Additional Points on the Threshold Theorem of Epidemiology

$$\boxed{x = (x_0 + y_0) - y + \frac{B}{A}\ln(y/y_0)} \quad \boxed{x \equiv x(y)}$$

i.) x has a turn (a maximum) when $\boxed{y = B/A}$.

ii.) When $y \to 0$, (i.e. $y \ll B/A$), $x \to -\infty$.

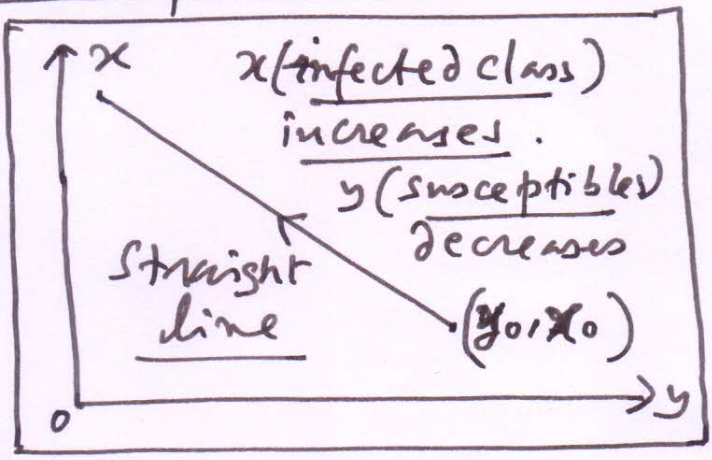
i.e. $\boxed{x \sim \frac{B}{A}\ln\left(\frac{y}{y_0}\right)}$. The logarithmic part dominates.

iii.) When $y \to \infty$ (i.e. $y \gg B/A$), then $\boxed{x \sim -y}$. The linear part dominates.

iv.) For $\boxed{B = 0}$,

$\boxed{\frac{dZ}{dt} = 0}$ (No recovered individual)

and $\boxed{x = (x_0 + y_0) - y}$.



In this case, starting at $t = 0$, all susceptibles become infected. No one recovers and no one is removed.

A Correction: $y_0 - y_\infty \simeq 2y_0\left(\frac{y_0}{\rho} - 1\right)$

Now $\boxed{y_0 = \rho + \epsilon} \Rightarrow \boxed{\frac{y_0}{\rho} - 1 = \frac{\epsilon}{\rho}}$. where $\boxed{\epsilon \ll \rho}$.

Hence, $\boxed{y_0 - y_\infty \simeq 2y_0\,\epsilon/\rho = 2(\rho + \epsilon)\,\epsilon/\rho}$.

$\Rightarrow y_0 - y_\infty \simeq \frac{2\cancel{\rho}\epsilon}{\cancel{\rho}} + \frac{2\epsilon^2}{\rho} \simeq 2\epsilon$ (neglecting ϵ^2)

$\Rightarrow \boxed{y_0 - y_\infty \simeq 2\epsilon \simeq 2(y_0 - \rho)}$ when y_0 is slightly greater than ρ