4–4 S-PARAMETERS AND SIGNAL FLOW GRAPHS

Any linear multiport network may be characterized by a set of coefficients known as *S-parameters*.[8] These coefficients are the elements of the scattering matrix described in Appendix D. For a two-port network, Eq. (D–8) reduces to

$$\begin{aligned} \mathbf{b}_1 &= S_{11}\mathbf{a}_1 + S_{12}\mathbf{a}_2 \\ \mathbf{b}_2 &= S_{21}\mathbf{a}_1 + S_{22}\mathbf{a}_2 \end{aligned} \tag{4–65}$$

where the incident (**a**) and scattered (**b**) waves are shown in Fig. 4–39 and defined in Eqs. (D–1), (D–2), and (D–7). With $Z_{01} = Z_{02} = Z_0$,

$$\begin{aligned} \mathbf{a}_k &\equiv \frac{\mathbf{V}_k^+}{\sqrt{Z_0}} = \frac{1}{2}\left(\frac{\mathbf{V}_k}{\sqrt{Z_0}} + \sqrt{Z_0}\,\mathbf{I}_k\right) \\ \mathbf{b}_k &\equiv \frac{\mathbf{V}_k^-}{\sqrt{Z_0}} = \frac{1}{2}\left(\frac{\mathbf{V}_k}{\sqrt{Z_0}} - \sqrt{Z_0}\,\mathbf{I}_k\right) \end{aligned} \tag{4–66}$$

where $\mathbf{V}_k$ and $\mathbf{I}_k$ represent the terminal voltage and current at port k. $\mathbf{V}_k^+$ and $\mathbf{V}_k^-$, on the other hand, are the incident and scattered voltage waves at port k. The S-parameters in Eq. (4–65) represent the reflection and transmission coefficients *when the network is match terminated.*[9] They are defined as

$$S_{11} \equiv \left.\frac{\mathbf{b}_1}{\mathbf{a}_1}\right|_{\mathbf{a}_2=0} = \begin{array}{l}\text{Input Reflection Coefficient} \\ \text{with } Z_L = Z_0.\end{array} \tag{4–67}$$

$$S_{22} \equiv \left.\frac{\mathbf{b}_2}{\mathbf{a}_2}\right|_{\mathbf{a}_1=0} = \begin{array}{l}\text{Output Reflection Coefficient} \\ \text{with } Z_G = Z_0 \text{ and } V_G = 0\end{array} \tag{4–68}$$

$$S_{21} \equiv \left.\frac{\mathbf{b}_2}{\mathbf{a}_1}\right|_{\mathbf{a}_2=0} = \begin{array}{l}\text{Forward Transmission Coefficient} \\ \text{with } Z_L = Z_0\end{array} \tag{4–69}$$

$$S_{12} \equiv \left.\frac{\mathbf{b}_1}{\mathbf{a}_2}\right|_{\mathbf{a}_1=0} = \begin{array}{l}\text{Reverse Transmission Coefficient} \\ \text{with } Z_G = Z_0 \text{ and } V_G = 0.\end{array} \tag{4–70}$$

Since $\mathbf{a}_k$ and $\mathbf{b}_k$ in Eq. (4–66) represent rms-phasor quantities, the power flow associated with the incident and scattered waves are given by

$$\begin{aligned} P_1^+ &= a_1^2 = \text{Power incident on the input port.} \\ P_2^+ &= a_2^2 = \text{Power incident on the output port.} \\ P_1^- &= b_1^2 = \text{Power reflected from the input port.} \\ P_2^- &= b_2^2 = \text{Power emanating from the output port.} \end{aligned} \tag{4–71}$$

where a_1, a_2, b_1, and b_2 are respectively the rms values of $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{b}_1$, and $\mathbf{b}_2$. For the circuit in Fig. 4–39, a_2^2 equals the power reflected by the load Z_L, while b_2^2 is the

[8] Networks may also be characterized in terms of their Z and Y matrices. The conversion between these matrices and the scattering matrix are given in Refs. 4–13 and 4–26.

[9] S-parameters are usually referenced to the impedance of the connecting lines. In this text, all S-parameter values and $\mathcal{T}$ matrix elements are referenced to Z_0, the characteristics impedance of the input and output lines.

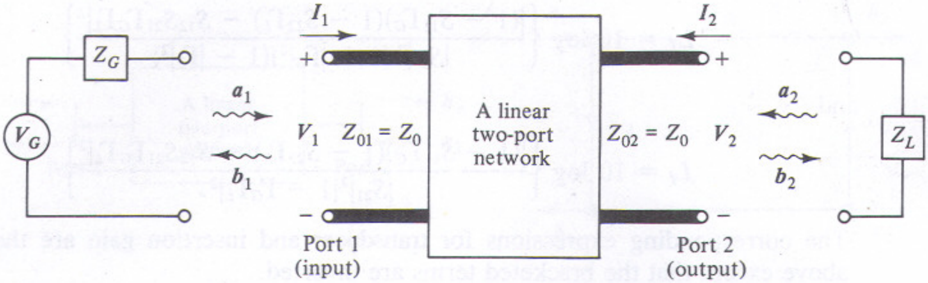

Figure 4–39 A linear two-port network and its associated input and output quantities.

power incident on the load. Thus with $\Gamma_L \equiv \mathbf{a}_2/\mathbf{b}_2$, $a_2^2 = |\Gamma_L|^2\, b_2^2$ and the net power delivered to the load is

$$P_L = b_2^2 - a_2^2 = (1 - |\Gamma_L|^2)b_2^2$$

Similarly, the net input power is

$$P_{\text{in}} = a_1^2 - b_1^2 = (1 - |\Gamma_{\text{in}}|^2)a_1^2$$

where $\Gamma_{\text{in}} \equiv \mathbf{b}_1/\mathbf{a}_1$. For a matched generator ($Z_G = Z_0$), a_1^2 represents the available generator power P_A.

Power ratios are easily measured at microwave frequencies. The following expressions relate the S-parameter magnitudes to measurable power ratios.

$$|S_{11}|^2 = \frac{\text{Power reflected from the input port}}{\text{Power incident on the input port}}\bigg|_{Z_L = Z_0} \tag{4–72}$$

$$|S_{22}|^2 = \frac{\text{Power reflected from the output port}}{\text{Power incident on the output port}}\bigg|_{\substack{Z_G = Z_0 \\ V_G = 0}} \tag{4–73}$$

$$|S_{21}|^2 = \frac{\text{Power delivered to matched load}}{\text{Power incident on the input port}}\bigg|_{Z_L = Z_0} \tag{4–74}$$

$$|S_{12}|^2 = \frac{\text{Power delivered to matched load at the input port}}{\text{Power incident on the output port}}\bigg|_{\substack{Z_G = Z_0 \\ V_G = 0}} \tag{4–75}$$

Since the incident power equals the available generator power when $Z_G = Z_0$, $|S_{21}|^2$ represents the forward transducer power gain ratio. Its reciprocal is the transducer power loss ratio. From the loss definitions given in Eqs. (4–41) and (4–47),

$$L_T = L_I = 10 \log \frac{1}{|S_{21}|^2} \tag{4–76}$$

when $Z_G = Z_L = Z_0$. For these same conditions, $|S_{12}|^2$ represents the *reverse* transducer power gain ratio and its reciprocal the power loss ratio.

For the general case (that is, arbitrary Z_G and Z_L) the forward transducer and insertion loss are related to the S-parameters by