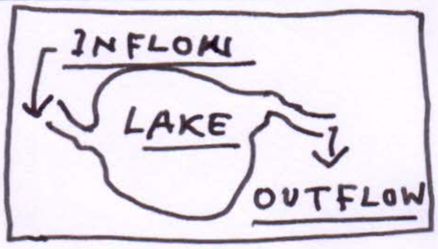
A Lake Pollution Model

Assumptions: 1/. Lake has constant volume.
2/. Pollution is uniform throughout (well-mixed)



i.) Amount of pollutant in the lake is $\boxed{M(t)}$ (mass).

ii.) If the lake has a constant volume V, then pollutant concentration is $\boxed{M(t)/V}$

iii.) To maintain constant volume, both inflow and outflow rate is F (constant). (volume per time).

iv.) The pollutant concentration in the inflow is C_{in}. Hence, amount of pollutant entering the lake is $\boxed{FC_{in}}$ (mass per unit time)

v.) After uniform mixing the amount of pollutant leaving the lake is $\boxed{F\,M(t)/V}$ (mass per unit time).

vi.) The dynamic balance is written as

$$\boxed{\frac{dM}{dt} = FC_{in} - F\frac{M(t)}{V}}$$

F and V are fixed (constants)

vii.) $\boxed{\text{Mass} = \text{Concentration} \times \text{volume}}$

$\Rightarrow$ $\boxed{M(t) = c(t)\,V}$. Dividing by V, gives

$$\boxed{\frac{d(M/V)}{dt} = \frac{F}{V}C_{in} - \frac{F}{V}(M/V)}$$

$$\Rightarrow \boxed{\frac{dc}{dt} = \frac{F}{V}C_{in} - \frac{Fc}{V}}$$

Dynamic equation in terms of concentration.

(P.T.O.)

viii)

Integral Solution : $\boxed{\frac{dc}{dt} = \frac{F}{V}C_{in} - \frac{F}{V}c}$

$\Rightarrow \boxed{\int \frac{dc}{C_{in} - c} = \frac{F}{V}\int dt}$ Separation of variables

In the form $\boxed{\dot{x} = a - bx}$

$\Rightarrow \int \frac{d(-c)}{C_{in} - c} = -\frac{F}{V}\int dt$

$\Rightarrow \boxed{\ln(C_{in} - c) = -\frac{Ft}{V} + A}$ A is the integration constant

When $\boxed{t = 0, \ c = C_0}$ (initial pollutant concentration in the lake).

$\Rightarrow \boxed{A = \ln(C_{in} - C_0)} \Rightarrow \boxed{C_{in} - c = (C_{in} - C_0)\, e^{-Ft/V}}$

Hence, $\boxed{C = C_{in} - (C_{in} - C_0)\exp\left(-\frac{Ft}{V}\right)}$.

i.) When $\boxed{t = 0, \ C = C_0}$, ii.) When $\boxed{t \to \infty, \ C \to C_{in}}$.

$\boxed{C(t) = C_0\, e^{-Ft/V} + \left(C_{in} - C_{in}\, e^{-Ft/V}\right)}$

iii.) The term $\boxed{C_0\, e^{-Ft/V}}$ is the contribution to the pollution from the initial condition.

iv.) The term $\boxed{C_{in}\left(1 - e^{-Ft/V}\right)}$ is the contribution to the pollution from the inflow.

v.) If $\boxed{C_{in} = 0}$, $\Rightarrow \boxed{C = C_0\, e^{-Ft/V}}$. Hence, pollution in the lake will reduce with time.

vi) Time taken for ~~[illegible]~~ this process is $\boxed{t = \frac{V}{F}\ln\left(\frac{C_0}{C}\right)}$.