4

Noise

4.1 Introduction

Noise, as commonly understood, is a disturbance one "hears," but in telecommunications the word noise is also used as a label for the electrical disturbances that give rise to audible noise in a system. These electrical disturbances also appear as interference in video systems, for example, the white flecks seen on a television picture when the received signal is weak, referred to as a "noisy picture."

Noise can arise in a variety of ways. One obvious example is when a faulty connection exists in a piece of equipment, which, if it is a radio receiver, results in an intermittent or "crackling" type of noise at the output. Such sources of noise can, in principle anyway, be eliminated. Noise also occurs when electrical connections that carry current are made and broken, as, for example, at the brushes of certain types of motors. Again in principle, this type of noise can be suppressed at the source.

Natural phenomena that give rise to noise include electric storms, solar flares, and certain belts of radiation that exist in space. Noise arising from these sources may be more difficult to suppress, and often the only solution is to reposition the receiving antenna to minimize the received noise, while ensuring that reception of the desired signal is not seriously impaired.

Noise is mainly of concern in receiving systems, where it sets a lower limit on the size of signal that can be usefully received. Even when precautions are taken to eliminate noise from faulty connections or that arising from external sources, it is found that certain fundamental sources of noise are present within electronic equipment that limit the receiver sensitivity. One might think that any signal, however small, could simply be amplified up to any desired level. Unfortunately, adding amplifiers to a receiving system also adds noise, and the signal-to-noise ratio, which is the significant quantity, may be degraded by the addition of the amplifiers. Thus the study of the fundamental sources of noise within equipment is essential if the effects of the noise are to be minimized.

4.2 Thermal Noise

It is known that the free electrons within an electrical conductor possess kinetic energy as a result of heat exchange between the conductor and its surroundings. The kinetic energy means that the electrons are in motion, and this motion in turn is randomized through collisions with imperfections in the structure of the conductor. This process occurs in all real conductors and is what gives rise to the conductors' resistance. As a result, the electron density throughout the conductor varies randomly, giving rise to a randomly varying voltage across the ends of the conductor (Fig. 4.2.1). Such a voltage may sometimes be observed in the flickerings of a very sensitive voltmeter. Since the noise arises from thermal causes, it is referred to as *thermal noise* (and also as *Johnson noise,* after its discoverer).

The average or mean noise voltage across the conductor is zero, but the root-mean-square value is finite and can be measured. (It will be recalled that a similar situation occurs for sinusoidal voltage, which has a mean value of zero and a finite rms value.) It is found that the mean-square value of the noise voltage is proportional to the resistance of the conductor, to its absolute temperature, and to the frequency bandwidth of the device measuring (or responding to) the noise. The rms voltage is of course the square root of the mean-square value.

Consider a conductor that has resistance R, across which a true rms measuring voltmeter is connected, and let the voltmeter have an ideal band-pass frequency response of bandwidth B_n as shown in Fig. 4.2.2. The subscript n signifies noise bandwidth, which for the moment may be assumed to be the same as the bandwidth of the ideal filter. The relationship between noise bandwidth and actual frequency response will be developed more fully later. The mean-square voltage measured on the meter is found to be

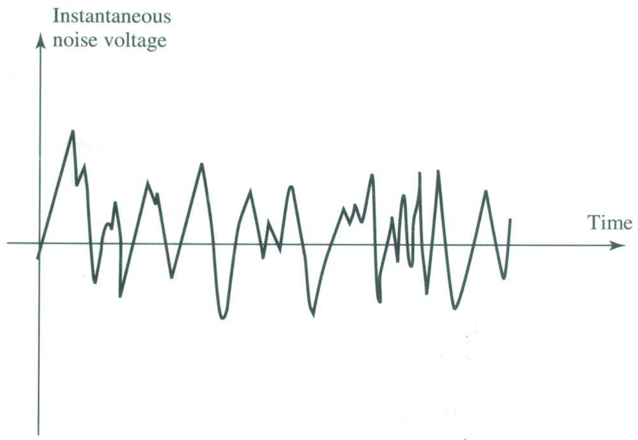

Figure 4.2.1 Thermal noise voltage.

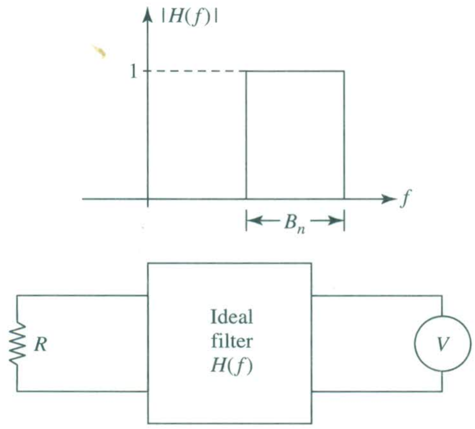

Figure 4.2.2 Measurement of thermal noise.

$$E_n^2 = 4RkTB_n \tag{4.2.1}$$

where E_n = root-mean-square noise voltage, volts
R = resistance of the conductor, ohms
T = conductor temperature, kelvins
B_n = noise bandwidth, hertz
k = Boltzmann's constant
= 1.38×10^{-23} J/K

The equation is given in terms of mean-square voltage rather than root mean square, since this shows the proportionality between the noise power (proportional to E_n^2) and temperature (proportional to kinetic energy).

The rms noise voltage is given by

$$E_n = \sqrt{4RkTB_n} \tag{4.2.2}$$

The presence of the mean-square voltage at the terminals of the resistance R suggests that it may be considered as a generator of electrical noise power. Attractive as the idea may be, thermal noise is not unfortunately a free source of energy. To abstract the noise power, the resistance R would have to be connected to a resistive load, and in thermal equilibrium the load would supply as much energy to R as it receives.

The fact that the noise power cannot be utilized as a free source of energy does not prevent the power being calculated. In analogy with any electrical source, the *available average power* is defined as the maximum average power the source can deliver. For a generator of emf E volts (rms) and internal resistance R, the available power is $E^2/4R$. Applying this to Eq. (4.2.1) gives for the available thermal noise power:

$$P_n = kTB_n \tag{4.2.3}$$

EXAMPLE 4.2.1

Calculate the thermal noise power available from any resistor at room temperature (290 K) for a bandwidth of 1 MHz. Calculate also the corresponding noise voltage, given that $R = 50\ \Omega$.

SOLUTION For a 1-MHz bandwidth, the noise power is

$$P_n = 1.38 \times 10^{-23} \times 290 \times 10^6$$
$$= 4 \times 10^{-15}\ \mathrm{W}$$
$$E_n^2 = 4 \times 50 \times 1.38 \times 10^{-23} \times 290$$
$$= 810^{-13}$$
$$\therefore E_n = \mathbf{0.895\ \mu V}$$

The noise power calculated in Example 4.2.1 may seem to be very small, but it may be of the same order of magnitude as the signal power present. For example, a receiving antenna may typically have an induced signal emf of 1 μV, which is of the same order as the noise voltage.

The thermal noise properties of a resistor R may be represented by the equivalent voltage generator of Fig. 4.2.3(a). This is one of the most useful representations of thermal noise and is widely used in determining the noise performance of equipment. It is best to work initially in terms of E_n^2 rather than E_n, for reasons that will become apparent shortly.

Norton's theorem may be used to find the equivalent current generator and this is shown in Fig. 4.2.3(b). Here, using conductance G (= $1/R$), the rms noise current I_n is given by

$$I_n^2 = 4GkTB_n \tag{4.2.4}$$

It will be recalled that the bandwidth is that of the external circuit, not shown in the source representations, and this must be examined in more detail. Suppose the resistance is left open circuited; then the bandwidth ideally would be infinite, and Eq. (4.2.3) suggests that the open-circuit noise voltage would also be infinite! Two factors prevent this from happening. The first relates to the derivation of the noise energy, which is based on classical thermodynamics and ignores quantum mechanical effects. The quantum mechanical derivation shows that the energy drops off with increasing frequency, and this therefore sets a fundamental limit to the noise power available. However, quantum mechanical effects only become important at

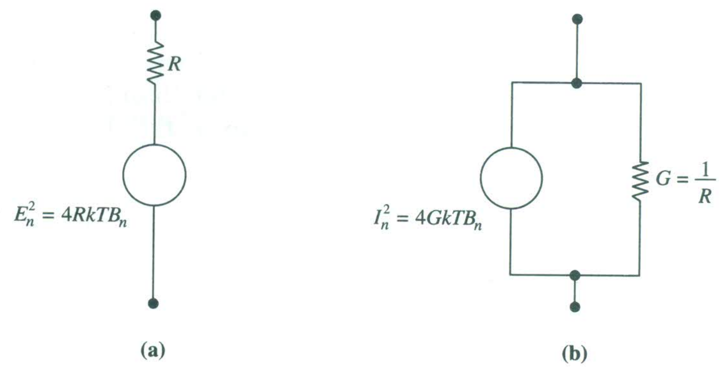

Figure 4.2.3 Equivalent sources for thermal noise: (a) voltage source and (b) current source.

frequencies well into the infrared region. The second and more significant practical factor from the circuit point of view is that *all* real circuits contain reactance (for example, self-inductance and self-capacitance), which sets a finite limit on bandwidth. In the case of the open-circuited resistor, the self-capacitance sets a limit on bandwidth, a situation that is covered in more detail later.

