Logistic Modelling of IBM's Economic Data

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This lab is about the effectiveness of the logistic function modeling of the given economic systems which is of IBM.The given data is based on the company data that is from IBM. We also analyse the behaviour which follows the power law.

I. INTRODUCTION

Logistic Equation and Dynamical Systems: Here the logistic equation is a fundamental model for first-order autonomous nonlinear dynamical systems and is used in field like socio-economics research. The mathematical representation of such system is given as $\frac{dx}{dt} = f(x)$, where $f(x)$ is a function determining the system's behavior. Below is the basic nonlinear function with a and b being fixed parameters, leading to the logistic equation as:

$$\frac{dx}{dt} = f(x) = ax - bx^2 \tag{1}$$

Now taking the initial condition of x(0) = x0, and $k = a/b$, this generates the integral solution of Eq.(1) as

$$x(t) = \frac{kx_0 e^{at}}{k + x_0(e^{at} - 1)} \tag{2}$$

Now using the logistic equation (1), the non-linear time scale is obtained by taking the derivative of Eq.(1) as

$$t_{nl} = \frac{1}{a} \ln\left(\frac{k}{x_0} - 1\right) \tag{3}$$

For the revenue R measured in US dollars and time t in years, the logistic model for revenue growth is given by:

$$\frac{dR}{dt} = R(R) = \rho_1 R - \rho_2 R^2 \tag{4}$$

Here, the parameters a and b in the logistic equation is written as ρ_1 and ρ_2 respectively, with $K_R = \rho_1/\rho_2$.

Similarly, for the Human Resource equation, logistic equation takes form as $H(t)$, and with a and b used as η_1 and η_2 respectively, also $K_H = \eta_1/\eta_2$. the logistic model for human resource growth is given by:

$$\frac{dH}{dt} = H(H) = \eta_1 H - \eta_2 H^2 \tag{5}$$

Now we analyze the correlated growth of R and H,which is a coupled autonomous dynamical system and is given by $\frac{dR}{dt} = R(H, R)$ and $\frac{dH}{dt} = H(H, R)$. By defining $U = H^{-1} - k^{-1}H$, $V = R^{-1} - k_R^{-1}H$, and $\beta = \rho/\eta_1$, the $H - R$ phase solutions are transformed to a compact power-law form as:

$$V \sim U^\beta \tag{6}$$

II. STATICAL ANALYSIS

Annual Revenue Growth

- Mean: 0.025

- Standard Deviation: 0.4870

Human Resource Growth

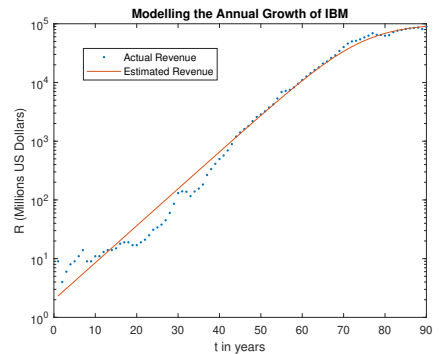- Mean: 0.0901

- Standard Deviation: 0.2980

III. RESULTS

FIG. 1: The given data is from 1914 to 2006. The parameter value of $\rho = 0.145 year^{-1}$ and $K_R = 100 billion$ The saturation of time scale starts from 75-80 years.

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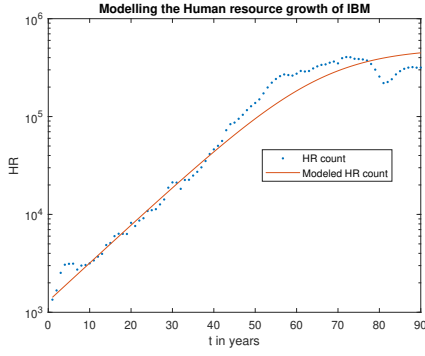[†]Electronic address: 202101522@daiict.ac.in

FIG. 2: Modeling the human resource growth of IBM, using the company data from 1914 to 2006 , the smooth dotted curve represents the logistic function given by Eq. (5). The parameter values chosen to fit the human resource growth are $\eta_1 = 0.09\,\text{year}^{-1}$ and $k_H = 500000$ The human resource graph exhibits a decline around 75-80 years.
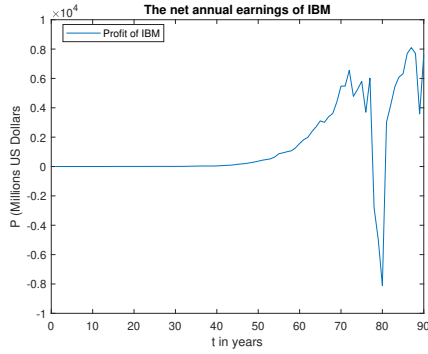

FIG. 3: The net annual earnings of IBM is a steady growth until in range 75-80 years, . During this period, IBM experienced significant losses in its net earnings, amounting to 8 billion in 1993 which is observed to be around 75-80 years.This suggests nonlinear saturation phase in revenue growth for IBM.
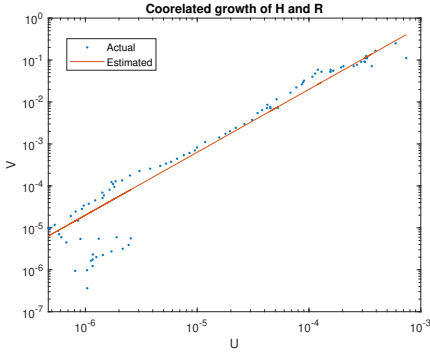

FIG. 4: From Eq.(6) the correlated growth of H and R with $\beta = 1.5$ (close to $\beta = \frac{\rho_1}{\eta_1} \approx 1.6$), the cusp observed in the data points at the bottom left is attributed to human resource loss around 75-80 years.

IV. CONCLUSIONS

The applicability of Logistic Equation is preferred mathematical tool for modelling the evolution of socio-economic system, despite being originally designed for the understanding the population dynamics founds to be effective in long term economic systems.

– There is saturation observed in the graph of annual revenue and human resource count around $10^1 1$ dollars which is decreased to 60 billion dollars according to year 2022.

– When IBM experienced loss, it happened around the same time that they also lost a significant number of employees. This shows a connection between the company's profits and its workforce.

– From Eq.(6) we decipher that annual revenue is proportional to human resource with slope $\beta \cong 1.6$

[1] Arnab K. Ray, *Logistic modeling of economic dynamics* (September 6, 2023).