Attitude Control of an Underactuated Satellite Using Two Reaction Wheels

A. Dynamic Model

The dynamic model is given by the Euler's rotational equation, in the body frame,

$$\dot{\boldsymbol{L}} + \boldsymbol{\omega} \times \boldsymbol{L} = 0 \tag{1}$$

where $\boldsymbol{\omega}$ is the angular velocity of the body, and the total angular momentum $\boldsymbol{L}$ for a satellite equipped with reaction wheels is given by

$$\boldsymbol{L} = I\boldsymbol{\omega} + \boldsymbol{h} \tag{2}$$

where I is the inertial matrix, and $\boldsymbol{h}$ is the angular momentum of the wheels.

Substituting Eq. (2) into Eq. (1), the dynamic model is given by

$$I\dot{\omega} = -\boldsymbol{\omega} \times (I\boldsymbol{\omega} + \boldsymbol{h}) + \boldsymbol{u} \tag{3}$$

where $\boldsymbol{u} = -\dot{\boldsymbol{h}}$ is the torque applied by the wheels to the satellite.

If the satellite is assembled with the wheels placed on the satellite's principal axes, $I = \mathrm{diag}\{I_1, I_2, I_3\}$. Also, consider a total failure, without loss of generality, of the wheel placed on the third coordinate, yielding to $\dot{h}_3 = 0$. According to [10], any equilibrium orientation can only be achieved under the zero total angular momentum condition

$$\boldsymbol{I}\boldsymbol{\omega} + \boldsymbol{h} = 0 \tag{4}$$

Since I is a diagonal matrix, it follows that $h_3 = -I_3\omega_3$. However, h_3 cannot change because $\dot{h}_3 = 0$. Then, for a three-axes stabilization, $\omega_3 = 0$ and $h_3 = 0$. Hence, the dynamic model is given by

$$\begin{cases} I_1\dot{\omega}_1 = u_1, \\ I_2\dot{\omega}_2 = u_2, \\ I_3\dot{\omega}_3 = 0 \end{cases}$$

As we have studied the attitude Kinematics using Quaternions in AE642, we got to a result that is as follows:-

$$\begin{cases} \dot{\boldsymbol{q}} = -\frac{1}{2}\boldsymbol{\omega} \times \boldsymbol{q} + \frac{1}{2}q_4\boldsymbol{\omega}, \\ \dot{q}_4 = -\frac{1}{2}\boldsymbol{\omega}^T\boldsymbol{q} \end{cases}$$

Expanding this Vector equation, we get:-

$$\dot{q}_1 = \frac{1}{2}\omega_1 q_4 - \frac{1}{2}\omega_2 q_3$$

$$\dot{q}_2 = \frac{1}{2}\omega_1 q_3 + \frac{1}{2}\omega_2 q_4$$

$$\dot{q}_3 = \frac{1}{2}\omega_2 q_1 - \frac{1}{2}\omega_1 q_2$$

$$\dot{q}_4 = -\frac{1}{2}\omega_1 q_1 - \frac{1}{2}\omega_2 q_2$$

Now, on basis of above equations, Horri and Palmer proposed a controller:-

$$\omega_{d1} = -kq_1 + g\,\frac{q_2 q_3}{q_1^2 + q_2^2}$$

$$\omega_{d2} = -kq_2 - g\,\frac{q_1 q_3}{q_1^2 + q_2^2}$$

Here, k and g are positive constants.

When $q_1 \to 0$ and $q_2 \to 0$, the fraction terms may reach high values. Therefore, for practical implementation, a saturated form of the controller in Eq. (8) can be used, as given by

$$\begin{aligned} \omega_{d1} &= -kq_1 + g\mathrm{sat}\left(\frac{q_2 q_3}{q_1^2 + q_2^2}, a_1\right), \\ \omega_{d2} &= -kq_2 - g\mathrm{sat}\left(\frac{q_1 q_3}{q_1^2 + q_2^2}, a_2\right) \end{aligned} \tag{9}$$

where $\mathrm{sat}(x, a)$ is the saturation function with the upper limit given by a and the lower limit given by $-a$.

Throughout the simulations, the SNLC will be used as a benchmark controller.

