RF and Antenna Engineering (CT 425)

1st In-Semester Examination

Closed Books and Closed Notes Examination

Date: 14 September 2022 **Time: 8:30am to 10:30 am**

Answer all questions.

1. (a) What RF stands for? Define RF in terms of type of waves and frequency range.

RF= Radiofrequency (1 mark)
Type of wave of RF = Electromagnetic wave (1 mark)
Frequency range = 3 kHz to 300 GHz ((1 mark)

(b) Why uniform plane wave is called transverse electromagnetic (TEM) wave? Define Poynting vector in terms and electric field intensity (E) and magnetic field intensity (H). What is represented by magnitude and direction of Poynting vector?

Uniform plane wave is TEM because electric field and magnetic field are perpendicular (1 mark) and both lie in plane perpendicular to the direction of propagation (1 marks).
$\vec{\mathrm{P}} = \vec{\mathrm{E}} \times \vec{\mathrm{H}}$ **or Poynting vector is cross product of electric and magnetic fields (1 mark)**
***M*agnitude and direction of Poynting Vector represent power density and direction of power flow. (1 mark)**

(c) An electromagnetic wave of frequency 300 MHz travels in the +z direction in an infinite, lossless medium having ε_R (relative permittivity) = 9 and μ_R (relative permeability) =1. The value of electric field is 100 volts/meter. Calculate phase constant (β), intrinsic impedance (η) and average power density (p_z).

$\beta = 2\pi / \lambda$, $\lambda = \lambda_o/\sqrt{\varepsilon\mu}$,
$\lambda_o = 3 \times 10^8 / 300 \times 10^6 = 1$ **meter,** $\lambda = 0.33$ **meter,** $\beta = 19.04$ **radians/meter (1 mark)**
$\eta = 377 / \sqrt{9} = 41.88$ **ohms (1.5 marks)**
$p_z = E^2 / 2\eta = 100^2 / (2 \times 41.88) = 118.388$ **W/m²** **(1.5 marks)**

(d) Define polarization of an electromagnetic wave. Define and explain (by using equations and figures) linear polarizations (horizontal and vertical) and circular polarizations (left-hand and right-hand) of electromagnetic waves. Use IEEE

(2)

notations to explain clockwise or counterclockwise rotation of the circular polarizations. (8 mark)

Polarization is direction of electric field in radiated electromagnetic wave or direction of the line traced out by the tip of the electric field vector determines the *polarization* of the wave. (1 mark)

(e) What is the frequency ranges of UHF and SHF bands?

UHF = 0.3 to 3 GHz (1 mark)
SHF = 3 to 30 GHz (1 mark)

2. (a) What is guiding surface waves in ground wave propagation? What is the highest frequency for ground wave propagation? (3 mark)

Surface waves are guided by surface of the earth. (1.5 marks)
Highest frequency for ground wave propagation is 3 MHz (1.5 marks)

(b) What is the frequency range of sky wave propagation? State the layer of ionosphere which gives day time as well night time coverage. (4 marks)

Frequency range of Sky Wave propagation = 3 to 30 MHz (2 marks)
Layer of ionosphere for day and night coverages = F layer (2 marks)

(c) For radio wave propagation, derive the fundamental equation for free-space transmission (Friis equation) relating transmitted power (P_T), received power (P_R), transmitting antenna gain (G_T), receiving antenna gain (G_R), distance (d) and wavelength (λ). (6 marks)

(d) Why radio horizon is greater than optical horizon in line-of-sight (space) radio propagation system?

Bending of radio waves due to curvature of earth. So, radio horizon > optical horizon. (3 marks)

(e) In a satellite communication system, free-space conditions are assumed. The satellite is at a height of 36000 km and the frequency used is 6000 MHz. Calculate free-space transmission loss.

Free-space loss (L) = 32.5 + 20 $\log_{10}$ (d in kilometer) and 20 $\log_{10}$ (f in MHz) = 32.5 +20 log (36000) + 20 log (4000) = 32.5 + 91.1 + 72.04 = 195. 6 dB (4 marks)

3. (a) State relationships of Z, Y, h and ABCD parameters in terms on V_1, V_2, I_1 and I_2 for two-port networks. At RF frequencies, why Z, Y, h and ABCD parameters are difficult to measure? (8 marks)

(b) State relationship between scattered wave amplitudes (b_1 and b_2) and incident wave amplitudes (a_1 and a_2) in a two-port RF network. Define all 4 S-parameters in terms of scattered and incident amplitudes with termination conditions at input and output. (8 marks)

(c) For dissipationless networks of 3-port network, what conditions must be satisfied?

