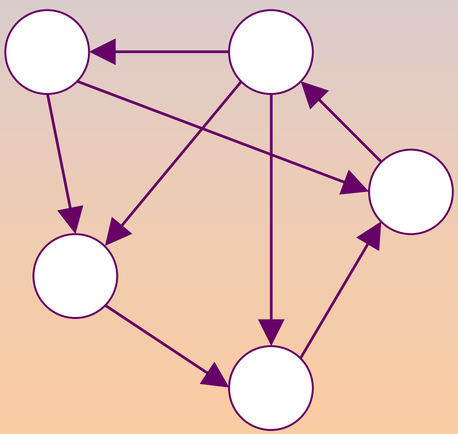
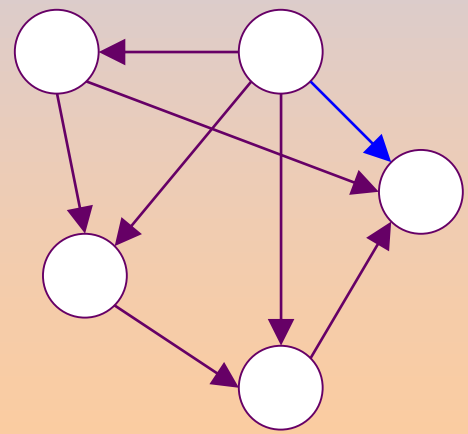
Strongly Connected Components

- Every pair of vertices are reachable from each other
- Graph *G* is ***strongly connected*** if, for every *u* and *v* in *V*, there is some path from *u* to *v* and some path from *v* to *u*.

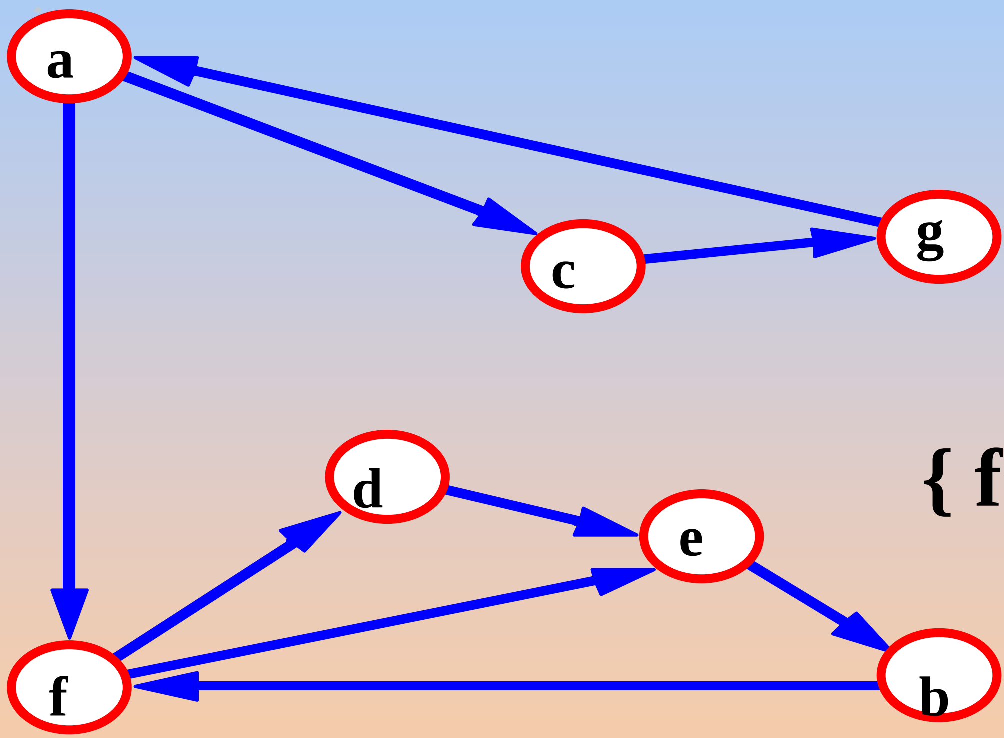
Strongly
Connected



Not Strongly
Connected



Example



{ a , c , g }

{ f , d , e , b }

Finding Strongly Connected Components

- Input: A directed graph G = (V,E)
- Output: a partition of V into disjoint sets so that each set defines a strongly connected component of G