Quaternion-Based Nonlinear Controller

The previous controller has terms that relate the nonactuated axis and the controlled ones. Intending to improve the time response of this controller, a control law is proposed called the quaternion-based nonlinear controller (QBNC), which deals with the relations between the actuated axes coordinates and the time derivative of the unactuated axis by the following:

$$\omega_{d1} = -kq_1 + g\frac{q_2 q_3}{q_1^2 + q_2^2} - k_D\frac{q_1 \dot{q}_3}{q_1^2 + q_2^2}$$

$$\omega_{d2} = -kq_2 - g\frac{q_1 q_3}{q_1^2 + q_2^2} - k_D\frac{q_2 \dot{q}_3}{q_1^2 + q_2^2}$$

Now we can substitute this w_desired in the above attitude kinematic equations:-

$$\begin{aligned}
\dot{q}_3 &= \frac{1}{2}\omega_{d2}q_1 - \frac{1}{2}\omega_{d1}q_2, \\
&= \frac{1}{2}\left(-kq_2 - g\frac{q_1 q_3}{q_1^2 + q_2^2} - k_D\frac{q_2 \dot{q}_3}{q_1^2 + q_2^2}\right)q_1 \\
&\quad -\frac{1}{2}\left(-kq_1 + g\frac{q_2 q_3}{q_1^2 + q_2^2} - k_D\frac{q_1 \dot{q}_3}{q_1^2 + q_2^2}\right)q_2, \\
&= \frac{1}{2}\left(-kq_2 q_1 + kq_1 q_2 - g\frac{q_1^2 q_3}{q_1^2 + q_2^2} - g\frac{q_2^2 q_3}{q_1^2 + q_2^2}\right. \\
&\quad \left. - k_D\frac{q_1 q_2 \dot{q}_3}{q_1^2 + q_2^2} + k_D\frac{q_1 q_2 \dot{q}_3}{q_1^2 + q_2^2}\right), \\
&= \frac{1}{2}\left(-gq_3\frac{q_1^2 + q_2^2}{q_1^2 + q_2^2}\right), \\
\dot{q}_3 &= -\frac{g}{2}q_3
\end{aligned}$$

$$\omega_{d1} = -kq_1 + g\frac{q_2 q_3}{q_1^2 + q_2^2} + \frac{gk_D}{2}\frac{q_1 q_3}{q_1^2 + q_2^2}$$

$$\omega_{d2} = -kq_2 - g\frac{q_1 q_3}{q_1^2 + q_2^2} + \frac{gk_D}{2}\frac{q_2 q_3}{q_1^2 + q_2^2}$$

Till now, we were doing somethings with the kinematics of the satellite, but when it comes to real actuation, it comes due to the torque, so Palmer and Horri gave this angular velocity control law:-

$$\boldsymbol{u} = I(-\boldsymbol{K}(\boldsymbol{\omega} - \boldsymbol{\omega}_d) + \dot{\boldsymbol{\omega}}_d)$$

Where K is a constant Gain Matrix:-

V. Simulations

Consider the inertia matrix of the UYS-1 nanosatellite [9] given by $I = \mathrm{diag}(0.1521, 0.1521, 0.0375)\ \mathrm{kg} \cdot \mathrm{m}^2$ and the following parameters adjusted by simulations: $a_1 = 0.2$, $a_2 = 0.2$, $k = 0.9$, $g = 3$, $K = \mathrm{diag}(10, 10, 0)$, and maximum torque $u_{\mathrm{MAX}} = 0.002\ \mathrm{N} \cdot \mathrm{m}$. Also consider the sampling time $\Delta t = 0.01\ \mathrm{s}$ and the initial conditions of the attitude $q_1(0) = q_2(0) = q_3(0) = 0.2236$, $q_4(0) = 0.9220$, and angular velocity $\boldsymbol{\omega}(0) = 0$.

