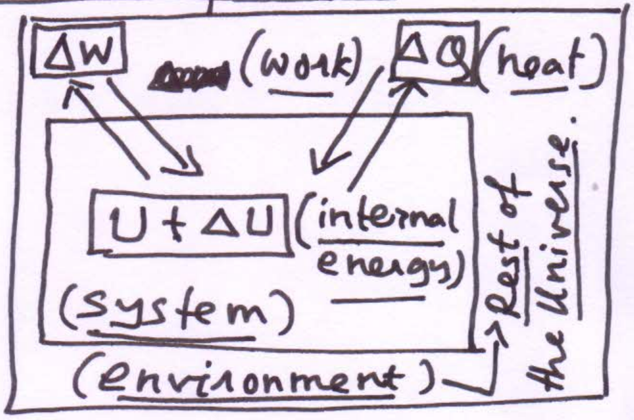
Thermodynamical Principles



1/. $\boxed{U}$ → Internal energy of the system.

2/. $\boxed{\Delta U}$ → Change in internal energy.

3/. $\boxed{\Delta W}$ → Work done. $\boxed{\Delta W > 0}$ Work done on the system, $\boxed{\Delta W < 0}$ Work done by the system.

4/. $\boxed{\Delta Q}$ → Heat exchanged between the system and the environment, due to temperature difference. $\boxed{\Delta Q > 0}$ Heat entering the system (is positive). $\boxed{\Delta Q < 0}$ Heat expelled by the system (is negative).

5/. Heat is energy in transit due to temperature difference. It is a spontaneous non-mechanical means of energy transfer.

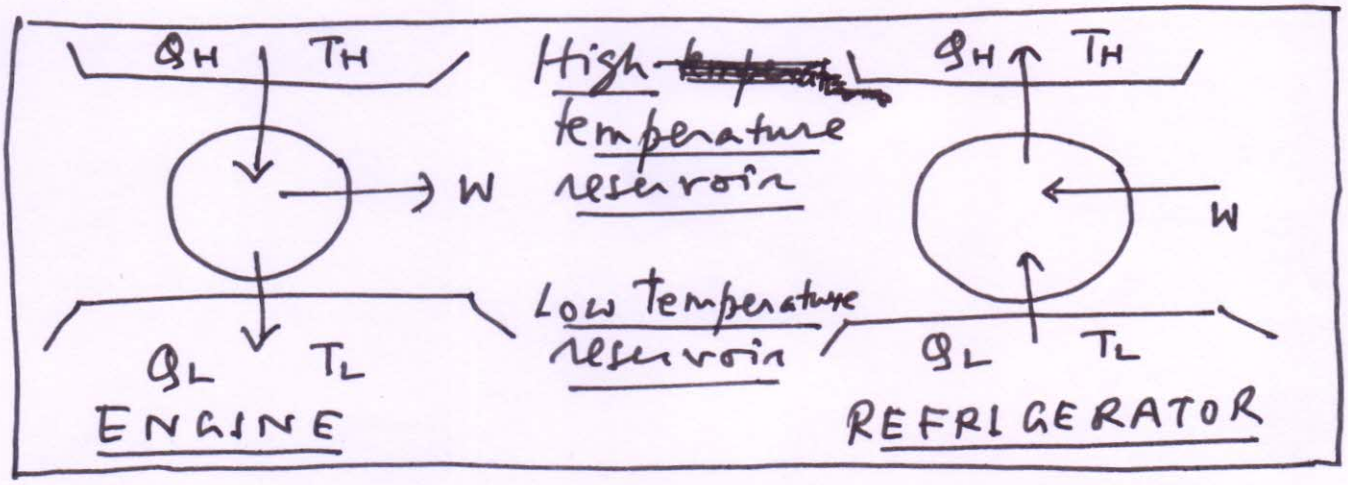
6/. Heat flow stops when the temperature of the system and that of the environment are in equilibrium. Temperature is the equilibrium quantity. Hence, heat flow restores thermal equilibrium. (Refer to the Zeroth Law of Thermodynamics).

[P.T.O.]

