CHAPTER 6

ARRAYS: LINEAR, PLANAR, AND CIRCULAR

6.1 INTRODUCTION

In the previous chapter, the radiation characteristics of single-element antennas were discussed and analyzed. Usually the radiation pattern of a single element is relatively wide, and each element provides low values of directivity (gain). In many applications it is necessary to design antennas with very directive characteristics (very high gains) to meet the demands of long distance communication. This can only be accomplished by increasing the electrical size of the antenna.

Enlarging the dimensions of single elements often leads to more directive characteristics. Another way to enlarge the dimensions of the antenna, without necessarily increasing the size of the individual elements, is to form an assembly of radiating elements in an electrical and geometrical configuration. This new antenna, formed by multielements, is referred to as an *array*. In most cases, the elements of an array are identical. This is not necessary, but it is often convenient, simpler, and more practical. The individual elements of an array may be of any form (wires, apertures, etc.).

The total field of the array is determined by the vector addition of the fields radiated by the individual elements. This assumes that the current in each element is the same as that of the isolated element. This is usually not the case and depends on the separation between the elements. To provide very directive patterns, it is necessary that the fields from the elements of the array interfere constructively (add) in the desired directions and interfere destructively (cancel each other) in the remaining space. Ideally this can be accomplished, but practically it is only approached. In an array of identical elements, there are five controls that can be used to shape the overall pattern of the antenna. These are:

1. the geometrical configuration of the overall array (linear, circular, rectangular, spherical, etc.)
2. the relative displacement between the elements
3. the excitation amplitude of the individual elements
4. the excitation phase of the individual elements
5. the relative pattern of the individual elements

The influence that each one of the above has on the overall radiation characterist will be the subject of this chapter. In many cases the techniques will be illustra with examples.

The simplest and one of the most practical arrays is formed by placing t elements along a line. To simplify the presentation and give a better physical inte pretation of the techniques, a two-element array will first be considered. The analys of an N-element array will then follow. Two-dimensional analysis will be the subje at first. In latter sections, three-dimensional techniques will be introduced.

6.2 TWO-ELEMENT ARRAY

Let us assume that the antenna under investigation is an array of two infinitesima horizontal dipoles positioned along the z-axis, as shown in Figure 6.1(a). The tota field radiated by the two elements, assuming no coupling between the elements, i equal to the sum of the two and in the y-z plane it is given by

$$\mathbf{E}_t = \mathbf{E}_1 + \mathbf{E}_2 = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l}{4\pi}\left\{\frac{e^{-j[kr_1-(\beta/2)]}}{r_1}|\cos\theta_1| + \frac{e^{-j[kr_2+(\beta/2)]}}{r_2}|\cos\theta_2|\right\} \tag{6-1}$$

where β is the difference in phase excitation between the elements. The magnitude excitation of the radiators is identical. Assuming far-field observations and referring to Figure 6.1(b),

$$\theta_1 \simeq \theta_2 \simeq \theta \tag{6-2a}$$

$$\left.\begin{aligned} r_1 &\simeq r - \frac{d}{2}\cos\theta \\ r_2 &\simeq r + \frac{d}{2}\cos\theta \end{aligned}\right\} \text{for phase variations} \tag{6-2b}$$

$$r_1 \simeq r_2 \simeq r \quad \text{for amplitude variations} \tag{6-2c}$$

Equation 6-1 reduces to

$$\mathbf{E}_t = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r}|\cos\theta|[e^{+j(kd\cos\theta+\beta)/2} + e^{-j(kd\cos\theta+\beta)/2}]$$

$$\mathbf{E}_t = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r}|\cos\theta|\, 2\cos\left[\frac{1}{2}(kd\cos\theta+\beta)\right] \tag{6-3}$$

It is apparent from (6-3) that the total field of the array is equal to the field of a single element positioned at the origin multiplied by a factor which is widely referred to as the *array factor*. Thus for the two-element array of constant amplitude, the array factor is given by

$$\mathrm{AF} = 2\cos[\tfrac{1}{2}(kd\cos\theta+\beta)] \tag{6-4}$$

which in normalized form can be written as

$$(\mathrm{AF})_n = \cos[\tfrac{1}{2}(kd\cos\theta+\beta)] \tag{6-4a}$$

The array factor is a function of the geometry of the array and the excitation phase. By varying the separation d and/or the phase β between the elements, the characteristics of the array factor and of the total field of the array can be controlled.