A. Nonsaturated and Noiseless Closed-Loop System Simulation

Kinematic Equations:-

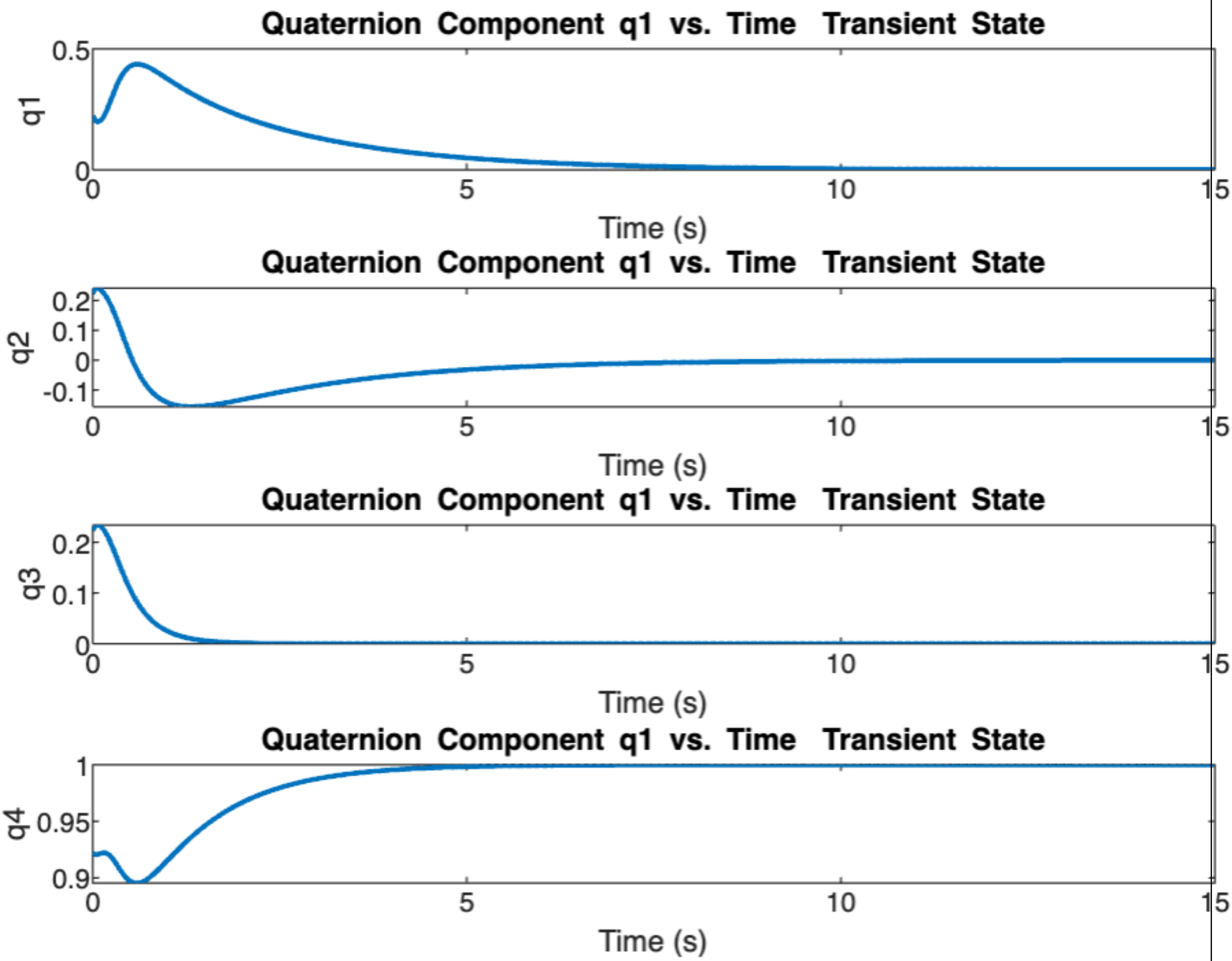
$$\boxed{\begin{aligned}\omega_{d1} &= -kq_1 + g\frac{q_2q_3}{q_1^2+q_2^2} + \frac{gk_D}{2}\frac{q_1q_3}{q_1^2+q_2^2}\\ \omega_{d2} &= -kq_2 - g\frac{q_1q_3}{q_1^2+q_2^2} + \frac{gk_D}{2}\frac{q_2q_3}{q_1^2+q_2^2}\end{aligned}}$$