Resistors in Series

Let R_{ser} represent the total resistance of the series chain, where $R_{ser} = R_1 + R_2 + R_3 + \cdots$; then the noise voltage of the equivalent series resistance is

$$\begin{aligned} E_n^2 &= 4R_{ser}kTB_n \\ &= 4(R_1 + R_2 + R_3 + \cdots)kTB_n \\ &= E_{n1}^2 + E_{n2}^2 + E_{n3}^2 + \cdots \end{aligned} \tag{4.2.5}$$

This shows that the total noise voltage *squared* is obtained by summing the mean-square values. Hence the noise voltage of the series chain is given by

$$E_n = \sqrt{E_{n1}^2 + E_{n2}^2 + E_{n3}^2 + \cdots} \tag{4.2.6}$$

Note that simply adding the individual noise voltages would have given the wrong result.

Resistors in Parallel

With resistors in parallel it is best to work in terms of conductance. Thus let G_{par} represent the parallel combination where $G_{par} = G_1 + G_2 + G_3 + \cdots$; then

$$\begin{aligned} I_n^2 &= 4G_{par}kTB_n \\ &= 4(G_1 + G_2 + G_3 + \cdots)kTB_n \\ &= I_{n1}^2 + I_{n2}^2 + I_{n3}^2 + \cdots \end{aligned} \tag{4.2.7}$$

Again, it is to be noted that the mean-square values are added to obtain the total mean-square noise current. Usually, it is more convenient to work in terms of noise voltage rather than current. This is most easily done by first determining the equivalent parallel resistance from $1/R_{par} = 1/R_1 + 1/R_2 + 1/R_3 + \cdots$ and using

$$E_n^2 = 4R_{par}kTB_n \tag{4.2.8}$$

EXAMPLE 4.2.2

Two resistors of 20 and 50 kΩ are at room temperature (290 K). For a bandwidth of 100 kHz, calculate the thermal noise voltage generated by (a) each resistor, (b) the two resistors in series, and (c) the two resistors in parallel.

SOLUTION (a) For the 20-kΩ resistor

$$E_n^2 = 4 \times (20 \times 10^3) \times (4 \times 10^{-21}) \times (100 \times 10^3)$$
$$= 32 \times 10^{-12}\ \mathrm{V}^2$$
$$\therefore E_n = \mathbf{5.66\ \mu V}$$

The voltage for the 50-kΩ resistor may be found by simple proportion:

$$E_n = 5.66 \times \sqrt{\frac{50}{20}}$$
$$= \mathbf{8.95\ \mu V}$$

(b) For the series combination, $R_{ser} = 20 + 50 = 70$ kΩ. Hence

$$E_n = 5.66 \times \sqrt{\frac{70}{20}}$$
$$= \mathbf{10.59\ \mu V}$$

(c) For the parallel combination, $R_{par} = \dfrac{20 \times 50}{20 + 50} = 14.29$ kΩ.

$$\therefore E_n = 5.66 \times \sqrt{\frac{14.29}{20}} = \mathbf{4.78\ \mu V}$$

Reactance

Reactances do not generate thermal noise. This follows from the fact that reactance cannot dissipate power. Consider an inductive or capacitive reactance connected in parallel with a resistor R (Fig. 4.2.4). In thermal equilibrium, equal amounts of power must be exchanged; that is, if the resistor supplies thermal noise power P_2 to the reactance, the reactance must supply thermal noise power $P_1 = P_2$ to the resistor. But since the reactance cannot dissipate power, the power P_2 must be zero, and hence P_1 must also be zero.

The effect of reactance on the noise bandwidth must, however, be taken into account, as shown in the next section.

Spectral Densities

Thermal noise falls into the category of power signals as described in Section 2.17, and hence it has a spectral density. As pointed out previously, the bandwidth B_n is a property of the external measuring or receiving system and is

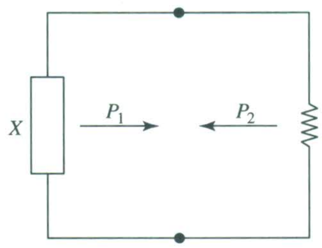

Figure 4.2.4 Power exchange between a reactance and a resistance is $P_1 = P_2 = 0$.

assumed flat so that, from Eq. (4.2.3), the available power spectral density, in watts per hertz, or joules, is

$$\begin{aligned} G_a(f) &= \frac{P_n}{B_n} \\ &= kT \end{aligned} \tag{4.2.9}$$

The spectral density for the mean-square voltage is also a useful function. This has units of volts^2 per hertz and is given by

$$\begin{aligned} G_v(f) &= \frac{E_n^2}{B_n} \\ &= 4RkT \end{aligned} \tag{4.2.10}$$

The spectral densities are flat, that is, independent of frequency, as shown in Fig. 4.2.5, and as a result thermal noise is sometimes referred to as *white noise,* in analogy to white light, which has a flat spectrum. When white noise is passed through a network, the spectral density will be altered by the shape of the network frequency response. The total noise power at the output is found by summing the noise contributions over the complete frequency range, taking into account the shape of the frequency response.

Consider a power spectral response as shown in Fig. 4.2.6. At frequency f_1, the available noise power for an infinitesimally small bandwidth δf about f_1 is $\delta P_{n1} = S_p(f_1)\delta f$. This is so because the bandwidth δf may be assumed flat about f_1, and the available power is given as the product of spectral density (watts/hertz) $\times$ bandwidth (hertz). The available noise power is therefore seen to be equal to the area of the shaded strip about f_1. Similar arguments can be applied at frequencies f_2, f_3, ..., and the total power, given by the sum of all these contributions, is equal to the sum of all these small areas, which is the total area under the curve. More formally, this is equal to the integral of the spectral density function over the frequency range $f = 0$ to $f = \infty$.

A similar argument can be applied to mean-square voltage. The spectral density curve in this case has units of V^2/Hz, and multiplying this by bandwidth δf Hz results in units of V^2, so the area under the curve gives the total mean-square voltage.

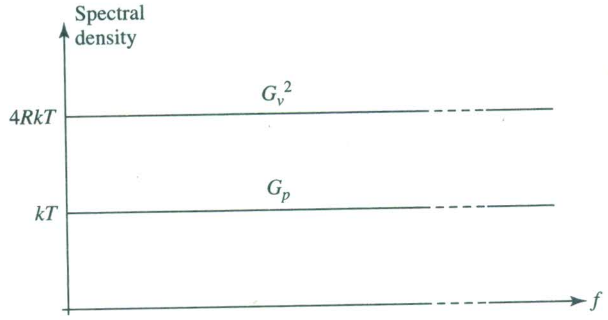

Figure 4.2.5 Thermal noise spectral densities.

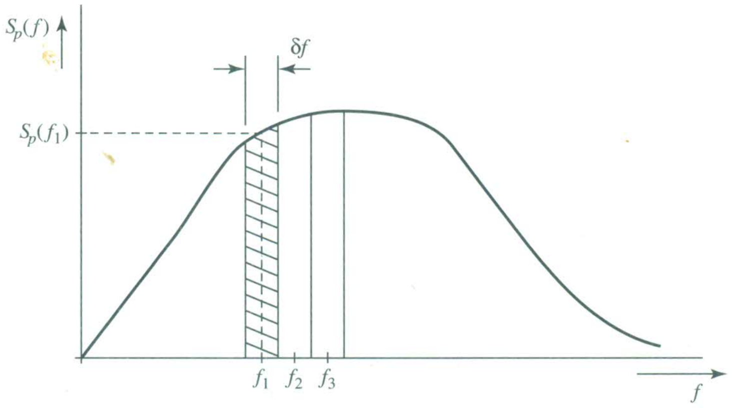

Figure 4.2.6 Nonuniform noise spectral density.