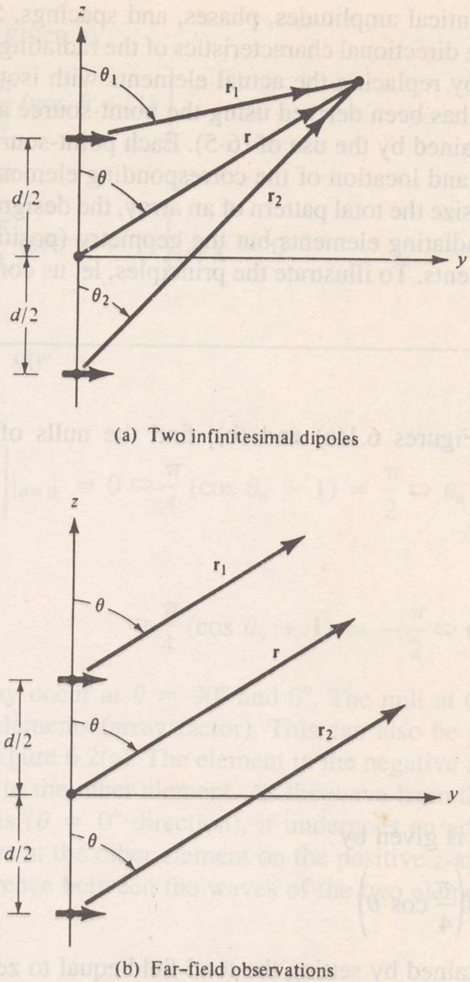

Figure 6.1 Geometry of a two-element array positioned along the z-axis.

It has been illustrated that the far-zone field of a uniform two-element array of identical elements is equal to the *product of the field of a single element, at a selected reference point* (*usually the origin*), *and the array factor of that array.* That is,

$$\boxed{\mathbf{E}(\text{total}) = [\mathbf{E}(\text{single element at reference point})] \times [\text{array factor}]} \quad (6\text{-}5)$$

This is referred to as *pattern multiplication* for arrays of identical elements, and it is analogous to the pattern multiplication of (4-59) for continuous sources. Although it has been illustrated only for an array of two elements, each of identical magnitude, it is also valid for arrays with any number of identical elements which do not necessarily have identical magnitudes, phases, and/or spacings between them. This will be demonstrated in this chapter by a number of different arrays.

Each array has its own array factor. The array factor, in general, is a function of the number of elements, their geometrical arrangement, their relative magnitudes, their relative phases, and their spacings. The array factor will be of simpler form if the elements have identical amplitudes, phases, and spacings. Since the array factor does not depend on the directional characteristics of the radiating elements themselves, it can be formulated by replacing the actual elements with isotropic (point) sources. Once the array factor has been derived using the point-source array, the total field of the actual array is obtained by the use of (6-5). Each point-source is assumed to have the amplitude, phase, and location of the corresponding element it is replacing.

In order to synthesize the total pattern of an array, the designer is not only required to select the proper radiating elements but the geometry (positioning) and excitation of the individual elements. To illustrate the principles, let us consider some examples.

