 (Example)

Problem of Sharks and Salmon

$$\boxed{\frac{dx}{dt} = ax - bx^2 - c}\ , \quad \boxed{a, b, c > 0}$$

(c is an additive constant)

$\Rightarrow \boxed{\frac{dx}{dt} = ax - bx^2 + (-c)}$. Now we already know that a system like $\boxed{\frac{dx}{dt} = ax - bx^2 + c}$ can be transformed to a form $\boxed{\frac{dy}{dt} = \alpha^2 - by^2}$, in which $\boxed{y = x - \frac{a}{2b}}$ and $\boxed{\alpha^2 = \frac{a^2}{4b} + c}$. We, thus replace all $\boxed{\text{"c" with "}-c\text{"}}$, i.e. $\boxed{c \longrightarrow -c}$, and rescale further by $\boxed{X = \frac{y}{\alpha/\sqrt{b}}}$ and $\boxed{T = \alpha\sqrt{b}\,t}$ to get $\boxed{\frac{dX}{dT} = 1 - X^2}$, whose integral solution is

$$\boxed{X = \frac{A - e^{-2T}}{A + e^{-2T}}}$$

A is an integra-tion constant

$\boxed{c = 0 \Rightarrow \text{logistic equation}}$

This is then written as,

$$\boxed{x = \frac{a}{2b} + \frac{\alpha}{\sqrt{b}}\left[\frac{A - e^{-2\alpha\sqrt{b}t}}{A + e^{-2\alpha\sqrt{b}t}}\right]}$$

When $t \to \infty$,

$$\boxed{x \longrightarrow \frac{a}{2b} + \frac{1}{\sqrt{b}} \cdot \sqrt{\frac{a^2}{4b} - c}} \Rightarrow \boxed{x \longrightarrow \frac{a}{2b}\left(1 + \sqrt{1 - \frac{4bc}{a^2}}\right)}$$

This limiting value of the population does not depend on the value of A.

(Example) Population of New York City: A Critical Case

$$\boxed{\frac{dx}{dt} = ax - bx^2 - c} \quad \boxed{a, b, c > 0}.$$

$\boxed{a \rightarrow \text{Growth parameter}. \quad b, c \rightarrow \text{decline parameters}}$

$$\boxed{a = \frac{1}{25} = 4\times10^{-2}, \quad b = \frac{1}{25\times10^{6}} = 4\times10^{-8}, \quad c = 10^{4}}$$

$$\therefore \boxed{\frac{4bc}{a^2} = \frac{4\times4\times10^{-8}\times10^{4}}{4\times4\times10^{-4}} = 1} \Rightarrow \boxed{a^2 = 4bc} \Rightarrow \boxed{\frac{a^2}{4b} - c = 0}$$

$$\Rightarrow \boxed{\alpha^2 = 0} \Rightarrow \boxed{\frac{dy}{dt} = -by^2} \Rightarrow \boxed{\int y^{-2}\,dy = -b\int dt}$$

$$\Rightarrow \boxed{-y^{-1} = -bt + \text{Constant}}. \Rightarrow \boxed{\frac{1}{y} = bt + A}$$ A is the integration constant.

$$\Rightarrow \boxed{y = \frac{1}{bt+A}} \Rightarrow \boxed{x = \frac{a}{2b} + \frac{1}{bt+A}}$$ Power-law convergence.

When $t \rightarrow \infty$, $y \rightarrow 0$, and $\boxed{x \rightarrow \frac{a}{2b}}$.

This is the limiting value of the population, $\boxed{x \rightarrow \frac{4\times10^{-2}}{2\times4\times10^{-8}}}$ $x \rightarrow 0.5$ million.

This convergence is slow as in a power-law. This happens in all critical phenomena, such as phase transitions. Power laws are also seen in gas laws, Zipf's law (GEORGE KINGSLEY ZIPF) and Pareto's law in income and wealth distributions (VILFREDO PARETO). They are SCALE-FREE.