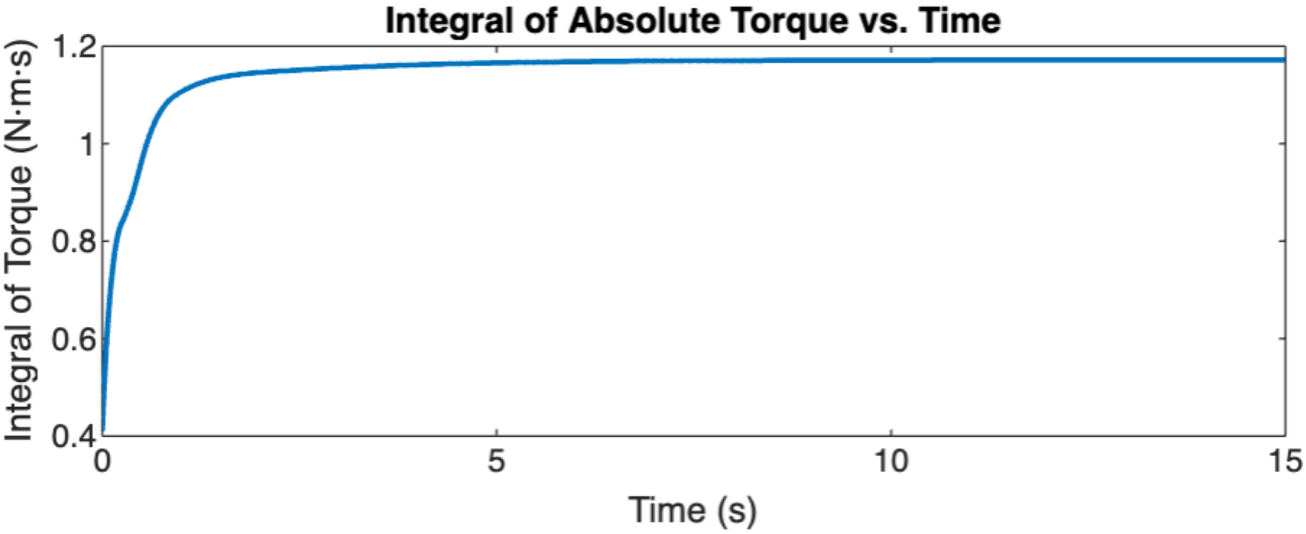
$$\boxed{\begin{aligned}\dot{q}_1 &= \frac{1}{2}\omega_1q_4 - \frac{1}{2}\omega_2q_3\\ \dot{q}_2 &= \frac{1}{2}\omega_1q_3 + \frac{1}{2}\omega_2q_4\\ \dot{q}_3 &= \frac{1}{2}\omega_2q_1 - \frac{1}{2}\omega_1q_2\\ \dot{q}_4 &= -\frac{1}{2}\omega_1q_1 - \frac{1}{2}\omega_2q_2\end{aligned}}$$