Example 6.1

Given the array of Figures 6.1(a) and (b), find the nulls of the total field when $d = \lambda/4$ and

(a) $\beta = 0$

(b) $\beta = +\dfrac{\pi}{2}$

(c) $\beta = -\dfrac{\pi}{2}$

SOLUTION

(a) $\beta = 0$

The normalized field is given by

$$E_{tn} = |\cos\theta| \cos\left(\frac{\pi}{4}\cos\theta\right)$$

The nulls are obtained by setting the total field equal to zero, or

$$E_{tn} = |\cos\theta| \cos\left(\frac{\pi}{4}\cos\theta\right)\Big|_{\theta=\theta_n} = 0$$

Thus

$$\cos\theta_n = 0 \Rightarrow \theta_n = 90^\circ$$

and

$$\cos\left(\frac{\pi}{4}\cos\theta_n\right) = 0 \Rightarrow \frac{\pi}{4}\cos\theta_n = \frac{\pi}{2}, -\frac{\pi}{2} \Rightarrow \theta_n = \text{does not exist}$$

The only null occurs at $\theta = 90^\circ$ and is due to the pattern of the individual elements. The array factor does not contribute any additional nulls because there is not enough separation between the elements to introduce a phase difference of 180° between the elements, for any observation angle.

(b) $\beta = +\dfrac{\pi}{2}$

The normalized field is given by

$$E_{in} = |\cos\theta| \cos\left[\frac{\pi}{4}(\cos\theta + 1)\right]$$

The nulls are found from

$$E_{in} = |\cos\theta| \cos\left[\frac{\pi}{4}(\cos\theta + 1)\right]\Big|_{\theta=\theta_n} = 0$$

Thus

$$\cos\theta_n = 0 \Rightarrow \theta_n = 90^\circ$$

and

$$\cos\left[\frac{\pi}{4}(\cos\theta + 1)\right]\Big|_{\theta=\theta_n} = 0 \Rightarrow \frac{\pi}{4}(\cos\theta_n + 1) = \frac{\pi}{2} \Rightarrow \theta_n = 0^\circ$$

and

$$\Rightarrow \frac{\pi}{4}(\cos\theta_n + 1) = -\frac{\pi}{2} \Rightarrow \theta_n = \text{does not exist}$$

The nulls of the array occur at $\theta = 90^\circ$ and 0°. The null at 0° is introduced by the arrangement of the elements (array factor). This can also be shown by physical reasoning, as shown in Figure 6.2(a). The element in the negative z-axis has an initial phase lag of 90° relative to the other element. As the wave from that element travels toward the positive z-axis ($\theta = 0^\circ$ direction), it undergoes an additional 90° phase retardation when it arrives at the other element on the positive z-axis. Thus there is a total of 180° phase difference between the waves of the two elements when travel is

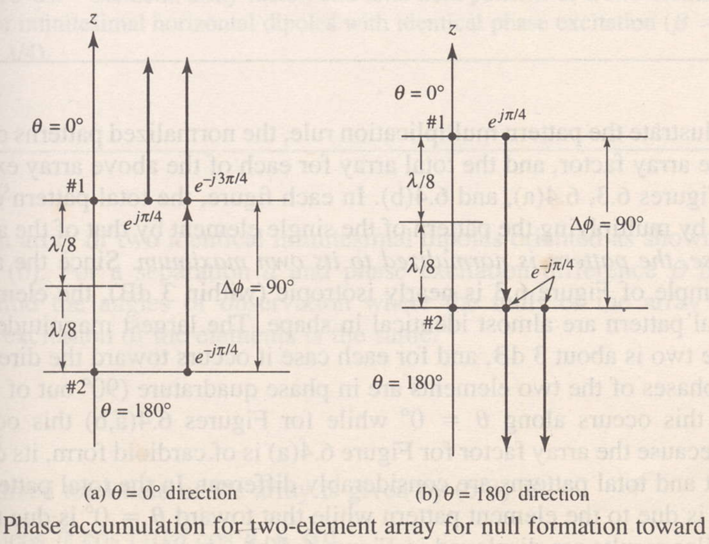

Figure 6.2 Phase accumulation for two-element array for null formation toward $\theta = 0^\circ$ and 180°.