Equivalent Noise Bandwidth

Suppose that a resistor R is connected to the input of an LC filter, as shown in Fig. 4.2.7(a). This represents an input generator of mean-square voltage spectral density $4RkT$ feeding a network consisting of R and the LC filter. Let the transfer function of the network including R be $H(f)$, as shown in Fig. 4.2.7(b). The spectral density for the mean-square output voltage is therefore $4RkT|H(f)|^2$. This follows since $H(f)$ is the ratio of output to input voltage, and here mean-square values are being considered.

From what was shown previously, the total mean-square output voltage is given by the area under the output spectral density curve

$$V_n^2 = \int_0^\infty 4RkT|H(f)|^2 df$$

$$= 4RkT \times (\text{area under } |H(f)|^2 \text{ curve}) \tag{4.2.11}$$

Now the total mean-square voltage at the output can be stated as $V_n^2 = 4RkTB_n$, and equating this with Eq. (4.2.11) gives, for the equivalent noise bandwidth of the network,

$$B_n = \int_0^\infty |H(f)|^2 df$$

$$= (\text{area under } |H(f)|^2 \text{ curve}) \tag{4.2.12}$$

As a simple example consider the circuit of Fig. 4.2.8, which consists of a resistor in parallel with a capacitor. The capacitor may in fact be the self-capacitance of the resistor, or an external capacitor, for example, the input capacitance of the voltmeter used to measure the noise voltage across R.

The transfer function of the RC network is

$$|H(f)| = \frac{1}{\sqrt{1 + (\omega CR)^2}} \tag{4.2.13}$$

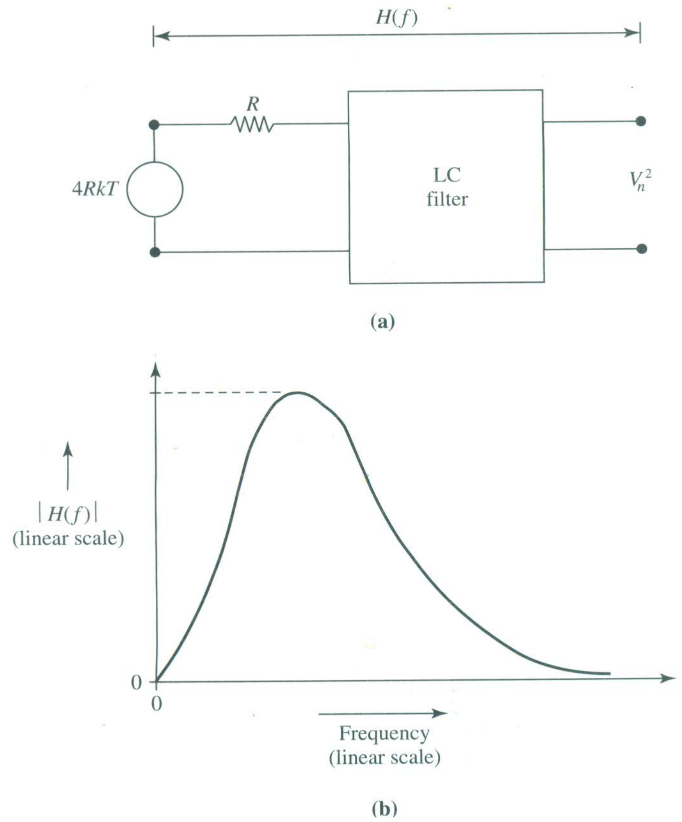

Figure 4.2.7 (a) Filtered noise and (b) the transfer function of the filter including R.

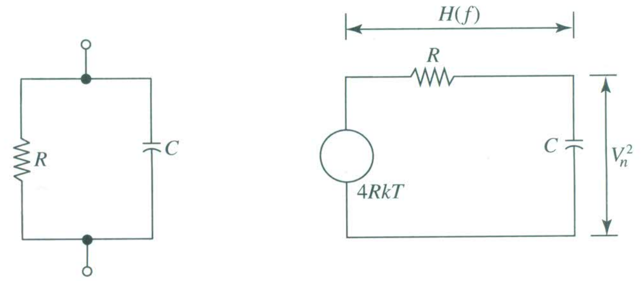

Figure 4.2.8 *RC* network and its transfer function used in determining noise bandwidth.

The equivalent noise bandwidth of the *RC* network is found using Eq. (4.2.12) as

$$B_n = \int_0^\infty |H(f)|^2 df$$

$$= \frac{1}{4RC} \tag{4.2.14}$$

(Details of the integration are left as an exercise for the reader.) The mean-square output voltage is given by

$$V_n^2 = 4RkT \times \frac{1}{4RC}$$

$$= \frac{kT}{C} \qquad (4.2.15)$$

This is a surprising result. It shows that the mean-square output voltage is independent of R, even though it originates from R, and it is inversely proportional to C, even though C does not generate noise.

A second example is that of the tuned circuit shown in Fig. 4.2.9. Here the capacitor is assumed lossless, and the inductor has a series resistance r that generates thermal noise.

The transfer function in this case is

$$|H(f)| = \left|\frac{X_c}{Z_s}\right| \qquad (4.2.16)$$

where $Z_s = r(1 + jyQ)$ is the impedance of the series tuned circuit as given by Eq. (1.3.10) and $X_c = 1/j\omega C$ is the reactance of C. As before, the equivalent noise bandwidth is found by solving Eq. (4.2.12).

Consider first the situation where the circuit is resonant at f_0, and the noise is restricted to a small bandwidth $\Delta f \ll f_0$ about the resonant frequency. The transfer function is then approximated by $|H(f)| \cong 1/\omega_0 Cr = Q$, and the area under the $|H(f)|^2$ curve over a small constant bandwidth δf is $Q^2\delta f$. Hence the mean-square noise voltage is

$$\begin{aligned} V_n^2 &= 4rkTB_n \qquad (4.2.17)\\ &= 4rQ^2kT\delta f \\ &= 4R_D kT\Delta f \end{aligned}$$

Here, use is made of the relationship $Q^2r = R_D$ developed in Section 1.4. This is an important result, because the bandwidth is often limited in practice to some small percentage about f_0. An example will illustrate this.

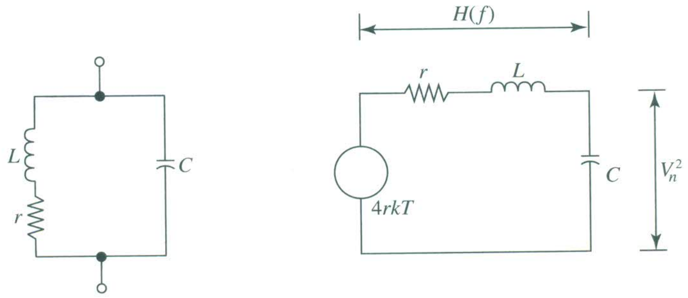

Figure 4.2.9 Tuned circuit and its transfer function used in determining noise bandwidth.

EXAMPLE 4.2.3

The parallel tuned circuit at the input of a radio receiver is tuned to resonate at 120 MHz by a capacitance of 25 pF. The Q-factor of the circuit is 30. The channel bandwidth of the receiver is limited to 10 kHz by the audio sections. Calculate the effective noise voltage appearing at the input at room temperature.

SOLUTION

$$R_D = \frac{Q}{\omega_o C}$$

$$= \frac{30}{2 \times \pi \times 120 \times 10^6 \times 25 \times 10^{-12}}$$

$$= 1.59 \text{ k}\Omega$$

$$\therefore V_n = \sqrt{4 \times 1.59 \times 10^3 \times 4 \times 10^{-21} \times 10^4}$$

$$= \mathbf{0.5\ \mu V}$$

Where the complete frequency range 0 to ∞ has to be taken into account, the integral becomes much more difficult to solve, and only the result will be given here. This is

$$B_n = \frac{1}{4R_D C} \tag{4.2.18}$$

where R_D is the dynamic resistance of the tuned circuit.