7/. $\boxed{\Delta U = \Delta Q + \Delta W}$ $\Rightarrow$ Energy is conserved.
(Refer to the First Law of Thermodynamics)

8/. When $\boxed{\Delta U = 0}$ and $\boxed{\Delta W > 0}$ $\Rightarrow$ $\boxed{\Delta Q = -\Delta W}$

All work done on the system is expelled as heat by the system. Hence all work can be converted to heat. However, not all heat can be converted to work.
(Refer to the Second Law of Thermodynamics)

9/. 

10/. Efficiency of an engine: $\boxed{\eta = \frac{W}{Q_H}}$

But $\boxed{Q_H = W + Q_L}$ $\Rightarrow$ $\boxed{\eta = \frac{Q_H - Q_L}{Q_H}}$ $\Rightarrow$ $\boxed{\eta = 1 - \frac{Q_L}{Q_H}}$

$\boxed{Q_L \neq 0}$ Some heat must always be rejected to the environment.

11/. For a Carnot Engine (reversible and quasi-static)

$\boxed{\frac{Q_H}{T_H} = \frac{Q_L}{T_L}}$ $\Rightarrow$ $\boxed{\frac{Q_L}{Q_H} = \frac{T_L}{T_H}}$ $\Rightarrow$ $\boxed{\eta = 1 - \frac{T_L}{T_H}}$.

(ENTROPY) A Carnot Engine is the most efficient between T_H and T_L. Entropy is unchanged.

[P.T.O.]

12/. Entropy $\boxed{S = \frac{Q}{T}}$ $\Rightarrow$ $\boxed{S_H = \frac{Q_H}{T_L} = \frac{Q_L}{T_L} = S_L}$

$\Rightarrow$ $\boxed{S_H = S_L}$ for a Carnot Engine.

A real engine is less efficient than a Carnot Engine, and expels more heat.

$\Rightarrow$ $\boxed{\frac{Q_L}{T_L} > \frac{Q_H}{T_H}}$ $\therefore$ $\boxed{S_L > S_H}$ A real engine increases entropy.

13/. $\boxed{\Delta S \geq 0}$ • Half-conserved quantity. (Equality is for a Carnot Engine).

14/. Entropy is a measure of disorder. Marks the arrow of time.

15/. Thermodynamics relates to a large aggregate system. Temperature, heat, internal energy and entropy are all measures of a large-aggregate system.

16/.

Zeroth Law $\rightarrow$ Temperature (Equilibrium)
First Law $\rightarrow$ Energy (Conservation)
Second Law $\rightarrow$ Entropy (Half-conserved)

17/. Extraction of work continuously increases the entropy of the Universe. Disorder increases.

18/. When the temperature of the Universe becomes uniform throughout $\Rightarrow$ $\boxed{\text{HEAT DEATH}}$

Compartment Models and Diagrams

Inputs into and (or) outputs from a compartment (over time).

1/. **Input and output:** The Atmospheric CO_2

Input CO_2 → [atmosphere] → Output CO_2

2/. **Only output:** Radioactivity / Radiator

[Radioactive material (U-238)] → α-particles

[Heat radiator] → Energy

3/. **Only input:** Greenhouse Effect (Atmosphere)

Heat → [Atmosphere] (Negligible (Small) heat dissipation compared to heat input.)

Advanced Examples (More than one compartment)

1/. Radioactive Series: (Art forgery case) (3 compartments)

[Ra-226 (radioactive)] → (1600 years) → [Pb-210 Radioactive] → (22 years) → [Pb-206 Non-radioactive.]

2/. Drug Administration: (Single Dose or a Course)

(Ingestion) → [GI tract] → (Digestion) → [Blood] → (To tissues)

(2 compartments)

Additive and Multiplicative Growth (or Decay)

Consider $\boxed{\frac{dx}{dt} = a_0 + a_1 x + a_2 x^2 + \cdots}$ (series)

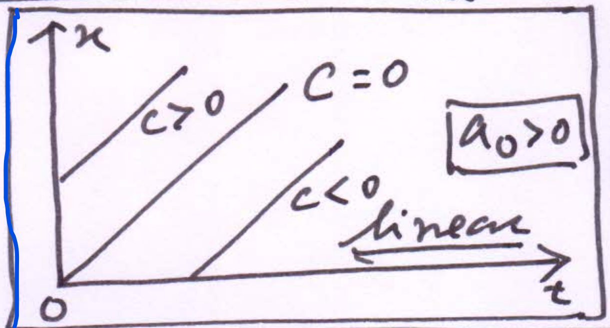
1/. **Additive Growth:** $\boxed{\frac{dx}{dt} = a_0}$ → only the zero order.