For scattering matrix, what is the necessary condition of N-port dissipationless network for any column p from 1 to N?

$\sum_{n=1}^{N} |S_{np}|^2 = 1$ **(2 marks)**

When p and q represent different columns of the scattering matrix, what is the constraining necessary condition? Assume N-port dissipationless network.

$\sum_{n=1}^{N} S_{np} S_{nq}^{*} = 0$ **(2 marks)**

4. (a) In a transmission line, what is the relationship between SWR and magnitude of reflection coefficient ($|\Gamma|$)?

SWR = (1 + $|\Gamma|$) / (1 - $|\Gamma|$) (4 marks)

(b) Define SWR in terms of voltage maximum (V_{max}) and voltage minimum (V_{min}) of standing wave pattern in a transmission line.

SWR = V_{max} / V_{min} (3 marks)

(c) What are the advantages of using Smith chart as compared to calculations in transmission line theory?

(i) **Reduction in computational effort for calculations involving transmission lines** **(2 marks)**

(ii) **Improved intuitive understanding of transmission line problems.** **(2 marks)**

(d) For transmission line parameters (R', L', G' and C'), assume ω is the radian frequency. State condition for low-loss lines in terms of transmission line parameters and ω.

$R' \ll \omega L'$ **(1.5 marks)**
$G' \ll \omega C'$ **(1.5 marks)**

(e) In Smith chart, there are two scales wavelength towards generator (wtg) and wavelength towards load (wtl). These scales show distances on transmission line in terms of wavelengths. Why these scales are from 0 to 0.5 wavelength?

Wtg and wtl scales in Smith chart is from 0 to 0.5 wavelengths because one complete revolution around Smith chart is a half wavelength. **(1 mark)**

$2\,\beta\, d = 2\,\pi$, So, $d = \lambda / 2$ **(2 marks)**

(f) In Smith chart, draw sketches of polar chart of reflection constant values in terms of normalized impedances, constant resistance circles and constant reactance lines in Smith chart. (3 marks)

1(d) 5

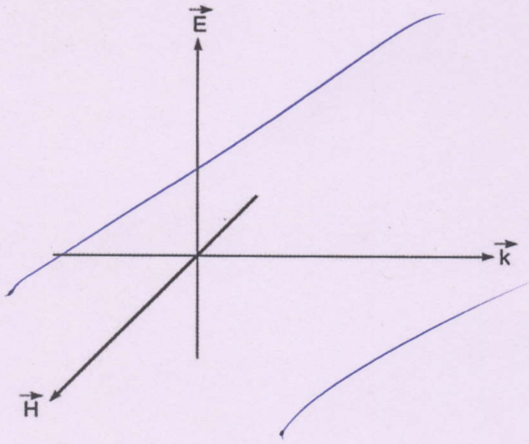

Figure 5.1 Vector diagram for a transverse electromagnetic (TEM) wave.

to consider the electric field in this discussion. The tip of the **E** vector may trace out a straight line, in which case the polarization is referred to as *linear.* Other forms of polarization, specifically elliptical and circular, will be introduced later.

In the early days of radio, there was little chance of ambiguity in specifying the direction of polarization in relation to the surface of the earth. Most transmissions utilized linear polarization and were along terrestrial paths. Thus *vertical polarization* meant that the electric field was perpendicular to the earth's surface, and *horizontal polarization* meant that it was parallel to the earth's surface. Although the terms vertical and horizontal are used with satellite transmissions, the situation is not quite so clear. A linear polarized wave transmitted by a geostationary satellite may be designated vertical if its electric field is parallel to the earth's polar axis, but even so the electric field will be parallel to the earth at the equator. This situation will be clarified shortly.

Suppose for the moment that horizontal and vertical are taken as the x and y axes of a right-hand set, as shown in Fig. 5.2*a*. A vertically polarized electric field can be described as

$$\mathbf{E}_y = \hat{a}_y E_y \sin \omega t \tag{5.1}$$

1

where $\hat{a}_y$ is the unit vector in the vertical direction and E_y is the *peak value* or *amplitude* of the electric field. Likewise, a horizontally polarized wave could be described by

$$\mathbf{E}_x = \hat{a}_x E_x \sin \omega t \tag{5.2}$$

1

These two fields would trace out the straight lines shown in Fig. 5.2*b*. Now consider the situation where both fields are present simultaneously.

