Phys 225 Akshar Mehrotra

HW #9

Q1 a) $|\vec{A}+\vec{B}| = \sqrt{(\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B})} = \sqrt{|A|^2+|B|^2+2\vec{A}\cdot\vec{B}} = \sqrt{A^2+B^2+2AB\cos\theta}$

b) $|B| = 2|A|$

$|\vec{A}+\vec{B}| = \sqrt{(\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B})} = \sqrt{\vec{A}\cdot\vec{A}+\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{A}+\vec{B}\cdot\vec{B}}$

$= \sqrt{|A|^2+4|A|^2+2\cdot|A|\cdot 2|A|\cos 45^\circ}$

$= \sqrt{5|A|^2+4|A|\cdot\frac{1}{\sqrt{2}}}$

c) $\vec{A} = \sum_{i=1} A\hat{i}$, $\vec{B} \sum_{i=1} B\hat{i}$, where $\hat{i}$'s are the orthonormal basis.

$\vec{A}\cdot\vec{B} = \left(\sum_{i=1} A\hat{i}\right)\cdot\left(\sum_{i=1} B\hat{i}\right)$

$= (A\hat{1}+A\hat{2}+A\hat{3}\ldots)\cdot(B\hat{1}+B\hat{2}+B\hat{3}\ldots)$

$= (A\hat{1}\cdot B\hat{1} + A\hat{2}B\hat{2} + A\hat{3}B\hat{3}+\ldots)$

Because Basis are othogonal

$= A_1B_1 + A_2B_2 \ldots$

$\vec{A}\cdot\vec{B} = \sum_{i=1} A_iB_i$

d) $|\vec{A}| = \sqrt{\vec{A}\cdot\vec{A}}$, $\vec{A} = \sum_{i=1} A\hat{i}$

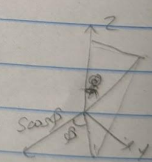
from part (c) we know

$$\vec{A}\cdot\vec{A} = A_1A_1 + A_2A_2 \dots$$
$$= A_1^2 + A_2^2 \dots$$
$$= \sum_{i=1} (A_i)^2$$

$$\Rightarrow |\vec{A}| = \sqrt{\sum_{i=1} A_i^2}$$

e) $$f(r,\theta,\phi) = |2\hat{r} + 3\hat{\theta}|$$
$$= \sqrt{(2\hat{r}+3\hat{\theta})\cdot(2\hat{r}+3\hat{\theta})}$$
$$= \sqrt{4 + \cancel{6}\,\hat{r}\cdot\hat{\theta} + 6\hat{r}\cdot\hat{\theta} + 9\hat{\theta}}$$
$$= \sqrt{13}$$

f) $$f(s,\phi,z) = |z\hat{z} + x\hat{s}|$$
$$= |z\hat{z} + s\cos\phi\,\hat{s}|$$
$$= \sqrt{(z\hat{z} + s\cos\phi\,\hat{s})\cdot(z\hat{z} + s\cos\phi\,\hat{s})}$$
$$= \sqrt{z^2 + s^2\cos^2\phi}$$



g) $$f(r,\theta,\phi) = |z\hat{z} + \hat{r}|$$
$$= |r\cos\theta\,\hat{z} + \hat{r}|$$
$$= \sqrt{r^2\cos^2\theta + 1}$$

h) $A = (3, \pi/2, \pi/2)$ $B = (4, 0, 0)$

$\Rightarrow$ ~~x=~~ $x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$A = (0, 3, 0)$ $B = (0, 0, 4)$

$|(\vec{B} - \vec{A})| = \sqrt{9 + 16} = 5$

i) $A = (3, 60°, 90°)$ $B = (4, 45, 0)$

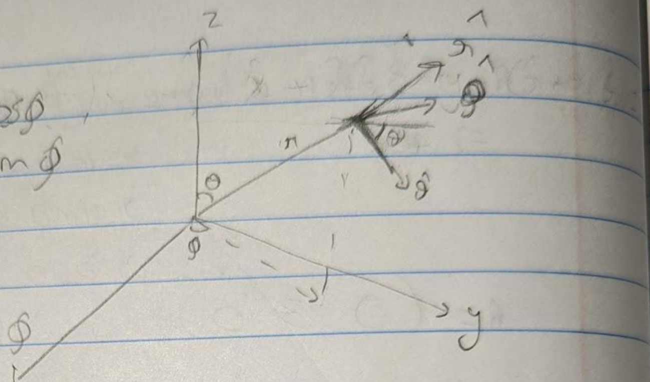
$A = \left(0, \frac{3}{2}\sqrt{3}, \frac{3}{2}\right)$ $B = (2\sqrt{2}, 0, 2\sqrt{2})$