The noise bandwidth can be expressed as a function of the −3-dB bandwidth of the circuit. From Eq. (1.3.17), $B_{3\text{ dB}} = f_o/Q$, and from Eq. (1.4.4) $R_D = Q/\omega_o C$. Combining these expressions along with that for the noise bandwidth gives

$$B_n = \frac{\pi}{2} B_{3\text{ dB}} \tag{4.2.19}$$

By postulating that the noise originates from a resistor R_D and is limited by the bandwidth B_n, the mean-square voltage at the output can be expressed as

$$V_n^2 = 4R_D kT \times \frac{1}{4R_D C}$$

$$= \frac{kT}{C} \tag{4.2.20}$$

In the foregoing, to simplify the analysis it was assumed that the Q-factor remained constant, independent of frequency. This certainly would not be true for the range zero to infinity, but the end result still gives a good indication of the noise expected in practice.

For most radio receivers the noise is generated at the front end (antenna input) of the receiver, while the output noise bandwidth is determined by the audio sections of the receiver. The equivalent noise bandwidth is equal to the area under the normalized power-gain/frequency curve for the low-frequency sections. By normalized is meant that the curve is scaled such that the maximum value is equal to unity. Usually this information is available in the form of a frequency response curve showing output in decibels relative to maximum and with frequency plotted on a logarithmic scale, as sketched in Fig. 4.2.10(a).

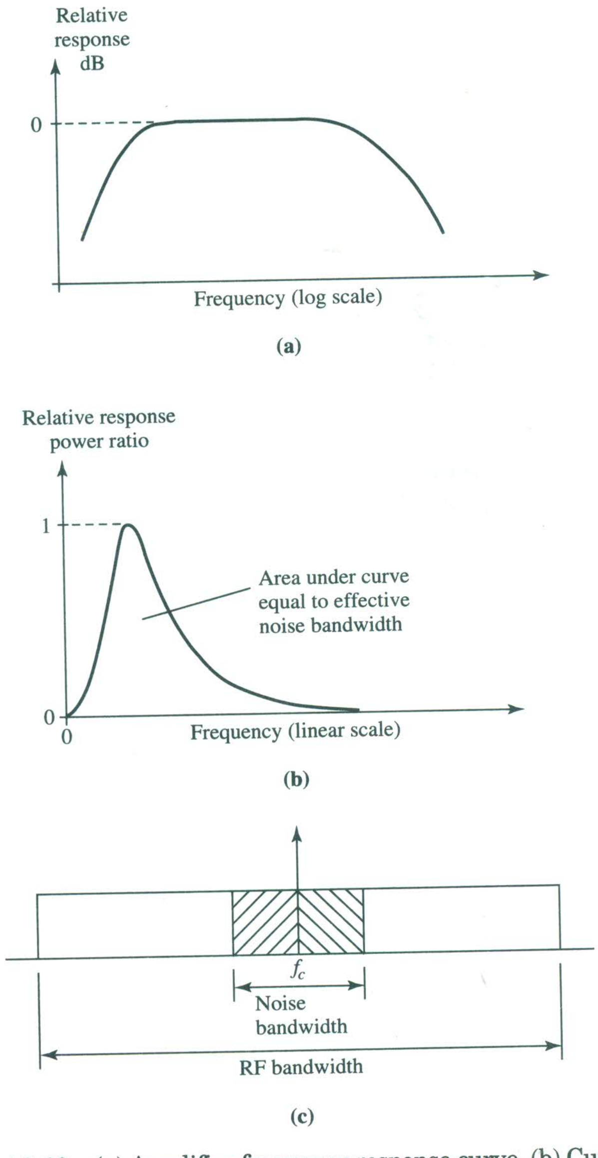

Figure 4.2.10 (a) Amplifier frequency response curve. (b) Curve of (a) using linear scales. (c) Noise bandwidth of a double-sideband receiver.

Before determining the area under the curve, the decibel axis must be converted to a linear power-ratio scale and the frequency axis to a linear frequency scale, as shown in Fig. 4.2.10(b). The equivalent noise bandwidth is then equal to the area under this curve for a single-sideband receiver. Where the receiver is of the double-sideband type, then the noise bandwidth appears on both sides of the carrier and is effectively doubled. This is shown in Fig. 4.2.10(c).

4.3 Shot Noise

Shot noise is a random fluctuation that accompanies any direct current crossing a potential barrier. The effect occurs because the carriers (holes and electrons in semiconductors) do not cross the barrier simultaneously, but rather with a random distribution in the timing for each carrier, which gives rise to a random component of current superimposed on the steady current. In the case of bipolar junction transistors, the bias current crossing the forward biased emitter–base junction carries shot noise. With vacuum tubes the electrons emitted from the cathode have to overcome a potential barrier that exists between cathode and vacuum. The name *shot noise* was first coined in connection with tubes, where the analogy was made between the electrons striking the plate and lead shot from a gun striking a target.

Although it is always present, shot noise is not normally observed during measurement of direct current because it is small compared to the dc value; however, it does contribute significantly to the noise in amplifier circuits. The idea of shot noise is illustrated in Fig. 4.3.1.

Shot noise is similar to thermal noise in that its spectrum is flat (except in the high microwave frequency range). The mean-square noise component is proportional to the dc flowing, and for most devices the mean-square, shot-noise current is given by

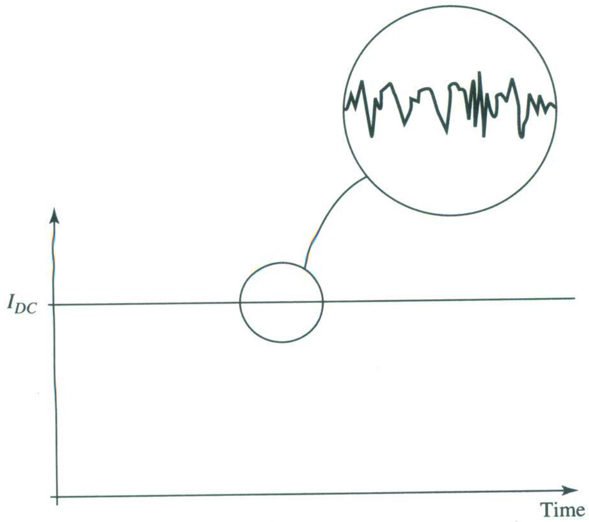

Figure 4.3.1 Shot noise.

$$I_n^2 = 2 I_{dc} q_e B_n \quad \text{amperes}^2 \tag{4.3.1}$$

where I_{dc} is the direct current in amperes, q_e the magnitude of electron charge ($= 1.6 \times 10^{-19}$ C), and B_n is the equivalent noise bandwidth in hertz.

EXAMPLE 4.3.1

Calculate the shot noise component of current present on a direct current of 1 mA flowing across a semiconductor junction, given that the effective noise bandwidth is 1 MHz.

SOLUTION

$$\begin{aligned} I_n^2 &= 2 \times 10^{-3} \times 1.6 \times 10^{-19} \times 10^6 \\ &= 3.2 \times 10^{-16}\ \text{A}^2 \\ \therefore I_n &= \mathbf{18\ nA} \end{aligned}$$

4.4 Partition Noise

Partition noise occurs wherever current has to divide between two or more electrodes and results from the random fluctuations in the division. It would be expected therefore that a diode would be less noisy than a transistor (other factors being equal) if the third electrode draws current (such as base or gate current). It is for this reason that the input stage of microwave receivers is often a diode circuit, although, more recently, gallium arsenide field-effect transistors, which draw zero gate current, have been developed for low-noise microwave amplification. The spectrum for partition noise is flat.

4.5 Low Frequency or Flicker Noise

Below frequencies of a few kilohertz, a component of noise appears, the spectral density of which increases as the frequency decreases. This is known as *flicker noise* (and sometimes as $1/f$ noise). In vacuum tubes it arises from slow changes in the oxide structure of oxide-coated cathodes and from the migration of impurity ions through the oxide. In semiconductors, flicker noise arises from fluctuations in the carrier densities (holes and electrons), which in turn give rise to fluctuations in the conductivity of the material. It follows therefore that a noise voltage will be developed whenever direct current flows through the semiconductor, and the mean-square voltage will be proportional to the square of the direct current. Interestingly enough, although flicker noise is a low-frequency effect, it plays an important part in limiting the sensitivity of microwave diode mixers used for Doppler radar systems. This is because, although the input frequencies to the mixer are in the microwave range, the Doppler frequency output is in the low (audio-frequency) range, where flicker noise is significant.