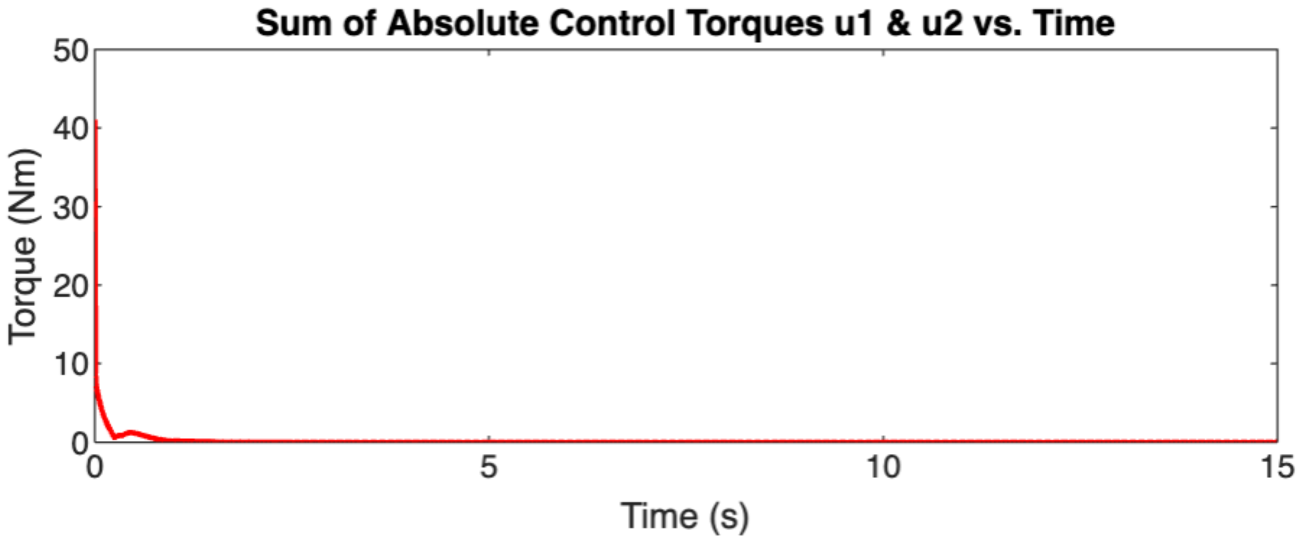
Dynamic Equations:-

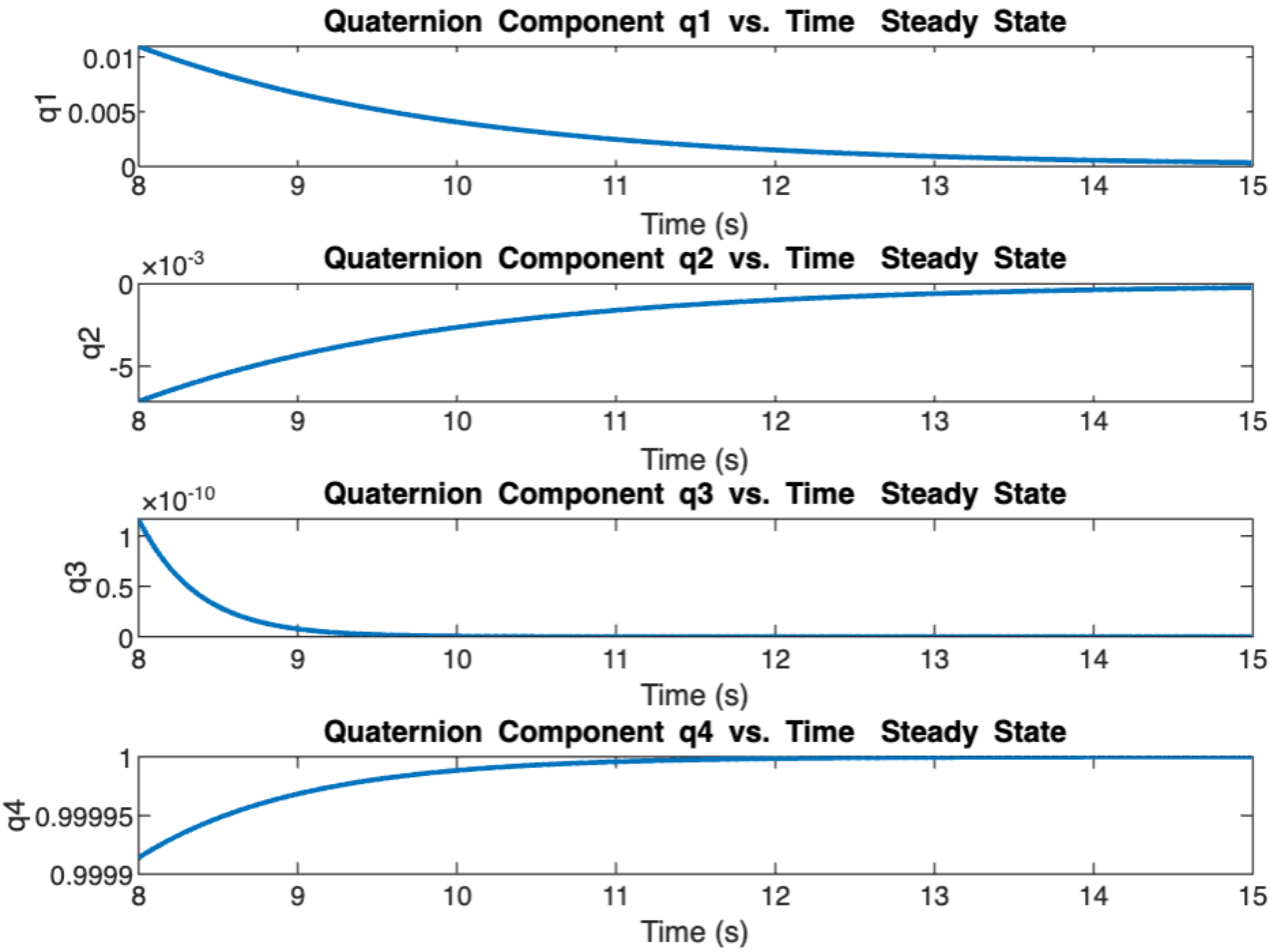
$$\boxed{\boldsymbol{u} = I(-\boldsymbol{K}(\boldsymbol{\omega} - \boldsymbol{\omega}_d) + \dot{\boldsymbol{\omega}}_d)}$$

