Modifications of the Logistic Equation

$$\boxed{\frac{dx}{dt} = ax - bx^2 + c} \text{ where } \boxed{a, b, c > 0}$$

(adding a constant to the right hand side)

$$\Rightarrow \frac{dx}{dt} = -\left(\sqrt{b}\,x\right)^2 + 2\sqrt{b}\,x\,\frac{a}{2\sqrt{b}} + c + \frac{a^2}{4b} - \frac{a^2}{4b}$$

$$\Rightarrow \boxed{\frac{dx}{dt} = -\left[\left(\sqrt{b}\,x\right)^2 - 2\left(\sqrt{b}\,x\right)\left(\frac{a}{2\sqrt{b}}\right) + \frac{a^2}{4b}\right] + \left(\frac{a^2}{4b} + c\right)}$$

$$\Rightarrow \boxed{\frac{dx}{dt} = \left(\frac{a^2}{4b} + c\right) - \left(\sqrt{b}\,x - \frac{a}{2\sqrt{b}}\right)^2}$$

$\rightarrow$ This term is a perfect square

$$\Rightarrow \boxed{\frac{dx}{dt} = \left(c + \frac{a^2}{4b}\right) - b\left(x - \frac{a}{2b}\right)^2}$$

define $\boxed{\alpha^2 = \frac{a^2}{4b} + c}$ and $\boxed{y = x - \frac{a}{2b}}$,

to get. $\boxed{\frac{dy}{dt} = \alpha^2 - by^2}$ $\boxed{\text{Since, } \frac{dx}{dt} = \frac{dy}{dt}}$

$$\Rightarrow \boxed{\frac{1}{\alpha^2}\frac{dy}{dt} = 1 - \frac{y^2}{\alpha^2/b}}$$

, Now define $\boxed{X = \frac{y}{\alpha/\sqrt{b}}}$

$$\Rightarrow \boxed{\frac{1}{\alpha^2}\cdot\frac{\alpha}{\sqrt{b}}\,\frac{dX}{dt} = 1 - X^2} \Rightarrow \boxed{\frac{dX}{dT} = 1 - X^2},$$

when $\boxed{T = \alpha\sqrt{b}\,t}$. The solution of this equation, as earlier is

$$\boxed{\frac{1+X}{1-X} = Ae^{2T}} \Rightarrow \boxed{X = \frac{Ae^{2T} - 1}{Ae^{2T} + 1}}$$

A is an integration constant