlled by the directive properties of the antenna and the desired accuracy. The method is most accurate for broad beam antennas. However, with the computer facilities and the numerical methods now available, this method is very attractive even for highly directional antennas. To maintain a given accuracy, the number of sampling points must increase as the pattern becomes more directional. The pattern data is recorded digitally on tape, and it can be entered to a computer at a later time. If on-line computer facilities are available, the measurements can be automated to provide essentially real-time computations.

The above discussion assumes that all the radiated power is contained in a single polarization, and the measuring probe possesses that polarization. If the antenna is polarized such that the field is represented by both θ and ϕ components, the *partial directivities* $D_\theta(\theta, \phi)$ and $D_\phi(\theta, \phi)$ of (2-17)–(2-17b) must each be found. This is accomplished from pattern measurements with the probe positioned, respectively, to sample the θ and ϕ components. The *total directivity* is then given by (2-17)–(2-17b), or

$$D_0 = D_\theta + D_\phi \tag{16-21}$$

where

$$D_\theta = \frac{4\pi U_\theta}{(P_{\text{rad}})_\theta + (P_{\text{rad}})_\phi} \tag{16-21a}$$

$$D_\phi = \frac{4\pi U_\phi}{(P_{\text{rad}})_\theta + (P_{\text{rad}})_\phi} \tag{16-21b}$$

U_θ, $(P_{\text{rad}})_\theta$ and U_ϕ, $(P_{\text{rad}})_\phi$ represent the radiation intensity and radiated power as contained in the two orthogonal θ and ϕ field components, respectively.

The same technique can be used to measure the field intensity and to compute the directivity of any antenna that possess two orthogonal polarizations. Many antennas have only one polarization (θ or ϕ). This is usually accomplished by design and/or proper selection of the coordinate system. In this case, the desired polarization is defined as the *primary polarization*. Ideally, the other polarization should be zero. However, in practice, it is non-vanishing, but it is very small. Usually it is referred to as the *cross-polarization*, and for good designs it is usually below -40 dB.

The directivity of circularly or elliptically polarized antennas can also be measured. Precautions must be taken [7] as to which component represents the primary polarization and which the cross-polarization contribution.

16.6 RADIATION EFFICIENCY

The radiation efficiency is defined as the ratio of the total power radiated by the antenna to the total power accepted by the antenna at its input terminals during radiation. System factors, such as impedance and/or polarization mismatches, do not contribute to the radiation efficiency because it is an inherent property of the antenna.

The radiation efficiency can also be defined, using the direction of maximum radiation as reference, as

$$\text{radiation efficiency} = \frac{\text{gain}}{\text{directivity}} \tag{16-22}$$

where directivity and gain, as defined in Sections 2.5 and 2.7, imply that they are measured or computed in the direction of maximum radiation. Using techniques that were outlined in Sections 16.4 and 16.5 for the measurements of the gain and directivity, the radiation efficiency can then be computed using (16-22).

If the antenna is very small and simple, it can be represented as a series network as shown in Figures 2.21(b) or 2.22(b). For antennas that can be represented by such a series network, the radiation efficiency can also be defined by (2-90) and it can be computed by another method [50]. For these antennas, the real part of the input impedance is equal to the total antenna resistance which consists of the radiation resistance and the loss resistance.

The radiation resistance accounts for the radiated power. For many simple antennas (dipoles, loops, etc.), it can be found by analytically or numerically integrating