1(d)

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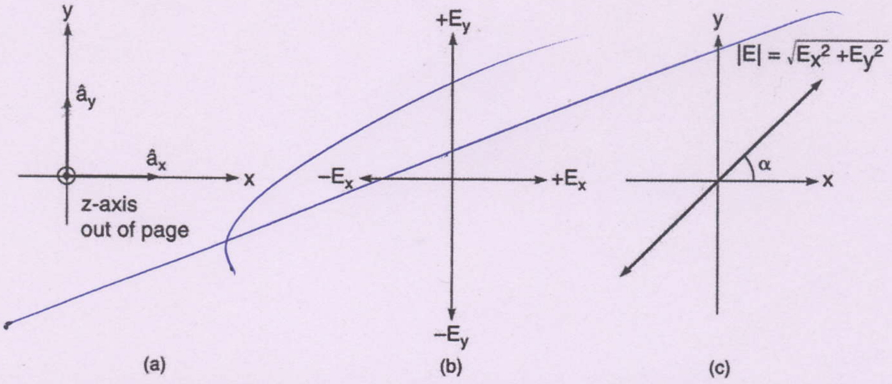

Figure 5.2 Horizontal and vertical components of linear polarization.

These would add vectorially, and the resultant would be a vector **E** (Fig. 5.2*c*) of amplitude $\sqrt{E_x^2 + E_y^2}$, at an angle to the horizontal given by

$$\alpha = \arctan \frac{E_y}{E_x} \tag{5.3}$$

E varies sinusoidally in time in the same manner as the individual components. It is still linearly polarized but cannot be classified as simply horizontal or vertical. Arguing back from this, it is evident that **E** can be resolved into vertical and horizontal components, a fact which is of great importance in practical transmission systems. The power in the resultant wave is proportional to the voltage $\sqrt{E_x^2 + E_y^2}$, squared, which is $E_x^2 + E_y^2$. In other words, the power in the resultant wave is the sum of the powers in the individual waves, which is to be expected.

More formally, $\mathbf{E}_y$ and $\mathbf{E}_x$ are said to be *orthogonal*. The dictionary definition of orthogonal is at *right angles*, but a wider meaning will be attached to the word later.

Consider now the situation where the two fields are equal in amplitude (denoted by E), but one leads the other by 90° in phase. The equations describing these are Okay.

$$\mathbf{E}_y = \hat{a}_y E \sin \omega t \tag{5.4a}$$

$$\mathbf{E}_x = \hat{a}_x E \cos \omega t \tag{5.4b}$$

1

Applying Eq. (5.3) in this case yields $\alpha = \omega t$. The tip of the resultant electric field vector traces out a circle, as shown in Fig. 5.3*a*, and the resultant wave is said to be *circularly polarized*. The amplitude of the resultant vector is E. The resultant field in this case does not go through zero. At $\omega t = 0$, the y component is zero and the x component is E. At

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1 (a)

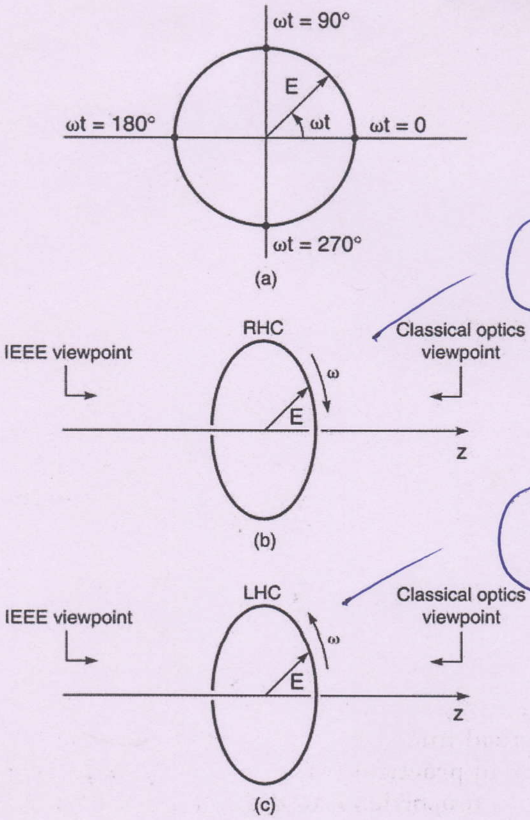

Figure 5.3 Circular polarization.

$\omega t = 90°$, the y component is E and the x component is zero. Compare this with the linear polarized case where at $\omega t = 0$, both the x and y components are zero, and at $\omega t = 90°$, both components are maximum at E. Because the resultant does not vary in time, the power must be found by adding the powers in the two linear polarized, sinusoidal waves. This gives a resultant proportional to $2E^2$.

