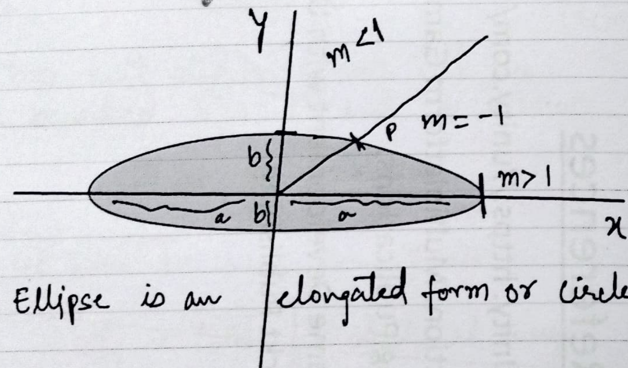
①

Mid-Point Ellipse Drawing Algorithm.



↳ Ellipse is an elongated form or circle.

The radius is (m > 1) which is near to x-axis and (m < 1) which is far end near to y-axis.

Here, $\boxed{\text{Major axis} = 2a \quad \text{Minor axis} = 2b}$

②

Difference of circle and ellipse

Circle :- ① 8-way symmetry

② calculate 1 octant and rest of the octant is generated.

Ellipse :- ① 4-way symmetry.

② calculate atleast 2 region to complete the calculation.

Ellipse General formula

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

and $\boxed{f(x,y) \Rightarrow x^2b^2 + y^2a^2 - a^2b^2 = 0}$

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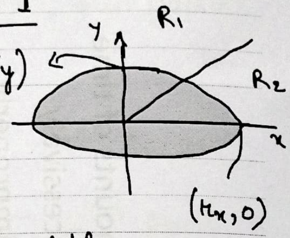
Quadrant 1 :- Region 1

* start Point = $(0, r_y)$



* slope of the curve = [< -1]

* Take unit step in x-axis till the boundary b/w 'x' and 'y' is not reached $(x \to x+1)$

* we have to check the Y axis

$$\left.\begin{array}{l} y_1 = y_0 + 1 \\ \text{or} \\ y_1 = y_0 \end{array}\right\} \text{check at every iteration}$$

Quadrant 2 :- {Region 2}

* slope of curve = [≥ -1]

* Take unit step in y-axis direction till end of the quadrant $(y_1 = y_0 - 1)$

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Proof :- slope of curve

$$x^2 b^2 + y^2 a^2 - a^2 b^2 = 0 \longrightarrow (1)$$

$a = r_x$ and $b = r_y$

$$r_y^2 x^2 + y^2 r_x^2 - r_x^2 r_y^2 = 0 \longrightarrow (11)$$

Differentiate the eq^n (11) in terms of 'x' and 'y'.

$$r_y^2 \frac{d x^2}{dx} + r_x^2 \frac{d y^2}{dy} - \frac{d}{dx}(r_x^2 r_y^2) = 0$$

$$(2 r_y^2 x)\, dy = (-2 r_x^2 x)\, dx$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{-2 r_x^2 x}{2 r_y^2 y}}$$

* Stop at this condition and move to next region $(m = -1)$.

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Region 1 :- check for the two point

Mid-point value.

(x_k+1, y_k) ← 1^{st} Point

(x_k+1, y_k+1) ← 2^{nd} point

Take the sum in x-axis and y-axis and divide by '2' to find Mid-point.

$$\frac{x_k+1 + x_k+1}{2} = [x_k+1]$$

$$\frac{y_k + y_k - 1}{2} = \left[y_k - \frac{1}{2}\right]$$

$\therefore$ Mid-point $= \left[x_k+1, y_k - \frac{1}{2}\right]$

Mid-Point value.

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Mid-Point Ellipse

→ Initialize (0,0) center.