Algorithm

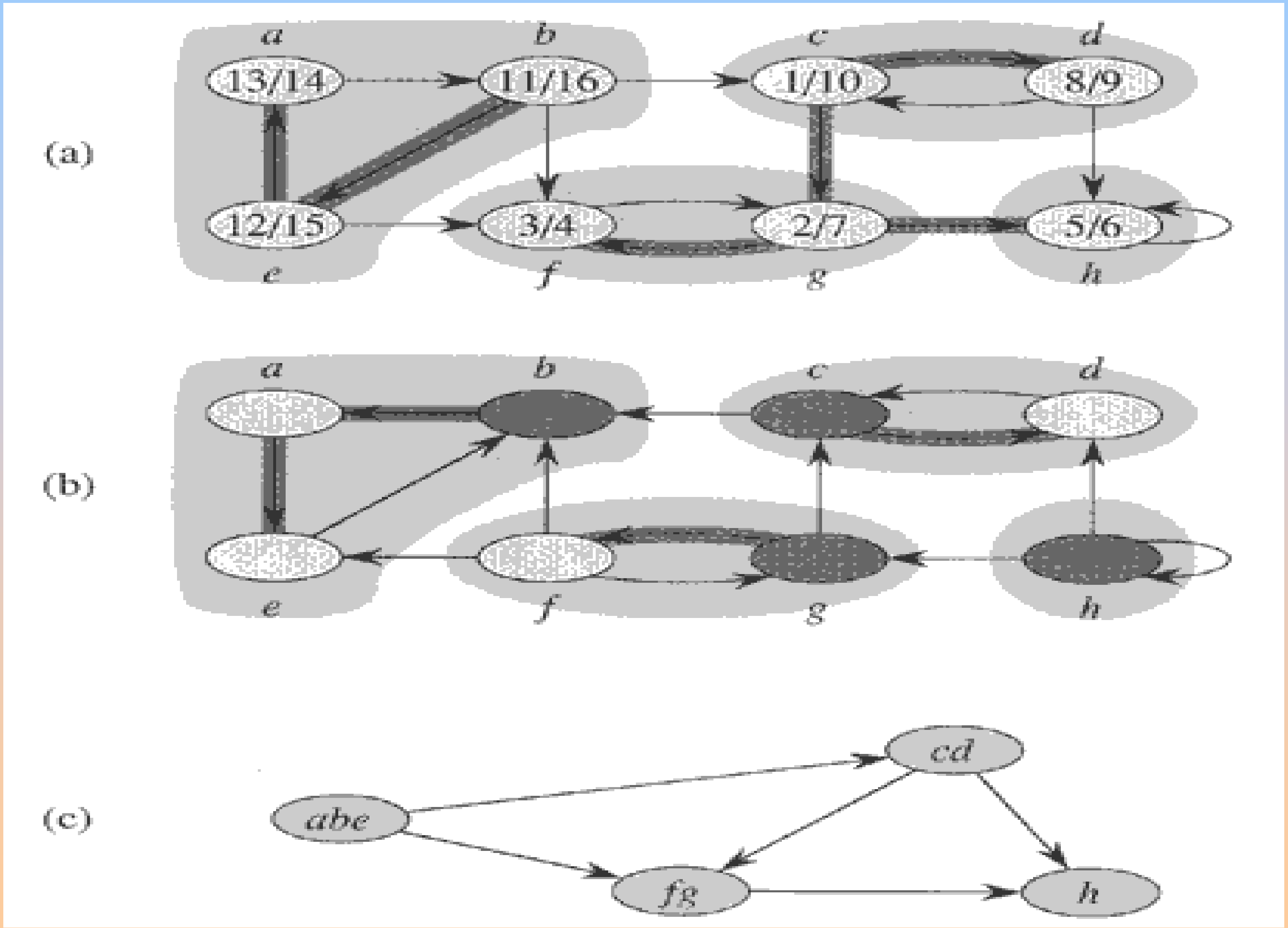
Strongly-Connected-Components(G)

1. call DFS(G) to compute finishing times f[u] for each vertex u. Cost: O(E+V)
2. compute G^T Cost: O(E+V)
3. call DFS(G^T), but in the main loop of DFS, consider the vertices in order of decreasing f[u] Cost: O(E+V)
4. output the vertices of each tree in the depth-first forest of step 3 as a separate strongly connected component.

The graph G^T is the transpose of G, which is visualized by reversing the arrows on the digraph.

- Cost: O(E+V)

Example



(a)
a
b
c
d
13/14
11/16
1/10
8/9
12/15
3/4
2/7
5/6
e
f
g
h
(b)
a
b
c
d
e
f
g
h
(c)
cd
abe
fg
h

Questions

- Let $C=\{C_1,C_2,\ldots,C_n\}$ be the set of strongly connected components of $G=(V,E)$.
  Let,
  $G^{SCC}=(V^{SCC}, E^{SCC})$ where $V^{SCC}=\{v_1,v_2,\ldots,v_n\}$ where each vertex v_i in V^{SCC} represents the strongly connected component C_i in C. There is a directed edge (v_i, v_k) in E^{SCC} if there is a directed edge $(x,y) \in E$ such that $x \in C_i$ and $y \in C_k$.
  Then, $((G^T)^{SCC})^T = G^{SCC}$ where G^T denotes the transpose graph of G.

True/False ?

- Consider a directed graph G=(V,E) where each node is initially colored white. What should be the minimum number of nodes that we should change to red such that for each white node *v* in G, there exists at least one red node *r* such that there is a directed path from *r* to *v* and *v* to *r*. Assume that for every node there is at least one other node it is reachable from and can reach to.