The direction of circular polarization is defined by the sense of rotation of the electric vector, but this also requires that the way the vector is viewed must be specified. The *Institute of Electrical and Electronics Engineers* (IEEE) defines *right-hand circular* (RHC) *polarization* as a rotation in the clockwise direction when the wave is viewed along the direction of propagation, that is, when viewed from "behind," as shown in Fig. 5.3*b*. *Left-hand circular* (LHC) *polarization* is when the rotation is in the counterclockwise direction when viewed along the direction of propagation, as shown in Fig. 5.3*c*. LHC and RHC polarizations are orthogonal. The direction of propagation is along the $+z$ axis.

1 (a)

8

As a caution it should be noted that the classical optics definition of circular polarization is just the opposite of the IEEE definition. The IEEE definition will be used throughout this text.

For a right-hand set of axes (Fig. 5.1) and with propagation along the $+z$ axis, then when viewed along the direction of propagation (from "behind") and with the $+y$ axis directed upward, the $+x$ axis will be directed toward the left. Consider now Eq. (5.4). At $\omega t = 0$, E_y is 0 and E_x is a maximum at E along the $+x$ axis. At $\omega t = 90°$, E_x is zero and E_y is a maximum at E along the $+y$ axis. In other words, the resultant field of amplitude E has rotated from the $+x$ axis to the $+y$ axis, which is a clockwise rotation when viewed along the direction of propagation. Equation (5.4) therefore represents RHC polarization.

Given that Eq. (5.4) represents RHC polarization, it is left as an exercise to show that the following equations represent LHC polarization:

$$\mathbf{E}_y = \hat{a}_y E \sin\omega t \qquad (5.5a)$$

$$\mathbf{E}_x = -\hat{a}_x E \cos\omega t \qquad (5.5b)$$

1

In the more general case, a wave may be *elliptically polarized*. This occurs when the two linear components are

$$\mathbf{E}_y = \hat{a}_y E_y \sin\omega t \qquad (5.6a)$$

$$\mathbf{E}_x = \hat{a}_x E_x \sin(\omega t + \delta) \qquad (5.6b)$$

Here, E_y and E_x are not equal in general, and δ is a fixed phase angle. It is left as an exercise for the student to show that when $E_y = 1$, $E_x = 1/3$, and $\delta = 30°$, the polarization ellipse is as shown in Fig. 5.4.

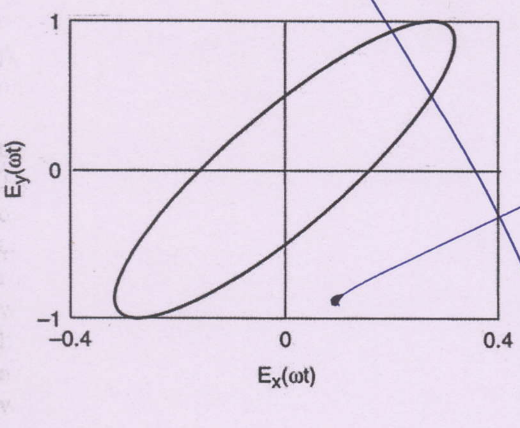

Figure 5.4 Elliptical polarization.

2 (c)

9

15

Radio-wave Propagation

15.1 Introduction

Radio communications use electromagnetic waves propagated through the earth's atmosphere or space to carry information over long distances without the use of wires. Radio waves with frequencies ranging from about 100 Hz in the ELF band to well above 300 GHz in the EHF band have been used for communications purposes, and more recently radiation in and near the visible range (near 1000 THz, or 10^{15} Hz) have also been used. Figure 15.8.1 shows the frequency-band designations in common use.

Some of the basic properties of a *transverse electromagnetic* (TEM) wave are described in Appendix B. Although the electric and magnetic fields exist simultaneously, in practice, antennas are designed to work through one or other of these fields. Antennas are described in Chapter 16. Basically, to launch an electromagnetic wave into space, an electric charge has to be accelerated, which in practice means that the current in the radiator must change with time (for example, be alternating). In this chapter, sinusoidal or cosinusoidal variations will be assumed unless stated otherwise.

15.2 Propagation in Free Space

Mode of Propagation

Consider first an average power P_T, assumed to be radiated equally in all directions (isotropically). This will spread out spherically as it travels away from the source, so that at distance d, the power density in the wave, which is the power per unit area of wavefront, will be

2 (c)

16

$$P_{Di} = \frac{P_T}{4\pi d^2} \quad \text{W/m}^2 \tag{15.2.1}$$

1

This is so because $4\pi d^2$ is the surface area of the sphere of radius d, centered on the source. P_{Di} stands for isotropic power density.