$|B - A| = \sqrt{(2\sqrt{2})^2 + \left(\frac{3}{2}\sqrt{3}\right)^2 + \left(2\sqrt{2} - \frac{3}{2}\right)^2}$

$= 4.0638$

HW#9

Q2



$\hat{\theta}\cdot\hat{x} = \cos\theta\cos\phi$

$\hat{\theta}\cdot\hat{y} = \cos\theta\sin\phi$

$\hat{\theta}\cdot\hat{z} = -\sin\theta$

$\hat{r}\cdot\hat{x} = \sin\theta\cos\phi$

$\hat{r}\cdot\hat{y} = \sin\theta\sin\phi$

$\hat{r}\cdot\hat{z} = \cos\theta$

$\hat{\phi}\cdot\hat{x} = -\sin\phi$

$\hat{\phi}\cdot\hat{y} = \cos\phi$

$\hat{\phi}\cdot\hat{z} = 0$

$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$

$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$

$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$

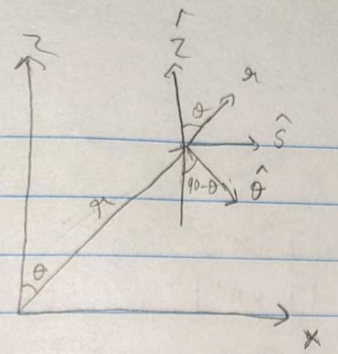
b)

$\hat{x} = \cos\theta\cos\phi\,\hat{\theta} + \sin\theta\cos\phi\,\hat{r} - \sin\phi\,\hat{\phi}$

$\hat{y} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}$

$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

c)



$\hat{z}\cdot\hat{r} = \cos\theta$

$\hat{z}\cdot\hat{\theta} = -\sin\theta$

$\hat{z}\cdot\hat{\phi} = 0$

$\cancel{\hat{\phi}}\cdot$

$\hat{s}\cdot\hat{r} = \sin\theta$

$\hat{s}\cdot\hat{\theta} = \cos\theta$

$\hat{s}\cdot\hat{\phi} = 0$

$\hat{\phi}\cdot\hat{r} = 0$

$\hat{\phi}\cdot\hat{\theta} = 0$

$\hat{\phi}\cdot\hat{\phi} = 1$

$\hat{r} = \cos\theta\,\hat{z} + \sin\theta\,\hat{s} + 0\,\hat{\phi}$

$\hat{\theta} = -\sin\theta\,\hat{z} + \cos\theta\,\hat{s}$

$\hat{\phi} = \hat{\phi}$

$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

$\hat{s} = \sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}$

$\hat{\phi} = \hat{\phi}$

d)

$$\frac{\partial\hat{r}}{\partial\theta} = \frac{\partial(\sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z})}{\partial\theta}$$

$$= \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} + -\sin\theta\,\hat{z}$$

$$= \hat{\theta}$$

$$\frac{\partial\hat{\theta}}{\partial\theta} = \frac{\partial(\cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z})}{\partial\theta}$$

$$= -\sin\theta\cos\phi\,\hat{x} - \sin\theta\sin\phi\,\hat{y} - \cos\theta\,\hat{z}$$

$$= -\hat{r}$$

$$\frac{\partial\hat{\phi}}{\partial\theta} = \partial(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y})/\partial\theta$$

$$= \cancel{-\cos\theta}\ 0$$

Q3

a) $$\vec{B}(\vec{r}) = \frac{\mu_0}{2\pi} \frac{\vec{I} \times \vec{r}}{|\hat{I} \times \vec{r}|^2}$$

$\vec{B}(\vec{r})$ $\vec{r} = z\hat{z} + s\hat{s}$

$$\vec{B}(r) = \frac{\mu_0}{2\pi} \frac{\vec{I} \times (z\hat{z} + s\hat{s})}{|\hat{I} \times (z\hat{z} + s\hat{s})|^2}$$

$$= \frac{\mu_0}{2\pi} \frac{I\hat{z} \times (z\hat{z} + s\hat{s})}{|\hat{z} \times (z\hat{z} + s\hat{s})|^2}$$

$$= \frac{\mu_0}{2\pi} \frac{I \cdot s \cdot \hat{z} \times \hat{s}}{|s\hat{z} \times \hat{s}|^2} = \frac{\mu_0 I s \hat{\phi}}{2\pi |s\hat{\phi}|^2}$$

$$= \frac{\mu_0 I \cancel{s} \hat{\phi}}{2\pi s^{\cancel{2}}} = \frac{\mu_0 I}{2\pi s} \hat{\phi}$$

