NCPC 2016
Presentation of solutions

The Jury

2016-10-08

NCPC 2016 Jury

- Per Austrin (KTH Royal Institute of Technology)
- Pål Grønås Drange (Statoil ASA)
- Antti Laaksonen (CSES)
- Ulf Lundström (Excillum)
- Jimmy Mårdell (Spotify)
- Lukáš Poláček (Google)
- Mathias Rav (Aarhus University)
- Pehr Söderman (Kattis)
- Jon Marius Venstad (Yahoo!)

J — Jumbled Compass

Problem

Simple problem with solutions by the jury in all languages available in the contest.

Some solution (guess the language)

```
solve(N1, N2) :-
    (N2-N1 >  180 -> Ans is N2-N1-360;
     N2-N1 > -180 -> Ans is N2-N1;
                     Ans is N2-N1+360),
    write(Ans), nl.
```

Statistics: X submissions, Y accepted, first after HH:MM

D - Daydreaming Stockbroker

Problem

Play the stock market when knowing the future.

Solution (guess the language)

```
fscanf(STDIN, "%d", $days);
$money = 100;
$prev = 1<<30;
for ($i = 0; $i < $days; ++$i) {
    fscanf(STDIN, "%d", $cur);
    if ($cur > $prev)
       $money += min(floor($money/$prev), 100000)*($cur-$prev);
    $prev = $cur;
}
echo $money;
```

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G - Game Rank

Problem

Simulate ranking system of some vaguely familiar game.

Solution

1. Read and understand the rules.
2. Keep track of current rank, current number of stars and current number of consecutive wins.
3. Update accordingly.
4. Don't try to be clever.

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F - Fleecing the Raffle

Problem

When drawing p items out of $n + x$ items, what is probability that *exactly one* out of the first x items is drawn?

What is the maximum such probability over all x?

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$$\frac{\binom{x}{1} \cdot \binom{n}{p-1}}{\binom{n+x}{p}} = \{\ldots\text{some calculations}\ldots\} = \frac{x \cdot p}{n+1} \cdot \prod_{i=2}^{x} \frac{n-p+i}{n+i}$$

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2. When going from $x-1$ to x, probability changes by factor
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$$\frac{x}{x-1} \cdot \frac{n-p+x}{n+x}$$
3. Some calculus $\Rightarrow$ increase if $x < \frac{n}{p-1}$, decrease otherwise $\Rightarrow$ max happens at $x = \lfloor n/(p-1) \rfloor$.

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Linear $O(n/p)$ time solution:

```
int n, p;
scanf("%d%d", &n, &p);
int x = n/(p-1);
double res = double(x*p) / (n+1);
for (int i = 2; i <= x; ++i)
    res *= double(n-p+i) / (n+i);
printf("%.9lf\n", res);
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Solution

Constant time solution:

