Drug (Medicine) Dosage (Compartment Model)

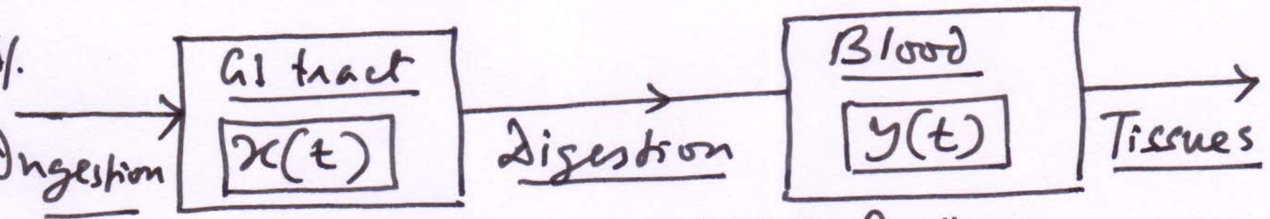
Single Dose Administration

1/. A single pill is ingested in the gastro-intestinal tract (GI tract).

2/. The drug is dissolved in the GI tract, and is then diffused into the blood stream.

3/. From the blood stream the drug is then absorbed into the tissues.

4/.



Compartment Modelling

i.) $\boxed{x(t)}$ is the amount of drug in the GI tract.

ii.) $\boxed{y(t)}$ is the amount of drug in the blood.

iii.) Equations: $\boxed{\dfrac{dx}{dt} = -k_1 x}$, $\boxed{x(0) = x_0}$ and

$\boxed{\dfrac{dy}{dt} = k_1 x - k_2 y}$, $\boxed{y(0) = 0}$ $\boxed{k_1, k_2 > 0}$ are the rate constants.

iv.) Solution of $x(t)$: $\displaystyle\int \frac{dx}{x} = -k_1 \int dt \Rightarrow \boxed{\ln x = -k_1 t + A_1}$

$A_1 \to$ Integration Constant.

when $\boxed{t = 0,\ x = x_0}$ (initial amount of drug)

$\Rightarrow \boxed{A_1 = \ln x_0} \Rightarrow \boxed{x = x_0 e^{-k_1 t}} \to$ Exponential decay

(P.T.O.)

v.) <u>Solution of $y(t)$</u> : $\boxed{\dfrac{dy}{dt} + k_2 y = k_1 x}$

<u>By the method of integrating factors</u> $e^{k_2 t}\dfrac{dy}{dt} + k_2 y e^{k_2 t} = k_1 x \times e^{k_2 t}$

$$\Rightarrow \boxed{\frac{d}{dt}\left(ye^{k_2 t}\right) = k_1 x e^{k_2 t}} \Rightarrow \boxed{ye^{k_2 t} = \int k_1 x e^{k_2 t} dt}$$

Now $\boxed{x = x_0 e^{-k_1 t}}$ $\therefore\ ye^{k_2 t} = k_1 x_0 \int e^{(k_2 - k_1)t} dt$

$$\Rightarrow \boxed{ye^{k_2 t} = \frac{k_1 x_0}{k_2 - k_1} e^{(k_2 - k_1)t} + A_2}$$ $A_2 \rightarrow$ <u>Integration Constant</u>

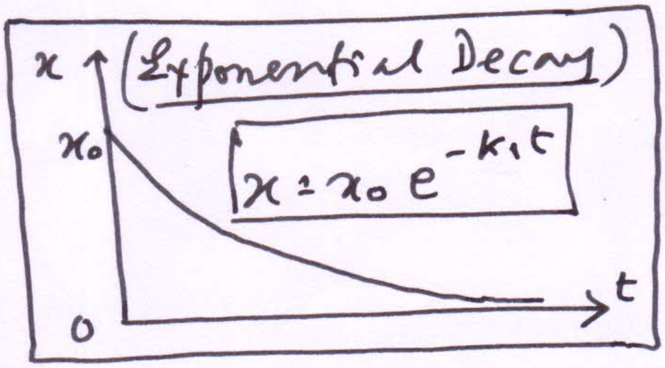
When $\boxed{t = 0,\ y = 0}$ $\Rightarrow$ $\boxed{A_2 = -\dfrac{k_1 x_0}{k_2 - k_1}}$

$$\Rightarrow \boxed{y = \frac{k_1 x_0}{k_2 - k_1}\left(e^{-k_1 t} - e^{-k_2 t}\right)}$$

1.) <u>When $t = 0$, $y = 0$</u>
2.) <u>When $t \to \infty$ $y \to 0$</u>.

vi.) <u>Plotting of $x(t)$</u> :

The <u>drug dissolves</u> and is <u>diffused from</u> the <u>GI tract</u> according to an <u>exponential decay</u>.



vii.) <u>Plotting of $y(t)$</u> : 1.) When $\boxed{t > 0,\ y > 0}$.

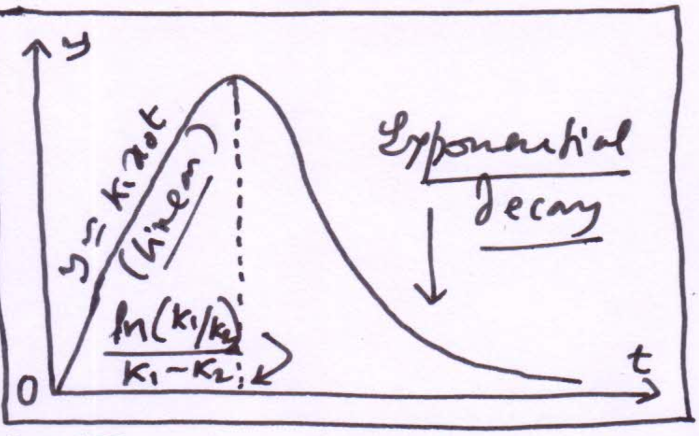
2.) When $\boxed{t = 0,\ y = 0}$, 3.) When $\boxed{t \to \infty,\ y \to 0}$.

