Chapter 6

MICROWAVE NETWORK THEORY AND PASSIVE DEVICES

6.1 INTRODUCTION

A microwave network is formed when several microwave devices and components such as sources, attenuators, resonators, filters, amplifiers, etc., are coupled together by transmission lines or waveguides for the desired transmission of a microwave signal. The point of interconnection of two or more devices is called a *junction*.

For a low frequency network, a port is a pair of terminals whereas for a microwave network, a port is a reference plane transverse to the length of the microwave transmission line or waveguide. At low frequencies the physical length of the network is much smaller than the wavelength of the signal transmitted. Therefore, the measurable input and output variables are voltage and current which can be related in terms of the impedance Z-parameters, or admittance Y-parameters, or hybrid h-parameters, or $ABCD$ parameters. For a two port network as shown schematically in Fig. 6.1, these relationships are given by

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{6.1}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{6.2}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \tag{6.3}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \tag{6.4}$$

where Z_{ij}, Y_{ij}, and A, B, C and D are suitable constants that characterise the junction. A, B, C and D parameters are convenient to represent each junction when a number of circuits are connected together in cascade. Here the resultant matrix, which describes the complete cascade connection, can be obtained by multiplying the matrices describing each junction:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdots \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \tag{6.4a}$$

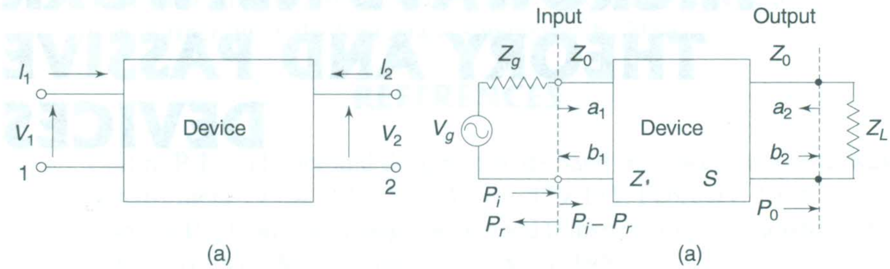

Fig. 6.1 *A two-port network*

These parameters can be measured under short or open circuit condition for use in the analysis of the circuit.

At microwave frequencies the physical length of the component or line is comparable to or much larger than the wavelength. Thus the voltage and current are not well-defined at a given point for a microwave circuit, such as a waveguide system. Furthermore, measurement of Z, Y, h and $ABCD$ parameters is difficult at microwave frequencies due to following reasons.

1. Non-availability of terminal voltage and current measuring equipment.
2. Short circuit and especially open circuit are not easily achieved for a wide range of frequencies.
3. Presence of active devices makes the circuit unstable for short or open circuit.

Therefore, microwave circuits are analysed using scattering or S-parameters which linearly relate the reflected waves' amplitude with those of incident waves. However, many of the circuit analysis techniques and circuit properties that are valid at low frequencies are also valid for microwave circuits. Thus, for circuit analysis S-parameters can be related to the Z or Y or $ABCD$ parameters. The properties of the parameters are described in the following sections.

6.2 SYMMETRICAL *Z* AND *Y* MATRICES FOR RECIPROCAL NETWORK

In a reciprocal network, the impedance and the admittance matrices are symmetrical and the junction media are characterised by scalar electrical parameters μ and ε. For a multiport (N ports) network, let the incident wave amplitudes V_n^+ be so chosen that the total voltage $V_n = V_n^+ + V_n^- = 0$ at all ports $n = 1, 2, \ldots, N$, except the ith port where the fields are $\mathbf{E}_i$, $\mathbf{H}_i$. Similarly, let $V_n = 0$ at all ports except jth one where the fields are $\mathbf{E}_j$, $\mathbf{H}_j$. Then from the Lorentz reciprocity theorem

$$\int_S (\mathbf{E}_i \times \mathbf{H}_j - \mathbf{E}_j \times \mathbf{H}_i) \cdot \mathrm{d}S = 0 \tag{6.5}$$

where S is the closed surface area of the conducting walls enclosing the junction and N ports in the absence of any source. Since the integral over the perfectly conducting walls vanishes, the only non-zero integrals are those taken over the reference planes of the corresponding ports, so that

$$\sum_{n=1}^{N} \int_{t_n} (\mathbf{E}_i \times \mathbf{H}_j - \mathbf{E}_j \times \mathbf{H}_i) \cdot dS = 0 \tag{6.6}$$

Since all V_n except V_i and V_j are zero, $\mathbf{E}_{ti} = \mathbf{n} \times \mathbf{E}_i$ and $\mathbf{E}_{tj} = \mathbf{n} \times \mathbf{E}_j$ are zero on all reference planes at the corresponding ports except t_i and t_j, respectively. Therefore, Eq. 6.6 reduces to

$$\int_{t_i} (\mathbf{E}_i \times \mathbf{H}_j) \cdot dS = \int_{t_j} (\mathbf{E}_j \times \mathbf{H}_i) \cdot dS \tag{6.7}$$

or, $$P_{ij} = P_{ji} \tag{6.8}$$

where P_{ij} represents the power at reference plane i due to an input voltage at plane j.