4.6 Burst Noise

Another type of low-frequency noise observed in bipolar transistors is known as *burst noise,* the name arising because the noise appears as a series of bursts at two or more levels (rather like noisy pulses). When present in an audio system, the noise produces popping sounds, and for this reason is also known as "popcorn" noise. The source of burst noise is not clearly understood at present, but the spectral density is known to increase as the frequency decreases.

4.7 Avalanche Noise

The reverse-bias characteristics of a diode exhibit a region where the reverse current, normally very small, increases extremely rapidly with a slight increase in the magnitude of the reverse-bias voltage. This is known as the *avalanche region* and comes about because the holes and electrons in the diode's depletion region gain sufficient energy from the reverse-bias field to ionize atoms by collision. The ionizing process means that additional holes and electrons are produced, which in turn contribute to the ionization process, and thus the descriptive term *avalanche.*

The collisions that result in the avalanching occur at random, with the result that large noise spikes are present in the avalanche current. In diodes such as zener diodes, which are used as voltage reference sources, the avalanche noise is a nuisance to be avoided. However, avalanche noise is put to good use in noise measurements, as described in Section 4.19. The spectral density of avalanche noise is flat.

4.8 Bipolar Transistor Noise

Bipolar transistors exhibit all the sources of noise discussed previously, that is, thermal, shot, partition, flicker, and burst noise. The thermal noise is generated by the bulk or extrinsic resistances of the electrodes, but the only significant component is that generated by the extrinsic base resistance. It should be emphasised at this point that the small-signal equivalent resistances for the base–emitter and the base–collector junctions do not generate thermal noise, but they do enter into the noise calculations made using the small-signal equivalent circuit for the transistor.

The bias currents in the transistor show shot noise and partition noise, and, in addition, the flicker and burst noise components are usually associated with the base current.

4.9 Field-effect Transistor Noise

In field-effect transistors (both JFETs and MOSFETs), the main source of noise is the thermal noise generated by the physical resistance of the drain–source channel. Flicker noise also originates in this channel. Additionally, there will be shot noise associated with the gate leakage current. This will develop a noise component of voltage across the signal–source impedance and is only significant where this impedance is very high (in the megohm range).

4.10 Equivalent Input Noise Generators and Comparison of BJTs and FETs

An amplifier may be represented by the block schematic of Fig. 4.10.1(a), in which a noisy amplifier is shown and where the source and load resistances generate thermal noise. The circuit may be redrawn as shown in Fig. 4.10.1(b) in which the amplifier itself is considered to be noiseless, the amplifier noise being represented by *fictitious noise generators* $V_{na} = \sqrt{4R_n kT_o B_n}$ and $I_{na} = \sqrt{2q_e I_{EQ} B_n}$ at the input. Here, B_n is the equivalent noise bandwidth of the amplifier in hertz, T_o is room temperature in kelvins, k is Boltzmann's constant $= 1.38 \times 10^{-23}$ J/K, and $q_e = 1.6 \times 10^{-19}$ C is the magnitude of the electron charge. These terms have all been defined previously. What is new here is the *fictitious* resistance R_n ohms, known as the *equivalent input noise resistance* of the amplifier, and I_{EQ} amperes, the *equivalent input shot noise current*. Both these parameters have to be calculated or specified for a transistor under given operating conditions.

The noise generated by the load resistance R_L is generally very small compared to the other sources and is assumed to be negligible, so this is dropped from the equivalent circuit. The thermal noise generated by the signal–source resistance R_s is generally significant and must be taken into account as shown in Fig. 4.10.1(b).

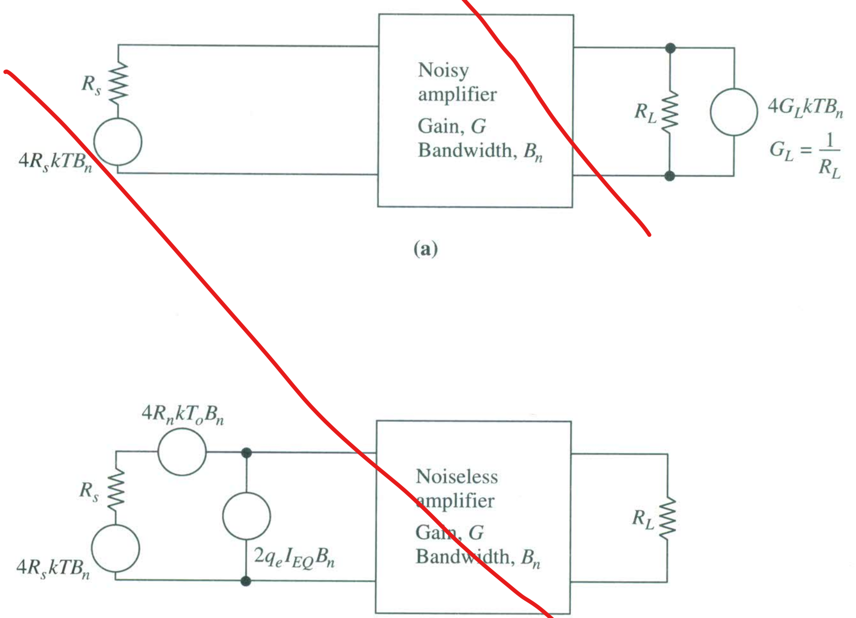

Figure 4.10.1 (a) Noisy amplifier and (b) the equivalent input noise generators.

The total noise voltage at the input to the amplifier is found as follows. Referring to the equivalent circuit of Fig. 4.10.2(a), the noise sources are

$$V_{ns}^2 = 4R_s kT_o B_n \tag{4.10.1}$$

$$V_{na}^2 = 4R_n kT_o B_n \tag{4.10.2}$$

$$I_{na}^2 = 2q_e I_{EQ} B_n \tag{4.10.3}$$

As a first step in simplifying this, the emf sources can be combined as shown in Fig. 4.10.2(b). Next, the Thevenin equivalent circuit can be obtained by combining the current source as an equivalent emf as shown in Fig. 4.10.2(c). Thus the equivalent noise voltage at the input to the amplifier is

$$V_n = \sqrt{V_{ns}^2 + V_{na}^2 + (I_{na}R_s)^2} \tag{4.10.4}$$

Throughout, it is assumed that all the noise sources are uncorrelated. Correlation actually exists between V_{na} and I_{na}, but this is only significant at high frequencies, where the analysis must take correlation into account.

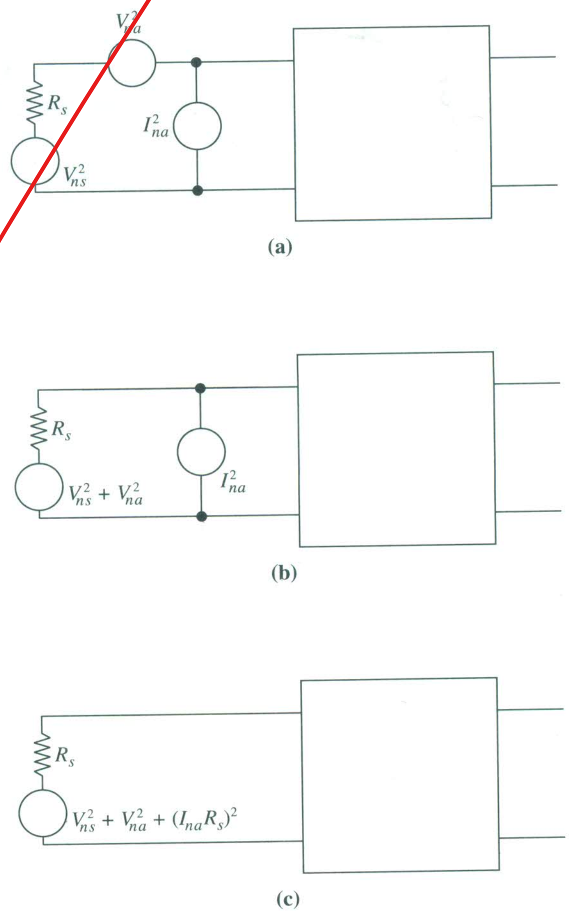

Figure 4.10.2 (a) Equivalent input noise generators; (b) the voltage sources combined; (c) all sources combined.

