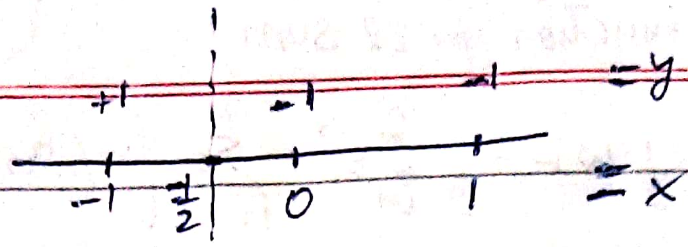
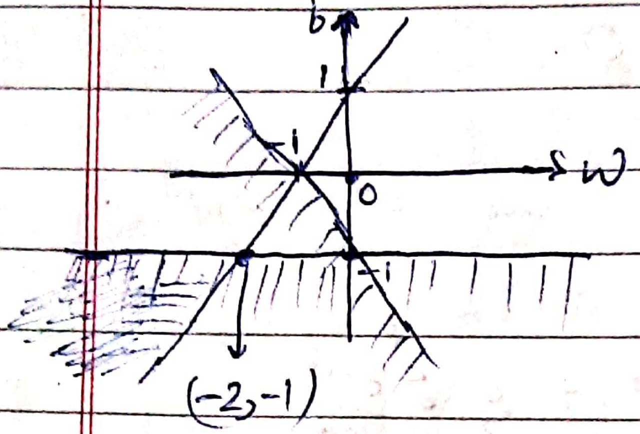
1.



1) $L = \min\limits_{w} \frac{1}{2}\|w\|^2$, s.t. $-w+b \geq 1$

$-b \geq 1$

$-w-b \geq 1$



$w = -2,\ b = -1$

$wx + b = 0 \implies x = \frac{-1}{2}$ - decision boundary

2) $\text{Dual} = \max\limits_{\alpha} \sum \alpha_i - \frac{1}{2} \sum\sum \alpha_i \alpha_j y_i y_j x_i^T x_j$, st $\sum \alpha_i y_i = 0$

$\alpha_1 - \alpha_2 - \alpha_3 = 0 \implies \alpha_2 = \alpha_1 - \alpha_3$

$\text{Max Dual} = 2\alpha_1 - \frac{1}{2}(\alpha_1^2 + \alpha_3^2 + 2\alpha_1\alpha_3) = 2\alpha_1 - \frac{1}{2}(\alpha_1 + \alpha_3)^2$

$\implies \alpha_1 = \alpha_2 = 2,\ \alpha_3 = 0$

3) • $w = \sum \alpha_i y_i x_i = -2$

• $y_i(w^T x_i + b) \geq 1 \implies b = -1$

+1	−1	+1	= y
−1	0	1	= x

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2. Kernel $= (p^T q + 1)^2$

1) Max dual $= \sum \alpha_i - \frac{1}{2} \sum \sum \alpha_i \alpha_j y_i y_j (x_i^T x_j + 1)^2 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 4 \end{bmatrix}$

S.t $\sum \alpha_i y_i = 0 \implies \alpha_2 = \alpha_1 + \alpha_3$

2) $= 2(\alpha_1 + \alpha_3) - \frac{1}{2}\left[4\alpha_1^2 + 4\alpha_3^2 + \alpha_2^2 - 2\alpha_2(\alpha_1 + \alpha_3)\right]$

$= 2(\alpha_1 + \alpha_3) - 2\alpha_1^2 - 2\alpha_3^2 + \frac{1}{2}(\alpha_1 + \alpha_3)^2$

3) $\frac{\partial \, dual}{\partial \alpha_1} = 0 \implies 2 - 3\alpha_1 + \alpha_3 = 0$ — ①

$\frac{\partial \, dual}{\partial \alpha_2} = 0 \implies 2 - 3\alpha_3 + \alpha_1 = 0$ — ②

$\therefore \alpha_1 = \alpha_3 = 1, \ \alpha_2 = 2$

4) $\text{sign}\left(\sum \alpha_i y_i K(x_i, x) + b\right)$, $\boxed{y_i(w^T \phi(x_i) + b) \geq 1 \implies b = -1}$

$= 2x^2 - 1$

$\therefore x^2 \geq \frac{1}{2}$ for +ve class else −ve

x_1	x_2	y
-1	-1	-1
-1	+1	+1
+1	-1	+1
+1	+1	-1

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3. ExOR Problem $x_1, x_2 \in \{-1, 1\}$

1) kernel $(b^T q + 1)^2$

$$\begin{bmatrix} 9 & 1 & 1 & 1 \\ 1 & 9 & 1 & 1 \\ 1 & 1 & 9 & 1 \\ 1 & 1 & 1 & 9 \end{bmatrix}$$

2) $\max_{\alpha} \text{dual} = \sum \alpha_i - \frac{1}{2} \sum\sum \alpha_i \alpha_j y_i y_j K(x_i, x_j)$

$= \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \frac{1}{2} \left[9(\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2) - 2\alpha_1\alpha_2 - 2\alpha_1\alpha_3 + 2\alpha_1\alpha_4 \right.$

$\left. + 2\alpha_2\alpha_3 - 2\alpha_2\alpha_4 - 2\alpha_3\alpha_4 \right]$

3) $\frac{\partial \text{dual}}{\partial \alpha} = 0 \Rightarrow$
$9\alpha_1 - \alpha_2 - \alpha_3 + \alpha_4 = 1$

