The Law of Proportionate Effect | Robert Gibrat

In discrete time $t = 1, 2, 3, \ldots$ (units), at a given time, t, the change in a variable, x, is a random fraction of a function of x in the previous instant. $\Rightarrow$ $\boxed{x_t - x_{t-1} = \epsilon_t f(x_{t-1})}$.

i/ $\epsilon_t \rightarrow$ Random and a fraction. $(|\epsilon_t| < 1)$

ii/ ϵ_t are the outcomes of uncorrelated random process and do not depend on x.

If $\boxed{f(x) = x}$ $\rightarrow$ Law of Proportionate Effect (Robert Gibrat).

$$\therefore \boxed{x_t - x_{t-1} = \epsilon_t x_{t-1}} \Rightarrow \boxed{\Delta x = \epsilon_t x}$$

$$\Rightarrow \boxed{\frac{\Delta x}{x} = \epsilon_t} \Rightarrow \boxed{\Delta(\ln x) = \epsilon_t}$$

In the continuum limit $\boxed{d(\ln x) = \epsilon_t}$

Since ϵ_t is random $\ln x$ is the random variable.

[P.T.O.]

Also $\boxed{x_t = x_{t-1} + \epsilon_t x_{t-1}}$ Initially $\boxed{x = x_0}$

$\Rightarrow$ $\boxed{x_t = x_{t-1}(1+\epsilon_t)}$ $\therefore$ $\boxed{x_1 = x_0(1+\epsilon_1)}$

Similarly $\boxed{x_2 = x_1(1+\epsilon_2) = x_0(1+\epsilon_1)(1+\epsilon_2)}$

Likewise $\boxed{x_n = x_0(1+\epsilon_1)(1+\epsilon_2)\cdots(1+\epsilon_n)}$

$$\Rightarrow \boxed{\ln x_n = \ln x_0 + \sum_{i=1}^{n} \ln(1+\epsilon_i)}.$$

In a short time interval, $\boxed{|\epsilon_i| << 1}$.

$\because$ $\boxed{\ln(1+z) \simeq z}$ when $z << 1$, we get

$$\boxed{\ln x_n \simeq \ln x_0 + \sum_{i=1}^{n} \epsilon_i}$$

ϵ_i is random $\Rightarrow$ $\ln x_n$ is random

1/. The random walk is in $\ln x$. This is a multiplicative random walk. There can be negative stock prices in an additive random walk.

2/. On short time steps, this distribution shows larger fluctuations than in a Gaussian.

3/. Used to study distribution of companies by size, bulk of personal income distribution, and data of agriculture, commerce, and industries in metallurgy, explosives, electrochemicals.