Simulation Results:-




Integral of Absolute Torque vs. Time
Integral of Torque (N·m·s)
1.2
1
0.8
0.6
0.4
0
5
10
15
Time (s)


Sum of Absolute Control Torques u1 & u2 vs. Time
Torque (Nm)
50
40
30
20
10
0
0
5
10
15
Time (s)



Quaternion Component q1 vs. Time Steady State
q1
0.01
0.005
0
8
9
10
11
12
13
14
15
Time (s)
Quaternion Component q2 vs. Time Steady State
×10^-3
q2
0
-5
Time (s)
Quaternion Component q3 vs. Time Steady State
×10^-10
q3
1
0.5
0
Time (s)
Quaternion Component q4 vs. Time Steady State
q4
1
0.99995
0.9999
Time (s)

B. Saturated and Noiseless Closed-Loop System Simulation

Kinematic Equations:-

$$\dot{q}_1 = \frac{1}{2}\omega_1 q_4 - \frac{1}{2}\omega_2 q_3$$

$$\dot{q}_2 = \frac{1}{2}\omega_1 q_3 + \frac{1}{2}\omega_2 q_4$$

$$\dot{q}_3 = \frac{1}{2}\omega_2 q_1 - \frac{1}{2}\omega_1 q_2$$

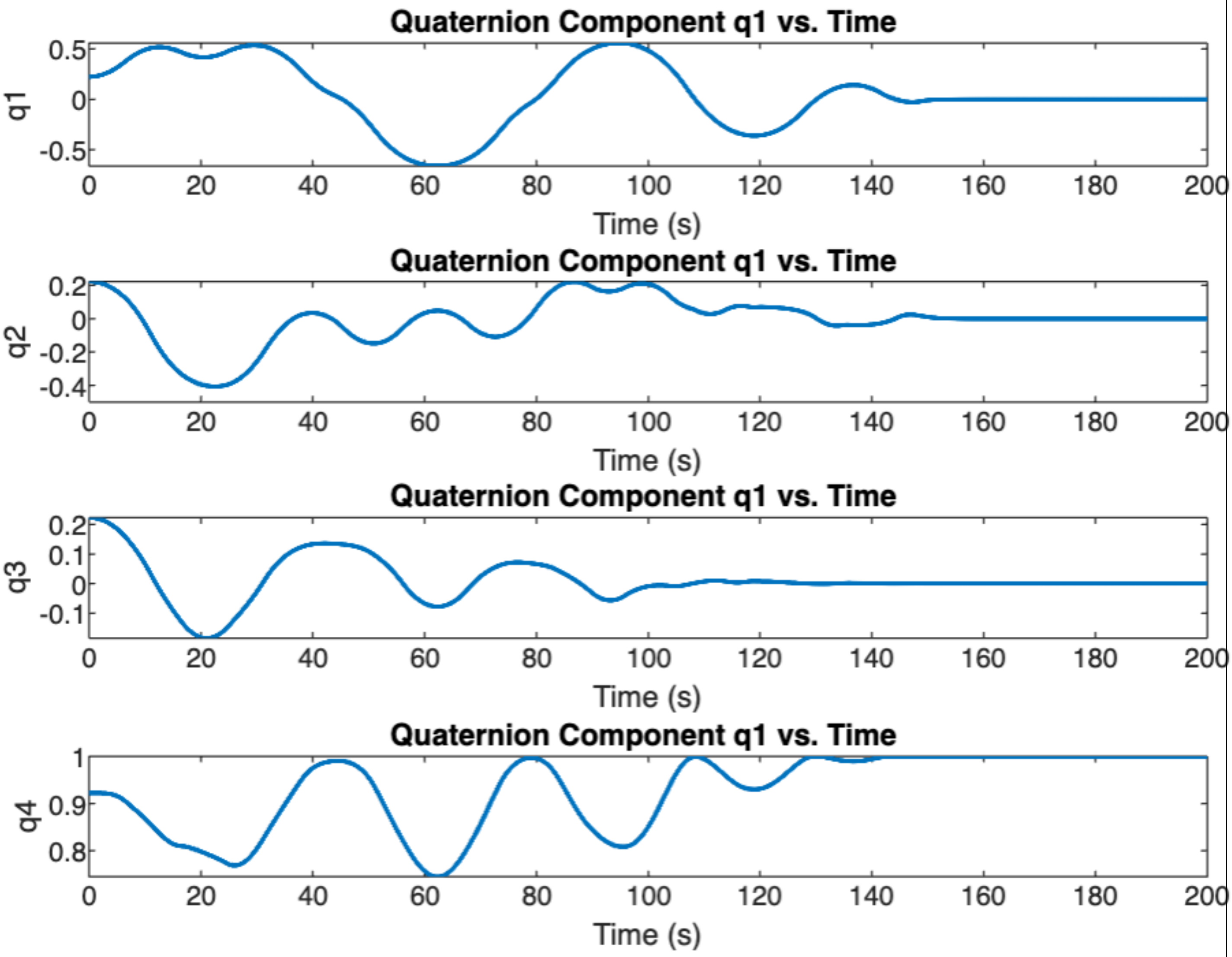
$$\dot{q}_4 = -\frac{1}{2}\omega_1 q_1 - \frac{1}{2}\omega_2 q_2$$