A detailed comparison of the performance of BJT and FET amplifiers is too involved to be included here, but the following general remarks may be made. R_n is generally smaller and I_{EQ} larger for BJTs compared to FETs. For input signal sources with low resistances, where the noise voltage $I_{na}R_s$ is small enough to be neglected, the BJT will produce lower noise because of its smaller value of R_n. Where, however, R_s is large such that the $I_{na}R_s$ voltage is significant, the FET will produce lower noise than the BJT because of its lower I_{EQ}. There will be an intermediate range for R_s where, in fact, the thermal noise generated by R_s itself dominates, and the type of transistor may have little bearing on the overall noise performance of the amplifier.

4.11 Signal-to-Noise Ratio

In a communications link it is the signal-to-noise ratio, rather than the absolute value of noise, that is important. Signal-to-noise is defined as a power ratio, and since at a given point in a circuit power it is proportional to the square of the voltage, then

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{P_s}{P_n} = \frac{V_s^2}{V_n^2} \tag{4.11.1}$$

EXAMPLE 4.11.1

The equivalent noise resistance for an amplifier is 300 Ω, and the equivalent shot noise current is 5 μA. The amplifier is fed from a 150-Ω, 10-μV rms sinusoidal signal source. Calculate the individual noise voltages at the input and the input signal-to-noise ratio in decibels. The noise bandwidth is 10 MHz.

SOLUTION Assume room temperature so that $kT = 4 \times 10^{-21}$ J and $q_e = 1.6 \times 10^{-19}$ C. The shot noise current is $I_{na} = \sqrt{2q_e I_{EQ} B_n} = 4$ nA. The noise voltage developed by this across the source resistance is $I_{na}R_s =$ **0.6 μV.**

Note that the shot noise current *does not* develop a voltage across R_n. The noise voltage generated by R_n is $V_{na} = \sqrt{4R_n kT_o B_n} = 6.93$ μV. The thermal noise voltage from the source is

$$V_{ns} = \sqrt{4R_s kT_o B_n} = 4.9\ \mu\text{V}$$

The total noise voltage at the input to the amplifier is

$$V_n = \sqrt{4.9^2 + 6.93^2 + .6^2} = \mathbf{8.51\ \mu V}$$

The signal-to-noise ratio in decibels is

$$\frac{S}{N} = 20 \log \frac{V_s}{V_n} = \mathbf{1.4\ dB}$$

4.12 S/N Ratio of a Tandem Connection

In an analog telephone system it is usually necessary to insert amplifiers to make up for the loss in the telephone cables, the amplifiers being known as repeaters. As shown in Fig. 4.12.1, if the power loss of a line section is L, then the amplifier power gain G is chosen so that $LG = 1$. A long line will be divided into sections that are near enough identical, and each repeater adds its own noise, so the noise accumulates with the signal as it travels along the system.

Consider the situation where the input signal power to the first section of the line is P_s, and at this point the input noise may be assumed negligible. After traveling along the first section of line, the signal is attenuated by a factor L. At the output of the first repeater the signal power is again P_s since the gain G exactly compensates for the loss L. The noise at the output of the first repeater is shown as P_{n1} and consists of the noise added by the line section and amplifier, or what is termed the *first link* in the system.

As the signal progresses along the links, the power output at each repeater remains at P_s because $LG = 1$ for each link. However, the noise powers are additive, and the total noise at the output of the Mth link is $P_n = P_{n1} + P_{n2} + \cdots + P_{nM}$. If the links are identical such that each link contributes P_n, the total noise power becomes $P_{nM} = MP_n$. The output signal-to-noise ratio in this case is

$$\left(\frac{\mathrm{N}}{\mathrm{S}}\right)_o \mathrm{dB} = 10 \log \frac{P_s}{MP_n}$$

$$= \left(\frac{\mathrm{S}}{\mathrm{N}}\right)_1 \mathrm{dB} - (M)\,\mathrm{dB} \tag{4.12.1}$$

where (S/N) is the signal-to-noise ratio of any one link, and (M) dB is the number of links expressed as a power ratio in decibels (that is, in decilogs).

EXAMPLE 4.12.1

Calculate the output signal-to-noise ratio in decibels for three identical links, given that the signal-to-noise ratio for any one link is 60 dB.

SOLUTION $(\mathrm{S/N})_o = 60 - 10 \log 3 = \mathbf{55.23\ dB}$

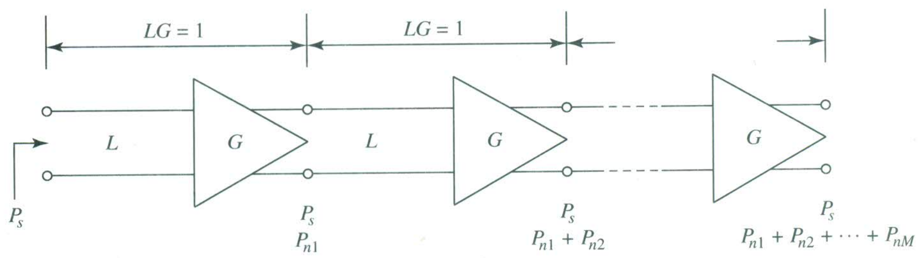

Figure 4.12.1 Tandem connection of repeaters.

If the S/N ratio of any one link is much worse than the others, that link will determine the overall S/N ratio. Suppose for example that the S/N ratio of the first link is much lower than the others; then the $(N/S)_1$ ratio will be much greater than the other noise-to-signal ratios. Hence

$$\begin{aligned}\left(\frac{N}{S}\right)_o &= \frac{P_{n1}}{P_s} + \frac{P_{n2}}{P_s} + \cdots \\ &= \left(\frac{N}{S}\right)_1 + \left(\frac{N}{S}\right)_2 + \cdots \\ &\cong \left(\frac{N}{S}\right)_1 \end{aligned} \tag{4.12.2}$$

EXAMPLE 4.12.2

Calculate the output signal-to-noise ratio in decibels for three links, the first two of which have S/N ratios of 60 dB and the third an S/N of 40 dB.

SOLUTION The noise-to-power ratio of the first two links is −60 dB, or a power ratio of 10^{-6}, while that of the third link is −40 dB, or a power ratio of 10^{-4}. The overall noise-to-signal ratio is

$$\begin{aligned}\left(\frac{S}{N}\right)_o &= 10^{-6} + 10^{-6} + 10^{-4} \\ &\cong 10^{-4}\end{aligned}$$

Thus the output signal-to-noise is approximately **40 dB.**

This example shows that the S/N ratio is approximately equal to that of the worst link, and the old saying that "a chain is no stronger than its weakest link" applies here also!

4.13 Noise Factor

Consider a signal source at room temperature $T_o = 290$ K providing an input to an amplifier. As explained in Section 4.2, the available noise power from such a source would be $P_{ni} = kT_oB_n$. Let the available signal power from the source be denoted by P_{si}; then the available signal-to-noise ratio from the source is

$$\left(\frac{S}{N}\right)_{in} = \frac{P_{si}}{kT_oB_n} \tag{4.13.1}$$

With the source connected to an amplifier, this represents the available input signal-to-noise ratio and hence the use of the subscript *in*. If now the amplifier has an available power gain denoted by G, the available output signal power would be $P_{so} = GP_{si}$, and if the amplifier was entirely noiseless, the available output noise power would be $P_{no} = GkT_oB_n$, as shown in Fig. 4.13.1(a).

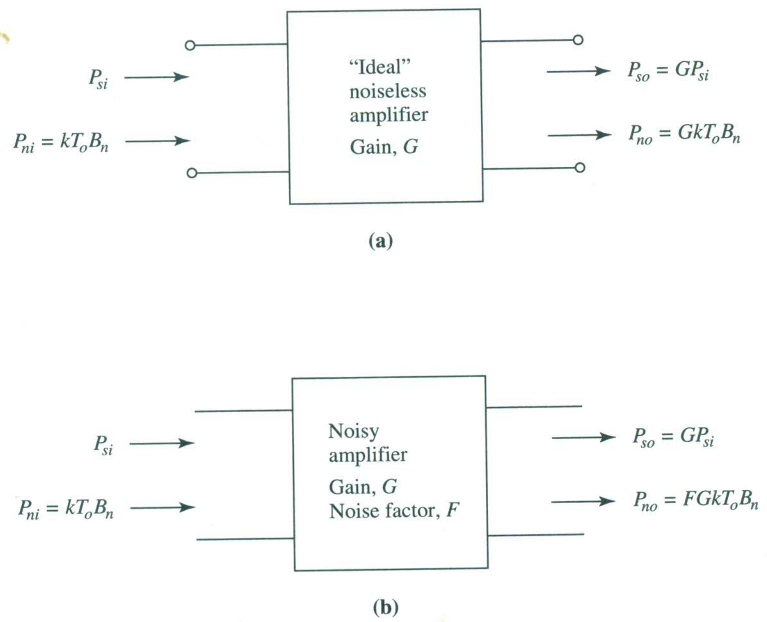

Figure 4.13.1 Noise factor F.