```
int n, p;
scanf("%d%d", &n, &p);
int x = n++/(p-1);
printf("%.9lf\n", x*p*exp(lgamma(n-p+x)-lgamma(n-p+1)
                           -lgamma(n+x)+lgamma(n));
```

(But in order to do this in languages that don't provide full ISO C support, one may have to implement the Γ function oneself)

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C - Card Hand Sorting

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What is minimum number of cards to move to get list of cards in some form of order?

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1. Try all $4! = 24$ possible ways of ordering the 4 suits.
2. Try all $2^4 = 16$ possible ways of choosing ascending/descending order within suits.

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Two dogs move around along straight line segments, what is the closest they get to each other?

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4. Time complexity: $O(n)$.

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Solution 2 (more or less the same but different perspective)

1. Split the walks into intervals during which the two dogs don't switch line segments.
2. In an interval where dogs walk from P to $P + \Delta P$ and Q to $Q + \Delta Q$, square dist. after fraction $t \in [0, 1]$ of the time is

$$\|P - Q + t(\Delta P - \Delta Q)\|_2^2 = \|P - Q\|_2^2 + 2t\langle P - Q, \Delta P - \Delta Q\rangle + t^2\|\Delta P - \Delta Q\|_2^2$$

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3. Minimum happens at $t = \frac{-\langle P - Q, \Delta P - \Delta Q\rangle}{\|\Delta P - \Delta Q\|_2^2}$ (basic calculus)
   Truncate to $t \in [0, 1]$, be careful with $\Delta P = \Delta Q$.

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B - Bless You Autocorrect!

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Type a word using autocorrect.

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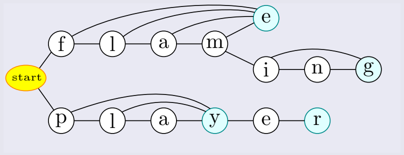

Graph for dictionary “flame”, “flaming”, “play”, “player”

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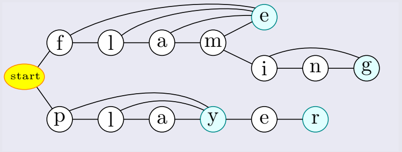

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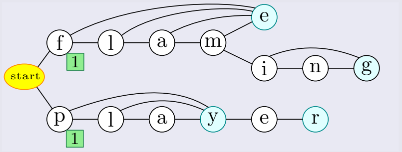

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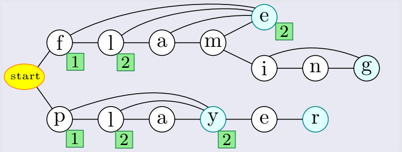

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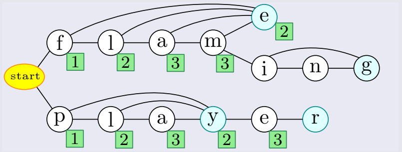

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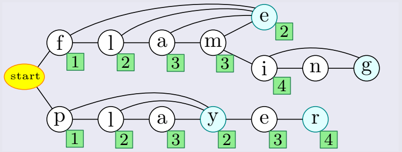

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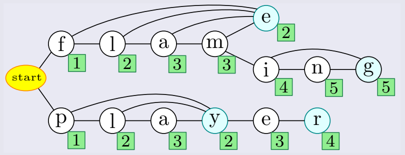

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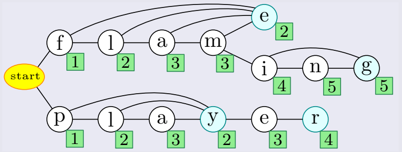

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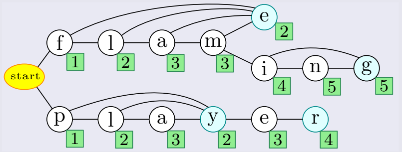

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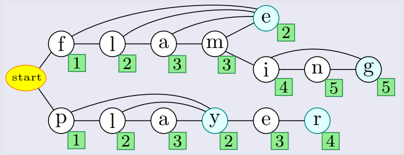

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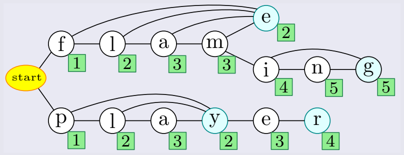

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A - Artwork

Problem

Fill horizontal and vertical blocks of squares in an initially empty grid, and output the number of unfilled connected components after each operation.

Solution

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Fill horizontal and vertical blocks of squares in an initially empty grid, and output the number of unfilled connected components after each operation.

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1. The problem can be modelled as a graph where each node represents a square in the grid, and two nodes are connected if the squares belong to the same component.

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E - Exponial

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Compute $f(n) \bmod m$, where $f(1) = 1$, $f(n) = n^{f(n-1)}$.

Solution

1. If $n \le 5$: just compute $f(n-1)$ and then $n^{f(n-1)} \bmod m$ with modular exponentiation.
2. If $n > 5$: ??????????

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Lemma

For all n and m, and $e \geq log_2(m)$ it holds that

$$n^e \bmod m = n^{\phi(m)+e \bmod \phi(m)} \bmod m.$$

($\phi(m)$ = Euler's totient function.)

Proof: ugly and does not fit on slide. (Boils down to Chinese Remainder Theorem and ϕ being multiplicative.)

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    - each recursive call dominated by time to compute $\phi(m)$: $O(\sqrt{m})$ using naive factorization.
