Date:
Page:

Quiz-Y:

Q2).

a). Let $f(x) = x$; let $g(x) = x^2$.

~~det~~

$f(x)$ and $g(x)$ are defined $\forall \mathbb{R}$

$$\int_a^b [f(x)\, g(x)]\, dx = \int_a^b x \cdot x^2\, dx = \int_a^b x^3\, dx$$

$$= \left.\frac{x^4}{4}\right]_a^b = \frac{b^4 - a^4}{4} = \text{LHS}.$$

$$\text{RHS} = \int_a^b f(x)\,dx \cdot \int_a^b g(x)\,dx$$

$$= \int_a^b x\,dx \cdot \int_a^b x^2\,dx. = \left.\frac{x^2}{2}\right]_a^b \cdot \left.\frac{x^3}{3}\right]_a^b$$

$$= \frac{b^2 - a^2}{2} \cdot \frac{b^3 - a^3}{3} = \frac{(b^2 - a^2)(b^3 - a^3)}{6}$$

LHS $\neq$ RHS.

$\therefore$ The statement is false.

b). $\int_{-2}^{1} 1/x^4 \, dx = \int_{-2}^{1} x^{-4} \, dx = \left. \frac{x^{-3}}{-3} \right]_{-2}^{1}$

$= -1/3x^3 \Big]_{-2}^{1} = \frac{-1}{3} \left[1^{-3} - (-2)^{-3} \right]$

$= \frac{-1}{3} \left[1 - (-0.125) \right] = \frac{-1.125}{3} = \frac{-3}{8}$ //

True//

Date:
Page:

Q3). i) $\int_0^{\pi/2} e^x \cos x \, dx$.

$= \cancel{\int_0^{\pi/2}}$ Let $dv = e^x dx$; $u = \cos x$.

Integrating by parts.

$\int_0^{\pi/2} e^x \cos x \, dx = \cancel{\int} uv - \int_0^{\pi/2} v \, du$.

$I = e^x \cos x - \int_0^{\pi/2} e^x . (-\sin x) dx$.

~~$I = e^x \cos x =$~~

Again, integrating by parts with $u = \sin x$, $dv = e^x dx$

$I = e^x \cos x + \left[e^x \sin x - \int_0^{\pi/2} e^x \cos x \, dx . \right]$.

$I = e^x \cos x + e^x \sin x - I$.

$\therefore I = \left. \frac{e^x \cos x + e^x \sin x}{2} \right]_0^{\pi/2}$.

$I = \frac{e^{\pi/2} \cos \pi/2 - e^0 \cos 0 + e^{\pi/2} \sin \pi/2 - e^0 \sin 0}{2}$.

$I = \frac{e^{\pi/2} . 0 - 1.1 + e^{\pi/2} . 1 - 1.0}{2}$.

$I = \frac{e^{\pi/2} - 1}{2}$ //.

Date:
Page:

ii]. $\int \sin^3 x \, dx.$

$= \int \sin^2 x \, dx \cdot (\sin x) = \int (1-\cos^2 x) \sin x \, dx.$

Let $u = \cos x.$

$\therefore du = -\sin x \, dx.$

$\therefore -\int (1-u^2) \, du = \int (u^2-1) \, du.$

By sum rule

$= \int u^2 du - \int du = \frac{u^3}{3} - u + C.$

substituting $u = \cos x.$

$= \frac{\cos^3 x}{3} - \cos x + C$ //.

Date:
Page:

Q4 a) $\int \frac{x^2+8x-3}{x^3+3x^2}\,dx$

$= \int \frac{x^2+8x-3}{x^2(x+3)}\,dx = \cancel{\int \frac{x^2}{x^2(x+3)}dx + 8\int \frac{x}{x^2(x+3)}dx - 3\int \frac{dx}{x^2(x+3)}}$

$\cancel{= \int \frac{1}{x+3}dx + 8\int \frac{1}{x(x+3)}dx - 3\int \frac{dx}{x^2(x+3)}}$

By partial fractions.

$$\frac{x^2+8x-3}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$$

$$\frac{A(x)(x+3) + B(x+3) + Cx^2}{x^2(x+3)} = \frac{x^2+8x-3}{x^2(x+3)}.$$

$Ax^2+3Ax+Bx+3B+Cx^2 = x^2+8x-3$

$(A+C)x^2 = x^2$; $(3A+B)x = 8x$; $3B = -3$

$\therefore B = -1$; $A = (8+1)/3 = 3$; $C = 1-3 = -2$.

$$\therefore \int \frac{x^2+8x-3}{x^2(x+3)} = \int \frac{3}{x}dx + \int \frac{-1}{x^2}dx + \int \frac{-2}{x+3}dx.$$

$$= 3\ln|x| - \frac{1}{x} - 2\ln|x+3| + C.$$

$\left[\because \cancel{\int \frac{1}{f(x)} = \ln(f(x))}\quad \int \frac{1}{x+c} = \ln|x+c| \right.$