Hence the available output signal-to-noise ratio would be the same as that at the input since the factor G would cancel for both signal and noise.

However, it is known that all real amplifiers contribute noise, and the available output signal-to-noise ratio will be less than that at the input. The noise factor F is defined as

$$F = \frac{\text{available } S/N \text{ power ratio at the input}}{\text{available } S/N \text{ power ratio at the output}} \tag{4.13.2}$$

In terms of the symbols, this can be written as

$$\begin{aligned} F &= \frac{P_{si}}{kT_oB_n} \times \frac{P_{no}}{GP_{si}} \\ &= \frac{P_{no}}{GkT_oB_n} \end{aligned} \tag{4.13.3}$$

It follows from this that the available output noise power is given by

$$P_{no} = FGkT_oB_n \tag{4.13.4}$$

This is shown in Fig. 4.13.1(b). F can be interpreted as the factor by which the amplifier increases the output noise, for, if the amplifier were noiseless, the output noise would be GkT_oB_n.

A few comments are in order here regarding the definitions. Available power gain G is used because it can be defined unambiguously; that is, it does not depend on the load impedance. It may be thought that this definition requires the input to be matched for maximum power transfer, but this is not so. The available output power depends on the actual input power delivered to the amplifier and hence takes into account any input mismatch that may be present. It must also be noted that noise factor is defined for the source at room temperature $T = 290$ K.

Noise factor is a measured parameter and will usually be specified for a given amplifier or network (the definition given applies for any linear network). It is usually specified in decibels, when it is referred to as the *noise figure*. Thus

$$\text{noise figure} = (F)\,\text{dB} = 10 \log F \tag{4.13.5}$$

EXAMPLE 4.13.1

The noise figure of an amplifier is 7 dB. Calculate the output signal-to-noise ratio when the input signal-to-noise ratio is 35 dB.

SOLUTION From the definition of noise factor it follows that

$$\begin{aligned}(S/N)_o &= (S/N)_{in} - (F)\,\text{dB} \\ &= 35 - 7 \\ &= \mathbf{28\ dB}\end{aligned}$$

4.14 Amplifier Input Noise in Terms of F

Amplifier noise is generated in many components throughout the amplifier, but it proves convenient to imagine it to originate from some equivalent power source at the input of the amplifier. (This is somewhat similar to the equivalent input generator approach described in Section 4.10.) From Eq. (4.13.4), the total available input noise is

$$\begin{aligned}P_{ni} &= \frac{P_{no}}{G} \\ &= FkT_oB_n\end{aligned} \tag{4.14.1}$$

This is illustrated in Fig. 4.14.1.

The source contributes an available power kT_oB_n and hence the amplifier must contribute an amount P_{na}, where

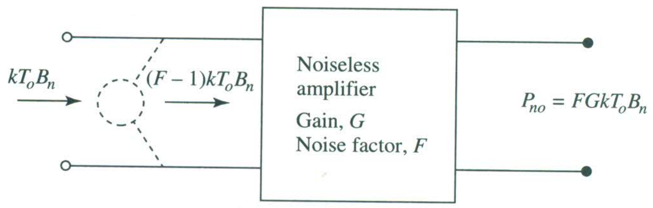

Figure 4.14.1 Equivalent input noise power source for an amplifier.

$$\begin{aligned} P_{na} &= FkT_oB_n - kT_oB_n \\ &= (F-1)kT_oB_n \end{aligned} \tag{4.14.2}$$

EXAMPLE 4.14.1

An amplifier has a noise figure of 13 dB. Calculate the equivalent amplifier input noise for a bandwidth of 1 MHz.

SOLUTION 13 dB is a power ratio of approximately 20 : 1. Hence

$$P_{na} = (20-1)4 \times 10^{-21}10^{6} = \mathbf{1.44\ pW}$$

It will be noted in the example that the noise figure must be converted to a power ratio F to be used in the calculation.

4.15 Noise Factor of Amplifiers in Cascade

Consider first two amplifiers in cascade as shown in Fig. 4.15.1. The problem is to determine the overall noise factor F in terms of the individual noise factors and available power gains.

The available noise power at the output of amplifier 1 is $P_{no1} = F_1G_1kT_oB_n$ and this is available to amplifier 2. Amplifier 2 has noise $(F_2-1)kT_oB_n$ of its own at its input, and hence the total available noise power at the input of amplifier 2 is

$$P_{ni2} = F_1G_1kT_oB_n + (F_2-1)kT_oB_n \tag{4.15.1}$$

Now since the noise of amplifier 2 is represented by its equivalent input source, the amplifier itself can be regarded as being noiseless and of available power gain G_2, so the available noise output of amplifier 2 is

$$\begin{aligned} P_{no2} &= G_2P_{ni2} \\ &= G_2(F_1G_1kT_oB_n + (F_2-1)kT_oB_n) \end{aligned} \tag{4.15.2}$$

The overall available power gain of the two amplifiers in cascade is $G = G_1G_2$, and let the overall noise factor be F; then the output noise power can also be expressed as [see Eq. 4.13.4]

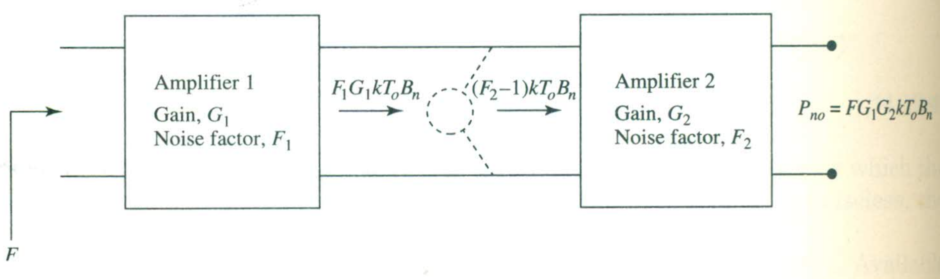

Figure 4.15.1 Noise factor of two amplifiers in cascade.

$$P_{no} = FGkT_oB_n \tag{4.15.3}$$

Equating the two expressions for output noise and simplifying yields

$$F = F_1 + \frac{F_2 - 1}{G_1} \tag{4.15.4}$$

This equation shows the importance of having a high-gain, low-noise amplifier as the first stage of a cascaded system. By making G_1 large, the noise contribution of the second stage can be made negligible, and F_1 must also be small so that the noise contribution of the first amplifier is low.

The argument is easily extended for additional amplifiers to give

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \cdots \tag{4.15.5}$$

This is known as *Friis's formula.*

There are two particular situations where a low-noise, front-end amplifier is employed to reduce noise. One of these is in satellite receiving systems, and this is discussed more fully in Chapter 19. The other is in radio receivers used to pick up weak signals, such as short-wave receivers. In most receivers, a stage known as the *mixer stage* is employed to change the frequency of the incoming signal, and it is known that mixer stages have notoriously high noise factors. By inserting an RF amplifier ahead of the mixer, the effect of the mixer noise can be reduced to negligible levels. This is illustrated in the following example.

EXAMPLE 4.15.1

A mixer stage has a noise figure of 20 dB, and this is preceded by an amplifier that has a noise figure of 9 dB and an available power gain of 15 dB. Calculate the overall noise figure referred to the input.