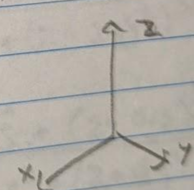
b) $$B(\vec{r}) = \frac{\mu_0}{2\pi} \frac{\vec{I} \times \vec{r}}{|\hat{I} \times \vec{r}|^2}$$

$$= \frac{\mu_0}{2\pi} \frac{I\hat{z} \times (r\hat{r})}{|\hat{z} \times r\hat{r}|^2} = \frac{\mu_0 I r}{2\pi} \frac{\hat{z} \times \hat{r}}{|r\, \hat{z} \times \hat{r}|^2}$$

Now $\hat{z} = \cos\theta\, \hat{r} - \sin\theta\, \hat{\theta}$ (from Q2)

$$\Rightarrow \frac{\mu_0}{2\pi} \frac{I R\, (\cos\theta\, \hat{r} - \sin\theta\, \hat{\theta}) \times \hat{r}}{R^2\, |\hat{z} \times \hat{r}|^2}$$

$$= \frac{\mu_0 I R \sin\theta\, \hat{\phi}}{2\pi R^2 \sin^2\theta} = \frac{\mu_0 I\, \hat{\phi}}{2\pi R \sin\theta}$$



c) $\vec{I} = I\hat{x}$

$$B(\vec{r}) = \frac{\mu_0}{2\pi} \frac{\vec{I} \times \vec{r}}{|\hat{I} \times \vec{r}|^2}$$

$$= \frac{\mu_0}{2\pi} \frac{I\hat{x} \times (x\hat{x} + y\hat{y} + z\hat{z})}{|\hat{x} \times (x\hat{x} + y\hat{y} + z\hat{z})|^2}$$

$$= \frac{\mu_0 I (y\hat{z} + -z\hat{y})}{2\pi \, |y\hat{z} - z\hat{y}|^2}$$

$$= \frac{\mu_0 I (y\hat{z} - z\hat{y})}{2\pi (y^2 + z^2)}$$

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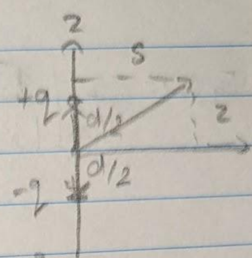
Q4 $\vec{E}(\vec{r}) = \frac{q}{4\pi\epsilon_0} \frac{\vec{r} - \vec{r}_q}{|\vec{r} - \vec{r}_q|^3}$



$$E_{q_1}(\vec{r}) = \frac{q(s\hat{s} + z\hat{z} - z_1\hat{z})}{4\pi\epsilon_0 |s\hat{s} + z\hat{z} - z_1\hat{z}|^3}$$

$$E_{q_2}(\vec{r}) = \frac{q(s\hat{s} + z\hat{z} + z_2\hat{z})}{4\pi\epsilon_0 |s\hat{s} + z\hat{z} + z_2\hat{z}|^3}$$

$$E = E_1 + E_2 = \frac{q}{4\pi\epsilon_0}\left[\frac{(s\hat{s} + (z - z_1)\hat{z})}{|s\hat{s} + (z - z_1)\hat{z}|^3} + \frac{(s\hat{s} + (z + z_2)\hat{z})}{|s\hat{s} + (z + z_2)\hat{z}|^3}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{s\hat{s} + (z - z_1)\hat{z}}{(s^2 + (z - z_1)^2)^{3/2}} + \frac{s\hat{s} + (z + z_2)\hat{z}}{(s^2 + (z + z_2)^2)^{3/2}}\right]$$

$$= \frac{p}{d\,4\pi\epsilon_0}\left[\frac{s\hat{s} + (z - d/2)\hat{z}}{(s^2 + (z - \frac{d}{2})^2)^{3/2}} + \frac{s\hat{s} + (z + d/2)\hat{z}}{(s^2 + (z + \frac{d}{2})^2)^{3/2}}\right]$$

$$= \frac{p}{d\,4\pi\epsilon_0}\left[\frac{s\cos\phi\,\hat{x} + (z - d/2)\hat{z}}{(s^2 + (z - \frac{d}{2})^2)^{3/2}}\right]$$