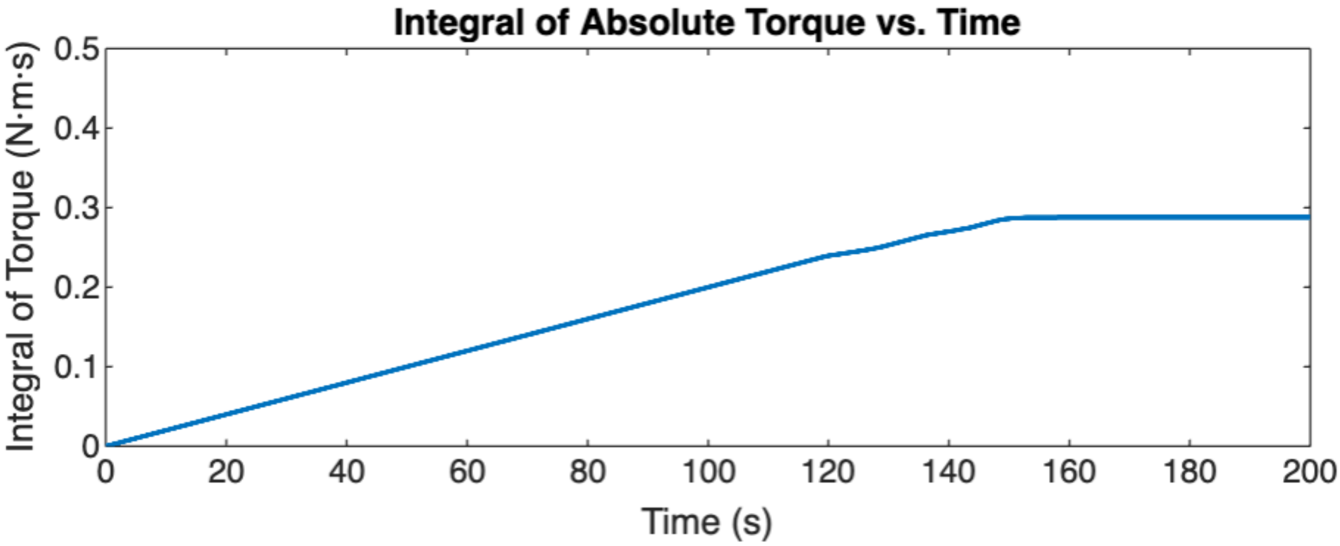
$$\begin{aligned} \omega_{d1} &= -kq_1 + g\mathrm{sat}\left(\frac{q_2 q_3}{q_1^2 + q_2^2}, a_1\right) + \frac{gk_D}{2}\mathrm{sat}\left(\frac{q_1 q_3}{q_1^2 + q_2^2}, a_2\right), \\ \omega_{d2} &= -kq_2 - g\mathrm{sat}\left(\frac{q_1 q_3}{q_1^2 + q_2^2}, a_2\right) + \frac{gk_D}{2}\mathrm{sat}\left(\frac{q_2 q_3}{q_1^2 + q_2^2}, a_1\right) \end{aligned} \quad (14)$$