It is known that all practical antennas have directional characteristics; that is, they radiate more power in some directions at the expense of less in others. The directivity gain is the ratio of actual power density along the main axis of radiation of the antenna to that which would be produced by an isotropic antenna at the same distance fed with the same input power. Let G_T be the *maximum* directivity gain of the transmitting antenna; then the power density along the direction of maximum radiation will be

$$\begin{aligned} P_D &= P_{Di} G_T \\ &= \frac{P_T G_T}{4\pi d^2} \end{aligned} \tag{15.2.2}$$

1

A receiving antenna can be positioned so that it collects maximum power from the wave. When so positioned, let P_R be the power delivered by the antenna to the load (receiver) under matched conditions; then the antenna can be considered as having an effective area (or aperture) A_{eff}, where

$$\begin{aligned} P_R &= P_D A_{eff} \\ &= \frac{P_T G_T}{4\pi d^2} A_{eff} \end{aligned} \tag{15.2.3}$$

1

It can be shown that for any antenna, the ratio of maximum directivity gain to effective area is

$$\frac{A_{eff}}{G} = \frac{\lambda^2}{4\pi} \tag{15.2.4}$$

1

Here, λ is the wavelength of the wave being radiated. Letting G_R be the maximum directivity gain of the receiving antenna, we have

$$\frac{P_R}{P_T} = G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2 \tag{15.2.5}$$

2

This is the fundamental equation for free-space transmission. Usually it is expressed in terms of frequency f, in megahertz, and distance d, in kilometers. As shown in Appendix B, $\lambda f = c$, and on substituting this in Eq. (15.2.5) and doing the arithmetic, which is left as an exercise for the reader, the result obtained is

11

3 (a)

Chapter 6

MICROWAVE NETWORK THEORY AND PASSIVE DEVICES

6.1 INTRODUCTION

A microwave network is formed when several microwave devices and components such as sources, attenuators, resonators, filters, amplifiers, etc., are coupled together by transmission lines or waveguides for the desired transmission of a microwave signal. The point of interconnection of two or more devices is called a *junction*.

For a low frequency network, a port is a pair of terminals whereas for a microwave network, a port is a reference plane transverse to the length of the microwave transmission line or waveguide. At low frequencies the physical length of the network is much smaller than the wavelength of the signal transmitted. Therefore, the measurable input and output variables are voltage and current which can be related in terms of the impedance Z-parameters, or admittance Y-parameters, or hybrid h-parameters, or $ABCD$ parameters. For a two port network as shown schematically in Fig. 6.1, these relationships are given by

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{6.1}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{6.2}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \tag{6.3}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \tag{6.4}$$

3 (a)

12

where Z_{ij}, Y_{ij}, and A, B, C and D are suitable constants that characterise the junction. A, B, C and D parameters are convenient to represent each junction when a number of circuits are connected together in cascade. Here the resultant matrix,which describes the complete cascade connection, can be obtained by multiplying the matrices describing each junction:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdots \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \tag{6.4a}$$

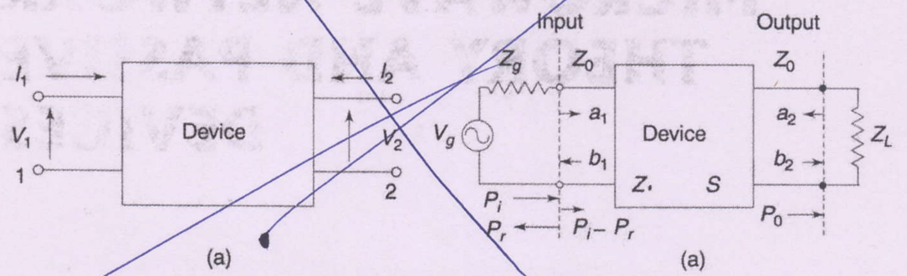

Fig. 6.1 *A two-port network*

These parameters can be measured under short or open circuit condition for use in the analysis of the circuit.

At microwave frequencies the physical length of the component or line is comparable to or much larger than the wavelength. Thus the voltage and current are not well-defined at a given point for a microwave circuit, such as a waveguide system. Furthermore, measurement of Z, Y, h and $ABCD$ parameters is difficult at microwave frequencies due to following reasons.

1. Non-availability of terminal voltage and current measuring equipment. 1
2. Short circuit and especially open circuit are not easily achieved for a wide range of frequencies. 2
3. Presence of active devices makes the circuit unstable for short or open circuit. 1

Therefore, microwave circuits are analysed using scattering or S-parameters which linearly relate the reflected waves' amplitude with those of incident waves. However, many of the circuit analysis techniques and circuit properties that are valid at low frequencies are also valid for microwave circuits. Thus, for circuit analysis S-parameters can be related to the Z or Y or $ABCD$ parameters. The properties of the parameters are described in the following sections.