$\Rightarrow \boxed{x = a_0 t + c}$ when at $\boxed{t = 0,\ x = 0}$ ($\Rightarrow c = 0$),

$\Rightarrow \boxed{\frac{dx}{dt} \neq 0}$ $\therefore$ for $\boxed{a_0 > 0}$ growth can still happen even when $\boxed{x = 0 \text{ at } t = 0}$. ($\because \frac{dx}{dt} > 0$)

This is additive growth (or arithmetic growth)

For $\boxed{a_0 < 0}$, there is decay.



2/. **Multiplicative Growth:** $\boxed{\frac{dx}{dt} = a_1 x}$ → only the first order

When $\boxed{x = 0}$, at $\boxed{t = 0}$ $\Rightarrow \boxed{\frac{dx}{dt} = 0}$ $\therefore$ The system will remain $\boxed{\text{static}}$ at $\boxed{x = 0}$ for all $\boxed{t > 0}$ thereafter $\Rightarrow$ Trivial Solution.

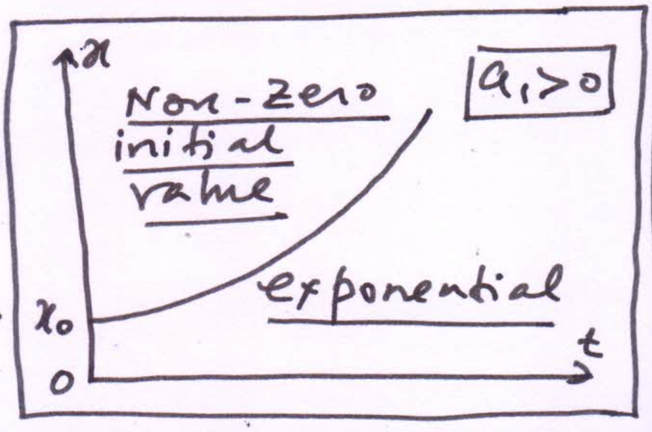
For growth, $\boxed{a_1 > 0}$ and $\boxed{x \neq 0}$ at $\boxed{t = 0}$

$\Rightarrow \boxed{x = x_0 e^{a_1 t}}$ when $\boxed{t = 0}$, $\boxed{x = x_0 > 0}$

This is multiplicative growth (or geometric growth).

If $\boxed{x_0 = 0}$ there will be No growth.

For $\boxed{a_1 < 0}$, there is decay.



Discrete Changes and Continuous Differentia-bility

1/. Population size changes in a discrete step size of unity (± 1) (integer steps)

2/. If a population size is x, and it changes (grows) by Δx, then the per capita growth (relative growth) is $\boxed{\frac{\Delta x}{x}}$.

3/. The per capita growth rate is $\boxed{\frac{1}{x}\frac{\Delta x}{\Delta t}}$, in which Δt is the time taken for the growth.

4/. Now, when $\boxed{\Delta x \to 0}$ we replace the discrete change by a continuous differential, i.e. $\boxed{\Delta x \to dx}$.

5/ However, $\boxed{|\Delta x| = 1}$. Hence, we look now at the ~~largeness~~ largeness of x. If $\boxed{x \to \infty}$ then $\boxed{\frac{\Delta x}{x} \to 0}$ because $\boxed{\Delta x \ll x}$.

The discrete Δx can be replaced by dx due to the large background value of x.

6/. $\Rightarrow \boxed{\frac{1}{x}\frac{\Delta x}{\Delta t} \equiv \frac{1}{x}\frac{dx}{dt}}$ by which x is now continuously differentiable with respect to t (approximation is valid for large x)