From the admittance matrix representation $[I] = [Y][V]$ and power relation $P = V\,I$, Eq. 6.8 reduces to

$$V_i\, V_j\, Y_{ij} = V_j\, V_i\, Y_{ji}$$

or, $$Y_{ij} = Y_{ji} \tag{6.9}$$

and $$Z_{ij} = Z_{ji} \tag{6.10}$$

This proves that the impedance and admittance matrices are symmetrical for a reciprocal junction.

6.3 SCATTERING OR *S*-MATRIX REPRESENTATION OF MULTIPORT NETWORK

As discussed in Sec. 6.1 the incident and reflected amplitudes of microwaves at any port are used to characterise a microwave circuit. The amplitudes are normalised in such a way that the square of any of these variables gives the average power in that wave in the following manner

Input power at the nth port, $$P_{in} = 1/2\ |a_n|^2 \tag{6.11}$$

Reflected power at the nth port, $$P_{rn} = 1/2\ |b_n|^2 \tag{6.12}$$

where a_n and b_n represent the normalised incident wave amplitude and normalised reflected wave amplitude at the nth port.

In a two-port network we can express the normalised waves by.

$$a_1 = \frac{V_{i1}}{\sqrt{Z_0}} = \frac{V_1 + V_{r1}}{\sqrt{Z_0}}, a_2 = \frac{V_{i2}}{\sqrt{Z_0}} = \frac{V_2 - V_{r2}}{\sqrt{Z_0}} \quad (6.13)$$

$$b_1 = \frac{V_{r1}}{\sqrt{Z_0}} = \frac{V_1 - V_{i1}}{\sqrt{Z_0}}, b_2 = \frac{V_{r2}}{\sqrt{Z_0}} = \frac{V_2 - V_{i2}}{\sqrt{Z_0}} \quad (6.14)$$

where a's represents normalised incident wave and b's represent normalised reflected wave at the corresponding ports. Here the total voltage wave is the sum of incident and reflected voltage waves V_i and V_r, respectively

$$V_1 = V_{i1} + V_{r1} \quad (6.15)$$

$$V_2 = V_{i2} + V_{r2} \quad (6.16)$$

The numeric suffices represent the port number. The total or net power flow into any port is given by

$$P = P_i - P_r = 1/2\,(|a|^2 - |b|^2) \quad (6.17)$$

Therefore, in this normalisation process, the characteristic impedance is normalised to unity. For a two-port network (Fig. 6.1) the relation between incident and reflected waves are expressed in terms of scattering paramters S_{ij}'s

$$b_1 = S_{11}\, a_1 + S_{12}\, a_2 \quad (6.18)$$

$$b_2 = S_{21}\, a_1 + S_{22}\, a_2 \quad (6.19)$$

The normalisation process leads to a symmetrical scattering matrix for reciprocal structures. The physical significance of S-parameters can be described as follows:

$S_{11} = b_1/a_1 \mid a_2 = 0$ = reflection coefficient Γ_1 at port 1 when port 2 in terminated with a matched load ($a_2 = 0$)

$S_{22} = b_2/a_2 \mid a_1 = 0$ = reflection coefficient Γ_2 at port 2 when port 1 in terminated with a matched load ($a_1 = 0$)

$S_{12} = b_1/a_2 \mid a_1 = 0$ = attenuation of wave travelling from port 2 to port 1

$S_{21} = b_2/a_1 \mid a_2 = 0$ = attenuation of wave travelling from port 1 to port 2.

In general, since the incident and reflected waves have both amplitude and phase, the S-parameters are complex numbers.

For a multiport (N) networks or components, the S-parameters equations are expressed by

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ \vdots \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ \vdots \\ a_N \end{bmatrix} \quad (6.20)$$

In microwave devices or circuits it is important to express several losses in terms of S-parameters when the ports are matched terminated. In a two port network if power fed at port 1 is P_i, power reflected at the same port is P_r and the output power at port 2 is P_o, then following losses are defined in terms of S-parameters

$$\text{Insertion loss (dB)} = 10 \log \frac{P_i}{P_o} = 10 \log \frac{|a_1|^2}{|b_2|^2}$$

$$= 20 \log \frac{1}{|S_{21}|}$$

$$= 20 \log \frac{1}{|S_{12}|} \tag{6.21}$$

$$\text{Transmission loss or attenuation (dB)} = 10 \log \frac{P_i - P_r}{P_o}$$

$$= 10 \log \frac{1 - |S_{11}|^2}{|S_{12}|^2} \tag{6.22}$$

$$\text{Reflection loss (dB)} = 10 \log \frac{P_i}{P_i - P_r}$$

$$= 10 \log \frac{1}{1 - |S_{11}|^2} \tag{6.23}$$

$$\text{Return loss (dB)} = 10 \log P_i / P_r$$

$$= 20 \log \frac{1}{|\Gamma|}$$

$$= 20 \log \frac{1}{|S_{11}|} \tag{6.24}$$

6.3.1 Properties of S-Parameters

In general the scattering parameters are complex quantities having the following properties for different characteristics of the microwave network.

(a) Zero diagonal elements for perfect matched network

For an ideal N-port network with matched termination, $S_{ii} = 0$, since there is no reflection from any port. Therefore, under perfect matched conditions the diagonal elements of $[S]$ are zero.

(b) Symmetry of $[S]$ for a reciprocal network

A reciprocal device has the same transmission characteristics in either direction of a pair of ports and is characterised by a symmetric scattering matrix,

$$S_{ij} = S_{ji} \ (i \neq j) \tag{6.25}$$