4.) $\boxed{\dfrac{dy}{dt} = \dfrac{k_1 x_0}{k_2 - k_1}\left(-k_1 e^{-k_1 t} + k_2 e^{-k_2 t}\right)}$ <u>Turning Point</u> $\downarrow$

When $\boxed{\dfrac{dy}{dt} = 0}$ $\Rightarrow$ $\boxed{\dfrac{k_1}{k_2} = e^{(k_1 - k_2)t}}$ $\Rightarrow$ $\boxed{t = \dfrac{\ln(k_1/k_2)}{k_1 - k_2}}$

(P.T.O.)

vii.) Plotting of $y(t)$:



5.) $\frac{d^2y}{dt^2} = k_1 \frac{dx}{dt} - k_2 \frac{dy}{dt}$

When $\boxed{\frac{dy}{dt} = 0}$ (turning point)

$$\Rightarrow \boxed{\frac{d^2y}{dt^2} = k_1 \frac{dx}{dt} = -k_1^2 x < 0}$$

→ Turning point is maximum.

6.) When $t \to 0$, $y(t) = \frac{k_1 x_0}{k_2 - k_1}\left(\cancel{1} - k_1 t + \cdots - \cancel{1} + k_2 t + \cdots\right)$

$$\Rightarrow y(t) \simeq \frac{k_1 x_0}{\cancel{(k_2 - k_1)}}\cancel{(k_2 - k_1)}\, t \Rightarrow \boxed{y(t) \simeq k_1 x_0 t}$$

⇒ Early growth is linear.

Special Case: $\boxed{k_1 = k_2 = k}$

$$y(t) = \frac{k_1 x_0}{k_2 - k_1}\left(e^{-k_1 t} - e^{-k_2 t}\right) = \frac{k_1 x_0 e^{-k_2 t}}{(k_2 - k_1)} \times \left[e^{(k_2 - k_1)t} - 1\right]$$

Now $\boxed{k_1 \to k_2 = k}$ $\Rightarrow \boxed{(k_2 - k_1) \to 0}$

$$\therefore \boxed{e^{(k_2 - k_1)t} = 1 + (k_2 - k_1)t + \frac{(k_2 - k_1)^2}{2!}t^2 + \cdots}$$

$$\Rightarrow \boxed{e^{(k_2 - k_1)t} - 1 \simeq (k_2 - k_1)t}$$ (linear approximation)

$$\Rightarrow y(t) \simeq \frac{k_1 x_0}{\cancel{(k_2 - k_1)}} e^{-k_2 t} \times \cancel{(k_2 - k_1)}\, t \simeq k_1 x_0 e^{-k_2 t}\, t$$

Since $\boxed{k_1 \to k_2 = k}$ $\boxed{y(t) = k x_0 t e^{-kt}}$

The turning point: $\boxed{t = \frac{1}{k_2}\frac{\ln[1 + (k_1/k_2) - 1]}{[(k_1/k_2) - 1]} \simeq \frac{1}{k_2}\frac{\cancel{(k_1/k_2) - 1}}{\cancel{(k_1/k_2) - 1}} \simeq \frac{1}{k}}$

Solutions for Equal Rate Constants

$\boxed{k_1 = k_2 = k}$ $\Rightarrow$ $\boxed{\frac{dx}{dt} = -kx}$ $\boxed{x(0) = x_0}$

$\Rightarrow$ $\boxed{x = x_0 e^{-kt}}$ $\rightarrow$ Exponential Decay (as before).

$\boxed{\frac{dy}{dt} = kx - ky}$ $\boxed{y(0) = 0}$ $\Rightarrow$ $\boxed{\frac{dy}{dt} + ky = kx}$

$\Rightarrow e^{kt}\frac{dy}{dt} + kye^{kt} = kx_0 \cancel{e^{-kt}}\,\cancel{e^{kt}} = kx_0$

$\Rightarrow$ $\boxed{\frac{d}{dt}\left(e^{kt}y\right) = kx_0}$ $\Rightarrow$ $\boxed{ye^{kt} = kx_0 t + C}$

$C \rightarrow$ Integration constant.

When $\boxed{t = 0,\ y = 0}$ $\Rightarrow$ $\boxed{C = 0}$

$\Rightarrow$ $\boxed{y(t) = kx_0 t e^{-kt}}$ (As known for the case of $k_1 \rightarrow k_2$)

Plotting of $y(t)$: 1/. $\boxed{y = 0 \text{ at } t = 0}$.

2/. $\boxed{t \rightarrow \infty,\ y \rightarrow 0}$ since $\boxed{y(t) = \frac{kx_0 t}{e^{kt}}}$

3/. When $\boxed{t \rightarrow 0}$, $\boxed{y(t) \simeq kx_0 t\left[1 - kt + \cdots\right] \simeq kx_0 t}$

(turning point) (early growth is linear)

4/. $\boxed{\frac{dy}{dt} = k(x - y) = 0}$

$\Rightarrow$ $\boxed{kx_0(1 - kt)e^{-kt} = 0}$.

$\Rightarrow$ $\boxed{t = 1/k}$ (As known when $k_1 \rightarrow k_2$)

5/. $\boxed{\frac{d^2y}{dt^2} = k\frac{dx}{dt} - k\frac{dy}{dt} = -k^2x < 0}$

(maximum)

6/. At $\boxed{t = 1/k}$ $\boxed{y = x_0/e}$