6.2 SYMMETRICAL *Z* AND *Y* MATRICES FOR RECIPROCAL NETWORK

In a reciprocal network, the impedance and the admittance matrices are symmetrical and the junction media are characterised by scalar electrical parameters μ and ε. For a multiport (N ports) network, let the incident wave amplitudes V_n^+ be so chosen that the total voltage $V_n = V_n^+ + V_n^- = 0$ at all ports $n = 1, 2, \ldots, N$, except the ith port where the fields are $\mathbf{E}_i$, $\mathbf{H}_i$. Similarly, let $V_n = 0$ at all ports

3 (6)

13

4–4 S-PARAMETERS AND SIGNAL FLOW GRAPHS

Any linear multiport network may be characterized by a set of coefficients known as *S-parameters*.[8] These coefficients are the elements of the scattering matrix described in Appendix D. For a two-port network, Eq. (D–8) reduces to

$$\left.\begin{aligned}\mathbf{b}_1 &= S_{11}\mathbf{a}_1 + S_{12}\mathbf{a}_2\\ \mathbf{b}_2 &= S_{21}\mathbf{a}_1 + S_{22}\mathbf{a}_2\end{aligned}\right\} \text{— 2} \qquad (4\text{–}65)$$

where the incident (**a**) and scattered (**b**) waves are shown in Fig. 4–39 and defined in Eqs. (D–1), (D–2), and (D–7). With $Z_{01} = Z_{02} = Z_0$,

$$\mathbf{a}_k \equiv \frac{\mathbf{V}_k^+}{\sqrt{\mathbf{Z}_0}} = \frac{1}{2}\left(\frac{\mathbf{V}_k}{\sqrt{\mathbf{Z}_0}} + \sqrt{\mathbf{Z}_0}\,\mathbf{I}_k\right)$$
$$\mathbf{b}_k \equiv \frac{\mathbf{V}_k^-}{\sqrt{\mathbf{Z}_0}} = \frac{1}{2}\left(\frac{\mathbf{V}_k}{\sqrt{\mathbf{Z}_0}} - \sqrt{\mathbf{Z}_0}\,\mathbf{I}_k\right) \qquad (4\text{–}66)$$

where $\mathbf{V}_k$ and $\mathbf{I}_k$ represent the terminal voltage and current at port k. $\mathbf{V}_k^+$ and $\mathbf{V}_k^-$, on the other hand, are the incident and scattered voltage waves at port k. The S-parameters in Eq. (4–65) represent the reflection and transmission coefficients *when the network is match terminated*.[9] They are defined as

$$S_{11} \equiv \left.\frac{\mathbf{b}_1}{\mathbf{a}_1}\right|_{\mathbf{a}_2=0} = \begin{array}{l}\text{Input Reflection Coefficient}\\ \text{with } Z_L = Z_0.\end{array} \text{— 2} \qquad (4\text{–}67)$$

$$S_{22} \equiv \left.\frac{\mathbf{b}_2}{\mathbf{a}_2}\right|_{\mathbf{a}_1=0} = \begin{array}{l}\text{Output Reflection Coefficient}\\ \text{with } Z_G = Z_0 \text{ and } V_G = 0\end{array} \text{— 1.} \qquad (4\text{–}68)$$

$$S_{21} \equiv \left.\frac{\mathbf{b}_2}{\mathbf{a}_1}\right|_{\mathbf{a}_2=0} = \begin{array}{l}\text{Forward Transmission Coefficient}\\ \text{with } Z_L = Z_0\end{array} \text{— 2} \qquad (4\text{–}69)$$

$$S_{12} \equiv \left.\frac{\mathbf{b}_1}{\mathbf{a}_2}\right|_{\mathbf{a}_1=0} = \begin{array}{l}\text{Reverse Transmission Coefficient}\\ \text{with } Z_G = Z_0 \text{ and } V_G = 0.\end{array} \text{— 1} \qquad (4\text{–}70)$$

Since $\mathbf{a}_k$ and $\mathbf{b}_k$ in Eq. (4–66) represent rms-phasor quantities, the power flow associated with the incident and scattered waves are given by

$$\begin{aligned}P_1^+ &= a_1^2 = \text{Power incident on the input port.}\\ P_2^+ &= a_2^2 = \text{Power incident on the output port.}\\ P_1^- &= b_1^2 = \text{Power reflected from the input port.}\\ P_2^- &= b_2^2 = \text{Power emanating from the output port.}\end{aligned} \qquad (4\text{–}71)$$

where a_1, a_2, b_1, and b_2 are respectively the rms values of $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{b}_1$, and $\mathbf{b}_2$. For the circuit in Fig. 4–39, a_2^2 equals the power reflected by the load Z_L, while b_2^2 is the

[8] Networks may also be characterized in terms of their Z and Y matrices. The conversion between these matrices and the scattering matrix are given in Refs. 4–13 and 4–26.

