Taylor Expansion in Multiple Variables

I/. One variable: $\boxed{f \equiv f(x)}$ expanded about $\boxed{x = x_c}$:

$$\Rightarrow \boxed{f = f(x_c) + \left.\frac{df}{dx}\right|_{x_c}(x - x_c) + \frac{1}{2!}\left.\frac{d^2f}{dx^2}\right|_{x_c}(x - x_c)^2 + \cdots}$$

II/. Two Variables: $\boxed{f \equiv f(x, y)}$ about $\boxed{(x_c, y_c)}$.

$$\Rightarrow \boxed{\begin{aligned} f =\ & f(x_c, y_c) \longrightarrow \text{1 zero-order term } (2^0) \\ & + \left.\frac{\partial f}{\partial x}\right|_{x_c, y_c}(x - x_c) + \left.\frac{\partial f}{\partial y}\right|_{x_c, y_c}(y - y_c) \longrightarrow \text{2 first-order terms } (2^1) \\ & + \frac{1}{2!}\left.\frac{\partial^2 f}{\partial x^2}\right|_{x_c, y_c}(x - x_c)^2 + \frac{1}{2!}\left.\frac{\partial^2 f}{\partial y \partial x}\right|_{x_c, y_c}(x - x_c)(y - y_c) \\ & + \frac{1}{2!}\left.\frac{\partial^2 f}{\partial x \partial y}\right|_{x_c, y_c}(y - y_c)(x - x_c) + \frac{1}{2!}\left.\frac{\partial^2 f}{\partial y^2}\right|_{x_c, y_c}(y - y_c)^2 + \cdots \\ & \longrightarrow \text{4 second-order terms } (2^2) \end{aligned}}$$

III/. Three Variables: $\boxed{f \equiv f(x, y, z)}$ about $\boxed{(x_c, y_c, z_c)}$.

$$\boxed{\begin{aligned} \Rightarrow f =\ & f(x_c, y_c, z_c) \longrightarrow \text{1 zero-order term } (3^0) \\ & + \left.\frac{\partial f}{\partial x}\right|_{x_c, y_c, z_c}(x - x_c) + \left.\frac{\partial f}{\partial y}\right|_{x_c, y_c, z_c}(y - y_c) + \left.\frac{\partial f}{\partial z}\right|_{x_c, y_c, z_c}(z - z_c) \longrightarrow \text{3 first-order terms } (3^1) \\ & + \frac{1}{2!}\left.\frac{\partial^2 f}{\partial x^2}\right|_{x_c, y_c, z_c}(x - x_c)^2 + \frac{1}{2!}\left.\frac{\partial^2 f}{\partial y^2}\right|_{x_c, y_c, z_c}(y - y_c)^2 + \frac{1}{2!}\left.\frac{\partial^2 f}{\partial z^2}\right|_{x_c, y_c, z_c}(z - z_c)^2 \\ & + \frac{2}{2!}\left.\frac{\partial^2 f}{\partial x \partial y}\right|_{x_c, y_c, z_c}(x - x_c)(y - y_c) + \frac{2}{2!}\left.\frac{\partial^2 f}{\partial y \partial z}\right|_{x_c, y_c, z_c}(y - y_c)(z - z_c) \\ & + \frac{2}{2!}\left.\frac{\partial^2 f}{\partial z \partial x}\right|_{x_c, y_c, z_c}(z - z_c)(x - x_c) + \cdots \longrightarrow \text{9 second-order terms } (3^2), \\ & \text{with 6 mixed terms.} \end{aligned}}$$

Additional Discussions on the Spread of Industrial Innovations (E. Mansfield)

$\boxed{\lambda = f\left(p, s, \frac{x}{N}\right)}$. Following a Taylor expansion we are able to write $\boxed{\lambda = (a_4 + a_8 p + a_9 s)\frac{x}{N}}$.

In λ, we have p and s as variables.

Writing $\boxed{\lambda = k(x/N)}$, where $\boxed{k = a_4 + a_8 p + a_9 s}$, we use it in $\boxed{\frac{dx}{dt} = k\frac{x}{N}(N - x)}$. In this equation, $\boxed{k = k(p, s)}$ has p and s as free parameters, with their values fixed at the beginning.

Nonlinear Time Scale in Mansfield's Equation

Given $\boxed{x = \frac{N}{1 + (N-1)e^{-k(t-t_0)}}}$, which is the solution of the logistic equation, we set $\boxed{x = N/2}$, the scale of nonlinearity in time, $\boxed{(t - t_0)|_{nl}}$.

$$\therefore \boxed{\frac{N}{2} = \frac{N}{1 + (N-1)e^{-k(t-t_0)|_{nl}}}} \Rightarrow 2 = 1 + (N-1)e^{-k(t-t_0)|_{nl}}$$

$$\Rightarrow \boxed{(N-1)\, e^{-k(t-t_0)|_{nl}} = 1} \Rightarrow \boxed{(N-1) = e^{k(t-t_0)|_{nl}}}.$$

$$\therefore k(t-t_0)|_{nl} = \ln(N-1) \Rightarrow \boxed{(t-t_0)|_{nl} = \frac{1}{k}\ln(N-1)}.$$

The nonlinear time.