→ Initialize (r_x, r_y)

Region 1

step 1 :- Initial Point $(0, r_y)$

step 2 :- Decission Parameter

$$\boxed{P_k = r_y^2 + \frac{1}{4} r_x^2 - r_x^2 r_y}$$

step 3 :- if $[P_k \geq 0]$ and $[P_k < 0]$

Case 1 :- $[P_k < 0]$

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k$$

$$P_{k+1} = P_k + r_y^2\left(1+2(x_k+1)\right)$$

Case II :- $[P_k \geq 0]$

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k - 1$$

$$P_{k+1} = P_k + r_y^2\left(1+2(x_k+1)\right) + 2r_x^2\left(1-y_k\right)$$

step 4 :- stoping condition

$$\boxed{2r_y^2 x \geq 2r_x^2 y}$$

step 5 :- Move to Region 2

↳ Decission parameter

$$\boxed{P_k = r_y^2\left(x_k + \frac{1}{2}\right)^2 + r_x^2\left(y_k - 1\right)^2 - r_x^2 r_y^2}$$

step 6 :- $[P_k \geq 0]$ and $[P_k < 0]$

Case I :- if $[P_k < 0]$

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k - 1$$

$$\boxed{P_{k+1} = P_k + 2r_y^2\left(x_k+1\right) + r_x^2\left(1-2(y_k-1)\right)}$$

Case II :- if $[P_k > 0]$

$$x_{k+1} = x_k$$

$$y_{k+1} = y_k - 1$$

$$\boxed{P_{k+1} = P_k + r_x^2\left(1 - 2(y_k - 1)\right)}$$

Question :- Center (0,0)

$r_x = 8$ and $r_y = 6$

Solution :-

Step 1 :- Initial Point

$(0, r_y) = (0, 6)$

Step 2 :- Decission Parameter

$$P_k = r_y^2 + \frac{1}{4} r_x^2 - r_x^2 r_y$$

$$= (6)^2 + 1/4\,(8)^2 - 8^2 \times 6$$

$$= 52 - 384$$

$$= -332$$

Here, $P_k < 0$

$$x_{k+1} = x_k + 1 = 0 + 1 = 1$$

$$y_{k+1} = y_k = 6$$

$$P_{k+1} = P_k + r_y^2\left(1 + 2(x_k + 1)\right)$$

$$= -332 + (6)^2(1 + 2 \times 1)$$

$$= -332 + 36 \times 3$$

$$= -224$$

Here $P_k < 0$

$$x_{k+1} = x_k + 1 = 1 + 1 = 2$$

$$y_{k+1} = y_k = 6$$

$$P_{k+1} = P_k + r_y^2\left(1 + 2(x_k + 1)\right)$$

$$= -224 + 6^2(1 + 2 \times 2)$$

$$= -44$$

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Here $[P_k < 0]$

$x_{k+1} = x_k + 1 = 2+1 = 3$

$y_{k+1} = y_k = 6$

$P_{k+1} = P_k + r_y^2 (1 + 2(x_k + 1))$

$= -44 + 6^2 (1 + 2 \times 3)$

$= -44 + 36 \times 7$

$= 208$

Here $P_k > 0$

$x_{k+1} = x_k + 1 = 3 + 1 = 4$

$y_{k+1} = y_k - 1 = 6 - 1 = 5$

$P_{k+1} = P_k + r_y^2 (1 + 2(x_k + 1)) + 2r_x^2 (1 - y_k)$

$= 208 + 6^2 (1 + 2 \times 4) + 2 \times 8^2 \times (-5)$

$= 208 + 324 - 640$

$= -108.$

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Here, $[P_k < 0]$

$x_{k+1} = x_k + 1 = 4 + 1 = 5$

$y_{k+1} = y_k = 5 = 5$

$P_{k+1} = P_k + r_y^2 (1 + 2(x_k + 1))$

$= -108 + 6^2 (1 + 2 \times 5)$

$= -108 + 36 \times 11$

$= 288$

Here $[P_k > 0]$

$x_{k+1} = x_k + 1 = 5 + 1 = 6$

$y_{k+1} = y_k - 1 = 5 - 1 = 4$

$P_{k+1} = P_k + r_y^2 (1 + 2(x_k + 1)) + 2r_x^2 (1 - y_k)$

$= 288 + 6^2 (1 + 2 \times 6) + 2 \times 8^2 (1 - 5)$

$= 288 + (36 \times 13) + (128 \times -4)$

$= 288 + 468 - 512$

$= 244$

Here, $[P_k > 0]$

$x_{k+1} = x_k + 1 = 6 + 1 = 7$

$y_{k+1} = y_k + 1 = 4 - 1 = 3$

$P_{k+1} = P_k + r_y^2(1 + 2(x_k + 1)) + 2r_x^2(1 - y_k)$

$= 244 + 6^2(1 + 2\times 7) + 2\times 8^2 \times (-3)$

$= 244 + 540 - 384$

$= 400.$

step 4:- wait and check for the stopping condition

$\boxed{2y r_x^2 \geq 2x r_y^2}$

K	P_k	x_{k+1}, y_{k+1}	$2x r_y^2$	$2y r_x^2$
0	-332	0,6	0	768
1	-224	1,6	72	768
2	-44	2,6	144	768
3	208	3,6	216	768
4	-108	4,5	288	640
5	288	5,5	360	640
6	244	6,4	432	512
7	400	7,3	504	384