[9] S-parameters are usually referenced to the impedance of the connecting lines. In this text, all S-parameter values and $\mathcal{T}$ matrix elements are referenced to Z_0, the characteristics impedance of the input and output lines.

4 (f)

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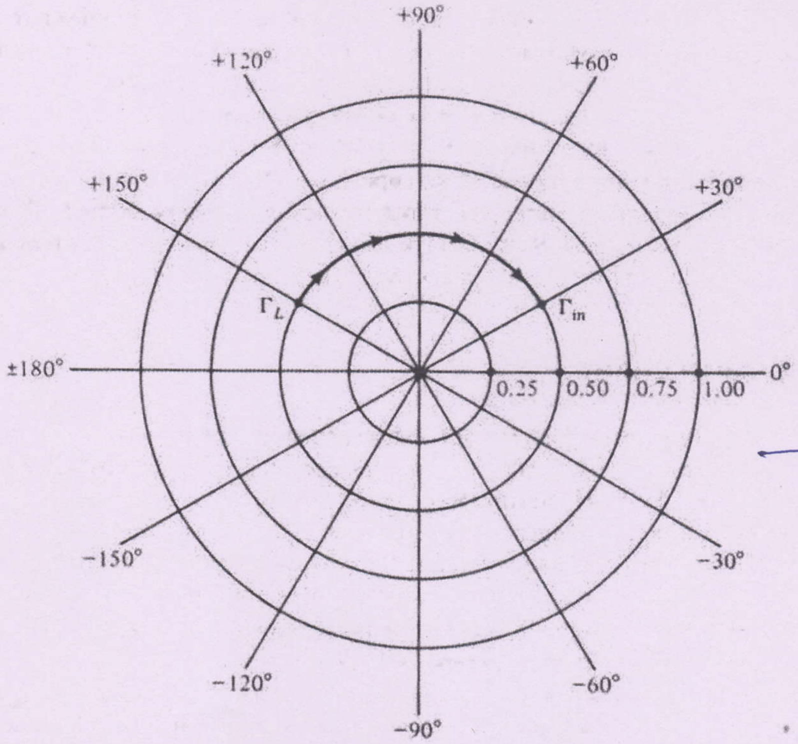


Figure 3–17 A polar chart for plotting complex reflection coefficients, $|\Gamma|\underline{/\phi}$.

1

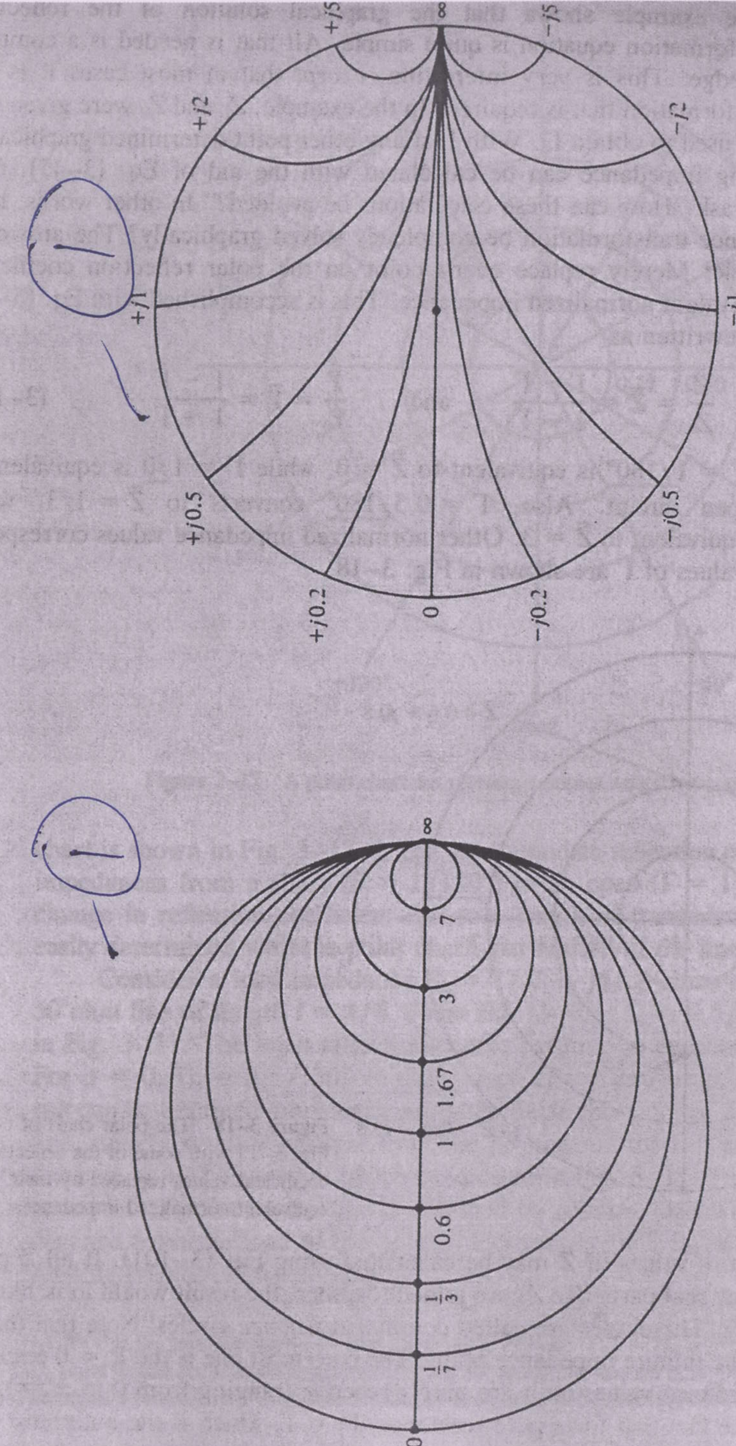
chart is shown in Fig. 3–17. It can accommodate reflection coefficient values for all impedances from a short ($\Gamma = 1\underline{/180°}$) to an open ($\Gamma = 1\underline{/0}$). Furthermore, the change in reflection coefficient due to a length of transmission line (Eq. 3–43) is easily determined with the polar chart, particularly if the line is lossless.