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     Lemma: $\phi(\phi(\cdots\phi(m)))$ reaches 1 after $O(\log m)$ iterations
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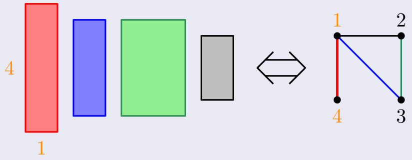
H - Highest Tower

Problem

Given a set of rectangles, build a tower of maximum height.

Solution

1. Make a graph: vertices = lengths, edges = given rectangles.



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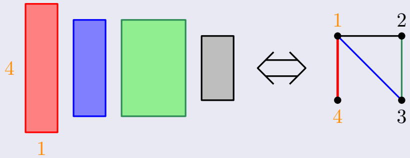
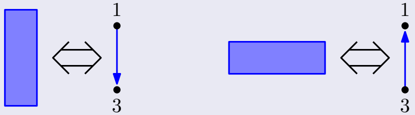
1. Make a graph: vertices = lengths, edges = given rectangles.



2. Encode orientation of a rectangle as direction of an edge.



H - Highest Tower

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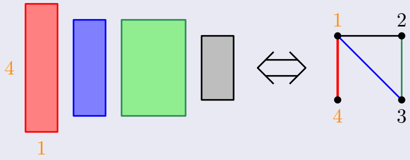

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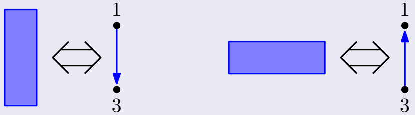

3. Orientation of rectangles gives a valid tower if no width occurs more than once $\Leftrightarrow$ all nodes have outdegree ≤ 1

H - Highest Tower

Problem **Reformulated**

Given an undirected graph, direct edges so that each node has at most one out-going edge and maximize $\sum_{v \in V} \text{value}(v) \cdot \text{indeg}(v)$

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H - Highest Tower

Problem Reformulated

Given an undirected graph, direct edges so that each node has at most one out-going edge and maximize $\sum_{v \in V} \text{value}(v) \cdot \text{indeg}(v)$

Solution

1. In connected component with v vertices and e edges, average out-degree is e/v, so we must have $e \leq v$
   Each component is a tree, or a tree plus one edge.

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I - Interception

Problem

Given large set of paths in large graph with very special structure, find minimum set of edges that hit all paths.

Solution 1

I - Interception

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4. **Case 3, don't use crossings**: main challenge to handle.
   - In order to improve on Case 1, can use at most 1 extra device for the sides.
   - One side must use minimum number of crossings.

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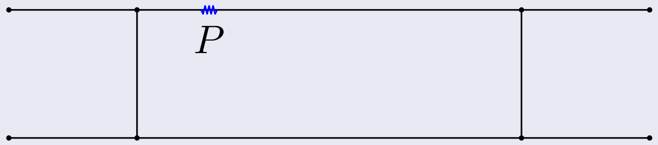

Guess first position P to use after first crossing, $O(n)$ choices.

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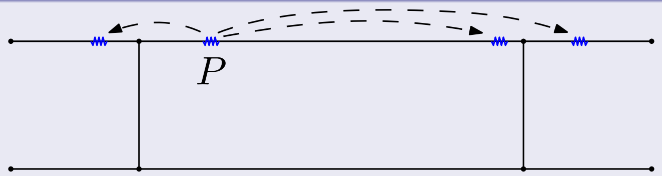

Try to squeeze the rest as close as possible to the crossing, $O(1)$ choices given P.

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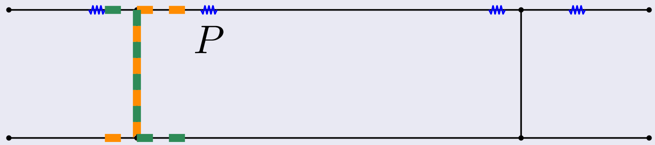

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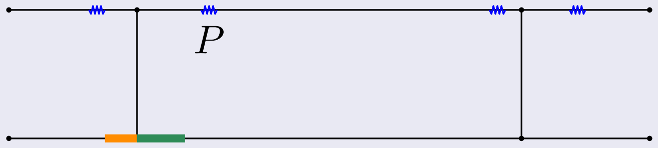

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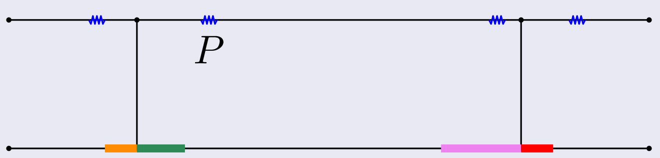

Now want optimal solution to the other side with up to 4 extra intervals added – there are $O(1)$ choices to try for the key positions.

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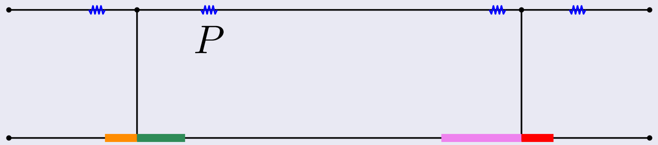

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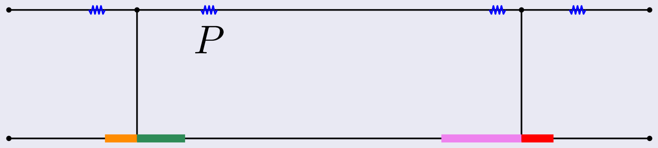

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Time complexity: $O((n+m)\log n)$ (though can be made linear at cost of making implementation even more horrible)

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If you prefer to remain sane and don't want to take the mildly masochistic path of solution 1...

Solution 2

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1. Cover all paths contained in one of the four "tails" optimally, get first device as close to beginning as possible. *Greedy.*

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Random numbers

XYZ teams

XYZ contestants

XYZ total number of submissions

XYZ number of lines of code used in total by the shortest **team** solutions to solve the entire problem set.

482 number of lines of code used in total by the shortest **jury** solutions to solve the entire problem set.

Random facts

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- The jury wrote Python solutions for almost all problems.
  Exceptions: C (**Card Hand Sorting**) for no good reason, and I (**Interception**) because painful.

What now?

- Northwestern Europe Contest: November 18 in Bath (UK). Teams from Nordic, Benelux, Germany, UK, Ireland.

- Each university sends up to three(?) teams to fight for spot in World Finals (May, in Rapid City, South Dakota, USA)