Region 2 :- Initialize

$x = 7, \ y = 3$

Decission Parameter (P_k)

$P_k = r_y^2(x_k + \frac{1}{2})^2 + r_x^2(y_k - 1)^2 - r_x^2 r_y^2$

$P_{k+1} = 6^2(7 + \frac{1}{2})^2 + 8^2(3-1)^2 - 6^2 \times 8^2$

$= (36 \times 56.25) + (64 \times 4) - 2304$

$= 2025 + 256 - 2304$

$= -23$

Here, $[P_k < 0]$

$x_{k+1} = x_k + 1 = 7 + 1 = \underline{\underline{8}}$

$y_{k+1} = y_k - 1 = 3 - 1 = \underline{\underline{2}}$

$P_{k+1} = P_k + 2r_y^2 (x_k + 1) + r_x^2 (1 - 2(y_k - 1))$

$= -23 + 2 \times 6^2 \times 8 + 8^2 (1 - 2 \times 2)$

$= -23 + 576 - 64 \times 3$

$= -23 + 576 - 192$

$= \underline{\underline{361}}$

Here, $[P_k > 0]$

$x_{k+1} = x_k = 8$

$y_{k+1} = y_k - 1 = 2 - 1 = 1$

$P_{k+1} = P_k + r_x^2 (1 - 2(y_k - 1))$

$= 361 + 8^2 (1 - 2(1))$

$= 361$

Here, $[P_k > 0]$

$x_{k+1} = x_k = \underline{\underline{8}}$

$y_{k+1} = y_k - 1 = 2 - 1 = \underline{\underline{1}}$

$P_{k+1} = P_k + r_x^2 (1 - 2(y_k - 1))$

$= 361 + 8^2 (1 - 2 \times 1)$

$= 361 - 64$

$= \underline{\underline{297}}.$

Here, $[P_k > 0]$

$x_{k+1} = x_k = \underline{\underline{8}}$

$y_{k+1} = y_k - 1 = 1 - 1 = \underline{\underline{0}}$

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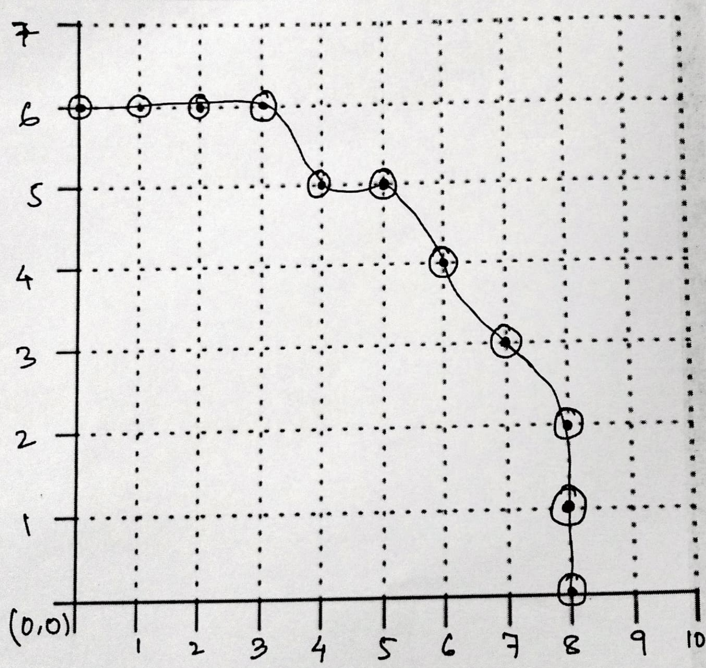
stop the simulation ones its satisfy the $(x,y) = (8,0)$



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K	P_K	x_{k+1}	y_{k+1}
0	—	7	3
1	-23	8	2
2	361	8	1
3	297	8	0