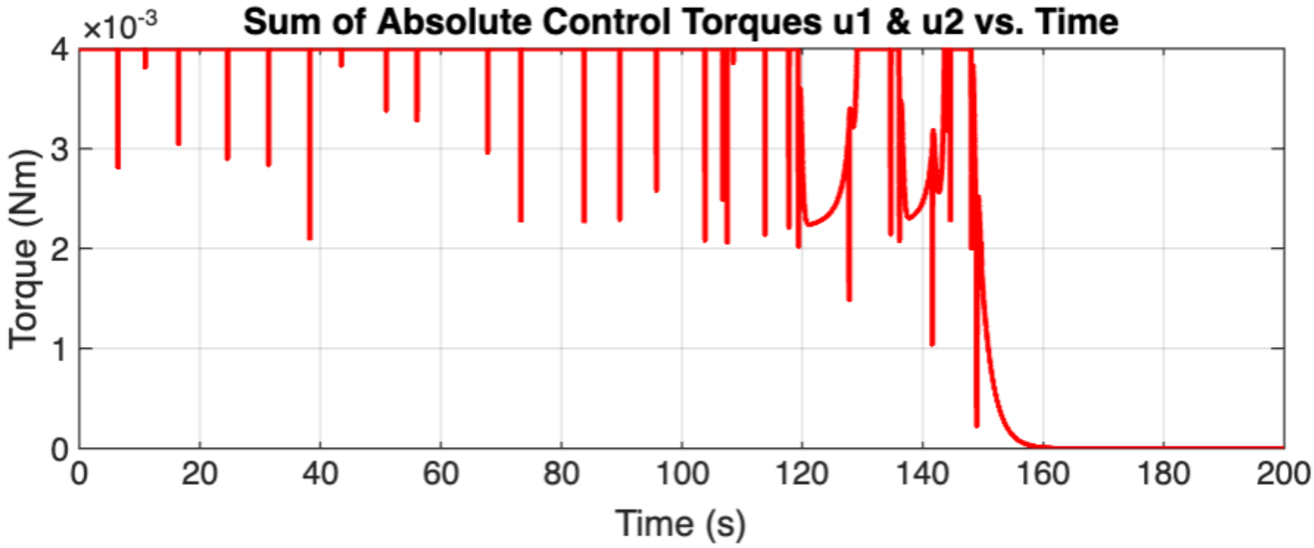
Dynamics Equations:-

$$\boldsymbol{u} = I(-\boldsymbol{K}(\boldsymbol{\omega} - \boldsymbol{\omega}_d) + \dot{\boldsymbol{\omega}}_d)$$

Simulations:-




Integral of Absolute Torque vs. Time
Integral of Torque (N·m·s)
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
140
160
180
200
Time (s)


Sum of Absolute Control Torques u1 & u2 vs. Time
×10^-3
Torque (Nm)
4
3
2
1
0
0
20
40
60
80
100
120
140
160
180
200
Time (s)

Conclusions:-

A controller was introduced that uses only two reaction wheels in order to deal with the case of failure of one of three of them. The controller has a parameter that allows consideration of the effect of the derivative of the unactuated component on the other ones. To analyze the tuning effect for this extra parameter, the controller was compared with two tuning situations, one of which reduces to a well-known controller of the literature (SNLC).

Compared to the SNLC, simulations show that the proposed algorithm, for situations with saturation, external disturbances, and noisy measurements, may diverge at the beginning of the transitory response and slightly increase the torque required to maintain an attitude in order to significantly reduce the system settling time and steady-state error. Also, the overall integrated torque comparison shows that some performance enhancement is obtained with some savings in the integrated torque.