$\left. \int x^a dx = \frac{x^{a+1}}{a+1} \right]$

$$\therefore \int \frac{x^2+8x-3}{x^3+3x^2}dx = 3\ln|x| - \frac{1}{x} - 2\ln|x+3| + C$$

Date:
Page:

Q4).
b). $\int \frac{x^4+x}{x-1}\,dx = \int \frac{x^4}{x-1}\,dx + \int \frac{x}{x-1}\,dx$. By sum rule

first solving $\int \frac{x^4}{x-1}\,dx$.

$$\frac{x^4}{x-1} = \begin{array}{r|l} & x^3+x^2+x+1 \\ \hline x-1 & x^4. \\ & -x^4+x^3 \\ \hline & x^3 \\ & -x^3+x^2 \\ \hline & x^2 \\ & -x^2+x \\ \hline & x \\ & -x+1 \\ \hline & 1. \end{array}$$

$\therefore \frac{x^4}{x-1} = x^3+x^2+x+1+\frac{1}{x-1}$.

$\therefore \int \frac{x^4}{x-1}\,dx = \int x^3\,dx + \int x^2\,dx + \int x\,dx + \int dx + \int \frac{1}{x-1}\,dx$.

$= \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1|$

.... $\left[\because \int x^a = \frac{x^{a+1}}{a+1} \text{ and } \int \frac{1}{x+a} = \ln|x+a|\right]$.

And $\int \frac{x}{x-1}\,dx = \int \frac{x-1}{x-1}\,dx + \int \frac{1}{x-1}\,dx$.

$= \int dx + \int 1/x-1\,dx = x + \ln|x-1|$.

$\therefore \int \frac{x^4+x}{x-1}\,dx = \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\ln|x-1| + C$ //.

Date:
Page:

Q5
a). $\int \frac{\sqrt{16-x^2}}{x^2}dx$ ₹

Let $x = 4\sin y$. $dx = 4\cos y\,dy$.

$= \int \frac{\sqrt{16-16\sin^2 y}}{16\sin^2 y} 4\cos y\,dy$.

$= \int \frac{\sqrt{16(1-\sin^2 y)}}{16\sin^2 y} 4\cos y\,dy = \int \frac{4\cos y}{16\sin^2 y} 4\cos y\,dy$.

$= \int \cancel{16} \cot^2 y\,dy = \int \operatorname{cosec}^2 y\,dy - \int 1\,dy$.

$= -\cot y - y \ldots [\because \int \operatorname{cosec}^2 u\,du = -\cot u]$.

substitute $y = \cancel{\frac{x}{4}} \sin^{-1}\left(\frac{x}{4}\right)$.

$-\cot\left(\sin^{-1}\left(\frac{x}{4}\right)\right) - \sin^{-1}\left(\frac{x}{4}\right) + c$.

By using ~~hypotenuse for~~ pythagoras theorem to find. base when opposite $= x$ and hypo $= 4$.

base $= \sqrt{16-x^2}$

$\therefore \cot\left(\sin^{-1}\left(\frac{x}{4}\right)\right) = \frac{\text{base}}{\text{opp}} = \frac{\sqrt{16-x^2}}{x}$.

₹ $\int \frac{\sqrt{16-x^2}}{x^2}dx = -\frac{\sqrt{16-x^2}}{x} - \sin^{-1}\left(\frac{x}{4}\right) + c$ //.

Date:
Page:

Q5.b). $\int_0^3 \sqrt{2x+1}\, dx.$

Let $u = 2x+1$. $\therefore du = 2\,dx$

$$\int_{\cancel{0}}^{\cancel{3}} \frac{\sqrt{u}}{2}\, du$$

$$= \frac{1}{2}\int_{\cancel{0}}^{\cancel{3}} \sqrt{u}\, du = \frac{1}{2}\, u \cdot \cancel{3}\sqrt{u}$$

~~Substitute $u = 2x+1$.~~

$$\cancel{= \frac{1}{2}\left[2x+1\right]\sqrt{2x+1}\Big]_0^3}$$

$$\cancel{\frac{1}{2}[}$$

~~Ato~~ $U = 2x+1$; when $x = 0$
$u = 2(0)+1 = 1$.
when $x = 3$; $u = 2(3)+1 = 7$.

$$\int_0^3 \sqrt{2x+1}\, dx = \int_1^7 \frac{\sqrt{u}}{2}\, du = \frac{2}{2}\,\frac{u\sqrt{u}}{3}\Big]_1^7.$$

$$= \frac{1}{3}\cancel{\frac{2}{2}}\left[7\sqrt{7} - 1\right] = \frac{7\sqrt{7}}{3} - \frac{1}{3} \;//$$