Industrial Innovations: (Edwin Mansfield)

(Study on Coal, iron and steel, brewing and railroads).

i) Total number of firms in an industry is N.
ii) $x(t) \rightarrow$ Number of firms that have adopted a technological innovation.

$\boxed{\Delta x \propto \Delta t}$ and $\boxed{\Delta x \propto (N-x)}$ $\Rightarrow$ $\boxed{\Delta x \propto (N-x)\Delta t}$

Jointly, we write $\boxed{\Delta x = \lambda (N-x) \Delta t}$,

in which $\lambda \rightarrow$ proportional factor (not constant)

$\boxed{\lambda \equiv \lambda\left(p, s, \frac{x}{N}\right)}$, in which (with N being fixed)

i) $p \rightarrow$ profitability in investing in an innovation.
ii) $s \rightarrow$ Investing ability to acquire innovation, as a percentage of the total assets.
iii) $\frac{x}{N} \rightarrow$ Percentage of firms that have already adopted the innovation.

Edwin Mansfield's Study: (To determine λ).

i) Carry out a Taylor expansion of λ about some equilibrium values of p, s and x/N, represented with a subscript c $\left(p_c, s_c, \frac{x}{N}\Big|_c\right)$.
ii) Limit the Taylor expansion only up to the second order, (i.e. orders of p^2, s^2, $\left(\frac{x}{N}\right)^2$.).
iii) Gather all the coefficients of zeroth, first and second orders.

Accordingly $\boxed{\lambda = f\left(p, s, \frac{x}{N}\right)}$ is Taylor expanded as,

$$\boxed{\begin{aligned}
\lambda = {} & f\left(p_c, s_c, \tfrac{x}{N}\Big|_c\right) \\
& + \frac{\partial f}{\partial p}\Big|_c (p - p_c) + \frac{\partial f}{\partial s}\Big|_c (s - s_c) + \frac{\partial f}{\partial (x/N)}\Big|_c \left(\frac{x}{N} - \frac{x}{N}\Big|_c\right) \\
& + \frac{1}{2!}\frac{\partial^2 f}{\partial p^2}\Big|_c (p - p_c)^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial s^2}\Big|_c (s - s_c)^2 + \frac{\partial^2 f}{\partial^2 (x/N)}\Big|_c \left(\frac{x}{N} - \frac{x}{N}\Big|_c\right)^2 \\
& + \frac{2}{2!}\frac{\partial^2 f}{\partial p \partial s}\Big|_c (p - p_c)(s - s_c) + \frac{2}{2!}\frac{\partial^2 f}{\partial p \partial (x/N)}\Big|_c (p - p_c)\left(\frac{x}{N} - \frac{x}{N}\Big|_c\right) \\
& + \frac{2}{2!}\frac{\partial^2 f}{\partial s \partial (x/N)}\Big|_c (s - s_c)\left(\frac{x}{N} - \frac{x}{N}\Big|_c\right) + \cdots
\end{aligned}}$$

In deriving the above expression we have used the mathematical principle $\boxed{\dfrac{\partial^2 \psi}{\partial x \partial y} = \dfrac{\partial^2 \psi}{\partial y \partial x}}$.

we get, on collecting ~~Collecting~~ all the terms of the same order,

$$\boxed{\begin{aligned}
\lambda = {} & a_1 + a_2 p + a_3 s + a_4\left(\frac{x}{N}\right) + a_5 p^2 + a_6 s^2 + a_7 p s \\
& + a_8 p\left(\frac{x}{N}\right) + a_9 s\left(\frac{x}{N}\right) + a_{10}\left(\frac{x}{N}\right)^2
\end{aligned}},$$

in which all a_i are constants, depending on the equilibrium values of p, s and $\frac{x}{N}$, and their derivatives.

Edwin Mansfield's study shows $\boxed{a_{10} = 0}$ and

$$\boxed{a_1 + a_2 p + a_3 s + a_5 p^2 + a_6 s^2 + a_7 p s = 0}.$$

The remaining terms $\boxed{\lambda = (a_4 + a_8 p + a_9 s)\frac{x}{N}}$

Define $\boxed{k = a_4 + a_8 p + a_9 s} \Rightarrow \boxed{\lambda = k\frac{x}{N}}$.

$\boxed{K \equiv k(p,s)}$, i.e, k depends on profitability and investing power. (k is not to be confused with the carrying capacity in the logistic equation)

$$\therefore \quad \boxed{\Delta x = k \frac{x}{N}(N-x)\Delta t} \Rightarrow \boxed{\frac{\Delta x}{\Delta t} = k\frac{x}{N}(N-x)}$$

$$\Rightarrow \boxed{\frac{dx}{dt} = k\frac{x}{N}(N-x)} \Rightarrow \boxed{\frac{d(x/N)}{dt} = k\frac{x}{N}\left(1-\frac{x}{N}\right)}$$

Define $\boxed{X = \frac{x}{N}}$ and $\boxed{T = kt}$, to get

$$\boxed{\frac{dX}{dT} = X(1-X)}$$, which is the logistic equation.

The solution is $\boxed{X = \frac{1}{1+A^{-1}e^{-T}}} \Rightarrow \boxed{x = \frac{N}{1+A^{-1}e^{-kt}}}$.

Initial condition: when $\boxed{t = t_0,\ x = 1}$.

$$\Rightarrow 1 = \frac{N}{1+A^{-1}e^{-kt_0}} \Rightarrow \boxed{A^{-1} = (N-1)e^{kt_0}}$$

$$\Rightarrow \boxed{x = \frac{N}{1+(N-1)e^{-k(t-t_0)}}}$$ The integral solution for spread of industrial innovations.

This solution was used to study:

i.) The spread of twelve innovations such as the shuttle car, trackless mobile loaders, mining machines, coke ovens, wide strip mills, etc.

ii.) Across four major industries like coal, iron and steel, brewing and railroads.