Consider a load impedance $Z_L = 17.7 + j11.8$ ohms connected to a lossless 50 ohm line of length $l = \lambda/6$. From Eq. (3–45), $\Gamma_L = 0.5\underline{/150°}$, which is plotted in Fig. 3–17. The input reflection coefficient may be determined from Eq. (3–43). For $\alpha = 0$, $\Gamma_{in} = 0.5\underline{/150° - 120°}$, since $2\beta l = 2\pi/3$ rad or 120°. This same result can be obtained graphically using the polar chart. Since $|\Gamma_{in}| = |\Gamma_L|$ for a lossless line, merely rotate *clockwise* from the Γ_L point on the 0.5 radius circle an angular distance of 120° (namely, $2\beta l$), as indicated in Fig. 3–17. The reflection coefficient at any other point on the line is obtained by rotating clockwise $2\beta d$ where d is the distance from the load to the point.[14] Conversely, if Γ is known, Γ_L is obtained by rotating *counterclockwise* $2\beta d$.

[14] If the line is lossy ($\alpha \neq 0$), $|\Gamma| = |\Gamma_L|e^{-2\alpha d}$ and $|\Gamma_{in}| = |\Gamma_L|e^{-2\alpha l}$. In the example given, the Γ_{in} point would occur at the intersection of the 30° radial line and the $0.5e^{-2\alpha l}$ circle. Thus the locus of Γ points as a function of d would be a spiral of decreasing radius starting at Γ_L and ending at the Γ_{in} point. For low-loss lines, the approximation $\alpha = 0$ is usually valid.

15

4 (f)



(a) Constant resistance circles

(b) Constant reactance lines

Figure 3–19 Constant resistance circles and constant reactance lines. (Note: The reactance lines are portions of circles.)

on the chart. Joining all poi
the curved lines shown in p
lines and in fact are portion
tween the zero and infinity p
nates with values ranging fr
$\bar{R} = \infty$. The impedance gri
and the constant reactance
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wavelength since for $d = \lambda/$
shown in Sec. 3–6a that imp
chart in Fig. 3–20, the uppe
impedance values have an i
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nates." The reason for this is
admittance coordinates. For
$(\bar{Y} \equiv Y/Y_0)$ is 180° away on
changing its sign) converts $\bar{Z}$
for $\bar{Z}$ and $\bar{Y}$ in Eq. (3–101).
comments should be kept in

1. The $\bar{Y} = 0$ point corres
   sponds to a short circui
2. The resistance coordina
   coordinates become sus
   ceptance value (top half
   a negative value denot
   chart.
3. When using admittance
   scale must be rotated 18
   the values on the top h
   bottom half.

[15] The proof that the locus of
circles is given in many texts. See, f