SOLUTION It is first necessary to convert all decibel values to the equivalent power ratios:

$$F_2 = 20\text{ dB} = 100 : 1 \text{ power ratio}$$

$$F_1 = 9\text{ dB} = 7.94 : 1 \text{ power ratio}$$

$$G_1 = 15\text{ dB} = 31.62 : 1 \text{ power ratio}$$

$$\begin{aligned} F &= F_1 + \frac{F_2 - 1}{G_1} \\ &= 7.94 + \frac{100 - 1}{31.62} \\ &= 11.07 \end{aligned}$$

This is the overall noise factor. The overall noise figure is

$$(F)\text{ dB} = 10 \log 11.07$$
$$= \mathbf{10.44\ dB}$$

4.16 Noise Factor and Equivalent Input Noise Generators

The noise factor is a function of source resistance as well as amplifier input noise. Referring once again to Fig. 4.10.2, the total mean-square input noise voltage is V_n^2, while the noise from the source alone is V_{ns}^2. In terms of these quantities, the noise factor is

$$F = \frac{V_n^2}{V_{ns}^2} \tag{4.16.1}$$

Substituting from Eqs. (4.10.1) through (4.10.3) and simplifying gives

$$F = 1 + \frac{R_n}{R_s} + \frac{q_e I_{EQ} \cdot R_s}{2kT_o}$$
$$= 1 + \frac{R_n}{R_s} + \frac{I_{EQ} R_s}{2V_T} \tag{4.16.2}$$

Here, $V_T = kT_o/q_e = 26$ mV is a constant.

The second term is inversely proportional to R_s, and the third term is proportional to R_s, which means that there must be an optimum value for R_s that minimizes F. This can be found by differentiating Eq. (4.16.2) and equating to zero. After simplifying, this results in

$$R_{s\text{ opt}} = \frac{V_{na}}{I_{na}}$$
$$= \sqrt{\frac{2R_n V_T}{I_{EQ}}} \tag{4.16.3}$$

Thus, knowing the input generator parameters allows the optimum value of source resistance to be determined. An input transformer coupling circuit may be necessary in order to transform the actual source resistance to the optimum value.

4.17 Noise Factor of a Lossy Network

When a signal source is matched through a lossy network, such as a connecting cable, the available signal power at the output of the network is reduced by the insertion loss of the network. The output noise remains unchanged at kT_oB_n

(assuming source and network to be at room temperature), since available noise power is independent of source resistance. In effect, the network attenuates the source noise, but at the same time adds noise of its own. The S/N ratio is therefore reduced by the amount that the output power is attenuated.

Denoting the power insertion loss ratio as L, the output S/N ratio will be $1/L$ times the input S/N ratio, and, from the definition of noise factor given by Eq. (4.13.2),

$$\begin{aligned} F &= \frac{\text{available } S/N \text{ power ratio at the input}}{\text{available } S/N \text{ power ratio at the output}} \\ &= L \end{aligned} \tag{4.17.1}$$

In Section 1.3 the insertion loss IL was defined in terms of currents. In terms of power, the power insertion loss is $L = (\mathrm{IL})^2$. Alternatively, specifying the insertion loss in decibels, which apply equally to current and power ratios, also specifies the noise figure in decibels.

EXAMPLE 4.17.1

Calculate the noise factor of an attenuator pad that has an insertion loss of 6 dB.

SOLUTION The insertion loss is 6 dB, and therefore the noise figure is 6 dB. This is equivalent to a noise factor of **4.**

The available power gain of a lossy network is $1/L$, and therefore when a lossy network, such as a connecting cable, is placed ahead of an amplifier, Friis's formula gives for the overall noise factor

$$\begin{aligned} F &= F_{nw} + \frac{F_a - 1}{G_{nw}} \\ &= L + (F_a - 1) \cdot L \end{aligned} \tag{4.17.2}$$

The subscript nw refers to the lossy network and a to the amplifier. It will be seen therefore that the loss L adversely affects the overall noise factor in two ways: by its direct contribution and by increasing the effect of the amplifier noise.

Alternatively, if the amplifier is placed ahead of the network, the overall noise factor is

$$\begin{aligned} F &= F_a + \frac{F_{nw} - 1}{G_a} \\ &= F_a + \frac{L - 1}{G_a} \end{aligned} \tag{4.17.3}$$

In this case, provided the amplifier has high gain, the overall noise factor of the system is essentially that of the amplifier alone. This situation is met with in satellite receiving systems (see Problem 4.42 and Chapter 19).

4.18 Noise Temperature

The concept of noise temperature is based on the available noise power equation given in Section 4.2, which is repeated here for convenience:

$$P_n = kT_a B_n \tag{4.18.1}$$

Here, the subscript a has been included to indicate that the noise temperature is associated only with the available noise power. In general, T_a will not be the same as the physical temperature of the noise source. As an example, an antenna pointed at deep space will pick up a small amount of cosmic noise. The equivalent noise temperature of the antenna that represents this noise power may be a few tens of kelvins, well below the physical ambient temperature of the antenna. If the antenna is pointed directly at the sun, the received noise power increases enormously, and the corresponding equivalent noise temperature is well above the ambient temperature.

When the concept is applied to an amplifier, it relates to the equivalent noise of the amplifier referred to the input. If the amplifier noise referred to the input is denoted by P_{na}, the equivalent noise temperature of the amplifier referred to the input is

$$T_e = \frac{P_{na}}{kB_n} \tag{4.18.2}$$

In Section 4.14, it was shown that the equivalent input power for an amplifier is given in terms of its noise factor by $P_{na} = (F - 1)kT_o B_n$. Substituting this in Eq. (4.18.2) gives for the equivalent input noise temperature of the amplifier

$$T_e = (F - 1)T_o \tag{4.18.3}$$

This shows the proportionality between T_e and F, and knowing one automatically entails knowing the other. In practice, it will be found that noise temperature is the better measure for low-noise devices, such as the low-noise amplifiers used in satellite receiving systems, while noise factor is a better measure for the main receiving system.

Friis's formula can be expressed in terms of equivalent noise temperatures. Denoting by T_e the overall noise of the cascaded system referred to the input, and by T_{e1}, T_{e2}, and so on, the noise temperatures of the individual stages, then Friis's formula is easily rearranged to give

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots \tag{4.18.4}$$

EXAMPLE 4.18.1

A receiver has a noise figure of 12 dB, and it is fed by a low-noise amplifier that has a gain of 50 dB and a noise temperature of 90 K. Calculate the noise temperature of the receiver and the overall noise temperature of the receiving system.

SOLUTION 12 dB represents a power ratio of 15.85 : 1. Hence

$$T_{em} = (15.85 - 1) \times 290 \cong \mathbf{4306\ K}$$

The 50-dB gain represents a power ratio of 10^5 : 1. Hence

$$T_e = 90 + \frac{4306}{10^5} \cong \mathbf{90\ K}$$

This example shows the relatively high noise temperature of the receiver, which clearly cannot be its physical temperature! It also shows how the low-noise amplifier controls the noise temperature of the overall receiving system. In this example, the cable connecting the low-noise amplifier and the receiver is assumed to contribute negligible noise. In satellite receiving systems the connecting cable can contribute significantly to the noise, and this is discussed in Chapter 19.

4.19 Measurement of Noise Temperature and Noise Factor

Noise temperature (and noise factor) can be measured in a number of ways, the method selected depending largely on the range of values expected. For normal receiving systems, an avalanche diode noise source is commonly employed, and this method will be described. (Older noise-figure-meters made use of the shot-noise generated by a vacuum tube diode.)

When operated in the avalanche mode, the diode generates a comparatively large amount of noise and can be considered as a source of noise power at some equivalent "hot" temperature T_h. With the reverse bias switched off, the diode reverts to normal noise output and generates noise at some equivalent "cold" temperature T_c. The *excess noise ratio* ENR is defined as

$$\text{ENR (dB)} = 10 \log \frac{T_h - T_c}{T_c} \tag{4.19.1}$$

The cold temperature is normally taken as room temperature $T_c = T_o =$ 290 K. The ENR for the source is normally printed on the diode enclosure and is specified by the manufacturer for a range of frequencies. Knowing the ENR and T_c, the hot temperature T_h can be found.

Now let the diode source be matched to the input of the amplifier under test, and let the (unknown) equivalent input noise temperature of the amplifier be denoted by T_e. The amplifier output noise is measured for two conditions, one with the diode in the avalanche mode, denoted by P_h, and one with the reverse bias switched off, denoted by P_c. The two equations for the noise output are

$$P_h = Gk(T_h + T_e)B_n \tag{4.19.2}$$

$$P_c = Gk(T_c + T_e)B_n \tag{4.19.3}$$