$-\alpha_1 + 9\alpha_2 + \alpha_3 - \alpha_4 = 1$

$-\alpha_1 + \alpha_2 + 9\alpha_3 - \alpha_4 = 1$

$\alpha_1 - \alpha_2 - \alpha_3 + 9\alpha_4 = 1$

4) $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \frac{1}{8}$

5) $\phi(x) = [1, x_1^2, \sqrt{2}x_1x_2, x_2^2, \sqrt{2}x_1, \sqrt{2}x_2]^T$

$$w = \sum \alpha_i y_i \phi(x_i) = -\frac{1}{8}\begin{bmatrix} 1 \\ 1 \\ \sqrt{2} \\ 1 \\ -\sqrt{2} \\ -\sqrt{2} \end{bmatrix} + \frac{1}{8}\begin{bmatrix} 1 \\ 1 \\ -\sqrt{2} \\ 1 \\ -\sqrt{2} \\ \sqrt{2} \end{bmatrix} + \frac{1}{8}\begin{bmatrix} 1 \\ 1 \\ -\sqrt{2} \\ 1 \\ \sqrt{2} \\ -\sqrt{2} \end{bmatrix} - \frac{1}{8}\begin{bmatrix} 1 \\ 1 \\ \sqrt{2} \\ 1 \\ \sqrt{2} \\ \sqrt{2} \end{bmatrix} = \frac{-1}{8}\begin{bmatrix} 0 \\ 0 \\ -4\sqrt{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

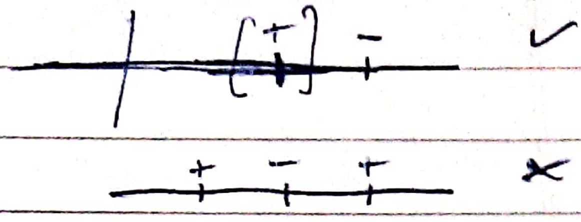
$w^T\phi(x) = -x_1x_2$

x_1	x_2	y	$-x_1x_2$
-1	-1	-1	-1
-1	1	1	1
1	-1	1	1
1	1	-1	-1

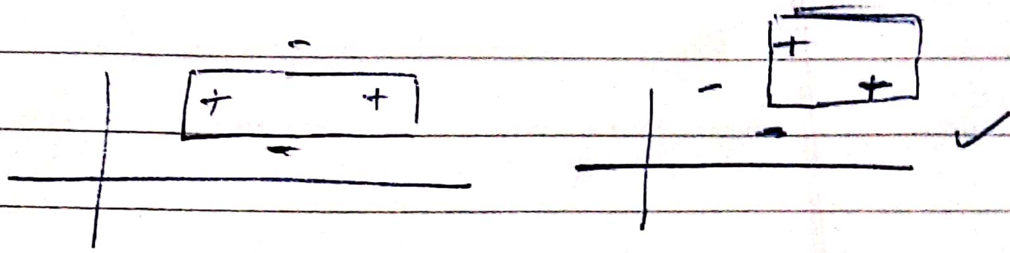
As the table shows, $(-x_1x_2)$ has same sign as ~~t~~ true y.

$\therefore$ $\text{sign}(w^T\phi(x) + b)$ is able to solve ExOR problem.

4. 1) VC dim = 2

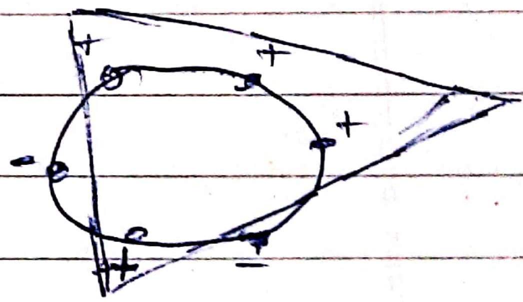


2) VC dim = 4

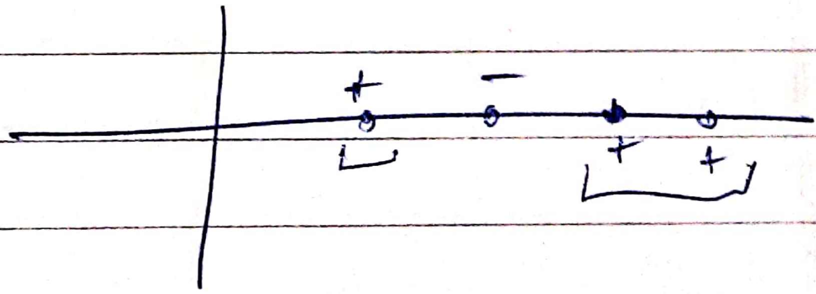


3) VC dim = ∞

Points on Circle



4) VC dim = ∞



Derive dual function for L2 svm

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S. $L(w,b,\alpha,\varepsilon) = \frac{1}{2} w^T w + \frac{c}{2} \sum_{i=1}^{N} \varepsilon_i^2 - \sum_{i=1}^{N} \alpha_i \left[y_i (w^T x_i + b) - 1 + \varepsilon_i \right]$

$\frac{\partial L}{\partial w} = 0 \Rightarrow w = \sum_{i=1}^{N} \alpha_i y_i x_i$

$\frac{\partial L}{\partial b} = 0 \Rightarrow \sum_{i=1}^{N} \alpha_i y_i = 0$

$\frac{\partial L}{\partial \varepsilon_i} = 0 \Rightarrow \varepsilon_i = \frac{\alpha_i}{c}$ for $i = 1, \ldots, N$.

Dual $= J(\alpha) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j x_i^T x_j + \frac{c}{2} \sum_{i=1}^{N} \frac{\alpha_i^2}{c^2}$

$- \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j x_i^T x_j - b \sum_{i=1}^{N} \cancel{\alpha_i y_i} + \sum_{i=1}^{N} \alpha_i - \sum_{i=1}^{N} \frac{\alpha_i^2}{c}$

$= \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j x_i^T x_j - \frac{1}{2c} \sum_{i=1}^{N} \alpha_i^2$